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DIFFRACTION AT A SURFACE IMPEDANCE DISCONTINUITY - APPLICATION TO SEA-LAND RADIOWAVE PROPAGATION.

First Annual Technical Report

by

Giorgio Franceschetti and Vittorio G. Vaccaro (°)

September 1979

EUROPEAN RESEARCH OFFICE
United States Army
London England

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(°) Istituto Elettrotecnico, Università di Napoli, Italy

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The field produced by prescribed sources can be synthesized using a plane wave expansion and then applying results of the scattering analysis.

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For practical application to radiowave propagation over the Earth, the field close to the surface is of interest. This is represented as incident, reflected, surface and extra term field, this last characteristic of the surface impedance discontinuity. In addition, a cylindrical wave, scattered by the discontinuity is present.

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1. Introduction

Groundwave propagation is one of the oldest studied topics in applied electromagnetics. Several models of the propagation medium have been considered, e.g., flat or spherical Earth, homogeneous, stratified or anisotropic ground, corrugated surfaces, single or mixed path propagation. An excellent summary of problems and modern solutions to this subject is given in [1], which contains also a large number of references.

A canonical problem in mixed-path propagation is that of an abrupt discontinuity in ground parameters along a straight line, as depicted in Fig.1 : the ground exhibits different properties, e.g., conductivity and/or permittivity, for $y > 0$ and $y < 0$. The canonical problem is the following : for a given incident plane wave, compute the field at point P. If the plane wave solution is needed to synthesize the field produced by localized sources, the angle ϕ_o should be allowed to be complex, and the incidence should not be restricted to the normal case (two-dimensional problem).

A central role in the solution to this problem is played by the surface impedance concept, i.e., by definition, the ratio of the tangential components E_t and H_t of the fields at the interface $x = 0$. For the problem of an incident plane wave with a fixed angle ϕ_o, the use of an impedance type boundary condition is an exact approach. However, when using plane wave expansions (as in the solution of the just mentioned canonical problem) the use of an angle independent surface impedance implies an approximation to the exact boundary conditions.

A discussion about the validity of the surface impedance concept is given in [2] and, more recently, in [3]. We want also to mention a few of the excellent agreements between experimental and theoretical results using impedance type boundary conditions: see [4,5] for the case of scattering by a half-plane with two face impedances, and [6-8] for the case of laboratory models of groundwave propagation. Additional results are listed in [1]. Accordingly, the use of impedance type boundary conditions seems quite adequate to the case at hand.

The solution to the canonical problem depicted in Fig.1 is known essentially for the case of normal incidence

[9-17] ; when oblique incidence is considered, only an approximate matching of tangential components of fields at the impedance discontinuity line is provided. We note that the problem of scattering, under normal incidence, by a wedge of arbitrary angle with two different face impedances has been rigorously solved since 1958 by Maliuzhinets [20]. When the wedge angle is made equal to π , the problem of Fig.1 is recovered. Maliuzhinets solution has been recently extended to the case of oblique incidence, for the half-plane case [21] ; we are using essentially the same technique to construct the solution to the problem of Fig.1 for the case of oblique incidence.

Before considering the extension of Maliuzhinets' solution and its modifications, appropriate to groundwave propagation problems, it is worth remembering that an alternative convenient way to study mixed-path propagation problems is obtained by using the compensation theorem [22]. This procedure has been originally exploited by Wait [23-24] and then applied to oblique propagation of groundwaves across a coastline [25-27]. Extensive theoretical and experimental work, using essentially this procedure is referred to in [1]. Use of the compensation theorem is very attractive since it allows you to consider not only the simple canonical problem of Fig.1, but, in principle, any geometry of surface discontinuity, therefore rendering the procedure very powerful from the application viewpoint. However, the resulting integral equation should be solved numerically, even when some simplifying approximations are made. Accordingly the rigorous solution of the canonical problem, e.g., the one depicted in Fig.1, is worth a physical description of the refraction phenomena at the impedance discontinuity.

In the following we will present this solution for both normal and oblique incidence, using an uniform asymptotic evaluation of the diffraction integral in order to obtain a field description bounded everywhere. This is not a trivial problem when the observation point is close to the surface, and the incident wave is at grazing angle, since as many as three poles cluster around the "saddle point". The difficulty has been overcome using an "ad hoc" modified version of Bleistein 's procedure [31], thus obtaining pole type contributions in addition to the usual saddle-point contributions (cylindrical wave emanating from the surface impedance discontinuity). The pole-type contribution consists of the incident, reflected, surface and an extra-term field, this last characteristic of the surface impedance discontinuity.

The field produced by prescribed sources can be synthesyzed using a plane wave expansion and then applying results of the scattering analysis. Evaluation of the resulting integral is made difficult by the existence of a rapidly varying Fresnel integral in the integrand

2. Scattering under normal incidence. Application of Maliuzhinets' theory

Let us consider the geometry depicted in Fig.1, wherein a plane wave is normally incident at a (possibly complex) angle ϕ_o on a two region half-space. No z-variation of the field is assumed (two-dimensional problem). Accordingly, we can consider two (decoupled) types of field polarization- the E-type, wherein the only component of the electric field is $E_z(\rho,\phi)$, and the H-type, wherein the only component of the magnetic field is $H_z(\rho,\phi)$. In order to simplyfy the notation we will use the symbol F for the field assuming $F_e = E_z$, and $F_h = H_z$ and using the suscripts only whenever necessary.

We will assume the boundary conditions at $\phi = \pm\pi/2$ to be of impedance type, e.g.,

$$\frac{1}{\rho}\frac{\partial F}{\partial \phi} \mp ik \sin\theta^{\pm} F = 0 \qquad \text{at} \quad \phi = \pm\pi/2 \tag{2.1}$$

where:

$$\sin\theta_e^{\pm} = \zeta/Z^{\pm} \quad , \quad \sin\theta_h^{\pm} = \zeta/Z^{\pm} \tag{2.2}$$

$k = \omega\sqrt{\varepsilon\mu}$ and is the free-space propagation constant,
$\zeta = \sqrt{\mu/\varepsilon}$ is the free space intrinsic impedance;
$Z^{\pm}$ are the surface impedances at $\phi=\pm\pi/2$ respectively, and a time dependance $\exp(-i\omega t)$ is understood and suppressed.

Let us now express the field as a superposition of plane waves, hence :

$$F(\rho,\phi) = \frac{1}{2\pi i}\int_{\Gamma} \hat{F}(\rho+\phi)\exp(-ik\rho\cos\alpha)\, d\alpha \tag{2.3}$$

wherein Γ is the two-branch Sommerfeld contour of integration, as depicted in Fig 2. Forcing boundary conditions (2.1) in (2.3) gives, upon integration by parts,

$$\int_{\Gamma} (\sin\alpha \pm \sin\theta^{\pm})\hat{F}(\alpha \pm \pi/2)\exp(-ik\rho\cos\alpha)\,d\alpha = 0 \qquad (2.4)$$

A necessary and sufficient condition for the integral (2.4) to vanish is the integrand to be an even function of α, hence :

$$(\sin\alpha \pm \sin\theta^{\pm})\hat{F}(\alpha \pm \pi/2) = (-\sin\alpha \pm \sin\theta^{\pm})\hat{F}(-\alpha \pm \pi/2) \qquad (2.5)$$

due to the symmetry of the integration contour Γ . Eq. (2.5) can be solved by means of the method of logarithmic derivative and Fourier transform. Only the general solution is quoted hereafter, under the assumption that a single pole at $\alpha=\phi_o$ does exist in the strip $-\pi/2 < \alpha' < \pi/2$:

$$\hat{F}(\alpha) = \left[\frac{A}{\sin\alpha - \sin\phi_o} + \sum_{n=0}^{\infty} B_n \sin^n\alpha\right]\psi(\alpha) \qquad (2.6)$$

wherein A and B_n are arbitrary constants and the function $\psi(\alpha)$ is introduced and discussed in Appendix A.

As discussed in [29], the value of the field F for $\rho \longrightarrow 0$ is related to the asymptotic behaviour of (2.6) as $|\alpha''| \longrightarrow \infty$ In particular , $F(\rho,\phi)$ is regular for $\rho \longrightarrow 0$ if $\hat{F}(\alpha)$ is bounded for $|\alpha''| \longrightarrow \infty$. Since from results of Appendix A

$$\psi(\alpha) \sim O(\exp(|\alpha''|)) \qquad (2.7)$$

as $|\alpha''| \longrightarrow \infty$ it follows that $B_n = 0$, thus [20] :

$$\hat{F}(\alpha) = F_o^i \frac{\cos\phi_o}{\sin\alpha - \sin\phi_o} \frac{\psi(\alpha)}{\psi(\phi_o)} \qquad (2.8)$$

wherein F_o^i is a constant (corresponding to the intensity of the incident field at $\rho=0$, see (2.10)).

The integral (2.3) shows two saddle-points at $\alpha_s = \pm\pi$. For an asymptotic evaluation of the integral for $k\rho \longrightarrow \infty$ is therefore convenient to deform the original integration contour Γ into the two-branch steepest-descent path γ (see Fig.2) :

$$\alpha' = \pm\pi - sg(\alpha'')\cos^{-1}\left(\frac{1}{\cosh\alpha''}\right) \qquad (2.9)$$

then :

$$F(\rho,\phi) = \frac{F_o^i}{2\pi i} \int_\gamma \frac{\cos\phi_o}{\sin(\alpha+\phi)-\sin\phi_o} \frac{\psi(\alpha+\phi)}{\psi(\phi_o)} \exp(-ik\rho\cos\alpha)d\alpha$$

$$+ \sum (\text{Residues of } \hat{F}(\alpha+\phi) \text{ in } \Gamma-\gamma\ ) \qquad (2.10)$$

Eq. (2.10) provides the (Maliuzhinets) solution to the problem of the plane wave normal incidence of the scattering geometry depicted in Fig.1. Discussion about the location of poles, and evaluation of corresponding residues, is deferred to Appendix B. Hereafter only conclusions are summarized, with reference to radiowave propagation over the Earth, so that H polarized waves are of interest, and

$$\sin\theta_h = \frac{1}{\sqrt{\varepsilon_r + i\sigma/\omega\varepsilon_o}} \qquad (2.11)$$

wherein ε_r and σ are the appropriate relative permittivity and conductivity (different for $y > 0$ and $y < 0$) of the Earth.

With reference to Fig.3 (wherein the angle ϕ_o is assumed real, for simplicity), residues of $\hat{F}(\alpha+\phi_o)$ correspond to the incident wave

$$F^i(\rho,\phi) = F_o^i \exp(-ik\rho\cos(\phi-\phi_o)) \qquad (2.12)$$

the reflected wave for $y > 0$

$$F_+^r(\rho,\phi) = \frac{\cos\phi_o - \sin\theta^+}{\cos\phi_o + \sin\theta^+} F_o^i \exp(ik\rho\cos(\phi+\phi_o)) \qquad (2.13)$$

existing only for $\phi > \phi_o$; the reflected wave for $y > 0$

$$F_-^r(\rho,\phi) = \frac{\cos\phi_o - \sin\theta^-}{\cos\phi_o + \sin\theta^-} F_o^i \exp(ik\rho\cos(\phi+\phi_o)) \qquad (2.14)$$

existing only for $\phi < -\phi_o$ .The integrand, however, possesses other poles, which reside outside the closed contour $\Gamma-\gamma$ but can approach the saddle points at $\pm\pi$ so to influence the asymptotic evaluation of the integral in (2.10). This evaluation is deferred to in Sect. 4, after the solution (2.10) is extended to the oblique incidence case.

3. Scattering under oblique incidence. Extension of Maliuzhinets'theory.

Let us now extend the two-dimensional solution discussed under Sect.2 to the three dimensional problem of oblique incidence, wherein the incident wave is a plane wave of components of the type:

$$F(\rho,\phi,z) = F_o^i \exp(ikz\cos\beta)\exp(-ik\rho\cos(\phi-\phi_o)) \qquad (3.1)$$

depicted in Fig.4. Without loss of generality we will assume: $0 < \mathrm{Re}(\beta) \leq \pi/2$, $0 \leq \mathrm{Re}(\phi_o) \leq \pi/2$ and will drop the z-dependence for the field components.

Boundary conditions are now expressed as

$$\underline{E}-\hat{x}\hat{x}\cdot\underline{E} = Z^{\pm}\hat{x}\times\underline{H} \qquad \text{at } \phi=\pm\pi/2 \qquad (3.2)$$

or, equivalently,

$$\underline{H}-\hat{x}\hat{x}\cdot\underline{H} = (1/Z^{\pm})\underline{E}\times\hat{x} \qquad \text{at } \phi=\pm\pi/2 \qquad (3.3)$$

By taking the divergence of (3.2 and 3) and using Maxwell's equations, we get :

$$\begin{aligned} &\frac{1}{\rho}\frac{\partial E_x}{\partial\phi} \mp ik\sin\theta_h^{\pm} E_x = 0 \qquad \text{at } \phi=\pm\pi/2 \\ &\frac{1}{\rho}\frac{\partial H_x}{\partial\phi} \mp ik\sin\theta_{\bar{h}}^{\pm} H_x = 0 \qquad \text{at } \phi=\pm\pi/2 \end{aligned} \qquad (3.4)$$

respectively. Eq.s (3.4 and 5) are symilar to (2.1) and suggest that results of Sect. 2 could possibly be extended to the oblique incidence case by operating upon the x-components of the field. It will be shown in the following that this extension is not straightforward.

Let:

$$\left\{\begin{aligned} E_x(\rho,\phi) &= \frac{1}{2\pi i}\int_{\Gamma} \hat{E}_x(\alpha+\phi)\exp(-ik\rho\sin\beta\cos\alpha)d\alpha \\ H_x(\rho,\phi) &= \frac{1}{2\pi i}\int_{\Gamma} \hat{H}_x(\alpha+\phi)\exp(-ik\rho\sin\beta\cos\alpha)d\alpha \end{aligned}\right. \qquad (3.5)$$

Then, forcing boundary conditions (3.4) we get as in Sect.2

$$(\sin\alpha \pm \sin\theta_h^{\pm})\hat{E}_x(\alpha \pm \pi/2) = (-\sin\alpha \pm \sin\theta_h^{\pm})\hat{E}_x(-\alpha \pm \pi/2) \qquad (3.6a)$$

$$(\sin\alpha \pm \sin\theta_h^{\pm})\hat{H}_x(\alpha \pm \pi/2) = (-\sin\alpha \pm \sin\theta_h^{\pm})\hat{H}_x(-\alpha \pm \pi/2) \qquad (3.6b)$$

wherein, at variance of (2.2),

$$\sin\theta_h^{\pm} = \frac{Z^{\pm}}{\xi \sin\beta} \qquad \sin\theta_e^{\pm} = \frac{\xi}{Z^{\pm}\sin\beta} \qquad (3.7)$$

The general solution of (3.6a or b)under the condition that a single pole exists at $\alpha = \phi_o$ in the strip $-\pi/2 < < \alpha < \pi/2$ is the same as (2.6). However, E_x and H_x are no more necessarily regular for $\rho \longrightarrow 0$, so we cannot apply the arguments of Sect.2 . It is therefore convenient to express $\hat{E}_z$, $\hat{H}_z$ in terms of $\hat{E}_x$, $\hat{H}_x$ (for details, see Appendix C), hence:

$$\begin{cases} \hat{E}_z(\alpha) = \dfrac{\cos\beta\cos\alpha\hat{E}_x(\alpha) - \sin\alpha\,\zeta\hat{H}_x(\alpha)}{1 - \cos^2\alpha\sin^2\beta}\sin\beta \\[2ex] \zeta\hat{H}_z(\alpha) = \dfrac{\zeta\cos\beta\cos\alpha\hat{H}_x(\alpha) + \sin\alpha\,\hat{E}_x(\alpha)}{1 - \cos^2\alpha\sin^2\beta}\sin\beta \end{cases} \qquad (3.8)$$

Now, $(\hat{E}_z, \hat{H}_z)$ should approach a constant value, as $|\alpha''| \longrightarrow \infty$. Accordingly, from (2.6):

$$\begin{cases} \hat{E}_x(\alpha) = \left[\dfrac{E^i_{ox}\cos\phi_o}{\sin\alpha - \sin\phi_o} + B_h\right]\dfrac{\psi_h(\alpha)}{\psi_h(\phi_o)} \\[2ex] \zeta\hat{H}_x(\alpha) = \left[\dfrac{\zeta H^i_{ox}\cos\phi_o}{\sin\alpha - \sin\phi_o} + B_e\right]\dfrac{\psi_e(\alpha)}{\psi_e(\phi_o)} \end{cases} \qquad (3.9)$$

wherein E^i_{ox}, H^i_{ox}, B_h and B_e are constants, and $\psi_h(\alpha) = \psi(\alpha, \theta_h)$, $\psi_e = \psi(\alpha, \theta_e)$ (see Appendix A). (E^i_{ox}, E^i_{oy}) are the values of the x-components of the incident field at $\rho = 0$, and are easily computed forcing the residues of the spectral representation for (E_z, H_z) at $\rho = 0$ to coincide with the values of the incident components (E^i_{oz}, H^i_{oz}) at $\rho = 0$, hence:

$$\begin{cases} E^{i}_{ox} = \dfrac{E^{i}_{oz}\cos\beta\cos\phi_o + \zeta H^{i}_{oz}\sin\phi_o}{\sin\beta} \\ \zeta H^{i}_{ox} = \dfrac{\zeta H^{i}_{oz}\cos\beta\cos\phi_o - E^{i}_{oz}\sin\phi_o}{\sin\beta} \end{cases} \qquad (3.10)$$

For evaluating the remaining constants (B_e, B_h) we note that the denominator of (3.8) vanishes for

$$\cos^2\alpha = (\sin^2\beta)^{-1} \qquad (3.11)$$

For β real eq.(3.11) is verified for

$$\alpha = \alpha_n^{\pm} = n\pi \pm i \cosh^{-1}\left(\frac{1}{\sin\beta}\right) \qquad (3.12)$$

Solutions of (3.11) for complex β are provided in Appendix D, but they are not relevant to the discussion which follows.

When the integration path Γ of the integrals (3.5) is deformed onto the steepest descent path γ, the two poles at $\alpha = \alpha_o^{\pm} - \phi$ are crossed. The corresponding residues will describe (inhomogeneous) plane waves of the type

$$\exp(-ik\rho\cos\phi)\exp(\pm k\rho\sin\phi\cos\beta) \qquad (3.13)$$

It is evident that such two waves are diverging in $\phi_o \gtrless 0$

and therefore should be cancelled if the solution is required to be bounded everywhere in space. This can be accomplished by forcing the numerators of (3.8) to vanish for $\alpha_o = \alpha_o^{\pm}$, hence:

$$\hat{E}_x(\alpha_o^{\pm}) \pm i\zeta\hat{H}_x(\alpha_o^{\pm}) = 0 \qquad (3.14)$$

In deriving (3.14) note that

$$\sin\alpha_o^{\pm} = \pm i \operatorname{cotg}\beta \qquad (3.15)$$

as it follows (3.11)

Eqs (3.14) are a system of two equations in the two unknowns (B_e, B_h) and the solution is:

$$\left\{\begin{array}{l} B_h = \dfrac{\cos\phi_o \sin\beta}{1-\sin^2\beta\cos^2\phi_o} \cdot E^i_{ox}(\sin\phi_o\sin\beta - iQ\cos\beta) - H^i_{ox}S_e\cos\beta \\ B_e = \dfrac{\cos\phi_o \sin\beta}{1-\sin^2\beta\cos^2\phi_o} \cdot \mathcal{J}H^i_{ox}(\sin\phi_o\sin\beta + iQ\cos\beta) + E^i_{ox}S_h\cos\beta \end{array}\right. \quad (3.16)$$

wherein:

$$Q = \frac{\psi_e(\alpha_o^+)\psi_h(\alpha_o^-) - \psi_e(\alpha_o^-)\psi_h(\alpha_o^+)}{\psi_e(\alpha_o^+)\psi_h(\alpha_o^-) + \psi_e(\alpha_o^-)\psi_h(\alpha_o^+)} \quad (3.17)$$

$$S_e = \frac{2\,\psi_e(\alpha_o^+)\psi_e(\alpha_o^-)}{\psi_e(\alpha_o^+)\psi_h(\alpha_o^-) + \psi_e(\alpha_o^-)\psi_h(\alpha_o^+)} \quad (3.18)$$

$$S_h = \frac{2\,\psi_h(\alpha_o^+)\psi_h(\alpha_o^-)}{\psi_e(\alpha_o^+)\psi_h(\alpha_o^-) + \psi_e(\alpha_o^-)\psi_h(\alpha_o^+)} \quad (3.19)$$

4. Asymptotic evaluation of the field.

With reference to the two-dimensional case considered in Sect. 2, the explicit evaluation of the field requires computation of the integral which appears in (2.10). When $k\rho$ is large, asymptotic techniques can be applied.

In order to have a uniform asymptotic evaluation of the integral, care should be taken concerning the possible location of poles of the integrand nearby the two saddle points

$$\alpha_s = \pm\,\pi$$

Poles of the integrand do appear at (see Appendix B):

$$\alpha_n = -\phi + (-1)^n\phi_o + n\pi \quad , \quad n = 0;\pm 1;\pm 2;\ldots.$$

$$\alpha_\pm = -\phi \pm (3\pi/2 + \theta^\pm)$$

It is evident that $\alpha_1 \to \pi$ and $\alpha_{-1} \to -\pi$ when $\phi+\phi_o \to 0$.

Furthermore, when the incidence is at grazing angle, i.e., $\phi_o \sim \pi/2$, and the observation point is close to the surface $y < 0$ i.e., $\phi \sim \pi/2$, then $\alpha_0 \to \pi$, $\alpha_2 \to -\pi$.
Under these circunstances, and $\theta^- \to 0$, also $\alpha_- \to -\pi$.
Accordingly, the conclusion is drawn that, for H - polatized waves and radiowave propagation over the Earth surface, the

poles α_0,α_1 approach the saddle point $\alpha_s=\pi$, while the poles $\alpha_{-2},\alpha_{-1},\alpha_-$ cluster around the saddle point $\alpha_s=\pi$.

On the other hand, this is a very important situation for practical application, when both the source and the observation points are close to the Earth surface, for $y>0$ and $y<0$ respectively.

Accordingly, we will present in this section an asymptotic evaluation of the integral which appears in (2.10) for an H-polarized incident field and $\phi_0\sim\pi/2, \phi_0\sim-\pi/2$, $\theta^- << 1$.

Let us first consider the integral

$$I_1 = \int_{\gamma_1} \frac{\cos\phi_0}{\sin(\alpha+\phi)-\sin\phi_0} \frac{\psi(\alpha+\phi)}{\psi(\phi_0)} \exp(-ik\rho\cos\alpha)d\alpha \qquad (4.1)$$

whose integration contour γ_1, is the branch of γ passing through the saddle point $\alpha=\alpha_s=\pi$, with nearby singularities at $\alpha=\alpha_0$ and $\alpha=\alpha_1$.

A simple procedure for the asymptotic evaluation of the integral (4.1) will now be presented; a more complete one, a modified version of Bleistein method [31] (which allows the error estimate) is given in Appendix F.

Let us expand

$$\frac{\cos\phi_0}{\sin(\alpha+\phi)-\sin\phi_0} = \frac{-\frac{1}{2}\sin\alpha_0}{\cos^2\frac{1}{2}\alpha-\cos^2\frac{1}{2}\alpha_0} + \frac{\frac{1}{2}\sin\alpha_1}{\cos^2\frac{1}{2}\alpha-\cos^2\frac{1}{2}\alpha_1} + B(\alpha) \qquad (4.2)$$

Wherein $B(\alpha)$ is regular in the neighbourhood of $\alpha=\pi$.

When (4.2) is substituted in (4.1) the main contribution to the integral is provided by the first factor, when $\phi\sim-\pi/2$, $\phi_0\sim\pi/2$; therefore, we can neglect the contribution arising from the $B(\alpha)$ term. Then we are reduced to consider the canonical integral

$$\int_{\gamma_1} \frac{\frac{1}{2}\sin\bar{\alpha}}{\cos^2\frac{1}{2}\alpha-\cos^2\frac{1}{2}\bar{\alpha}} \frac{\psi(\alpha+\phi)}{\psi(\phi_0)} \exp(-ik\rho\cos\alpha)d\alpha \qquad (4.3)$$

whose solution is given in Appendix E.

The total field $H(\rho,\phi)$ is then represented as a sum of five terms, which will be discussed hereafter.

The first term is the sum of the direct field (2.12) and the contribution of the integral (4.11) at $\alpha=-\phi+\phi_0$:

$$H^i(\rho,\phi) = H_0^i \exp(-ik\rho\cos(\phi_0-\phi)) \frac{F(-\sqrt{2k\rho}\cos\frac{1}{2}(\phi_0-\phi))}{2F(0)}$$

$$= H_0^i \exp(ik\rho\cos(\delta_0+\delta)) \frac{F(-\sqrt{2k\rho}\,\mathrm{sen}\frac{1}{2}(\delta_0+\delta))}{2F(0)} \qquad (4.4)$$

where the two convenient (positive) angles δ , δ_o :

$$\phi_o = \pi/2-\delta_o \; ; \; \phi = -\pi/2+\delta \qquad (4.5)$$

have been introduced. The field constituent (4.4) is the incident wave (2.12) "corrected" by the (normalized) Frèsnel integral, and is symbolically sketched in the first line of fig. 5.

The second term is the sum of the reflected field (2.13) at $y>0$ and the contribution of the integral (4.1) at $\alpha_1 = -(\phi_o+\phi)+\pi$:

$$H_r^+(\rho,\phi) = \frac{\sin\delta_o - \sin\theta^+}{\sin\delta_o+\sin\theta^-} H_o^i \exp(ik\rho\cos(\delta_o-\delta)) \frac{F(\sqrt{2k\rho}\,\sin\tfrac{1}{2}(\delta_o-\delta))}{2F(0)} \qquad (4.6)$$

Where θ^+ is a shorthand notation for θ_n^+.
The field constituent (4.6) is the reflected wave (2.13) with the appropriate (normalized) Frésnel transition function which renders this field continous for $-\pi/2\leq\phi\leq\pi/2$ and is symbolically sketched in the second line of Fig. 5.

The third term is the sum of the reflector field (2.14) at $y<0$ and the contribution of the integral (4.1) at $\alpha_{-1} = -\pi-(\phi+\phi_o)$:

$$H_r^-(\rho,\phi) = \frac{\sin\delta_o - \sin\theta^-}{\sin\delta_o+\sin\theta^-} H_o^i \exp(ik\rho\cos(\delta_o-\delta)) \frac{F(\sqrt{2k\rho}\,\sin\tfrac{1}{2}(\delta-\delta_o))}{2F(0)} \qquad (4.7)$$

The field constituent (4.7) is the reflected wave (2.14) with the appropriate (normalized) Frèsnel transition function which renders this field continous for $-\pi/2\leq\phi\leq\pi/2$, and is symbolically sketched in the third line of Fig. 5.

The fourth term is the contribution of the integral (4.1) at $\alpha_- = -\phi-3\pi/2-\theta_h^-$:

$$H_-(\rho,\phi) = \frac{2\sin\delta_o\,\mathrm{tg}\,\theta^-}{\cos\delta_o-\cos\theta^-}\,\frac{\psi(\pi/2+\theta^-)}{\psi(\pi/2-\delta_o)}\,H_o^i \exp(ik\rho\cos(\delta+\theta_h^-)) \frac{F(\sqrt{2k\rho}\,\sin\tfrac{1}{2}(\delta+\theta_h^-))}{2F(0)}$$

(4.8)

The field constituent (4.8) is the surface wave excited on the half-plane $y<0$ " corrected" by the (normalized) Frèsnel integral and symbolically sketched in the fourth line of Fig. 5.

The fifth term is the contribution of the integral (4.1) at $\alpha_{-2} = -\phi+\phi_o-2\pi$:

$$H_e(\rho,\phi) = \frac{\sin\delta_o - \sin\theta^+}{\sin\delta_o + \sin\theta^+} \frac{\sin\delta_o + \sin\theta^-}{\sin\delta_o - \sin\theta^-} H_o^i \exp(ik\rho\cos(\delta+\delta_o))$$

$$\frac{F(\sqrt{2k\rho}\sin\tfrac{1}{2}(\delta_o+\delta_1))}{2F(0)} \qquad (4.9)$$

The field constituent (4.9) shows the same symbolic representation as the field constituent (4.4), i.e., the first line of Fig. 5. In order to understand its physical meaning, let us sum and substract the incident field (2.12) to the total field. Then.

$$H(\rho,\phi) = H_o^i \exp(ik\rho\cos(\delta_o+\delta)) + R_+H_o^i \exp(ik\rho\cos(\delta_o-\delta)) \frac{F(\sqrt{2k\rho}\sin\tfrac{1}{2}(\delta_o-\delta))}{2F(0)}$$

$$+ R_-H_o^i \exp(ik\rho\cos(\delta-\delta_o)) \frac{F(\sqrt{2k\rho}\sin\tfrac{1}{2}(\delta-\delta_o))}{2F(0)} +$$

$$\frac{2\sin\delta_o \operatorname{tg}\theta^-}{\cos\delta_o - \cos\theta^-} \frac{\psi(\pi/2+\theta^-)}{\psi(\pi/2-\delta_o)} H_o^i \exp(ik\rho\cos(\delta+\theta^-)) \frac{F(\sqrt{2k\rho}\sin\tfrac{1}{2}(\delta+\theta))}{2F(0)} +$$

$$\frac{2\sin\delta_o(\sin\theta^- - \sin\theta^+)}{(\sin\delta_o+\sin\theta^+)(\sin\delta_o-\sin\theta^-)} H_o^i \exp(ik\rho\cos(\delta_o-\delta)) \frac{F(\sqrt{2k\rho}\sin\tfrac{1}{2}(\delta_o+\delta))}{2F(0)} \qquad (4.10)$$

Accordingly, the total field for $\phi \sim -\pi/2$, $\phi_o \sim \pi/2$ is the sum of the incident, the two reflected, (with appropriate transition function) the surface field and an extra (pole-type) field due to the discontinuity of the surface impedance at $y = 0$. In general, also the diffracted wave (saddle-point type) due to the discontinuity should be considered (see Appendix F).

5. Field produced by a line source over a mixed-path plane Earth.

Let us now consider an uniform magnetic line source parallel

to the surface impedance discontinuity, as depicted in Fig.6 , and located at the point $S(x_o, y_o)$.
The free-space magnetic field radiated by the source in $P(x,y)$ would be given by

$$H^i(x,y) = H^i(\rho,\phi) = -\frac{kI_m\omega}{4\zeta} H_o^{(1)}(kd) = CH_o^{(1)}(kd) \qquad (5.1)$$

wherein I_m is the magnetic current.
By using the integral representation for the Hankel function we have

$$H^i(\rho,\delta) = C\frac{1}{\pi}\int_{S_o} \exp(ikd\cos(\delta_o-\eta))d\delta_o$$

$$= \frac{C}{\pi}\int_{S_o}\exp(ik\rho_o\cos(\delta_o-\eta_o))\exp(ik\rho\cos(\delta_o+\delta))d\delta_o \qquad (5.2)$$

wherein Γ_o is the Sommerfeld integration contour as depicted in Fig. 7 and we have assumed, for convenience, $y_o > y$.

Eq. (5.2) represents the field radiated by the line source as a superposition of plane waves with spectral distribution $\exp(ik\rho_o\cos(\delta_o-\eta_o))$.Accordingly,the field H^S radiated by the source in the presence of the mixed path configuration of Fig.6 is given,by superposition,as

$$H^S(\rho,\delta) = \frac{C}{\pi}\int_{S_o} H(\pi,\delta,\delta_o)\exp(ik\rho_o\cos(\delta_o-\eta_o))d\delta_o \qquad (5.3)$$

wherein $H(\rho,\delta,\delta_o)$ is given by (4.10) plus the possible contribution from the field scattered by the discontinuity at $y=0$.
The perturbation in the field excitation due to the impe-

dance discontinuity is of interest. The change in the spectrum can easily be obtained by assuming $\theta^{+}=\theta^{-}$ (the surface impedance is now costant and equal to Z^{-} everywhere) in (4.10); and then subtracting resulting expression from (4.10), hence:

$$\Delta H = 2\sin\delta_o \frac{\sin\theta^{-}-\sin\theta^{+}}{\sin\delta_o+\sin\theta^{+}} H_o^i \left\{ \frac{\exp(ik\rho\cos(\delta_o-\delta))}{\sin\delta_o+\sin\theta^{-}} \frac{F(\sqrt{2k\rho}\,\sin\frac{1}{2}(\delta_o-\delta))}{2F(0)} + \frac{\exp(ik\rho\cos(\delta_o+\delta))}{\sin\delta_o-\sin\theta^{-}} \frac{F(\sqrt{2k\rho}\sin\frac{1}{2}(\delta_o+\delta))}{2F(0)} \right. \tag{5.4}$$

For $\delta=0$ (observation point upon the Earth's surface), expression (5.4) further simplifies as

$$\Delta H = \frac{4\sin^2\delta_o(\sin\theta^{-}-\sin\theta^{+})}{(\sin\delta_o+\sin\theta^{+})(\sin^2\delta_o-\sin^2\theta^{-})} H_o^i \exp(ik\rho\cos\delta_o) \frac{F(\sqrt{2k\rho}\sin\frac{1}{2}(\delta_o-\delta))}{2F(0)} \tag{5.5}$$

Since $\rho_o\cos(\delta_o-\eta_o)+\rho\cos\delta_o=d\cos(\delta_o-\eta)$, we have the integral representation for ΔH^s.

$$\Delta H^s = \frac{4C(\sin\theta^{-}-\sin\theta^{+})}{\pi} H_o^i \int_{S_c} \frac{\sin^2\delta_o}{(\sin\delta_o+\sin\theta^{+})(\sin^2\delta_o-\sin^2\theta^{-})} \frac{F(\sqrt{2k\rho}\sin\frac{1}{2}\delta_o)}{2F(0)} \exp(ikd\cos(\delta_o-\eta))d\delta_o \tag{5.6}$$

i.e. for the perturbation of the magnetic field on the Earth surface due to a discontinuity in the surface impedance and excited by a magnetic line source.

Analytical evaluation of the integral (5.6) is not an easy task. The integrand presents poles at $\delta_o=-\theta^{+}; -\theta^{-}$, clustering around the saddle point $\delta_o=\eta$ (when $\eta<<1$). This complication can be overcome using the evaluation techniques presented in Sect. 4. However, the integrand is also rapidly varying due to the presence of the Fresnel integral, when $k\rho>>1$. This point requires further study to be overcome.

6. Conclusions and recommendations

The problem of plane-wave scattering by a surface impedance discontinuity over a plane has been rigorously solved. The solution is attractive inasmuch it provides a physical description of the plane-wave scattered constituents. These consist of the pole-type contributions, essentially plane waves with attached Frèsnel transition functions; and saddle-point type contribution, essentially a cylindrical wave scattered by the discontinuity. For an observation point close to the surface and an incident wave at a grazing angle, a novel result is the appearance of a new type of (pole-type) contribution, in addition to the conventional incident, reflected and surface fields. The role of the (cylindrical) wave scattered by the discontinuity should be better analysed, and graphs pertinents to a pratical situation computed.

The field excited by prescribed sources can be computed, in principle, from the plane-wave expansion of the incident field and then application of results of the scattering analysis. However, the resulting integral representation contains a rapidly varyng term (Frèsnel integral) in the neighbourhood of the saddle point, and also as many as three poles. This is a point which needs further attention and the use of appropriate (if available) mathematical techniques of asymptotic evaluation of integrals.

Appendix A : The function $\psi(\alpha)$.

The function $\psi(\alpha)$ is defined as follows [20]:

$$\psi(\alpha) = N(\alpha+\pi-\theta^{+})N(\alpha+\theta^{+})N(\alpha-\pi+\theta^{-})N(\alpha-\theta^{-}) \qquad (A.1)$$

or equivalently:

$$\psi(\alpha) = N^{2}(\alpha/2)\,\frac{N(\alpha+\theta^{+})N(\alpha-\theta^{-})}{N(\alpha-\theta^{+})N(\alpha+\theta^{-})}\,\cos\frac{\alpha-\theta^{+}+\pi/2}{2}\,\cos\frac{\alpha+\theta^{-}-\pi/2}{2} \qquad (A.2)$$

The function $N(\alpha)$ is given by

$$N(\alpha) = \exp\left(\frac{1}{4\pi}\int_{0}^{\alpha}\frac{2t-\pi\sin t}{\cos t}\,dt\right) \qquad (A.3)$$

and is a particular determination of a most general class of function introduced by Maliuzhinets [20] with reference to the problem of impedance wedge scattering. The function $N(\alpha)$ is easy to compute numerically and, in any case, it is simply related to a tabulated function [21,28]. Its main properties are hereafter summarized:

$$N(\alpha) = N(-\alpha) \qquad (A.4)$$

$$N(\alpha+\pi/2)\,N(\alpha-\pi/2) = N^{2}(\pi/2)\cos\alpha/2 \qquad (A.5)$$

$$\lim_{|\alpha''|\to\infty} N(\alpha) \sim O(\exp|\alpha''|/4) \qquad (A.6)$$

The location of poles of $\psi(\alpha)$ within the strip

$$-3\pi/2 < \alpha' < 3\pi/2$$

is of interest. From (A.3) it is noted that

$$\int_0^{\alpha} \frac{2t - \pi \sin t}{\cos t} dt \qquad (A.7)$$

is finite for $\alpha = \pi/2$, diverges toward negative values for $\alpha = 3\pi/2$ and toward positive values for $\alpha = 5\pi/2$ Since the divergence of (A.7) is of logarithmic type, the function $N(\alpha)$ has a simple pole at $\alpha = \pm 5\pi/2$. Accordingly, the poles of $\psi(\alpha)$ in the strip of interest are, from (A.1):

$$\alpha_{\pm} = \pm(3\pi/2+\theta^{\pm}) \qquad (A.8)$$

Appendix B: Poles of the spectrum of the field and residues of the integral representation.

Poles of the trigonometric factor in (2.8) do occour at

$$\alpha_n = -\phi + (-1)^n \phi_o + n\pi \quad , \quad n = 0 \; ; \pm 1 \; ; \; \ldots. (B.1)$$

For n=0, the residue

$$F^i(\rho,\phi) = F_o^i \exp(-ik\rho\cos(\phi-\phi_o))U(\pi-\phi+\xi_o)U(\pi+\phi-\xi_o) \qquad (B.2)$$

describes the incident field, wherein:

$$\xi_o = \phi_o' + gd(\phi_o'') \; , \; \phi_o = \phi_o' + i\phi_o'' \; , \; \phi_o'' \geq 0 \qquad (B.3)$$

U(t) is the Heaviside unit step function and gd (x) is the Gudermanian function

$$gd(x) = sn(x) \cos^{-1}\left(\frac{1}{\cosh x} \right) = \sin^{-1}(tgh\, x)$$

For $n = \pm 1$, the residues (B.4)

$$F_{\pm}^r(\rho,\phi) = -F_o^i \frac{\psi(\pm\pi-\phi_o)}{\psi(\phi_o)} \exp(ik\rho\cos(\phi+\phi_o)) \cdot$$

$$\cdot U(2\pi\mp(\xi_o+\phi))U(\pm(\xi_o+\phi)) \qquad (B.5)$$

describe the reflected fields upon the two half-planes $\phi = \pm \pi/2$ respectively. Use of (2.5) shows that:

$$-\frac{\psi(\pm\pi-\phi_o)}{\psi(\phi_o)} = \frac{\cos\phi_o - \sin\theta^{\pm}}{\cos\phi_o + \sin\theta^{\pm}} = R^{\pm} \qquad (B.6)$$

Accordingly, the excitation factor for the reflected waves is identified with the Frèsnel reflection coefficient $R^{\pm}$ appropriate to the two surfaces $\phi = \pm \pi/2$, as it should be expected. For an incident H-polarized surface wave at the Brewster angle, i.e., $\phi_o = \pi/2 - \theta_h^{+}$ there is no reflected wave at $\phi = \pi/2$; the reflection coefficient at $\phi = -\pi/2$ takes the simple form

$$\frac{\sin\theta_h^{+} - \sin\theta_h^{-}}{\sin\theta_h^{+} + \sin\theta_h^{-}} \qquad (B.7)$$

Poles of the other factor in (2.8) do occour at

$$\alpha^{\pm} = -\phi \pm (3\pi/2 + \theta^{\pm}) \qquad (B.8)$$

as follows from (A.7). The corresponding residues :

$$F^{\pm}(\rho,\phi) = F_o^{i} \frac{2\cos\phi_o \operatorname{tg}\theta^{\pm}}{\sin\phi_o \pm \cos\theta^{\pm}} \frac{\psi[\mp(\theta^{\pm} + \pi/2)]}{\psi(\phi_o)} .$$

$$\cdot \exp\left[\mp ik\rho \sin(\phi \mp \theta^{\pm})\right] U(\pm\phi - \xi^{\pm} - \pi/2) \qquad (B.9)$$

$$\xi = \theta' + gd(\theta'') \ , \ \theta = \theta' + i\,\theta'' \ , \ \theta' \geq 0 \qquad (B.10)$$

correspond to surface waves, one (upper sign) with phase velocity in the direction $\pi/2+\theta^{+}$; and the other (lower sign) with phase velocity in the direction $-\pi/2 - \theta^{-}$
In computing (B.9) use has been made of (2.5). These surface waves do exist in the angular range

$$\pm\phi > \pi/2 + \xi^{\pm} \qquad (B.11)$$

Then, a necessary condition for such waves to exist is that

$$\xi = \theta' + gd(\theta'') < 0$$

Letting $\sin\theta = r + ix$, tedious algebra shows that this is is equivalent to the condition :

$$-x > \frac{r}{\sqrt{1+r^2}} \quad (B.12)$$

For radiowave propagation over the Earth, H - polarized waves are of interest and

$$\sin\theta_h = \frac{1}{\sqrt{\varepsilon_r + \frac{i\sigma}{\omega\varepsilon_0}}} \quad (B.13)$$

wherein ε_r and σ are the relative permitivity and conductivity of the Earth. Then $x < 0$ and (B.12) transforms as follows:

$$\varepsilon_r^{-1} > 4\left[1 + \left(\frac{\omega\varepsilon_0\varepsilon_r}{\sigma}\right)^2\right] \quad (B.14)$$

The conclusion is drawn that surface waves (B.9) are not excited, since the poles (B.8) are not crossed upon deformation of the intergration contour from Γ to γ .

Appendix C: Spectral relations between longitudinal and transverse field components.

Let:

$$\begin{cases} E_z(\rho,\phi,z) = \dfrac{\exp(ikz\cos\beta)}{2\pi i}\displaystyle\int_\Gamma \hat{E}_z(\alpha+\phi)\exp(-ik\rho\sin\beta\cos\alpha)d\alpha \\ H_z(\rho,\phi,z) = \dfrac{\exp(ikz\cos\beta)}{2\pi i}\displaystyle\int_\Gamma \hat{H}_z(\alpha+\phi)\exp(-ik\rho\sin\beta\cos\alpha)d\alpha \end{cases} \quad (C.1)$$

It is known (30) that the total field, ($\underline{E}$, $\underline{H}$), can be expressed in terms of (E_z , H_z):

$$\begin{cases} \underline{E} = k^{-2}\sin^{-2}\beta\ (\nabla\times\nabla\times E_z\ \hat{z} + i\omega\mu\ \times H_z\ \hat{z}) \\ \underline{H} = k^{-2}\sin^{-2}\beta\ (\nabla\times\nabla\times H_z\ \hat{z} - i\omega\varepsilon\ \times E_z\ \hat{z}) \end{cases} \tag{C.2}$$

Substituting (C,1) in (3.2) , projecting on the x - axis and equating the corresponding spectral components, we get

$$\begin{cases} \hat{E}_x(\alpha) = \dfrac{\cos\beta\ \cos\alpha\ \hat{E}_z(\alpha) + \sin\alpha\ \zeta\hat{H}_z(\alpha)}{\sin\beta} \\ \zeta\,\hat{H}_x(\alpha) = \dfrac{\cos\beta\cos\alpha\ \zeta\hat{H}_z(\alpha) - \sin\alpha\ \hat{E}_z(\alpha)}{\sin\beta} \end{cases} \tag{C.3}$$

The system (C.3) can now be solved for ($\hat{E}_z$, $\hat{H}_z$), hence :

$$\begin{cases} \hat{E}_z = \dfrac{\cos\alpha\ \cos\beta\ \hat{E}_x - \zeta\ \sin\alpha\ \hat{H}_x}{1 - \cos^2\alpha\ \sin^2\beta}\sin\beta \\ \hat{H}_z = \dfrac{\cos\beta\cos\alpha\ \zeta\hat{H}_x + \sin\alpha\ \hat{E}_x}{1 - \cos^2\alpha\ \sin^2\beta}\sin\beta \end{cases} \tag{C.4}$$

. Appendix D - Spurious poles in the case of oblique incidence

Poles of (3.11) are given by

$$\cos\alpha = \pm(\sin\beta)^{-1} \tag{D.1}$$

Letting

$$(\sin\beta)^{-1} = a + i\,b \tag{D.2}$$

Eq. (D.1) becomes

$$\begin{cases} \cos\alpha'\ \cosh\alpha'' = \pm\, a \\ \sin\alpha'\ \sinh\alpha'' = \mp\, b \end{cases} \tag{D.3}$$

After squaring, substitution and solution, we get

$$\left\{\begin{array}{l} \cosh^2 \alpha'' = \dfrac{a^2+b^2+1}{2} + \sqrt{\left(\dfrac{a^2+b^2+1}{2}\right)^2 - a^2} \\ \\ \sin^2 \alpha' = \dfrac{b^2}{\dfrac{a^2+b^2-1}{2} + \sqrt{\left(\dfrac{a^2+b^2-1}{2}\right)^2 - a^2}} \end{array}\right. \qquad (D.4)$$

Appendix E : Solution for the canonical integral .

$$I = -\frac{1}{2}\int_{\gamma_1} \frac{\sin\alpha_o}{\cos^2\frac{1}{2}\alpha - \cos^2\frac{1}{2}\alpha_o} \frac{\psi(\alpha+\phi)}{\psi(\phi_o)} \exp(-ik\rho\cos\alpha)d\alpha$$

$$= \frac{1}{2}\int_{\gamma_o} \frac{\sin\beta_o}{\sin^2\frac{1}{2}\beta - \sin^2\frac{1}{2}\beta_o} \frac{\psi(\pi+\phi+\beta)}{\psi(\phi_o)} \exp(-ik\rho\cos\beta)\, d\beta \qquad (E.1)$$

where $\beta = \alpha-\pi$, $\beta_o = \phi_o-\phi-\pi$ and the integration contour γ_o runs parallel to γ through the saddle point $\beta = \beta_s = 0$.
With the new transformation of variables

$$\sqrt{2}\exp(i\pi/4)\sin\tfrac{1}{2}\beta = t \qquad (E.2)$$

The contour γ_o is transformed into the real axis and we get for the integral :

$$I = \frac{i}{\sqrt{2}} \exp\left[i(k\rho-\pi/4)\right]\int_{-\infty}^{+\infty} \frac{\psi(\pi+\phi+\beta)}{\psi(\phi_o)} \frac{\sin\beta_o}{\cos\frac{1}{2}\beta} \frac{\exp(-k\rho t^2)}{t^2-2i\sin^2\frac{1}{2}\beta_o}\, dt$$

$$\simeq i\exp\left[i(k\rho-\pi/4)\right]\sqrt{2}\sin\tfrac{1}{2}\beta \int_{-\infty}^{+\infty} \frac{\exp(-k\rho t^2)}{t^2-2i\sin^2\frac{1}{2}\beta_o}\, dt$$

$$= i\pi\exp(ik\rho\cos\beta_o)\, \frac{F(n\sqrt{2k\rho}\,\sin\frac{1}{2}\beta_o)}{nF(0)} \qquad (E.3)$$

Where F (u) is the Frèsnel integral

$$F(u) = \int_u^\infty \exp(it^2)dt \tag{E.4}$$

$$\eta = \pm 1 \quad \text{according to} \quad \beta_o' + g\alpha(\beta_o'') \gtrless 0 \tag{E.5}$$

and the property

$$F(u) + F(-u) = 2\,F(o) = \sqrt{\pi}\,\exp i\pi/4 \tag{E.6}$$

has been used. Note that

$$\int_{-\infty}^{+\infty} \frac{\exp(-\Omega t^2)}{t^2 - iz^2}\,dt = \exp(-i\Omega z^2)\,\frac{2\sqrt{\pi}}{\sqrt{\eta\zeta}}\,F(\sqrt{\Omega}z) \tag{E.7}$$

The other integrals can be computed similarly.

Appendix F : Modified Bleistein's method for the asymptotic evaluation of the integral.

Let us consider the function B(α) defined in(4.2) :

$$B(\alpha) = \frac{\cos\phi_o}{\sin(\alpha+\phi)-\sin\alpha_o} + \frac{\frac{1}{2}\sin\alpha_o}{\cos^2\frac{1}{2}\alpha-\cos^2\frac{1}{2}\alpha_o} - \frac{\frac{1}{2}\sin\alpha_1}{\cos^2\frac{1}{2}\alpha-\cos^2\frac{1}{2}\alpha_1} \tag{F.1}$$

and let us expand it as follows

$$B(\alpha) = a + \cos\tfrac{1}{2}\alpha\,B_1(\alpha) \tag{F.2}$$

wherein B(α) is not only regular around $\alpha = \alpha_s = \pi$, as B (α), but also attains a zero value for $\alpha = \alpha_s = \pi$.
Accordingly in the asymptotic evaluation of the integral

$$I_2 = \int_{\gamma_1} \frac{\psi(\phi+\alpha)}{\psi(\phi_o)} B(\alpha) \exp(-ik\rho\cos\alpha)\, d\alpha \qquad (F.3)$$

the dominant **contribution** will be provided by the first factor at the right hand side of (F.2).
Letting $\alpha = \pi$, we can easily compute a; hence:

$$a = \frac{-\cos\phi_o}{\sin\phi + \sin\phi_o} + \frac{\frac{1}{2}\sin(\phi_o-\phi)}{\cos^2\frac{1}{2}(\phi_o-\phi)} - \frac{\frac{1}{2}\sin(\phi_o-\phi)}{\cos^2\frac{1}{2}(\phi_o+\phi)}$$

$$= \frac{-3\cos\phi_o}{\sin\frac{1}{2}(\phi_o+\phi)\cos\frac{1}{2}(\phi_o-\phi)} \qquad (F.4)$$

then,

$$I_2 \sim a \int_{\gamma_1} \frac{\psi(\alpha+\phi)}{\psi(\phi_o)} \exp(-ik\rho\cos\alpha)\, d\alpha$$

$$= a \int_{\gamma_0} \frac{\psi(\pi+\phi+\beta)}{\psi(\phi_o)} \exp(ik\rho\cos\beta)\, d\beta$$

$$= \frac{ia}{\sqrt{2}} \exp[i(k\rho-\pi/4)] \int_{-\infty}^{+\infty} \frac{\psi(\beta+\pi+\phi)}{\psi(\phi_o)} \frac{\exp(-k\rho t^2)}{\cos\frac{\beta}{2}}\, dt$$

$$\sim \frac{ia}{\sqrt{2}} \exp[i(k\rho-\pi/4)] \frac{\psi(\phi+\pi)}{\psi(\phi_o)} \int_{-\infty}^{+\infty} \exp(-k\rho t^2)\,dt$$

$$= ia\sqrt{\frac{2}{\pi k\rho}} \exp[i(\phi-\pi/4)] \frac{\psi(\phi+\pi)}{\psi(\phi_o)} \qquad (F.5)$$

This corresponds to a cylindrical wave, scattered by the discontinuity at $y=0$, which decays with distance as

$$\frac{-3\cos\phi_o}{\sqrt{k\rho}\,\sin\frac{1}{2}(\phi+\phi_o)\cos\frac{1}{2}(\phi+\phi_o)} \qquad (F.6)$$

Accordingly, its contribution to the total field cannot in general be neglected.

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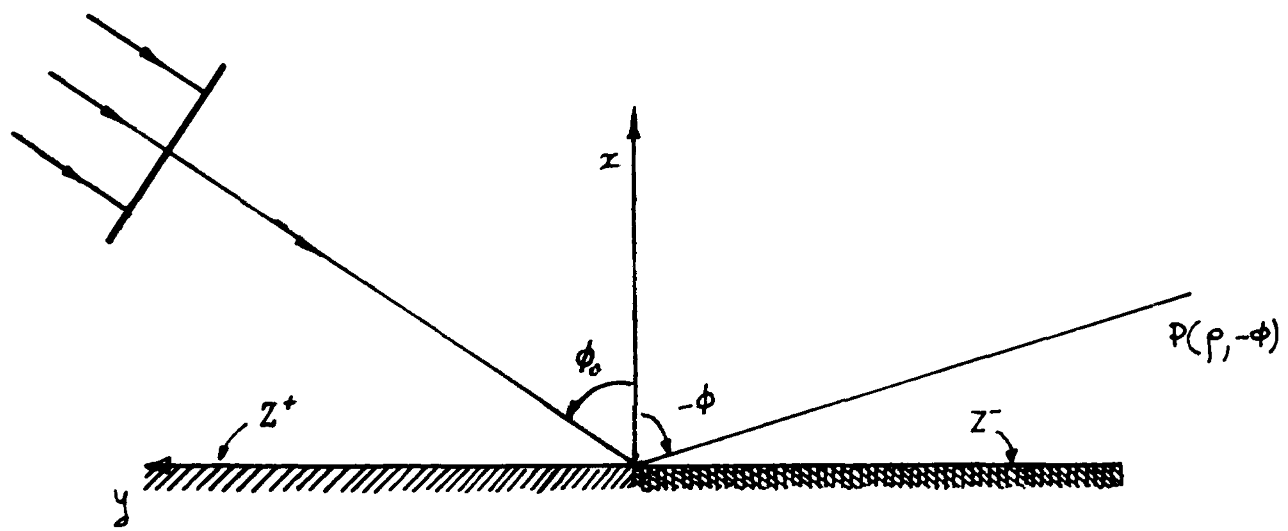

Fig.1. Geometry of the scattering problem under consideration.
Normal incidence.

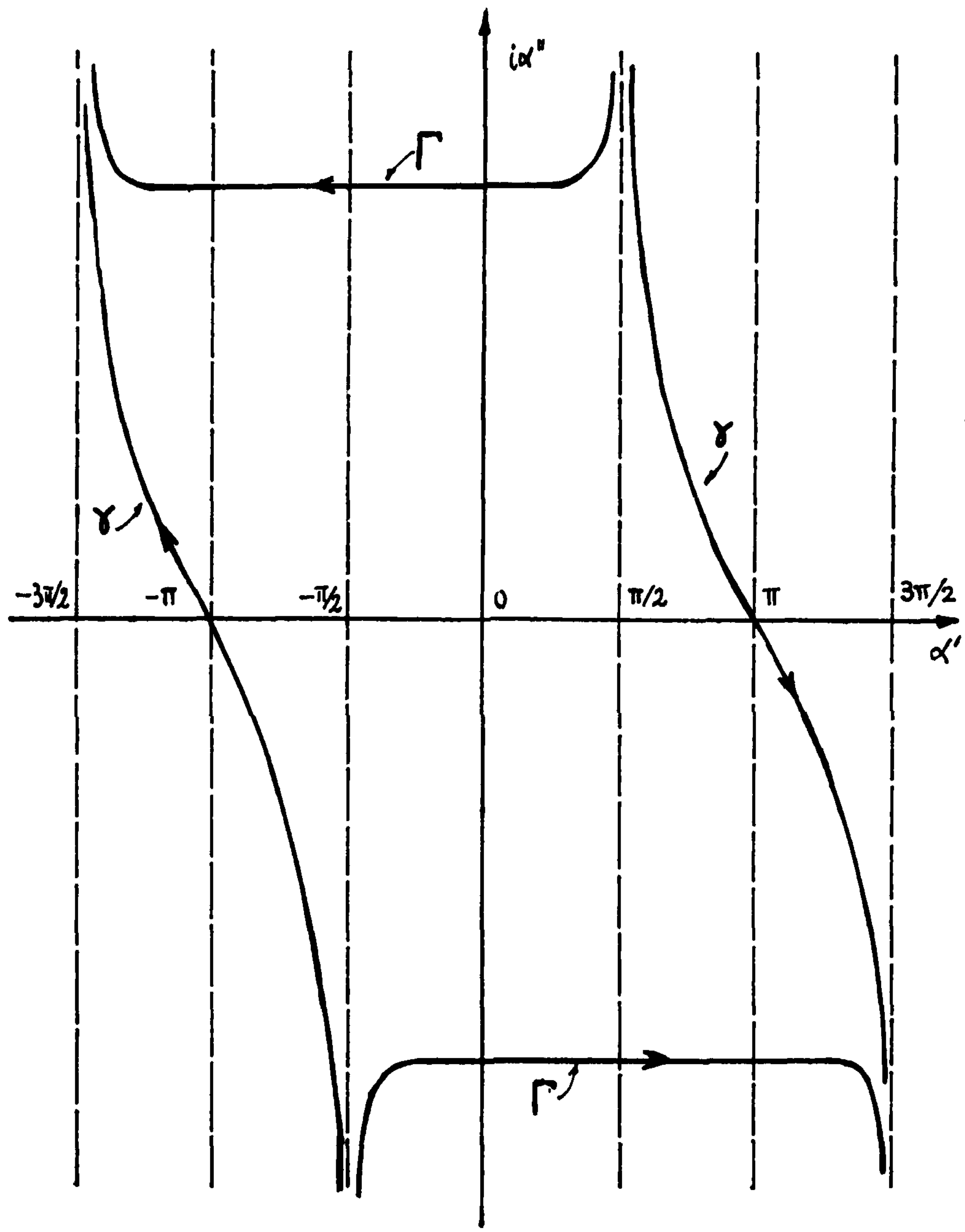

Fig.2. Integration contour in the complex α plane.

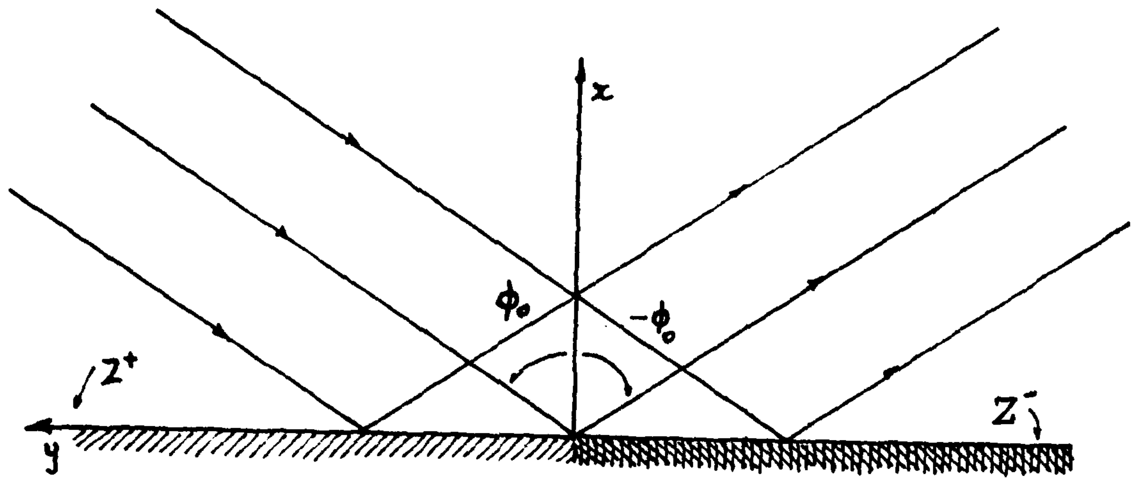

Fig.3. Geometry of incident and reflected rays.

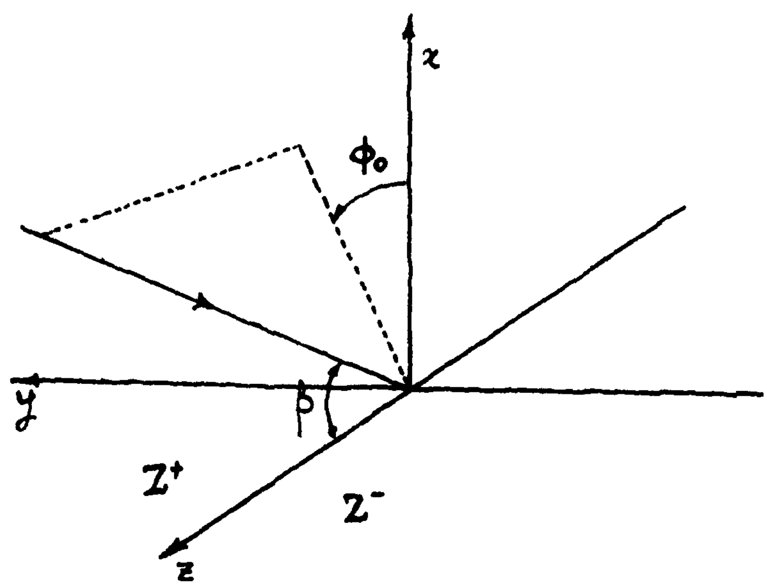

Fig.4. Geometry of the scattering problem under consideration. Oblique incidence.

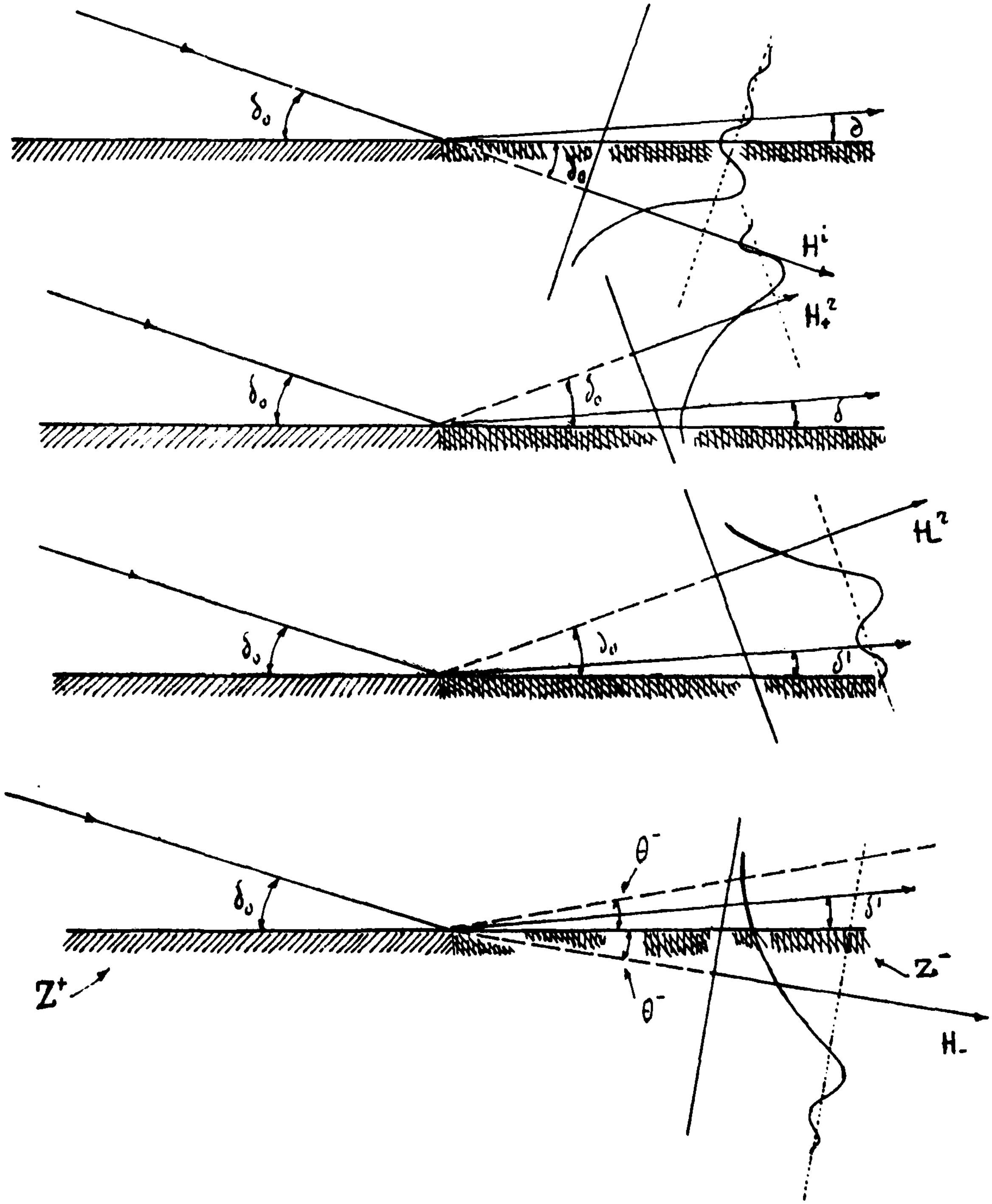

Fig.5. Symbolic sketch of several constituents of the magnetic field close to the surface.

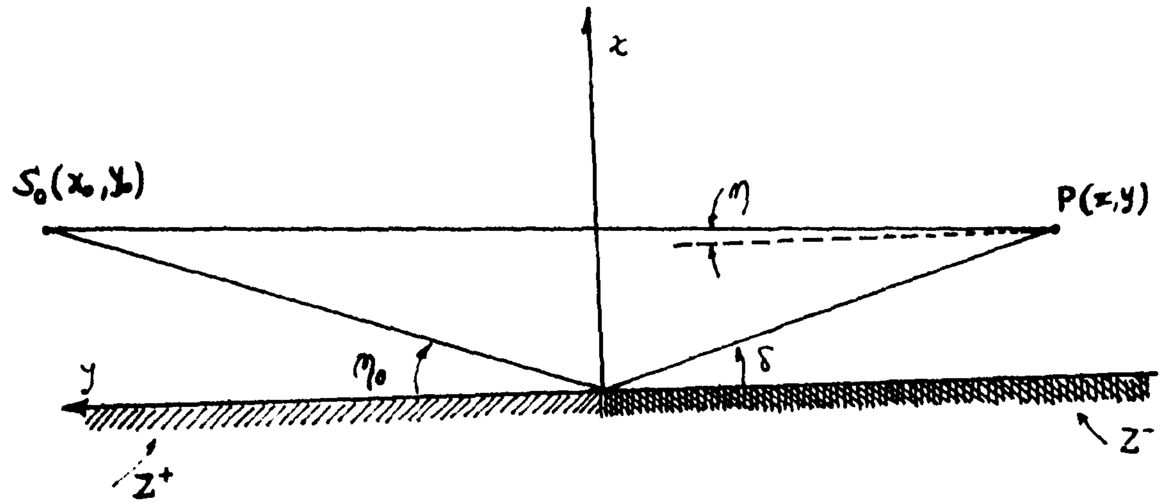

Fig.6. Geometry of an uniform magnetic line source over a mixed-path Earth.

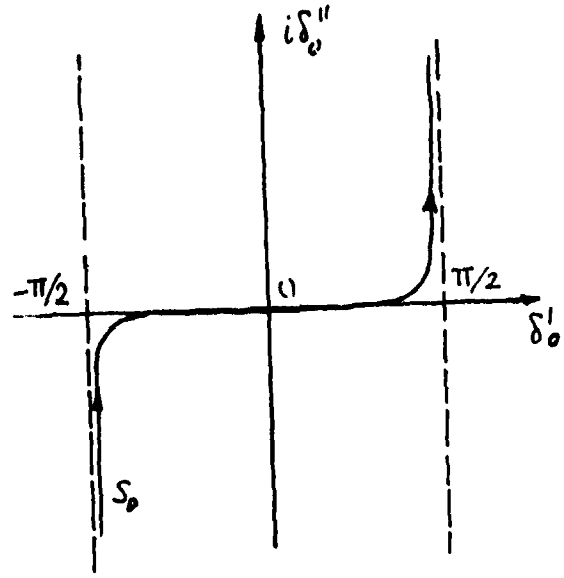

Fig.7. Sommerfeld contour of integration for the expansion of the Hankel function $H_0^{(1)}(kd)$.