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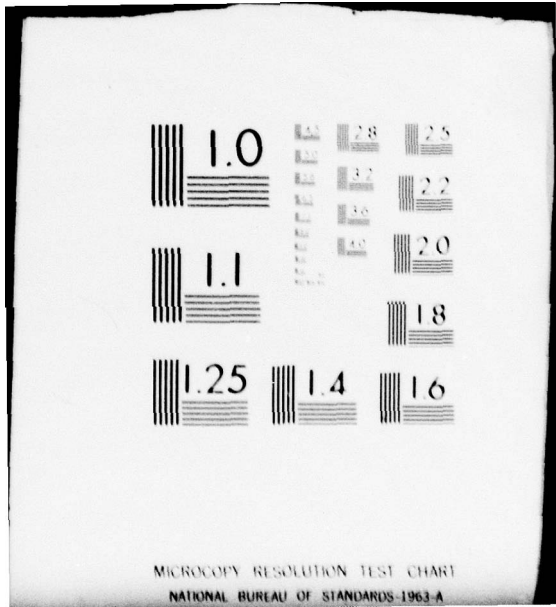
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## THESIS

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EVALUATION OF A PROPOSED MODIFIED  
LOG-GAMMA CONFIDENCE BOUND METHOD  
FOR FLEET MISSILE SYSTEM RELIABILITY

by

Peter Allen Craig

September 1979

Thesis Advisor: M. Woods

Approved for public release; distribution unlimited.

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Evaluation of a Proposed Modified Log-Gamma Confidence Bound Method for Fleet Missile System Reliability,		5. TYPE OF REPORT & PERIOD COVERED Master's Thesis September 1979
7. AUTHOR(s) Peter Allen Craig		6. PERFORMING ORG. REPORT NUMBER
8. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Postgraduate School Monterey, California 93940		9. CONTRACT OR GRANT NUMBER(s) 13 472
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Postgraduate School Monterey, California 93940		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		12. REPORT DATE Sep 79
		13. NUMBER OF PAGES 46
		14. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Fleet Missile System Reliability Log-Gamma Confidence Bound		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) A statistical method is evaluated to determine its accuracy for estimating lower confidence bounds on system reliability of a mixture of missile configurations using component data.		

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#20 - ABSTRACT - CONTINUED

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EVALUATION OF A PROPOSED MODIFIED LOG-GAMMA CONFIDENCE  
BOUND METHOD FOR FLEET MISSILE SYSTEM RELIABILITY

by

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Submitted in partial fulfillment of the  
requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

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ABSTRACT

A statistical method is evaluated to determine its accuracy for estimating lower confidence bounds on system reliability of a mixture of missile configurations using component data. Monte Carlo simulations are used to establish the accuracy of these bounds.

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## I. INTRODUCTION

A statistical method has been proposed which obtains a lower confidence bound on system reliability. It is a modified log-gamma procedure developed to measure fleet missile system reliability. Monte Carlo simulations were performed to evaluate its accuracy as an estimate for system reliability. Five hundred simulations were run for each of twelve cases examined at 80% and 90% confidence levels. The results of these simulations are included in this paper. Additional simulations were performed with minor modifications to the proposed log-gamma method. These changes are documented and the results are included. A comparison was made between the two versions on their accuracy for estimating the lower confidence bound on system reliability.

The reliability equations were applied to a hypothetical fleet missile system configuration and analyzed for changes in test sample sizes, component reliabilities and weighting factors. The proposed procedure was determined to be significantly inaccurate for small and large amounts of accumulated test data on missile components. It also has the distracting defect that larger lower confidence bounds are obtained from data sets with one failure than those obtained from data sets with zero failures.

## II. MODIFIED LOG-GAMMA METHOD

The log-gamma method, in its more general form, can apply to nonseries as well as to fleet-mixture populations. The underlying theory is contained in [Ref. 1]. Examples of cases where it is suspect have been included in the following chapter. The procedure below describes the proposed modified log-gamma method as it is applied to a series system.

Assume that in a series system there are  $k$  components each with a sample size  $n_i$ , where  $i = 1, 2, \dots, k$ . Let the number of failures be  $f_i$  for  $i = 1, 2, \dots, k$ . Consider first the case when there is at least one failure in the system. Thus  $\sum f_i > 0$ .

Let

$$\hat{R}_i = 1 - \frac{f_i}{n_i} \quad (2.1)$$

be the point estimates of the  $i$ -th component reliability. Then the equation

$$\hat{R} = \prod_{i=1}^k \hat{R}_i \quad (2.2)$$

is the point estimate of the system reliability. Define

$$\bar{R} = R^{1/k} \quad (2.3)$$

and

$$\hat{V} = (1 - \bar{R}) \sum_{i=1}^k \frac{1}{n_i} \quad (2.4)$$

$\hat{V}$  is used as an estimate of the variance of  $-\ln \hat{R}$ . It is assumed that the distribution of  $-\ln \hat{R}$  can be approximated by a gamma distribution as follows

$$f(z) = \frac{z^{L-1} e^{-Lz/\ln R}}{\left(\frac{-\ln R}{L}\right) \Gamma(L)}, \quad z \geq 0 \quad (2.5)$$

where  $z = -\ln \hat{R}$  and  $L$  and  $\left(\frac{-\ln R}{L}\right)$  are parameters.

Let

$$L^* = \frac{(-\ln \hat{R})^2}{\hat{V}} \quad (2.6)$$

and

$$\hat{L} = L^* + 2.25 \quad (2.7)$$

$L^*$  is the method-of-moments estimate of the shape parameter. A constant term 2.25 is added to  $L^*$ , the shape parameter estimate in the proposed modified log-gamma procedure. The lower  $(1-\alpha)$  confidence bound,  $\underline{R}(1-\alpha)$  is given by solving the equation

$$\underline{R}(1-\alpha) = \hat{R}^{(2\hat{L}/\chi_{2\hat{L},\alpha}^2)} \quad (2.8)$$

where  $\chi_{2\hat{L},\alpha}^2$  is the lower  $\alpha$ -quantity of the chi-square distribution with  $2\hat{L}$  degrees of freedom. Interpolation is required if  $2\hat{L}$  is noninteger.

If there are zero failures in the system ( $\sum f_i = 0$ ),  
let

$$N^* = \frac{k}{\sum_{i=1}^k \frac{1}{n_i}} \quad (2.9)$$

where  $N^*$  is defined to be the effective sample size. Then the lower  $1 - \alpha$  confidence bound  $\underline{R}(1 - \alpha)$  is computed according to a binomial confidence bound based on zero failures out of  $N^*$  trials (i.e.,  $\underline{R}(1 - \alpha) = \sqrt[N^*]{1 - \alpha}$ ). If  $N^*$  is noninteger then linear interpolation is recommended in the proposed procedure but it is not necessary because the same formula could be used for  $N^*$  an integer.

The modified log-gamma method has been described here for both zero failures and one or more failures in series. The more general form of this method was applied to an actual missile system configuration to determine the lower confidence bounds. The program used to evaluate its accuracy has been included in Appendix B. The complete listing and definitions of the variables used in the program are listed in Appendix A. A description of the more generalized method is described as it was applied to the specific missile system simulated.

In the fleet missile system examined there were different groups of missiles with different configurations. The population was therefore not homogeneous and weights were

assigned to the different groups. There were 14 components in the system modeled and eight mixture weights for the subgroups. The input data consisted of  $f_i$  (the number of failures in the  $i$ -th component),  $n_i$  (the sample size for the  $i$ -th component),  $M_i$  (the exponent of each component) and  $C_j$  (the weights applied to each subgroup). Point estimates for this system were defined as follows

$$\hat{R}_i = 1 - \frac{f_i}{n_i} \quad (2.10)$$

$$\hat{P}_R = \prod_{i=1}^5 R_i^{M_i} \quad (2.11)$$

$$\hat{P}_N = \prod_{i=6}^{10} R_i^{M_i} \quad (2.12)$$

with the subgroup reliability point estimates being

$$\begin{aligned} \hat{R}^{(1)} &= \hat{P}_R \hat{R}_{11} \hat{R}_{12} \\ \hat{R}^{(2)} &= \hat{P}_N \hat{R}_{11} \hat{R}_{12} \\ \hat{R}^{(3)} &= \hat{P}_R \hat{R}_{13} \hat{R}_{14} \\ \hat{R}^{(4)} &= \hat{P}_N \hat{R}_{13} \hat{R}_{14} \\ \hat{R}^{(5)} &= \hat{P}_R \hat{R}_{13} \hat{R}_{12} \\ \hat{R}^{(6)} &= \hat{P}_N \hat{R}_{13} \hat{R}_{12} \end{aligned} \quad (2.13)$$

$$\hat{R}^{(7)} = \hat{p}_R \hat{R}_{11} \hat{R}_{14}$$

$$\hat{R}^{(8)} = \hat{p}_N \hat{R}_{11} \hat{R}_{14}$$

and

$$\hat{R} = \sum_{j=1}^8 c_j \hat{R}^{(j)} \quad (2.14)$$

The variance of  $-\ln \hat{R}$  is then estimated by  $\hat{V}$  given by equation (2.15)

$$\hat{V} = \frac{1}{\hat{R}^2} \sum_{i=1}^8 \sum_{j=1}^8 c_i c_j \hat{R}^{(i)} \hat{R}^{(j)} S_{ij} \quad (2.15)$$

where  $S_{ij}$  estimates the  $\text{cov}(z^{(i)}, z^{(j)})$  and where  $z^{(i)} = -\ln \hat{R}^{(i)}$ . The estimates  $S_{ij}$  are found by solving the following equations.

$$z_i = -\ln \hat{R}_i \quad (2.16)$$

$$\bar{R} = \exp\left(-\frac{\sum_{i=1}^{14} M_i z_i}{\sum_{i=1}^{14} M_i}\right) \quad (2.17)$$

and

$$V_R = (1 - \bar{R}) \sum_{i=1}^5 \frac{M_i^2}{n_i} \quad (2.18)$$

$$V_N = (1 - \bar{R}) \sum_{i=6}^{10} \frac{M_i^2}{n_i} \quad (2.19)$$

$$V_i = (1 - \bar{R})/n_i, \quad i = 11, \dots, 14 \quad (2.20)$$

Then the  $S_{ij}$ 's are solved by the equations listed in the program in Appendix B and repeated below.

$$\begin{array}{ll}
 S(1,1) = V_R + V_{11} + V_{12} & S(2,6) = V_N + V_{12} \\
 S(2,2) = V_N + V_{11} + V_{12} & S(2,7) = V_{11} \\
 S(3,3) = V_R + V_{13} + V_{14} & S(2,8) = V_N + V_{11} \\
 S(4,4) = V_N + V_{13} + V_{14} & S(3,4) = V_{13} + V_{14} \\
 S(5,5) = V_R + V_{13} + V_{12} & S(3,5) = V_R + V_{13} \\
 S(6,6) = V_N + V_{13} + V_{12} & S(3,6) = V_{13} \\
 S(7,7) = V_R + V_{11} + V_{14} & S(3,7) = V_R + V_{14} \\
 S(8,8) = V_N + V_{11} + V_{14} & S(3,8) = V_{14} \\
 S(1,2) = V_{11} + V_{12} & S(4,5) = V_{13} \\
 S(1,3) = V_R & S(4,6) = V_N + V_{13} \quad (2.21) \\
 S(1,4) = 0 & S(4,7) = V_{14} \\
 S(1,5) = V_R + V_{12} & S(4,8) = V_N + V_{14} \\
 S(1,6) = V_{12} & S(5,6) = V_{13} + V_{12} \\
 S(1,7) = V_R + V_{11} & S(5,7) = V_R \\
 S(1,8) = V_{11} & S(5,8) = 0 \\
 S(2,3) = 0 & S(6,7) = 0 \\
 S(2,4) = V_N & S(6,8) = V_N \\
 S(2,5) = V_{12} & S(7,8) = V_{11} + V_{14}
 \end{array}$$

Finally,

$$\hat{L} = \frac{(-\ln \hat{R})^2}{\hat{V}} + 2.25 \quad (2.22)$$



and DF, the degrees of freedom, is equal to

$$DF = 2\hat{L} \quad (2.23)$$

Thus

$$\underline{R}(1 - \alpha) = \hat{R}^{(DF/\chi_{DF, \alpha}^2)} \quad (2.24)$$

### III. EVALUATION PROCEDURE

The equation for system reliability is

$$R_s = \sum_{j=1}^L w_j \prod_{i=1}^k p_i^{M_i} \quad (3.1)$$

where

- L = number of subsystems
- $w_j$  = the weighting factor of the j-th subsystem
- k = the number of components
- $p_i$  = the reliability of the i-th component
- $M_i$  = the exponent of the i-th component

The computer program modeled a system that had 8 subsystems and 14 components. System reliability (RS) was determined for each case and a lower confidence bound for  $\alpha = .1$  and  $\alpha = .2$  was computed. Random numbers were drawn from a shuffled random number generator [Ref. 3]. Inverse chi-square values were determined using the international mathematical and statistical library (IMSL) routine called MDCHI. All computations were done in single precision arithmetic, coded in FORTRAN, using an IBM 360 computer.

#### A. ZERO FAILURE VS ONE FAILURE CASE

An examination of two cases revealed a shortcoming and a motivation for evaluating the modified log-gamma procedure. These two examples are considered below.

Example 1.

Let  $k$ , the number of components in the system, be 14 and let  $R_i$ , the component reliabilities, all equal .99. The sample sizes (mission trials) and failures for each component are listed in Table I. The lower 90% confidence limit on system reliability is desired.

Table I

	Component													
	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$M_i$ : # mission trials	10	10	10	10	500	10	10	10	10	500	10	10	10	10
$f_i$ : # failures	0	0	0	0	0	0	0	0	0	0	0	0	0	0

When the  $\sum f_i = 0$  the modified log-gamma procedure defines  $N^*$ , the effective sample size, as

$$N^* = \frac{k}{\sum_{i=1}^k \frac{1}{n_i}} \quad (3.2)$$

For the data given in the table above  $N^*$  is equal to 11.628. For this procedure the lower  $1-\alpha$  confidence bound  $\underline{R}(1-\alpha)$  is computed according to a binomial confidence bound based on zero failures out of  $N^*$  trials. The value obtained for  $\underline{R}(1-\alpha)$  was .820.

Example 2.

Let sample sizes and  $f_i$  (failures for each component) be given in Table II. Again the lower 90% confidence limit on system reliability is desired.

Table II

	Component													
	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$M_i$ : # mission trials	10	10	10	10	500	10	10	10	10	500	10	10	10	10
$f_i$ : # failures	0	0	0	0	1	0	0	0	0	0	0	0	0	0

When  $\sum f_i \neq 0$  the modified log-gamma method solves for  $\underline{R}(1-\alpha)$  (the lower confidence bound) by the following procedure.

Let

$$\hat{R}_i = 1 - \frac{f_i}{n_i},$$

$$\hat{R}_5 = .998, \quad (3.3)$$

$$\hat{R}_i = 1 \text{ for } i \neq 5$$

be the point estimate of the  $i$ -th component reliability.

Then

$$\hat{R} = \prod_{i=1}^k \hat{R}_i = .998 \quad (3.4)$$

is the point estimate for system reliability. Define

$$\bar{R} = \hat{R}^{1/k} = .99986 \quad (3.5)$$

$$\hat{V} = (1-\bar{R}) \sum_{i=1}^k \frac{1}{n_i} = .000172 \quad (3.6)$$

where  $\hat{V}$  estimates the variance of  $-\ln \hat{R}$ .

Let

$$L^* = \frac{(-\ln \hat{R})^2}{\hat{V}} = .02328 \quad (3.7)$$

and define

$$\hat{L} = L^* + 2.25 = 2.27328 \quad (3.8)$$

where 2.25 is the correction term and  $L^*$  is the method-of-moments estimate of the shape parameter. Then the lower  $1-\alpha$  confidence bound,  $\underline{R}(1-\alpha)$  is computed by solving

$$\underline{R}(1-\alpha) = \hat{R}^{(2\hat{L}/\chi_{2\hat{L},\alpha}^2)} \quad (3.9)$$

where  $\chi_{2\hat{L},\alpha}^2$  is the lower  $\alpha$  quantity of the chi-square distribution with  $2\hat{L}$  degrees of freedom. In example 2  $\underline{R}(1-\alpha)$  is equal to .993

These two examples have shown the shortcoming of this method. The lower confidence bound for one failures is higher than the lower confidence bound for zero failures.

## B. SIMULATION RESULTS

The lower confidence bound values obtained for the twelve cases studied have been listed in Table III.  $R_S$  is the system reliability, ACV is the actual confidence value computed by the modified log-gamma method and  $R_{(1-\alpha)*500}$  is the percentile value of the 500 ordered  $R_{(1-\alpha)}$  estimates for  $\alpha = .1$  and  $\alpha = .2$ .  $N(I)$ ,  $RI(I)$  and  $W(I)$  are the respective sample sizes, reliabilities and weights assigned to each case.

For example, in case number 3 the number of components  $k$ , is equal to 14 with the sample sizes equal to 50 for  $i \neq 5$  or 10 and 250 for  $i = 5$  or 10. The reliabilities of each component is .99 and the 8 weights are all equal to .125. System reliability,  $R_S$ , was computed to be .816 and for  $\alpha = .1$  the 450-th value in the ordered 500 LCL estimates was .895. The  $R_S$  value of .816 was the 35th of the 500 ordered LCL estimates yielding an actual confidence level of 7.8%. Likewise for  $\alpha = .2$  the 400-th value in the ordered 500 LCL estimates was .898. The  $R_S$  value of .816 was the 13-th of the 500 ordered LCL estimates yielding an actual confidence level of 2.8%. In only one case (case 8) did the actual confidence value approach that of the system reliability as a lower bound.

An examination of the MLG (modified log-gamma) procedure questioned the inclusion of the correction term 2.25. Additional simulations were run on the same twelve cases when this correction term was removed and the degrees of freedom bounded

below by 1.0. The results obtained from this modification, while an improvement, were still far from providing accurate lower bounds on the system. The values determined from these runs are listed in Table IV. ACV values of 100% indicate that the system reliability was greater than all 500 estimates.

It would appear that in order to generate more estimates less than RS the exponent,  $2L/\chi_{2L,\alpha}^2$ , needs to take on larger values. Adding a constant term such as 2.25 yields more values for  $R(1-\alpha)$  that are greater than RS. Indeed, Tables III and IV did show this to be the case. As the exponent becomes larger (the chi-squared value smaller) the confidence level decreases. The estimate for  $\hat{L}$  used in generating the values listed in Table IV seem more accurate when used in the modified log-gamma procedure.

This modification still left much room for improvement. A closer reivev of the MLG method pointed to the estimate of the shape parameter as a possible cause of the extreme results. Since  $Z = -\ln \hat{R}$  its distribution was approximated by a two-parameter gamma distribution. Then

$$f(z) = \frac{z^{L-1} e^{-Lz/\ln R}}{\left(-\frac{\ln R}{L}\right)^L \Gamma(L)} , \quad z \geq 0 \quad (3.10)$$

where  $L$  and  $\left(-\frac{\ln R}{L}\right)$  are the parameters. Then

$$E(z) = L \cdot \frac{(-\ln R)}{L} = -\ln R \quad (3.11)$$

and

$$\text{Var}(z) = L \left( \frac{-\ln R}{L} \right)^2 = \frac{\ln^2 R}{L} \quad (3.12)$$

Note:

$$L = \frac{\ln^2 R}{\text{Var}(z)} = \frac{[E(z)]^2}{\text{Var}(z)} \quad (3.13)$$

The proposed estimator  $\hat{L}$  for  $L$  is

$$\hat{L}_1 = \frac{z^2}{\widehat{\text{Var}}(z)} \quad (3.14)$$

Since  $L = \frac{[E(z)]^2}{\text{Var}(z)}$  it would appear that  $\hat{L} = \frac{[\widehat{E}(z)]^2}{\widehat{\text{Var}}(z)}$  would

be a better estimator for  $L$ . Since

$$[E(z)]^2 = E(z^2) - \text{Var}(z)$$

we have

$$\hat{L} = \frac{\widehat{E}(z^2) - \widehat{\text{Var}}(z)}{\widehat{\text{Var}}(z)} \quad (3.15)$$

and since  $z^2$  is unbiased for  $E(z^2)$  we get

$$\hat{L} = \frac{z^2 - \widehat{\text{Var}}(z)}{\widehat{\text{Var}}(z)} = \frac{z^2}{\widehat{\text{Var}}(z)} - 1 \quad (3.16)$$



Note that this is a departure from  $\hat{L}_1$  in the proposed method. Thus the shape parameter  $L$  can be estimated by Eq. 3.16 above. This estimate is different from the original version of the MLG method.

Substituting this new value for  $L$  and bounding the degrees of freedom by 1.0, so as not to obtain a negative value, the results show a little more improvement. The results obtained from this second modification are listed in Table V.

TABLE III

Accuracy of  $R(1-\alpha)$  as a  $100(1-\alpha)\%$  Lower Confidence Limit for RS  
 (Correction Term Equal to 2.25)

CASE NO.	K	N(I), R(I) AND W(J)	RS	$\alpha$ ALPHA	P OF $(1-\alpha)*500$	ACV	STANDARD DEVIATION OF $R(1-\alpha)$
1	14	N(I)=10, I=1,2,...,14 EXCEPT N(5)=50, N(10)=50	.816	.1	.872	66.4%	.068
						29.4%	.062
2	14	R(I)=.99, I=1,2,...,14 W(J)=.125, J=1,2,...,8 N(I)=20, I=1,2,...,14 EXCEPT N(5)=100 AND N(10)=100	.816	.2	.900 .903	40.2%	.052
						20.8%	.048
3	14	R(I)=.99, I=1,2,...,14 W(J)=.125, J=1,2,...,8 N(I)=50, I=1,2,...,14 EXCEPT N(5)=250 AND N(10)=250	.816	.1	.895 .898	7.8%	.029
						2.8%	.028
4	14	R(I)=.99, I=1,2,...,14 W(J)=.125, J=1,2,...,8 N(I)=10, I=1,2,...,14 EXCEPT N(5)=80, N(10)=80	.816	.2	.903 .889	43.4%	.067
						27.2%	.061

TABLE III (Continued)

CASE NO.	K	N(I), RI(I) AND W(J)	RS	ALPHA	P OF (1- $\alpha$ )*500	ACV	STANDARD DEVIATION OF R (1- $\alpha$ )
5	14	N(I)=20 EXCEPT I=1,2,...,14 N(5)=160 AND N(10)=160	.816	.1 .2	.858 .900	30.6% 12.2%	.048 .044
6	14	RI(I)=.99, I=1,2,...,14 W(J)=.125, J=1,2,...,8 N(I)=50 EXCEPT I=1,2,...,14 N(5)=400 AND N(10)=400	.816	.1 .2	.855 .898	5.6% 1.0%	.026 .025
7	14	RI(I)=.99, I=1,2,...,14 W(J)=.125, J=1,2,...,8 N(I): 5 4 7 RI(I): .998 .99 .995 N(I): 9 20 5 4 RI(I): .98 .957 .958 .99 N(I): 7 9 20 5 RI(I): .955 .958 .998 .99 N(I): 4 7 9 RI(I): .993 .98 .99 W(J): .0625 .125 .125 .125 .0625 .250 .125 .125	.683	.1 .2	.523 .878	83.0% 82.8%	.090 .070

TABLE III (Continued)

CASE NO.	K	N(I), RI(I) AND W(J)	RS	ALPHA	R OF (1- $\alpha$ )*500	ACV	STANDARD DEVIATION OF R (1- $\alpha$ )
8	14	N(I): 20 16 28 36	.883	.1 .2	.531 .538	44.0% 26.4%	.041 .036
		RI(I): .998 .99 .995 .98					
		N(I): 80 20 16 28					
		RI(I): .997 .998 .99 .995					
		N(I): 36 80 20 16					
		RI(I): .998 .998 .99 .993					
N(J): 28 36	.883	.1 .2	.931 .937	20.6% 5.4%	.023 .021		
RI(I): .98 .99							
W(J): .0625 .125 .125 .125							
RI(I): .998 .99 .995 .98							
N(I): 50 40 70 90							
RI(I): .997 .998 .99 .995							
9	14	N(I): 50 200 50 40	.883	.1 .2	.931 .937	20.6% 5.4%	.023 .021
		RI(I): .998 .998 .99 .993					
		N(I): 70 90					
		RI(I): .98 .99					
		W(J): .0625 .125 .125 .125					
		RI(I): .998 .998 .99 .993					

TABLE III (Continued)

CASE NO.	K	N(I), R(I) AND W(J)	RS	ALPHA	F CF (1- $\alpha$ )*500	ACV	STANDARD DEVIATION OF R (1- $\alpha$ )
10	14	N(I): 15 5C 20 30	.821	.1	.886	38.4%	.049
		RI(I): .995 .95 .957 .98					
		N(I): 100 5 20 10					
		RI(I): .998 .99 .993 .99					
11	14	N(I): 20 100 15 30	.665	.2	.787	23.4%	.057
		RI(I): .997 .953 .97 .99					
		N(I): 8 7					
		RI(I): .98 .97					
12	14	W(J) = .125, J=1,2,....,8	.907	.2	.939	61.6%	.031
		N(I): 15 5C 20 30					
		RI(I): .985 .98 .987 .97					
		N(I): 100 5 20 10					
11	14	N(I): 20 100 15 30	.665	.2	.801	27.4%	.055
		RI(I): .987 .983 .96 .98					
		N(I): 8 7					
		RI(I): .97 .96					
12	14	W(J) = .125, J=1,2,....,8	.907	.2	.536	61.6%	.038
		N(I): 15 50 20					
		RI(I): .9975 .995 .9985					
		N(I): 30 100 5 20					
11	14	N(I): 99 .995 .995 .9965	.907	.2	.939	43.2%	.031
		RI(I): .99 .995 .995 .9965					
		N(I): 10 20 100					
		RI(I): .995 .985 .9965					
12	14	W(J) = .125, J=1,2,....,8	.907	.2	.939	43.2%	.031
		N(I): 15 30 8 7					
		RI(I): .585 .955 .59 .985					
		W(J) = .125, J=1,2,....,8					

TABLE IV

Accuracy of  $R(1-\alpha)$  as a  $100(1-\alpha)\%$  Lower Confidence Limit for RS  
 (Correction term equal to 0.0,  $DF \geq 1.0$ )

CASE NO.	K	N(I), RI(I) AND W(J)	RS	ALPHA	P OF (1-a)*500	ACV	STANDARD DEVIATION OF R (1-a)
1	14	N(I)=10, I=1,2,...,14 EXCEPT N(5)=50, N(10)=50	.816	.1 .2	.816 .859	100.2 69.42	.148 .074
3	14	RI(I)=.99, I=1,2,...,14 W(J)=.125, J=1,2,...,8	.816	.1 .2	.886 .894	10.02 2.22	.028 .027
2	14	RI(I)=.99, I=1,2,...,14 W(J)=.125, J=1,2,...,8	.816	.1 .2	.818 .857	86.42 59.42	.157 .074

TABLE IV (Continued)

CASE NO.	K	N(I), RI(I) AND W(J)	RS	ALPHA	R OF (1- $\alpha$ )*500	ACV	STANDARD DEVIATION OF R (1- $\alpha$ )
5	14	N(I)=20, I=1,2, ..., 14 EXCEPT N(5)=160 AND N(10)=160	.816	.1 .2	.663 .882	54.8% 19.6%	.051 .041
6	14	RI(I)=.99, I=1,2, ..., 14 W(J)=.125, J=1,2, ..., 8	.816	.1 .2	.888 .894	7.4% 1.2%	.025 .024
7	14	N(I)=50, I=1,2, ..., 14 EXCEPT N(5)=400 AND N(10)=400	.883	.1 .2	.655 .775	100.0% 100.0%	.241 .130
		RI(I)=.99, I=1,2, ..., 14 W(J)=.125, J=1,2, ..., 8					
		N(I): 5 4 99 .995 RI(I): .998 .99 .997 .998 .99					
		N(I): 9 20 5 4 RI(I): .98 .997 .998 .99					
		N(I): 7 9 20 5 RI(I): .995 .998 .998 .999					
		N(I): 4 7 9 RI(I): .993 .98 .99					
		W(J): .0625 .125 .125 .125 .0625 .250 .125 .125					

TABLE IV (Continued)

CASE NO.	K	N(I), RI(I) AND W(J)	RS	$\alpha$ ALPHA	R OF (1- $\alpha$ )*500	ACV	STANDARD DEVIATION OF R (1- $\alpha$ )
8	14	N(I): 20 16 28 36	.883	.1 .2	.887 .506	88.0% 61.8%	.146 .057
		RI(I): .998 .99 .995 .98					
		N(I): 80 20 16 28					
		RI(I): .997 .998 .99 .995					
9	14	N(I): 36 80 20 16	.883	.1 .2	.917 .928	43.0% 8.2%	.022 .020
		RI(I): .998 .998 .99 .993					
		N(I): 28 36					
		RI(I): .98 .99					
9	14	W(J): .0625 .125 .125 .125	.883	.1 .2	.917 .928	43.0% 8.2%	.022 .020
		PI(I): .50 40 70 90					
		N(I): 200 50 40 70					
		RI(I): .597 .958 .59 .995					
9	14	N(I): 50 200 50 40	.883	.1 .2	.917 .928	43.0% 8.2%	.022 .020
		RI(I): .598 .998 .99 .993					
		N(I): 70 90					
		RI(I): .98 .99					
9	14	W(J): .0625 .125 .125 .125	.883	.1 .2	.917 .928	43.0% 8.2%	.022 .020
		PI(I): .50 40 70 90					



TABLE IV (Continued)

CASE NO.	K	N(I), RI(I) AND W(J)	RS	ALPHA	R CF (1- $\alpha$ )*500	ACV	STANDARD DEVIATION OF R (1- $\alpha$ )
10	14	N(I): 15 50 20 30	.821	.1 .2	.860 .874	74.2% 50.4%	.148 .075
		RI(I): .995 .99 .557 .98					
		N(I): 100 5 20 10					
		RI(I): .998 .99 .993 .99					
11	14	N(I): 20 100 15 30	.665	.1 .2	.759 .776	40.6% 11.0%	.062 .053
		RI(I): .997 .993 .97 .99					
		N(I): 8 7					
		RI(I): .98 .97					
12	14	W(J) = .125, J = 1, 2, ..., 8	.507	.1 .2	.886 .906	55.8% 80.6%	.245 .094
		N(I): 15 50 20 30					
		RI(I): .985 .98 .987 .97					
		N(I): 100 5 20 10					
12	14	N(I): 20 100 15 30	.507	.1 .2	.886 .906	55.8% 80.6%	.245 .094
		RI(I): .987 .983 .96 .98					
		N(I): 8 7					
		RI(I): .97 .96					
12	14	W(J) = .125, J = 1, 2, ..., 8	.507	.1 .2	.886 .906	55.8% 80.6%	.245 .094
		N(I): 15 50 20 30					
		RI(I): .9975 .995 .9985					
		N(I): 30 100 5 20					
12	14	N(I): 10 20 100	.507	.1 .2	.886 .906	55.8% 80.6%	.245 .094
		RI(I): .99 .995 .9965					
		N(I): 15 30 8 7					
		RI(I): .985 .995 .99 .985					
12	14	W(J) = .125, J = 1, 2, ..., 8	.507	.1 .2	.886 .906	55.8% 80.6%	.245 .094
		N(I): 15 50 20 30					
		RI(I): .9975 .995 .9985					
		N(I): 30 100 5 20					
12	14	N(I): 10 20 100	.507	.1 .2	.886 .906	55.8% 80.6%	.245 .094
		RI(I): .99 .995 .9965					
		N(I): 15 30 8 7					
		RI(I): .985 .995 .99 .985					
12	14	W(J) = .125, J = 1, 2, ..., 8	.507	.1 .2	.886 .906	55.8% 80.6%	.245 .094
		N(I): 15 50 20 30					
		RI(I): .9975 .995 .9985					
		N(I): 30 100 5 20					
12	14	N(I): 10 20 100	.507	.1 .2	.886 .906	55.8% 80.6%	.245 .094
		RI(I): .99 .995 .9965					
		N(I): 15 30 8 7					
		RI(I): .985 .995 .99 .985					
12	14	W(J) = .125, J = 1, 2, ..., 8	.507	.1 .2	.886 .906	55.8% 80.6%	.245 .094
		N(I): 15 50 20 30					
		RI(I): .9975 .995 .9985					
		N(I): 30 100 5 20					
12	14	N(I): 10 20 100	.507	.1 .2	.886 .906	55.8% 80.6%	.245 .094
		RI(I): .99 .995 .9965					
		N(I): 15 30 8 7					
		RI(I): .985 .995 .99 .985					

TABLE V

Accuracy of  $R(1-\alpha)$  as a  $100(1-\alpha)\%$  Lower Confidence Limit for RS  
 (Correction term equal to  $-1.0$ ;  $DF > 1.0$ )

CASE NO.	K	$N(I), RI(I)$ AND $W(J)$	RS	$\alpha$ ALPHA	P OF $(1-\alpha)^{*500}$	ACV	STANDARD DEVIATION OF R $(1-\alpha)$
1	14	$N(I)=10, I=1,2,\dots,14$ EXCEPT $N(5)=50, N(10)=50$	.816	.1 .2	.816 .752	100.8% 88.8%	.242 .155
		$RI(I)=.99, I=1,2,\dots,14$ $W(J)=.125, J=1,2,\dots,8$					
2	14	$N(I)=20, I=1,2,\dots,14$ EXCEPT $N(5)=100$ AND $N(10)=100$	.816	.1 .2	.833 .833	86.6% 53.8%	.180 .078
		$RI(I)=.59, I=1,2,\dots,14$ $W(J)=.125, J=1,2,\dots,8$					
3	14	$N(I)=50, I=1,2,\dots,14$ EXCEPT $N(5)=250$ AND $N(10)=250$	.816	.1 .2	.880 .891	11.8% 3.2%	.027 .026
		$RI(I)=.99, I=1,2,\dots,14$ $W(J)=.125, J=1,2,\dots,8$					
4	14	$N(I)=10, I=1,2,\dots,14$ EXCEPT $N(5)=80, N(10)=80$	.816	.1 .2	.700 .799	94.6% 86.4%	.245 .147
		$RI(I)=.99, I=1,2,\dots,14$ $W(J)=.125, J=1,2,\dots,8$					

TABLE V (Continued)

CASE NO.	K	N(I), R(I) AND W(J)	RS	ALPHA	F CF (1- $\alpha$ )*500	ACV	STANCARC DEVIATION OF R (1- $\alpha$ )
5	14	N(I)=20, I=1,2,...,14 EXCEPT N(5)=160 AND N(10)=160	.816	.1 .2	.848 .867	74.4% 33.6%	.137 .069
		RI(I)=.99, I=1,2,...,14 W(J)=.125, J=1,2,...,8					
6	14	N(I)=50, I=1,2,...,14 EXCEPT N(5)=400 AND N(10)=400	.816	.1 .2	.884 .892	7.8% 1.2%	.024 .024
		RI(I)=.99, I=1,2,...,14 W(J)=.125, J=1,2,...,8					
7	14	N(I): 5 4 7 RI(I): .998 .95 .995	.883	.1 .2	.695 .775	100.0% 100.0%	.303 .225
		N(I): 5 20 5 4 RI(I): .98 .997 .998 .99 N(I): 7 9 20 5 RI(I): .995 .998 .998 .99 N(I): 4 7 9 RI(I): .993 .98 .99 W(J): .0625 .125 .125 .125 .0625 .250 .125 .125					

TABLE V (Continued)

CASE NO.	K	N(I), RI(I) AND W(J)	RS	ALPHA	R OF (1- $\alpha$ )*500	ACV	STANDARD DEVIATION OF R (1- $\alpha$ )
8	14	N(I): 20 16 28 36	.883	.1	.866	97.8%	.255
		RI(I): .598 .95 .995 .98					
		N(I): 80 20 16 28					
		RI(I): .597 .958 .59 .995					
9	14	N(I): 36 80 20 16	.883	.2	.908	62.0%	.652
		RI(I): .998 .958 .59 .993					
		N(I): 28 36					
		RI(I): .98 .99					
9	14	W(J): .0625 .125 .125 .125	.883	.1	.919	13.6%	.626
		RI(I): .998 .99 .995 .98					
		N(I): 50 40 70 90					
		RI(I): .997 .998 .99 .995					
9	14	N(I): 200 50 40 70	.883	.2	.919	13.6%	.626
		RI(I): .997 .998 .99 .995					
		N(I): 90 200 50 40					
		RI(I): .998 .958 .59 .993					
9	14	N(I): 70 90	.883	.2	.919	13.6%	.626
		RI(I): .58 .99					
		W(J): .0625 .125 .125 .125					
		RI(I): .998 .99 .995 .98					

TABLE V (Continued)

CASE NO.	K	N(I), RI(I) AND W(J)	RS	ALPHA	P OF (1- $\alpha$ )*500	ACV	STANDARD DEVIATION OF R (1- $\alpha$ )
10	14	N(I): 15 50 20 30	.821	.1	.854	86.0%	.294
		RI(I): .995 .99 .997 .98					
		N(I): 100 5 20 10					
		RI(I): .998 .99 .993 .99					
11	14	N(I): 20 100 15 30	.669	.2	.744	56.6%	.056
		RI(I): .997 .993 .97 .99					
		N(I): 8 7					
		RI(I): .98 .97					
12	14	W(J) = .125, J=1,2,3,4,5,6,7,8	.907	.1	.854	100.0%	.334
		N(I): 15 50 20 30					
		RI(I): .985 .98 .987 .97					
		N(I): 100 5 20 10					
12	14	PI(I): .988 .98 .983 .98	.907	.2	.856	95.8%	.163
		N(I): 20 100 15 30					
		RI(I): .987 .983 .96 .98					
		N(I): 8 7					
12	14	RI(I): .97 .96	.907	.2	.856	95.8%	.163
		W(J) = .125, J=1,2,3,4,5,6,7,8					
		N(I): 15 50 20 30					
		RI(I): .9975 .995 .9985					
12	14	N(I): 30 100 5 20	.907	.2	.856	95.8%	.163
		RI(I): .99 .999 .995 .9965					
		N(I): 10 20 100					
		RI(I): .995 .985 .9965					
12	14	N(I): 15 30 8 7	.907	.2	.856	95.8%	.163
		RI(I): .985 .995 .99 .985					
		N(I): 8 7					
		RI(I): .97 .96					
12	14	W(J) = .125, J=1,2,3,4,5,6,7,8	.907	.2	.856	95.8%	.163
		N(I): 15 50 20 30					
		RI(I): .985 .98 .987 .97					
		N(I): 100 5 20 10					

#### IV. CONCLUSIONS

Additional simulations on many more cases would be required to determine the particular conditions under which this modified log-gamma method is reasonably accurate. For the cases examined here the proposed procedure remains suspect in estimating lower confidence bounds on system reliability.

## APPENDIX A

AA	CORRECTION TERM EQUAL TO 2.25 IN THE MODIFIED LOG-GAMMA METHOD
AB	VARIABLE THAT STORES THE DIFFERENCE BETWEEN RS (SYSTEM RELIABILITY) AND RR(400)-- THE 80TH PERCENTILE POINT WHEN ALPHA=0.2
ABS	ABSOLUTE VALUE
AC	VARIABLE THAT STORES THE DIFFERENCE BETWEEN RS (SYSTEM RELIABILITY) AND R(450)-- THE 90TH PERCENTILE POINT WHEN ALPHA=0.1
ALOG	NATURAL LOGARITHM SUBROUTINE
ALPHA	VARIABLE ASSIGNED A VALUE OF 0.1
ALPHA A	VARIABLE ASSIGNED A VALUE OF 0.2
AM	ARRAY THAT STORES THE EXPONENTS---M SUB I
BLHAT	VARIABLE THAT STORES THE L HAT VALUE
CA	ACTUAL CONFIDENCE LEVEL FOR ALPHA=0.1
CALL	FORTRAN CODE FOR ACCESSING SUBROUTINES
CB	ACTUAL CONFIDENCE LEVEL FOR ALPHA=0.2
CONTINUE	FORTRAN CODE TO CLOSE EACH DO LOOP
CA	DUMMY VARIABLE USED TO DETERMINE THE ACTUAL CONFIDENCE LEVEL
DDF	DEGREES OF FREEDOM
DIMENSION	FORTRAN CODE REQUIRED FOR DIMENSIONING ARRAYS
DO	FORTRAN CODE USED TO BEGIN LOOPS
DUM	DUMMY VARIABLE
EA	DUMMY VARIABLE USED TO DETERMINE ACTUAL CONFIDENCE VALUE
EFFN	EFFECTIVE SAMPLE SIZE
END	FORTRAN CODE REQUIRED TO END PROGRAM
EXP	EXPONENTIAL SUBROUTINE
FA	DUMMY VARIABLE USED TO DETERMINE THE ACTUAL CONFIDENCE VALUE
FLOAT	FORTRAN CODE USED TO CHANGE INTEGERS TO DECIMAL VALUES
FORMAT	FORTRAN STATEMENT
GA	DUMMY VARIABLE
GB	DUMMY VARIABLE USED TO DETERMINE THE ACTUAL CONFIDENCE VALUE
GO	FORTRAN CODE USED IN THE --GO TO--STATEMENT
HISTG	SUBROUTINE WHICH GENERATES A HISTOGRAM OF THE DATA AND SUMMARY STATISTICS

I INDEX VARIABLE  
 IER ERROR VARIABLE IN SUBROUTINE MDCHI  
 IF VARIABLE USED TO STORE THE NUMBER OF FAILURES  
 (ALSO PART OF THE FORTRAN --IF-- STATEMENT)  
 II INDEX VARIABLE  
 ISEED VARIABLE THAT STORES THE INITIAL VALUE FOR CALLING  
 RANDOM NUMBERS  
 J INDEX VARIABLE  
 JF VARIABLE THAT STORES THE NUMBER OF FAILURES  
 PER COMPONENT  
 JJ INDEX VARIABLE  
 JM INDEX VARIABLE  
 K VARIABLE THAT STORES THE NUMBER OF COMPONENTS  
 (ALSO USED AS AN INDEXING VARIABLE)  
 KJ INDEX VARIABLE  
 L VARIABLE THAT STORES THE NUMBER OF SUBSYSTEMS  
 (ALSO USED AS AN INDEXING VARIABLE)  
 MC COUNTER VARIABLE  
 MDCHI INVERSE CHI SQUARE SUBROUTINE  
 MM INDEX VARIABLE  
 N ARRAY THAT STORES THE K SAMPLE SIZES  
 NC COUNTER VARIABLE  
 NCASE VARIABLE THAT STORES THE NUMBER OF CASES  
 NN INDEX VARIABLE  
 CVFLOW SUBROUTINE REQUIRED FOR RANDOM NUMBER GENERATION  
 P ARRAY THAT STORES THE UNIFORM RANDOM NUMBERS  
 PD VARIABLE THAT STORES THE ALPHA VALUE OF .1  
 PN VARIABLE THAT STORES A POINT ESTIMATE  
 PR VARIABLE THAT STORES A POINT ESTIMATE  
 R ARRAY THAT STORES THE LOWER CONFIDENCE BOUND  
 VALUE WHEN ALPHA=0.1  
 RB VARIABLE THAT STORES THE R(450) VALUE  
 RBAR VARIABLE THAT STORES R BAR  
 RBN NUMBER OF REENTRY PODS PER MISSILE  
 RCEN DUMMY VARIABLE USED TO COMPUTE RBAR  
 READ FORTRAN STATEMENT  
 REFFN VARIABLE THAT STORES THE INVERSE OF THE EFFECTIVE  
 SAMPLE SIZE  
 RHAT VARIABLE THAT STORES THE SUM OF THE WEIGHTED  
 SUBGROUP RELIABILITY ESTIMATES



RI	ARRAY THAT STORES THE INPUTED RELIABILITY VALUES
RIHAT	ARRAY THAT STORES THE COMPUTED RELIABILITY VALUES
RMEAN	VARIABLE THAT STORES THE MEAN OF THE R ARRAY
RNUM	DUMMY VARIABLE USED TO COMPUTE RBAR
RR	ARRAY THAT STORES THE LOWER CONFIDENCE BOUNDS WHEN ALPHA=0.2
RRB	VARIABLE THAT STORES THE RR(400) VALUE
RRMEAN	VARIABLE THAT STORES THE MEAN OF THE RR ARRAY
RRVAR	VARIABLE THAT STORES THE VARIANCE OF THE RR ARRAY
RS	VARIABLE THAT STORES THE TOTAL SYSTEM RELIABILITY
RUHAT	ARRAY THAT STORES THE SUBGROUP RELIABILITY ESTIMATES
RVAR	VARIABLE THAT STORES THE VARIANCE OF THE F ARRAY
S	ARRAY THAT STORES THE VAR/COV MATRIX
SCR	VARIABLE THAT STORES THE STANDARD DEVIATION OF THE R ARRAY
SDRR	VARIABLE THAT STORES THE STANDARD DEVIATION OF THE RR ARRAY
SGRT	SUBROUTINE THAT SOLVES SQUARE ROOTS
SRAND	SUBROUTINE THAT IS THE SHUFFLED RANDOM NUMBER GENERATOR
STOP	FORTRAN REQUIRED CODE
SUM	DUMMY VARIABLE USED THROUGHOUT THE PROGRAM
TO	PART OF THE FORTRAN --GO TO-- STATEMENT
V	ARRAY THAT STORES 4 VARIANCE ESTIMATES FOR COMPONENTS 11 THROUGH 14
VHAT	VARIABLE THAT STORES THE VARIANCE ESTIMATE FOR -LN(RFAT)
VN	VARIABLE THAT STORES THE VARIANCE ESTIMATE FOR -LN(PN)
VR	VARIABLE THAT STORES THE VARIANCE ESTIMATE FOR -LN(PR)
VX	DUMMY VARIABLE USED IN THE MDCHI SUBROUTINE
VY	DUMMY VARIABLE USED IN THE MDCHI SUBROUTINE
W	ARRAY THAT STORES THE WEIGHTED VALUES OF EACH SUBSYSTEM
WRITE	FORTRAN STATEMENT
Z	DUMMY VARIABLE USED TO DETERMINE RBAR

APPENDIX B

```

DIMENSION R(500),RI(50),W(50),N(50),RIHAT(50),AM(50),
RIJHAT(50),V(4),S(8,8),RR(500),P(500)
CALL CVFLCW
NCASE = 0
ISEED = 134869

READING IN THE INPUT PARAMETERS K,L,RBN,A,ALPHA AND
ALPHAA

READ (5,330) K,L,RBN,AA,ALPHA,ALPHAA

READING IN N(I) --- THE NUMBER OF RANDOM NUMBERS PER
COMPONENT
20 READ (5,340) (N(I),I=1,K)

READING IN THE COMPONENT/FUNCTION RELIABILITIES

READ (5,350) (RI(I),I=1,K)
NCASE = NCASE+1
IF (ISEED.GT.134869) GO TO 30

READING IN THE EXPONENTS M SUB I

READ (5,360) (AM(I),I=1,K)

READING IN THE WEIGHTS FOR EACH SUBSYSTEM/GRUOP
30 READ (5,370) (W(I),I=1,L)

STARTING THE MAIN LCCP FOR 500 SIMULATIONS

DO 190 I=1,500
  LOOPING FOR EACH COMPONENT
  AND DRAWING THE RANDOM NUMBERS
  IF = 0
  DO 50 J=1,K
    JJ = N(J)
    CALL SRAND (ISEED,P,JJ)
    JF = 0
    DO 40 JM=1,JJ
      IF (P(JM).GT.RI(J)) JF=JF+1
40 CONTINUE
    IF = IF+JF
  VARIABLE "IF" COUNTS THE FAILURES
  RIHAT(J) = 1.-(FLOAT(JF)/FLOAT(JJ))
50 CONTINUE

```

```

C
C      IF (IF.EQ.0) GO TO 170
C      COMPUTING THE POINT ESTIMATES
C      PN = 1.
C      PR = 1.
C
C      DO 60 J=1,5
C      FR = (RIHAT(J)**AM(J))*FR
C      PN = (RIHAT(J+5)**AM(J+5))*PN
50  CONTINUE
C
C      COMPUTING THE SUBGROUP RELIABILITY ESTIMATES
C
C      RUHAT(1) = PR*RI(11)*RI(12)
C      RUHAT(2) = PN*RI(11)*RI(12)
C      RUHAT(3) = PR*RI(13)*RI(14)
C      RUHAT(4) = PN*RI(13)*RI(14)
C      RUHAT(5) = PR*RI(13)*RI(12)
C      RUHAT(6) = PN*RI(13)*RI(12)
C      RUHAT(7) = PR*RI(11)*RI(14)
C      RUHAT(8) = PN*RI(11)*RI(14)
C
C      COMPUTING RHAT AND CALLING IT BY THE SAME NAME--RHAT
C      RHAT = 0.
C      DO 70 J=1,L
C      RHAT = (W(J)*RUHAT(J))+RHAT
70  CONTINUE
C
C      ESTIMATING THE VARIANCE OF -LN(RHAT)-----VHAT
C
C      STEP 1: DETERMINING RBAR
C      RNUM = 0.
C      RDEN = 0.
C
C      DO 80 J=1,K
C      RNUM = (-ALOG(RIHAT(J))*AM(J))+RNUM
C      RDEN = AM(J)+RDEN
80  CONTINUE
C
C      Z = (-RNUM)/RDEN
C      IF (Z.LT.0) GO TO 90
C      RBAR = EXP(Z)
C      GO TO 100
90  RBAR = 1./EXP(ABS(Z))
C
C      STEP 2: DETERMINING THE VARIANCE ESTIMATES
C
100 VR = 0.
C      VN = 0.
C
C      DO 110 J=1,5
C      VR = (AM(J)**2/FLOAT(N(J)))+VR
C      VN = (AM(J+5)**2/FLOAT(N(J+5)))+VN
110 CONTINUE
C
C      VR = (1.-RBAR)*VR
C      VN = (1.-RBAR)*VN
C
C      DO 120 J=1,4
C      V(J) = (1.-RBAR)/FLCAT(N(J+10))

```

120 CONTINUE

STEP 3: FINAL SOLUTIONS FOR COVARIANCE ESTIMATES

S(1,1) = VR+V(1)+V(2)  
S(2,2) = VN+V(1)+V(2)  
S(3,3) = VR+V(3)+V(4)  
S(4,4) = VN+V(3)+V(4)  
S(5,5) = VR+V(3)+V(2)  
S(6,6) = VN+V(3)+V(2)  
S(7,7) = VR+V(1)+V(4)  
S(8,8) = VN+V(1)+V(4)  
S(1,2) = V(1)+V(2)  
S(1,3) = VR  
S(1,4) = 0.  
S(1,5) = VR+V(2)  
S(1,6) = V(2)  
S(1,7) = VR+V(1)  
S(1,8) = V(1)  
S(2,3) = 0.  
S(2,4) = VN  
S(2,5) = V(2)  
S(2,6) = VN+V(2)  
S(2,7) = V(1)  
S(2,8) = VN+V(1)  
S(3,4) = V(3)+V(4)  
S(3,5) = VR+V(3)  
S(3,6) = V(3)  
S(3,7) = VR+V(4)  
S(3,8) = V(4)  
S(4,5) = V(3)  
S(4,6) = VN+V(3)  
S(4,7) = V(4)  
S(4,8) = VN+V(4)  
S(5,6) = V(3)+V(2)  
S(5,7) = VR  
S(5,8) = 0.  
S(6,7) = 0.  
S(6,8) = VN  
S(7,8) = V(1)+V(4)

FILLING IN THE REST OF THE VAR/COVAR MATRIX

DO 140 MM=1,L

DO 130 NN=1,L  
S(NN,MM) = S(MM,NN)

130 CONTINUE

140 CONTINUE

SOLVING THE OVERALL EQUATION FOR VHAT

VHAT = 0.

DO 160 J=1,L

DO 150 KJ=1,L

VHAT = W(J)\*W(KJ)\*RUHAT(J)\*RUHAT(KJ)\*S(J,KJ)+VHAT

150 CONTINUE

160 CONTINUE

VHAT = VHAT/(RHAT\*\*2)



```
C      SUM = 0.
      DUM = 0.
C
      DJ 220 I=1,500
      SUM = R(I)+SUM
      DUM = RR(I)+DUM
220  CONTINUE
C
      RMEAN = SUM/500.
      RRMEAN = DUM/500.
      SUM = 0.
      DJM = 0.
C
      DJ 230 I=1,500
      SUM = (R(I)-RMEAN)**2+SUM
      DJM = (RR(I)-RRMEAN)**2+DJM
230  CONTINUE
C
      RVAR = SUM/499.
      RRVAR = DJM/499.
      SDR = SQRT(RVAR)
      SDRR = SQRT(RRVAR)
C
C
C
C      DETERMINING THE ACTUAL CONFIDENCE VALUE
C
C      MC = 0
C
      DC 250 I=1,500
      IF (RS.GT.R(I)) GO TO 240
      DA = R(I)-RS
      EA = RS-R(MC)
      IF (DA.LT.EA) GO TO 260
      CA = FLOAT(MC)/500.
      GJ TO 270
240  MC = MC+1
250  CONTINUE
C
260  CA = FLOAT(I)/500.
270  CA = CA*100.
C
C
C
C      NC = 0
C
      DJ 290 I=1,500
      IF (RS.GT.RR(I)) GO TO 280
      FA = RR(I)-RS
      GA = RS-RR(NC)
      IF (FA.LT.GA) GO TO 300
      CB = FLOAT(NC)/500.
      GJ TO 310
280  NC = NC+1
290  CONTINUE
C
300  CB = FLOAT(I)/500.
310  CB = CB*100.
C
C
C
C      PRINTING THE FINAL RESULTS FOR EACH CASE
C
      WRITE (6,380) NCASE
      WRITE (6,390)
      WRITE (6,400) RS,RB,AC,SDR,CA
      WRITE (6,410)
      WRITE (6,420) RS,RRB,AB,SDRR,CE
      ISEED = ISEED+27
      IF (NCASE.EQ.8) GO TO 320
      GJ TO 20
320  STOP
```

C

```
330 FJRMAT (I2,I2,F5.2,F4.2,2F3.1)
340 FJRMAT (I4I3)
350 FJRMAT (10F6.4/4F6.4)
360 FJRMAT (14F4.2)
370 FJRMAT (8F6.4)
380 FJRMAT ('1',T62,'CASE ',I2)
390 FJRMAT ('0',///'0',T30,'ALPHA=.1')
400 FJRMAT ('0',T35,'RS=',F10.8,T55,'R(450)=',F10.8,T75,
1,'RS-R(450)=' ,F10.8///'0',T35,'STANDARD DEVIATION=' ,
1,F10.8///'0',T35,'ACTUAL CONFIDENCE VALUE=' ,F6.2,' %')
410 FJRMAT ('0',///'0',T30,'ALPHA=.2')
420 FJRMAT ('0',T35,'RS=',F10.8,T55,'RR(400)=' ,F10.8,T75
1,'RS-RR(400)=' ,F10.8///'0',T35,'STANDARD DEVIATION=' ,
1,F10.8///'0',T35,'ACTUAL CONFIDENCE VALUE=' ,F6.2,' %')
END
```

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