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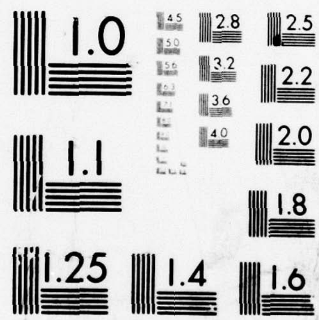
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LEVEL II

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Monterey, California

OPTIMALLY LOADED ELECTROHYDRODYNAMIC
POWER GENERATOR

O. Biblarz and T. H. Gawain

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By requiring that the local electrical field strength in an electrohydrodynamic (EHD) power generator be everywhere uniformly close to the critical limit, an optimum channel shape can be established. This shape is based on the one-dimensional flow of a compressible fluid into which are injected unipolar charged particles of negligible mobility. The maximum electrical work output per unit mass of fluid that can be attained in this way is shown to be greater by a factor of two than that of the best comparable channel of uniform area.

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1. Introduction

This paper pertains to a device for generating useful electrical power by means of an electrohydrodynamic (EHD) process. The device utilizes the flow of a fluid, normally a gas, such as air or water vapor, in which are entrained a very large number of very fine and well distributed solid or liquid particles, for example, water droplets as in an aerosol spray. The particles are electrically charged, either positively or negatively, as may be convenient in a particular case. The gas is caused to flow through a nozzle-like channel by the imposition of a suitable pressure drop. Let station 0 be the inlet to the nozzle. The charged particles are introduced into the stream of gas by a suitable injector at some upstream location, call it station i, and are removed from the stream by a suitable collector grid at some downstream location, call it station 2. An important intermediate location, station 1, is at the throat of the nozzle. The electrical charges which are transported downstream along with the particles on which they reside constitute an electrical current. These charges move through an electrical field which exerts forces upon them in a direction and sense opposed to the general fluid motion. By proper design of the injector, the size of the water droplets or particles may be so regulated that these droplets have low mobility, that is, they move approximately with the surrounding gas with negligible relative slip produced by the opposing electrical forces.

The gas stream does work on the charged particles in moving them against the resistance of the electrical forces. In this process the gas stream undergoes a corresponding decrease in total enthalpy. The work done upon the charged particles creates a difference in electrical potential between stations i and 2. These stations are connected by an external circuit which includes a useful electrical load. Thus the enthalpy drop of the gas is ultimately converted into a useful electrical power output from the external circuit.

The above scheme accomplishes the primary conversion of thermodynamic energy directly into electrical power without the use of any major rotating or reciprocating components such as large turbines or electrical generators. Nevertheless, there still exists a requirement for a small pump to recirculate the condensate and a fan to recirculate the carrier fluid. A condenser and a boiler are also normally a part of the complete system, but these various auxiliaries are not considered here in any detail as our present concern is primarily with the nozzle in which the basic thermo-electric power conversion occurs.

The above concepts are well known and can be found in the technical literature. See References 1, 2 and 3, for example. They are reviewed in this introduction merely to provide a proper background for our result which is quite specific and which is a consequence of the following analysis.

The performance that can be achieved by an EHD device is limited by, among other things, the maximum electrical field strength that can be sustained at the most critical point in the field without inducing electrical breakdown, through a spark discharge, of the carrier medium. The electrical field that is present in most EHD devices has its highest value at the point of injection. See References 1-4, for example. In this regard there is experimental evidence that breakdown field strength in any region varies linearly with the quantity $R\rho$ where R is the gas constant and where ρ is the fluid density in the region considered. This experimentally established limit depends of course on the type of gas/aerosol combination that is involved.

But the presence of a maximum field at the beginning of the EHD conversion channel limits performance severely because the rest of the channel must operate below maximum capacity or, in other words, the electric pressure is at its allowable limit in only a small fraction of the conversion channel.

The above considerations suggest that an optimum design would be one in which the local field strength is everywhere uniformly close to the critical limit. Such a design would achieve greater electric work output per unit mass of fluid than would any other. The essence of this paper is the recognition that a uniform maximum loading through the channel is the optimum loading and that this condition can indeed be achieved at least to an acceptable approximation. A secondary aim of this paper is to derive the characteristic geometrical features of the optimum design and certain associated performance parameters and limits.

The geometrical, electrical, and thermodynamic features of such a power generator are governed by various physical laws of which one of the most significant in the present context is Poisson's equation as it applies to an electric field. In its full generality, Poisson's equation is three dimensional in nature; by restricting attention to configurations having polar symmetry we can simplify this to a two-dimensional form. Moreover, by further restricting the application to configurations whose largest radial dimensions are small in comparisons with their axial length, we may finally simplify Poisson's equation to a one-dimensional approximation. Despite certain limitations in accuracy, this one-dimensional version can be more enlightening than the more elaborate two-dimensional analysis because it shows basic trends so much more simply and clearly. It is this one-dimensional approximation on which the present paper is based.

The analysis must deal not only with Poisson's equation for electrical fields, but also with the laws of fluid flow. It is consistent with the foregoing commentary to write these laws also in their common one-dimensional forms. At this level of idealization it is also appropriate to treat the flow through the channel as isentropic. We also consider the fluid as a perfect gas for which γ, the ratio of specific heats, is constant.

One other special assumption is involved. This is based on the fact that the drop in total temperature through the channel is normally very small in comparison with the absolute total temperature T_o at the nozzle inlet, station 0. Hence it is permissible for certain analytical purposes to neglect this temperature drop and to approximate the flow through the channel as an isentropic flow of constant total temperature. Calculations to be presented later show that the actual change of total temperature amounts to less than one percent for a typical case.

Future progress in EHD power generation might increase electric power output by an order of magnitude over levels that seem currently feasible. In that case the drop in total temperature through an EHD nozzle could become significant. Fortunately, the present analysis can be revised and generalized to account rigorously for such variation in total temperature when and if necessary. Under present circumstances, however, it is not warranted to complicate the analysis to include this refinement since at low power outputs it has no appreciable effect on the final calculated results.

2. Analysis

It is convenient for the purposes of the present analysis to introduce a parameter σ which is used to distinguish between the two distinct cases of positively and negatively charged particles. Specifically, we set $\sigma = +1$ for the case of positively charged particles, and we set $\sigma = -1$ for the case of negatively charged particles. It is then appropriate to denote the electrical charge per unit mass by means of the product σq where q has units of coulombs/kg and is always positive by definition while σ is a dimensionless factor as defined previously.

Using the above notation we may write the following two expressions for the electric current i and the mass flow rate $\dot{m}$ through the channel, namely,

$$i = \sigma q \rho A v = \text{constant along channel} \tag{2.1}$$

$$\dot{m} = \rho A v = \text{constant along channel} \tag{2.2}$$

where

i = electric current, amps

$\dot{m}$ = mass flow rate through channel, kg/sec

q = electric charge per unit mass, positive by definition, coulomb/kg

ρ = fluid density, kg/m^3 (variable)

A = cross-sectional area of channel at any given streamwise station, m^2 (variable)

v = mean axial velocity at any given streamwise station, m/sec (variable)

Note that Eq. (2.1) is based on the previously stated assumption that size of the fluid particles and the charge on each particle can be so regulated that the mobility of the particle with respect to the surrounding medium is essentially negligible.

By dividing Eq. (2.1) by Eq. (2.2) we also find that

$$\frac{i}{\dot{m}} = \sigma q = \text{constant along channel} \tag{2.3}$$

The one-dimensional version of Poisson's equation can now be written in the form

$$\left(\frac{d^2\phi}{dz^2}\right) = -\frac{\sigma}{\varepsilon} q\rho \tag{2.4}$$

where z = axial coordinate, m

ϕ = electrical potential, volts

$\varepsilon = \varepsilon_o$ = electrical permittivity of free space

$= 8.854 \times 10^{-2}$ farad/m

The local electrical field strength at an arbitray axial station z is defined as usual

$$E = - \left(\frac{d\phi}{dz} \right) \tag{2.5}$$

where E = field strength, volts/m

As will be seen here, an optimum EHD generator has an electric field which does not change sign from inlet to outlet. Under these circumstances the electric field is always negative for positive space charged and positive for negative space charge. Hence we may write

$$E = - \sigma |E| \tag{2.6}$$

It has been established by experiment that over a broad (but not unlimited) range, the field strength at breakdown is well represented by the simple linear law (see Reference 2).

$$|E_B| = C_o + C_B R \rho \tag{2.7}$$

where C_o and C_B are characteristic constants of the medium. R is the gas constant in units of Joule/kg^{o}K . Experimental measurements show that constants C_o and C_B happen to have the same numerical values for both air and steam. However, we defer to later pages of this paper any reference to actual experimental values. Of course, the present analysis is restricted to that specific range of conditions for which Eq. (2.7) is in fact a valid approximation.

If we now impose the constraint that the local field strength is everywhere just equal to its critical value at impending breakdown, we readily find from the last three equations that

$$\left(\frac{d\phi}{dz}\right) = \sigma(C_o + C_B R \rho) \tag{2.8}$$

Now differentiating this gives

$$\left(\frac{d^2\phi}{dz^2}\right) = \sigma C_B R \frac{d\rho}{dz} \tag{2.9}$$

Upon equating the right sides of Eqs. (2.4) and (2.9) and simplifying, we obtain the important relation

$$\frac{1}{\rho}\left(\frac{d\rho}{dz}\right) = -\frac{q}{\varepsilon C_B R} \tag{2.10}$$

The reciprocal of the quantity on the right side of Eq. (2.10) (momentarily disregarding the negative sign) now identifies a significant characteristic length; let us denote it by symbol λ.

$$\lambda = \frac{\varepsilon C_B R}{q} \quad \text{(meters)} \tag{2.11}$$

It is very instructive to rewrite Eq. (2.11) in the following alternative form

$$\lambda q = \lambda_{min} q_{max} = \varepsilon C_B R = \text{constant} \tag{2.12}$$

Notice that the quantity $\varepsilon C_B R$ is a characteristic property of the medium. Once the medium is chosen the designer has no further control of the value of this constant. Eq. (2.12) now tells us that in order to maintain the electrical loading at incipient breakdown, the product λq must remain constant. Thus any increase in q must be accompanied by a corresponding decrease in λ, or vice versa.

Experience up to now indicates, however, that there exists some practical upper limit q_{max} to the value of q that can actually be achieved for any specific type of design, and hence there exists some corresponding lower limit λ_{min} on the accompanying longitudinal characteristic length. Of course, a value of q less than q_{max} can always be employed if necessary in which case the corresponding value of λ will be greater than λ_{min}.

The situation is shown schematically in Fig. 2.1 in the form of a log-log plot. The solid line represents the locus of points all of which produce incipient breakdown. Incidentally, it will also be shown later that all of these points correspond to the same theoretical power output from the generator. Note that the line continues on indefinitely toward decreasing values of q and increasing values of λ. At the right end, however, it terminates at the point corresponding to q_{max} and λ_{min}.

Some typical values of these various characteristic constants based on published data are summarized in Table 2.1 for reference purposes.

Upon substituting λ into Eq. (2.10), we can easily integrate the simple expression that is thereby obtained. The result is

$$\boxed{\left(\frac{\rho}{\rho_o}\right) = e^{-\frac{z}{\lambda}}} \qquad (2.13)$$

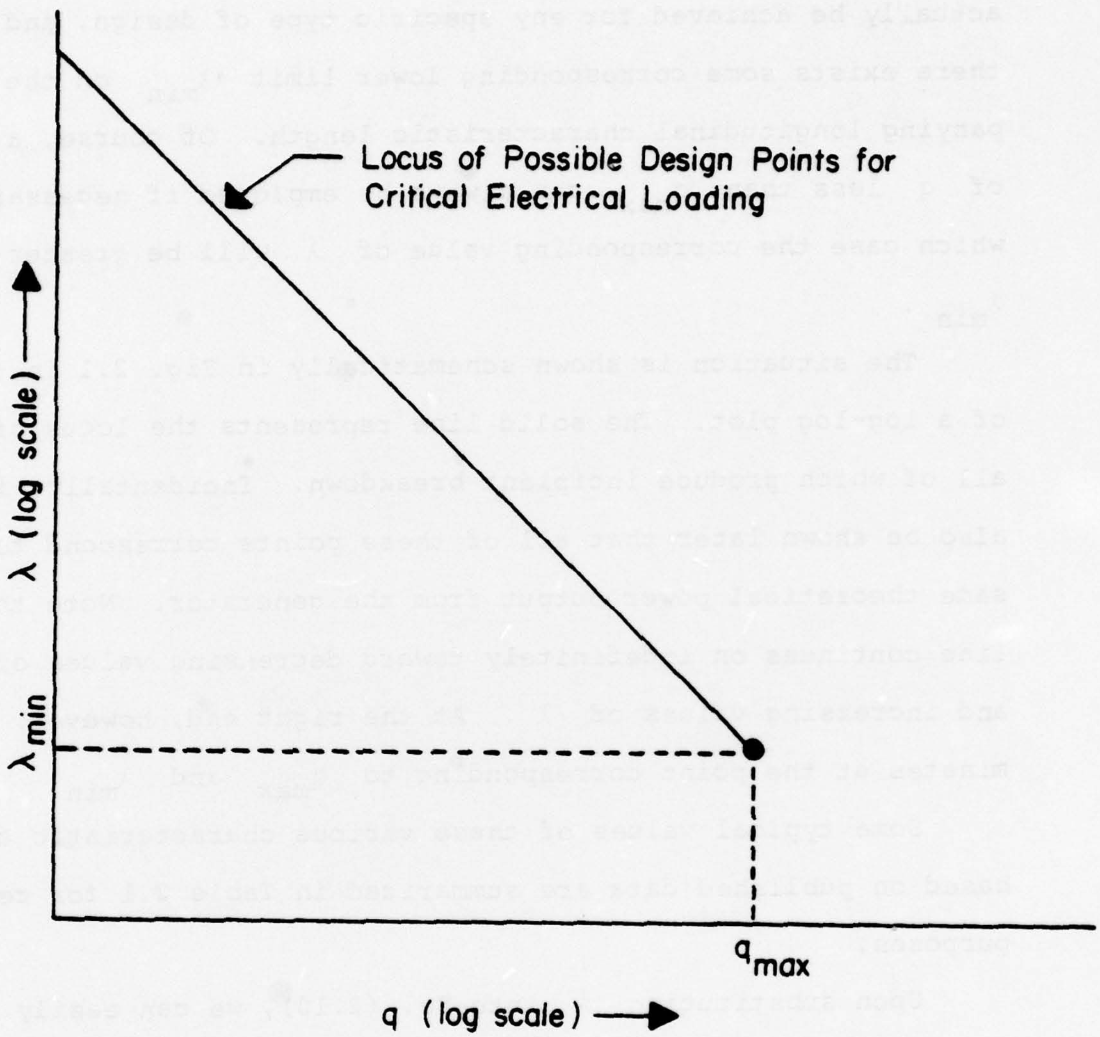

Fig. 2.1 Design Points and Design Limits for an Optimum Slender EHD Nozzle

Table 2.1 Typical EHD Properties

Source of Data		Ref. 1	Ref. 2	Ref. 3
Medium		Water droplets in steam	Water droplets in air	Water droplets in air
Quantity	Units			
C_o	volts/m	8.63×10^5	8.63×10^5	8.63×10^5
C_B	$m^2\ {}^oK/C$	9.49×10^3	9.49×10^3	9.49×10^3
$\varepsilon\ C_B\ R = \lambda q$	C m/kg	3.9×10^{-5}	2.4×10^{-5}	2.4×10^{-5}
q_{max}	C/kg	2.5×10^{-3}	3×10^{-2}	3×10^{-2}
λ_{min}	m	1.5×10^{-2}	8×10^{-4}	8×10^{-4}

This important result is the key to all remaining details of the analysis.

Eq. (2.13) reveals that the density continues to drop monotonically with increasing distance downstream. Thus the flow corresponds to the known isentropic flow through a converging-diverging nozzle for which the walls are so designed that the density drops exponentially as specified by Eq. (2.13). Let subscript 1 denote the throat of the nozzle. Then the following relations are known to apply

$$\left(\frac{\rho}{\rho_o}\right) = \left[1 + \frac{\gamma - 1}{2} M^2\right]^{-\frac{1}{(\gamma - 1)}} \qquad (2.14)$$

$$\left(\frac{r}{r_1}\right) = \left[\frac{1 + \frac{\gamma - 1}{2} M^2}{\frac{\gamma + 1}{2}}\right]^{\frac{(\gamma + 1)}{4(\gamma - 1)}} \frac{1}{\sqrt{M}} \tag{2.15}$$

where γ = ratio of specific heats

M = Mach number at station z

r = nozzle radius at station z

r_1 = nozzle radius at throat

It is readily deduced from Eqs. (2.13) and 2.14) that

$$\left(\frac{z}{\lambda}\right) = \frac{1}{(\gamma - 1)} \ln \left[1 + \frac{\gamma - 1}{2} M^2\right] \tag{2.16}$$

Eqs. (2.15) and (2.16) now constitute a pair of parametric equations for $(\frac{r}{r_1})$ as a function of $(\frac{z}{\lambda})$ with Mach number M as parameter. These equations define the shape of the optimum nozzle having uniform loading just below breakdown all along its length. A tabulation and a curve of $(\frac{r}{r_1})$ versus $(\frac{z}{\lambda})$ are given in the next section, using $\lambda = 1.3$. This value of λ is representative for steam. See Table 4.1 and Fig. 4.1.

At the throat of the nozzle $M = M_1 = 1$. Then Eq. (2.16) gives

$$\left(\frac{z_1}{\lambda}\right) = \frac{1}{(\gamma - 1)} \ln \left(\frac{\gamma + 1}{2}\right) \tag{2.17}$$

For $\gamma = 1.3$ this yields

$$\left(\frac{z_1}{\lambda}\right) = 0.466 \tag{2.18}$$

Let z_i denote the axial station which locates the injector that introduces unipolar electrical charges into the flow. We do not stipulate in advance any particular fixed location for the injector because the nature of the charged particles may require one location or another depending on certain supersaturation conditions that might be needed for the formation of the aerosol droplets (see Reference 1). Moreover, the converging-diverging nozzle does not have to be of the shape specified in this analysis upstream of the injector location.

Of great importance is the theoretical gross power output of such an optimum device. To determine this we must first find the current from Eq. (2.1). It is advantageous to evaluate the terms on the right side of Eq. (2.1) at the nozzle throat, station 1. Thus,

$$i = \sigma q \rho_1 A_1 v_1 \tag{2.19}$$

Now letting a_o and a_1 denote sonic velocity at stations 0 and 1, respectively, we have

$$v_1 = a_1 = a_o \left(\frac{a_1}{a_o}\right) \tag{2.20}$$

Upon combining the above two relations we finally obtain the current in the form

$$i = \sigma q \rho_o a_o A_1 \left[\left(\frac{\rho_1}{\rho_o}\right)\left(\frac{a_1}{a_o}\right)\right] \tag{2.21}$$

Note, however, that from the previous analysis of the isentropic nozzle flow, it follows that

$$\left[\left(\frac{\rho_1}{\rho_o}\right)\left(\frac{a_1}{a_o}\right)\right] = \left[\frac{2}{\gamma + 1}\right]^{\frac{\gamma + 1}{2(\gamma - 1)}} \tag{2.22}$$

Moreover, it is useful to eliminate q from Eq. (2.21) by means of Eq. (2.12) and to rearrange the result in dimensionless form. In this way we finally obtain the dimensionless current in the form

$$\left(\frac{\lambda i}{\varepsilon C_B R \rho_o a_o A_1}\right) = \sigma \left[\frac{2}{\gamma + 1}\right]^{\frac{(\gamma + 1)}{2(\gamma - 1)}} \tag{2.23}$$

The next step is to find the change in electric potential between stations i and 2. This is accomplished by integrating Eq. (2.8) between these limits. We again rearrange the result in dimensionless form. The result is

$$\frac{(\phi_2 - \phi_i)}{\rho_o C_B R \lambda} = \sigma \left\{\left(\frac{C_o}{C_B R \rho_o}\right)\left(\frac{z_2 - z_i}{\lambda}\right) + \left[e^{-\frac{z_i}{\lambda}} - e^{-\frac{z_2}{\lambda}}\right]\right\} \tag{2.24}$$

Finally, the theoretical power output follows from the relation

$$P_e = i(\phi_2 - \phi_i) \tag{2.25}$$

where P_e = theoretical gross electrical power, watts

The desired power can be obtained in dimensionless form by direct multiplication of Eqs. (2.23) and (2.24). After some minor rearrangement we obtain

$$\left(\frac{P_e}{\varepsilon\, C_B^{\,2}\, R^2\, \rho_o^{\,2}\, a_o\, A_1}\right) = \left\{\left(\frac{C_o}{C_B\, R\, \rho_o}\right)\left(\frac{z_2 - z_i}{\lambda}\right) + \left[e^{-\frac{z_i}{\lambda}} - e^{-\frac{z_2}{\lambda}}\right]\right\}\left[\frac{2}{\gamma + 1}\right]^{\frac{(\gamma + 1)}{2(\gamma - 1)}} \tag{2.26}$$

This result is fundamental. It shows just how the theoretical gross power output of the ideal nozzle depends on the various geometrical and physical parameters of the problem.

It is also instructive to observe from Eqs. (2.23), (2.24) and (2.26) how current, voltage and power vary with the characteristic length parameter λ of the nozzle. In other words we hold nozzle shape constant and simply vary the absolute size. Thus the quantities z_i/λ and z_2/λ remain constant. Then over the range $\lambda \geq \lambda_{min}$,

$$i \sim \lambda^{-1} \tag{2.27}$$

$$(\phi_2 - \phi_i) \sim \lambda \tag{2.28}$$

$$P_e \sim \lambda^{o} = \text{independent of } \lambda \tag{2.29}$$

Hence current is inversely proportional to size, voltage is directly proportional to size, and power is independent of size! This comes about, of course, because of the limitations imposed by the electrical breakdown phenomenon; see also Ref. 2.

Voltages tend to be inconveniently high in EHD devices. The above scaling rules suggest that this problem can be alleviated by making EHD devices small, with λ as close to λ_{min} as feasible. To the extent that it can be practically achieved, this method has the advantage that it decreases the size of the device without any corresponding decrease in its power output.

Incidentally, it should not be overlooked that Eq. (2.26), being based on a purely one-dimensional approximation, has high accuracy only for sufficiently low values of the ratio r_2/z_2.

3. Optimization

It is convenient to simplify Eq. (2.26) by introducing the following auxiliary notation. Let

$$\alpha = \left(\frac{C_o}{C_B \; R \; \rho_o} \right) \tag{3.1}$$

$$\zeta = \frac{z}{\lambda} \tag{3.2}$$

and let Eq. (2.26) be rewritten in the form

$$\left(\frac{P_e}{\varepsilon \, C_B^{\,2} \, R^2 \, \rho_o^{\,2} \, a_o \, A_1} \right) = F_e = \left[\alpha(\zeta_2 - \zeta_i) + e^{-\zeta_i} - e^{-\zeta_2} \right] \left(\frac{2}{\gamma + 1} \right)^{\frac{(\gamma + 1)}{2(\gamma - 1)}} \tag{3.3}$$

Another quantity of fundamental importance is the exit kinetic power of the jet. This is defined as follows

$$P_j = \left[\rho_2 \, A_2 \, V_2 \right] \left(\frac{V_2^{\,2}}{2} \right) \tag{3.4}$$

By using the various well known relations for isentropic compressible flow in a channel, we can readily reduce this to the following dimensionless form

$$\left(\frac{P_j}{\rho_o a_o^3 A_1}\right) = F_j(M_2) \qquad (3.5)$$

where function $F_j(M_2)$ is as follows:

For $M_2 \leq 1$

$$F_j(M_2) = \frac{M_2^{\,3}}{2}\left[1 + \frac{\gamma - 1}{2} M_2^{\,2}\right]^{-\frac{(3\gamma - 1)}{2(\gamma - 1)}} \qquad (3.6)$$

For $M_2 \geq 1$

$$F_j(M_2) = \frac{1}{2}\left[\frac{2}{\gamma + 1}\right]^{\frac{(\gamma + 1)}{2(\gamma - 1)}} \left[1 + \frac{\gamma - 1}{2} M_2^{\,2}\right]^{-1} M_2^{\,2} \qquad (3.7)$$

At $M_2 = 1$ these two expressions both give the result

$$F_j(1) = \frac{1}{2}\left[\frac{2}{\gamma + 1}\right]^{\frac{(3\gamma - 1)}{2(\gamma - 1)}} \qquad (3.8)$$

It will also be useful later to have the value of F_j in the limit as M_2 increases without bound. From Eq. (3.7) we may deduce that this is

$$F_j(\infty) = \frac{1}{(\gamma - 1)}\left[\frac{2}{\gamma + 1}\right]^{\frac{(\gamma + 1)}{2(\gamma - 1)}} \qquad (3.9)$$

Consider an EHD power generator which operates as a closed system. The fluid which discharges from the nozzle exit must be recirculated back to the nozzle inlet. This involves decelerating the flow, cooling the fluid, condensing and separating the liquid droplets, and so on. It involves restoring the pressure of the fluid by means of a suitable fan, compressor and/or pump, reheating the liquid and completing the thermodynamic cycle. These processes require power. The basic compressor and/or pump power requirement is inherent in the thermodynamic cycle itself and cannot be eliminated. Additional power demands arise from the various irreversibilities that inevitably occur in any real physical system; this additional power cannot be completely eliminated but it can be minimized by careful engineering design. Fortunately, some of the power required can actually be recovered from the exit kinetic power of the jet itself. Inevitably, however, there will be a net power demand, call it δP_j, which must be subtracted from the gross electric power output P_e. Consequently, the net useful electrical power becomes

$$P_{e\ net} = P_e - \delta P_j \tag{3.10}$$

Yet another jet power term is very useful. This is the jet power developed at the throat of the nozzle, call it $P_j(1)$. This makes a useful dimensional reference parameter which may be used to normalize the net power output $P_{e\ net}$.

This reasoning leads us to define the following useful net power coefficient, namely,

$$\frac{P_{e\,net}}{P_j(l)} = \chi = \frac{(P_e - \delta P_j)}{P_j(l)} \tag{3.11}$$

Upon substituting into this definition the expressions for P_e , P_j and $P_j(l)$ as previously developed, and upon simplifying, we can reduce the result to the form

$$\boxed{\chi = \frac{1}{F_j(l)} \left\{ \beta\, F_e - \delta\, F_j \right\}} \tag{3.12}$$

where, for a perfect gas,

$$\beta = \left(\frac{\varepsilon\, C_B{}^2\, R^2\, \rho_o}{a_o{}^2} \right) = \left(\frac{\varepsilon\, C_B{}^2\, p_o}{\gamma T_o{}^2} \right) \tag{3.13}$$

It is therefore clear that we should maximize inlet stagnation pressure p_o while keeping T_o no higher than necessary to obtain proper droplet size in the medium. Moreover it might be possible to eliminate this restriction on T_o by injecting solid or liquid charged particles rather than condensing droplets. In this case T_o could be sharply reduced thus greatly incresing the value of the crucial parameter β .

Eq. (3.12) for the power coefficient χ is the key result of this section. The optimization of an EHD generator can be based in part on an effort to maximize this quantity. In this connection, however, another quantity of interest is a power ratio η which we define as follows:

$$\left(\frac{P_{e\,net}}{P_e} \right) = \eta = 1 - \frac{\delta\, F_j}{\beta\, F_e} \tag{3.14}$$

Notice that the quantity η as defined above does not allude to the net heat input to the cycle. Hence it should not be confused with overall thermal efficiency which will always be some very much lower figure.

The various power terms that occur in Eq. (3.12) can also be used to determine the ratio of ΔT_o , the drop in total temperature through the nozzle, T_o , the inlet total temperature. This works out to be simply

$$\frac{\Delta T_o}{T_o} = \frac{\beta F_e}{F_j(\infty)} \tag{3.15}$$

Recall that the present analysis is restricted to cases in which $\Delta T_o/T_o$ is small compared with unity. Eq. (3.15) may be used to check whether this restriction is in fact satisfied in amy particular case.

It is evident from Eq. (3.12) that the value of the net power coefficient χ that can be attained is governed by the two key parameters β and δ . The coefficient β controls the maximum gross power that can be generated without electrical breakdown, and the factors that govern β in turn are shown in Eq. (3.13). Coefficient δ describes the power loss inherent in the thermodynamic cycle itself, as augmented by the various irreversibilities that must inevitably occur in such a system.

4. Summary of Key Relations and Results

The maximum specific charge q that can be employed in an ideal, slender EHD nozzle without causing electrical breakdown is governed by the relation

$$\lambda q = \lambda_{min}\, q_{max} = \varepsilon\, C_B\, R = \text{constant} \tag{4.1}$$

The proper geometric form for an optimum EHD nozzle is defined by the following pair of parametric equations

$$\left(\frac{r}{r_1}\right) = \left[\frac{1 + \frac{\lambda - 1}{2} M^2}{\frac{\gamma + 1}{2}}\right]^{\frac{(\gamma + 1)}{4(\gamma - 1)}} \frac{1}{\sqrt{M}} \tag{4.2}$$

$$\zeta = \left(\frac{z}{\lambda}\right) = \frac{1}{(\gamma - 1)}\, \ell n \left[1 + \frac{\gamma - 1}{2} M^2\right] \tag{4.3}$$

The value of ζ_2 corresponding to any chosen value of exit Mach number M_2 may be calculated from Eq. (4.3). Then we set

$$\alpha = \left(\frac{C_o}{C_B\, R\, \rho_o}\right) \tag{4.4}$$

and find

$$F_e = \left[\alpha(\zeta_2 - \zeta_i) + e^{-\zeta_i} - e^{-\zeta_2}\right]\left(\frac{2}{\gamma + 1}\right)^{\frac{(\gamma + 1)}{2(\gamma - 1)}} \tag{4.5}$$

where ζ_i denotes the location of the injector.

The following functions and constants may next be found.

For $M_2 \leq 1$

$$F_j = \frac{{M_2}^3}{2}\left[1 + \left(\frac{\gamma - 1}{2}\right) {M_2}^2\right]^{-\frac{(3\gamma - 1)}{2(\gamma - 1)}} \tag{4.6}$$

From $M_2 \geq 1$

$$F_j = \frac{1}{2}\left[\frac{2}{\gamma + 1}\right]^{\frac{(\gamma + 1)}{2(\gamma - 1)}} \left[1 + \left(\frac{\gamma - 1}{2}\right) M_2{}^2\right]^{-1} \tag{4.7}$$

also

$$F_j(1) = \frac{1}{2}\left[\frac{2}{\gamma + 1}\right]^{\frac{(\gamma - 1)}{2(\gamma - 1)}} \tag{4.8}$$

and

$$F_j(\infty) = \frac{1}{(\gamma - 1)}\left[\frac{2}{\gamma + 1}\right]^{\frac{(\gamma + 1)}{2(\gamma - 1)}} \tag{4.9}$$

Finally

$$\beta = \left(\frac{\varepsilon\, C_B{}^2\, P_o}{\gamma\, T_o{}^2}\right) \tag{4.10}$$

$$\chi = \frac{1}{F_j(1)}\left\{\beta\, F_e - \delta\, F_j\right\} \tag{4.11}$$

$$\eta = 1 - \frac{\delta\, F_j}{\beta\, F_e} \tag{4.12}$$

$$\frac{\Delta T_o}{T_o} = \frac{\beta\, F_e}{F_j(\infty)} \tag{4.13}$$

Values of χ , η and $\Delta T_o/T_o$ versus M_2 are listed in Table 4.2 for a typical case which is defined by the specified values of γ , ζ_i , α , β and δ . The value $\beta = 0.1$ corresponds to air or steam at about 250 atmospheres; this example illustrates the extreme measures that are required to lift performance to levels which are even marginally acceptable. The assumed value $\delta = 0.1$ implies that pump work and all fluid losses amount to only 10 percent of the kinetic power of the jet; this is a very optimistic hypothetical example, but it suffices to show certain trends. Reference (5) gives more detailed data on one particular type of loss that may be involved, namely, losses in pipe or channel bends.

Table 4.2 shows clearly the general effects of varying exit Mach number M_2 . It also shows the trade-off between performance parameters χ and η ; generally speaking an increase in either of these can only be obtained at the expense of a decrease in the other. Note that excessively high values of M_2 lead to negative values of output!

The last column of Table 4.2 reveals that, over the range of greatest interest, the value of $\Delta T_o/T_o$ is smaller than 0.01. This confirms that one of the basic restrictions of the theoretical analysis is indeed satisfied in this case.

The optimum EHD device disclosed herein is useful not only as a possible real physical system but also as a hypothetical standard against which the performance of any other real or hypothetical EHD system may be compared.

The analysis of this section could be greatly improved by treating the optimum EHD channel as just one element of a complete thermodynamic cycle. By working with such a complete cycle in detail the true value of parameter δ , instead of being

TABLE 4.1

OPTIMUM SLENDER EHD NOZZLE

$\gamma = 1.3$

M	z/λ	r/r_1
0.20	0.0199	1.7305
0.40	0.0791	1.2658
0.60	0.1753	1.0924
0.80	0.3056	1.0196
1.00	0.4659	1.0000
1.20	0.6519	1.0159
1.40	0.8591	1.0596
1.60	1.0833	1.1275
1.80	1.3203	1.2182
2.00	1.5667	1.3316
2.20	1.8194	1.4682
2.40	2.0757	1.6290
2.60	2.3337	1.8153
2.80	2.5916	2.0289
3.00	2.8481	2.2715
3.20	3.1020	2.5451
3.40	3.3526	2.8518
3.60	3.5992	3.1938
3.80	3.8416	3.5734
4.00	4.0793	3.9930
4.20	4.3121	4.4551
4.40	4.5400	4.9621
4.60	4.7629	5.5166
4.80	4.9808	6.1214
5.00	5.1938	6.7791

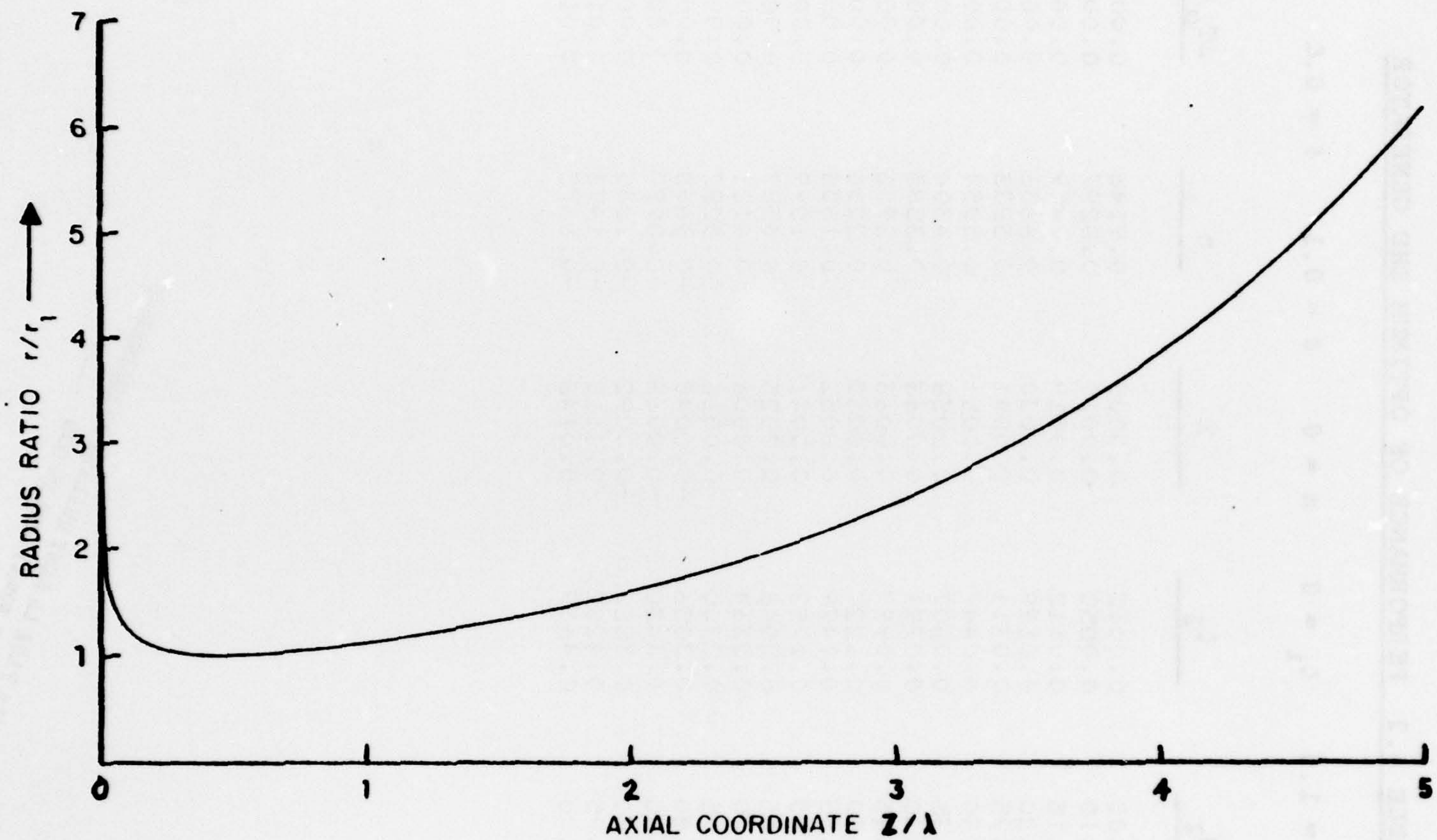

FIG. 4.1 SHAPE OF OPTIMUM SLENDER EHD NOZZLE

TABLE 4.2 PERFORMANCE OF OPTIMUM EHD GENERATOR

$\gamma = 1.3$ $\zeta_i = 0$ $\alpha = 0$ $\beta = 0.1$ $\delta = 0.1$

M_2	ζ_2	χ	η	$\Delta T_o/T_o$
0.05	0.0012	0.0003	0.9146	0.0000
0.10	0.0050	0.0010	0.8297	0.0001
0.15	0.0112	0.0019	0.7459	0.0003
0.20	0.0199	0.0030	0.6636	0.0006
0.25	0.0311	0.0041	0.5833	0.0009
0.30	0.0447	0.0051	0.5054	0.0013
0.35	0.0607	0.0058	0.4304	0.0018
0.40	0.0791	0.0063	0.3585	0.0023
0.45	0.0997	0.0063	0.2903	0.0028
0.50	0.1227	0.0060	0.2260	0.0035
0.55	0.1479	0.0052	0.1658	0.0041
0.60	0.1753	0.0041	0.1099	0.0048
0.65	0.2048	0.0025	0.0587	0.0056
0.70	0.2364	0.0006	0.0121	0.0063
0.75	0.2700	-0.0016	-0.0297	0.0071
0.80	0.3056	-0.0040	-0.0668	0.0079
0.85	0.3430	-0.0066	-0.0990	0.0087
0.90	0.3822	-0.0092	-0.1265	0.0095
0.95	0.4232	-0.0118	-0.1493	0.0104
1.00	0.4659	-0.0143	-0.1675	0.0112

merely assumed, could be actually calculate for each case. Also instead of working with relative indices of performance like χ and η, we could calculate actual overall thermodynamic efficiency. Moreover, the effect on overall thermodynamic efficiency of varying various characteristics of the thermodynamic cycle could then be systematically evaluated. Time limitations have precluded such a more ambitious and informative effort. The present report concentrates instead primarily on the concept of the optimum EHD channel itself.

5. Comparison of Optimum Channel with Uniform Channel

In EHD power generation, a uniform channel has the obvious merit of simplicity. In this case the gas density remains practically constant. However, the electrical field is always a maximum at the entrance to the working section, and gradually decreases thereafter. Maximum power output is obtained if the length be so chosen that the electrical field is zero at the exit from the working section.

It is well known that the electrical power output of such a uniform channel is

$$P_e = \frac{1}{2} \varepsilon E_B^2 AV \qquad \text{(watts)} \qquad (5.1)$$

where

E_B = breakdown strength of medium, volts/m

A = area of uniform channel, m^2

V = constant mean velocity in channel, m/sec

For the purpose of the present section we divide P_e by the mass flow rate in thus fixing the electrical work per unit mass of fluid (Joule/kg). It is also convenient to nondimensionalize the resulting quantity by dividing through by $a^{*2}/2$ (Joule/kg) where a^* is the sonic velocity which corresponds to a Mach number of unity. We also utilize Eq. (2.7) and make use of the fact that the constant C_o is negligible in comparison with the quantity $C_B R\rho$. In addition we employ various standard perfect gas relations. In this way we reduce Eq. (5.1) to obtain a dimensionless power coefficient C_e as follows:

$$\frac{P_e}{\dot{m}\, a^{*2}/2} = C_e = \frac{1}{2}\,\frac{\varepsilon (C_B^{\,2}\, R^2\, \rho^2) AV}{(\rho AV)\left[\left(\frac{\gamma}{\gamma+1}\right) RT_o\right]} = \left(\frac{\gamma+1}{2\gamma}\right)\left(\frac{\varepsilon\, C_B^{\,2}\, R\, \rho}{T_o}\right) \tag{5.2}$$

$$= \left(\frac{\gamma+1}{2\gamma}\right)\left(\frac{\varepsilon\, C_B^2\, p_o}{T_o^{\,2}}\right)\left(\frac{\rho}{\rho_o}\right) = \left(\frac{\gamma+1}{2\gamma}\right)\left(\frac{\varepsilon\, C_B^{\,2}\, p_o}{T_o^{\,2}}\right)\left[1 + \frac{\gamma-1}{2} M^2\right]^{-\frac{1}{(\gamma-1)}}$$

This shows clearly that reducing the Mach number M increases the electric work per unit mass of fluid. The maximum value that can be achieved in this way is

$$\left.\frac{P_e}{\dot{m}\, a^{*2}/2}\right)_{max} = \left. C_e\right)_{max} = \left(\frac{\gamma+1}{2\gamma}\right)\left(\frac{\varepsilon\, C_B^{\,2}\, p_o}{T_o^{\,2}}\right) \tag{5.3}$$

Now let us consider the optimally loaded configuration. Eq. (2.26) provides a convenient starting point. As in the previous case we neglect the quantity $(C_o/C_B\, R\, p_o)$ thus eliminating the corresponding term from the equation. The maximum power is obtained by setting $z_1/\lambda = 0$ and $z_2/\lambda = \infty$. The result can then be nondimensionalized in much the same way as for the previous case. Notice that in this case sonic velocity occurs at the throat of the nozzle so that $a^* = a_1$. The required result can now be developed as follows

$$\left(\frac{P_e}{\dot{m}\ a^{*2}/2}\right)_{opt} = C_e\Big)_{opt} = \frac{(\varepsilon\ C_B{}^2\ R^2\ \rho_o{}^2\ a_o\ A_1)}{(\rho_1\ A_1\ a_1)(a_1{}^2/2)}\left[\frac{2}{\gamma + 1}\right]^{\frac{(\gamma+1)}{2(\gamma-1)}}$$

$$= \frac{\varepsilon\ C_B{}^2\ (R\rho_o)\ R(\frac{\rho_o}{\rho_1})(\frac{a_o}{a_1})}{(\frac{\gamma}{\gamma+1})\ RT_o}\left[\frac{2}{\gamma + 1}\right]^{\frac{(\gamma+1)}{2(\gamma-1)}} \tag{5.4}$$

$$= \left(\frac{\gamma + 1}{\gamma}\right)\left(\frac{\varepsilon\ C_B{}^2\ p_o}{T_o{}^2}\right)\left[\frac{\gamma + 1}{2}\right]^{\left[\frac{1}{(\gamma-1)} + \frac{1}{2} - \frac{(\gamma+1)}{2(\gamma-1)}\right]}$$

The bracketed exponent turns out to be zero whereupon the final result becomes simply

$$\frac{P_e}{\dot{m}\ a^{*2}/2}\Bigg)_{opt} = C_e\Big)_{opt} = \left(\frac{\gamma + 1}{\gamma}\right)\left(\frac{\varepsilon\ C_B{}^2\ p_o}{T_o{}^2}\right) \tag{5.5}$$

Inspection of Eqs. (5.3) and (5.5) now reveals that the optimally loaded channel produces twice as much electric work per unit mass of fluid as does the uniform channel. These formulas are of fundamental importance because they show the basic performance limits of these two types of EHD channels. Substitution of typical numerical values into these formulas will also show how stringent these performance limits really are.

An improvement in performance by a factor of 2 is unfortunately not sufficient in itself to bring EHD power generation to practical reality. A further improvement by a factor of somewhere between 5 and 50 is probably still needed to achieve that goal. Moreover a factor of 2 improvement may not be sufficiently profitable under the circumstances to warrant the extra geometrical and other complexities involved. Nevertheless, the two performance limits developed above have considerable theoretical interest and value.

Incidentally, recall that another simple but important result of the present paper is its identification of the key dimensionless scaling parameter λ . Eqs. (2.27) through (2.29) show the basic significance of this quantity.

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