AD-A076 550 NAVAL POSTGRADUATE SCHOOL MONTEREY CA F/G 13/13
THE CONSISTENT SECOND-ORDER THEORY OF WAVE/STRUCTURE INTERACTIO--ETC(U)
SEP 79 C J GARRISON NSF-ENG73-04019
UNCLASSIFIED NPS-69-79-010 NL

1 OF 1
AD-
A076550

END
DATE
FILMED
12 79
DDC

2 LEVEL II

NPS-69-79-010

NAVAL POSTGRADUATE SCHOOL

Monterey, California

THE CONSISTENT SECOND-ORDER THEORY OF WAVE/STRUCTURE INTERACTION

by

C. J. Garrison

September 1979

Approved for public release; distribution unlimited.

Prepared for: National Science Foundation
Washington, DC 20550

NAVAL POSTGRADUATE SCHOOL
Monterey, California

THE CONSISTENT SECOND-ORDER THEORY
OF WAVE/STRUCTURE INTERACTION

The consistent second-order theory of the interaction of regular gravity waves with a fixed object in water of finite depth is developed. The theory is carried out for the most general case of a body of arbitrary shape which may extend through the free-surface or be completely immersed. The incident wave evolves in the development as a second-order Stokes' wave. Boundary-value problems are established for both the first- and second-order velocity potentials and a numerical method based on the Green's function is outlined.

The work reported herein has been partially supported by the National Science Foundation, Washington, D.C. 20550.

C. J. Garrison

C. J. Garrison
Associate Professor of
Mechanical Engineering

Approved by:

P. J. Marto

Paul J. Marto, Chairman
Department of
Mechanical Engineering

William M. Tolles

William M. Tolles
Dean of Research

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER 14 NPS-69-79-010	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) 6 THE CONSISTENT SECOND-ORDER THEORY OF WAVE/ STRUCTURE INTERACTION,		5. TYPE OF REPORT & PERIOD COVERED 9 Final rep[redacted]t, Sept. 1979
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) 10 C. J. Garrison		8. CONTRACT OR GRANT NUMBER(s) NSF Grant No. 15 NSF-ENG-73-04019 [redacted]
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Postgraduate School Monterey, CA 93940		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 12 45
11. CONTROLLING OFFICE NAME AND ADDRESS National Science Foundation Washington, DC 20550		12. REPORT DATE 11 Sep[redacted]79
		13. [redacted]BER OF PAGES
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

wave/body interaction
wave forces
nonlinear wave forces
Green's function

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The consistent second-order theory of the interaction of regular gravity waves with a fixed object in water of finite depth is developed. The theory is carried out for the most general case of a body of arbitrary shape which may extend through the free-surface or be completely immersed. The incident wave evolves in the development as a second-order Stokes' wave. Boundary-value problems are established for both the first- and second-order velocity potentials and a numerical method based on the Green's function is outlined.

THE CONSISTENT SECOND-ORDER THEORY OF WAVE/STRUCTURE INTERACTION

by

C. J. Garrison

ABSTRACT

The consistent second-order theory of the interaction of regular gravity waves with a fixed object in water of finite depth is developed. The theory is carried out for the most general case of a body of arbitrary shape which may extend through the free-surface or be completely immersed. The incident wave evolves in the development as a second-order Stokes' wave. Boundary-value problems are established for both the first- and second-order velocity potentials and a numerical method based on the Green's function is outlined.

INTRODUCTION

The determination of forces exerted by gravity waves on large structures immersed in the sea has become of great practical interest in recent years. For example, in the design of bottom-mounted oil storage facilities or large ocean caissons, the wave-induced horizontal and up-lift forces and overturning moments are factors of primary importance. The effect of large amplitude waves in particular is of importance in the determination of the permanence of an ocean structure and, therefore, a higher-order theory appears to have significant practical value. For example, Apelt and Macknight (1976) found measured forces on a ocean caisson model in fairly large-amplitude shallow-water waves to be considerably in excess of calculations based on linear diffraction theory.

With this application in mind then, the solution through the second-order in wave height is developed herein for regular wave interaction with a fixed object of arbitrary shape in water of finite depth. The theoretical development is carried out in a mathematically consistent manner in that all terms through the second-order in wave height are included in the analysis, and, as such, represents the extension of the linear theory presented previously by Garrison and Seetharama Rao (1971). The problem is recognized as a regular perturbation problem in the small parameter (wave height/characteristic body dimension) and the incident wave appears within this framework as the second-order Stokes' wave. A numerical method for solving the resulting equations suitable for digital computer evaluation is outlined.

REVIEW OF LITERATURE

The solution of the linear wave/structure interaction problem is now fairly well-developed for infinite depth as well as for the finite depth case. A number of papers have appeared in the literature dealing with the oscillation of two-dimensional bodies on a free surface in water of infinite depth, a problem which is mathematically similar to the fixed-body/wave interaction problem. Examples include the work of Ursell (1949), Porter (1960), Vugts (1968), and Paulling and Richardson (1962). Dean and Ursell (1959) made both an experimental and theoretical study of small amplitude wave interaction with a fixed, semi-submerged circular cylinder in deep water. Yu and Ursell (1961) treated the problem of a semi-submerged circular cylinder oscillating vertically in water of finite depth.

Somewhat less attention has been given to solving the corresponding three-dimensional problems. Apparently the first analysis of the linear wave interaction with a fixed body was carried out by Havelock (1940) for the case of a pile in water of infinite depth and MacCamy and Fuchs (1954) solved the same problem for water of finite depth. Havelock (1959) also evaluated the added mass and damping coefficients for a heaving semi-immersed sphere in deep water, and Wang (1966) extended this to include the finite depth case. Haskind's relations (Haskinds (1957)) as discussed by Newman (1962) may be used to determine the linear or first-order forces acting on the fixed body from a knowledge of the damping coefficients for the same body oscillating in still water. Garrison (1974) has made this evaluation using Havelock's results for the hemisphere and the results were in good agreement with his direct calculation of the heave force based on the Green's function approach. Kim (1965, 1966) has applied the Green's function approach to compute the excitation forces, added mass and damping coefficients for a semi-immersed ellipsoid in deep water.

Most work on wave/body hydrodynamics reported in the literature treats configurations and conditions which have primary application to the study of ship motion; only recently an interest in large, fixed offshore structures has developed. Even though, probably the first potential flow (or diffraction) solution to a wave force problem having direct application to large ocean structures was obtained over twenty years ago by MacCamy and Fuchs (1954), namely, the small amplitude wave interaction with a fixed vertical circular cylinder (pile) in water of finite depth. This diffraction solution represents the only closed form solution of its type available in the literature and, therefore, is of primary importance to this general problem. Little work of this type having application to large ocean structures has appeared since that time until recently when Garrison and Seetharama Rao (1971) applied the Green's function approach to calculate the first-order pressure distribution and resulting forces acting on a bottom-mounted hemisphere. Their experimental results for this configuration compared well with the theory. Garrison and Chow (1972) then extended this analysis to include fixed bodies of arbitrary shape and compared their theoretical results with experimental results corresponding to two different submerged oil storage vessel configurations. Milgram and Halkyard (1971) also have developed a linear theory and applied their method to certain axisymmetric bodies in deep water. Garrison (1974, 1977) has more recently given details of a practical method based on the Green's function to evaluate added mass and damping coefficients for floating bodies of arbitrary shape in water of finite depth, and at the same time calculate the linear wave forces.

It may be stated that, in general, linear theory is quite well developed and good agreement between theory and experiment has been obtained for wave forces corresponding to small amplitude waves in both the two- and three-dimensional cases. This is true as well for bodies oscillating in a free surface in still water. Experience has also shown that linear theory gives good wave force results when the body is fairly deeply submerged even if the amplitude of the incident wave is not small. However, if the object is surface piercing or not deeply submerged and the water is shallow, nonlinear effects caused by waves of finite amplitude become pronounced. In such cases linear theory is inadequate, and a higher order wave/structure interaction theory becomes necessary.

Nonlinear potential solutions to wave/structure interaction problems are much more limited than linear solutions. Olgilvie (1963) solved for the first- and second-order forces on a horizontal, submerged circular cylinder in water of infinite depth, the wave crests being parallel to the cylinder axis. However, his second-order forces included only the time-average part and, as a consequence, the first-order potential only was needed and obtained. This work, therefore, did not actually represent a complete second-order solution. However, a consistent second-order theory for a horizontal cylinder in still water has been developed by Lee(1968) and Potash(1970). Garrison and Smith(1977) have treated the corresponding wave/fixed-body interaction problem.

Recently there has been some interest in developing analyses for nonlinear wave interaction with three dimensional fixed bodies in water of finite depth. Chakrabarti (1972) made an attempt at the extension of MacCamy and Fuch's (1954) linear solution for the vertical circular cylinder (pile) to the fifth order. However, he treated the free-surface boundary condition improperly so that the results are mathematically inconsistent and, consequently, of little value for making judgements regarding the magnitude of nonlinear diffraction effects. More recently Raman, et.al. (1976, 1977) have also treated this problem although the method of solution is extremely complex and there appears to be no simple means of checking the results.

Isaacson (1977) has argued that the second-order solution for a body with a circular waterline curve does not exist because the free-surface and kinematic boundary condition on the cylinder are not consistent at the mean waterline on the cylinder. However, this inconsistency does not preclude the existance of a solution; it appears that such inconsistencies at the point where two different boundary conditions join is rather common.

In this report the solution through the second-order in wave height is developed for bodies of arbitrary shape held fixed in regular waves in water of finite depth. The problem is formulated as a regular perturbation problem, and the incident wave is shown to represent a second-order Stoke's wave. It is also shown that the second-order problem is the same type of boundary-value problem as the first-order problem, the primary difference being that the second-order free surface boundary condition is nonhomogeneous. The solution is formulated for both the first- and second-order potentials in terms of a Green's function, and the numerical results are presented for several configurations.

FORMULATION OF THE PROBLEM

The problem under consideration is depicted in Figure 1. A rigid object having a characteristic dimension $\bar{a}$ is immersed in water of depth $\bar{h}$, and a train of regular waves propagates in the positive $\bar{x}$-direction. The potential solution is sought to the interaction of the incident wave with

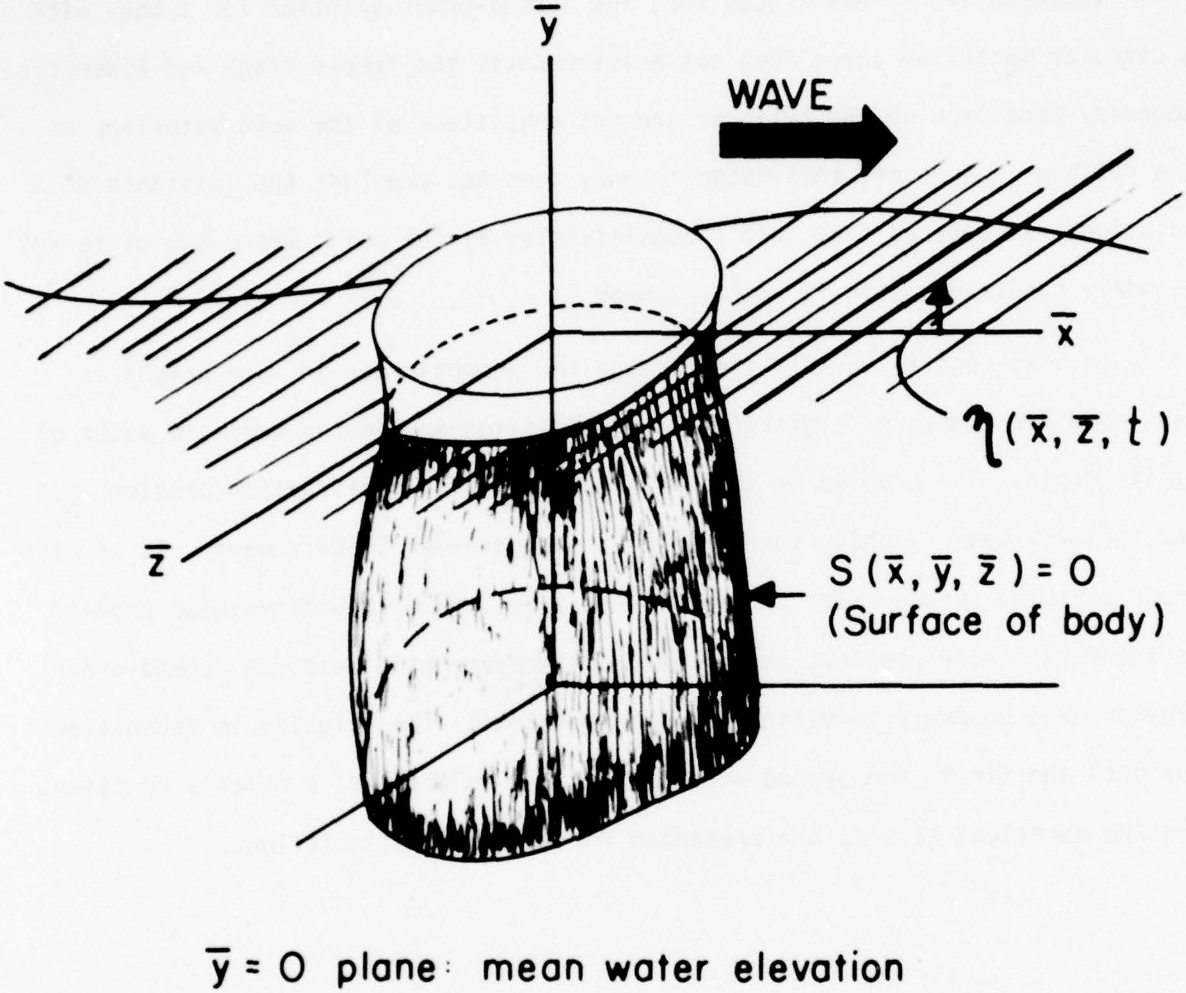

FIGURE 1 DEFINITIONS

the fixed, rigid body through the second-order in the wave amplitude, or more precisely, in the ratio of the wave amplitude to characteristic dimension of the object, $\bar{a}$.

The fluid is assumed to be irrotational so that the velocity potential, ϕ, may be defined as

$$\vec{\bar{q}} = \vec{\bar{\nabla}}\Phi(\bar{x}, \bar{y}, \bar{z}, \bar{t}) \tag{1}$$

where $\vec{\bar{q}}$ denotes the fluid velocity vector. Assuming the fluid to be incompressible, it follows that the velocity potential must satisfy

$$\bar{\nabla}^2\Phi(\bar{x}, \bar{y}, \bar{z}, \bar{t}) = 0 \tag{2}$$

within the fluid region. (The barred quantities denote dimensional quantities.)

On the bottom the kinematic condition specifying zero normal velocity is

$$\bar{\Phi}_{\bar{y}}(\bar{x}, -\bar{h}, \bar{z}, \bar{t}) = 0 \tag{3}$$

where $\bar{h}$ denotes the mean fluid depth. On the rigid, wetted surface of the body, specified by $S(\bar{x}, \bar{y}, \bar{z}) = 0$, the kinematic boundary condition describing the zero normal velocity condition is given by

$$\vec{\bar{\nabla}}\Phi \cdot \vec{\bar{\nabla}}S = 0 \tag{4}$$

The elevation of the free surface above the $\bar{y} = 0$ plane is denoted by $\bar{\zeta}(\bar{x}, \bar{z}, t)$ and the kinematic boundary condition,

$$\frac{D}{Dt}\left[\bar{\eta}(\bar{x},\bar{z},\bar{t}) - \bar{y}\right]_{\bar{y}=\bar{\eta}} = 0 \tag{5}$$

must be satisfied on $\bar{y} = \bar{\eta}$. Moreover, on the free surface the pressure is set equal to zero, and, accordingly, Bernoulli's equation supplies the dynamic free surface boundary condition,

$$\bar{\Phi}_{\bar{t}}(\bar{x},\bar{\eta},\bar{z},\bar{t}) + \frac{1}{2}\left[\bar{\Phi}_{\bar{x}}(\bar{x},\bar{\eta},\bar{z},\bar{t})^2 + \bar{\Phi}_{\bar{y}}(\bar{x},\bar{\eta},\bar{z},\bar{t})^2 + \bar{\Phi}_{\bar{z}}(\bar{x},\bar{\eta},\bar{z},\bar{t})^2\right] + g\bar{\eta}(\bar{x},\bar{z},\bar{t}) = g\bar{h}^* \tag{6}$$

where $\bar{h}^*$ denotes the Bernoulli constant.

It is convenient to cast the boundary value problem thus far established in dimensionless form, and for this purpose introduce the following dimensionless parameters:

$$\left.\begin{aligned} &x = \bar{x}/\bar{a}\ , \quad y = \bar{y}/\bar{a}\ , \quad z = \bar{z}/\bar{a}\ , \quad d = \bar{d}/\bar{a}\ , \quad h = \bar{h}/\bar{a} \\ &H = \bar{H}/2\bar{a}\ , \quad h^* = \bar{h}^*/\bar{a}\ , \quad \nu = \sigma^2\bar{a}/g\ , \quad t = \sigma\bar{t} \\ &\Phi = \sigma\bar{\Phi}/g\bar{a} \end{aligned}\right\} \tag{7}$$

where $\sigma = 2\pi/T$, T being the wave period. The dimensionless time is denoted by t, and h^* denotes a dimensionless Bernoulli constant. As indicated in Equation (7), all the coordinates and length scales are made dimensionless with the characteristic dimension of the fixed object, $\bar{a}$.

Using these parameters, Eqs. (2-6) which define the boundary value problem may be rewritten concisely in dimensionless form as:

$$\nabla^2\Phi(x,y,z,t) = 0 \qquad \text{in the fluid} \tag{8}$$

$$\Phi_y(x,-h,z,t) = 0 \tag{9}$$

$$\Phi_n(x,y,z,t) = 0 \quad on \quad S(x,y,z)=0 \tag{10}$$

$$\Phi_x(x,\zeta,z,t)\,\zeta_x(x,z,t) + \Phi_z(x,\zeta,z,t)\,\zeta_z(x,z,t) - \Phi_y(x,\zeta,z,t) + \nu\,\zeta_t(x,z,t) = 0 \tag{11}$$

$$\Phi_t(x,\zeta,z,t) + \frac{1}{2\nu}\left[\Phi_x(x,\zeta,z,t)^2 + \Phi_y(x,\zeta,z,t)^2 + \Phi_z(x,\zeta,z,t)\right]^2 + \zeta(x,z,t) = h^* \tag{12}$$

Equations (11) and (12) are applicable outside the body on the free surface when the body is surface piercing.

PERTURBATION PROCEDURE

According to the methods of perturbation analysis, the potential function, free surface elevation, and Bernoulli constant may be expanded, respectively, in the small parameter, ϵ, as

$$\Phi(x,y,z,t) = \epsilon\,\phi_1(x,y,z,t) + \epsilon^2\,\phi_2(x,y,z,t) + O(\epsilon^3) \tag{13}$$

$$\zeta(x,z,t) = \epsilon\,\zeta_1(x,z,t) + \epsilon^2\,\zeta_2(x,z,t) + O(\epsilon^3) \tag{14}$$

$$h^* = \epsilon^2 h_2^* + O(\epsilon^3) \tag{15}$$

where ϵ relates to the wave amplitude in a yet undetermined manner.

It may be noted that in Eqs. (11) and (12) the surface elevation, ζ, and consequently, the small parameter, ϵ, does not appear explicitly. Accordingly, it is necessary to expand the particular functions involved so as to form functions which do contain ϵ explicitly. For example, substituting Eq. (14) into (13) and expanding each term in a Taylor series about $\epsilon = 0$ gives:

$$\Phi(x,\eta,z,t) = \epsilon\,\Phi_1(x,0,z,t) + \epsilon^2\left[\Phi_2(x,0,z,t) + \eta_1(x,z,t)\,\Phi_{1_y}(x,0,z,t)\right] + O(\epsilon^3) \quad (16)$$

Similar results can be obtained for the derivatives of ϕ which appear in Eqs. (11) and (12).

Substituting Eqs. (13-16) as well as expressions similar to Eq. (16) for the derivatives into the boundary-value problem given in Eqs. (8-12), the separate boundary-value problems for ϕ_1, ϕ_2, etc. are obtained. The first- and second-order problems may be isolated and are given, respectively, by:

First-order:

$$\nabla^2\Phi_1(x,y,z,t) = 0 \quad (17)$$

$$\Phi_{1_y}(x,-h,z,t) = 0 \quad (18)$$

$$\Phi_{1_n}(x,y,z,t) = 0 \quad \text{on} \quad S(x,y,z) = 0 \quad (19)$$

$$\Phi_{1_y}(x,0,z,t) - \nu\,\eta_{1_t}(x,z,t) = 0 \quad (20)$$

$$\eta_1(x,z,t) + \Phi_{1_t}(x,0,z,t) = 0 \quad (21)$$

Second-order:

$$\nabla^2\Phi_2(x,y,z,t) = 0 \quad (22)$$

$$\Phi_{2_y}(x,-h,z,t) = 0 \quad (23)$$

$$\phi_{2_n}(x,y,z,t) = 0 \qquad \text{on} \qquad S(x,y,z)=0 \tag{24}$$

$$\phi_{2_y}(x,0,z,t) - \nu \zeta_{2_t}(x,z,t) = -\zeta_1(x,z,t)\,\phi_{1_{yy}}(x,0,z,t) + \phi_{1_x}(x,0,z,t)\zeta_{1_x}(x,z,t) + \phi_{1_z}(x,0,z,t)\,\zeta_{1_z}(x,z,t) \tag{25}$$

$$\zeta_2(x,z,t) + \phi_{2_t}(x,0,z,t) = -\zeta_1(x,z,t)\,\phi_{1_{yt}}(x,0,z,t) - \frac{1}{2\nu}\Big[\phi_{1_x}(x,0,z,t)^2 + \phi_{1_y}(x,0,z,t)^2 + \phi_{1_z}(x,0,z,t)^2\Big] + h_2^* \tag{26}$$

Both ϕ_1 and ϕ_2 are, in addition, subject to a suitable radiation condition which limits the scattering disturbance to outgoing waves.

A great deal of similarity is evident between the first- and second-order problems. In fact, the only basic difference lies in the right hand side of the free surface boundary conditions; in contrast to the first-order free surface boundary conditions, the second-order boundary conditions are not homogeneous, the right hand sides of which being dependent on the first-order potential.

Before proceeding to the solution of the boundary-value problems developed, advantage may be taken of the fact that the first-order potential will be represented by a periodic function. Accordingly, we may define the complex potential $u_1(x,y,z)$ as

$$\phi_1(x,y,z,t) = a\,b\,R_e\left[i\,u_1(x,y,z)\,e^{-it}\right] \tag{27}$$

where R_e denotes the real part, and $a = 2\pi\bar{a}/\bar{L}$, $\bar{a}$ being the characteristic dimension of the body and $\bar{L}$ the wave length. The symbol b denotes an unknown real constant. It is also appropriate and common practice in linear interaction problems to express the potential ϕ_1 as the sum,

$$\Phi = \phi_1^I + \phi_1^s \tag{28}$$

where ϕ_1^I denotes the incident wave potential, and ϕ_1^S denotes the scattering potential which is due to the presence of the rigid body. The complex potentials u_1^I and u_1^S are then defined in relation to ϕ_1^I and ϕ_1^s according to the form of Eq. (27) so that it follows that

$$u_1 = u_1^I + u_1^s \tag{29}$$

It is recognized, moreover, that the incident wave potential, ϕ_1^I, must satisfy the first-order problem when no body is present, i.e., ϕ_1^I must satisfy Eqs. (17), (18), (20), and (21). These equations represent simply the boundary-value problem for the well-known, first-order progressive wave. The solution to this in terms of the present notation is given by:

$$u_1^I = -\frac{i}{a}\,\frac{\cosh[a(h+y)]}{\cosh(ah)}\,e^{iax} \tag{30a}$$

In Eq. (30a) a is defined in terms of ν by the familiar expression from linear wave theory

$$\nu = \sigma^2\bar{a}/g = a\ \tanh(ah) \tag{30b}$$

where h denotes the dimensionless water depth.

Now, substituting Eqs. (30a), (29), and (27) into the first-order problem defined by Eqs. (17-21), and eliminating η_1 between Eqs. (20) and (21), the following boundary-value problem is established for the first-order complex scattering potential:

$$\nabla^2 u_1^s(x,y,z) = 0 \tag{31}$$

$$u^s_{1,y}(x,-h,z) = 0 \tag{32}$$

$$u^s_{1,n}(x,y,z) = \frac{1}{\cosh(ah)}\Big[n_y \sinh[a(h+y)] + i n_x \cosh[a(h+y)]\Big] e^{iax} \tag{33}$$

$$\text{on} \quad S(x,y,z) = 0$$

$$u^s_{1,y}(x,0,z) - \nu u^s_1(x,0,z) = 0 \tag{34}$$

The components of the unit vector directed normal to the immersed surface into the fluid are defined as $\vec{n} = \vec{i} n_x + \vec{j} n_y + \vec{k} n_z$. In addition to Eqs. (31-34), $u^S_1(x,y,z)$ is also subject to the usual radiation condition which allows only outgoing scattered waves at a great distance from the body.

Before dealing with the solution to the first-order scattering problem, the second-order problem will first be established. Proceeding in this direction it is appropriate in view of the linearity to express the solution to the second-order problem, Eqs. (22-26), as the sum

$$\Phi_2 = \Phi_2^I + \Phi_2^S \tag{35}$$

where, similar to the first order problem, ϕ_2^I denotes the second-order incident wave potential and ϕ_2^S denotes the scattering potential.

We again envoke the condition that when no body is present, there is no scattered wave so that $\phi_1^S = \phi_2^S = 0$. Moreover, under these conditions the boundary conditions on the immersed surface, Eqs. (19) and (24), are not applicable. Accordingly, we may substitute $\phi_1 = \phi_1^I$ and $\phi_2 = \phi_2^I$, along with the Eq. (30) and (27), into Eqs. (22), (23), (25), and (26) and obtain, upon eliminating η_2 between Eqs. (25) and (26), the following boundary-value problem for the second-order incident wave potential:

$$\nabla^2 \phi_2^I(x,y,z,t) = 0 \tag{36}$$

$$\phi_{2_y}^I(x,-h,z,t) = 0 \tag{37}$$

$$\phi_{2_y}^I(x,0,z,t) + \nu\, \phi_{2_{tt}}^I(x,0,z,t) = -\frac{3}{2}\, b^2(a^2-\nu^2)\sin[2(ax-t)] \tag{38}$$

The periodic solution to the boundary-value problem specified by Eqs. (36-38) may be expressed as

$$\phi_2^I = \frac{3}{4}\, a b^2 \nu\, R_e\left[i\, U_2^I(x,y)\, e^{-i2t}\right] \tag{39}$$

where the second-order complex potential, u_2^I, is given by

$$U_2^I(x,y) = -\frac{1}{2a}\, \frac{\cosh[2a(h+y)]}{\sinh^4(ah)}\, e^{i2ax} \tag{40}$$

This expression is familiar in wave theory and is of exactly the same form as that given by Eagleson and Dean (1966) for second-order Stokes' waves.

To complete the establishment of the boundary-value problem for the second-order scattering potential, it is necessary to substitute the forms $\phi_1 = \phi_1^I + \phi_1^S$ and $\phi_2 = \phi_2^I + \phi_2^S$ into Eqs. (22-26) and eliminate η_2 between Eqs. (25) and (26). In addition, the known expressions for the incident wave potentials, Eqs. (30) with Eqs. (29) and (27) for the first-order potentials, and for the second-order potentials, Eqs. (40) with Eqs. (39) and (35), are utilized to obtain:

$$\nabla^2 \Phi_2^s(x,y,z,t) = 0 \tag{41}$$

$$\Phi_{2_y}^s(x,-h,z,t) = 0 \tag{42}$$

$$\Phi_{2_n}^s(x,y,z,t) = -\frac{3}{4}\frac{a b^2 \nu}{\sinh^4(ah)} R_e\left[\left\{n_x \cosh[2a(h+y)] - i n_y \sinh[2a(h+y)]\right\} e^{-i2(ax-t)}\right] \tag{43}$$

$$\Phi_{2_y}^s + \nu \Phi_{2_{tt}}^s = \frac{a^2 b^2}{2}\Big\{ R_e\Big[i\Big(\frac{1}{\nu} u_{1_y}^s u_{1_{yy}}^s + u_1^I(u_{1_{yy}}^s - 5\nu u_{1_y}^s) \tag{44}$$

$$+ \frac{1}{\nu} u_{1_y}^s(u_{1_{yy}}^I - \nu u_{1_y}^I) - 4 u_{1_x}^s u_{1_x}^I - 2 u_{1_x}^{s^2}$$

$$- 2 u_{1_z}^{s^2} - 3 u_{1_y}^{s^2}\Big) e^{-i2t}\Big]\Big\} + U(x,z)$$

In Eq. (44), U(x,z) is a time independent function which is generated by substitution of ϕ_1 into Eqs. (25) and (26). However, since the function is not needed in the second-order theory for purposes of evaluating the pressures and forces, it is for brevity's sake not written out in full here.

The boundary-value problem now established for ϕ_2^S as given in Eqs. (41-44) is linear, and except for U(x,z) occurring in Eq. (44), is time dependent like e^{-i2t}. Accordingly, it is appropriate to express the solution in the form

$$\Phi_2^s = \frac{3}{4} a b^2 \nu\, R_e\left[i\, u_2^s(x,y,z) e^{-i2t} + i\, \tilde{u}_2^s(x,y,z)\right] \tag{45}$$

where the second term denotes the part of the complex potential which is independent of time. By substitution of Eq. (45) into the boundary-value problem described by Eq. (41-44), separate boundary-value problems arise for u_2^S and $\tilde{u}_2^S$. However, as will be evident subsequently, $\phi_{2_t}^S$

only is needed in order to evaluate the hydrodynamic pressures and resulting forces to the second order and, consequently, the time independent part of ϕ_2^S is of no interest. We, therefore, dismiss it from further consideration and concentrate on the solution u_2^S.

Substituting Eq. (45) into Eq. (41-44) the following boundary value problem is established for the second-order scattering potential:

$$\nabla^2 u_2^s(x,y,z) = 0 \tag{46}$$

$$u^s_{2_y}(x,-h,z) = 0 \tag{47}$$

$$u^s_{2_n}(x,y,z) = \frac{1}{\sinh^4(ah)}\left[n_y \sinh[2a(h+y)] + i n_x \cosh[2a(h+y)]\right] e^{i2ax} \tag{48}$$

$$u^s_{2_y}(x,0,z) - 4\nu\, u_2^s(x,0,z) = f^*(x,z) \tag{49}$$

where

$$f^*(x,z) = \frac{2}{3}\frac{a}{\nu}\Big[\frac{1}{\nu} u^s_{1_y} u^s_{1_{yy}} + u^I_1 (u^s_{1_{yy}} - 5\nu u^s_{1_y}) + \frac{1}{\nu} u^s_{1_y} (u^I_{1_{yy}} - \nu u^I_{1_y}) \tag{50}$$

$$- 4 u^I_{1_x} u^s_{1_x} - 2 u^{s\,2}_{1_x} - 2 u^{s\,2}_{1_z} - 3 u^{s\,2}_{1_y}\Big]_{y=0}$$

The radiation condition limits the scattered waves to outgoing regular waves and is given by

$$\lim_{r \to \infty} \sqrt{r}\left(\frac{\partial}{\partial r} - i a_2\right) u_2^s = 0 \tag{51}$$

where a_2 denotes the dimensionless wave number for the second-order problem which is defined by

$$4\nu = a_2 \tanh(a_2 h) \tag{52}$$

INCIDENT WAVE

At this juncture it is appropriate to completely define the incident wave and specify the unknown constants b and h_2^*. Solving Eqs. (21) and (26) for η_1 and η_2, respectively, and substituting the results into the expression for the elevation of the free surface defined by Eq. (14) gives

$$\zeta(x,z,t) = -\epsilon\, \phi_{1_t}(x,0,z,t) + \epsilon^2 \Big\{ -\phi_{2_t}(x,0,z,t) + \phi_{1_t}(x,0,z,t)\, \phi_{1_{yt}}(x,0,z,t) - \frac{1}{2\nu}\Big[\phi_{1_x}(x,0,z,t)^2 + \phi_{1_y}(x,0,z,t)^2 + \phi_{1_z}(x,0,z,t)^2 \Big] + h_2^* \Big\} + O(\epsilon^3) \tag{53}$$

Equation (53) expresses the free surface elevation in terms of the total potentials and, consequently, includes the effect of the scattered wave as well as the incident wave. However, it is presently of interest to obtain an expression for the free surface elevation of the incident wave alone in the absence of the object. For this purpose, therefore, we set $\phi_1^S = \phi_2^S = 0$ and evaluate Eq. (53) using the known expressions for the incident wave potentials, Eqs. (30) with (27) and Eq. (40) with (39). The result is:

$$\zeta^I(x,z,t) = \epsilon\, b\, R_e\big[e^{i(ax-t)} \big] + \epsilon^2 \Big\{ \frac{b^2(\nu^2 - a^2)}{4\nu} + h_2^* + \frac{a b^2}{4} \cosh(ah) \frac{(2 + \cosh(2ah))}{\sinh^3(ah)} R_e\big[e^{i2(ax-t)} \big] \Big\} + O(\epsilon^3) \tag{54}$$

where η^I denotes the dimensionless surface elevation due to the incident wave in the absence of the body. However, the constant term in Eq. (54) must vanish so

$$h_2^* = \frac{b^2(a^2 - \nu^2)}{4\nu} \tag{55}$$

The remaining terms in Eq. (54) represent sinusoidal variations, the second-order contribution having twice the frequency of the first.

If we define the wave height as the elevation difference between the trough and crest of the incident wave, then ϵb must represent the wave amplitude since the second-order contribution to the surface elevation in Eq. (54) is the same at both the crest and trough, the difference being zero. Therefore, in terms of the dimensionless wave height as defined in Eq. (7) we have:

$$\epsilon b = H \equiv \bar{H}/2\bar{a} \tag{56}$$

where $\bar{H}$ denotes the elevation difference between the crest and trough. The incident wave profile is then specified by the dimensionless surface elevation as:

$$\zeta^I(x,z,t) = H\cos(ax-t) + H^2\frac{a}{4}\frac{\cosh(ah)}{\sinh^3(ah)}(2+\cosh(2ah)) \cdot \cos[2(ax-t)] + O(H^3) \tag{57}$$

Equation (57) agrees with the expression for the second-order Stokes' wave given by, for example, Eagleson and Dean (1966).

PRESSURE, FORCES AND MOMENT

The forces and moments acting on the immersed surface are determined by carrying out surface integrals of the pressure. For this purpose the pressure is obtained from Bernoulli's equation as

$$P(x,y,z,t) = -\rho\,\Phi_{t} - \frac{1}{2}\rho\left[\Phi_{x}^{2} + \Phi_{y}^{2} + \Phi_{z}^{2}\right] - \rho g\bar{y} + \rho g\bar{h} \tag{58}$$

This expression may be cast in dimensionless form by use of Eq. (7), (13), (14), (15), (27), (35), (45), and (56), and the result carried to the second-order in wave height. The resulting expression is then

$$p(x,y,z,t) = -y - Ha\,R_e\left[u_1 e^{-it}\right] - H^2\frac{a^2}{4\nu}\Big\{R_e\Big[\Big(\frac{6\nu^2}{a}u_2 - u_{1_x}^{2} - u_{1_y}^{2} - u_{1_z}^{2}\Big)e^{-i2t}\Big] + |u_{1_x}|^2 + |u_{1_y}|^2 + |u_{1_z}|^2 + \frac{\nu^2}{a^2} - 1\Big\} \tag{59}$$

The dimensionless pressure coefficient expressed by Eq. (59) is defined as

$$p = \frac{P}{\rho g \bar{a}} \tag{60}$$

in which P denotes the fluid pressure.

The first term in Eq. (59) denotes the hydrostatic pressure as y represents the dimensionless depth beneath the mean free surface. The second and third terms are harmonic, the second-order part having twice the frequency of the first-order part. The final second-order term in Eq. (59) is independent of time and gives rise to time-average or steady state components of force and moments.

The components of the dimensionless force and the moment vectors may now be expressed in terms of integrals of the pressure over the wetted surface area. That is, we may write

$$C_i(t) = \iint_S p\, g_i\, ds \quad , \quad i = 1, 2, \cdots 6 \tag{61}$$

where the dimensionless force coefficients associated with the x, y, and z directions are defined, respectively, as

$$C_1(t) = \frac{F_x(t)}{\rho g \bar{a}^3} \tag{62a}$$

$$C_2(t) = \frac{F_y(t)}{\rho g \bar{a}^3} \tag{62b}$$

$$C_3(t) = \frac{F_z(t)}{\rho g \bar{a}^3} \tag{62c}$$

in which $F_x(t)$, $F_y(t)$, and $F_z(t)$ denote the three components of force. Similarly, the moment coefficients are defined as

$$C_4(t) = \frac{M_x(t)}{\rho g \bar{a}^4} \tag{62d}$$

$$C_5(t) = \frac{M_y(t)}{\rho g \bar{a}^4} \tag{62e}$$

$$C_6(t) = \frac{M_z(t)}{\rho g \bar{a}^4} \tag{62f}$$

in which $M_x(t)$, $M_y(t)$, and $M_z(t)$ denote the three components of the wave-induced moment.

The functions g_i are defined as

$$g_1 = n_x \ , \qquad g_2 = n_y \ , \qquad g_3 = n_z \tag{63a}$$

$$g_4 = (d+y)\,n_z - z\,n_y \ , \quad g_5 = z\,n_x - x\,n_z \ , \quad g_6 = x\,n_y - (y+d)\,n_x \tag{63b}$$

The forces and moments in Eq. (62) are defined relative to a coordinate system parallel to O(x,y,z) but shifted downward a distance $\bar{d}$ as is evident from Eq. (63b). The unit vector directed normal to the wetted surface is defined by $\vec{n} = \vec{i}\,n_x + \vec{j}\,n_y + \vec{k}\,n_z$ and dS denotes an element of surface area made dimensionless with the characteristic dimension of the body squared, $\bar{a}^2$.

We may make an initial ordering of the force or moment coefficient as given in Eq. (61) in terms of the small parameter, ϵ, or equivalent H, as

$$C_i(t) = -\iint_S p_0\, g_i\, dS - H\iint_S p_1\, g_i\, dS - H^2\iint_S p_2\, g_i\, dS - O(H^3) \tag{64}$$

where the pressure coefficient is expressed as

$$p = p_0 + H\, p_1 + H^2\, p_2 + O(H^3) \tag{65}$$

and the coefficients, p_0, p_1, and p_2, are defined in an obvious manner by comparison of Eq. (65) with Eq. (59).

It may be noted at this point that, if the body is completely submerged, the surface integrals in Eq. (64) refer simply to the actual surface area of the body which is, of course, specified. However, if the object is surface piercing, S denotes the actual wetted area which is dependent on the instantaneous surface elevation of the water on the body. Accordingly, it is necessary to express the limits of integration on the wetted surface S in terms of η and carry out the expansion retaining terms with coefficients up through H^2.

For this purpose the dimensionless differential surface area may be written as dS = dc dl where dc denotes a dimensionless differential arc length on the wetted surface in the horizontal plane, and dl denotes a dimensionless differential length in an orthogonal direction. Thus, it is possible to express dl in terms of y so that

$$dS = dc\, d\ell = \frac{dc\, dy}{\sqrt{1 - n_y^2}} \qquad (66)$$

The integrals indicated in Eq. (64) are now carried out over the wetted surface with y running from y = -e to y = η. Accordingly, a typical integral in Eq. (64) may be expressed in the form

$$\iint_S p_i\, g_i\, dS = \iint_{y=-e}^{y=\frac{H}{b}\eta_1 + \frac{H^2}{b^2}\eta_2 + \cdots} \frac{p_i\, g_i}{\sqrt{1 - n_y^2}}\, dc\, dy \qquad (67)$$

where e denotes the lower limit of the immersed surface.

The integrals of the type given in Eq. (67) are next expanded in a Taylor series about H = 0 using Leibniz's formula for purposes of evaluating the derivatives of the integrals. Carrying out these expansions, substituting the results into Eq. (64), and retaining terms up to order H^2 yields:

$$C_i(t) = -\iint_{S_0} p_0\, g_i\, dS - H\iint_{S_0} p_1\, g_i\, dS - H^2\Bigg\{\iint_{S_0} p_2\, g_i\, dS$$

$$+ \int_{C_0}\left(p_1 - \frac{\eta_1/b}{2\sqrt{1-n_y^2}}\right)_{y=0} g_i\, \frac{\eta_1}{b}\, dc\Bigg\} + O(H^3) \tag{68}$$

where S_0 denotes the surface area below the plane $y = 0$ and C_0 denotes the closed waterline curve formed by the intersection of the $y = 0$ plane with the surface of the rigid object.

The first term in Eq. (68) represents the hydrostatic force or moment on the body where the associated displaced volume is defined as that beneath the $y = 0$ plane. However, since our interest is in the dynamic forces and moments, we may disregard this first term with the understanding that the buoyant force defined as such should be accounted for in order to determine the final total force or moment.

Finally, applying Eq. (21) and (27) as well as the definitions for p_0, p_1 and p_2 we obtain:

$$C_i(t) = Ha\iint_{S_0} R_e[u_1 e^{-it}]\, g_i\, dS + H^2\Bigg\{\frac{a^2}{4\nu}\iint_{S_0} R_e\Big[\Big(6\frac{\nu^2}{a}u_2 - u_{1x}^2$$

$$- u_{1y}^2 - u_{1z}^2\Big)e^{-i2t}\Big]\, g_i\, dS + \frac{a^2}{2}\int_{C_0}\left[\frac{1}{2\sqrt{1-n_y^2}} - 1\right]$$

$$\cdot R_e[u_1^2 e^{-2it}]\, g_i\, dc + \frac{a^2}{2}\int_{C_0}\left[\frac{1}{2\sqrt{1-n_y^2}} - 1\right]|u_1|^2\, g_i\, dc \tag{69 a}$$

$$+ \frac{a^2}{4\nu}\iint_{S_0}\left[|u_{1x}|^2 + |u_{1y}|^2 + |u_{1z}|^2 + \frac{\nu^2}{a^2} - 1\right] g_i\, dS\Bigg\}$$

$$+ O(H^3)$$

Equation (69a) may also be written in coefficient form as

$$C_i(t) = H\,F_{1_i}\cos(\delta_{1_i} - t) + H^2\left\{F_{2_i}\cos(\delta_{2_i} - 2t) + F_{2_i}^{ss}\right\} + O(H^3) \tag{69b}$$

where the first- and second-order force (or moment) coefficients and phase shift angles are defined by comparison of Eq. (69a) with (69b) as follows:

$$F_{1_i}\, e^{i\delta_{1_i}} = a\iint_{S_o} u_1\, g_i\, dS \tag{69c}$$

$$F_{2_i}\, e^{i\delta_{2_i}} = \frac{a^2}{4\gamma}\iint_{S_o}\left(\frac{6\gamma^2}{a}u_2 - u_{1_x}^2 - u_{1_y}^2 - u_{1_z}^2\right) g_i\, dS + \frac{a^2}{2}\int_{C_o}\left(\frac{1}{2\sqrt{1-n_y^2}} - 1\right) u_1^2\, g_i\, dC \tag{69d}$$

where the dimensionless force coefficients, F_{1_i} and F_{2_i}, are real. The steady state (nonperiodic) force (or moment) coefficient is also defined by comparison of Eqs. (69a) and (69b) as

$$F_{2_i}^{ss} = \frac{a^2}{2}\int_{C_o}\left(\frac{1}{2\sqrt{1-n_y^2}} - 1\right)|u_1|^2\, g_i\, dc + \frac{a^2}{4\gamma}\iint_{S_o}\left[|u_{1_x}|^2 + |u_{1_y}|^2 + |u_{1_z}|^2 + \frac{\gamma^2}{a^2} - 1\right] g_i\, dS \tag{69e}$$

The first term in Eq. (69b) represents the linear solution having the fundamental frequency, σ. The next term which represents a second-order contribution to the force or moment is also harmonic but at twice the fundamental frequency. The last second-order term represents the steady state contribution which is independent of time. This latter force is generally referred to as a drift force in ship hydrodynamics. (See, for example, Maruo (1960).) However, drift force is generally calculated by applying the momentum equation to the diffracted wave rather than by using near field results.

The evaluation of the force or moment coefficients defined by Eq. (69c-e) represents the primary object of this paper. However, in order to carry out the evaluations indicated, it is clear that, given the geometry of the rigid immersed object, it is necessary to evaluate u_1 and u_2 as well as the derivatives of u_1 on the surface area denoted by S_0 and along the waterline curve C_0.

SOLUTION TO THE FIRST-ORDER PROBLEM

Having now established the need for the potentials, u_1 and u_2, we return to the consideration of the solution to the boundary-value problem developed. The practical method of solution of the first-order problem given in Eqs. (31-34) is discussed in depth by Garrison(1974,1978) and, therefore, only the major steps will be outlined here. Following the Green's function method of solution we write u_1^s, as the integral over the immersed surface S_0 as:

$$u_1^s(x,y,z) = \frac{1}{4\pi}\iint_{S_0} f_1(\xi,\eta,\zeta)\; G(x,y,z;\xi,\eta,\zeta;\nu)\; dS \tag{70}$$

where (ξ,η,ζ) denotes points on the immersed surface, f_1 (ξ,η,ζ) denotes the unknown source strength and $dS = \overline{dS}/\overline{a}^2$ denotes the differential surface area on the immersed surface made dimensionless with the characteristic dimension of the object, $\overline{a}$. The function, G $(x,y,z;\xi,\eta,\zeta;\nu)$, denotes the Green's function which must satisfy the equation

$$\nabla^2 G(x,y,z;\xi,\eta,\zeta;\nu) = \delta(x-\xi)\,\delta(y-\eta)\,\delta(z-\zeta) \tag{71}$$

in which δ denotes the Derac delta function. The function, $G(x,y,z;\xi,\eta,\zeta;\nu)$ must also satisfy Eqs. (32), (34), and the radiation condition. Such a function is given by Wehausen and Laitone (1960) as

$$G(x,y,z;\xi,\eta,\zeta;\nu) = \frac{1}{R} + \frac{1}{R'}$$

$$+ 2\,PV\int_0^\infty \frac{(\mu+\nu)\,e^{-\mu h}\cosh[\mu(h+\eta)]\cosh[\mu(h+y)]}{\mu\sinh(\mu h) - \nu\cosh(\mu h)}\,J_0(\mu r)\,d\mu \tag{72a}$$

$$+ i\,\frac{2\pi(a_1^2-\nu^2)\cosh[a_1(\eta+h)]\cosh[a_1(y+h)]}{a_1^2 h - \nu^2 h + \nu}\,J_0(a_1 r)$$

in which

$$R = \left[(x-\xi)^2 + (y-\eta)^2 + (z-\zeta)^2\right]^{1/2} \tag{72b}$$

$$R' = \left[(x-\xi)^2 + (y+2h+\eta)^2 + (z-\zeta)^2\right]^{1/2} \tag{72c}$$

$$r = \left[(x-\xi)^2 + (z-\zeta)^2\right]^{1/2} \tag{72d}$$

The symbol, a_1, is defined in terms of h and ν as the solution of the equation

$$a_1 \tanh(a_1 h) - \nu = 0 \tag{72e}$$

In view of Eqs. (72e) and (30b) the symbol a_1 is clearly equivalent to a. However, in the case of the second-order Green's function, the equivalence does not hold and, therefore, separate notation is maintained. In Eq. (72a), P. V. denotes principal value of the integral.

An alternate series form of the Green's function is also given by Wehausen and Laitone (1960) as

$$G(x,y,z;\xi,\eta,\zeta;\nu) = \frac{2\pi(\nu^2 - a_1^2)\cosh[a_1(h+y)]\cosh[a_1(h+\eta)]}{a_1^2 h - \nu^2 h + \nu}\left[Y_0(a_1 r) - i J_0(a_1 r)\right]$$

$$+ 4\sum_{k=1}^{\infty} \frac{(\mu_k^2 + \nu^2)}{(\mu_k^2 h + \nu^2 h - \nu)} \cos[\mu_k(h+y)]\cos[\mu_k(h+\eta)]\, K_0(\mu_k r) \tag{73a}$$

where J_0 and Y_0 denote, respectively, Bessel functions of the first and second kind of order zero, and K_0 denotes the modified Bessel function of the second kind of order zero. In Eq. (73a) a_1 is defined by Eq. (72e) and the quantities μ_k are defined as real positive roots of the equation

$$\mu_k \tan(\mu_k h) + \nu = 0 \tag{73b}$$

The solution to the boundary-value problem stated in Eqs.(31-34) is given by Eq.(70); it remains, however, to determine the source strength function in order to evaluate the potential. This is accomplished by application of the boundary condition, Eq. (33), which results in the following integral equation from which f_1 may be determined:

$$\frac{1}{4\pi}\iint_{S_o} f_1(\xi,\eta,\zeta)\frac{\partial G}{\partial n}(x,y,z;\xi,\eta,\zeta;\nu)\,dS = \frac{e^{iax}}{\cosh(ah)}\left[n_y \sinh[a(h+y)] + i n_x \cosh[a(h+y)]\right] \qquad (74)$$

The normal derivative of G evaluated on the immersed surface, as required in Eq. (74), is determined in a straightforward manner by differentiation of either Eq. (72a) or (73a).

SOLUTION TO SECOND ORDER PROBLEM

The solution to the second-order problem may be formulated by application of Green's theorem to the fluid region denoted by $\forall$ in Figure 2. Applying

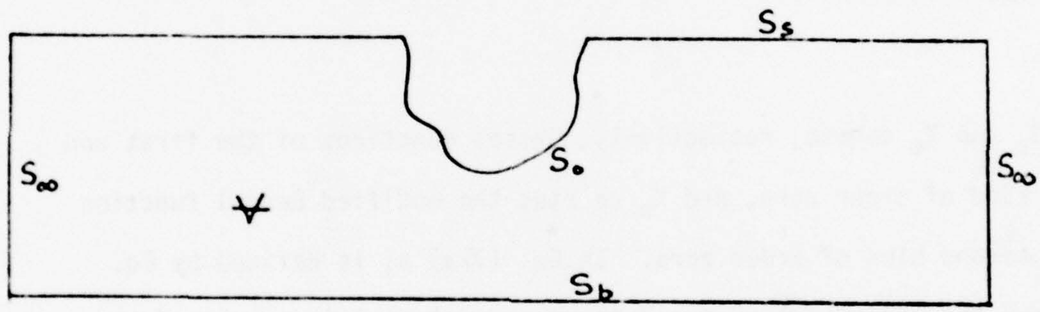

FIGURE 2 Region for application of Green's theorem.

Green's theorem with subjects u_2^s and G_2 (the second-order Green's function) and assuming that G_2 satisfies the equation,

$$\nabla^2 G_2(x,y,z;\xi,\eta,\zeta) = \delta(x-\xi)\,\delta(y-\eta)\,\delta(z-\zeta) \qquad (75)$$

as well as the kinematic condition on S_b given by Eq.(47), the radiation condition given by Eq.(51) on S_∞ and the homogeneous free surface condition,

$$G_{2_y}(x,0,z;\xi,\eta,\zeta) - 4\nu G_2(x,0,z;\xi,\eta,\zeta) = 0 \tag{76}$$

gives the result:

$$u_2^s(x,y,z) = \frac{1}{4\pi}\iint_{S_0} \left[u_2^s(\xi,\eta,\zeta)\, G_{2_n}(x,y,z;\xi,\eta,\zeta) - G_2(x,y,z;\xi,\eta,\zeta)\, u_{2_n}^s(\xi,\eta,\zeta)\right] dS$$

$$- \frac{1}{4\pi}\iint_{S_s} \left[u_{2_n}^s(\xi,0,\zeta) - 4\nu u_2^s(\xi,0,\zeta)\right] G_2(x,y,z;\xi,\eta,\zeta)\; dS \tag{77}$$

The first term in Eq.(77) may be considered to represent a source and doublet distribution over the immersed surface of the body. However, Lamb (1932) has shown for the case of an infinite fluid that the potential may be represented as either a source distribution only, a doublet distribution only, or some combination of both. This proof is easily extended to the present case allowing the first term to be written as a distribution of sources only. Then, if the free surface boundary condition, Eq.(49), is used in the second integral the result is

$$u_2^s(x,y,z) = \frac{1}{4\pi}\iint_{S_0} f_2(\xi,\eta,\zeta)\; G_2(x,y,z;\xi,\eta,\zeta)\; dS$$

$$- \frac{1}{4\pi}\iint_{S_s} f^*(\xi,\zeta)\; G_2(x,y,z;\xi,0,\zeta)\; dS \tag{78}$$

in which $f_2(\xi,\eta,\zeta)$ is the unknown source strength distribution.

The second-order scatter potential defined by Eq.(78) satisfies Eqs. (46, 47, 49-51). The remaining kinematic boundary condition on the surface of the body results in the integral equation

$$\frac{1}{4\pi}\iint_{S_0} f_2(\xi,\eta,\zeta)\,\frac{\partial G}{\partial n}(x,y,z;\xi,\eta,\zeta;4\nu,a_2)\,dS$$

$$= \frac{1}{\sinh^4(ah)}\left[n_y \sinh[2a(h+y)] + i n_x \cosh[2a(h+y)]\right]e^{i2ax} \tag{79}$$

$$+\ \frac{1}{4\pi}\iint_{S_s} f^*(\xi,\zeta)\,\frac{\partial G}{\partial n}(x,y,z;\xi,0,\zeta;4\nu,a_2)\,dS$$

which is to be satisfied for x,y,z on S_0 .

In both Eq.(78) and (79) a practical interpretation of the limits of the free surface area, S_s , should be understood. In deriving Eq.(78) through application of the Green's theorem the outer limit of S_s was considered to be large enough that the radiation condition was satisfied on S_∞ . In practical application this occurs rather rapidly.

In view of the conditions which were placed on the Green's function in the above development G_2 is clearly very similar to the Green's function used in formulating the first-order problem. In fact, if ν is replaced by 4ν and a_1 is replaced by a_2 in either Eq.(72) or (73) the resulting function is the appropriate Green's function for solution of the second-order problem. That is,

$$G_2(x,y,z;\xi,\eta,\zeta) = G(x,y,z;\xi,\eta,\zeta;4\nu,a_2) \tag{80}$$

where G is given by Eq.(72) and (73) and a_2 is defined through Eq.(72e)

$$a_2 \tanh a_2 h - 4\nu = 0 \tag{81a}$$

In the case of the alternate series form of G_2 given in Eq.(73), the roots are defined through Eq.(73b) as

$$\mu_k \tan(\mu_k h) + 4\nu = 0 \tag{81b}$$

An alternate form of Eq.(72a) for the sources on free surfaces is given by

$$G(x,y,z;\xi,0,\zeta;4\nu,a_2) = 2\ PV\int_0^\infty \frac{\mu \cosh \mu(h+y)\ J_0(\mu r)\ d\mu}{\mu \sinh \mu h - 4\nu \cosh \mu h}$$

$$+\ i\ \frac{2\pi\ a_2 \cosh[a_2(y+h)] \sinh(a_2 h)\ J_0(a_2 r)}{4\nu h + \sinh^2 a_2 h} \tag{82}$$

This if the form of the Green's function which is obtained by setting $\zeta = 0$ in Eq.(72a) and which was also given by Wehausen and Laitone (1960) for the case of a harmonic pressure distribution on the free surface. The nonhomogeneous form of the free surface boundary condition given in Eq.(49) is equivalent to the boundary condition for the first-order problem for a pressure distribution of the free surface.

NUMERICAL PROCEDURE

Eq.(69) clearly indicates that the determination of the forces and moment acting on the immersed object rests on the determination of the potentials u_1, u_2 and derivatives of u_1 at points on the immersed surface. The first-order scattering potential is specified by Eq.(70) in terms of the first-order source strength function, f_1. Thus, to determine u_1^s, it is first necessary to solve the integral equation, Eq.(74), for f_1.

We may develop a numerical solution to Eq.(74) beginning with the partitioning of the immersed surface S_0 into N subdivisions or facets of area ΔS_j, each with a nodal point at its center located at the point (x_j, y_j, z_j). Recognizing, moreover, that $f_1\ (\xi,\eta,\zeta)$ is a well-behaved function for smooth bodies we may define

$$\alpha_{ij}(\nu) = \frac{1}{4\pi}\iint_{\Delta S_j} \frac{\partial G}{\partial n}(x_i, y_i, z_i; \xi,\eta,\zeta;\nu)\ dS \tag{83}$$

$$h_i = \frac{1}{\cosh(ah)}\left[n_y(x_i,y_i,z_i) \sinh[a(h+y_i)] + i n_r(x_i,y_i,z_i) \cosh[a(h+y_i)]\right] e^{iax_i} \tag{84}$$

$$f_{1_j} = f_1(x_j, y_j, z_j) \tag{85}$$

and accordingly we may approximate Eq.(74) by the complex matrix equation:

$$\alpha_{ij}(\nu)\, f_{1_j} = h_i \qquad i,j = 1,2,\cdots N \tag{86}$$

Once $\alpha_{ij}(\nu)$ has been evaluated by use of Eq.(83) the solution of Eq.(86) may be carried out to determine f_{1j} at each nodal point on the surface of the body. A discussion of the details of the numerical evaluation of α_{ij} including the evaluation of the $1/R$ - singularity, has been given by Garrison (1978).

Having determined the source strength function, the first-order potential and its derivatives on the surface of the object may be determined from Eq.(70). For this purpose we may replace the surface integrals with summations, writing

$$u^s_{1_i} = \beta_{ij}(\nu)\, f_{1_j}, \qquad i,j = 1,2,\cdots N \tag{87}$$

$$u^s_{1_{x_i}} = \beta_{x_{ij}}(\nu)\, f_{1_j}, \qquad i,j = 1,2,\cdots N \tag{88}$$

$$u^s_{1_{y_i}} = \beta_{y_{ij}}(\nu)\, f_{1_j}, \qquad i,j = 1,2,\cdots N \tag{89}$$

$$u^s_{1_{z_i}} = \beta_{z_{ij}}(\nu)\, f_{1_j}, \qquad i,j = 1,2,\cdots N \tag{90}$$

where $u^s_{1_i}$, $u^s_{1_{x_i}}$, etc. denote functions evaluated at the i^{th} nodal point on the immersed surface, S_0. The complex matrices occurring in Eqs. (87-90) are given by

$$\beta_{ij}(\nu) = \frac{1}{4\pi}\iint_{\Delta S_j} G(x_i, y_i, z_i; \xi, \eta, \zeta; \nu)\, dS \tag{91}$$

$$\beta_{ij}(\nu) = \frac{1}{4\pi}\iint_{\Delta S_j} \frac{\partial G(x_i, y_i, z_i; \xi, \eta, \zeta; \nu)}{\partial x}\, dS \tag{92}$$

$$\beta_{y_{ij}}(\nu) = \frac{1}{4\pi}\iint_{\Delta S_j} \frac{\partial G}{\partial y}(x_i, y_i, z_i; \xi, \eta, \zeta; \nu)\, dS \tag{93}$$

$$\beta_{z_{ij}}(\nu) = \frac{1}{4\pi}\iint_{\Delta S_j} \frac{\partial G}{\partial z}(x_i, y_i, z_i; \xi, \eta, \zeta; \nu)\, dS \tag{94}$$

In evaluation of the integrals in Eqs.(83) and (91-98) it is generally adequate to simply evaluate G and its derivatives at the centroid of the panel and multiply by its area. However, when point i is either equal to j or near j it is necessary to take more care in integrating the 1/R-singularity in G. The procedures given by Garrison (1978) were used to evaluate the integrals on quadralateral panels.

To obtain a solution to the second-order problem it is necessary to first evaluate the function $f^*(\xi,\zeta)$ required in Eq.(49) and as defined by Eq.(50). For this purpose we proceed again numerically by dividing the mean free surface (or y = 0 plane) in the vacinity of the body into K area elements and evaluate f at the nodal points of these elemental areas. The numerical procedure is based on the use of Eq.s(87-94) with $y_i = 0$ for purposes of determination of $u^s_{1_i}$ and its derivatives at points on the free surface. The function f^* is then evaluated at the nodal points by use of Eq.(50).

The solution for the second-order source strength is obtained numerically through the integral equation, Eq.(79). That is, Eq.(79) is written as

$$\alpha_{ij}(4\nu)\, f_{2_j} = h_{2_i} + \alpha_{ik}(4\nu)\, f^*_k \qquad \begin{matrix} i = 1,2,\cdots N \\ j = 1,2,\cdots N \\ k = 1,2,\cdots K \end{matrix} \tag{95}$$

where $\alpha_{ij}(4\nu)$ is defined by replacing ν by 4ν in Eq.(83) and

$$h_{2_i} = \frac{1}{\sinh^4(ah)}\left[n_{y_i}\sinh[2a(h+y_i)] + i\,n_{x_i}\cosh[2a(h+y_i)]\right]e^{i2ax_i} \tag{96}$$

Once the α-matrices in (95) have been evaluated it may be solved for the source strength distribution on the immersed surface.

The hydrodynamic pressure and resulting forces carried to the second order as given in Eq.(59) and (69), respectively, require only the second-order potential rather than its derivatives. Once f_2 is known, u_2^s may be obtained through Eq.(78) which, when written in indicial form, becomes

$$u^s_{2_i} = \beta_{ij}(4\nu)\, f_{2_j} - \beta_{ik}(4\nu)\, f^*_k \qquad \begin{matrix} i, j = 1,2,\cdots N \\ k = 1,2,\cdots K \end{matrix} \tag{97}$$

where $\beta_{ij}(4\nu)$ is defined by Eq.(91).

NUMERICAL RESULTS

A computer code has been developed for the special case of a vertical circular cylinder (pile). MacCamy and Fuch's solution was used for the first-order solution and for purposes of evaluating f^*. The second-order solution was calculated on the basis of the distributed singularities as given by Eq.(78).

The particular example calculation was carried out for a cylinder placed in water one radii deep. The dimensionless frequency parameter was varied over the range $0 < \nu < 1.2$. The results for the first-order horizontal force coefficient, the second-order horizontal force coefficient, steady-state force coefficient and phase angles is shown in Figure 3. The results appear to indicate that the second-order effects are greatest at the lower frequency range, i.e., for $\nu < 0.6$.

In Figure 4 and 5 the dimensionless horizontal force is plotted for a complete wave cycle for two different frequencies (wave lengths). These results which correspond to rather steep wave show a rather sizeable non-linear effect. Also, the effect of including second-order effects is to shift the maximum force towards the phase of the wave crest.

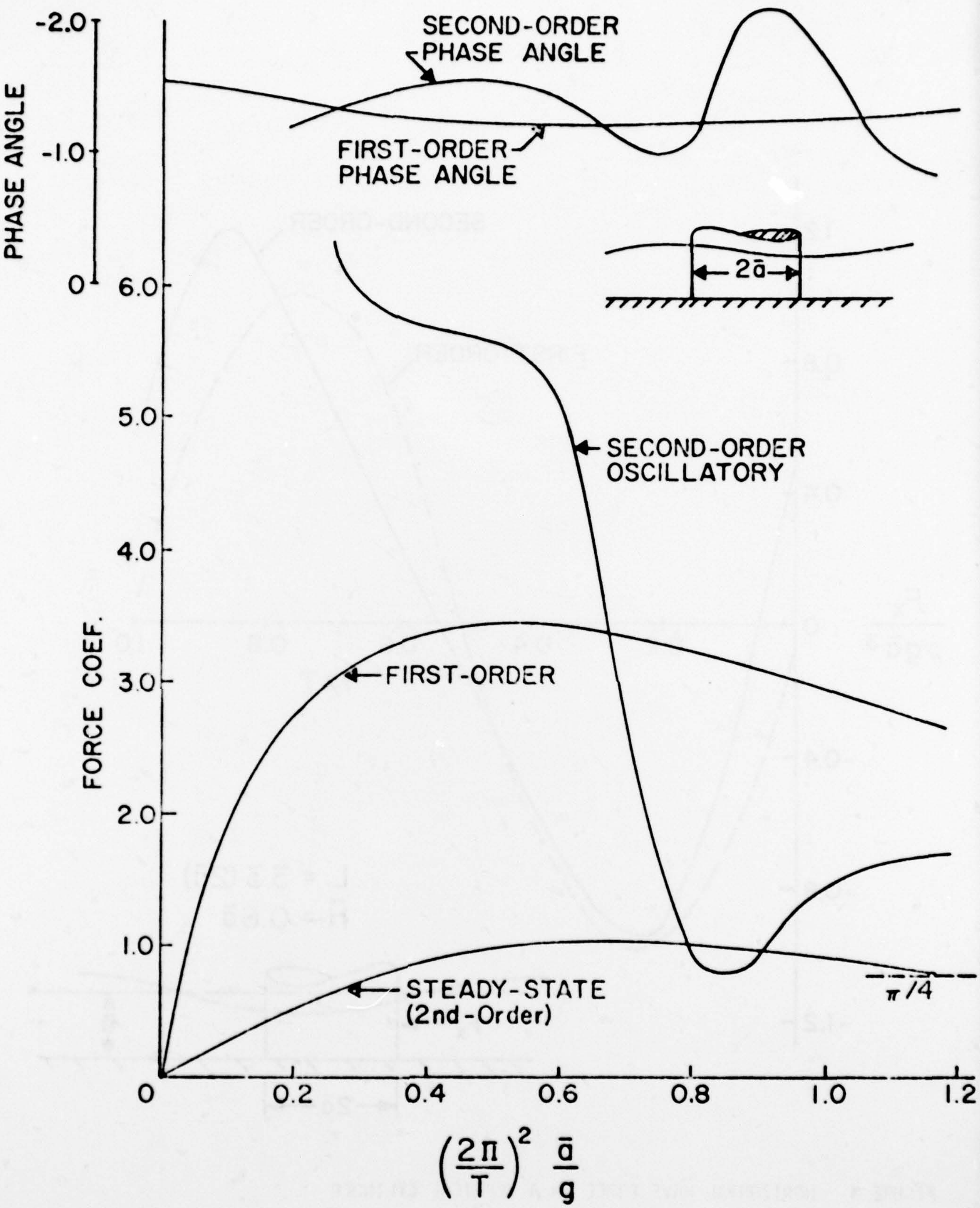

FIGURE 3 WAVE FORCE COEFFICIENTS FOR A VERTICAL CYLINDER

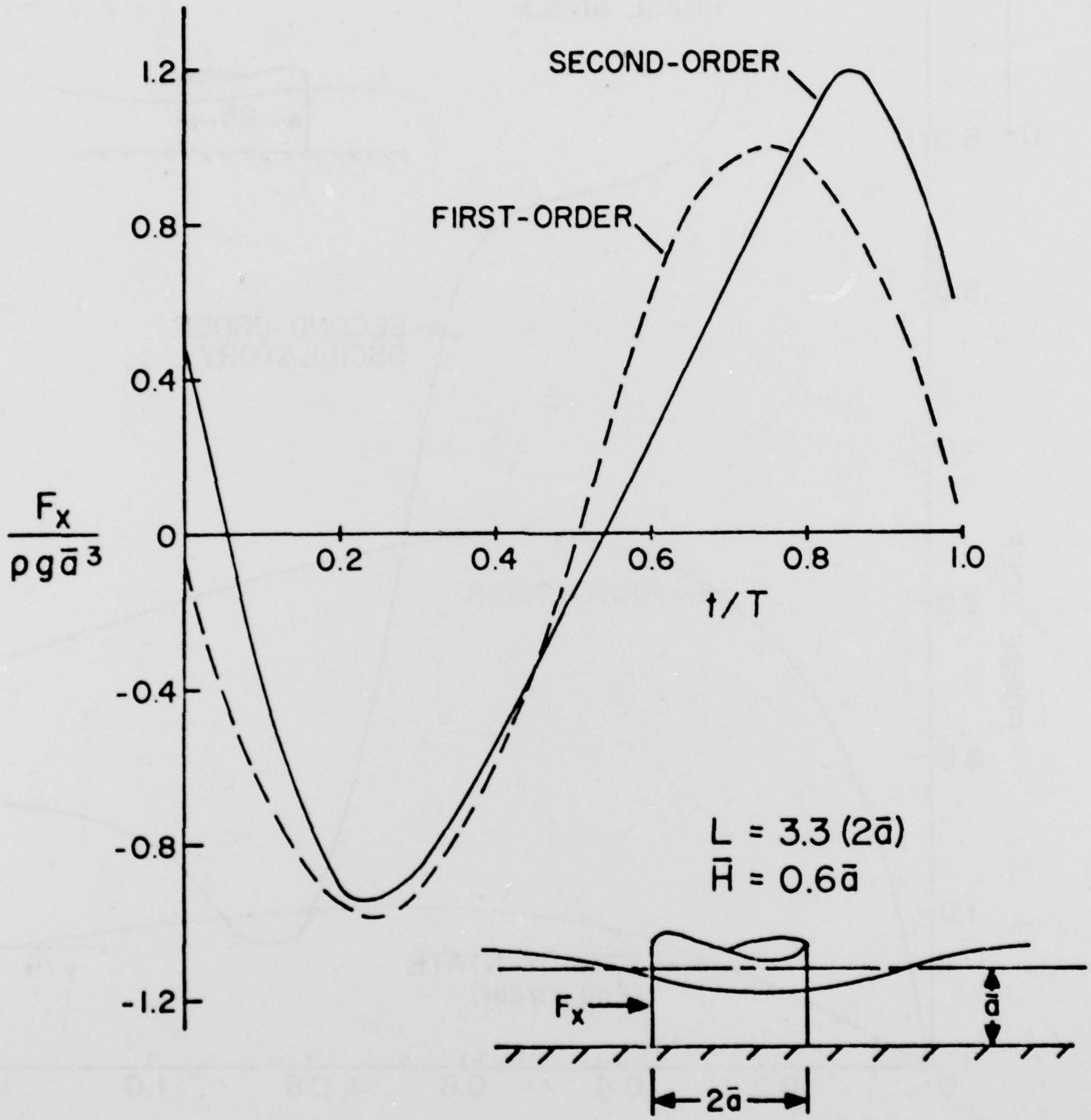

FIGURE 4 HORIZONTAL WAVE FORCE ON A VERTICAL CYLINDER

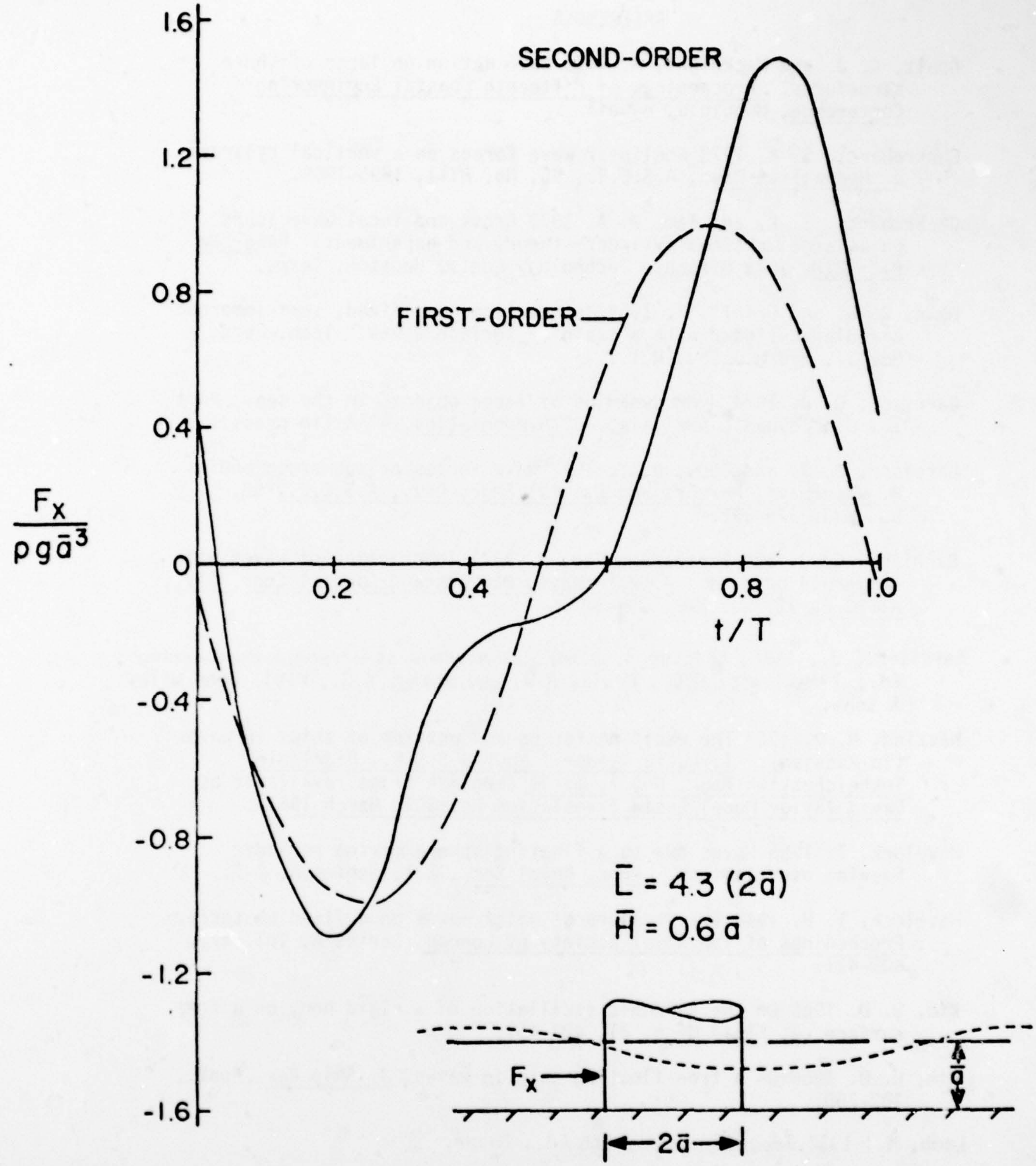

FIGURE 5 HORIZONTAL WAVE FORCE ON A VERTICAL CYLINDER

REFERENCES

Apelt, C. J. and Macknight, A. 1976 Wave action on large offshore structures. Proceedings of Fifteenth Coastal Engineering Conference, Honolulu, Hawaii

Chakrabarti, S. K. 1972 Nonlinear wave forces on a vertical cylinder. J. Hydraulics Div., A.S.C.E., 96, No. HY11, 1895-1909.

Chakrabarti, S. K. and Tam, W. A. 1973 Gross and local wave loads on a large vertical cylinder--theory and experiment. Paper No. OTC 1818, 1973 Offshore Technology Conf., Houston, Texas.

Dean, R. G. and Ursell, F. 1959 Interaction of a fixed, semi-immersed circular cylinder with a train of surface waves. Tech. Rept. No. 37, Hydro. Lab., M.I.T.

Garrison, C. J. 1974 Hydrodynamics of large objects in the sea. Part I., Hydrodynamic analysis. J. Hydronautics, AIAA (in press).

Garrison, C. J. and Chow, P. Y. 1972 Wave forces on submerged bodies. J. Waterways, Harbors and Coastal Engr. Div., A.S.C.E., 98, No. WW3, 375-392.

Garrison, C. J. and Seetharama Rao, V. 1971 Interaction of waves with submerged objects. J. Waterways, Harbors and Coastal Engr. Div., A.S.C.E. 97, No. WW2, 259-277.

Garrison,C.J., 1978, Chapter 3, *Numerical Methods in Offshore Engineering'*, Ed.; Zienkiewicz,O.C., Lewis, R.W. and Stagg, K.G., Publ. John Wiley & Sons.

Haskind, M. D. 1957 The exciting forces and wetting of ships in waves (in Russian). Izvestia Akademii Nauk S.S.S.R., Otdelenie Tekhnicheskikh Nauk, No. 7, 65-79 (English trans. available as David Taylor Model Basin Translation No. 307, March 1962).

Havelock, T. 1955 Waves due to a floating sphere making periodic heaving oscillations. Proc. Royal Soc., 231, Series A, 1-7.

Havelock, T. H. 1940 The pressure of water waves on a fixed obstacle. Proceedings of the Royal Society of London, Series A, Vol. 175, 409-421.

Kim, W. D. 1965 On the harmonic oscillation of a rigid body on a free surface. J. Fluid Mech. 21, 427-451.

Kim, W. D. 1966 On a free-floating ship in waves. J. Ship Res., Sept., 182-200.

Lamb, H., 1932. *Hydrodynamics*, 6th Ed., Dover.

Lee, C. M. 1968 The second-order theory of heaving cylinders in a free surface, J. Ship Res., Dec. 313-327.

MacCamy, R. C. and Fuchs, R. A. 1954 Wave forces on a pile: a diffraction theory. Beach Erosion Board, Tech. Memorandum, No. 69.

Maruo, H. 1960 The drift of a body floating on waves. J. Ship Res., 4, 1-10.

Milgram, J. H. and Halkyard, J. E. 1971 Wave forces on large objects in the sea. J. Ship Res., June, 115-124.

Newman, J. N. 1962 The exciting forces on fixed bodies in waves. J. Ship Res., 6, 10-17.

Paulling, J. R. and Richardson, R. K. 1962 Measurement of Pressure, forces, and radiating waves for cylinders oscillating in a free surface. Tech. Rept., Series 82, Issue 23, Inst. Engr. Res., University of California, Berkeley.

Porter, W. R. 1960 Pressure distribution, added-mass and damping coefficients for cylinders oscillating in a free surface. Tech. Rept., Series 82, Issue 16, Inst. Engr. Res., University of California, Berkeley.

Potash, R. L. 1970 Second-order theory of oscillating cylinders. Dissertation, University of California, Berkeley, California, p. 157.

Raman, H. and Venkatanarasaiah, P. 1976 Forces due to nonlinear waves on vertical cylinders. Journal of the Waterways, Harbors and Coastal Engineering Division, Proc. ASCE, Vol. 102, No. WW3.

Raman, H., Jothishankar, N. and Venkatanarasaiah, P. 1977 Nonlinear wave interaction with vertical cylinder of large diameter. *Journal of Ship Research*, SNAME Vol. 21, No. 2, 120-124.

Ursell, F. 1949 On the heaving motion of a circular cylinder on the surface of a fluid. Quart. J. Mech. Appl. Math. 2, 218-231.

Vugts, J. H. 1968 The hydrodynamic coefficients for swaying, heaving and rolling cylinders in a free surface. International Shipbuilding Progress. 15, No. 167, 251-276.

Wang, Shen, 1966 The hydrodynamic forces and pressure distributions for an oscillating sphere in a fluid of finite depth. Dissertation, Massachusetts Institute of Technology, Dept. of Naval Architecture and Marine Engineering, Cambridge, Massachusetts.

Wehausen, J. V. and Laitone, E. V. 1960 Surface waves. Encyclopedia of Physics, Vol. 9, Springer-Verlag, Berlin, 446-778.

Yu, Y. S. and Ursell, R. 1961 Surface waves generated by an oscillating circular cylinder on water of finite depth: theory and experiment. J. Fluid Mech., 11, 529-551.

INITIAL DISTRIBUTION LIST

National Science Foundation Washington, D.C. 20550	5
Defense Documentation Center Cameron Station Alexandria, Virginia 22314	2
Dean of Research Code 012 Naval Postgraduate School Monterey, California 93940	1
Library Naval Postgraduate School Monterey, California 93940	1
Professor Michael de St. Q. Isaacson Dept. of Civil Engineering University of British Columbia Vancouver, B.C., CANADA	1
Dr. Jared L. Black 231 Laurelwood Ave. Placentia, California 92670	1
Dr. Jon Buck Crestwood Bldg, Suite 402 5711 Sarvis Ave. Riverdale, Maryland 20840	1
Professor Robert Hudspeth Department of Civil Engineering Oregon State University Corvallis, Oregon 97330	1
Professor John Nath Department of Civil Engineering Oregon State University Corvallis, Oregon 97330	1
Professor Charles Sollitt Department of Civil Engineering Oregon State University Corvallis, Oregon 97330	1
Professor Tokuo Yamomoto Department of Civil Engineering Oregon State University Corvallis, Oregon 97330	1

Mr. Min-Chu Chen
Department of Civil Engineering 1
Oregon State University
Corvallis, Oregon 97330

Professor R. E. Nece
Department of Civil Engineering 1
University of Washington
Seattle, Washington 98105

Professor E. P. Richey
Department of Civil Engineering 1
University of Washington
Seattle, Washington 98105

Professor J. B. Herbich
Department of Civil Engineering 1
Texas A & M University
College Station, Texas 77840

Professor V. Seetharama Rao
Department of Civil Engineering
Virginia Polytechnic Institute & 1
State University
Blacksburg, Virginia 24060

Professor R. G. Dean
Department of Civil Engineering 1
University of Delaware
Newark, Delaware 19711

Professor T. Dalrymple
Department of Civil Engineering 1
University of Delaware
Newark, Delaware 19711

Professor C. J. Apelt
Department of Civil Engineering
University of Queensland 1
St. Lucia, Queensland 4067
AUSTRALIA

Professor C. J. Garrison
Department of Mechanical Engineering 20
Naval Postgraduate School
Monterey, California 93940

Deparment of Mechanical Engineering
Code 69 1
Naval Postgraduate School
Monterey, California 93940

Dr. G. R. Mogridge
Hydraulics Laboratory 1
National Research Council of Canada
Montreal Road
Ottawa, KIA OR6

Dr. W. W. Jamieson
Hydraulics Laboratory
National Research Council of Canada 1
Montreal Road
Ottawa, KIA OR6

Dr. Ping Chen
American Bureau of Shipping 1
45 Broad Street
New York, N.Y. 10004

Dr. Neil Hogben
National Maritime Institute 1
Faggo Road Feltham Middlesex TW14 OLQ
ENGLAND, U.K.

Mr. Alf Torum
Head, Department of Harbour &
Marine Technology
Vassdrags - Og Havnelaboriet 1
Klaebuvien 153
7000 Trondheim
NORWAY

Professor Odd Faltinsen
Institute for Shipshydrodynamikk
University Trondheim 1
Norges Tekniske Hogskole
N-7034 Trondheim
NORWAY

Dr. Valdas J. Grauzinis, President
Transpectrum Corporation 1
P.O. Box 82026
San Diego, California 92138

Professor Ing. K. Kokkinowrachos
Lehrgebiet
Grundlagen der Meerestechnik 1
Technische Hochschule Aachen
51 AA CHEN
Templergraben 55
GERMANY

Dr. Subrata K. Chakrabarti 1
Head, Analytical Group
Chicago Bridge & Iron Co.
Route 59
Plainfield IL 60544