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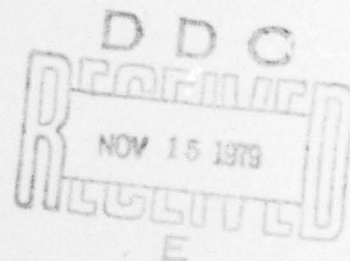
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**TARGET SELECTION ASSUMPTIONS
AND THEIR EFFECT ON AN
ASSESSMENT EQUATION**

Technical Paper 2-79



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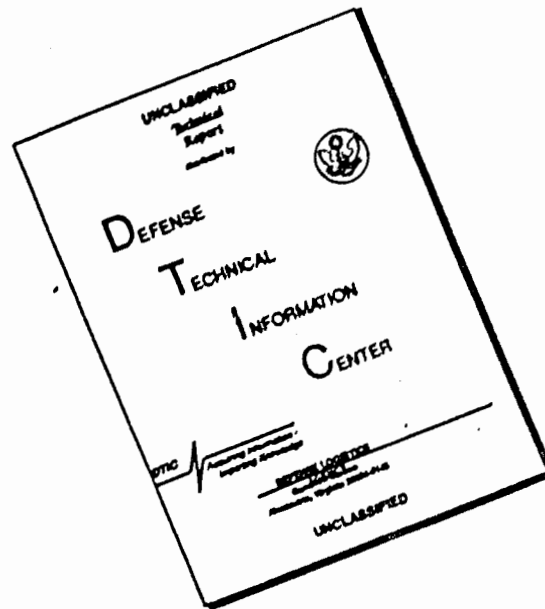
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US Army Combined Arms Combat Developments Activity
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6 TARGET SELECTION
ASSUMPTIONS AND THEIR EFFECT ON
AN ASSESSMENT EQUATION

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ABSTRACT

This paper compares two distinct assumptions of the techniques that an individual gunner might follow in target selection and the assessment of the effectiveness of several gunners. These two assumptions are:

- o Targets are selected at random with replacement after each engagement, or
- o Targets are selected at random without replacement after each engagement.

The first assumption leads to a widely used assessment equation.

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ASSESSMENT EQUATIONS

1. PURPOSE. This technical paper derives two assessment equations applicable to combat models. The underlying assumptions and predicted outcomes are compared.

2. INTRODUCTION. This paper compares two distinct assumptions of the technique that an individual gunner might follow in target selection and the assessment of the effectiveness of several gunners. These two assumptions are:

- o Targets are selected at random with replacement after each engagement, or
- o Targets are selected at random without replacement after each engagement.

As we show later, the first assumption leads to an assessment equation that has been used widely (see references a and d in paragraph 9). Paragraph 3 presents an example to illustrate the application of the two formulas. Then paragraph 4 develops a basic assessment equation depending on the expected kill capability of a weapon independent of the target selection process of the gunner. The succeeding paragraphs develop and compare the differences predicted by the selection processes.

3. EXAMPLE: Consider the situation of two targets of the same type, two firers with two rounds each, and distinct probabilities of kill P_1 and P_2 . We compute the expected number of kills by each target selection process.

a. Targets are selected with replacement after each round fired. The probability then that weapon one fires r_1 rounds at a given target and weapon two fires r_2 rounds at the same target is:

$$P_r(r_1, r_2) = \frac{1}{2^2 2^2} \frac{2!}{r_1! r_2!} \frac{2!}{(2-r_1)! (2-r_2)!}$$

The expected number of kills given r_1 and r_2 rounds fired at a given target is:

$$E(\text{kills}/r_1, r_2) = (1-(1-P_1)^{r_1})(1-(1-P_2)^{r_2}) + (1-(1-P_1)^{2-r_1})(1-(1-P_2)^{2-r_2})$$

The expected number of kills E_a is then:

$$E_a = E(\text{kills}) = E(E(\text{kills}/r_1, r_2)) =$$

$$\sum_{r_2=0}^2 \sum_{r_1=0}^2 P_r(r_1, r_2) E(\text{kills}/r_1, r_2) =$$

$$2 \left(1 - \left(1 - \frac{P_1}{2} \right)^2 \right) \left(1 - \frac{P_2}{2} \right)^2$$

The last equality requires multinomial algebraic manipulation.

b. Targets are selected without replacement after each round fired. The expectation of killing one target is the number of target kills (1), times the number of ways of choosing one target out of two (2), times the probability of killing exactly one target. This last probability is the probability that firer one kills neither target and firer two kills one but not the other. Thus, the expectation of killing one target is:

$$1 \cdot 2 \cdot (P_1(1-P_1)(1-P_2) + (1-P_1)^2 P_2(1-P_2))$$

The expectation of killing two targets is the number of target kills (2), times the number of ways of choosing two targets out of two (1), times the probability of killing exactly two targets. This last probability is the probability that firer one kills both targets, or that firer one kills one target but fails to kill the second target and firer two kills the second target, or firer one fails to kill either target and firer two kills both. Thus, the expectation of killing two targets is:

$$2 \cdot 1 \cdot (P_1^2 + 2P_1(1-P_1)P_2 + (1-P_1)^2 P_2^2)$$

In total, the expected number of kills is the sum of these equations and may be algebraically reworked to:

$$E_b = 2 \left(1 - (1-2P_1/2) \right) \left(1 - 2P_2/2 \right)$$

For a numerical example, if $P_1 = .6$ and $P_2 = .7$, then

$$\begin{aligned} \text{and } E_a &= 1.59 \\ E_b &= 1.76 \end{aligned}$$

Thus $E_b > E_a$ for this example.

4. ASSESSMENT EQUATION BASE. A "committee" modeling approach will be used to derive a base assessment equation that will consider no interaction between gunners in selecting targets. The next paragraph will particularize this base equation to the two target selection policies being considered. We will have B distinct firing weapons, with i the index of firing weapons. However, if some weapons are identical, then certain terms could be grouped in the final formula. We will later provide a formula with different target types. Assume then, that there are T targets of the same type and B firing weapons. This development will account for multiple kills of the same target by different weapons by using the "committee" approach. We associate the T targets with T people in a population from which committees are to be formed, and we allow one person to serve on more than one committee. We form B committees, with the i th committee having K_i members, each gunner (or committee former) selecting members without replacement from a uniform distribution. In other words, the members on the i th committee correspond to K_i target kills by the i th weapon. In reference c, page 165, the probability of having a total of M people on committees (i.e., a total of M target kills by at least one weapon) is given by:

$$\Pr(M / K_i's) = \binom{T}{M} \sum_{j=0}^M (-1)^{M-j} \binom{M}{j} \prod_{i=1}^B \frac{\binom{j}{K_i}}{\binom{T}{K_i}} .$$

where $0 \leq M \leq T$. Since M is a random variable, we must find its conditional expected value given K_i kills. The conditional expected value has been worked out in reference b. Using that paper, the expected value of M given the K_i is:

$$E(M / K_i) = T \left(1 - \prod_{i=1}^B (1 - K_i / T) \right) .$$

Now to obtain the expected number of total kills, the expectation must be taken with respect to the K_i . Since the K_i are independent, the expected number of total kills is:

$$E(E(M / K_i)) = T \left(1 - \prod_{i=1}^B (1 - E(K_i)/T) \right) \quad (1)$$

This last equation is the desired base assessment formula. The next paragraph derives the appropriate $E(K_i)$ for the two target selection assumptions.

5. PARTICULAR ASSESSMENT EQUATIONS.

a. We will now compute the expected number of kills by one weapon against T targets with a target selection process of randomly drawing a target for each after replacing any previous targets. Let P be the single shot kill probability of any one round against any of the identical targets, R the total number of rounds fired by the weapon, and R_t the number of rounds fired at target t , $t = 1, \dots, T$ ($\sum R_t = R$). Then the joint probability of the R_t is:

$$P_r(R_t = r_t, t=1, \dots, T) = \frac{1}{T^R} \frac{R!}{\prod_{t=1}^T r_t!}$$

Thus the expected number of kills given the r_t 's is

$$E(\text{kills}/r_t, t=1, \dots, T) = \sum_{t=1}^T (1 - (1-P)^{r_t})$$

The expected number of kills is the summation over all combinations of the r_t 's, of the product of the above two equations; i.e:

$$\begin{aligned} E(\text{kills}) &= \sum_{r_t} \dots \sum_{r_1} \frac{1}{T^R} \frac{R!}{\prod_{t=1}^T r_t!} \sum_{t=1}^T (1 - (1-P)^{r_t}) \\ &= \frac{1}{T^R} \sum_{t=1}^T \sum_{r_t} \dots \sum_{r_1} \frac{R!}{\prod_{t=1}^T r_t!} (1 - (1-P)^{r_t}) \end{aligned}$$

$$\begin{aligned}
&= \frac{1}{T^R} \sum_{t=1}^T (T^R - (T-P)^R) \\
&= \sum_{t=1}^T (1 - (1 - \frac{P}{T})^R) \\
&= T (1 - (1 - \frac{P}{T})^R)
\end{aligned}$$

This last expression is the $E(K_i)$ required in the base equation of paragraph 4. Substituting this value of $E(K_i)$ into the base equation we obtain K , the total number of kills by the B weapons, as:

$$\begin{aligned}
K &= T \left\{ 1 - \prod_{i=1}^B \left(1 - \frac{T (1 - (1 - \frac{P_i}{T})^{R_i})}{T} \right) \right\} \\
&= T \left(1 - \prod_{i=1}^B \left(1 - \frac{P_i^{R_i}}{T} \right) \right) \quad (2)
\end{aligned}$$

where:

P_i = single shot kill probability of weapon type i against the target type

R_i = total number of rounds fired by weapon type i against the target type.

This formula is the same type as the one used in references a and d.

b. The formula for the expected number of kills by one weapon (labeled i) against T targets, with a target selection process of randomly drawing a target without replacement for each engagement, will depend on the number of rounds fired in comparison to the number of targets. Let R_i be the total number of rounds fired by the weapon and P_i the single shot kill probability of any one round against any of the identical targets. If $R_i \leq T$, then the expected kills by using the binomial theorem is simply:

$$E(K_i) = R_i P_i$$

and thus the total number of expected kills is:

$$K = T \left(1 - \prod_{i=1}^B (1 - R_i P_i / T) \right) \quad (3)$$

If the time period or firing rate is such that $R_i > T$ is possible, then $E(K_i)$ is slightly more complicated. A more general formulation for $E(K_i)$ could involve assuming that any extra rounds (i.e. those in excess of targets) would be uniformly distributed over the targets. We will use the notation $g(x)$ to denote the greatest integer less than or equal to x . Then $E(K_i)$ is:

$$E(K_i) = \begin{cases} R_i P_i & \text{if } g\left(\frac{R_i}{T}\right) = 0 \\ \left((T(1 + g(\frac{R_i}{T})) - R_i) (1 - (1 - P_i)^{g(\frac{R_i}{T})}) + \right. \\ \quad \left. (R - g(\frac{R_i}{T})T) (1 - (1 - P_i)^{1 + g(\frac{R_i}{T})}) \right) & \text{otherwise} \end{cases}$$

6. IMPLEMENTATION: Both the assessment formula types are easily implemented on a computer. In fact, the formula for selecting targets with replacement is presently being used in the JIFFY Combat Model, reference a. For example, page 21 has a "Generalized Assessment Equation" the same as equation (2), namely:

$$K_k = \left\{ 1 - \prod_{\text{all } i} \left(1 - \frac{\text{SSKP}_{ik}}{T_k} \right)^{R_{ik}} \right\} T_k$$

where:

i is the index for the firing weapon type

k is the index for the target type

K_k is the number of target type k killed by all firers

T_k is the number of targets of type k engaged

R_{ik} is the number of rounds fired by all weapons of type i against target type k , and

SSKP_{ik} is the single shot kill probability of a single weapon of type i against a single target of type k .

The terms T_k and R_{ik} are determined internal to the model from such factors as terrain, line of sight, weather conditions, light and obscuration conditions, operational availability, firing rates, weapon loads, target massing, suppression, etc. To obtain the total kills, the JIFFY models sums the K_k over all target types.

7. COMPARISON OF EQUATIONS. The simple calculations of paragraph 3 suggest and detailed algebraic manipulations confirm, the selection without replacement formula yields more kills than the selection with replacement formula. This difference rests in the target selection assumptions imbedded in the derivations of the two equations. The target selection with replacement allows a weapon to repeatedly kill the same target. So for two shots against two targets with a probability of kill of .8,

a weapon has an overall expected number of kills of $2(1 - (1 - .8)^2) = 1.28$ by target replacement. However, if one shot each was fired at the two different targets, then the expected number of kills is $2(.8) = 1.6$. The difference, $.32 = (1/2)(.8)(.8)$, is the product of the probability the gunner picks as target two the same target as his first, times the probability of killing the first target, times the probability of killing his second target. Additionally, the target selection with replacement formula is insensitive to whether, for example, there are two gunners with 10 rounds each or 20 gunners with one round each, whereas the selection without replacement formula is sensitive to these differences. A remaining problem with both assessment equations is that of having large number of firers and targets; nevertheless in both formulas any firer can fire at any target independent of the geometry of the situation. When these equations are used in a combat model context, the overall battle should be decomposed into reasonable size sub-battles where the target selection assumptions of targets being uniformly picked by each gunner is appropriate.

8. SUMMARY: This paper has derived two assesement equations applicable to combat modeling. Each of these equations is built on a different assumption of gunner target selection.

a. A gunner selects targets at random with replacement, i.e, after firing he uses no prior knowledge of which targets he has already engaged and randomly picks any target, live or dead, or

b. A gunner selects targets at random without replacement, i.e. (in the case of firing fewer rounds than available targets in the engagement period) after firing, he randomly picks any target he has not yet engaged.

The modeler must choose the assumption he feels best represents his problem.

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