AD-A080 399 ARMY INVENTORY RESEARCH OFFICE PHILADELPHIA PA F/G 5/3
AN INVESTIGATION OF MINIMUM BUY POLICIES.(U)
AUG 79 S FRAZZA, A J KAPLAN
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FINAL REPORT
IRO REPORT NO. 269

AN INVESTIGATION OF MINIMUM BUY POLICIES

U.S. ARMY
INVENTORY
RESEARCH
OFFICE

AUGUST 1979

ROOM 800
U.S. CUSTOM HOUSE
2nd and Chestnut Streets
Philadelphia Pa. 19106

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

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14

1. REPORT NUMBER: IRO [illegible]-269
2. GOVT ACCESSION NO.
3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle): AN INVESTIGATION OF MINIMUM BUY POLICIES
5. TYPE OF REPORT & PERIOD COVERED: Final Report
6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s): SALLY FRAZZA AND ALAN J. KAPLAN
8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS: US Army Inventory Research Office, Room 800, US Custom House, 2nd & Chestnut Sts., Philadelphia, PA 19106
10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS: US Army Materiel Development & Readiness Command, 5001 Eisenhower Avenue, Alexandria, VA 22333
12. REPORT DATE: August 1979
13. NUMBER OF PAGES: 34
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)
15. SECURITY CLASS. (of this report): UNCLASSIFIED

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report): Approved for Public Release; Distribution Unlimited
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)
18. SUPPLEMENTARY NOTES: Information and data contained in this document are based on input available at the time of preparation. Because the results may be subject to change, this document should not be construed to represent the official position of the US Army Materiel Development & Readiness Command unless so stated.
19. KEY WORDS (Continue on reverse side if necessary and identify by block number):
    Non-stocked items
    EOQ
    Reorder Points
20. ABSTRACT (Continue on reverse side if necessary and identify by block number):

This study investigates the general economics of minimum buy for non-stocked items, and evaluates alternatives for determining the size of the minimum buy, including a form of EOQ tailored to non-stocked items. The study finds that minimum buy is economical, but that to take maximum advantage of it, some modification is required in how it is implemented in the standard computer system.

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CHAPTER I

INTRODUCTION

Until recently, a non-stocked centrally managed and procured item in the US Army wholesale supply system was purchased for direct delivery to the customer each time a requisition was received. This had two major drawbacks: purchase quantities were sometimes so low it was difficult to find a willing vendor; and procurement workload, and the corresponding costs, were inflated by the volume of small purchase actions.

At the urging of some of the commodity command personnel, the standard computer system, CCSS, was modified to permit minimum buys on non-stocked items. If the minimum buy parameters were $50, and only $20 worth of material were needed for the customer, $30 would be purchased for storage at a wholesale depot, to be used for any subsequent requisitions. To many this was a radical and dangerous concept, one which would lead to crowded warehouses and unnecessary excesses. The minimum buy procedure was for some time effectively bypassed at some of the commodity commands although the procedure was enthusiastically used at other commands. It is still controversial.

Those commands using the minimum buy concept recognized they had no good basis for selecting how much to buy for storage at the wholesale depot. This study was undertaken to investigate the general economics of minimum buy for non-stocked items, and to evaluate alternatives for determining the size of the minimum buy.

The basic research tool was empirical simulation based on actual demand histories for a large sample of items. The impact of minimum buy on administrative costs of procurement, supply performance, holding costs, number of items with stock on hand at the wholesale depot, and excess costs was investigated. Another study* is gathering data on the relationship between procurement quantity and unit price.

This study finds that minimum buy is economical, but that to take maximum advantage of it, some modification is required in how it is implemented in the standard computer system.

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*IRO Project 254, Implementation of Quantity Discount Procedures.

CHAPTER II

DATA

The IRO demand history file includes 11 years of requisitions and demand by quarter, accumulated from the AVSCOM Demand Return & Disposal (DRD) files from 1967 thru 1977. The file contains a sample of 20,865 items from all those in the system in 1966 and those which subsequently entered. In creating the data base originally we eliminated Supply Support Arrangement (SSA) and Grant Aid demands, (non-recurring), and eliminated items not purchased through central procurement, based on the last recorded IMPC code. Every attempt was made to drop items subject to logistical transfer.

For this project we reduced the data base, keeping only those items for which we would make a minimum buy and which were suitable for simulation. The first stage of this screening eliminated reparables (1322), items with unit price greater than $50 (4582), items with recorded unit price less than 0 or procurement lead time less than 1 quarter (1677), items with no demand for first five years (3268) and items which we considered stocked (5865). We considered an item stocked if it passed COSDIF [AR 710-1] based on the first two years of our data, which we used as a base period.

The second stage of the screening was applied to the remaining items at the time of the first requisition after the base period. At this time we reapply COSDIF and skip the items which pass at this point (736). We also skip items not yet provisioned (1022). We assume an item is not yet provisioned if there were no demands in the base period, yet this first requisition after the base is accompanied by at least four more in the next year. Remaining are 2393 items for which we make a minimum buy.

We also converted the quarterly data into a series of requisitions more appropriate for our simulation purposes, using a 2-step procedure.

Given demand quantity (D) and requisitions (R) for 1 quarter:

1. Distribute D into R cells, thereby determining the size of each of the R requisitions:

    a. Put 1 in each of R ($D \geq R$) cells

    b. Put surplus (D-R) into randomly chosen cell, one at a time.

2. Condense requisitions (cells) 3 thru R into requisition 3.

We then assumed each of the up to three requisitions occurred in a different month at the beginning of the month. The simulation is insensitive to just when in the quarter the requisitions really do occur. Little was lost by assuming a maximum of three requisitions per quarter as only .2% of the quarters in our original data had more than three requisitions.

CHAPTER III

SIMULATION AND EVALUATION

3.1 General Structure

The first two years of data (1967-68) are used as a base period only. For those items which qualify for minimum buy, the first subsequent requisition triggers the buy and we calculate a minimum buy according to the policy being tested. We deplete the assets as requisitions arrive and accumulate performance statistics for the item until one of the following ending conditions occurs:

a. The stock is depleted.

b. The item subsequently passes COSDIF and becomes stocked.

c. The item becomes obsolete.

d. The item's history ends.

An item is considered obsolete when coded as such by the IMPC code or when the item's demand history terminates with four years of no demands.

3.2 Cyclic Approach

The time from initiation of the minimum buy until an ending condition is reached is termed a cycle. Each item is tracked for just one cycle, and how long a cycle lasts may depend on how big the minimum buy is. With a bigger buy, it will take longer to deplete assets. Thus, total time simulated depends on the minimum buy policy. This is allowed for by reporting results as cost per requisition satisfied.

A full mathematical justification of this procedure is included as Appendix B. An alternative, more common approach, would have been to track all items until the end of their history, and compare total cost. The drawback of such a procedure is how to treat assets on hand at the end of the item's history. A major concern in adopting a minimum buy policy is the accumulation of unneeded assets, so that evaluation of assets bought but not used in the simulation becomes critical.

Under the cyclic approach we track only one buy, and if that does not occur by year 5 of the simulation, we exclude the item from the simulation. Any assets that remain at simulation end have therefore been held for at least six years. In such a period, there is a good chance the item will become obsolete, so we can write off the ending assets. Even if not obsolete, we have accumulated six years of holding cost, which is almost comparable in cost to writing the asset off completely.

In summary, the advantage of the cyclic approach is that all materiel bought can be tracked for a number of years. Under the more common approach the usefulness of materiel bought in the last years of the simulation cannot be evaluated properly.

3.3 Performance Statistics

For each minimum buy we record the dollar amount of the buy and any excess, i.e. the dollar amount remaining at the end of the cycle for items which become obsolete. The other performance statistics are accumulated over the duration of the cycle. These are:

a. Length of cycle in months.
b. Time weighted backorders (extended).
c. Number of item months stock was on hand at a depot.
d. Average dollar value of assets.
e. Requisitions satisfied.
f. Requisitions satisfied (extended).

Time weighted backorders relates to the number of backorders and the fraction of a year each requisition remains backordered. If a requisition appears before the minimum buy arrives, we compute the fraction of procurement lead time (PLT) remaining, and this fraction of a year is what gets added to the accumulation. This sum is not to be confused with the number of requisitions backordered. An item may receive two requisitions in the PLT: one with nine months left, the second with three months left. This item would contribute one time weighted backorder, i.e. one year of backorders. Note that the requisition which triggers the minimum buy always contributes one full PLT to the accumulation.

We call the backorders "extended" because for items whose cycle ends because they pass COSDIF we consider a backorder even after the end of the cycle. As long as there are remaining assets on hand or due-in, if an item passes COSDIF we look ahead the duration of one PLT and accumulate backorders in this period until the assets are depleted. We also accumulate all the requisitions, satisfied as well as backordered, for the same duration, but these are kept separately from the total requisitions within the cycle. When it comes time to compute costs per requisition, the backorder cost is divided by the extended requisitions while all the other costs are divided by only the requisitions in the cycle.

The rationale for this extension of backorders is that once an item passes COSDIF, the one significant remaining effect of the minimum buy policy pursued is that there may be stock on hand or due in for the PLT after the item passes COSDIF, minimizing backorders in this period.

Average dollar value of assets is used to compute holding costs. It is the sum of the assets for each month in the cycle divided by the number of months in the cycle. In computing assets we exclude any assets which eventually become excess, to avoid charging both excess and holding cost on the same stock.

The need for two counts of requisitions was explained under backorders. There is a complication in the treatment of partial fills. When the cycle ends because the stock is depleted, we do not include the last requisition as satisfied unless there are assets to cover the entire requisition. This treatment of partial fills is dictated by the math of the cyclic approach (see Appendix B). Also, when an item passes COSDIF, the requisition which causes the item to migrate to stocked is not included as part of the cycle, but is included under the extended definition of requisitions.

3.4 Costs Per Cycle

Costs per cycle are computed from the performance statistics and the appropriate cost parameters. They are:

a. Administrative Cost of Procurement: \$165 x 1 Buy per Cycle.

b. Excess Cost: \$ value of any excess.

c. Backorder Cost: \$350 x Time Weighted Backorders

d. Bin Cost: \$7.73 x Number of Item Months On Hand ÷ 12.

e. Holding Cost: Average \$ Value of Assets x (cycle length in months ÷ 12) x 13%

The values \$165, \$350, \$7.73 are all figures currently used by TSARCOM. These values were also used in determining if an item passed COSDIF.

In applying holding cost rate of 13%, we excluded the obsolescence component of holding cost. Obsolescence is separately measured as excess cost. In other words, when simulating, we do not need to assume some percent of our assets will become excess. We can see just what assets really do become excess.

We do not apply holding cost to assets excessed. Suppose a $10 item is held five years, then excessed. We have lost $10, plus whatever physical storage costs are involved. We have not lost $10 + ($10)(13%)(5) = $16.50. Looking at it another way, would we really have saved more than storage cost if we threw the assets away one year after we bought them, rather than five? Storage costs to hold $10 for five years are estimated to be ($7.73)(5), i.e., bin cost, plus ($10)(5)(1%), i.e., the storage cost component of holding cost. The latter is ignored on excessed assets as being relatively insignificant.

3.5 Costs Per Requisition

Costs per requisition are reported for five quasi-homogeneous classes of items: items with unit price in the intervals ($0,$1], ($1,$5], ($5,$10], ($10,$30], and ($30,$50]. An overall cost figure is also reported.

Costs are accumulated over all items in a class and then reduced to a per requisition basis by dividing each cost by the number of requisitions satisfied in the class. Recall that only the backorder cost is divided by the extended requisitions. When no minimum buy is made, the number of requisitions satisfied is equal to the number of buys, since each buy just covers the requisition which causes it. When a minimum buy is made, some additional number of requisitions may be satisfied before the end of the cycle.

Aggregate costs over all the classes are derived by weighting each class by the number of items in the class. Alternatively we might have weighted by the number of requisitions received for that class over a fixed period of time.

CHAPTER IV

POLICY TESTING AND FINDINGS

4.1 Policies

The policies we tested fall into three categories. Group I policies simply buy a minimum dollar amount of the item when it is demanded. We tried \$25, \$50, and \$100. For the \$25 policy, we buy at least \$25 worth of the item. If the demand is more than \$25 worth, then we buy only to satisfy the demand.

The Group II policies buy an additional dollar amount on top of the amount needed to satisfy the demand. We tried \$25, \$50, and \$100. In this case, for the \$25 policy, we buy to meet the demand plus \$25 worth more. If the demand is for \$10 worth we buy \$35 worth.

The dollar amount of an item in Policies I and II is truncated to an integer quantity. \$25 worth of a \$4 item is 6, since 7 exceeds \$25. For the higher priced items the \$100 limit on minimum buy severely restricts the quantity of the buy. No minimum buy can be made on an item costing more than \$50, and at most three \$30 items can be bought.

The Group III policies are the most sophisticated. They involve buying an amount Q in addition to the demand, where Q is calculated for each item based on system costs (e.g. cost to procure) and individual item parameters at the time of the demand. The Economic Q was calculated to minimize the overall costs. See Appendix A for the details of the calculation. We also ran the simulation using the Wilson Q, which is not as theoretically sound as the Economic Q, but is more familiar.

4.2 Results

Table 1 gives the various aggregate costs for the nine policies tested, including "no minimum buy". Table 2 shows the total cost for each of the policies broken down by unit price class. Table 3 gives the average amount placed on procurement for each policy, broken down by unit price class.

The aggregate costs in Table 1 show clearly that all the minimum buy policies reflect lower total cost than no minimum buy. The most significant contributing costs are the administrative cost of buys and the backorders, which are precisely the costs that the minimum buy concept strives to reduce. The Table 2 breakdown supports our intuitive expectations. While it appears from Table 1 that the costs keep decreasing as the size of the minimum buy increases, we see that this is not the case for the lower priced items. For unit prices less than $5, buying $100 over the demand costs more than buying $50 over the demand.

Additional results are reported in Appendix C. Some of these results were obtained to verify that the findings were not sensitive to the method used to express and aggregate total costs. The other results represent investigations of modifications in the Economic Q, including some fine tuning, and a modification designed to accommodate the Army computer system in which the policy would be embedded. The impact on costs of the fine tuning and modification is slight.

4.3 Conclusions

Use of the economic Q policy is most cost effective and most adaptable to changes in system parameters. Among the policies which work well, it is most conservative, i.e., buys the least. The Wilson Q does almost as well as the Economic Q and is easier to understand. Any of the minimum buy policies can be justified, even without regard to the lower unit prices to be realized from larger procurement quantities (not considered in this analysis).

In this analysis we limited buys to $100 plus demand. We believe a minimum buy policy could be more effectively applied to items in the $25-$50 unit price range and extended to higher priced items if this $100 limit on minimum buy were dropped. It is evident from Table 2 that the greatest savings are achieved for the lower priced items where the minimum buy concept is taking effect. To increase the limit on minimum buys to more than $100 would require no change in regulation or written policy, just a change in thinking.

TABLE 1: AGGREGATE COSTS PER REQUISITION

Group I Policies

	$25	$50	$100
Buys	141.91	127.76	107.67
Excess	2.89	8.22	20.94
Backorders	160.86	142.19	119.65
Bin	12.02	16.72	20.31
Holding	.69	2.05	5.65
Total	318.37	296.94	274.22

Group II Policies

	+$25	+$50	+$100
Buys	126.56	107.97	97.74
Excess	5.16	12.47	25.98
Backorders	144.10	123.39	108.01
Bin	17.48	21.23	21.59
Holding	1.33	3.44	7.37
Total	294.63	268.50	260.69

Group III Policies and No Minimum Buy

	Econ Q	Wilson Q	No Min Buy
Buys	98.80	98.07	165.00
Excess	16.65	19.84	0
Backorders	109.83	108.57	191.91
Bin	21.68	21.64	0
Holding	5.12	5.93	0
Total	252.08	254.05	356.91

TABLE 2: TOTAL COST PER REQUISITION BROKEN DOWN BY UNIT PRICE CLASS

	$UP<1$	$1<UP\leq 5$	$5<UP\leq 10$	$10<UP\leq 30$	$30<UP\leq 50$	Aggregate
Group I Policies						
\$25:	253.19	261.12	296.36	350.61	376.50	318.37
\$50:	254.60	253.12	264.06	308.63	376.50	296.94
\$100:	270.77	264.67	253.10	270.39	313.28	274.22
Group II Policies						
+\$25:	246.08	248.20	263.30	307.56	376.50	294.63
+\$50:	249.33	247.73	246.05	274.32	311.49	268.50
+\$100:	271.28	263.05	246.03	255.54	275.78	260.69
Group III Policies and No Minimum Buy						
Econ Q:	236.36	244.01	238.28	255.19	275.76	252.08
Wilson Q:	241.44	248.31	241.50	255.35	275.78	254.05
No Min Buy:	333.47	343.35	352.40	363.14	376.50	356.91
....						
Number of Items						
In Class	212	516	401	822	442	2393

TABLE 3: AVERAGE AMOUNT PLACED ON PROCUREMENT BROKEN DOWN BY UNIT PRICE CLASS

	$UP<1$	$1<UP\leq 5$	$5<UP\leq 10$	$10<UP\leq 30$	$30<UP\leq 50$	Aggregate
Group I Policies						
$25:	29.50	31.44	39.97	57.16	93.23	52.94
$50:	53.71	54.80	61.40	71.60	93.23	68.68
$100:	102.73	103.23	107.98	114.77	125.47	112.05
Group II Policies						
+$25:	33.05	38.29	51.53	71.67	93.23	61.66
+$50:	58.02	63.20	76.99	96.13	132.29	89.13
+$100:	107.97	113.24	127.02	146.29	178.38	138.47
Group III Policies						
Econ Q:	32.34	56.89	96.73	140.00	178.29	112.36
Wilson Q:	47.02	75.08	110.87	145.07	178.38	121.72
No Min Buy:	8.25	14.52	30.32	55.63	93.23	45.27

APPENDIX A

ECONOMIC Q

References:

a. Kaplan, A. "An Alternative to the Classifical Economic Order Quantity," IRO, 1974.

b. Kruse, W. Karl and Edward Bruckner "Calculation of Expected Depletion Time When Demand is Stuttering Poisson," IRO, 1968.

Notation:

C_p = cost to procure
UP = unit price
Q = amount to buy
λ = "Lambda," cost per requisition backordered per year
L = procurement lead time
OB = obsolescence rate
i = interest rate
D = yearly demand rate
R = yearly requisition rate
S = average requisition size or D/R
TC(Q) = total expected cost to manage an item over its expected lifetime if Q is bought whenever stock drops to -1.
K = (1-OB)/(1+i)

Discussion:

In reference a., it is shown that a discounted cash flow analogue to the classical Wilson EOQ is to find a Q to minimize:

$$(1) \qquad TC(Q) = [C_p+(UP)(Q)]\,\frac{1}{1-K^{Q/D}}$$

Although the classical EOQ may incorporate obsolescence into the holding cost, its derivation assumes that in fact demand will continue for many cycles, i.e., through many purchases of the item. The discounted cash flow Q does not. It allows for the probability there will be only 1 buy, 2 buys, and so on.

(These probabilities are based on the obsolescence rate). The discounted cash flow Q therefore seems more appropriate to non-stocked items.

Modifications were made to Equation (1) to account for policy of ordering only when net assets are below 0; to model impact of requisition sizes ≥ 1; and to model impact of Q on expected backorders. Understanding of these modifications requires review of Kaplan's original paper (ref a.).

Order Only When Stock is Below 0 (S = 1). Net stock when a buy is placed is -1. The buy is therefore for (Q+1) units. Net stock after the buy is made is Q units. The next buy is again made when net stock = -1, or equivalently, after total demand of (Q+1) units is received. Therefore Q must be replaced by (Q+1) in Equation 1.

Requisition Size $\geq$ 1. Given geometric requisition sizes, net stock when a buy is placed is -S. Interestingly, even if a requisition which is received when assets are > 0 triggers a buy, the expected value of the unfilled part of the requisition is -S. Hence expected amount per buy is Q+S. After a buy,assets are Q. Expected time to deplete at least Q+1 units, given geometric order sizes, is (Q+S)/D, as shown in ref b. In summary, then, we must replace Q with (Q+S) in equation 1.

Impact of Q on Backorders. An approximate backorder cost was incorporated.

$$\text{Backorder Cost} = (\lambda)(L) + (\lambda)(R)(L)\,\frac{L}{2}\,(K^{L/2})$$

The requisition which triggered the buy is backordered for the full lead time L; its cost is $(\lambda)(L)$. An additional (R)(L) backorders are expected in the lead time, each with average duration of L/2. Incorporating the discounting factor K gives a cost of $(\lambda)\ (\frac{L}{2})\quad K^{L/2}$ for the (R)(L) backorders in the lead time.

Incorporating the above three modifications into Equation (1) gives

$$TC(Q) = \frac{C_p + (UP)(Q+S) + (\lambda)(L) + (\lambda)\frac{(R)(L^2)}{2}\ (K^{L/2})}{1 - K^{(Q+S)/D}}$$

Optimum Q is found by a simple grid search. Values of Q between 1 and \$100/UP are tried in increments of max [1,\$5/UP]. The use of \$100 was designed to accommodate current thinking on reasonable minimum buys.

APPENDIX B

MATHEMATICAL JUSTIFICATION OF CYCLIC APPROACH

This approach progresses from consideration of a very simple model to the real world simulation for which the cyclic approach was used. Along the way, a more general model is discussed. The cyclic approach is valid for both the simple and more general models, but validity can be more concretely demonstrated for the simple model. The cyclic approach is only approximately correct within the context of the simulation, and the nature of approximation is discussed.

Simple Model

We are interested in lifetime costs as a function of minimum buy policy. An item's lifetime ends when either no future requisitions will occur, because the item becomes obsolete, or no future requisitions need be considered because the item is reclassified as stocked.

Requisitions are for 1 unit each. After each requisition, there is a probability of (1-p) that the item's lifetime will end. If assets are 0 and a requisition occurs, a quantity of n is bought. Whenever a buy is made, there are fixed costs, C(n), associated with it including the administrative and purchase costs of procurement, and cost of customer wait for unit to be direct shipped.

Cyclic costs are C(n). Cyclic demand is the total demand beginning with the requisition which triggers the minimum buy and ending with the last requisition which is satisfied from that buy, or the last requisition of the item's lifetime, whichever comes sooner.

Then, given there is at least one requisition,

A1:

$$\Pr(\geq 1 \text{ Buys in Lifetime}) = 1$$

$$\Pr(\geq 2 \text{ Buys in Lifetime}) = p^{n}$$

$$\Pr(\geq k \text{ Buys in Lifetime}) = p^{n(k-1)}$$

A2:

$$\text{Expected Lifetime Costs} = C(n) \sum_{k=1}^{\infty} p^{n(k-1)}$$

$$= C(n) \sum_{j=o}^{\infty} (p^{n})^{j}$$

$$= C(n) \frac{1}{1-p^{n}}$$

In computing expected requisitions per cycle, we note that there must be at least one requisition for a cycle to begin:

A3: Expected Requisitions per Cycle

$$= \sum_{i=1}^{n} (i)p^{i-1}(1-p) + np^{n}$$

where the first term is the sum of probabilities that there are exactly 1 to n requisitions in the items lifetime, and the second term is the probability of $>$ n requisitions.

$$= (1-p) \sum_{i=1}^{n} \frac{d}{dp} p^{i} + np^{n}$$

where $\frac{d}{dp}$ denotes derivative with respect to p

$$= (1-p) \frac{d}{dp} \left[\frac{1-p^{n+1}}{1-p} - 1\right] + np^{n}$$

$$= \frac{1-p^{n}}{1-p}$$

after taking the derivative and using algebra to simplify.

Therefore

A4:
$$\frac{\text{Expected Cost Per Cycle}}{\text{Expected Requisitions Per Cycle}} = C(n) \div \frac{1-p^{n}}{1-p}$$

$$= \frac{C(n)(1-p)}{1-p^{n}}$$

Comparing A4 and A2, the two measures are proportional to each other, with constant of proportionality (1-p) which is independent of n. In other words, any relative ranking of different policies, i.e. different n, by A2 is equivalent to a ranking by A4. The cyclic approach is validated provided we define cost per requisition as expected cycle cost $\div$ expected cyclic requisitions.

More General Model

The important changes in the model are that requisitions are for a variable number of units, and that cyclic costs depend on what happens during the cycle. In particular, instead of purchase costs, we include holding cost and the cost of assets remaining at the end of a cycle terminated by obsolescence; backorder costs include not only customer wait for the customer who triggered the minimum buy, but customer wait for any other requisitions delayed.

It is assumed that the distribution of requisition size is stationary, and that requisition sizes are independent. If a requisition can only be partially filled from existing assets, a minimum buy is made and the requisition is attributed to the cycle beginning with this buy. Initially we assume each minimum buy is for the amount to be direct shipped to the customer plus a fixed amount for the wholesale depot.

We consider the beginning of a cycle as a renewal event.* We imagine, for a moment, that an item has an infinite number of lifetimes, although of course we are only interested in its "first" lifetime. Each lifetime may have many cycles ending with depletion of assets and must have one cycle, the last, ending with obsolescence or migration of item to stocked status.

Given the model just described, we can make two claims:

(a) Number of requisitions and costs for each cycle are independent of requisitions and costs in other cycles.

(b) The event "end of a lifetime" is independent of requisitions and costs in subsequent cycle.

Both claims follow from the stationarity of demand, and the fact that each cycle begins with the same assets, and the same costs, namely the assets sent to the depot, the administrative cost of a buy, and backorder costs for one requisition to be direct delivered.

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*W. Karl Kruse first suggested it might be possible to use renewal theory to generalize the simple model.

The two claims made fulfill the conditions of Wald's Equation (cf Ross, Section 3.4), with these implications:

A5: $$\text{Ex (Lifetime Cost)} = \text{Ex (Number of Cycles/Lifetime)} \cdot \text{Ex (Cyclic Cost)}$$

$$\text{Ex (Lifetime Req)} = \text{Ex (Number of Cycles/Lifetime)} \cdot \text{Ex (Cyclic Req)}$$

By algebra, then,

A6: $$\frac{\text{Ex(Cyclic Cost)}}{\text{Ex (Cyclic Req)}} = \frac{\text{Ex (Lifetime Cost)}}{\text{Ex (Lifetime Req)}}$$

Therefore, any ranking of minimum buy policies by the measure on the left hand side of A6 is equivalent to a ranking by the right hand side. This validates use of cyclic costs, requisitions.

Extensions to General Model

Other Minimum Buy Policies. Suppose the minimum buy policy is to buy a fixed amount of stock, with the amount shipped to the depot the residual after stock for direct delivery is subtracted. Then if requisition sizes are variable, each cycle begins with a variable amount of assets. Wald's Equation can still be applied, without qualification, provided the probability distribution of beginning assets is independent of what transpired in other cycles.

Starting assets depend on what must be direct delivered. If the last cycle ended a lifetime, the direct delivery quantity is the size of one requisition. Otherwise, it can be the size of a requisition, but is often the size of "overshoot", e.g. if there are x assets, and a requisition for y is received, $y > x$, overshoot is $y - x$, and the next cycle begins with a partial fill. For Wald's Equation to completely apply, the distribution of overshoot quantity must be the same as distribution of requisition size. This will be true if and only if requisition size has a geometric distribution, or exponential if continuous distributions are considered.

Ranking By Demand. Instead of using cyclic requisitions in A6, it is possible to use total cyclic demand. In this case, when there is a partial fill, the demand equal to the overshoot is attributed to the next cycle, not the entire requisition quantity. Provided requisition size is geometric, Wald's Equation applies. Otherwise total demand in a cycle depends on how the last cycle ended.

Modification of Cyclic Definitions. In computing supply performance costs, we wish to include backorders and requisitions incurred up to a procurement lead time after an item migrated to stocked. There is some initial difficulty in applying Wald's Equation since it would seem we were double counting requisitions during the lead time, counting them in the last cycle of the "first lifetime" and first cycle of the next lifetime.

In fact, while the statement of Wald's Equation seems to require an infinite number of cycles, this is not necessary. Extending the last cycle without considering a new lifetime eliminates double counting. In justifying use of Wald's theorem we require that whether we begin a cycle is independent of what occurs in that cycle, and that requisitions and costs in cycles begun are independent of earlier requisitions and costs. Both these requirements are met in our situation.

Formally, in Ross' notation, redefine Y_n to be 1 if the n^{th} cycle is begun. The statement of the theorem is rewritten:

$$E \sum_{1}^{N} X_i = EN \cdot E(X|Y = 1)$$

where the conditional is expected value of X given it occurs before stopping. Then in Ross' proof of equation (12),

$$\sum_{n=1}^{N} E X_n = \sum_{n=1}^{\infty} E(X_n|Y_n = 1) \Pr(Y_n = 1) + 0$$

$$= E(X|Y = 1) \sum_{n=1}^{\infty} \Pr(Y_n = 1)$$

$$= E(X|Y=1) E(N)$$

Application to Simulation Results

Assumptions made which do not completely hold are:

(a) Geometric requisition sizes in evaluating some classes of minimum buy policies.

(b) Stationary demand.

(c) Migration to stocked independent of past cycles when size of requisitions in past cycles may impact on ability to pass COSDIF.

Failure of these assumptions to hold does not appear particularly worrisome.

Another problem is how to measure expected cyclic requisitions and costs. Application of the cyclic approach assumes this can be done accurately. The problem is that to get good estimates we require large (item) sample sizes, but items are not alike.

In the text we define groups of quasi-homogeneous items, but there is a definite tradeoff between size of group and its actual homogeneity.

Finally, in the text we terminated observation of extended requisitions/costs when assets were depleted, if this occurred earlier than a PLT after passing COSDIF. Technically, this was not correct, since it makes expected lifetime extended requisitions dependent, to a small degree, on the policy being evaluated.

APPENDIX C

ADDITIONAL RESULTS

Costs Per Dollar of Demand Satisfied

The largest costs in managing non-stocked items are requisition related, i.e. the costs of making buys and incurring backorders. For this reason it is more meaningful to talk of costs per requisition received than costs per dollar of demand received. Nevertheless, mathematically the cyclic approach can be used to compute the latter, and this was done as a check on the findings based on the per requisition computation.

The mathematics do require the assumption that requisition sizes are geometrically distributed (Appendix B). Also, when we run out of stock, we do include the dollar demand needed to get to 0, whereas a partially filled requisition is not included at all in the per requisition costs calculation.

Following are the results shown for costs per dollar value of demand satisfied. Tables 1A and 2A are defined as Tables 1 and 2 for the costs per requisition. Table 3, average amount placed on procurement is applicable to both sets of results and is not repeated.

Alternate Class Groupings

In addition to the unit price classes, we tried grouping the items by dollar value of demand. Included are the costs per requisition, Tables 1B and 2B, and the costs per dollar value of demand satisfied, Tables 1C and 2C. Table 3B is the average amount placed on procurement when the classes are determined by dollar value of demand satisfied.

The aggregate results for costs per requisition are relatively independent of the criteria used to classify the items. The costs shown here based on dollar demand classes are almost identical to the original results based on unit price classes.

The aggregate costs per dollar demand, however, are uniformly less when classified by dollar demand. The ranking of the various minimum buy policies remains the same regardless of our choice of quasi-homogeneous items.

Variations in Economic Q

Use of the Average Yearly Demand and Requisitions to compute minimum economic buys causes implementation problems. As one solution we ran the economic Q policy with an approximate Average Yearly Demand (AYD). For a conservative estimate of the AYD we assumed only one requisition occurred for two years. The AYD used was 1/2 the amount of the requisition which triggered the buy.

We also tried the economic Q policy for alternate values of K, the discounting factor. In Appendix A, K is defined as (1-OB)/(1+i) where OB is the obsolescence rate and i the interest rate. Taking both these rates to be 10%, which were accepted values, results in K equal to .8. We ran the economic Q policy for other values and found .7 worked the best. Table 4 shows the results of the economic Q policy with alternate K values with and without the approximate AYD.

TABLE 1A: AGGREGATE COSTS PER DOLLAR DEMAND

Group I Policies

	$25	$50	$100
Buys	3.40	3.15	2.73
Excess	.08	.21	.53
Backorders	4.03	3.77	3.34
Bin	.31	.43	.51
Holding	.02	.05	.14
Total	7.84	7.61	7.25

Group II Policies

	+$25	+$50	+$100
Buys	3.11	2.72	2.41
Excess	.13	.31	.63
Backorders	3.77	3.40	2.98
Bin	.44	.53	.53
Holding	.03	.09	.18
Total	7.48	7.05	6.73

Group III Policies and No Minimum Buy

	Econ Q	Wilson Q	No Min Buy
Buys	2.47	2.44	3.64
Excess	.42	.50	0
Backorders	3.06	3.01	4.24
Bin	.54	.54	0
Holding	.13	.15	0
Total	6.62	6.64	7.88

TABLE 2A: TOTAL COST PER DOLLAR DEMAND BROKEN DOWN BY UNIT PRICE CLASS

	UP<1	1<UP≤5	5<UP≤10	10<UP≤30	30<UP≤50	AGGREGATE
Group I Policies						
$25:	40.73	22.59	11.76	6.55	4.04	7.84
$50:	38.20	21.20	10.96	6.53	4.04	7.61
$100:	38.66	20.36	10.34	6.14	3.86	7.25
Group II Policies						
+$25:	36.78	20.72	10.79	6.42	4.04	7.48
+$50:	35.13	19.74	10.15	6.04	3.75	7.05
+$100:	35.08	19.24	9.76	5.72	3.48	6.73
Group III Policies and No Minimum Buy						
Econ Q:	31.93	18.85	9.56	5.70	3.48	6.62
Wilson Q:	32.30	18.75	9.63	5.71	3.48	6.64
No Min Buy:	40.42	23.64	11.62	6.53	4.04	7.88

TABLE 1B: AGGREGATE COSTS PER REQUISITION WHEN BROKEN DOWN BY DOLLAR DEMAND CLASSES

Group I Policies

	$25	$50	$100
Buys	142.78	128.77	108.93
Excess	2.78	7.98	20.84
Backorders	161.34	142.54	119.78
Bin	11.58	16.15	20.10
Holding	.67	2.00	5.59
Total	319.15	297.44	275.24

Group II Policies

	+$25	+$50	+$100
Buys	125.29	107.89	98.25
Excess	5.24	12.60	26.46
Backorders	140.59	121.44	107.07
Bin	17.81	21.39	21.83
Holding	1.37	3.46	7.42
Total	290.30	266.78	261.03

Group III Policies and No Minimum Buy

	Econ Q	Wilson Q	No Min Buy
Buys	99.43	98.62	165.00
Excess	16.58	19.95	0
Backorders	109.09	107.70	191.91
Bin	21.95	21.89	0
Holding	5.05	5.90	0
Total	252.10	254.06	356.91

TABLE 2B: TOTAL COST PER REQUISITION WHEN BROKEN DOWN BY DOLLAR DEMAND CLASSES

	DM< 3	3<DM≤10	10< DM≤25	25<DM≤50	50<DM	AGGREGATE
Group I Policies						
$25:	253.83	275.91	338.43	365.14	356.10	319.15
$ 50:	262.53	260.44	274.68	353.40	356.10	297.43
$100:	285.92	271.07	255.69	262.12	319.08	275.24
Group II Policies						
$+25:	254.20	266.34	285.80	326.74	318.27	290.30
+$50:	263.09	260.98	259.47	273.02	281.24	266.78
+$100:	286.05	272.81	256.46	241.75	251.32	261.04
Group III Policies and No Minimum Buy						
Econ Q:	251.73	261.24	254.98	241.75	251.48	252.10
Wilson Q:	256.30	265.48	256.26	241.75	251.32	254.06
No Min Buy:	343.21	357.76	359.38	365.14	356.10	356.91

TABLE 1C: AGGREGATE COSTS PER DOLLAR DEMAND (WEIGHTED BY DOLLAR DEMAND)

Group I Policies

	$25	$50	$100
Buys	2.99	2.59	2.06
Excess	.05	.14	.34
Backorders	3.40	2.91	2.34
Bin	.21	.29	.35
Holding	.01	.03	.09
Total	6.66	5.96	5.18

Group II Policies

	+$25	+$50	+$100
Buys	2.63	2.16	1.87
Excess	.10	.24	.47
Backorders	3.01	2.53	2.16
Bin	.35	.41	.40
Holding	.03	.07	.14
Total	6.12	5.41	5.04

Group III Policies and No Minimum Buy

	Econ Q	Wilson Q	No Min Buy
Buys	1.99	1.93	3.64
Excess	.34	.39	0
Backorders	2.28	2.22	4.24
Bin	.43	.42	0
Holding	.11	.12	0
Total	5.15	5.08	7.88

TABLE 2C: TOTAL COST PER DOLLAR DEMAND BROKEN DOWN BY DOLLAR DEMAND CLASSES (WEIGHTED BY DOLLAR DEMAND)

	$DM<3$	$3<DM\le 10$	$10<DM\le 25$	$25<DM\le 50$	$50<DM$	AGGREGATE
Group I Policies						
$25:	93.40	34.52	19.42	9.92	1.72	6.66
$50:	85.58	28.90	14.97	9.65	1.72	5.96
$100:	81.84	26.86	12.69	7.01	1.77	5.18
Group II Policies						
$+25:	91.34	31.40	15.58	9.26	1.79	6.12
+$50:	85.06	28.22	13.52	7.31	1.80	5.41
+$100:	81.66	26.80	12.35	6.24	1.82	5.04
Group III Policies and No Minimum Buy						
Econ Q:	91.12	27.71	12.37	6.24	1.83	5.15
Wilson Q:	85.11	27.00	12.35	6.24	1.82	5.08
No Min Buy:	181.66	49.18	20.14	9.92	1.72	7.88

TABLE 3B: AVERAGE AMOUNT PLACED ON PROCUREMENT WHEN BROKEN DOWN BY DOLLAR DEMAND CLASSES (WEIGHTED BY DOLLAR DEMAND)

	$DM<3$	$3<DM\leq 10$	$10< DM\leq 25$	$25<DM\leq 50$	$50<DM$	AGGREGATE
Group I Policies						
\$25:	24.29	22.09	20.36	36.81	206.47	45.27
\$50:	49.28	46.83	43.53	39.70	206.47	68.68
\$ 100:	99.25	97.06	92.88	87.96	215.48	112.05
Group II Policies						
\$+25:	26.19	29.36	37.04	43.25	216.19	61.66
+\$50:	51.17	54.10	61.38	76.36	246.76	89.13
+ \$100:	101.14	104.33	110.73	124.77	295.54	138.47
Group III Policies and No Minimum Buy						
Econ Q:	21.39	59.10	101.34	124.76	294.38	112.36
Wilson Q:	38.70	79.30	109.56	124.77	295.54	121.72
No Min Buy:	1.89	7.27	17.85	36.81	206.47	45.27

TABLE 4: AGGREGATE COSTS PER REQUISITION FOR VARIATIONS IN ECONOMIC Q POLICIES

ACTUAL AYD

	K = .6	K = .7	K = .8
Buys	103.20	100.08	98.80
Excess	10.24	13.33	16.65
Backorders	117.13	112.42	109.83
Bin	21.54	21.65	21.68
Holding	3.50	4.26	5.12
Total	255.61	251.74	252.08

APPROX AYD

	K = .6	K = .7	K = .8
Buys	104.99	100.60	98.82
Excess	9.22	12.54	16.04
Backorders	121.12	113.89	110.27
Bin	21.51	21.59	21.64
Holding	3.11	3.98	4.97
Total	259.95	252.60	251.74

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