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INFLUENCE OF FLUID LOADING ON THE TRANSMISSION ACROSS, RADIATIO--ETC(U)  
JAN 88 D CRIGHTON, T EISLER, & MAIDANIK  
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DAVID W. TAYLOR NAVAL SHIP  
RESEARCH AND DEVELOPMENT CENTER



Bethesda, Md. 20084

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INFLUENCE OF FLUID LOADING ON THE TRANSMISSION  
ACROSS, RADIATION FROM, AND REFLECTION  
BY RIBS ON A PANEL

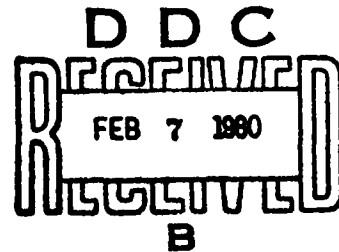
by

D. Crighton  
T. Eisler  
G. Maidanik

INFLUENCE OF FLUID LOADING ON THE TRANSMISSION ACROSS,  
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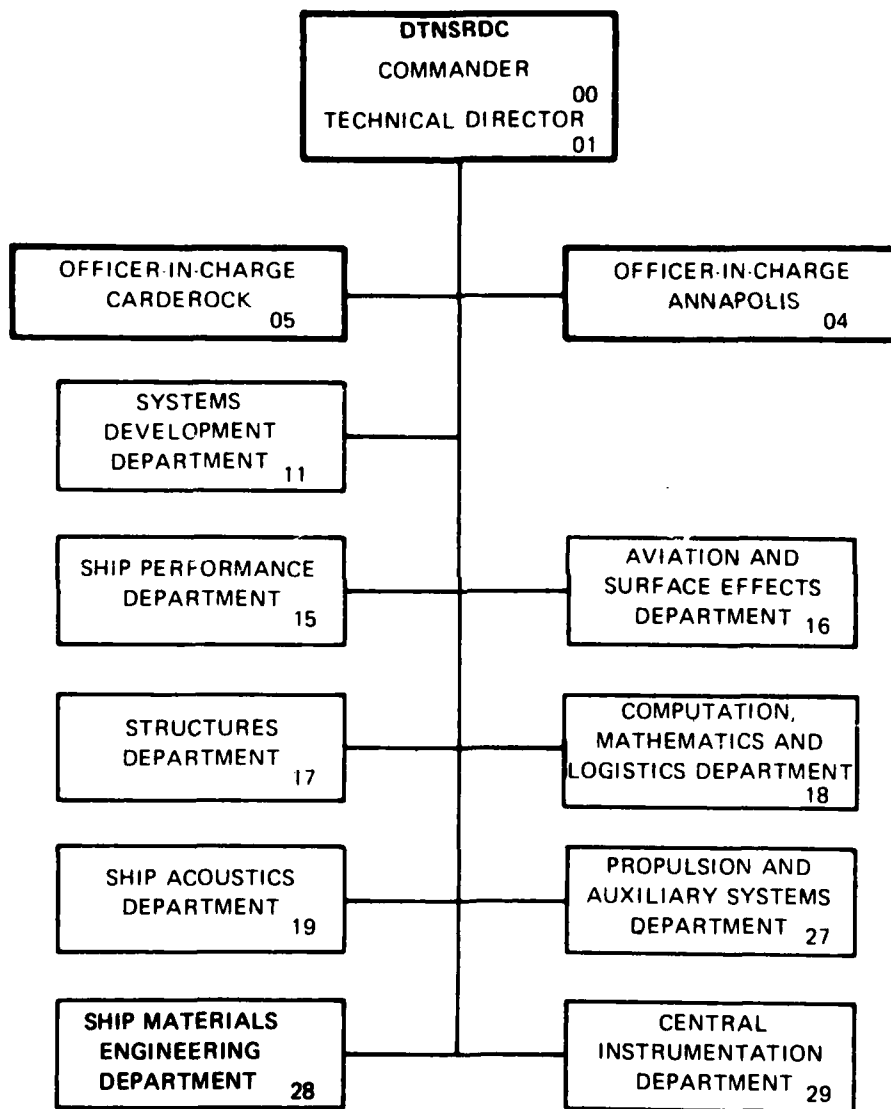
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TRANSMISSION ACROSS, RADIATION FROM, AND  
REFLECTION BY RIBS ON A PANEL .

ABSTRACT

A formalism of the response of an infinite, plane, and uniform panel is derived in terms of the impulse response function. The modification of the impulse response function caused by attaching parallel line mechanical constraints is considered. It is shown that the impulse response function of the so constrained panel can be cast in the form of two terms. The first is simply the impulse response of the unconstrained panel. The second is a functional of the impulse response function of the unconstrained panel and the impedances of the line mechanical constraints. It is argued that the formalism so cast is particularly suitable for ascertaining the modification to the response introduced by line mechanical constraints (ribs). The argument is exemplified by deriving the expressions for the transmission coefficient across the ribs, the radiation to the far field generated by the presence of the ribs, and the change in the reflective properties of the panel introduced by the ribs.

ADMINISTRATIVE INFORMATION

This report was prepared as part of the IR/IED Program at the David W. Taylor Naval Ship Research and Development Center under Task Area ZR0110801, Work Unit 1-1902-005, and was previously published as Ship Acoustics Department Technical Memorandum TM-1902-78-52. The report contains the texts of three papers presented at the joint meeting of the Acoustical Society of America and the Acoustical Society of Japan, held in Honolulu, Hawaii, December 1978.

TEXT

The dynamic system under consideration is composed of three basic parts: a panel, ribs which provide localized mechanical constraints to the motion of the panel, and a fluid medium which occupies the semi-infinite space above the panel. This dynamic system is sketched on Slide 1. The panel is characterized by a surface impedance operator which is designated by  $z(x,t)$ , the fluid is characterized by a characteristic impedance which is designated by  $(\rho c)$ , and a rib [the (j)th rib] is characterized by a line impedance which is designated by  $z_{\ell j}(x_2,t)$ ; the ribs are assumed to lie along the  $x_2$ -direction. [The spatial vector  $\underline{x} = \{x_1, x_2\}$  lies the plane of the panel,  $t$  is the temporal variable,  $x_3$  is the spatial variable normal to the panel,  $c$  is the speed of sound, and  $\rho$  is the density of the fluid medium.] It is assumed that the composite surface impedance operator in the plane of the panel is pure with respect to the spatial variable  $x_2$  and the temporal variable  $t$ ;  $\{k_2, \omega\}$  is the Fourier conjugate vector of the vector  $\{x_2, t\}$ . The equation of motion of the panel in terms of the composite surface impedance operator  $\tilde{z}_T(x_1, k_2, \omega)$  and the external drive  $\tilde{p}_e(x_1, k_2, \omega)$  is depicted on Slide 2. In this paper the motion of the dynamic system is limited to the  $\{x_1, x_3\}$ -plane so that  $k_2$  in the equation of motion is set equal to zero. Suppressing as obvious the dependence on frequency, only the dependence on  $x_1$  need remain explicit. Equation (1) can then be rewritten in the form of equation (2). The dependence of the composite surface impedance operator on the ribs can be termed out as depicted in equation (3); the impedance operator  $\tilde{z}(x_1)$  is that of the panel in the absence of the ribs but in the presence of the fluid medium. The surface impedance operator  $\tilde{z}(x_1)$  is the composite surface impedance

operator of the fluid loaded panel; this operator is considered, by definition, to be pure also with respect to the spatial variable  $x_1$ . In equation (4) the presence of the ribs is accounted for by adding a drive term to the external drive; the influence of the ribs on the motion of the panel can be described in terms of the applied drive  $-\tilde{p}_g(x_1)$ .

The equation of motion in terms of the composite surface impedance operator can, on occasions, be conveniently inverted to derive the equation of motion in terms of the composite (surface) impulse response function. The equation of motion in terms of the composite impulse response function is stated in equation (6) on Slide 3. Under the assumptions made herein the composite impulse response function can be cast so that it consists, as does the composite surface impedance operator, equation (3), of two terms; the first term is the impulse response function of the fluid loaded panel in the absence of ribs and the second term accounts for the presence of the ribs; see equation (7) on Slide 3. On Slide 4 is exemplified the situation for the case in which the panel is membranelike; membranelike panels are those which can support only forces and not moments. The expression for the second term in the composite surface impedance operator is stated in equation (8) and the corresponding expression for the second term in the composite impulse response function is stated in equation (9). In equation (10) the elements of the matrix  $\tilde{S}$

$$\tilde{S} = (S_{rj})$$

are given explicitly. It is observed that the term in the impulse response function which accounts for the ribs is a function of the line impedances of the ribs, and the line admittance and line transfer



admittance of the fluid loaded panel in the absence of the ribs. The latter admittance is evaluated for the various separations between pairs of ribs. Once these quantities are specified and computed it is a relatively simple matter to estimate the vibrational and acoustic properties of the fluid loaded ribbed panel. Of these quantities the transfer line admittance, especially for relatively small separations, is the more difficult to compute. A considerable simplification is gained if first order solutions suffice; this is indicated in equation (11) [1,2]. In first order solutions the transfer line admittance need not be computed which is a considerable relief.

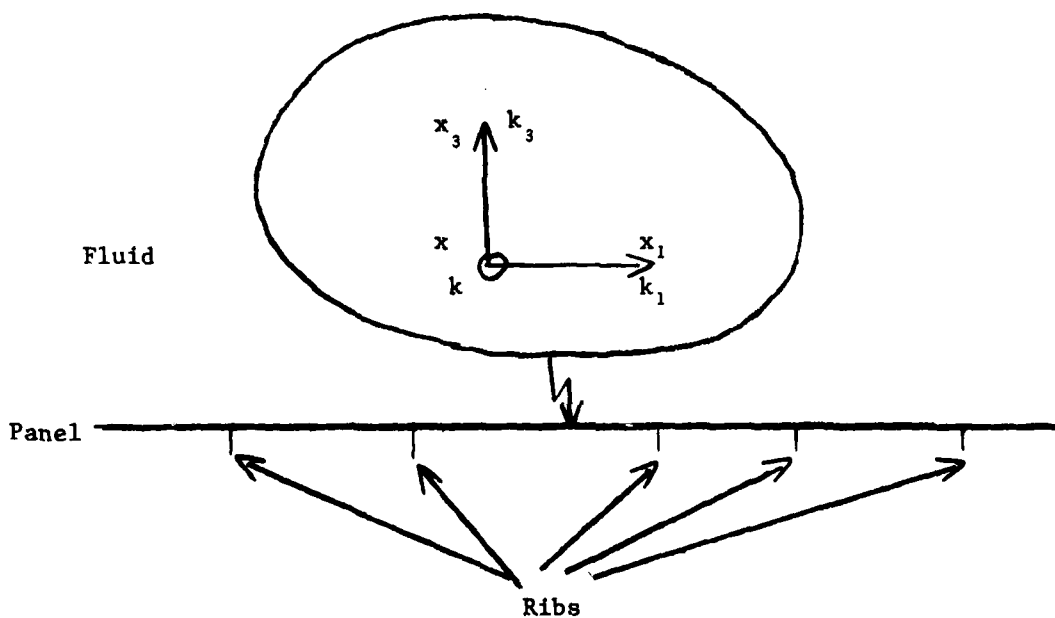
The usefulness of the formalism just presented is exemplified by accounting for three related, but distinct, phenomena. The first accounts for the transmission of free waves across ribs on a driven panel; the formalism is depicted on Slide 5 [3]. The second accounts for the acoustic radiation to the far field which is generated by the presence of the ribs on a driven panel; the formalism is depicted in Slide 6 [4]. The third accounts for the nonspecular reflection of plane incident acoustic pressure from the surface of a fluid loaded ribbed panel. The nonspecular reflection is contributed solely by the presence of the ribs; in the absence of ribs, nonspecular reflection cannot occur; a fluid loaded panel reflects only plane incident acoustic pressure specularly. The formalism of this phenomenon is depicted on Slide 7 [4]. Finally, on Slide 8 are given the explicit forms of the transmission coefficient of free waves across a single rib and across two ribs. Computations of the transmission efficiency,

$$\Gamma = |\text{transmissions coefficient } T|^2 ,$$

in these two cases are presented in the next two papers. The influence of fluid loading on the transmission efficiency is of particular interest.

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- [3] G. Maidanik, A.J. Tucker, and W.H. Vogel (1976) J. Sound Vib. 49, 445.
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Composite Dynamic System Consisting of Panel-Fluid-Ribs

Panel: Surface Impedance Operator  $z(x,t)$

Fluid: Characteristic Impedance  $\rho c$

Ribs: Line Impedance  $z_{lj}(x_2, t)$  located at

$$x_1 = x_j \quad ; \quad j = 1, 2, \dots, N$$

Slide 1

Formalism

Impedance Equation

$$\tilde{z}_T(x_1, k_2, \omega) \tilde{v}(x_1, k_2, \omega) = \tilde{p}(x_1, k_2, \omega) \quad (1)$$

$$\tilde{z}_T(x_1) \tilde{v}(x_1) = \tilde{p}(x_1) \quad (2)$$

$\tilde{z}_T(x_1)$ : Composite Surface Impedance Operator

$\tilde{v}(x_1)$ : Response (Velocity) of Panel

$\tilde{p}_e(x_1)$ : External Drive

$$\tilde{z}_T(x_1) = \tilde{z}(x_1) + \tilde{z}_s(x_1) \quad (3)$$

Impedance Equation Rewritten

$$\tilde{z}(x_1) \tilde{v}(x_1) = \tilde{p}_e(x_1) - \tilde{p}_s(x_1) \quad (4)$$

$$\tilde{p}_s(x_1) = \tilde{z}_s(x_1) \tilde{v}(x_1) \quad (5)$$

Slide 2

Impulse Response Equation (Inverse of the Impedance Equation)

$$\tilde{v}(x_1) = \int dx'_1 \tilde{g}_T(x_1 | x'_1) \tilde{p}_e(x'_1) \quad . \quad (6)$$

In special situations in which the ribbed fluid loaded panel is a valid example, the composite Impulse Response Function splits in the manner of the Composite Surface Impedance Operator so that

$$\tilde{g}_T(x_1 | x'_1) = \tilde{g}(x_1 | x'_1) - \tilde{g}_S(x_1 | x'_1) \quad , \quad (7)$$

where

$\tilde{g}(x_1 | x'_1)$  is the Impulse Response Function of the Fluid Loaded Panel in the absence of ribs.

Slide 3

EXAMPLE: Membranelike Panel

$$z_s(x_1) = \sum_j \tilde{z}_{lj} \delta(x_1 - x_j) \quad (8)$$

$$\tilde{g}_s(x_1 | x'_1) = \sum_r \sum_j \tilde{g}(x_1 | x_r) S_{rj} \tilde{g}(x_j | x'_1) \quad (9)$$

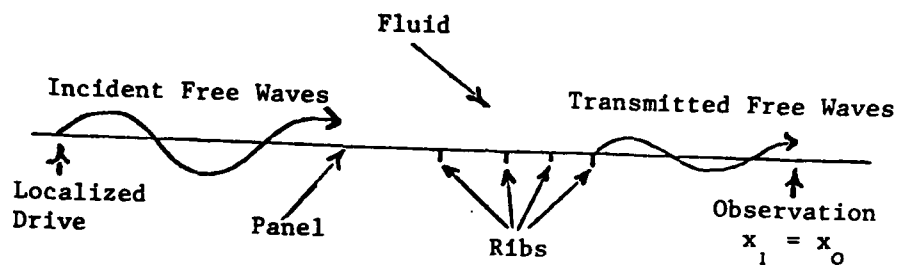
$$S_{rj} = \tilde{z}_{lr} H_{rj} \quad ; \quad H = (E)^{-1} \quad ; \quad E = \left( \delta_{ki} + \tilde{z}_{li} \tilde{g}(x_k | x_i) \right) \quad (10)$$

$$\tilde{g}_T(x_1 | x'_1) = \tilde{g}(x_1 | x'_1) - \sum_r \sum_j \tilde{g}(x_1 | x_r) S_{rj} \tilde{g}(x_j | x'_1) \quad (11)$$

This is a complete solution to the response of the ribbed membranelike panel

$$\tilde{v}(x_1) = \int dx'_1 \tilde{g}_T(x_1 | x'_1) p_e(x'_1)$$

Slide 4



$$\tilde{p}_e(x_1) = \tilde{p}_o \delta(x_1 - x_f)$$

$$\begin{aligned} \tilde{T} &= 1 - [\tilde{g}_s(x_o | x_f) / \tilde{g}(x_o | x_f)] \\ &= 1 - \sum_r \sum_j \tilde{g}(x_o | x_r) S_{rj} \tilde{g}(x_j | x_f) / \tilde{g}(x_o | x_f) \end{aligned}$$

$$\text{Assuming: } \tilde{g}(x_1 | x_1') = \bar{g}(\infty) \exp[-ik_s |x_1 - x_1'|]$$

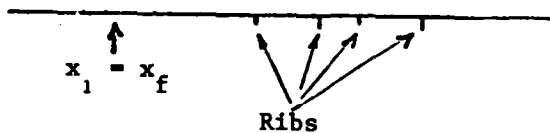
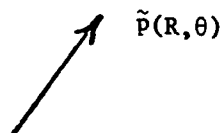
for  $|k_s(x_1 - x_1')| \gg 1$  ;  $|k_s|x_f \ll |k_s|x_r \ll |k_s|x_o$ , then

$$\tilde{T} = 1 - [\bar{g}(\infty) / \bar{g}(0)] \sum_r \sum_j [\bar{g}(0) \bar{S}_{rj}]$$

$$\bar{S}_{rj} = S_{rj} \exp[-ik_s(x_j - x_r)]$$

Slide 5

$$(R\omega/c) \gg 1$$



$$[\tilde{p}_l(R, \theta) / \tilde{p}_o(R, \theta)] = F(R, \theta)$$

$$F(R, \theta) = 1 - \sum_r \sum_j \exp[ik_{01}(x_r - x_f)] S_{rj} \tilde{g}(x_j | x_f)$$

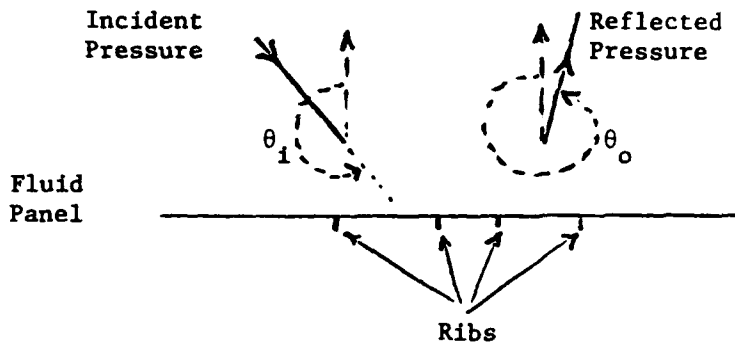
where  $k_{01} = (\omega/c) \sin(\theta)$       Assuming:  $|k_s|_{x_f} \ll |k_s|_{x_r}$

$$F(R, \theta) = 1 - [\bar{g}(\infty) / g(0)] \sum_r \sum_j [\tilde{g}(0) \bar{S}_{rj}]$$

$$\bar{S}_{rj} = S_{rj} \exp[ik_{01}(x_r - x_f) - ik_s(x_j - x_f)]$$

Slide 6





Reflection Due to Ribs [Largely Nonspecular Reflection  
Coefficient  $R_s(\theta_o|\theta_i)$ ]

$$R_s(\theta_o|\theta_i) = [Z_a(k_{o1})/\tilde{g}(0)] G(k_{o1}) G(k_{i1}) \sum_r \sum_j \tilde{g}(0) \tilde{S}_{rj}$$

$$\tilde{S}_{rj} = S_{rj} \exp[i(k_{o1} x_r + k_{i1} x_j)]$$

$$k_{o1} = (\omega/c) \sin(\theta_o) \quad k_{i1} = (\omega/c) \sin(\theta_i)$$

$$Z_a(k_{o1}) = \rho c / \cos(\theta_o) \quad ; \quad G(k_1) = (2\pi)^{-\frac{1}{2}} \int dx_1 \tilde{g}(x_1) \exp(ik_1 x_1)$$

Slide 7

EXAMPLE: Computing the Transmission Coefficient

$$\tilde{T} = 1 - AB \quad ; \quad A = \sum_r \sum_j \tilde{g}(0) S_{rj} \quad ; \quad B = g(\infty)/\tilde{g}(0)$$

Single Rib; rib  $\alpha$

$$A_\alpha = \tilde{z}_{l\alpha} \tilde{g}(0) [1 + \tilde{z}_{l\alpha} \tilde{g}(0)]^{-1}$$

Two Ribs; rib  $\alpha$  and rib  $\beta$

$$A_{\alpha\beta} = \{A_\alpha + A_\beta - 2A_\alpha A_\beta [\tilde{g}(x_\beta | x_\alpha) / \tilde{g}(0)] \cos[k_s(x_\beta - x_\alpha)]\} \\ \{1 - A_\alpha A_\beta [\tilde{g}(x_\beta | x_\alpha) / \tilde{g}(0)]^2\}^{-1}$$

First Order Value of  $A_{\alpha\beta}$ ; namely  $A_{\alpha\beta 0}$

$$A_{\alpha\beta 0} = A_\alpha + A_\beta - A_\alpha A_\beta B$$

$$\tilde{T}_{\alpha\beta 0} = (1 - A_\alpha B)(1 - A_\beta B) = \tilde{T}_\alpha \tilde{T}_\beta$$

Slide 8

EFFECTS OF FLUID LOADING ON THE TRANSMISSION  
OF FREE WAVES ACROSS A RIB

ABSTRACT

The line drive admittance and the transfer admittance have been evaluated in closed form for a plate or membrane. The evaluation takes into account the influence of fluid loading. Both admittances are necessary for the evaluation of the transmission of free waves across a rib. In this calculation the rib is characterized by an impedance, and the influence of fluid loading can thus be ascertained. Computations illustrating this effect in a number of cases of interest are presented and discussed. In the case of the membrane the phenomenon associated with the critical frequency is introduced by assuming the tension to be frequency dependent.

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## TEXT

In previous papers [1,2] exact expressions have been given for the drive admittances (for both point and line force and line moment excitation) of thin elastic plates under fluid loading. Analytical approximations were also given for those admittances at very low frequencies, these revealing the surprising result that the fluid acts as a stiffness for the point admittance, but as a mass for the line admittance. The present paper aims to take these analytical studies very much further, exemplifying the results by application to the problem of free surface wave transmission across a rib of arbitrary impedance on a panel though there are many other problems to which analytical expressions for the admittances could be applied.

We present results here only for a surface formed by a membrane for which the admittances are scalar quantities. The dispersive effects characteristic of plate dynamics are simulated by making the membrane tension vary appropriately with frequency. Now the dimensionless admittances are functions only of a frequency ratio

$$\Omega = \omega/\omega_g$$

and of a fluid loading parameter

$$\epsilon = \rho c/m\omega_g \quad ,$$

the surface specific mass  $m$  and the coincidence frequency  $\omega_g$  being assignable quantities. In almost all situations, the typical value of  $\epsilon$  is small, while values of  $\Omega$  of interest range from close to zero to unity and beyond. We take advantage of the assumed smallness of  $\epsilon$  to approximate the

admittances, but find that a singular perturbation problem results. No single approximation is valid over the whole frequency range of interest, and we have, in fact, to devise six different approximations to cover the whole range. To ensure that all frequencies are actually covered, we check that the various approximations overlap with each other, and that where they overlap they match one another in accordance with the asymptotic matching principle [3]. This insistence on matching serves as an essential check, not only on the consistency of our approximation procedure, but as an invaluable check on the algebraic detail of our working.

Slide 1 gives a definition sketch of the configuration envisaged, and defines the line drive admittance and line transfer admittance, and the transmission coefficient for normally incident surface waves on a rib of arbitrary impedance  $z_\ell$ . Slide 2 gives the exact expressions for the line and transfer admittances, and gives their dimensionless forms in terms of  $\Omega, \epsilon$ . We note that what is involved is simply the evaluation of the zeros  $\sigma_1, \sigma_2, \sigma_3$  of the cubic in  $(\sigma^2)$ ,  $P(\sigma) = 0$ , which lie in the upper half plane. It is then merely a matter of (excessively lengthy and tedious) algebra to find expansions for the admittances as  $\epsilon \rightarrow 0$ .

We start in Slide 3 by looking at fixed values of  $\Omega$ ,  $0 < \Omega < 1$ . The principal result quoted is that for the transmission coefficient across a rib of infinite mechanical impedance. This result is good if  $\Omega$  is not close to 0 or 1, and agrees with previous numerical studies. Note that  $T_\Omega = O(\epsilon)$ , i.e., the transmission is small unless  $\Omega$  is small, or close to unity.

Slide 4 shows the corresponding result for fixed values of  $\Omega > 1$ . The result is not valid if  $\Omega$  is very close to 1, but appears to hold up to

indefinitely large values of  $\Omega$ . It indicates that transmission is essentially perfect at all frequencies above coincidence.

Slide 5 shows how the previous results for  $\Omega < 1$  and  $\Omega > 1$  are smoothly joined by an expansion in the region  $[1-\Omega] = O(\epsilon^{2/3})$ . The formula for  $T$  increases smoothly from values  $O(\epsilon)$  when  $\Omega < 1$  towards the value 1 as  $\Omega$  increases through 1. Note that  $T$  has the value  $1/3$  at  $\Omega = 1$ .

To deal with low frequencies we need several expansions. We designate frequencies  $\Omega = O(\epsilon)$  as "intermediate," and give in Slide 6 the transmission coefficient for such frequencies. Thus  $T_{\Delta}$  matches the previously obtained  $T_{\Omega}$  in an appropriate way. Note that  $T_{\Delta}$  is clearly invalid at the still lower frequencies (where it predicts  $T_{\Delta} \rightarrow \infty$ ) and this points to the need for a further expansion for very low frequencies. Note also that for  $\Omega = O(\epsilon)$ ,  $T_{\Delta} = O(\epsilon^{1/2})$  so that the transmission coefficient is increasing as the frequency decreases.

The low frequency region is characterized by  $\Omega = O(\epsilon^2)$ ; here the transmission is  $O(1)$ , and in fact is equal to  $1/2$  at zero frequency. There is again perfect matching of the low frequency  $T_{\omega}$  to the intermediate frequency  $T_{\Delta}$ . Slide 7 gives some brief indication of the results, though the details are complicated.

One final nonuniformity remains to be corrected. That occurs around  $\Omega = (27/4) \epsilon^2$ , where we find that our low frequency approximations all break down. A separate analysis of the region is necessary, and the outcome is that the drive admittance becomes very large here, while the transfer admittance remains finite. Consequently, the free wave transmission coefficient rises essentially to the value 1 in a narrow region around this particular frequency. Numerical studies have so far not shown this

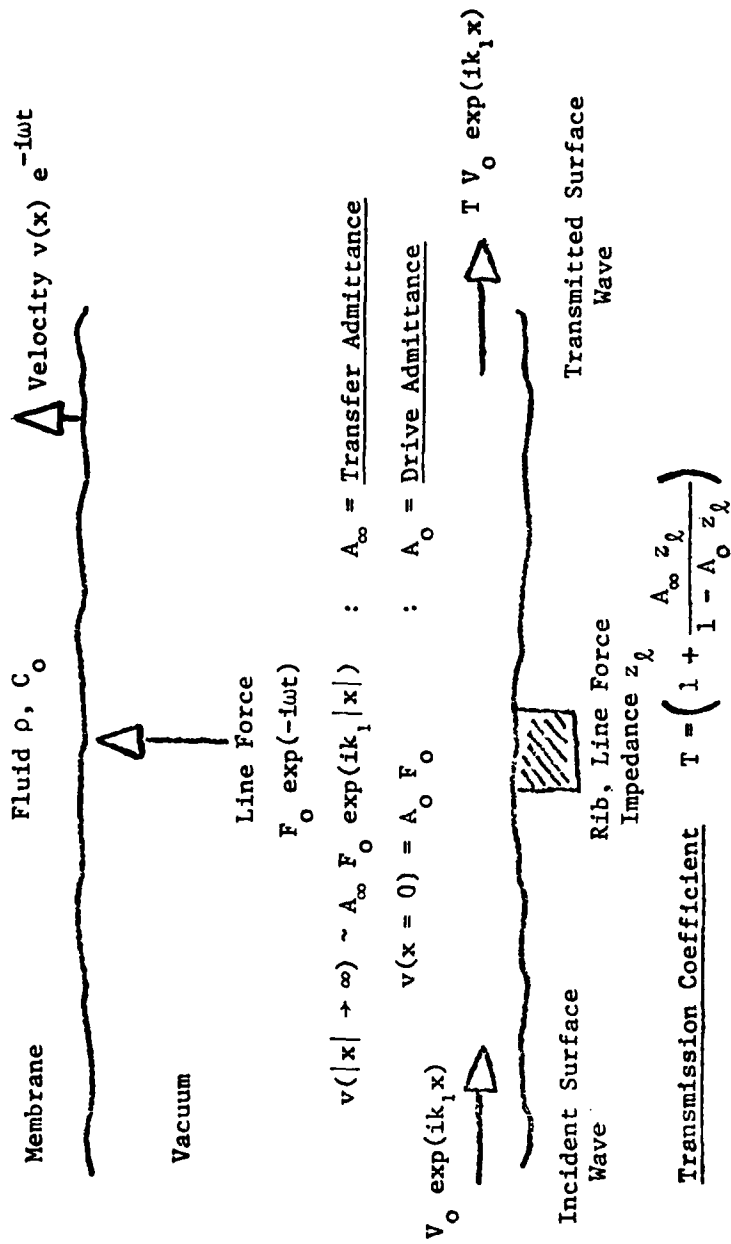
feature. We believe that is because of the presence of logarithmic terms ( $\ln \epsilon$ ) which only become dominant (and thus lead to  $T \approx 1$ ) when  $\epsilon$  is quite extraordinarily small;  $\epsilon = 10^{-10}$  might perhaps be small enough, whereas the typical value  $\epsilon = 10^{-1}$  used in numerical studies is certainly not small enough. Some confirmation of our predictions is, however, to be found in a low frequency bump in Figure 2 of reference [4]; the center of the bump is precisely at  $\Omega^{1/2} = 0.26$ , which indeed corresponds to  $\Omega = (27/4) \epsilon^2$  when  $\epsilon = 10^{-1}$ , as in [4].

Slide 8 sums up the results of these analytical predictions in a graphical plot of  $T$  vs  $\Omega$  for a typical small value of  $\epsilon$ , while Slide 9 attempts to pin down the various physical processes that are dominant in each of the different frequency ranges.

These analytical results are significantly different from published numerical results [4] when  $\Omega > 1$ , and they are also of interest in displaying the remarkably intricate mathematical structure of coupled acoustic waves-surface wave problems. It is only on the basis of results such as those given here that one can hope to build up a physical appreciation of such problems.

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- [4] G. Maidanik, A.J. Tucker, and W.H. Vogel (1976) J. Sound Vib. 49, 445.



Slide 1



EXACT  
EXPRESSIONS

$$A_{\infty} = \frac{\omega}{T} 2\mu k_m^2 \alpha_1 \gamma_1$$

$$A_0 = \frac{\omega}{T} \sum_{n=1}^3 \alpha_n \left\{ (k_n^2 - k_m^2)(k_n^2 - k_0^2) + \frac{2i}{\pi} \mu k_m^2 \gamma_n \ln \left( \frac{k_n + \gamma_n}{k_0} \right) \right\}$$

$$k_0 = \omega/c_0, k_m = (m\omega^2/T)^{\frac{1}{2}}, \quad \mu = \rho/m, \quad \gamma = (k^2 - k_0^2)^{\frac{1}{2}}$$

$k_1, k_2, k_3$  roots in upper half-plane of

$$P(k) = (k^2 - k_m^2)^2 (k^2 - k_0^2) - \mu^2 k_m^4 = 0$$

$$\text{and } \alpha_n^{-1} = P'(k_n)$$

DIMENSIONLESS VARIABLES     $\Omega = \omega/\omega_g, \quad \epsilon = \rho c_0/m\omega_g$

$$A_{\infty} = \beta_1 (\sigma_1^2 - \Omega)^{\frac{1}{2}}, \quad A_0 = \sum_{n=1}^3 \beta_n \left\{ (\sigma_n^2 - 1)(\sigma_n^2 - \Omega) + \frac{2i}{\pi} \frac{\epsilon}{\Omega^{\frac{1}{2}}} (\sigma_n^2 - \Omega)^{\frac{1}{2}} \ln \left( \frac{\sigma_n + (\sigma_n^2 - \Omega)^{\frac{1}{2}}}{\Omega^{\frac{1}{2}}} \right) \right\}$$

$$\beta_n^{-1} = P'(\sigma_n) \text{ and}$$

$$P(\sigma) \equiv (\sigma^2 - 1)^2 (\sigma^2 - \Omega) - \epsilon^2/\Omega$$

$$\underline{0 < \Omega < 1, \quad \epsilon \rightarrow 0}$$

$$T_{\Omega} = \epsilon \left( \frac{2 - \Omega}{2\Omega^{\frac{1}{2}} (1 - \Omega)^{3/2}} \right) \left\{ 1 + \frac{2i}{\pi} \frac{(1 - \Omega)^{\frac{1}{2}}}{(2 - \Omega)} \right. \\ \left. - \frac{2i}{\pi} \ln \left( \frac{1 + (1 - \Omega)^{\frac{1}{2}}}{\Omega^{\frac{1}{2}}} \right) \right\} + O(\epsilon^2)$$

N.B. (i)  $T_{\Omega} = O(\epsilon)$  SMALL

$$(ii) T_{\Omega} \sim \frac{\epsilon}{\Omega^{\frac{1}{2}}} \left( 1 + \frac{i}{\pi} - \frac{2i}{\pi} \ln \frac{2}{\Omega^{\frac{1}{2}}} \right) \rightarrow \infty \text{ As } \Omega \rightarrow 0$$

$$(iii) T_{\Omega} \sim \frac{\epsilon}{2(1 - \Omega)^{3/2}} \rightarrow \infty \text{ As } \Omega \rightarrow 1$$

Slide 3

$$\underline{1 < \Omega, \epsilon \rightarrow 0}$$

$$A_0 = \frac{1}{2} - \frac{\epsilon \Omega^{\frac{1}{2}}}{2\pi(\Omega-1)^{3/2}} \arctan(\Omega-1)^{\frac{1}{2}} - \frac{i\epsilon}{2\pi\Omega^{\frac{1}{2}}(\Omega-1)} + O(\epsilon^3)$$

$$A_\infty = \frac{\epsilon}{2\Omega(\Omega-1)^3} + O(\epsilon^3)$$

$$T_\Omega = 1 - \frac{2\epsilon^2}{\Omega^{3/2}(\Omega-1)^3} + O(\epsilon^3)$$

N.B. (i)  $T_\Omega \sim 1$  For ALL  $\Omega > 1$

(ii)  $T_\Omega \rightarrow \infty$  As  $\Omega \rightarrow 1$

Slide 4

STRUCTURE NEAR COINCIDENCE,  $|1 - \Omega| = 0(\epsilon^{2/3})$

$$1 - \Omega = \epsilon^{2/3} \lambda, \quad \lambda \text{ fixed, } \epsilon \rightarrow 0$$

$$T_\lambda = \frac{p_1(\lambda)}{(3p_1(\lambda) + 2\lambda)} + 0(\epsilon^{2/3})$$

where  $p_1$  is the real positive root of

$$p^3 + \lambda p^2 - 1 = 0$$

$$(p_1 = -\frac{\lambda}{3} + \frac{2\lambda}{3} \cos[1/3 \arccos(\frac{27}{2\lambda^3} - 1)]) \text{ when } \lambda > 0$$

N.B. (i)  $T_\lambda \rightarrow \frac{\epsilon}{2(1-\Omega)^{3/2}}$  As  $\lambda \rightarrow +\infty$  (matching with  $T_\Omega$  for  $\Omega < 1$ )

(ii)  $T_\lambda \rightarrow 1$  as  $\lambda \rightarrow -\infty$  (matching with  $T_\Omega$  for  $\Omega > 1$ )

(iii)  $T_\lambda = 0(1)$  significant when  $|1 - \Omega| = 0(\epsilon^{2/3})$

(iv)  $T_\lambda = 1/3$  at coincidence,  $\Omega = 1$ .

INTERMEDIATE FREQUENCIES  $\Omega = O(\epsilon)$

$\Omega = \epsilon\Delta$ ,  $\Delta$  Fixed,  $\epsilon \rightarrow 0$

$$T_{\Delta} = \frac{-\epsilon^{\frac{1}{2}}}{\Delta^{\frac{1}{2}}(1 + \Delta^2)^{\frac{1}{2}}} + \frac{\epsilon^{\frac{1}{2}}}{\Delta^{\frac{1}{2}}} + \frac{i\epsilon^{\frac{1}{2}}}{\pi\Delta^{\frac{1}{2}}} \left\{ 1 - 2\delta n \left( \frac{2}{(\epsilon\Delta)^{\frac{1}{2}}} \right) + \frac{2}{(1 + \Delta^2)^{\frac{1}{2}}} \delta n \left( \frac{(1 + (1 + \Delta^2)^{\frac{1}{2}})}{\Delta} \right) \right\} + O(\epsilon)$$

N.B. (i)  $T_{\Delta}$  matches  $T_{\Omega}$ .

(ii)  $T_{\Delta} = O(\epsilon^{\frac{1}{2}})$ , (larger than  $T_{\Omega} = O(\epsilon)$ )

(iii)  $T_{\Delta} \rightarrow \infty$  as  $\Delta \rightarrow 0$

LOW FREQUENCY ASYMPTOTICS  $\Omega = O(\epsilon^2)$

$\Omega = \epsilon^2 \tilde{\omega}$ ,  $\tilde{\omega}$  Fixed,  $\epsilon \rightarrow 0$

$$T_{\tilde{\omega}} = \frac{S}{S + \frac{i\pi}{p_1(3p_1 + 2)}} + O(\epsilon^2)$$

with  $S = \sum_{n=1}^3 \frac{k_n |p_n|}{p_n(3p_n + 2)}$ ,  $p^3 + p^2 - \frac{1}{\tilde{\omega}} = 0$ ,  $p_1 > 0$

N.B.

(i)  $T_{\tilde{\omega}}$  matches  $T_{\Delta}$

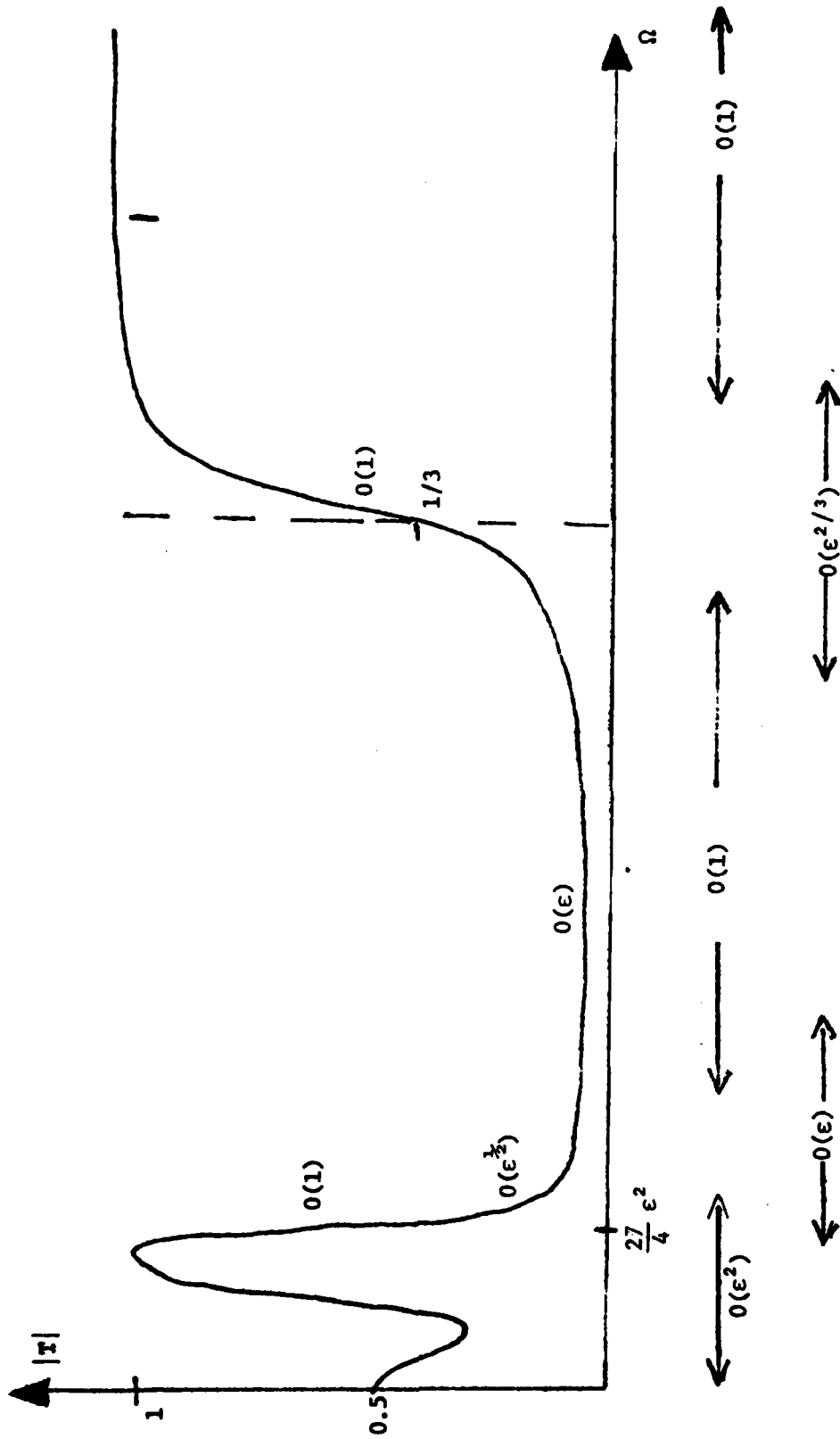
(ii) Expansion looks good down to zero frequency

(iii)  $T_{\tilde{\omega}} \rightarrow \frac{1}{2} e^{-\pi/3}$  as  $\Omega \rightarrow 0$

(iv)  $T_{\tilde{\omega}} = O(1)$  substantial transmission at frequencies  $O(\epsilon^2)$

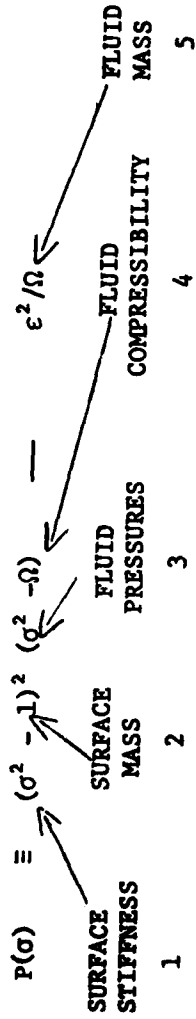
(v) Breakdown near  $\tilde{\omega} = 27/4$

Slide 7



AMPLITUDE OF TRANSMISSION COEFFICIENT FOR  $\epsilon \ll 1$ .

MECHANISMS RETAINED IN DIFFERENT RANGES



I.  $\Omega > 1$   $\sigma_1 : (3) + (4)$

$\sigma_2, \sigma_3 : (1) + (2)$

II.  $|1 - \Omega| = O(\epsilon^{2/3})$   
 $\sigma_1, \sigma_2, \sigma_3 : (1) + (2) + (3) + (4) + (5)$

III.  $0 < \Omega < 1$

$\sigma_1 : (1) + (2)$

$\sigma_2 : (1) + (2)$

$\sigma_3 : (3) + (4)$

IV.  $\Omega = O(\epsilon)$   $\sigma_1 : (1) + (2)$

$\sigma_2 : (1) + (2)$

$\sigma_3 : (3) + (4)$

V.  $\Omega = O(\epsilon^2)$   $\sigma_1, \sigma_2, \sigma_3 : (1) + (2) + (3) + (5)$

VI.  $\dot{\omega} \approx 27/4$   $\sigma_1, \sigma_2, \sigma_3 : (1) + (2) + (3) + (5)$

VII.  $\Omega \rightarrow 0$   $\sigma_1, \sigma_2, \sigma_3 : (1) + (3) + (5)$



EFFECTS OF FLUID LOADING ON THE TRANSMISSION OF  
FREE WAVES ACROSS TWO RIBS

ABSTRACT

The effects of fluid loading on the transmission of free waves across a single rib have been considered in the preceding paper. In this paper the transmission of free waves across two parallel ribs is considered. If the interaction between the two ribs is, or can be ignored, the evaluation of the transmission can be readily deduced from that of the single rib. However, of particular interest in this paper are the conditions concerning the characteristics of the ribs, the panel, and the fluid loading under which the interaction between the ribs is significant. Some computations illustrating this significance are presented.

## TEXT

In the preceding paper the transmission across a single rib was considered as an example of the response of a locally constrained fluid loaded panel to mechanical excitation. In this paper the consideration is briefly extended to a situation in which two parallel ribs are placed on an infinite panel. The analysis is limited to panels which are in the form of membranes, and tensions are limited to those below the critical tension; the equivalent thin plate frequency range is then limited to that below the critical frequency. This limitation is imposed to avoid discussing issues which arise from the conversion of one form of wave into another; the discussion of such issues is to be given in subsequent presentations.

The dynamic system under consideration is depicted on Slide 1. Also shown on this slide is the general expression for the transmission coefficient across parallel ribs on a fluid loaded membrane. On Slide 2 is shown the transmission coefficient across a single rib and two ribs. [See Slide 8 of the first paper.] In the preceding paper  $g^{(\infty)}$  and  $g(0)$  were evaluated. It is seen from Slide 2 that in the case of a single rib these evaluations are sufficient for the evaluation of the transmission coefficient, either  $\tilde{T}_\alpha$  or  $\tilde{T}_\beta$ . However, it is seen from Slide 2 that in the case of two (or more) ribs the transfer admittances between ribs on the membrane must also be evaluated. There are, nonetheless, two limiting situations in which the need to evaluate these transfer admittances can be dispensed with. On Slide 3 the first of these two situations is considered. It is assumed that the two ribs do not interact with each other so that first order solution is sought [1,2]. The conditions for the noninteraction are stated on top of Slide 3. Under these conditions the transmission

coefficient is simply related to the product of the transmission coefficient of each of the ribs separately, as could be anticipated. On Slide 4 the second of the two situations is considered. It is assumed that the two ribs are substantially coalescing. The conditions for the coalescence are stated on top of Slide 4. The transmission coefficient for this situation is stated on Slide 4. Conditions which are less restrictive than those just considered are stated on Slide 5. In this situation, interaction between the ribs is allowed; however, the ribs are assumed to be well separated,  $|k_s(x_\beta - x_\alpha)| \gg 1$ . If then, the transmission coefficient of each rib alone is small compared with unity, conditions may be prescribed for resonances and antiresonances in the transmission coefficient of the the two ribs system. These conditions are stated on Slide 5. The peaks correspond to the resonances in the transmission coefficient and the valleys correspond to the antiresonances in the transmission coefficient; the standard for the transmission coefficient in this consideration is the first order solution for this quantity. See Slide 5. From the preceding paper it can be deduced that for reasonable fluid loading

$$\epsilon = \rho c / m \omega_g < 0.2 ,$$

and ribs of high line impedances

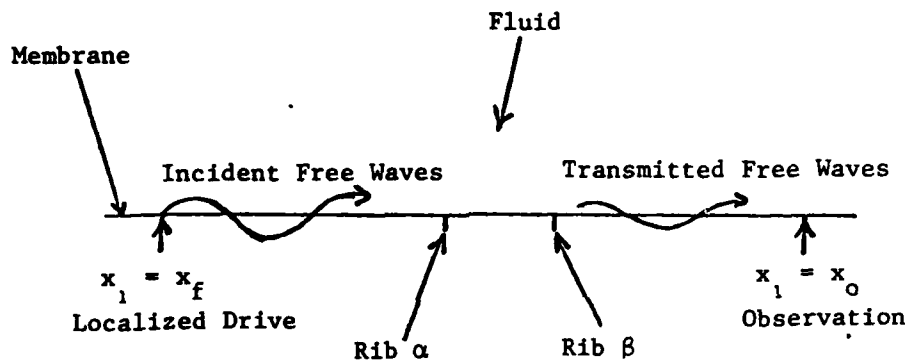
$$|\tilde{z}_{\ell j} \tilde{g}(0)| \gg 1 ,$$

the conditions for resonances and antiresonances are ripe. In the range of tension (or equivalently, frequency) under consideration,  $0(\epsilon) < \Omega = (\omega / \omega_g) < 1$ , the transmission coefficients can be approximated in accordance with the preceding paper in the manner indicated on Slide 6. In the specified range the transmission coefficients at the peaks and

valleys can be readily approximated; the approximate values are stated on Slide 6. Of particular interest is the fact that at the peaks the transmission coefficient is half that of the transmission coefficient of a single rib and at the valleys the transmission coefficient is half that of the first order solution of the two ribs system. The transmission coefficient oscillates between these two extreme values as tension (or equivalently, frequency) is monotonically changed.

#### REFERENCES

- [1] G. Maidanik (1976) J. Sound Vib. 44, 255.
- [2] G. Maidanik and A.J. Tucker (1976) J. Sound Vib. 44, 267.



$$\bar{T} = 1 - [\bar{g}(\infty)/\bar{g}(0)] \sum_r \sum_j [\bar{g}(0) \bar{S}_{rj}]$$

$$\bar{S}_{rj} = S_{rj} \exp[-ik_s(x_j - x_r)]$$

$$S_{rj} = \bar{z}_{\ell r} H_{rj} \quad ; \quad H_{rj} = E^{-1} \quad ;$$

$$E = \left( \delta_{ki} + \bar{z}_{\ell i} \bar{g}(x_k | x_i) \right)$$

Slide 1

Rib  $\alpha$  only

$$\tilde{T}_\alpha = 1 - A_\alpha B \quad ; \quad B = \bar{g}(\infty)/\tilde{g}(0)$$

$$A_\alpha = \tilde{z}_{\ell\alpha} \tilde{g}(0) [1 + \tilde{z}_{\ell\alpha} \tilde{g}(0)]^{-1}$$

Rib  $\beta$  only

$$\tilde{T}_\beta = 1 - A_\beta B \quad ; \quad A_\beta = \tilde{z}_{\ell\beta} \tilde{g}(0) [1 + \tilde{z}_{\ell\beta} \tilde{g}(0)]^{-1}$$

Rib  $\alpha$  and Rib  $\beta$

$$\tilde{T}_{\alpha\beta} = 1 - A_{\alpha\beta} B$$

$$A_{\alpha\beta} = \{A_\alpha + A_\beta - 2A_\alpha A_\beta [\tilde{g}(x_\beta|x_\alpha)/\tilde{g}(0)] \cos k_s(x_\beta - x_\alpha)\} \\ \{1 - A_\alpha A_\beta [\tilde{g}(x_\beta|x_\alpha)/\tilde{g}(0)]^2\}^{-1}$$

.. Slide 2

Noninteracting Ribs

$$\tilde{g}(x_\beta | x_\alpha) / \tilde{g}(0) \Rightarrow [\bar{g}(\infty) / \tilde{g}(0)] [\exp\{-ik_s |x_\beta - x_\alpha|\}]$$

$$\text{with } |x_\beta - x_\alpha| \Rightarrow \infty \text{ and Imaginary } (k_s) \neq 0$$

Under these conditions

$$A_{\alpha\beta 0} \Rightarrow A_\alpha + A_\beta - A_\alpha A_\beta B$$

$$A_\alpha = \tilde{z}_{\ell\alpha} \tilde{g}(0) [1 + \tilde{z}_{\ell\alpha} \tilde{g}(0)]^{-1}$$

$$A_\beta = \tilde{z}_{\ell\beta} \tilde{g}(0) [1 + \tilde{z}_{\ell\beta} \tilde{g}(0)]^{-1}$$

$$B = \bar{g}(\infty) \tilde{g}(0)$$

$$\begin{aligned} \tilde{T}_{\alpha\beta 0} &= 1 - A_{\alpha\beta} B = 1 - A_\alpha B - A_\beta B + A_\alpha A_\beta B^2 \\ &= (1 - A_\alpha B)(1 - A_\beta B) = \tilde{T}_\alpha \tilde{T}_\beta \end{aligned}$$

Slide 3

Coalescing Ribs

$$\tilde{g}(x_\beta | x_\alpha) / \tilde{g}(0) \Rightarrow 1$$

Under these conditions

$$\begin{aligned} A_{\alpha\beta c} &= (A_\alpha + A_\beta - 2A_\alpha A_\beta) (1 - A_\alpha A_\beta)^{-1} \\ &= (\tilde{z}_{\ell\alpha} + \tilde{z}_{\ell\beta}) \tilde{g}(0) [1 + (\tilde{z}_{\ell\alpha} + \tilde{z}_{\ell\beta}) \tilde{g}(0)]^{-1} \\ &= A_{(\alpha+\beta)} \end{aligned}$$

$$\tilde{T}_{\alpha\beta c} = 1 - A_{(\alpha+\beta)} B$$

Slide 4



$$\tilde{g}(x_\beta | x_\alpha) / \tilde{g}(0) \rightarrow [\bar{g}(\infty) / \tilde{g}(0)] [\exp\{-ik_s |x_\beta - x_\alpha|\}]$$

Under these conditions

$$A_{\alpha\beta} = \{A_{\alpha\beta 0} - A_{\alpha\beta} B \exp[-2ik_s |x_\beta - x_\alpha|]\}$$

$$\{1 - A_{\alpha\beta} B^2 \exp[-2ik_s |x_\beta - x_\alpha|]\}^{-1}$$

$$\tilde{T}_{\alpha\beta} = \tilde{T}_{\alpha\beta 0} \{1 - A_{\alpha\beta} B^2 \exp[-2ik_s |x_\beta - x_\alpha|]\}^{-1} ; \quad \tilde{T}_{\alpha\beta 0} = \tilde{T}_\alpha \tilde{T}_\beta$$

Peaks occur

$$A_{\alpha\beta} B^2 \rightarrow 1 \quad ; \quad \exp[-2ik_s |x_\beta - x_\alpha|] \rightarrow 1$$

Valleys occur

$$A_{\alpha\beta} B^2 \rightarrow 1 \quad ; \quad \exp[-2ik_s |x_\beta - x_\alpha|] \rightarrow -1$$

Slide 5

In the frequency range

$$0(\epsilon) < \Omega = (\omega/\omega_g) < 1$$

where  $\epsilon$  is the fluid loading parameter at the critical frequency

$\epsilon = \rho c / m \omega_g$ . Assume that  $\epsilon \lesssim 0.2$  and  $A_\alpha \rightarrow 1, A_\beta \rightarrow 1$ . Under these conditions

$$\tilde{T}_\alpha = 0(\epsilon) \quad ; \quad \tilde{T}_\beta = 0(\epsilon) \quad ; \quad \tilde{T}_{\alpha\beta_0} = [0(\epsilon)]^2$$

$$\tilde{T}_{\alpha\beta} = [0(\epsilon)]^2 \{1 - [1 - 20(\epsilon)] \exp[2ik_s |x_\beta - x_\alpha|]\}^{-1}$$

At peaks

$$\tilde{T}_{\alpha\beta} \approx 0(\epsilon)/2$$

At Valleys

$$\tilde{T}_{\alpha\beta} \approx [0(\epsilon)]^2/2$$

Slide 6

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