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AERO-ASTRONAUTICS REPORT NO. 146

AD A080445

FINAL REPORT

on

AIR FORCE GRANT No. AF-AFOSR-76-3075

NEW TECHNIQUES IN NUMERICAL ANALYSIS AND THEIR APPLICATION TO AEROSPACE SYSTEMS

by

ANGELO MIELE

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RICE UNIVERSITY

1979

(18) AFOSR

(19) TR-80-0058

(11) AF-

AERO-ASTRONAUTICS REPORT No. 146

(9) FINAL REPORT, 1 Sep 76 – 31 Mar 79

on

(15) AIR FORCE GRANT No. AF-AFOSR-76-3075

(6) NEW TECHNIQUES IN NUMERICAL ANALYSIS AND THEIR APPLICATION TO AEROSPACE SYSTEMS.

by

(10) ANGELO MIELE

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Final Report

on

Air Force Grant No. AF-AFOSR-76-3075

New Techniques in Numerical Analysis

and Their Applications to Aerospace Systems[1,2]

by

ANGELO MIELE[3]

Abstract. This document summarizes the research performed at Rice University during the period 1976-79 under Air Force Grant No. AF-AFOSR-76-3075 in several areas of numerical analysis of interest in aerospace systems theory, namely: solution of nonlinear equations, solution of differential equations, mathematical programming problems, and optimal control problems. The work summarized here is applicable to these areas of aerospace engineering: optimum atmospheric flight trajectories, optimum extra-atmospheric flight trajectories, optimum aerodynamic shapes, and optimum structures.

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[1]Period September 1, 1976 through December 31, 1979.

[2]This research was supported by the Office of Scientific Research, Office of Aerospace Research, United States Air Force, Grant No. AF-AFOSR-76-3075.

[3]Professor of Astronautics and Mathematical Sciences, Rice University, Houston, Texas.

Key Words. Aerospace engineering, applied mathematics, algorithm research, algorithm development; optimum atmospheric flight trajectories, optimum extra-atmospheric flight trajectories, optimum aerodynamic shapes, optimum structures.

Structural optimization, dynamic optimization, axial vibrations, frequency constraint, fundamental frequency constraint, cantilever beams, bars, rods.

Numerical analysis, numerical methods, computing methods, computing techniques, complexity of computation, philosophy of computation, comparison of algorithms, computational speed, measurement of computational speed, number of function evaluations, equivalent number of function evaluations, time-equivalent number of function evaluations, unconstrained minimization, mathematical programming.

One-dimensional search, cubic interpolation process, quadratic interpolation process, Lagrange interpolation scheme, modifications of the cubic interpolation process, alternatives to the cubic interpolation process, bisection process, interval of interpolation.

Differential equations, two-point boundary-value problems, multi-point boundary-value problems; method of particular solutions, quasilinearization algorithm, modified quasilinearization algorithm, restoration algorithm.

Gradient algorithms, conjugate gradient algorithms, gradient-restoration algorithms, conjugate gradient-restoration algorithms, sequential gradient-restoration algorithms, sequential conjugate gradient-restoration algorithms.

Optimal control, calculus of variations, differential constraints, nondifferential constraints, bounded state, bounded control, bounded time rate of change of the state; fixed initial state, free initial state, general boundary conditions; transformation techniques; state inequality constraints, linear state inequality constraints, partially linear state inequality constraints.

Contents

I. Introduction

This document summarizes the research performed at Rice University in the period 1976-79 under Air Force Grant No. AF-AFOSR-76-3075. The total duration of the grant was 40 months. The grant was successively monitored by Lt. Col. E.H. Ramirez, Dr. R.N. Buchal, and Captain C.E. Oliver.

The personnel participating in the research effort included the following people:

Faculty Personnel

Prof. A. Miele

Senior Personnel

Dr. A.V. Levy
Dr. A. Mangiavacchi

Junior Personnel

Mr. F. Bonardo
Mr. J.R. Cloutier
Ms. E. Coker
Mr. S. Gonzalez
Mr. G.T.C. Huang
Mr. C.J. Kao
Mr. E.J. Koo
Mr. C.Y. Leung
Mr. C.T. Liu
Mr. S. Meenaphant
Mr. B.P. Mohanty
Mr. A.K. Wu

As a partial result of the research performed under this grant, the following advanced degrees were awarded:

MS Degrees

J.R. Cloutier
A.K. Wu

PhD Degrees

J.R. Cloutier
S. Gonzalez
B.P. Mohanty
A.K. Wu

II. Research Achievements

The research undertaken under Grant No. AF-AFOSR-3075 over the past three years has spanned several areas of numerical analysis, namely: solution of nonlinear equations, solution of differential equations, mathematical programming problems, and optimal control problems. The principal results of this effort, summarized in 13 reports and 13 journal articles are described below.

(a) Development of criteria for testing and evaluation of algorithms for mathematical programming problems (Refs. 3 and 16).

(b) Modifications and alternatives to the cubic interpolation process for one-dimensional search (Refs. 4 and 23).

(c) Conversion of optimal control problems with free initial state into optimal control problems with fixed initial state (Ref. 15).

(d) Transformation technique for optimal control problems with linear or partially linear state inequality constraints (Refs. 8 and 22).

(e) Numerical determination of minimum mass structures with specified natural frequencies (Refs. 6, 7, 9, 18).

(f) Sequential conjugate gradient-restoration algorithm for optimal control problems with nondifferential constraints and given initial state (Refs. 1, 2, 19, 20, 21).

(g) Sequential ordinary gradient-restoration algorithm for optimal control problems with general boundary conditions, with and without nondifferential constraints (Refs. 10, 11, 17).

(h) Sequential conjugate gradient-restoration algorithm for optimal control problems with general boundary conditions, with and without nondifferential constraints (Refs. 12, 13, 25, 26).

In addition, two tutorial papers have been written, one dealing with some philosophical views on algorithms and computing methods in applied mathematics (Refs. 5 and 14) and one dealing with gradient algorithms for the optimization of dynamic systems (Ref. 24).

Areas of application are reviewed in Section III, and collaboration with Air Force personnel is reviewed in Section IV. A list of research reports is given in Section V, and a list of research papers is given in Section VI. Then, the abstracts of reports are given in Section VII, and the abstracts of papers are given in Section VIII.

III. Areas of Application

Attention to USAF technical personnel is called on the fact that the research summarized in Refs. 1-26 is eminently applicable to concrete problems of applied mathematics arising in several areas of aerospace engineering, namely: optimum atmospheric flight trajectories, optimum extra-atmospheric fligth trajectories, optimum aerodynamic shapes, and optimum structures. With particular reference to optimum structures, see Refs. 6, 7, 9, 18.

IV. Collaboration with Air Force Personnel

As a result of seminars given by the principal investigator at Wright-Patterson Air Force Base, Ohio, a collaboration has been undertaken with Dr. V.B. Venkayya, AFFDL. This collaboration has led to the employment of the sequential gradient-restoration algorithm and the modified quasilinearization algorithm in some problems of structural analysis (Refs. 6, 7, 9, 18). It is anticipated that this preliminary effort will lead to further research on optimum structures.

Further collaboration might be possible with Dr. E. Miller, AFFDL, on certain optimization problems of flight mechanics which arise in the reentry of hypervelocity vehicles.

V. Reports of the Aero-Astronautics Group

1. CLOUTIER, J.R., MOHANTY, B.P., and MIELE, A., Sequential Conjugate Gradient-Restoration Algorithm for Optimal Control Problems with Nondifferential Constraints, Part 1, Theory, Rice University, Aero-Astronautics Report No. 126, 1977.

2. CLOUTIER, J.R., MOHANTY, B.P., and MIELE, A., Sequential Conjugate Gradient-Restoration Algorithm for Optimal Control Problems with Nondifferential Constraints, Part 2, Examples, Rice University, Aero-Astronautics Report No. 127, 1977.

3. MIELE, A., GONZALEZ, S., and WU, A.K., On Testing Algorithms for Mathematical Programming Problems, Rice University, Aero-Astronautics Report No. 134, 1976.

4. MIELE, A., BONARDO, F., and GONZALEZ, S., Modifications and Alternatives to the Cubic Interpolation Process for One-Dimensional Search, Rice University, Aero-Astronautics Report No. 135, 1976.

5. MIELE, A., Some Philosophical Views on Algorithms and Computing Methods in Applied Mathematics, Rice University, Aero-Astronautics Report No. 136, 1976.

6. MIELE, A., Minimum Mass Structures with Specified Natural Frequencies, Rice University, Aero-Astronautics Memorandum No. WP-1, 1976.

7. MANGIAVACCHI, A., and MIELE, A., Some Qualitative Considerations on the Numerical Determination of Minimum Mass Structures with Specified Natural Frequencies, Rice University, Aero-Astronautics Memorandum No. WP-2, 1977.

8. MIELE, A., WU, A.K., and LIU, C.T., A Transformation Technique for Optimal Control Problems with Partially Linear State Inequality Constraints, Rice University, Aero-Astronautics Report No. 137, 1977.

9. MIELE, A., MANGIAVACCHI, A., MOHANTY, B.P., and WU, A.K., Numerical Determination of Minimum Mass Structures with Specified Natural Frequencies, Rice University, Aero-Astronautics Report No. 138, 1977.

10. GONZALEZ, S., and MIELE, A., Sequential Gradient-Restoration Algorithm for Optimal Control Problems with General Boundary Conditions, Rice University, Aero-Astronautics Report No. 142, 1978.

11. GONZALEZ, S., and MIELE, A., Sequential Gradient-Restoration Algorithm for Optimal Control Problems with

Nondifferential Constraints and General Boundary Conditions, Rice University, Aero-Astronautics Report No. 143, 1978.

12. WU, A.K., and MIELE, A., Sequential Conjugate Gradient-Restoration Algorithm for Optimal Control Problems with General Boundary Conditions, Rice University, Aero-Astronautics Report No. 144, 1978.

13. WU, A.K., and MIELE, A., Sequential Conjugate Gradient-Restoration Algorithm for Optimal Control Problems with Nondifferential Constraints and General Boundary Conditions, Rice University, Aero-Astronautics Report No. 145, 1978.

VI. Papers of the Aero-Astronautics Group

14. MIELE, A., Some Philosophical Views on Algorithms and Computing Methods in Applied Mathematics, Proceedings of the Workshop on Decision Information for Tactical Command and Control, Airlie, Virginia, 1976; Edited by R.M. Thrall, C.P. Tsokos, and J.C. Turner; Robert M. Thrall and Associates, Houston, Texas, pp. 192-208, 1976.

15. MIELE, A., MOHANTY, B.P., and WU, A.K., Conversion of Optimal Control Problems with Free Initial State into Optimal Control Problems with Fixed Initial State, Journal of the Astronautical Sciences, Vol. 25, No. 1, pp. 75-85, 1977.

16. MIELE, A., and GONZALEZ, S., On the Comparative Evaluation of Algorithms for Mathematical Programming Problems, Nonlinear Programming 3, Edited by O.L. Mangasarian, R.R. Meyer, and S.M. Robinson, Academic Press, New York, New York, pp. 337-359, 1978.

17. GONZALEZ, S., and MIELE, A., Sequential Gradient-Restoration Algorithm for Optimal Control Problems with General Boundary Conditions, Journal of Optimization Theory and Applications, Vol. 26, No. 3, pp. 395-425, 1978.

18. MIELE, A., MANGIAVACCHI, A., MOHANTY, B.P., and WU, A.K., Numerical Determination of Minimum Mass Structures with Specified Natural Frequencies, International Journal for Numerical Methods in Engineering, Vol. 13, No. 2, pp. 265-282, 1978.

19. MIELE, A., and CLOUTIER, J.R., Sequential Conjugate Cradient-Restoration Algorithm for Optimal Control Problems with Nondifferential Constraints, Applied Nonlinear Analysis, Edited by V. Lakshmikantham, J. Eisenfeld, and A.R. Mitchell, Academic Press, New York, New York, pp. 89-93, 1979.

20. MIELE, A., CLOUTIER, J.R., MOHANTY, B.P., and WU, A.K., Sequential Conjugate Gradient-Restoration Algorithm for Optimal Control Problems with Nondifferential Constraints, Part 1, International Journal of Control, Vol. 29, No. 2, pp. 189-211, 1979.

21. MIELE, A., CLOUTIER, J.R., MOHANTY, B.P., and WU, A.K., Sequential Conjugate Gradient-Restoration Algorithm for Optimal Control Problems with Nondifferential Constraints, Part 2, International Journal of Control, Vol. 29, No. 2, pp. 213-234, 1979.

22. MIELE, A., WU, A.K., and LIU, C.T., A Transformation Technique for Optimal Control Problems with Partially

Linear State Inequality Constraints, Journal of Optimization Theory and Applications, Vol. 28, No. 2, pp. 185-212, 1979.

23. MIELE, A., BONARDO, F., and GONZALEZ, S., Modifications and Alternatives to the Cubic Interpolation Process for One-Dimensional Search, Arabian Journal for Science and Engineering, Vol. 4, No. 2, pp. 121-128, 1979.

24. MIELE, A., Gradient Algorithms for the Optimization of Dynamic Systems, Advances in Control and Dynamic Systems: Theory and Applications, Vol. 16, Edited by C. T. Leondes, Academic Press, New York, New York, 1979.

25. WU, A.K., and MIELE, A., Sequential Conjugate Gradient-Restoration Algorithm for Optimal Control Problems with Nondifferential Constraints and General Boundary Conditions, Part 1, Optimal Control Application and Methods, Vol. 1, No. 1, 1980.

26. WU, A.K., and MIELE, A., Sequential Conjugate Gradient-Restoration Algorithm for Optimal Control Problems with Nondifferential Constraints and General Boundary Conditions, Part 2, Optimal Control Application and Methods, Vol. 1, No. 2, 1980.

VII. Abstracts of Reports

1. CLOUTIER, J.R., MOHANTY, B.P., and MIELE, A., Sequential Conjugate Gradient-Restoration Algorithm for Optimal Control Problems with Nondifferential Constraints, Part 1, Theory, Rice University, Aero-Astronautics Report No. 126, 1977.

Abstract. A sequential conjugate gradient-restoration algorithm is developed in order to solve optimal control problems involving a functional subject to differential constraints, nondifferential constraints, and terminal constraints. The algorithm is composed of a sequence of cycles, each cycle consisting of two phases, a conjugate gradient phase and a restoration phase.

The conjugate gradient phase involves a single iteration and is designed to decrease the value of the functional, while satisfying the constraints to first order. During this iteration, the first variation of the functional is minimized, subject to the linearized constraints. The minimization is performed over the class of variations of the control and the parameter which are equidistant from some constant multiple of the corresponding variations of the previous conjugate gradient phase. For the special case of a quadratic functional subject to linear constraints, various orthogonality and conjugacy conditions hold.

The restoration phase involves one or more iterations and is designed to restore the constraints to a predetermined accuracy, while the norm of the variations of the control and the parameter is minimized, subject to the linearized constraints. The restoration phase is terminated whenever the norm of the constraint error is less than some predetermined tolerance.

The sequential conjugate gradient-restoration algorithm is characterized by two main properties. First, at the end of each conjugate gradient-restoration cycle, the trajectory satisfies the constraints to a given accuracy; thus, a sequence of feasible suboptimal solutions is produced. Second, the conjugate gradient stepsize and the restoration stepsize can be chosen so that the restoration phase preserves the descent property of the conjugate gradient phase; thus, the value of the functional at the end of any cycle is smaller than the value of the functional at the beginning of that cycle. Of course, restarting the algorithm might be occasionally necessary.

To facilitate numerical integration, the interval of integration is normalized to unit length. Variable-time terminal conditions are transformed into fixed-time terminal conditions. Then, the actual time at which the terminal boundary is reached becomes a component of a vector parameter being optimized.

Convergence is attained whenever both the norm of the constraint error and the norm of the error in the optimality conditions are less than some predetermined tolerances. Several numerical examples illustrating the theory of this paper are given in Part 2 (see Ref. 2).

Key Words. Optimal control, gradient methods, conjugate gradient methods, numerical methods, computing methods, gradient-restoration algorithms, sequential gradient-restoration algorithms, sequential conjugate gradient-restoration algorithms, nondifferential constraints.

2. CLOUTIER, J.R., MOHANTY, B.P., and MIELE, A., Sequential Conjugate Gradient-Restoration Algorithm for Optimal Control Problems with Nondifferential Constraints, Part 2, Examples, Rice University, Aero-Astronautics Report No. 127, 1977.

Abstract. In Part 1 (see Ref. 1), Cloutier, Mohanty, and Miele developed the sequential conjugate gradient-restoration algorithm for minimizing a functional subject to differential constraints, nondifferential constraints, and terminal constraints. In this report, sixteen numerical examples are presented, four pertaining to a quadratic functional subject to linear constraints and twelve pertaining to a nonquadratic functional subject to nonlinear constraints. These examples demonstrate the feasibility as well as the convergence characteristics of the sequential conjugate gradient-restoration algorithm.

Key Words. Optimal control, gradient methods, conjugate gradient methods, numerical methods, computing methods, gradient-restoration algorithms, sequential gradient-restoration algorithms, sequential conjugate gradient-restoration algorithms, nondifferential constraints.

3. MIELE, A., GONZALEZ, S., and WU, A.K., <u>On Testing Algorithms for Mathematical Programming Problems</u>, Rice University, Aero-Astronautics Report No. 134, 1976.

Abstract. This paper considers the comparative evaluation of algorithms for mathematical programming problems. It is concerned with the measurement of computational speed and examines critically the concept of equivalent number of function evaluations N_e. Does this quantity constitute a fair way of comparing different algorithms?

The answer to the above question depends strongly on whether or not analytical expressions for the components of the gradient and the elements of the Hessian matrix are available. It also depends on the relative importance of the computational effort associated with algorithmic operations vis-a-vis the computational effort associated with function evaluations.

Both theoretical considerations and extensive numerical examples carried out in conjunction with the Fletcher-Reeves algorithm, the Davidon-Fletcher-Powell algorithm, and the quasilinearization algorithm suggest the following: the N_e concept, while accurate in some cases, has drawbacks in other cases; indeed, it might lead to a distorted view of the relative importance of an algorithm with respect to another.

The above distortion can be corrected through the

introduction of a more general parameter $\tilde{N}_e$. This generalized parameter is constructed so as to reflect accurately the computational effort associated with function evaluations and algorithmic operations.

From the analyses performed and the results obtained, it is inferred that, due to the weaknesses of the N_e concept, the use of the $\tilde{N}_e$ concept is advisable. In effect, this is the same as stating that, in spite of its obvious shortcomings, the direct measurement of the CPU time is still the more reliable way of comparing different minimization algorithms.

Key Words. Numerical analysis, numerical methods, computing methods, computing techniques, complexity of computation, philosophy of computation, comparison of algorithms, computational speed, measurement of computational speed, number of function evaluations, equivalent number of function evaluations, time-equivalent number of function evaluations, unconstrained minimization, mathematical programming.

4. MIELE, A., BONARDO, F., and GONZALEZ, S., Modifications and Alternatives to the Cubic Interpolation Process for One-Dimensional Search, Rice University, Aero-Astronautics Report No. 135, 1976.

Abstract. In this paper, the numerical solution of the problem of minimizing a unimodal function $f(\alpha)$ is considered, where α is a scalar. Two modifications of the cubic interpolation process are presented, so as to improve the robustness of the method and force the process to converge in a reasonable number of iterations, even in pathological cases. Modification M1 includes the nonoptional bisection of the interval of interpolation at each iteration of the process. Modification M2 includes the optional bisection of the interval of interpolation: this depends on whether the slopes $f_\alpha(\hat{\alpha}_o)$ and $f_\alpha(\alpha_o)$ at the terminal points $\hat{\alpha}_o$ and α_o of two consecutive iterations have the same sign or opposite sign.

An alternative to the cubic interpolation process is also presented. This is a Lagrange interpolation scheme in which the quadratic approximation to the derivative of the function is considered. The coefficients of the quadratic are determined from the values of the slope at three points: α_1 , α_2 , and $\alpha_3 = (\alpha_1+\alpha_2)/2$, where α_1 and α_2 are the endpoints of the interval of interpolation. The proposed alternative is investigated in two versions, Version A1 and

Version A2. They differ in the way in which the next interval of interpolation is chosen; for Version A1, the choice depends on the sign of the slope $f_\alpha(\alpha_o)$; for Version A2, the choice depends on the signs of the slopes $f_\alpha(\alpha_o)$ and $f_\alpha(\alpha_3)$.

Twenty-nine numerical examples are presented. The numerical results show that both modifications of the cubic interpolation process improve the robustness of the process. They also show the promising characteristics of Version A2 of the proposed alternative. Therefore, the one-dimensional search schemes described here have potential interest for those minimization algorithms which depend critically on the precise selection of the stepsize, namely, conjugate gradient methods.

Key Words. One-dimensional search, cubic interpolation process, quadratic interpolation process, Lagrange interpolation scheme, modifications of the cubic interpolation process, alternatives to the cubic interpolation process, bisection process, mathematical programming, interval of interpolation, numerical analysis, numerical methods, computing methods, computing techniques.

5. MIELE, A., Some Philosophical Views on Algorithms and Computing Methods in Applied Mathematics, Rice University, Aero-Astronautics Report No. 136, 1976.

Abstract. This paper summarizes some of the work done by the Aero-Astronautics Group of Rice University in the area of numerical methods and computing methods. It describes some of the philosophical thoughts that have guided this work throughout the years. Recommendations are offered concerning allocation of funds and distribution of funds. Additional recommendations are offered in order to bridge the gap between the top management of government agencies and the academic community.

Key Words. Aerospace engineering, applied mathematics, numerical methods, algorithm research, algorithm development.

6. MIELE, A., Minimum Mass Structures with Specified Natural Frequencies, Rice University, Aero-Astronautics Memorandum No. WP-1, 1976.

Abstract. The problem of the axial vibration of a cantilever beam is investigated numerically. The mass distribution that minimizes the total mass for a given fundamental frequency constraint is determined using both the sequential ordinary gradient-restoration algorithm (SOGRA) and an ad hoc modification of the modified quasilinearization algorithm (MQA).

Key Words. Structural optimization, cantilever beams, axial vibrations, fundamental frequency constraint, numerical methods, sequential ordinary gradient-restoration algorithm, modified quasilinearization algorithm.

7. MANGIAVACCHI, A., and MIELE, A., Some Qualitative Considerations on the Numerical Determination of Minimum Mass Structures with Specified Natural Frequencies, Rice University, Aero-Astronautics Memorandum No. WP-2, 1977.

Abstract. The problem of the axial vibration of a cantilever beam is investigated analytically. The range of values of the frequency parameter having technical interest is determined.

Key Words. Structural optimization, cantilever beams, axial vibrations, fundamental frequency constraint.

8. MIELE, A., WU, A.K., and LIU, C.T., A Transformation Technique for Optimal Control Problems with Partially Linear State Inequality Constraints, Rice University, Aero-Astronautics Report No. 137, 1977.

Abstract. This paper considers optimal control problems involving the minimization of a functional subject to differential constraints, terminal constraints, and a state inequality constraint. The state inequality constraint is of a special type, namely, it is linear in some or all of the components of the state vector.

A transformation technique is introduced, by means of which the inequality constrained problem is converted into an equality constrained problem involving differential constraints, terminal constraints, and a control equality constraint. The transformation technique takes advantage of the partial linearity of the state inequality constraint so as to yield a transformed problem characterized by a new state vector of minimal size. This concept is important computationally, in that the computer time per iteration increases with the square of the dimension of the state vector.

In order to illustrate the advantages of the new transformation technique, several numerical examples are solved by means of the sequential gradient-restoration algorithm for

optimal control problems involving nondifferential constraints. The examples show the substantial savings in computer time for convergence, which are associated with the new transformation technique.

Key Words. Optimal control, numerical methods, computing methods, transformation techniques, sequential gradient-restoration algorithm, nondifferential constraints, state inequality constraints, linear state inequality constraints, partially linear state inequality constraints.

9. MIELE, A., MANGIAVACCHI, A., MOHANTY, B.P., and WU, A.K., Numerical Determination of Minimum Mass Structures with Specified Natural Frequencies, Rice University, Aero-Astronautics Report No. 138, 1977.

Abstract. The problem of the axial vibration of a cantilever beam is investigated both analytically and numerically. The mass distribution that minimizes the total mass for a given value of the frequency parameter β is determined using both the sequential ordinary gradient-restoration algorithm (SOGRA) and the modified quasilinearization algorithm (MQA). Concerning the minimum value of the mass, SOGRA leads to a solution precise to at least 4 significant digits and MQA leads to a solution precise to at least 6 significant digits.

Comparison of the optimal beam (a variable-section beam) with a reference beam (a constant-section beam) shows that the weight reduction depends strongly on the frequency parameter β. This weight reduction is negligible for $\beta \to 0$, is 11.3% for $\beta = 1$, is 55.3% for $\beta = 1.4$, and approaches 100% for $\beta \to \pi/2$.

Key Words. Structural optimization, dynamic optimization, axial vibrations, frequency constraint, fundamental frequency constraint, optimal structures, cantilever beams, bars, sequential gradient-restoration algorithm, modified quasi-linearization algorithm, numerical methods, computing methods.

10. GONZALEZ, S., and MIELE, A., Sequential Gradient-Restoration Algorithm for Optimal Control Problems with General Boundary Conditions, Rice University, Aero-Astronautics Report No. 142, 1978.

Abstract. This paper considers the numerical solution of the problem of minimizing a functional I subject to differential constraints and general boundary conditions. It consists of finding the state $x(t)$, the control $u(t)$, and the parameter π so that the functional I is minimized, while the constraints and the boundary conditions are satisfied to a predetermined accuracy.

The approach taken is a sequence of two-phase cycles, composed of a gradient phase and a restoration phase. The gradient phase involves one iteration and is designed to decrease the value of the functional, while the constraints are satisfied to first order. The restoration phase involves one or more iterations and is designed to force constraint satisfaction to a predetermined accuracy, while the norm squared of the variations of the control, the parameter, and the missing components of the initial state is minimized.

The principal property of the algorithm is that it produces a sequence of feasible suboptimal solutions: the functions obtained at the end of each cycle satisfy the constraints to a predetermined accuracy. Therefore, the values

of the functional I corresponding to any two elements of the sequence are comparable.

The stepsize of the gradient phase is determined by a one-dimensional search on the augmented functional J, while the stepsize of the restoration phase is obtained by a one-dimensional search on the constraint error P. The gradient stepsize and the restoration stepsize are chosen so that the restoration phase preserves the descent property of the gradient phase. Therefore, the value of the functional I at the end of any complete gradient-restoration cycle is smaller than the value of the same functional at the beginning of that cycle.

The algorithm presented here differs from those of Refs. 1 and 2, in that it is not required that the state vector be given at the initial point. Instead, the initial conditions can be absolutely general. In analogy with Refs. 1 and 2, the present algorithm is capable of handling general final conditions; therefore, it is suited for the solution of optimal control problems with general boundary conditions. Its importance lies in the fact that many optimal control problems involve initial conditions of the type considered here.

Ten numerical examples are presented in order to illustrate the performance of the algorithm. The numerical results show the feasibility as well as the convergence characteristics of the present algorithm.

Key Words. Optimal control, numerical methods, computing methods, gradient methods, gradient-restoration algorithms, sequential gradient-restoration algorithms, general boundary conditions.

11. GONZALEZ, S., and MIELE, A., Sequential Gradient-Restoration Algorithm for Optimal Control Problems with Nondifferential Constraints and General Boundary Conditions, Rice University, Aero-Astronautics Report No. 143,1978.

Abstract. This paper considers the numerical solution of the problem of minimizing a functional I subject to differential constraints, nondifferential constraints, and general boundary conditions. It consists of finding the state x(t), the control u(t), and the parameter π so that the functional I is minimized, while the constraints and the boundary conditions are satisfied to a predetermined accuracy.

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The algorithm presented here differs from those of Refs. 1 and 2, in that it is not required that the state vector be given at the initial point. Instead, the initial conditions can be absolutely general. In analogy with Refs. 1 and 2, the present algorithm is capable of handling general final conditions; therefore, it is suited for the solution of optimal control problems with general boundary conditions. Its importance lies in the fact that many optimal control problems involve initial conditions of the type considered here.

Fourteen numerical examples are presented in order to illustrate the performance of the algorithm. The numerical results show the feasibility as well as the convergence characteristics of the present algorithm.

Key Words. Optimal control, numerical methods, computing methods, gradient methods, gradient-restoration algorithms, sequential gradient-restoration algorithms, general boundary conditions, nondifferential constraints, bounded control, bounded state.

12. WU, A.K., and MIELE, A., Sequential Conjugate Gradient-Restoration Algorithm for Optimal Control Problems with General Boundary Conditions, Rice University, Aero-Astronautics Report No. 144, 1978.

Abstract. This paper considers the numerical solution of the problem of minimizing a functional I subject to differential constraints and general boundary conditions. It consists of finding the state x(t), the control u(t), and the parameter π so that the functional I is minimized, while the constraints and the boundary conditions are satisfied to a predetermined accuracy.

The approach taken is a sequence of two-phase cycles, composed of a conjugate gradient phase and a restoration phase. The conjugate gradient phase involves one iteration and is designed to decrease the value of the functional, while the constraints are satisfied to first order. During this iteration, the first variation of the functional is minimized, subject to the linearized constraints. The minimization is performed over the class of variations of the control, the parameter, and the missing components of the initial state which are equidistant from some constant multiple of the corresponding variations of the previous conjugate gradient phase. The sequence of conjugate gradient phase generated by the algorithm is such that, for the special case of a quadratic

functional subject to linear constraints, various orthogonality and conjugacy conditions hold. The restoration phase involves one or more iterations and is designed to force constraint satisfaction to a predetermined accuracy, while the norm squared of the variations of the control, the parameter, and the missing components of the initial state is minimized.

The principal property of the algorithm is that it produces a sequence of feasible suboptimal solutions: the functions obtained at the end of each cycle satisfy the constraints to a predetermined accuracy. Therefore, the values of the functional I corresponding to any two elements of the sequence are comparable.

The stepsize of the conjugate gradient phase is determined by a one-dimensional search on the augmented functional J, while the stepsize of the restoration phase is obtained by a one-dimensional search on the constraint error P. The conjugate gradient stepsize and the restoration stepsize are chosen so that the restoration phase preserves the descent property of the conjugate gradient phase. Therefore, the value of the functional I at the end of any complete conjugate gradient-restoration cycle is smaller than the value of the same functional at the beginning of that cycle. Of course, restarting the algorithm might be occasionally necessary.

The sequential conjugate gradient-restoration algorithm presented here differs from that of Refs. 3 and 4, in that it is not required that the state vector be given at the initial point. Instead, the initial conditions can be absolutely general. In analogy with Refs. 3 and 4, the present algorithm is capable of handling general final conditions; therefore, it is suitable for the solution of optimal control problems with general boundary conditions. Its importance lies in the fact that many optimal control problems involve initial conditions of the type considered here.

Nine numerical examples are presented in order to illustrate the performance of the algorithm. The numerical results show the feasibility as well as the convergence characteristics of the present algorithm.

Key Words. Optimal control, numerical methods, computing methods, gradient methods, gradient-restoration algorithms, sequential gradient-restoration algorithms, conjugate gradient-restoration algorithm, sequential conjugate gradient-restoration algorithms, general boundary conditions.

13. WU, A.K., and MIELE, A., Sequential Conjugate Gradient-Restoration Algorithm for Optimal Control Problems with Nondifferential Constraints and General Boundary Conditions, Rice University, Aero-Astronautics Report No. 145, 1978.

Abstract. This paper considers the numerical solution of the problem of minimizing a functional I subject to differential constraints, nondifferential constraints, and general boundary conditions. It consists of finding the state x(t), the control u(t), and the parameter π so that the functional I is minimized, while the constraints and the boundary conditions are satisfied to a predetermined accuracy.

The approach taken is a sequence of two-phase cycles, composed of a conjugate gradient phase and a restoration phase. The conjugate gradient phase involves one iteration and is designed to decrease the value of the functional, while the constraints are satisfied to first order. During this iteration, the first variation of the functional is minimized, subject to the linearized constraints. The minimization is performed over the class of variations of the control, the parameter, and the missing components of the initial state which are equidistant from some constant multiple of the corresponding variations of the previous conjugate gradient phase. The sequence of conjugate gradient phases

generated by the algorithm is such that, for the special case of a quadratic functional subject to linear constraints, various orthogonality and conjugacy conditions hold. The restoration phase involves one or more iterations and is designed to force constraint satisfaction to a predetermined accuracy, while the norm squared of the variations of the control, the parameter, and the missing components of the initial state is minimized.

The principal property of the algorithm is that it produces a sequence of feasible suboptimal solutions: the functions obtained at the end of each cycle satisfy the constraints to a predetermined accuracy. Therefore, the values of the functional I corresponding to any two elements of the sequence are comparable.

The stepsize of the conjugate gradient phase is determined by a one-dimensional search on the augmented functional J, while the stepsize of the restoration phase is obtained by a one-dimensional search on the constraint error P. The conjugate gradient stepsize and the restoration stepsize are chosen so that the restoration phase preserves the descent property of the conjugate gradient phase. Therefore, the value of the functional I at the end of any complete conjugate gradient-restoration cycle is smaller than the value of the same functional at the beginning of that cycle. Of course, restarting the algorithm might be occasionally necessary.

The sequential conjugate gradient-restoration algorithm presented here differs from that of Refs. 3 and 4, in that it is not required that the state vector be given at the initial point. Instead, the initial conditions can be absolutely general. In analogy with Refs. 3 and 4, the present algorithm is capable of handling general final conditions; therefore, it is suitable for the solution of optimal control problems with general boundary conditions. Its importance lies in the fact that many optimal control problems involve initial conditions of the type considered here.

Twelve numerical examples are presented in order to illustrate the performance of the algorithm. The numerical results show the feasibility as well as the convergence characteristics of the present algorithm.

Key Words. Optimal control, numerical methods, computing methods, gradient methods, gradient-restoration algorithms, sequential gradient-restoration algorithms, conjugate gradient-restoration algorithms, sequential conjugate gradient-restoration algorithms, nondifferential constraints, bounded control, bounded state, general boundary conditions.

VIII. Abstract of Papers

14. MIELE, A., Some Philosophical Views on Algorithms and Computing Methods in Applied Mathematics, Proceedings of the Workshop on Decision Information for Tactical Command and Control, Airlie, Virginia, 1976; Edited by R.M. Thrall, C.P. Tsokos, and J.C. Turner; Robert M. Thrall and Associates, Houston, Texas, pp. 192-208, 1976.

Abstract. This paper summarizes some of the work done by the Aero-Astronautics Group of Rice University in the area of numerical methods and computing methods. It describes some of the philosophical thoughts that have guided this work throughout the years. Recommendations are offered concerning allocation of funds and distribution of funds. Additional recommendations are offered in order to bridge the gap between the top management of government agencies and the academic community.

Key Words. Aerospace engineering, applied mathematics, numerical methods, computing methods, algorithm research, algorithm development.

15. MIELE, A., MOHANTY, B.P., and WU, A.K., Conversion of Optimal Control Problems with Free Initial State into Optimal Control Problems with Fixed Initial State, Journal of the Astronautical Sciences, Vol. 25, No. 1, pp. 75-85, 1977.

Abstract. This note considers optimal control problems involving the minimization of a functional subject to differential constraints, initial conditions, and final conditions. The initial conditions can be partly fixed and partly free. Transformation techniques are suggested, by means of which problems with free initial state are converted into problems with fixed initial state. Thereby, it becomes possible to employ, without change, some of the gradient algorithms already developed for optimal control problems with fixed initial state (for instance, the sequential gradient-restoration algorithm).

The transformations introduced are two: (i) a linear transformation and (ii) a nonlinear transformation. In the linear-quadratic case, the former preserves unchanged the basic structure of the optimization problem, while this is not the case with the latter.

The application of these transformations to a problem of interest in the aerodynamics of a nonslender, axisymmetric body in Newtonian hypersonic flow is shown. It consists

of minimizing the pressure drag for given values of the length and the volume, with the nose radius and the base radius being free. After transformations (i) and (ii) are introduced, this problem is solved by means of the sequential ordinary gradient-restoration algorithm (SOGRA) and the sequential conjugate gradient-restoration algorithm (SCGRA).

Key Words. Optimal control, numerical methods, computing methods, transformation techniques, sequential ordinary gradient-restoration algorithm, sequential conjugate gradient-restoration algorithm, problems with free initial state, applied aerodynamics, optimum aerodynamic shapes.

16. MIELE, A., and GONZALEZ, S., On the Comparative Evaluation of Algorithms for Mathematical Programming Problems, Nonlinear Programming 3, Edited by O.L. Mangasarian, R.R. Meyer, and S.M. Robinson, Academic Press, New York, New York, pp. 337-359, 1978.

Abstract. This paper considers the comparative evaluation of algorithms for mathematical programming problems. It is concerned with the measurement of computational speed and examines critically the concept of equivalent number of function evaluations N_e. Does this quantity constitute a fair way of comparing different algorithms?

The answer to the above question depends strongly on whether or not analytical expressions for the components of the gradient and the elements of the Hessian matrix are available. It also depends on the relative importance of the computational effort associated with algorithmic operations vis-a-vis the computational effort associated with function evaluations.

Both theoretical considerations and extensive numerical examples carried out in conjunction with the Fletcher-Reeves algorithm, the Davidon-Fletcher-Powell algorithm, and the quasilinearization algorithm suggest the following: the N_e concept, while accurate in some cases, has drawbacks in other cases; indeed, it might lead to a distorted view of the

relative importance of an algorithm with respect to another.

The above distortion can be corrected through the introduction of a more general parameter, the time-equivalent number of function evaluations $\tilde{N}_e = T/\tau_0$, where T denotes the CPU time required to solve a particular problem on a particular computer and τ_0 denotes the CPU time required to evaluate the objective function once on that computer. This generalized parameter is constructed so as to reflect accurately the computational effort associated with function evaluations and algorithmic operations.

From the analyses performed and the results obtained, it is inferred that, due to the weaknesses of the N_e concept, the use of the $\tilde{N}_e$ concept is advisable. In effect, this is the same as stating that, in spite of its obvious shortcomings, the direct measurement of the CPU time is still the more reliable way of comparing different minimization algorithms.

Key Words. Numerical analysis, numerical methods, computing methods, computing techniques, complexity of computation, philosophy of computation, comparison of algorithms, computational speed, measurement of computational speed, number of function evaluations, equivalent number of function evaluations, time-equivalent number of function evaluations, unconstrained minimization, mathematical programming.

17. GONZALEZ, S., and MIELE, A., Sequential Gradient-Restoration Algorithm for Optimal Control Problems with General Boundary Conditions, Journal of Optimization Theory and Applications, Vol. 26, No. 3, pp. 395-425, 1978.

Abstract. This paper considers the numerical solution of two classes of optimal control problems, called Problem P1 and Problem P2 for easy identification.

Problem P1 involves a functional I subject to differential constraints and general boundary conditions. It consists of finding the state x(t), the control u(t), and the parameter π so that the functional I is minimized, while the constraints and the boundary conditions are satisfied to a predetermined accuracy. Problem P2 extends Problem P1 to include nondifferential constraints to be satisfied everywhere along the interval of integration. Algorithms are developed for both Problem P1 and Problem P2.

The approach taken is a sequence of two-phase cycles, composed of a gradient phase and a restoration phase. The gradient phase involves one iteration and is designed to decrease the value of the functional, while the constraints are satisfied to first order. The restoration phase involves one or more iterations and is designed to force constraint satisfaction to a predetermined accuracy, while the norm

squared of the variations of the control, the parameter, and the missing components of the initial state is minimized.

The principal property of both algorithms is that they produce a sequence of feasible suboptimal solutions: the functions obtained at the end of each cycle satisfy the constraints to a predetermined accuracy. Therefore, the values of the functional I corresponding to any two elements of the sequence are comparable.

The stepsize of the gradient phase is determined by a one-dimensional search on the augmented functional J, while the stepsize of the restoration phase is obtained by a one-dimensional search on the constraint error P. The gradient stepsize and the restoration stepsize are chosen so that the restoration phase preserves the descent property of the gradient phase. Therefore, the value of the functional I at the end of any complete gradient-restoration cycle is smaller than the value of the same functional at the beginning of that cycle.

The algorithms presented here differ from those of Refs. 1 and 2, in that it is not required that the state vector be given at the initial point. Instead, the initial conditions can be absolutely general. In analogy with Refs. 1 and 2, the present algorithms are capable of handling general final conditions; therefore, they are suited for the solution of optimal control problems with general boundary conditions.

Their importance lies in the fact that many optimal control problems involve initial conditions of the type considered here.

Six numerical examples are presented in order to illustrate the performance of the algorithms associated with Problem P1 and Problem P2. The numerical results show the feasibility as well as the convergence characteristics of these algorithms.

Key Words. Optimal control, numerical methods, computing methods, gradient methods, gradient-restoration algorithms, sequential gradient-restoration algorithms, general boundary conditions, nondifferential constraints, bounded control, bounded state.

18. MIELE, A., MANGIAVACCHI, A., MOHANTY, B.P., and WU, A.K., Numerical Determination of Minimum Mass Structures with Specified Natural Frequencies, Internatinal Journal for Numerical Methods in Engineering, Vol. 13, No. 2, pp. 265-282, 1978.

Abstract. The problem of the axial vibration of a cantilever beam is investigated both analytically and numerically. The mass distribution that minimizes the total mass for a given value of the frequency parameter β is determined using both the sequential ordinary gradient-restoration algorithm (SOGRA) and the modified quasilinearization algorithm (MQA). Concerning the minimum value of the mass, SOGRA leads to a solution precise to at least 4 significant digits and MQA leads to a solution precise to at least 6 significant digits.

Comparison of the optimal beam (a variable-section beam) with a reference beam (a constant-section beam) shows that the weight reduction depends strongly on the frequency parameter β. This weight reduction is negligible for $\beta \to 0$, is 11.3% for $\beta = 1$, is 55.3% for $\beta = 1.4$, and approaches 100% for $\beta = \pi/2$.

Key Words. Structural optimization, dynamic optimization, axial vibrations, frequency constraint, fundamental frequency constraint, optimal structures, cantilever beams, bars, rods, sequential gradient-restoration algorithm, modified quasilinearization algorithm, numerical methods, computing methods.

19. MIELE, A., and CLOUTIER, J.R., Sequential Conjugate Gradient-Restoration Algorithm for Optimal Control Problems with Nondifferential Constraints, Applied Nonlinear Analysis, Edited by V. Lakshmikantham, J. Eisenfeld, and A.R. Mitchell, Academic Press, New York, New York, pp. 89-93, 1979.

Abstract. A sequential conjugate gradient-restoration algorithm is developed in order to solve optimal control problems involving a functional subject to differential constraints, nondifferential constraints, and terminal constraints. The algorithm is composed of a sequence of cycles, each cycle consisting of two phases, a conjugate gradient phase and a restoration phase.

The conjugate gradient phase involves a single iteration and is designed to decrease the value of the functional, while satisfying the constraints to first order. During this iteration, the first variation of the functional is minimized, subject to the linearized constraints. The minimization is performed over the class of variations of the control and the parameter which are equidistant from some constant multiple of the corresponding variations of the previous conjugate gradient phase. For the special case of a quadratic functional subject to linear constraints, various orthogonality and conjugacy conditions hold.

The restoration phase involves one or more iterations and is designed to restore the constraints to a predetermined accuracy, while the norm of the variations of the control and the parameter is minimized, subject to the linearized constraints. The restoration phase is terminated whenever the norm of the constraint error is less than some predetermined tolerance.

The sequential conjugate gradient-restoration algorithm is characterized by two main properties. First, at the end of each conjugate gradient-restoration cycle, the trajectory satisfies the constraints to a given accuracy; thus, a sequence of feasible suboptimal solutions is produced. Second, the conjugate gradient stepsize and the restoration stepsize can be chosen so that the restoration phase preserves the descent property of the conjugate gradient phase; thus, the value of the functional at the end of any cycle is smaller than the value of the functional at the beginning of that cycle. Of course, restarting the algorithm might be occasionally necessary.

To facilitate numerical integrations, the interval of integration is normalized to unit length. Variable-time terminal conditions are transformed into fixed-time terminal conditions. Then, the actual time at which the terminal boundary is reached becomes a component of a vector parameter being optimized.

Convergence is attained whenever both the norm of the constraint error and the norm of the error in the optimality conditions are less than some predetermined tolerances.

Several numerical examples are presented, some pertaining to a quadratic functional subject to linear constraints and some pertaining to a nonquadratic functional subject to nonlinear constraints. These examples illustrate the feasibility as well as the convergence characteristics of the sequential conjugate gradient-restoration algorithm.

Key Words. Optimal control, gradient methods, conjugate gradient methods, numerical methods, computing methods, gradient-restoration algorithms, sequential gradient-restoration algorithms, sequential conjugate gradient-restoration algorithms, nondifferential constraints.

20. MIELE, A., CLOUTIER, J.R., MOHANTY, B.P., and WU, A.K., Sequential Conjugate Gradient-Restoration Algorithm for Optimal Control Problems with Nondifferential Constraints, Part 1, International Journal of Control, Vol. 29, No. 2, pp. 189-211, 1979.

Abstract. A sequential conjugate gradient-restoration algorithm is developed in order to solve optimal control problems involving a functional subject to differential constraints, nondifferential constraints, and terminal constraints. The algorithm is composed of a sequence of cycles, each cycle consisting of two phases, a conjugate gradient phase and a restoration phase.

The conjugate gradient phase involves a single iteration and is designed to decrease the value of the functional, while satisfying the constraints to first order. During this iteration, the first variation of the functional is minimized, subject to the linearized constraints. The minimization is performed over the class of variations of the control and the parameter which are equidistant from some constant multiple of the corresponding variations of the previous conjugate gradient phase. For the special case of a quadratic functional subject to linear constraints, various orthogonality and conjugacy conditions hold.

The restoration phase involves one or more iterations

and is designed to restore the constraints to a predetermined accuracy, while the norm of the variations of the control and the parameter is minimized, subject to the linearized constraints. The restoration phase is terminated whenever the norm of the constraint error is less than some predetermined tolerance.

The sequential conjugate gradient-restoration algorithm is characterized by two main properties. First, at the end of each conjugate gradient-restoration cycle, the trajectory satisfies the constraints to a given accuracy; thus, a sequence of feasible suboptimal solutions is produced. Second, the conjugate gradient stepsize and the restoration stepsize can be chosen so that the restoration phase preserves the descent property of the conjugate gradient phase; thus, the value of the functional at the end of any cycle is smaller than the value of the functional at the beginning of that cycle. Of course, restarting the algorithm might be occasionally necessary.

To facilitate numerical integrations, the interval of integration is normalized to unit length. Variable-time terminal conditions are transformed into fixed-time terminal conditions. Then, the actual time at which the terminal boundary is reached becomes a component of a vector parameter being optimized.

Convergence is attained whenever both the norm of the constraint error and the norm of the error in the optimality conditions are less than some predetermined tolerances. Several numerical examples illustrating the theory of this paper are given in Part 2 (see Ref. 21).

Key Words. Optimal control, gradient methods, conjugate gradient methods, numerical methods, computing methods, gradient-restoration algorithms, sequential gradient-restoration algorithms, sequential conjugate gradient-restoration algorithms, nondifferential constraints.

21. MIELE, A., CLOUTIER, J.R., MOHANTY, B.P., and WU, A.K., Sequential Conjugate Gradient-Restoration Algorithm for Optimal Control Problems with Nondifferential Constraints, Part 2, International Journal of Control, Vol. 29, No. 2, pp. 213-234, 1979.

Abstract. In Part 1 (see Ref. 20), Miele et al developed the sequential conjugate gradient-restoration algorithm for minimizing a functional subject to differential constraints, nondifferential constraints, and terminal constraints. In this paper, several numerical examples are presented, some pertaining to a quadratic functional subject to linear constraints and some pertaining to a nonquadratic functional subject to nonlinear constraints. These examples demonstrate the feasibility as well as the convergence characteristics of the sequential conjugate gradient-restoration algorithm.

Key Words. Optimal control, gradient methods, conjugate gradient methods, numerical methods, computing methods, gradient-restoration algorithms, sequential gradient-restoration algorithms, sequential conjugate gradient-restoration algorithms, nondifferential constraints.

22. MIELE, A., WU, A.K., and LIU, C.T., A Transformation Technique for Optimal Control Problems with Partially Linear State Inequality Constraints, Journal of Optimization Theory and Applications, Vol. 28, No. 2, pp. 185-212, 1979.

Abstract. This paper considers optimal control problems involving the minimization of a functional subject to differential constraints, terminal constraints, and a state inequality constraint. The state inequality constraint is of a special type, namely, it is linear in some or all of the components of the state vector.

A transformation technique is introduced, by means of which the inequality constrained problem is converted into an equality constrained problem involving differential constraints, terminal constraints, and a control equality constraint. The transformation technique takes advantage of the partial linearity of the state inequality constraint so as to yield a transformed problem characterized by a new state vector of minimal size. This concept is important computationally, in that the computer time per iteration increases with the square of the dimension of the state vector.

In order to illustrate the advantages of the new transformation technique, several numerical examples are solved by means of the sequential gradient-restoration algorithm for

optimal control problems involving nondifferential constraints. The examples show the substantial savings in computer time for convergence, which are associated with the new transformation technique.

Key Words. Optimal control, numerical methods, computing methods, transformation techniques, sequential gradient-restoration algorithm, nondifferential constraints, state inequality constraints, linear state inequality constraints, partially linear state inequality constraints.

33. MIELE, A., BONARDO, F., and GONZALEZ, S., Modifications and Alternatives to the Cubic Interpolation Process for One-Dimensional Search, Arabian Journal for Science and Engineering, Vol. 4, No. 2, pp. 121-128, 1979.

Abstract. The numerical solution of the problem of minimizing a unimodal function is considered. Modifications and alternatives to the cubic interpolation process are presented, so as to improve robustness and force convergence in a reasonable number of iterations, even in pathological cases.

Modification M1 includes the nonoptional bisection of the interval of interpolation. Modification M2 includes the optional bisection of the interval of interpolation. Alternatives A1 and A2 are Lagrange interpolation schemes in which the quadratic approximation to the derivative of the function is considered. They differ from one another in the technique employed for choosing the next interval of interpolation.

Several numerical examples are presented, and the numerical results show the promising characteristics of the proposed modifications and alternatives. Therefore, the one-dimensional search schemes described here have potential interest for those minimization algorithms which depend critically on the precise selection of the stepsize, namely, conjugate gradient methods.

Key Words. One-dimensional search, cubic interpolation process, quadratic interpolation process, Lagrange interpolation scheme, modifications of the cubic interpolation process, alternatives to the cubic interpolation process, bisection process, mathematical programming, interval of interpolation, numerical analysis, numerical methods, computing methods, computing techniques.

24. MIELE, A., Gradient Algorithms for the Optimization of Dynamic Systems, Advances in Control and Dynamic Systems: Theory and Applications, Vol. 16, Edited by C.T. Leondes, Academic Press, New York, New York, 1979.

Abstract. In every branch of science, engineering, and economics, there exist systems which are controllable, that is, they can be made to behave in different ways depending on the will of the operator. Every time the operator of a system exerts an option, a choice in the distribution of the quantities controlling the system, he produces a change in the distribution of the states occupied by the system and, hence, a change in the final state. Therefore, it is natural to pose the following question: Among all the admissible options, what is the particular option which renders the system optimum? As an example, what is the option which minimizes the difference between the final value and the initial value of an arbitrarily specified function of the state of the system? The body of knowledge covering problems of this type is called calculus of variations or optimal control theory. As stated before, applications occur in every field of science, engineering, and economics.

It must be noted that only a minority of current problems can be solved by purely analytical methods. Hence, it is important to develop numerical techniques enabling one to

solve optimal control problems on a digital computer. These numerical techniques can be classified into two groups: first-order methods and second-order methods. First-order methods (or gradient methods) are those techniques which employ at most the first derivatives of the functions under consideration. Second-order methods (or quasilinearization methods) are those techniques which employ at most the second derivatives of the functions under consideration.

Both gradient methods and quasilinearization methods require the solution of a linear, two-point or multi-point boundary-value problem at every iteration. This being the case, progress in the area of numerical methods for differential equations is essential to the efficient solution of optimal control problems on a digital computer.

In this paper, we review recent advances in the area of gradient methods for optimal control problems. Because of space limitations, we make no attempt to cover every possible technique and every possible approach, a material impossibility in view of the large number of publications available. Thus, except for noting the early work performed by Kelley and Bryson, we devote the body of the paper to a review of the work performed in recent years by the Aero-Astronautics Group of Rice University.

Also because of space limitations, we treat only single-subarc problems. More specifically, we consider two classes of optimal control problems, called Problem P1 and Problem P2 for easy identification.

Problem P1 consists of minimizing a functional I, which depends on the n-vector state x(t), the m-vector control u(t), and the p-vector parameter π. The state is given at the initial point. At the final point, the state and the parameter are required to satisfy q scalar relations. Along the interval of integration, the state, the control, and the parameter are required to satisfy n scalar differential equations. Problem P2 differs from Problem P1 in that the state, the control, and the parameter are required to satisfy k additional scalar relations along the interval of integration. Algorithms of the sequential gradient-restoration type are given for both Problem P1 and Problem P2.

The approach taken is a sequence of two-phase cycles, composed of a gradient phase and a restoration phase. The gradient phase involves one iteration and is designed to decrease the value of the functional, while the constraints are satisfied to first order. The restoration phase involves one or more iterations and is designed to force constraint satisfaction to a predetermined accuracy, while the norm squared of the variations of the control and the parameter is minimized, subject to the linearized constraints.

The principal property of the algorithms presented here is that a sequence of feasible suboptimal solutions is produced. In other words, at the end of each gradient-restoration cycle, the constraints are satisfied to a predetermined accuracy. Therefore, the values of the functional I corresponding to any two elements of the sequence are comparable.

The stepsize of the gradient phase is determined by a one-dimensional search on the augmented functional J, while the stepsize of the restoration phase is obtained by a one-dimensional search on the constraint error P. The gradient stepsize and the restoration stepsize are chosen so that the restoration phase preserves the descent property of the gradient phase. As a consequence, the value of the functional I at the end of any complete gradient-restoration cycle is smaller than the value of the same functional at the beginning of that cycle.

A time normalization is used in order to simplify the numerical computations. Specifically, the actual time θ is replaced by the normalized time $t = \theta/\tau$, which is defined in such a way that $t = 0$ at the initial point and $t = 1$ at the final point. The actual final time τ, if it is free, is regarded as a component of the vector parameter π to be optimized. In this way, an optimal control problem with variable final time is converted into an optimal control problem with fixed final time.

Section 2 contains the statements of Problem P1 and

Problem P2. Section 3 gives a description of the sequential gradient-restoration algorithm. Section 4 discusses the determinations of the basic functions for the gradient phase and the restoration phase. Section 5 considers the determination of the stepsizes for the gradient phase and the restoration phase. A summary of the sequential gradient-restoration algorithm is presented in Section 6. The experimental conditions are given in Section 7. The numerical examples for Problem P1 are given in Section 8, and the numerical examples for Problem P2 are given in Section 9. Finally, the discussion and the conclusions are presented in Section 10.

Key Words. Optimal control, numerical methods, computing methods, gradient methods, gradient-restoration algorithms, sequential gradient-restoration algorithms, survey papers.

25. WU, A.K., and MIELE, A., Sequential Conjugate Gradient-Restoration Algorithm for Optimal Control Problems with Nondifferential Constraints and General Boundary Conditions, Part 1, Optimal Control Applications and Methods, Vol. 1, No. 1, 1980.

Abstract. In this paper, a new member of the family of sequential gradient-restoration algorithms for the solution of optimal control problems is presented. This is an algorithm of the conjugate gradient type and solves two classes of optimal control problems, called Problem P1 and Problem P2 for easy identification.

Problem P1 involves minimizing a functional I subject to differential constraints and general boundary conditions. It consists of finding the state x(t), the control u(t), and the parameter π so that the functional I is minimized, while the constraints and the boundary conditions are satisfied to a predetermined accuracy. Problem P2 extends Problem P1 to include nondifferential constraints to be satisfied everywhere along the interval of integration.

The approach taken is a sequence of two-phase cycles, composed of a conjugate gradient phase and a restoration phase. The conjugate gradient phase involves one iteration and is designed to decrease the value of the functional, while the con-

straints are satisfied to first order. The restoration phase involves one or more iterations; each restorative iteration is designed to force constraint satisfaction to first order, while the norm squared of the variations of the control, the parameter, and the missing components of the initial state is minimized. The resulting algorithm has several properties: (i) it produces a sequence of feasible solutions; (ii) each feasible solution is characterized by a value of the functional I which is smaller than that associated with any previous feasible solution; and (iii) for the special case of a quadratic functional subject to linear constraints, the variations of the state, control, and parameter produced by the sequence of conjugate gradient phases satisfy various orthogonality and conjugacy conditions.

The algorithm presented here differs from those of Refs. 1-4, in that it is not required that the state vector be given at the initial point. Instead, the initial conditions can be absolutely general. In analogy with Refs. 1-4, the present algorithm is capable of handling general final conditions; therefore, it is suitable for the solution of optimal control problems with general boundary conditions.

The importance of the present algorithm lies in that many optimal control problems either arise naturally in the present format or can be brought to such a format by means of suitable transformation (Ref. 5). Therefore, a great variety of optimal control problems can be handled. This includes: (i) problems with control equality constraints, (ii) problems with state equality constraints, (iii) problems with state-derivative equality constraints, (iv) problems with control inequality constraints, (v) problems with state inequality constraints, (vi) problems with state-derivative inequality constraints, and (vii) Chebyshev minimax problems.

Several numerical examples are presented in Part 2 (see Ref. 26) in order to illustrate the performance of the algorithm associated with Problem P1 and Problem P2. The numerical results show the feasibility as well as the convergence characteristics of the present algorithm.

Key Words. Optimal control, gradient methods, conjugate-gradient methods, numerical methods, computing methods, gradient-restoration algorithms, sequential gradient-restoration algorithms, sequential conjugate gradient-restoration algorithms, nondifferential constraints, general boundary conditions, bounded control, bounded state.

26. WU, A.K., and MIELE, A., Sequential Conjugate Gradient-Restoration Algorithm for Optimal Control Problems with Nondifferential Constraints and General Boundary Conditions, Part 2, Optimal Control Application and Methods, Vol. 1, No. 2, 1980.

Abstract. In Part 1 (see Ref. 25), Wu and Miele developed the sequential conjugate gradient-restoration algorithm for minimizing a functional subject to differential constraints, with or without nondifferential constraints, and general boundary conditions. In this paper, several numerical examples are presented, some pertaining to a quadratic functional subject to linear constraints and some pertaining to a nonquadratic functional subject to nonlinear constraints. These examples demonstrate the feasibility as well as the convergence characteristics of the sequential conjugate gradient-restoration algorithm.

Key Words. Optimal control, gradient methods, conjugate-gradient methods, numerical methods, computing methods, gradient-restoration algorithms, sequential gradient-restoration algorithms, sequential conjugate gradient-restoration algorithms, nondifferential constraints, general boundary conditions, bounded control, bounded state.

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REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER AFOSR-TR- 80-0050	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) NEW TECHNIQUES IN NUMERICAL ANALYSIS AND THEIR APPLICATION TO AEROSPACE SYSTEMS		5. TYPE OF REPORT & PERIOD COVERED Final
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Angelo Miele		8. CONTRACT OR GRANT NUMBER(s) AFOSR 76-3075
9. PERFORMING ORGANIZATION NAME AND ADDRESS Rice University Dept. of Mech. Engineering and Materials Science Houston, TX 77001		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 61102F 2304/A3
11. CONTROLLING OFFICE NAME AND ADDRESS Air Force Office of Scientific Research/NM Bolling AFB, Washington, DC 20332		12. REPORT DATE 1979
		13. NUMBER OF PAGES 70
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Aerospace engineering, applied mathematics, algorithm research, algorith, development, optimum atmospherice flight trajectories, optimum extra-atmospheric flight trajectories, optimum aerodynamic shapes, optimum structures; numerical analysis, numerical methods, computing methods, computing techniques; search techniques, mathematical programming, optimal control, differential equations. OVER

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This document summarizes the research performed at Rice University during the period 1976-79 under AFOSR 76-3075 in several areas of numerical analysis of interest in aerospace systems theory, namely: solution of nonlinear equations, solution of differential equations, mathematical programming problems, and optimal control problems. The work summarized here is applicable to these areas of aerospace engineering: optimum atmospheric flight trajectories, optimum extra-atmospheric flight trajectories, optimum aerodynamic shapes, and optimum structures.

DD FORM 1 JAN 73 1473

UNCLASSIFIED

19. Key Words cont.

gradient algorithms, quasilinerarization algorithms, transformation techniques.