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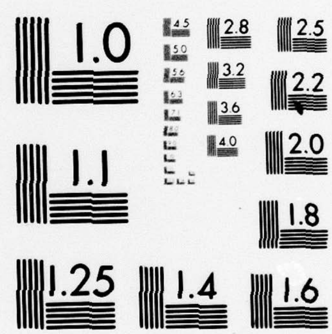

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THREE-DIMENSIONAL LAMINAR SOLUTION OF THE NAVIER-STOKES EQUATIONS USING BODY-FITTED COORDINATE SYSTEMS

by

C. W. MASTIN,
J. F. THOMPSON,
A. GHOSH, AND
T. H. FU

Final Report-Part II

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ABSTRACT

Numerical methods are developed for constructing body-fitted curvilinear coordinate systems for the region surrounding an arbitrary three-dimensional body. Finite difference schemes are investigated for solving the Navier-Stokes equations on body-fitted coordinates. Solutions are computed for flow about a sphere and a finite wing.

1. Introduction

The research efforts of the past three years on three-dimensional laminar flow can be divided into two areas. The first task was the generation of a body-fitted coordinate system by solving a system of elliptic equations. The theoretical basis for the coordinate generation scheme appeared in a paper by Mastin and Thompson [1]. Although the method was developed for the region about a single body, the capability exists for generating coordinate systems for nearly any bounded three-dimensional region. It may be necessary to partition the physical region into simpler subregions. This partitioning, which is essential for very complicated regions, has proven to be easily implemented in the coordinate generation scheme.

The second area of research, which depended on the generation of a suitable coordinate system, was the numerical solution of the Navier-Stokes equations for viscous flow about an arbitrary body. Two implicit methods have been developed. Both are generalizations to curvilinear coordinates of methods which have been developed for rectangular coordinate systems. The use of SOR iteration in solving the Navier-Stokes equations on three-dimensional body-fitted coordinates was first reported by Mastin and Thompson [2]. More extensive computations and comparisons with known experimental results for viscous flow about a sphere will appear in the dissertation by Fu [3].

In an effort to develop a more efficient method on the larger computational fields needed for high Reynolds number calculations, a fractional step method was investigated. The basic algorithm and preliminary computational results have appeared in a paper by Mastin, Ghosh, and Thompson [4]. In the problems considered so far, the physical region was partitioned and hence, an iteration process was needed with the fractional step algorithm so that the equations are satisfied across the cuts. While this allows the user to handle larger and more complex regions, it decreased the efficiency of the method. The accuracy of the fractional step method has been about the same as the SOR method for flow about a sphere.

The following sections will describe the major accomplishments of this research project. A more detailed description can be found by referring to the references given above.

2. Body-Fitted Coordinate System

Curvilinear coordinate systems have been generated about various three-dimensional bodies. The physical region is truncated at a finite distance and the region between the surface of the body and the surface at infinity is partitioned into subregions which can be transformed to rectangular computational regions. For low Reynolds number problems, three subregions were used as in Figure 1. For higher Reynolds numbers, the central section was divided into four smaller subregions to give a total of six subregions of approximately the same size. The partitioning of the region served a dual purpose. First of all, it allowed for a nearly equal distribution of mesh points on the surface of the body. This would not be the case for a spherical or cylindrical type coordinate system where mesh points would be clustered along an axis where the transformation would be singular. Secondly, it allowed for a much larger computational mesh since it was only necessary to keep the data for one region in the core memory of the computer.

The transformation from the physical xyz-space to the computational $\mu\nu\xi$-space was obtained by solving the elliptic system

$$\nabla^2\mu = f(\mu,\nu,\xi)$$
$$\nabla^2\nu = g(\mu,\nu,\xi)$$
$$\nabla^2\xi = h(\mu,\nu,\xi) \tag{1}$$

The homogeneous equations ($f = g = h = 0$) give a smooth transformation with a nonvanishing Jacobian. In some cases a nonzero value of h was used so that more mesh points were concentrated near the body. The generality of the transformation method has been verified by constructing coordinate systems about a multitude of bodies of varying shapes. These include a sphere, ellipsoids, cylinders with hemispherical caps, cylinders with conical caps, finite wings, and a flat plate.

A difficult problem in the generation of coordinate systems for three-dimensional bodies is the specification of the boundary correspondence for the system (1). In order to simplify the determination of coordinates for mesh points on the body, these mesh points were chosen to lie on cross-sections of the body. This also aided in the plotting of the flow field data.

The initial velocity and pressure distribution for the solution of the Navier-Stokes equations were usually obtained from a potential flow calculation. The potential function can be easily calculated during the solution of the system (1) in the computational regions.

3. SOR Methods for the Navier-Stokes Equations

Successive overrelaxation (SOR) iterative methods have been successfully used for solving the Navier-Stokes equations on many two-dimensional regions for a wide range of Reynolds numbers. A straightforward generalization to three dimensions has worked equally well although the method has proven expensive in terms of computer time and storage.

The form of the Navier-Stokes equations most frequently used was

$$\nabla \cdot \vec{v} = 0 \tag{2a}$$

$$\dot{\vec{v}} + (\vec{v} \cdot \nabla)\vec{v} = - \nabla p + \frac{1}{R} \nabla^2 \vec{v} \tag{2b}$$

where $\vec{v} = u\vec{i} + v\vec{j} + w\vec{k}$ is the velocity, p is the pressure, and R is the Reynolds number. A few calculations were made with the advective terms in conservation form. The result was a longer run time with no appreciable difference in the solution. The most efficient manner of differencing the momentum equation (2b) was

$$\vec{v}^{n+1} = \vec{v}^{n} - \nabla t[(\vec{v}^{n} \cdot \nabla)\vec{v}^{n} + \nabla p^{n+1} - \frac{1}{R} \nabla^2 \vec{v}^{n+1}]. \tag{3}$$

where n is the time step index and ∇t the step size. In this formulation, the viscous and pressure terms are implicit while the advective terms are explicit. The stability criteria imposed by the explicit terms was quite mild due to the small velocity components near the body where the mesh spacing was also small. In fact, with a fully implicit scheme which was also coded, a smaller time step size was needed to maintain SOR convergence than was required for the stability of the above scheme.

The Poisson equation used for the pressure calculation was

$$\nabla^2 p^{n+1} = 2[\frac{\partial(u,v)}{\partial(x,y)} + \frac{\partial(u,w)}{\partial(x,z)} + \frac{\partial(v,w)}{\partial(y,z)}]^n + \frac{1}{\nabla t} D^n - (D^n)^2 \tag{4}$$

where $D = \nabla \cdot \vec{v}$. It was necessary to delete some terms in the pressure equation derived from (3) so that the resulting system of four equations could be solved by SOR iteration in the transformed regions to obtain values of $\vec{v}$ and p at time step n+1.

In all computations a unit free stream velocity was imposed on the surface at infinity. On the body a no-slip velocity condition was assumed and a normal derivative condition for the pressure was computed from the equation

$$\nabla p^{n+1} = \frac{1}{R} \nabla^2 \vec{v}^{n+1} \tag{5}$$

A constant pressure on the surface at infinity was used for most of the computations. One could argue that the same normal derivative condition should hold on the surface at infinity. Thus condition was tried and gave essentially the same pressure distribution on the body. It does, however, require the determination of the pressure level at infinity in order to plot the pressure coefficient.

The above scheme gave realistic solutions to fluid flow problems and the velocity and pressure values compared well with known analytic and numerical results. The calculated value of D was large near the body where an impulsive start was used and decreased slowly as time increased. It was also observed that the velocity values converged much faster than the pressure when the system was solved by SOR. Consequently, a second SOR method was developed to solve the Navier-Stokes equations. An auxiliary vector $v^{n+1/2}$ was introduced in two steps.

$$\vec{v}^{n+1/2} = \vec{v}^{n} - \nabla t[(\vec{v}^{n} \cdot \nabla)\vec{v}^{n} - \frac{1}{R} \nabla^2 \vec{v}^{n+1/2}] \tag{6a}$$

$$\vec{v}^{n+1} = \vec{v}^{n+1/2} - \nabla t \quad \nabla p^{n+1} \tag{6b}$$

The pressure is computed after the first step by solving the Poisson equation

$$\nabla^2 p^{n+1} = \frac{1}{\nabla t} \nabla \cdot \vec{v}^{n+1/2} . \tag{7}$$

As with many splitting techniques, instabilities can arise if correct boundary values are not chosen for the auxiliary functions computed at the intermediate steps. For simplicity in programming, the first value used was

$$\vec{v}^{n+1/2} = \vec{v}^{n}$$

on the boundary. This value worked well in all our calculations with no indication of instability. Two normal derivative conditions for pressure were used. Both were obtained by considering the limiting value of the pressure gradient at the surface of the body. If only the second equation (6b) is considered we have

$$\nabla p^{n+1} = 0 , \tag{8}$$

while if both equations are considered we arive at

$$\nabla p^{n+1} = \frac{1}{R} \nabla^2 \vec{v}^{n+1/2} . \tag{9}$$

For this type of splitting, the first alternative (8) has been used by others in the theoretical analysis of the method and has also been used in most of our calculations. The normal component of the pressure gradient in (9) would appear to give a solution in closer agreement with the unsplit method where (5) is used. However, both boundary conditions gave essentially the same results in the problems which have been considered in our work.

This two step method was successfully applied and when the results of both methods were compared, the difference between the two solutions was less than the overall error in the approximation of the true solution. The two step method was preferred because it was faster and gave a smaller value of the divergence D. Unfortunately, neither method was able to reduce the maximum absolute value of D below about $10.\nabla t$ for most problems. For high Reynolds number calculations, the two step method was used with a zero normal pressure derivative. The one step method did not work as well on the highly contracted coordinate systems needed for high Reynolds number calculations.

4. A Fractional Step Method for the Navier-Stokes Equations

The fractional step method used in this project involved a splitting of the pressure term from the momentum equations as in the two step method discussed earlier and a splitting of the remaining terms according to the derivative direction in computational space. In order to solve the pressure equation in the same manner, a parabolic pressure equation is needed. Thus the continuity equation (2a) is replaced by the artificial compressibility equation.

$$\varepsilon p_t + \nabla \cdot \vec{v} = 0 \tag{10}$$

It was discovered that better results could be obtained by taking a variable ε which was $O(\Delta t^2)$.

The same type of partitioning was used in the physical region as with the SOR methods. An iteration scheme was introduced at each fractional step so that all equations are satisfied on the surfaces which partition the region. In this iteration process the nonlinear coefficients and mixed derivative terms are also updated so that all terms are treated implicitly.

The splitting of the velocity derivative terms in the momentum equations according to coordinate directions in the computational regions can be expressed as

$$(\vec{v} \, . \, \nabla)\vec{v} - \frac{1}{R} \nabla^2\vec{v} = A_\mu\vec{v} + A_\nu\vec{v} + A_\xi\vec{v} \, . \tag{11}$$

The mixed derivative terms must be included in some fashion. In our computations one mixed derivative term was included in each term on the right hand side of (11). A locally one-dimensional (LOD) implicit scheme for the momentum equations which also incorporates the splitting of the pressure term can now be formulated as follows.

$$\vec{v}^{n+1/4} + \Delta t \, A_\mu \vec{v}^{n+1/4} = \vec{v}^n \tag{12a}$$

$$\vec{v}^{n+1/2} + \Delta t \, A_\nu \vec{v}^{n+1/2} = \vec{v}^{n+1/4} \tag{12b}$$

$$\vec{v}^{n+3/4} + \Delta t \, A_\xi \vec{v}^{n+3/4} = \vec{v}^{n+1/2} \tag{12c}$$

$$\vec{v}^{n+1} = \vec{v}^{n+3/4} - \Delta t \, \nabla p^{n+1} \tag{12d}$$

When the derivatives are approximated by finite differences, the first three equations require the solution of a tridiagonal system. For the physical regions which have been considered, the body and surface at infinity were ξ = constant surfaces. Thus no intermediate boundary values were needed for $\vec{v}^{n+1/4}$ and $\vec{v}^{n+1/2}$. In other problems where it might be necessary to impose physical boundary conditions on μ = constant or ν = constant surfaces, one could resort to the more complex ADI or approximate factorization splitting techniques. The question of appropriate boundary values for $\vec{v}^{n+3/4}$ was handled in the same manner as the two step SOR method in the previous section with similar results. The parabolic pressure equation derived from (2b) and (10) is approximated using a similar LOD formulation. The Laplacian of the pressure function is written in terms of derivatives with respect to the computational variables as

$$\nabla^2 p = B_\mu p + B_\nu p + B_\xi p \, .$$

The splitting of the pressure equation can be expressed as follows.

$$\varepsilon(p^{n+1/3} - p^n) - (\Delta t)^2 B_\mu p^{n+1/3} = 0 \tag{13a}$$

$$\varepsilon(p^{n+2/3} - p^{n+1/3}) - (\Delta t)^2 B_\nu p^{n+2/3} = 0 \quad (13b)$$

$$\varepsilon(p^{n+1} - p^{n+2/3}) + \Delta t(\nabla.\vec{v})^{n+3/4} - (\Delta t)^2 B_\xi p^{n+1} = 0 \quad (13c)$$

The divergence term has been included in the last equation for simplicity. As with the two step SOR scheme, there are two possible boundary conditions for the body pressure which can be derived by considering (12d) alone or by summing (12a)-(12d). The above equations (13a)-(13c) are solved by a tridiagonal algorithm in an iterative sequence which is used to assure that all equations are satisfied on the surfaces which partition the region. The pressure boundary condition is also applied in the iterative solution of (13c). Even for a single computational region with no partitioning it may be necessary to implement the normal derivative condition for the pressure function iteratively to avoid stability problems.

This scheme has been used to compute viscous flow about a sphere at low and intermediate Reynolds numbers. It is comparable to the SOR methods in both accuracy and computer time used. However, the full potential of the method has not been realized in the present configuration with the partitioned regions. A direct implementation of the method, with no iterations, would be an order of magnitude faster. The fractional step method also has an advantage when core storage is considered. At each fractional step only one third of the derivative coefficients of the transformed equations are required for the computations. The main difficulty has been the choice of a proper value for the parameter ε. A value of ε which is too large leaves the pressure practically unchanged and the velocity divergence becomes large. On the other hand, a value of ε which is too small causes the solution to become unbounded after very few time steps.

5. Computational Results

The principle objective of this research has been algorithm development rather than the development of production codes. Thus most of the Navier-Stokes solutions have been computed for flow about a sphere where valid experimental and numerical results are available for comparison. The present methods worked best for the intermediate Reynolds numbers between 40 and 1,000, however, solutions were obtained for Reynolds numbers between 0.5 and 100,000.

The coordinate system about a sphere was constructed using the scheme described in Section 2. Figures 2 and 3 contain plots of a crossection of the coordinte surfaces and a perspective plot of one coordinate surface surrounding the sphere. For viscous flow about a sphere, drag calculations compared well with experimental results as can be seen in Figure 4. Both the SOR and fractional step schemes gave clearly formed vortices at the rear of the sphere. A crossection of the steady-state velocity field for the SOR computation at a Reynolds number of 40 appears in Figure 5. The corresponding pressure coefficient is plotted in Figure 6. As the Reynolds number increases, the velocity in the vortex region increases. This can be seen by examining Figure 7 which shows the same crossection with a Reynolds number of 290. The vectors have been scaled to one-fourth their actual length. The drop in pressure at the forward stagnation point can be noted from Figure 8. This second solution was computed using the fractional step method.

The practical application of body-fitted coordinate systems has not been ignored in our research. Flow about finite wings have been computed for wings having various lengths, cord to thickness ratios, camber, and tapering. Some of the large scale characteristics of the flow can be observed even though the wake region was not properly modeled due to the relatively small number of mesh points downstream of the wing. For example, consider the wing in Figure 9 which is perpendicular to the free stream and at a 10 degree angle of attack. An SOR scheme was used with a Reynolds number of 40. A rear view of the velocity near the trailing edge, illustrated in Figure 10, indicates a vortex formed at the wing tip and downwash immediately behind the wing. Due to symmetry, only half the wing was plotted.

6. Conclusions

Body-fitted coordinate systems have proven to be a valuable tool for solving the Navier-Stokes equations for viscons flow about arbitrary bodies. Both SOR and fractional step methods have been implemented on body-fitted coordinates. Thus the programs for solving the Navier-Stokes equations are essentially independent of the coordinate system even if several rectangular computational regions are used. The accuracy of the methods has been tested by computing viscous flow about a sphere for a wide range of Reynolds numbers. Despite the ability to use relatively large time steps, the solution of the Navier-Stokes equations starting with potential flow and marching to the steady-state solution has required large amounts of computer time. However, realistic solutions of viscous flow problems about a variety of body shapes and a wide range of Reynolds numbers can be computed by the methods developed under this project.

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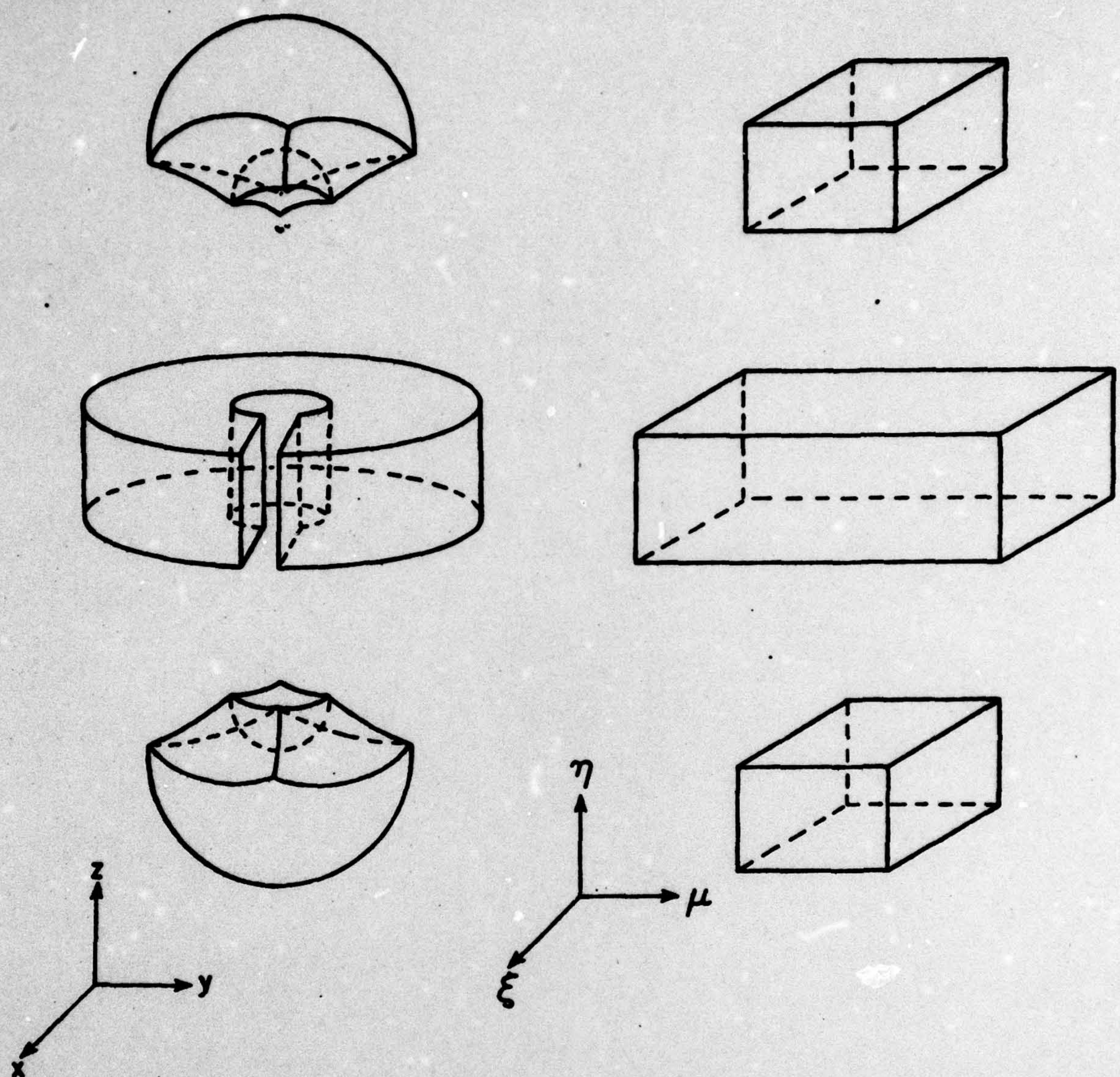

Figure 1. Transformation from Physical to Computational Regions.

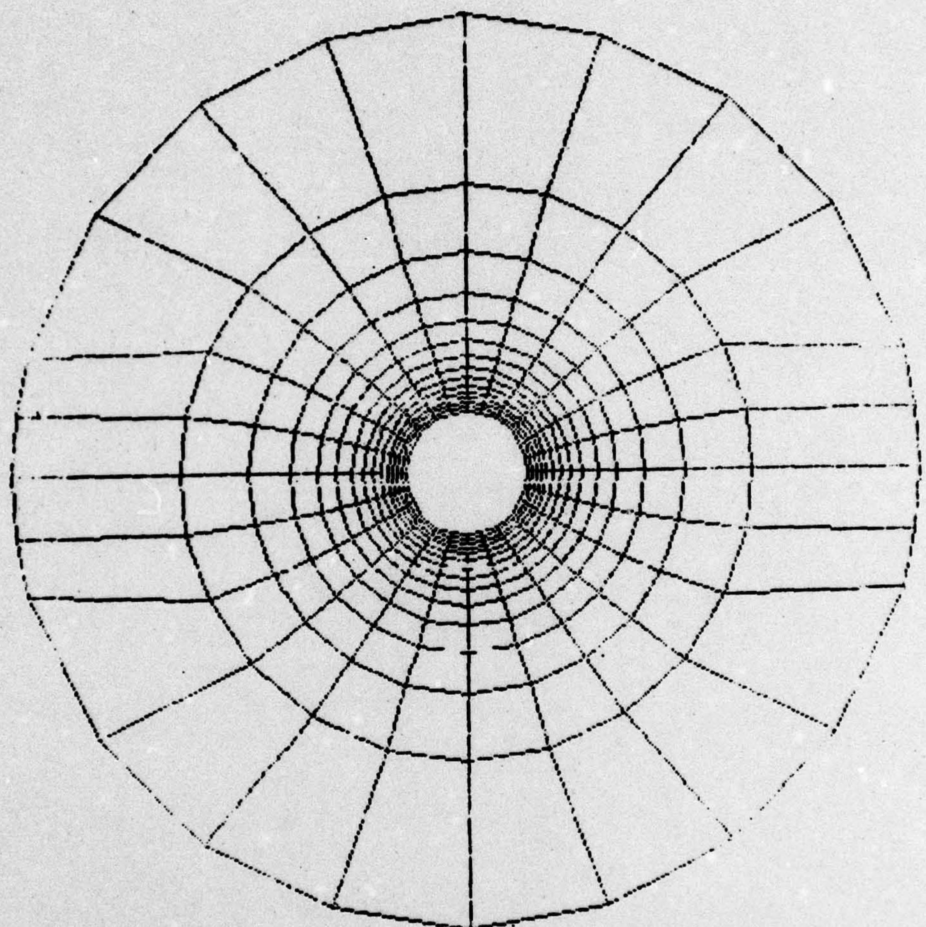

Figure 2. Cross-Section of Coordinate Surfaces About a Sphere.

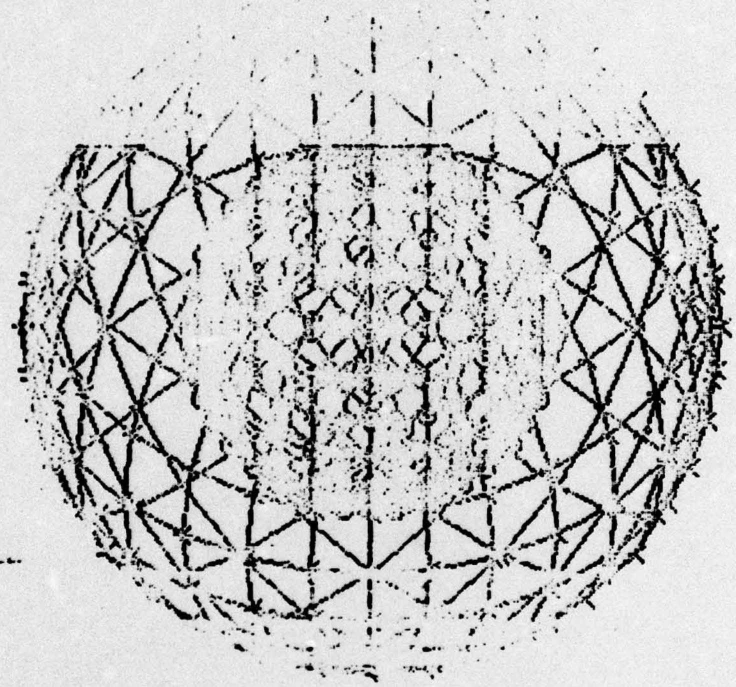

Figure 3. Perspective Plot of a Coordinate Surface Surrounding a Sphere.

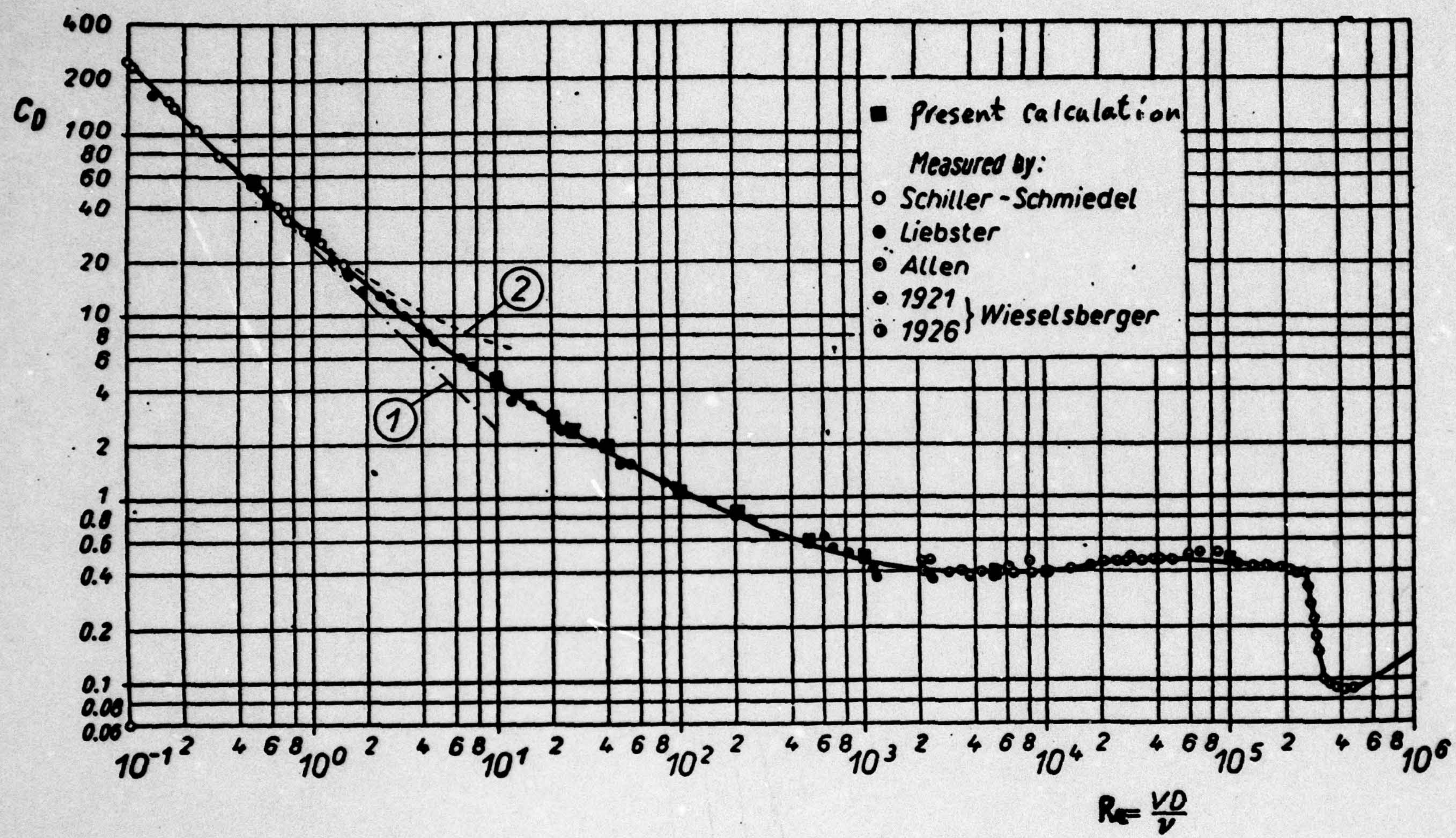

Figure 4. **Drag coefficient for spheres as a function of the Reynolds number**

Curve (1): Stokes's theory, curve (2): Oseen's theory.

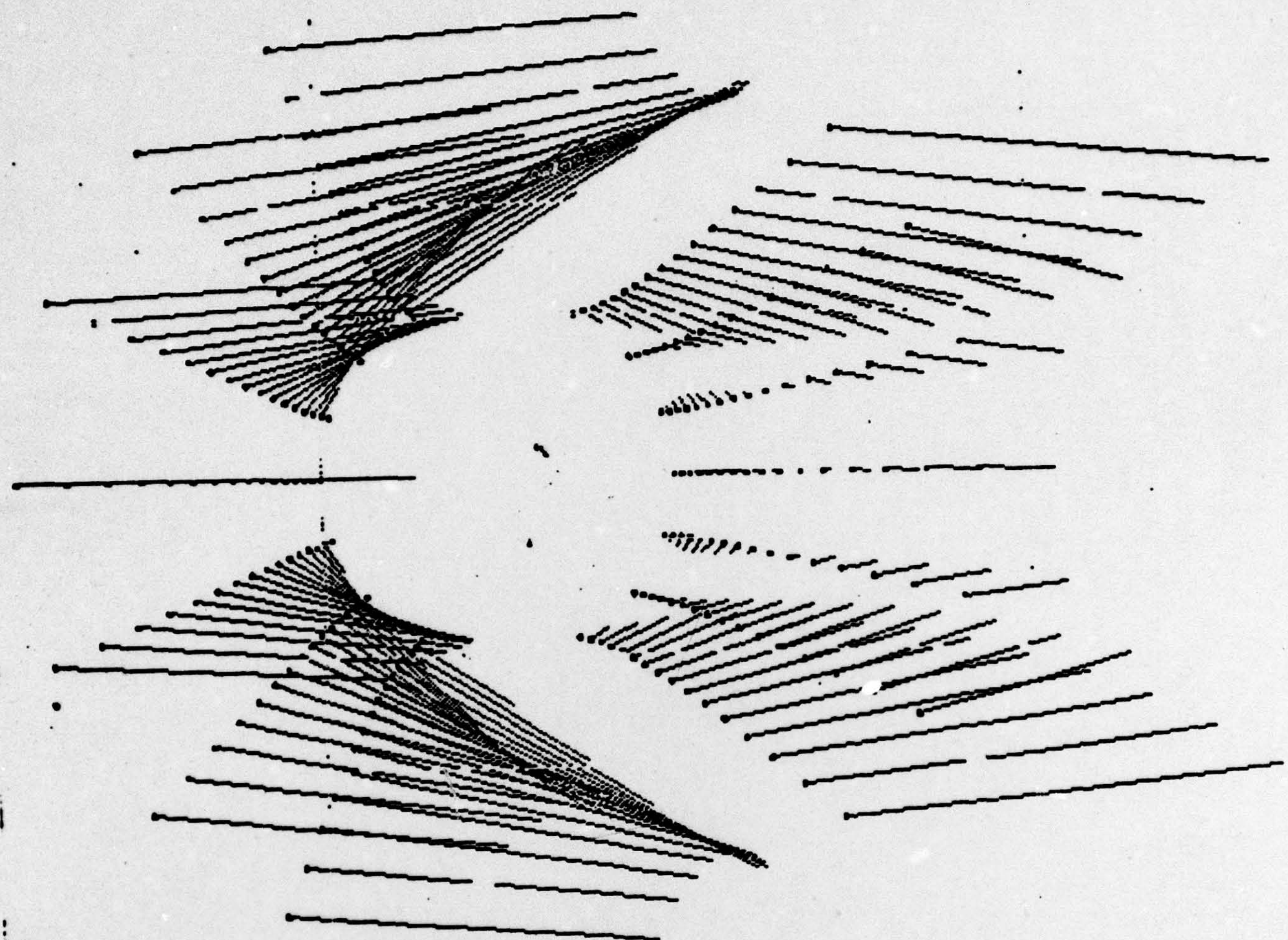

Figure 5. Velocity on Cross-Section, Re = 40.

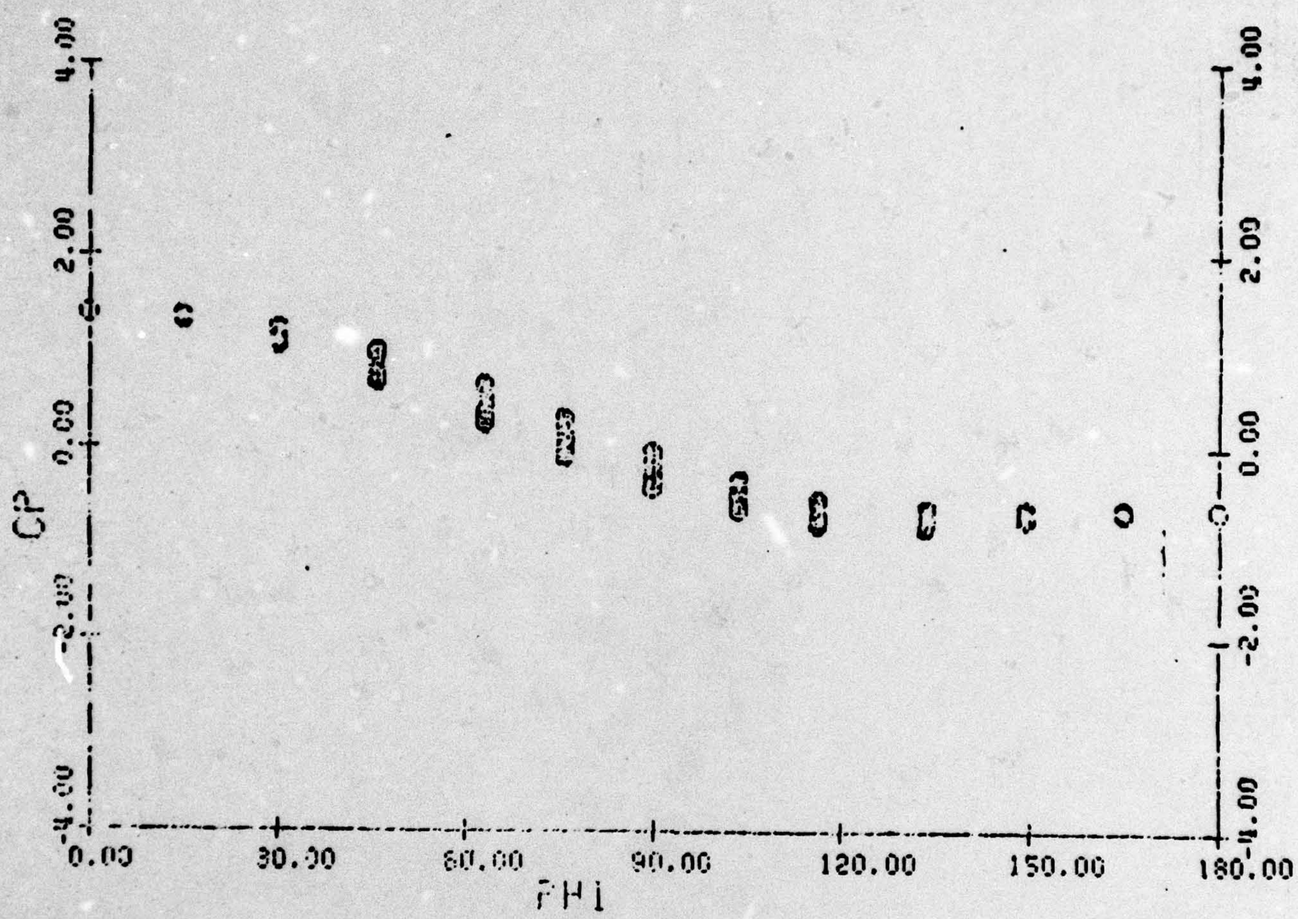

Figure 6. Pressure Coefficient As a Function of Angular Measure from Forward Stagnation Point, Re = 40.

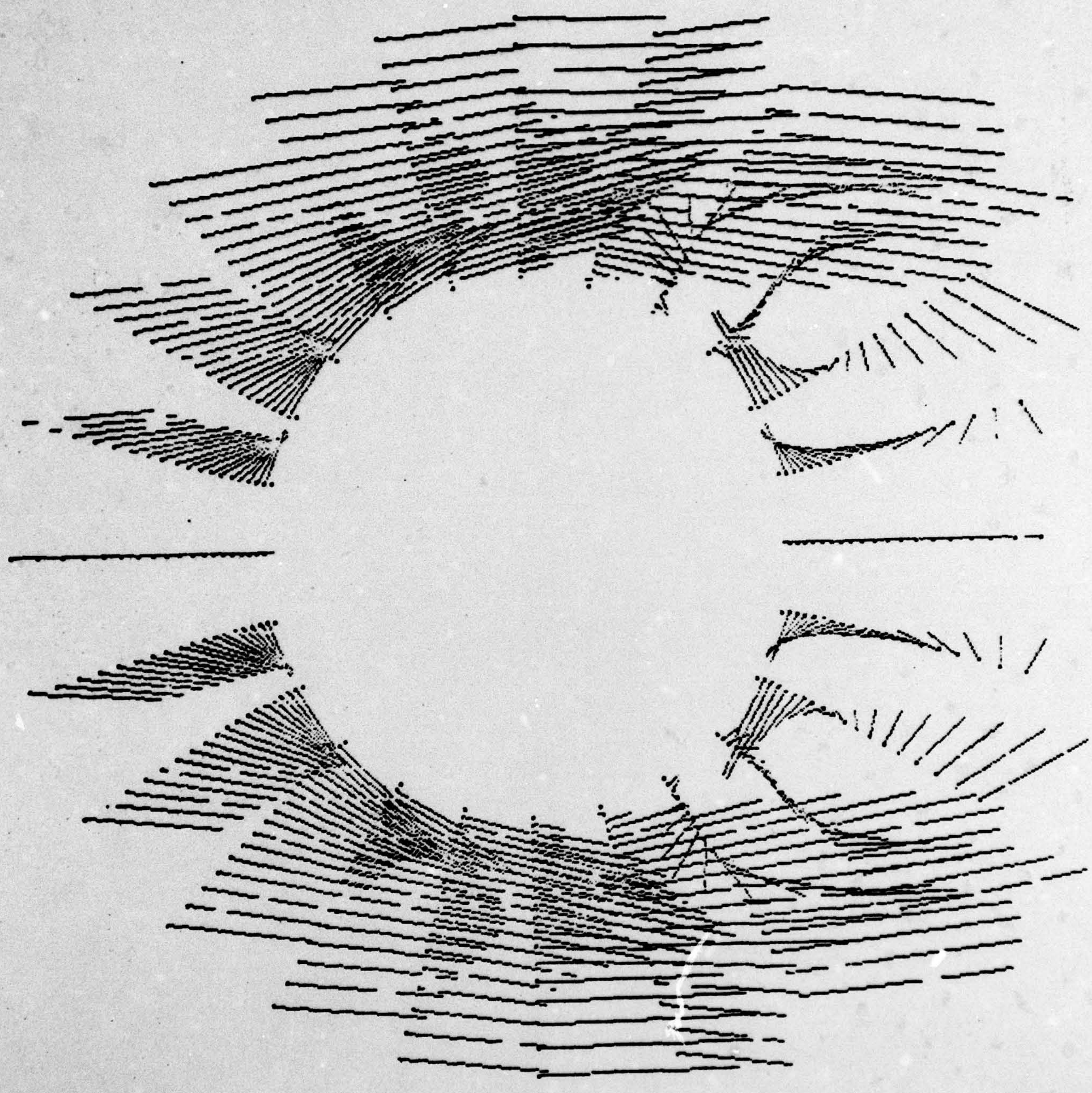

Figure 7. Velocity on Cross-Section, Re = 290.

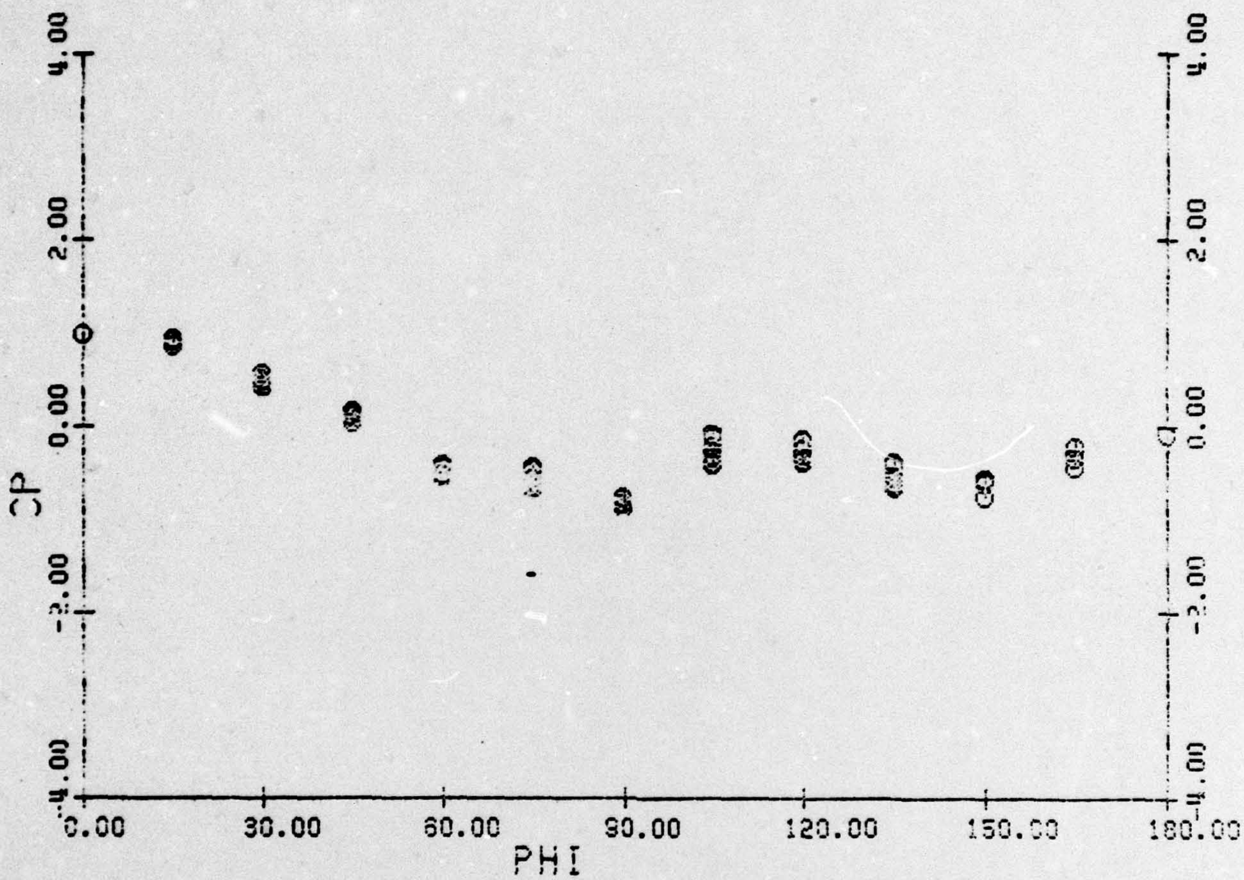

Figure 8. Pressure Coefficient, Re = 290.

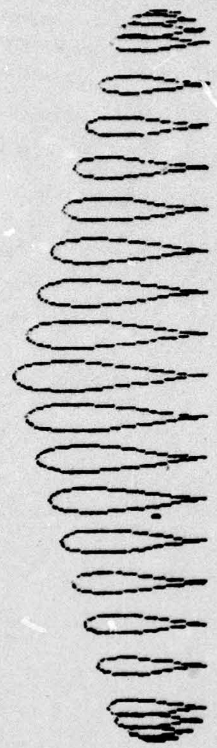

Figure 9. Tapered Wing.

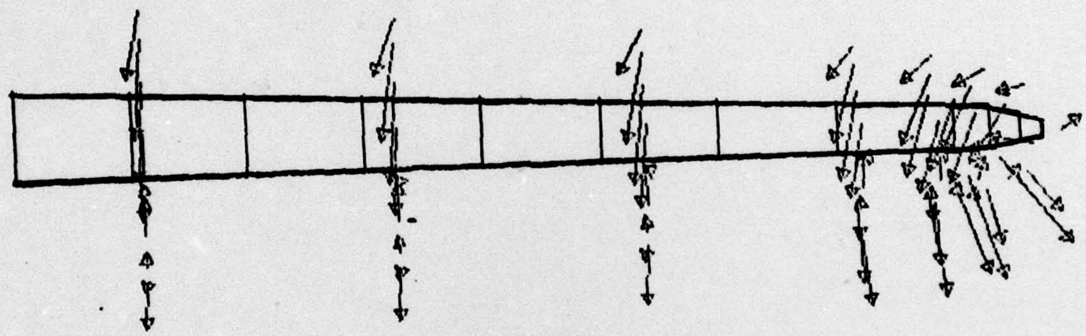

Figure 10. Velocity Near the Trailing Edge, Re = 40.