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REMOTE MEASUREMENT OF WATER CURRENTS USING CORRELATION SONAR. (U)  
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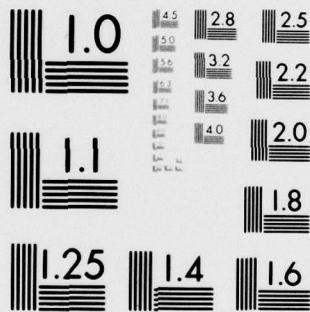
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Summary  Acoustic backscatter measurement has certain advantages over other means of obtaining current velocity profiles in water. A complete profile can be obtained essentially instantaneously and platform motion is less restricted than with techniques requiring the use of mechanical probes. Doppler sonar is one approach, in use today, but it has some important limitations. These relate to the requirement for three receiving beams to measure the three components of velocity and the limitation on measurement accuracy when short duration signals are used to provide high resolution in depth. Correlation sonar is an alternate approach which has the capability of measuring the magnitude and direction of the mean velocity vector in a bounded resolution volume using a single transmitting/receiving array. Furthermore, measurement accuracy in a correlation system is enhanced rather than degraded by the use of wideband signals. This paper reviews the basic theory of correlation velocity measurement, and discusses system design constraints and operating parameters and their relationship to such performance measures as volume and depth resolution, profiling range, and measurement accuracy. In addition, results from a limited number of in-water measurements are presented.  This paper was presented at the 98th meeting of the Acoustical Society of America and an abstract is published in J. Acoust. Soc. Am. Suppl., Vol 66, p 557 paper CCI, Fall 1979.					

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## SECTION I INTRODUCTION

The use of acoustic backscatter measurements to remotely estimate the velocity of a water mass offers a number of potential advantages over measurement techniques requiring mechanical or electromagnetic sensors mounted at fixed points in space or mounted on a free-falling or ascending platform. These include the ability to quickly measure a complete velocity profile extending from the surface to the bottom from a single-sensor platform, and the ability to readily move this platform from one area of interest to another, for example by mounting it on, or towing it from a surface ship.

In a remote acoustic sensing system the water volume of interest is insonified by an acoustic transmission, and velocity information is obtained from measurements of the back-scattered signals (i. e. , volume reverberation) from that volume. These signals are modulated by the motion of the individual scattering mechanisms within the volume. The received signals provide a measure of the mean velocity of these scatterers, which in most cases will be equal to the mean velocity of the water mass.

The principle of acoustic backscatter measurements is illustrated by the sketch in Figure 1-1. This shows an insonified volume bounded in angle by a round-trip beam pattern of width  $\theta_b$ . At any instant of time, scattered signals will be received from a range increment equal to one-half the sound velocity multiplied by the duration of the transmitted signal pulse designated  $T_p$ . The range to this volume is determined by the timing of a receiving range gate or integration window. During this integration window the insonified volume will advance a distance  $1/2 CT_i$ . Thus the range resolution of the system is determined by the shape and duration of both the transmitted pulse and the integration window. Normally these would be set equal. The depth resolution, which is of primary interest in a velocity profiling operation, will be determined by the shape and dimensions of this resolution volume and its orientation relative to the vertical.

Other geometries are, of course, possible. For example, in a bistatic geometry it is possible to bound the resolution volume in range by the intersection of the two beam patterns independent of the pulse duration and integration time. This, of course, limits the system flexibility.

Velocity estimates may be obtained from either Doppler or correlation measurements. This document deals with the correlation approach believed to have a number of advantages in this application. However, assuming that most of the readers are more familiar with the Doppler approach, both systems will be referenced in order to point out similarities and differences between the two approaches.

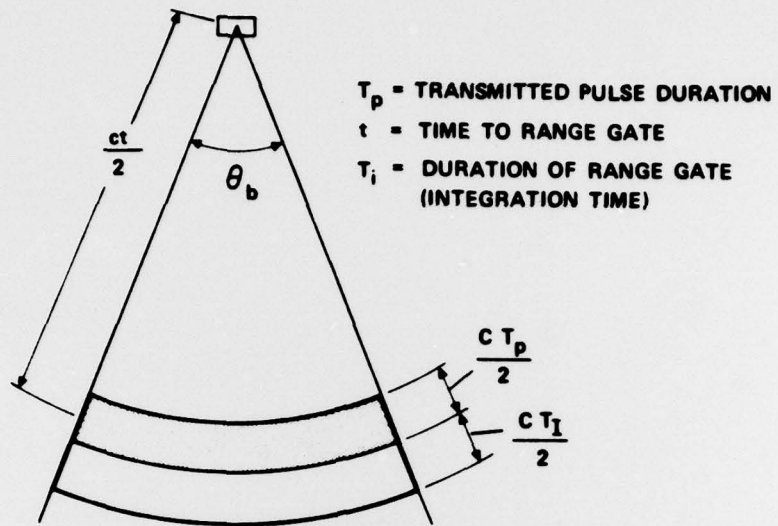


Figure 1-1. Volume Resolution Element in a Remote Acoustic Velocity Sensing System.

## SECTION II OPERATING PRINCIPLES

In a Doppler system, velocity information is obtained from the frequency shifts in the signal returned by each scatterer due to its velocity relative to the source and receiver. Each receiving beam measures only one velocity component (in the mean propagation direction), and in order to completely define the velocity vector, at least three receiving beams are required. This leads to a configuration such as is shown in Figure 2-1 in which three round-trip beam patterns are used to obtain velocity components in three different directions each of which is associated with a different water mass. To interpret the data obtained, one must assume that the mean water velocity varies only with depth. This difficulty can be circumvented by employing a multistatic configuration similar to that used in atmospheric probing. This is illustrated in Figure 2-2 in which a single transmitting beam is intersected from different directions by two receiving beam patterns.

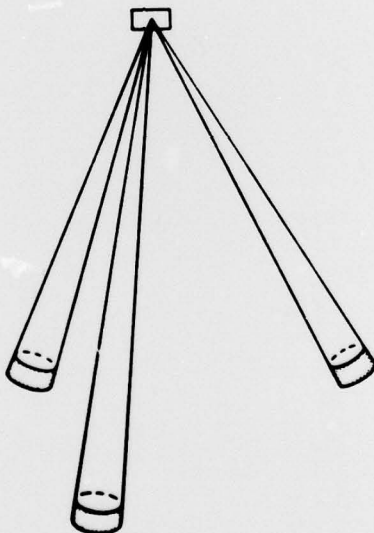


Figure 2-1. Doppler System Employing Three Round-Trip Beam Patterns to Obtain Three Components of Velocity Vector.

In a correlation system a pair of identical pulses separated by a relatively short time period are transmitted, and the waveforms received at several different elements in the receiving array are monitored when these pulses pass through the resolution volume of interest. The timing relationships are illustrated in Figure 2-3. It can be shown (see Appendix A) that the waveform received at one hydrophone from the first pulse will be repeated on the second

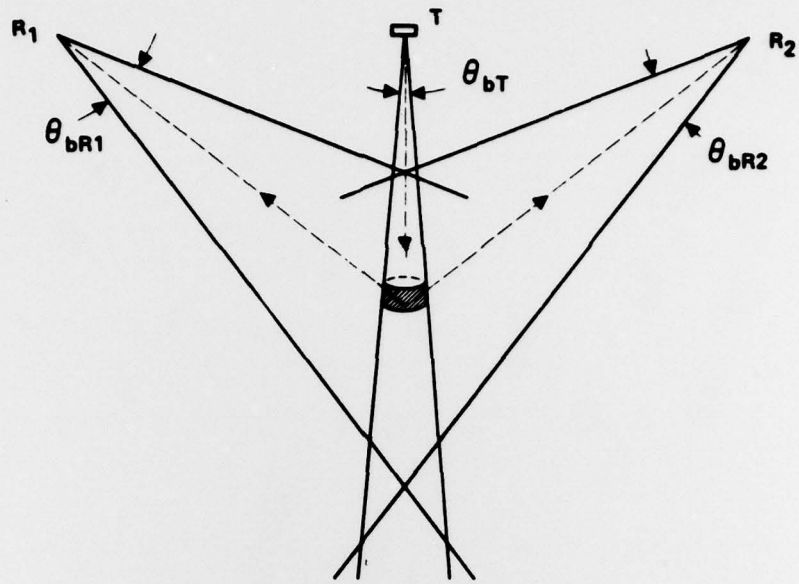


Figure 2-2. Multistatic Doppler System for Estimating Velocity Vector for a Single Resolution Volume.

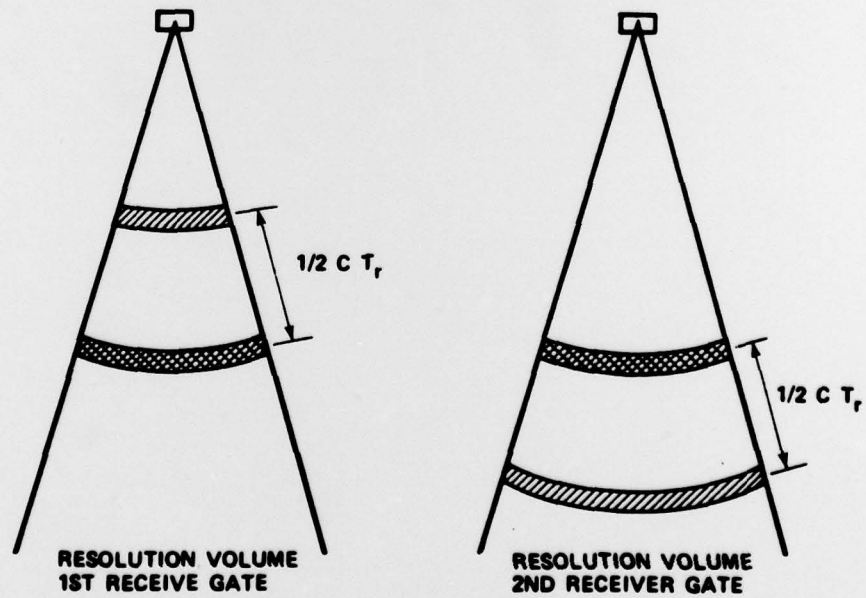


Figure 2-3. Resolution Volumes for Two Receiving Gates in a Correlation Velocity Measurement System.

pulse at a second hydrophone when the displacement between the two hydrophones is exactly equal to twice the distance moved by the volume scatterers during the interpulse period. The geometry is illustrated in Figure 2-4. This gives us the fundamental velocity relationship for the correlation sonar.

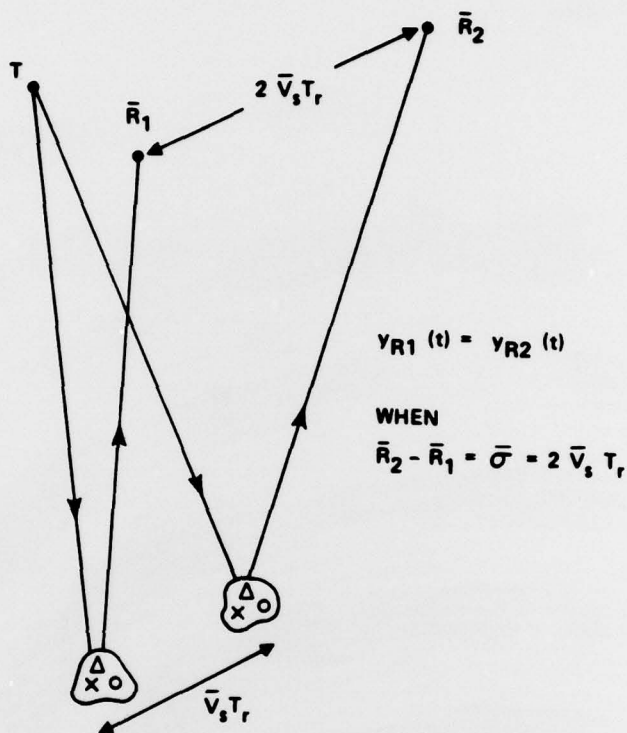


Figure 2-4. Waveform Invariance Geometry

$$\bar{R}_2 - \bar{R}_1 = \bar{\sigma} = 2\bar{v}_s T_r \tag{2-1}$$

Here  $\bar{R}_1$  and  $\bar{R}_2$  are the vector positions of the two hydrophones,  $\bar{\sigma}$  is the separation vector, and  $T_r$  is the interpulse period. We note that this relationship does not involve the local sound velocity. In practice, the received waveforms will not be identical because of differences in background noise, the presence of scattered energy from outside the volume of interest (from the other pulse), and the possibility that all of the scatterers may not have the same instantaneous velocity. However the cross correlation will be a maximum when the relationships of Equation (2-1) are satisfied. Thus, the basic approach in correlation sonar is to measure the cross correlations for pairs of elements having different vector separations and then to search for the vector separation for which the correlation is a maximum. The search may be accomplished, for example, by using appropriate interpolation or curve-fitting algorithms.

In a practical receiving array this can be done only in two dimensions, viz. the plane of the array. To meet this limitation, the correlation delay time can be adjusted to compensate for the time compression in the interpulse period associated with motion in the direction of acoustic propagation. Thus, a search in time delay for a correlation maximum is equivalent to a search in space in a direction normal to the array plane. Thus, using a single two-dimensional array, it is possible to estimate three orthogonal velocity components for the resolution volume of interest. This is believed to be a significant advantage for many applications.

To further compare the correlation and Doppler approaches to velocity measurement, note that the statistical properties of an acoustic field may be described either in terms of a space-time correlation function or of a frequency-direction (or frequency-wave number) spectrum. Within the region where the field may be assumed to be stationary and homogeneous, these representations form a three-dimensional Fourier transform pair.

$$R(\bar{\sigma}, \tau) = \int \int P(f, \bar{n}) \epsilon^{j2\pi f(\tau + \frac{\bar{n} \cdot \bar{\sigma}}{c})} df d\phi(\bar{n})$$

$$P(f, \bar{n}) = \left(\frac{f}{c}\right)^3 \int \int R(\bar{\sigma}, \tau) \epsilon^{-j2\pi f(\tau + \frac{\bar{n} \cdot \bar{\sigma}}{c})} d\tau dv(\bar{\sigma}) \quad (2-2)$$

Here  $\bar{n}$  is a unit vector defining a direction in space and  $d\phi(\bar{n})$  is a differential solid angle in that direction.  $\bar{\sigma}$  is the spatial separation vector and  $dv(\bar{\sigma})$  is a differential volume formed by the components of  $\bar{\sigma}$ .

In a Doppler system velocity information is encoded in the frequency-direction spectral domain, and resolution and accuracy is achieved by using narrowband signals and narrow beam patterns. In a correlation system velocity information is encoded in the space-time correlation domain, and wideband signals and relatively wide beam patterns are desired for good resolution and accuracy. This comparison identifies another important advantage of the correlation system for the velocity profiling application. In this application a short pulse-duration is required to provide good depth resolution. This is compatible with the wide signal bandwidth desired for velocity resolution in the correlation system. However, in the Doppler system time-bandwidth product constraints will limit our ability to provide both depth resolution and velocity resolution (unless, of course, depth resolution is provided by the beam patterns in a bistatic configuration).

### SECTION III DESIGN CONSIDERATIONS

As has been pointed out, depth resolution is determined by both the pulse duration and the duration of the integration window with these normally set equal to each other. At long ranges the curvature of the surfaces bounding the resolution volume over the beam width will also add to the depth dimension and decrease the resolution obtained. The number of resolution volumes for which a velocity estimate can be obtained on each transmission will depend on the computational capacity provided, and, when this is sufficient, a complete profile can be obtained on each transmission.

The maximum depth at which measurements can be made will depend on the acoustic power levels provided, the propagation losses, and the volume scattering strengths. In this regard, the correlation approach has an advantage in that since wide beam patterns can be used, one can use low frequencies to limit the absorption losses without requiring an unduly large sonar array. No fundamental limitation is seen on the depth to which velocity profiles can be measured. However, low frequency, high power, and some increase in the depth resolution dimension will be required for operation in the deep ocean areas.

## SECTION IV ACCURACY LIMITATIONS

One can identify a number of error sources which will limit the accuracy of the water velocity measurements which can be obtained with an acoustic sensing system. These include:

1. Corrections for the cyclical components of the sensor platform motion.
2. Geometric tolerances in the spacing and response of the individual receiving elements and in the angular alignment of the array structure.
3. Biases related to the interpolation and curve fitting algorithms used to estimate velocity components.
4. Statistical fluctuations in the (noise like) signals and limits on the smoothing which can be used.

With a mobile platform, as for example a ship-mounted installation, it is anticipated that the first of these will constitute the dominant source of error. This will be true for both correlation and Doppler sensor systems. Effectively, one must estimate the water current velocity by subtracting the sensor velocity relative to the water mass from the velocity relative to the bottom, both of which are likely to be much larger than the velocity difference of interest and to have errors proportional to the larger velocity values.

If the ship contains an inertial navigation system of sufficient accuracy, these subtractions can be made on an instantaneous basis for each depth increment. Otherwise, one can compare the acoustic measurements relative to the water volume with similar measurements made relative to the bottom. Due to propagation time differences, these two velocities will be measured at different times in the velocity cycle of the sensor platform. Changes in the sensor velocity and orientation during this time interval will appear as "sampling time" errors in the velocity estimates of the water current. These can be removed by averaging over a sufficiently large number of acoustic transmission cycles. Ideally, one should convert each measurement to a stable coordinate system before averaging, using sensor attitude information from the ship's gyro.

If the acoustic sensor is mounted on a stable platform, for example a bottom-mounted, upward-looking installation, the first error source will be absent and only the last three need be considered. Array orientation and element location uncertainties and element sensitivity differences may be viewed as imposing design tolerances which must be met if specified accuracy goals are to be met. The same is essentially true of the computational algorithms which convert acoustic measurements to velocity estimates.

The final error source arises from the fact that one is estimating the statistical properties of random signals from a sample of finite duration. It can be shown that the stan-



standard deviation of the velocity estimate due to this effect is directly proportional to the width of the spatial correlation function and inversely proportional to the interpulse correlation delay time, the maximum signal correlation and the square root of the bandwidth-integration time product. The relationship can be expressed by

$$\delta v = \frac{W}{2\pi^{1/2} K_v T_r (BT_I)^{1/2}} \left(1 + \frac{P_n}{P_s}\right) \quad (4-1)$$

Here  $W$  is the width of the spatial correlation function. It is approximately equal to the dimension of the transmitting aperture<sup>s</sup>, and is related to the round-trip beam pattern by

$$W = \frac{c}{f\theta_b} \quad (4-2)$$

$B$  is the signal bandwidth, and  $T_I$  is the integration time, which, on a single transmission is limited by the depth resolution requirement.  $T_r$  is the interpulse delay period.  $K_v$  represents the signal decorrelation which occurs with nonuniform scatterer velocities within the resolution volume. If it is assumed that the velocity component in the propagation direction of the individual scatterers has a Gaussian probability distribution about its mean value with a standard deviation  $\sigma_{vn}$ ,  $K_v$  is given by

$$K_v = \epsilon^{-2} \left( \frac{2\pi f \sigma_{vn} T_r}{c} \right)^2 \quad (4-3)$$

This limits the value of  $T_r$  which can be used in a given environment. If the value of  $T_r$  is chosen for which the product  $K_v T_r$  is a maximum, we obtain for the statistical fluctuation

$$\delta v = 2(\pi\epsilon)^{1/2} \frac{1}{\theta_b} \left( \frac{1}{BT_I} \right)^{1/2} \sigma_{vn} \left(1 + \frac{P_n}{P_s}\right) \quad (4-4)$$

Thus, the measurement error is seen to be determined by the velocity variations within the resolution volume and the smoothing provided, i. e., the angular extent of the volume probed and the bandwidth integration time product.

In a Doppler system, velocity variations within the resolution volume will limit the measurement accuracy by increasing the bandwidth of the received signals. However, the signal bandwidth will also be broadened by the use of short pulses to provide the required range resolution and by mean Doppler differences across the beam pattern. Which of these effects dominates will depend on the system design and the environmental parameters.

SECTION V  
OPERATING EXPERIENCE

Most of the operational experience with correlation sonar has involved its use as an instrument for accurately measuring the velocity of a ship relative to the ocean bottom or to a bounded water volume. Its potential for that application has, it is believed, been successfully demonstrated.\*

It is presently planned to modify a correlation sonar to provide a measure of velocity relative to two different water volumes on each acoustic transmission. This will provide a measure of current shear which can be compared with measurements from other sensors in an open water test. It was hoped to have some results to report at this time, but this has been precluded by schedule slippages. Hopefully some results will be reported at a future time.

However, some velocity measurements have been made which provide an indirect measure of a small water current at the Cayuga Lake Sonar Test Facility. These were obtained during an evaluation of a correlation sonar velocity log designed to track the bottom to depths of about 1000 ft, and provided with a variable receiving gate to track a water volume at shorter ranges. The transducer for this system, shown in Figure 5-1, contains a projector and six hydrophones in an area less than two inches square.

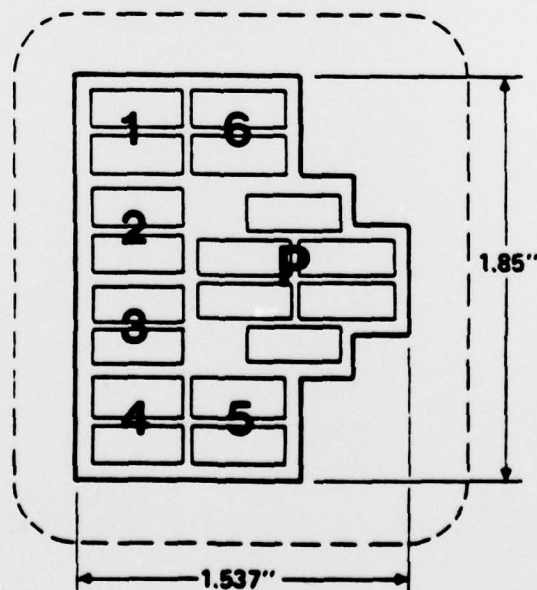


Figure 5-1. Correlation Sonar Velocity Log Transducer Layout

\*"Velocity Measurement Using Correlation Sonar," Frank R. Dickey, Jr., and John A. Edward, IEEE 1978 Position Location and Navigation Symposium Record, Nov. 6-9, 1978, pp. 255-264.

Tests were made at Cayuga Lake in which sonar measurements relative to the bottom and relative to a water volume were compared with data obtained by a survey team on shore using a tracking theodolite and a tellurometer.

Runs were made toward and away from the tellurometer for distances of about 3000 yds at speeds near 4 knots. Survey positions and sonar velocity readings were recorded at 30 second intervals during each run. The sonar also provided a readout of integrated distance along course and across course from which average velocities could be computed for each run and compared with values obtained from the survey end points. For 5 water track runs, 2 in one direction and 3 in the other, average velocity estimates indicated the presence of a water current of 0.06 knots. When the sonar measurements were corrected for this value of water current, the average difference between the sonar and survey velocity estimates was 0.016 knots.

We were not equipped to make independent measurements to confirm the presence of such a current, nor were the tests repeated to obtain a larger statistical data base. However, it is believed these tests are at least indicative of the capabilities of an acoustic sensor based on correlation measurements.

## SECTION VI SUMMARY

Correlation sonar offers an approach to remote acoustic velocity sensing in which three orthogonal components of the mean velocity vector for a single resolution volume can be measured using a single transmitting/receiving array. Velocity components parallel to the array surface (i. e., horizontal components in the usual system geometry) can be measured without a knowledge of the local sound velocity. Wideband signals may be used to provide smoothing of statistical fluctuations and to meet the range resolution requirements. Wide beam patterns are preferred for best system accuracy, maximum values being determined by volume resolution requirements. This makes it feasible to use low frequencies and modest array sizes to achieve deep water operation.

APPENDIX A

WAVEFORM INVARIANCE RELATIONSHIP FOR AN ENSEMBLE OF MOVING SCATTERERS

Referring to Figure A-1, consider a transmitter at  $\bar{T}$  and a receiver at  $\bar{R}_1$ . A signal pulse of relatively short duration is transmitted from  $\bar{T}$  and received at  $\bar{R}_1$  after being reflected by scattering mechanisms within the insonified volume. The signal received at  $\bar{R}_1$  can be expressed as the convolution of the transmitted signal waveform and an impulse response function for the medium.

$$y_{R_1}(t) = \int y_T(t-\tau)W_1(\tau)d\tau \tag{A-1}$$

At each value of  $\tau$ , the function  $W_1(\tau)$  will be determined by those scatterers for which the round-trip propagation time from  $\bar{T}$  and back to  $\bar{R}_1$  is equal to  $\tau$ . These scatterers lie on a surface designated  $S_1$  which, in a homogeneous medium, is an ellipsoid of revolution having foci at  $\bar{T}$  and  $\bar{R}_1$ . In the far field this ellipsoid of revolution will very nearly coincide with a sphere having its center at  $\bar{O}_1$  midway between  $\bar{T}$  and  $\bar{R}_1$ .

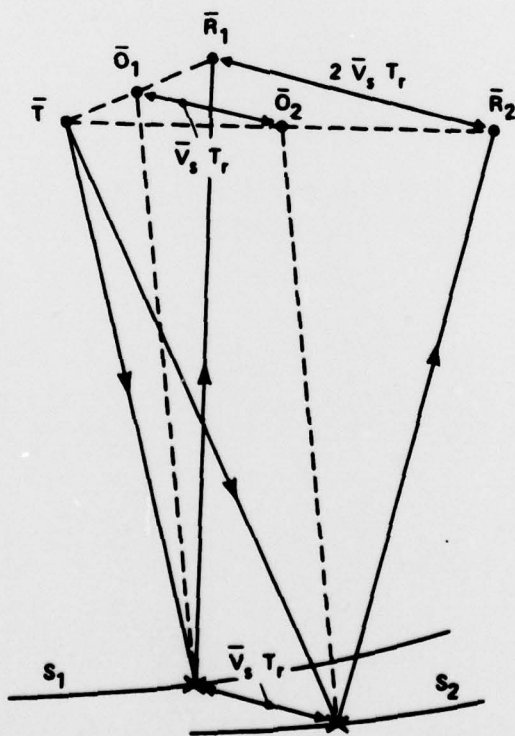


Figure A-1. Waveform Invariance Geometry

Now, wait a period of time  $T_r$  and transmit a second pulse from  $\bar{T}$  identical in waveform to the first. During the interpulse period assume that each scatterer on the surface  $S_1$  has the same velocity  $\bar{v}_s$  so that  $S_1$  moves a distance  $\bar{v}_s T_r$  to  $S_2$ . The surface  $S_2$  is very nearly spherical with center at  $\bar{O}_2$ , displaced from  $\bar{O}_1$  by

$$\bar{O}_2 - \bar{O}_1 = \bar{v}_s T_r \quad (\text{A-2})$$

Again using the far-field approximation, one can match  $\bar{S}_2$  to an ellipsoid of revolution having foci at  $\bar{T}$  and  $\bar{R}_2$  where  $\bar{R}_2$  is located such that the midpoint between  $\bar{T}$  and  $\bar{R}_2$  coincides with  $\bar{O}_2$ .

The signal received at  $\bar{R}_2$  corresponding to the second pulse can be expressed by

$$y_{R_2}(t) = \int y_T(t-\tau) W_2(\tau) d\tau$$

where  $W_2(\tau)$  is determined by the scatterers located on  $S_2$ . Under the assumed conditions,  $S_2$  and  $S_1$  represent the same scatterers with the same relative geometry, and thus one may conclude that  $W_2$  and  $W_1$  will be equal giving the same received waveforms  $y_{R_1}$  and  $y_{R_2}$ .

From the geometry and Equation (A-2), it can be seen that the required conditions are met whenever

$$\bar{R}_2 - \bar{R}_1 = 2\bar{v}_s T_r \quad (\text{A-3})$$

independent of the transmitter location  $\bar{T}$ .

In practice, of course, the same waveforms are not received because of the presence of added noise and scattered signals received from outside the resolution volume of interest (the other pulse), and Equation (A-3) represents a condition for maximum correlation.

While this model ignores a number of second-order effects such as refraction, weighting associated with the beam patterns, nonuniform scatterer velocity, etc., it is adequate to illustrate the fundamental principle on which correlation velocity measurements are based.