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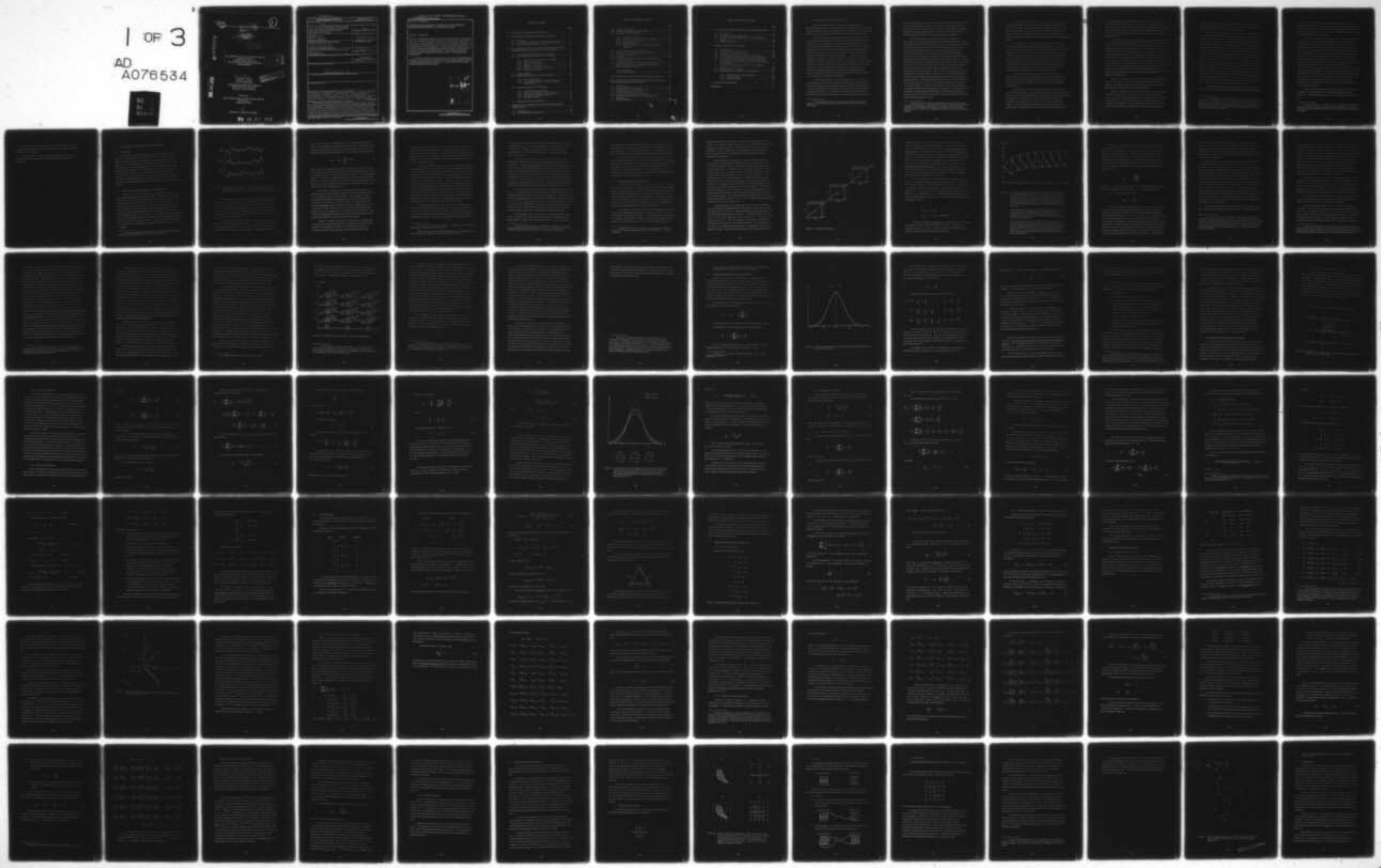

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THE EXPANDED OCEAN THERMAL-STRUCTURE ANALYSIS SYSTEM: A DEVELOPMENT BASED ON THE FIELDS BY INFORMATION BLENDING METHODOLOGY.

by

Manfred M. Holl
Michael J. Cuming
Bruce R. Mendenhall

Meteorology International Incorporated
2600 Garden Road, Suite 145
Monterey, California 93940

Prepared for

Naval Ocean Research and Development Activity
NSTL Station
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The Expanded Ocean Thermal-Structure (EOTS) analysis system, an application of the general-purpose Fields by Information Blending (FIB) methodology, is an advanced, comprehensive and flexible system for the four-dimensional analysis of thermal structure from the ocean surface to the bottom. Using surface (ships, buoys, satellites) and sounding observations and information carried and accrued along the time axis from previous analyses and climatologies, analyses may be performed for any region and for any grid resolution in space (continued)

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Blending analysis methodology. Blending by Weighted Spreading. Alternating Parallel Analysis. Information processing.

Block 20. (continued)

and time. Significant variabilities in the vertical profile are represented by a set of thermal-structure parameters--absolute values, gradients and curvatures. Special provision is made for fine-resolution of thermal structure in the vicinity of the Primary Layer Depth and for restricting flow of information across land barriers. EOTS may be used for real-time synoptic analyses and, utilizing available data bases, for production of historical sequences and climatologies. A wide variety of optional outputs includes horizontal and vertical sections of temperature and sound speed.

This Report provides a comprehensive account of the FIB information-processing concepts and formulations upon which EOTS is based. Samples of EOTS-generated output are included. A Users Manual for the EOTS analysis system also is available.

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1. SUMMARY AND GENERAL INTRODUCTION

Computer software developed by Meteorology International Incorporated (MII) includes unique systems for the analysis of meteorological and oceanographic data. These analysis systems are based on one principal underlying analysis methodology--Fields by Information Blending (FIB).

The information-processing concepts and formulations upon which the FIB analysis methodology is founded were documented by Holl in 1971 in association with a system for the analysis of sea-level pressure [1]. FIB has progressed rapidly from these early concepts to its present level of development. The Fields by Information Blending methodology is now a comprehensive technique for the objective analysis of scalar and vector fields with a wide range of realized and potenti[illegible] applications. For the analysis of a specific environmental parameter FIB should be considered as equal or superior to any currently available technique. However no other technique is based on fundamental and generalized concepts of information processing. These concepts, expressed in terms of formulations and algorithms designed specifically to facilitate their implementation as computer software,[1] provide FIB with unique capabilities for the analysis of a wide range of environmental parameters.

To illustrate the diversity of FIB applications, the U. S. Navy's Fleet Numerical Weather Central (FNWC) currently utilizes FIB analysis systems to produce routine operational analyses of sea-level pressure for the Northern and Southern Hemispheres; fine-mesh sea-level pressure and surface wind analyses for the Greater Mediterranean; upper-air contour height analyses for both hemispheres; sea-surface temperature analyses for

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[1] All relevant formulations in this Report are expressed in finite-difference form; analysis algorithms involve extensive use of iterative techniques.

both hemispheres, and for several sub-regions on finer scales of resolution; ocean thermal-structure analyses for the Northern Hemisphere, and for several sub-regions on finer scales of resolution; and Northern Hemisphere analyses of significant wave height. All these systems[1] are based on a common analysis methodology--Fields by Information Blending.

As FIB evolved and its range of applications increased, the development of a new analysis capability often revealed methods by which earlier applications could be improved. However, in general terms, contract requirements for the documentation of FIB-based analysis systems have focussed on the practical aspects of program maintenance and system operation; documentation of the underlying principles and associated formulations has not always been consolidated, especially where an earlier capability has been modified to encompass later developments. As a result, the information needed to comprehend a particular current FIB analysis system generally has to be sought among a large number of memoranda, progress reports, technical reports, and program documentations such as users guides and maintenance manuals. Until this Report there existed no single publication for a current analysis capability which traced the development of FIB from first principles and explained its application to that particular analysis system. Without appropriate documentation neither the operator of the analysis system nor the user of its output can have a full appreciation and understanding of the system and its products; FIB thus has acquired a reputation for being excessively abstruse and esoteric. In addition the lack of technical documentation has prevented the capabilities of FIB-based analysis systems from being disseminated among the environmental analysis and forecasting community at large.

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[1] Perhaps the most advanced FIB capability is that for determining full-scan clear radiance components from measured cloud-contaminated VTPR radiances for the enhanced resolution of atmospheric thermal-structure variabilities [2]. However this capability is not yet used in an operational context.

One of the earliest applications of FIB was to the analysis of sea-surface temperature distributions on behalf of FNWC. This operational capability later was extended to encompass the added dimension of depth, thus providing FNWC with a real-time system for the analysis of ocean thermal-structure. Development continued and by 1975 an advanced analysis capability known as FIB/OTS was providing routine thermal-structure analyses.

In the context of U. S. Navy requirements, perhaps the most important application of ocean thermal-structure analyses is to provide an input to programs which determine sound propagation conditions; real-time knowledge of these conditions has a direct bearing on the surface and sub-surface capabilities of the U. S. Navy. In order to keep pace with operational demands for improved ocean thermal-structure analyses, in 1976 a plan was produced for providing FNWC with a very advanced, comprehensive and flexible analysis capability for the 4-dimensional analysis of ocean thermal-structure; this capability is known as the Expanded Ocean Thermal-Structure (EOTS) analysis system.

By 1977 development of EOTS had advanced to the point of operational utility although EOTS products remained in an "operational evaluation" phase. A preliminary version of a Users Guide/System Description was written, updated in early 1978 to reflect the growing EOTS capability.

In September 1978 a formal Users Manual for the EOTS analysis system [3] was produced. This Manual was designed in such a manner that subsequent additions and modifications to EOTS easily could be documented by amendment. This version of the Manual provides all information necessary to effectively use the system in its current configuration. However, although containing a comprehensive list of references, the Manual does not provide the User with an account of the principles and information-processing concepts of the FIB analysis methodology upon which the EOTS analysis system is based. Accordingly, in early 1979 under the sponsorship of the Naval Ocean Research and Development Activity (NORDA), work was begun on preparing a comprehensive Technical Report intended to provide the necessary theoretical background. The result of that work is this publication.

In this Technical Report most concepts and formulations of relevance to the EOTS analysis system have been gathered together, thus avoiding the need to seek essential background information from the large number of references given in the Users Manual. This policy, of course, has resulted in a publication of some considerable length. In addition, although the many formulations could have been presented with little explanation, relying on the insight of the reader for their interpretation, such a procedure has not been followed. To make the subject more easily readable, most formulations are accompanied by a discussion explaining their purpose and the underlying concepts. Where possible without loss of direct relevance to the EOTS analysis system, concepts and formulations are presented in general terms to facilitate an appreciation of their application to the analysis of other environmental parameters.

Section 2 of this Report discusses, in a largely qualitative manner, some of the factors to be taken into account by any effective analysis system. The Section also is intended to introduce certain FIB concepts and associated terminology in an appropriate context.

Section 3 provides many of the generalized information-processing concepts and formulations upon which the FIB analysis methodology is based. Commencing with a discussion of the reliability of observations as information, it is shown that every piece of information which can contribute to an analysis has an associated and quantifiable reliability. The remainder of Section 3 essentially is concerned with methods for combining independent information-contributions in such a manner as to provide the best estimate of the "true" field of the analysis-parameter that can be obtained from the total information available.

Although all FIB analysis systems are based on a common analysis methodology, a system for the analysis of a particular environmental parameter has to be tailored to suit the characteristics of that parameter. For example, a system for the analysis of sea-level pressure cannot be directly used for the analysis of surface wind merely by substituting one input-data set for another. Section 4 uses the concepts and formulations of Sections 2 and 3 to develop an analysis system for Sea-Surface Temperature (SST)--the required system is relatively uncomplicated, and in addition, SST is one of the parameters analyzed by the EOTS system. By the end of Section 4.9 all FIB concepts and formulations necessary to comprehend the analysis of SST distributions in space <u>and</u> time have been presented. Following a discussion of the modular structure of FIB-based analysis systems and their implementation, Section 4.11 shows the information flow and processing in a FIB system for SST analysis. Section 4.12 provides an account of Alternating Parallel Analysis (APA), a method (not yet implemented) for assimilating remotely-sensed satellite data into a FIB analysis system in a manner which circumvents calibration bias and drift.[1] Examples of SST analyses are given in Section 4.13.

This Report documents the concepts underlying the EOTS analysis system. Although self-contained in that familiarity with the EOTS system

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[1] Where appropriate, all FIB analysis systems can utilize satellite-derived data. For example the EOTS analysis system uses satellite-derived estimates of sea-surface temperature and can also assimilate gradient information subjectively derived from satellite imagery (HRIR supported by visible). However less sophisticated techniques than APA currently are used to assimilate this information.

is not essential to comprehend the concepts upon which EOTS is based, this Report and the Users Manual [3] are complementary to a considerable extent. On completion of Section 4 it is recommended that Section 2 of the Users Manual be studied. Section 3 of the Users Manual should be studied on completing Section 5 of this Report.

Section 5 is concerned with the extension of the analysis system described in Section 4 to encompass significant temperature variations in the vertical structure. As a preliminary it is pointed out that the analysis system described in Section 4 does not necessarily have to be applied to the temperature at the surface; it could equally well be applied to the temperature at any depth or to the temperature on a variable-depth surface, e.g., the Primary or Principal Layer Depth (PLD). However a simple system based on independent horizontal (or quasi-horizontal) temperature analyses at various depths is entirely inadequate for determining significant 4-dimensional ocean thermal-structure variabilities. The procedure adopted is to parameterize the vertical profile in terms of "ocean thermal-structure parameters"--measures of profile temperature, gradient and curvature at various fixed and "floating"[1] levels. The analysis system described in Section 4 is used to analyze each of these parameters. (The EOTS analysis system automatically sets various tunable constants encompassed by the analysis algorithm to reflect the characteristics of each analyzed parameter.)

Having completed analysis of the set of ocean thermal-structure parameters, the next step is that of vertical blending, described in Section 5.7; vertical blending completes the 3-D analysis at a point in time.

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[1]For example PLD is a "floating" level. This depth is determined by analysis and is one of the thermal-structure parameters. A number of other parameters are defined relative to the PLD and thus also are "floating" parameters.

Section 6 provides a brief review of the EOTS analysis system and is included to make this Report self-contained; a more detailed account is given in the Users Manual.

Section 7 contains a number of miscellaneous (but nevertheless significant) topics of relevance to the EOTS analysis system and its utilization.

2. PRELIMINARY CONSIDERATIONS AND TERMINOLOGY

2.1 Introduction

This Section is intended to reveal, in a largely qualitative manner, many of the considerations that should be taken into account by any effective analysis system. By discussing certain characteristics of observed data, and describing the analysis process primarily by reference to the simple and subjective technique often still used for the analysis of environmental parameters (i.e., the drawing of isopleths by hand), many of the concepts underlying the objective FIB analysis methodology, together with associated terminology, are introduced. How these concepts are expressed mathematically and combined into a system for the objective analysis of ocean thermal-structure parameters is described in subsequent Sections.

2.2 Observations as Estimates of Representative Values

Suppose that, at a fixed location, we measure the value of an environmental parameter as a function of time. The method of measurement and presentation of the results would depend very much on the time scale in which we were interested--the "object scale of resolution".[1] For example if we were concerned with small-scale atmospheric turbulence then instruments with very short response times would be required to resolve the rapid temporal changes of the parameters to be measured. Such instruments were used to produce Fig. 1 which shows horizontal wind speed, vertical wind speed and air temperature on the turbulence range of scale for a period of about 40 seconds. Note the relatively large and rapid changes in air temperature for example--up to 3°C in as little as 2 seconds.

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[1]As discussed later in this Section, an "object scale of resolution" usually involves considerations of both space and time.

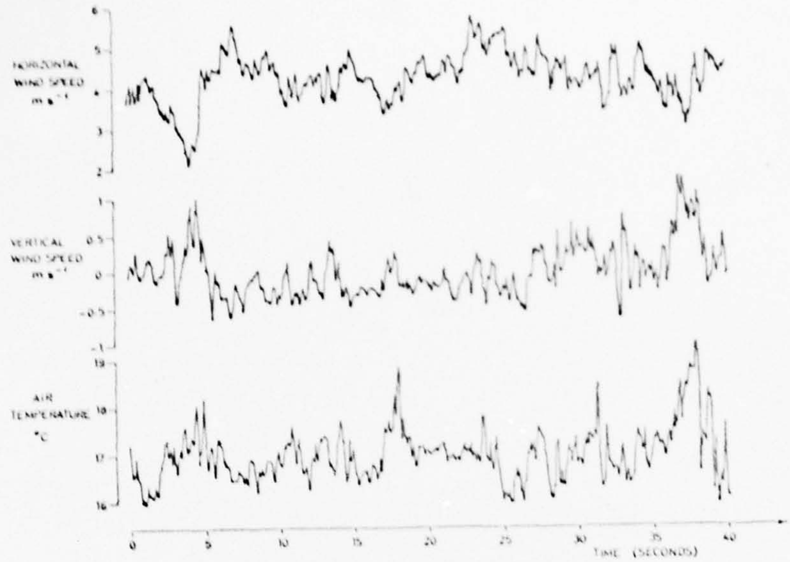

Figure 1   A short section of a record of horizontal wind speed, vertical wind speed and temperature at a height of two meters at the meteorological field site of Reading University (after Ibbetson [4]).

However suppose that we were concerned not with atmospheric turbulence but with the variation in air temperature over a period of 24 hours at the same location. Clearly we are no longer interested in the degree of resolution in time which can be provided by the instrument used to produce Fig. 1 (a fine-wire resistance thermometer). Our object scale of resolution is quite different. Assume though that this thermometer has to be used for the 24 hour period.

If the trace for 24 hours were shown on a chart of the same length as Fig. 1, the 40-second period shown in Fig. 1 would be represented by a segment less than 0.002" long--in effect a "point". Plotting a series of points for the 24-hour period would provide an apparently continuous trace. But what value of temperature should be plotted at each point? For each 40-second period we require a "representative" temperature which could be obtained by computing a time-averaged mean of the temperature recorded by the resistance thermometer. Thus, for this particular example,

$$T_{40} = \frac{1}{40} \int_{-20}^{+20} T(\tau)d\tau \qquad (1)$$

where T_{40} is the representative temperature appropriate to an object scale of resolution in time of 40 seconds and $T(\tau)$ is the temperature T observed as a function of time τ on the finer scale of resolution. (An "instantaneous" reading of T from the resistance thermometer is itself a time-averaged mean, the averaging process being due to the finite response time of the instrument. Closer examination would show fluctuations on time scales even shorter than indicated by Fig. 1.)

The degree of resolution in time provided by 40-second mean temperatures still is far greater than needed to show significant variations in air temperature over a 24-hour period. The variability between successive 40-second mean temperatures would not provide any useful information in the context of a 24-hour period. To obtain variabilities which are significant, the time-span over which the mean temperature is determined should be chosen so as to "smooth out" fluctuations which, though real, are not significant to the object scale of resolution. Ten-minute mean temperatures probably would be more than adequate to show significant variabilities occurring over a 24-hour period.

However, suppose that instead of being able to compute time-averaged means over an appropriate time span we are only able to obtain relatively "instantaneous" values of T--in other words a value of T time-averaged over a markedly shorter period than required to show variabilities which are significant to the object scale of resolution. For example, for the period shown in Fig. 1, an instantaneous observation might be anywhere between about 16° and 19° C whereas the representative value is, say, 17.0° C. Similarly, a 40-second mean temperature measured within a 10-minute time-span might differ by several degrees from the 10-minute mean temperature.

This example illustrates a point which should be taken into account by any analysis[1] scheme (but which is usually ignored)--an observation of any environmental parameter which is representative on one (i.e., the observed) range-of-scale can only provide an estimate of the value of that parameter on another range-of-scale. Such an observation provides a "sample" of the representative value. If, as is usually the case in producing analyses of environmental parameters, observations are made on a finer range-of-scale than the object range-of-scale of the analysis, then in general any observation contains an information component which is sub-scale to the desired representative value. This inherent and unavoidable sub-scale component introduces uncertainty concerning the reliability that can be attributed to any observation unless it is known to be representative of the object scale of analysis resolution.[2]

The difference between a "true" value and a "representative" value should be appreciated. Assuming that no errors are made by the observer and that the instrument is accurate, then all observations are true in the sense that they actually did occur at the time and place the observation was made. Given this assumption a representative value

[1]The meaning of the term "analysis" as applied to environmental parameters is discussed in Section 2.3.

[2]There are other factors which contribute to the uncertainty of an observation; these factors are discussed in Section 3.1.

is equal to the (observed) true value _if_ the range-of-scale for which a representative value is desired corresponds to the range-of-scale of observation. In general, however, although an observation may be representative on one range-of-scale, it is not representative on any other range-of-scale.

For convenience the terms "object scale of resolution" and associated "representative value" have been discussed by considering a time series. However the _local_ small-scale variability in space associated with an observation of an environmental parameter also must be taken into account. For example suppose that at any instant while Fig. 1 was being recorded other instruments were used to obtain a continuous field of air temperature over an area of, say, 150 meters x 150 meters around the first instrument.[1] Over a few meters in any direction the temperature would probably vary between about 16° and 19° C. However the space-averaged mean (the representative temperature) would probably be close to 17° C.

Now suppose that instead of being able to compute a space-averaged mean for a particular time we were only able to obtain one reading of temperature somewhere in the area under consideration. This observation might be anywhere between 16° and 19° C whereas the representative value is, say, 17.0° C. Once again the observation has been made on a finer range-of-scale than appropriate to the object range-of-scale (in this case an area of 150m x 150m) and so the observation contains an information component which is sub-scale to the desired representative value.

In the example used above the space and time scales involved are much smaller than those considered by synoptic meteorology and oceanography. However similar arguments apply. Thus, for example,

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[1] How this could be done in practice is not clear. Fine wire resistance thermometers arranged in a grid with a grid-spacing of a few centimeters would provide an effective, if impractical, simulation.

an observation of sea-surface temperature made using a mercury-in-glass thermometer may be "true" in local space and time. However measuring the temperature a few minutes earlier or later, or a few hundred meters away, could provide an observation perhaps differing by several degrees.

It may be concluded that, in general, an observation of any environmental parameter, even if "true" in the sense that the observed value actually did occur at the time and place of observation, can only be regarded as an estimate of the local representative value of that parameter on another range-of-scale in space and/or time. The consequences of this inherent lack of reliability of observations when used in an analysis will become apparent later.

2.3 Synoptic Analyses in Space and Time

The Glossary of Meteorology [5] defines an analysis as follows: "In synoptic[1] meteorology, a detailed study of the state of the atmosphere based on actual observations, usually including a separation of the entity into its component patterns and involving the drawing of families of isopleths for various elements. Thus the analysis of synoptic charts may consist, for example, of the drawing and interpretation of the patterns of wind, pressure, pressure change, temperature, clouds and hydrometeors, all based on actual observations made simultaneously."

This definition, of course, refers to all environmental parameters which may be separately analyzed but studied as a set in order to comprehend a given synoptic situation. (The definition may be extended to encompass oceanographic as well as meteorological parameters.) The purpose of the analysis process is to arrive at an analysis of a single

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[1] Synoptic--"relating to or displaying atmospheric and weather conditions as they exist simultaneously over a broad area". (Webster's Dictionary).

specified parameter which, in FIB, is termed the "object parameter" of the analysis. The analysis process, adjusted to suit the particular characteristics of each environmental parameter, may be applied to a selection of object parameters in order to produce the set of analyses required to represent a total synoptic situation in accordance with the definition given by the Glossary of Meteorology.

To aid discussion, the simple and subjective analysis procedure envisaged by the Glossary--"the drawing of families of isopleths"--may be outlined as follows. Briefly, all observations of an object parameter P made at a common (i.e., synoptic) time τ are plotted on a chart. Isopleths, usually integer, are then drawn with some appropriate "contour interval", thus providing lines of constant value of the object parameter. Since observed values of P do not correspond, in general, to isopleth values, observed values are interpolated subjectively to determine the location of isopleths. If isopleths can be drawn which agree reasonably well with an observed value then the observation is judged to be "good"; if the isopleths cannot be manipulated to fit an observed value (usually because of nearby conflicting observed values), then the observation is judged to be in error and is largely ignored. This procedure provides an analysis of P for time τ. Repeating the process for time $\tau+1$, $\tau+2$, etc., provides a synoptic sequence of analyses which is illustrated in Fig. 2.

In drawing isopleths for a particular synoptic time, the analysis could be produced by considering only observations for that time. However the analysis for, say, time $\tau+1$ may be enhanced by taking into account the features of the previous analysis at time τ. A common technique for doing this is to use a light table with the plotted chart for $\tau+1$ on top of the analyzed chart for τ. The $\tau+1$ analysis is then drawn, subjectively assimilating the features of the previous analysis. Thus for example, in the case of a sea-level pressure analysis, a depression shown on the analysis for time τ may be subjectively moved, modified and shown on the

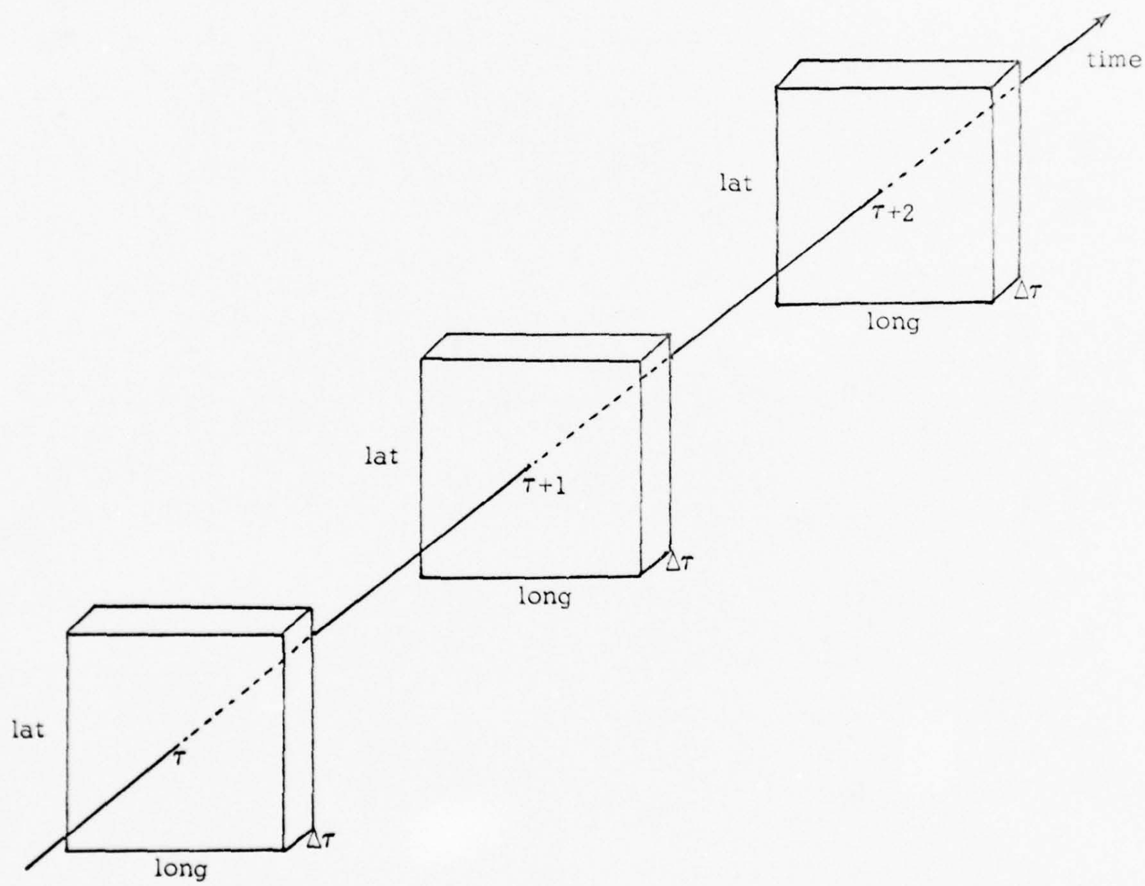

Figure 2 A sequence of analyses.

analysis for time $\tau+1$ even though there are no $\tau+1$ observations to support the feature. This subjective technique is termed "maintaining continuity between analyses" and, in effect, carries information along the time axis. If this technique is employed the analysis frequency must be such that the information in one analysis has relevance to the next analysis. For example it would not be realistic to expect synoptic-scale features such as rapidly-moving depressions and anticyclones to be of any significant relevance to an analysis carried out two days later. Information decays with lapsed time. For a normal synoptic sequence in a meteorological context, the time interval between analyses is usually 3, 6 or 12 hours; this analysis frequency generally is adequate for carrying synoptic-scale information along the time axis.

To further illustrate the potential utility of information carried along the time axis, suppose that at every analysis time the available new observations provide a fixed information contribution I. (In practice I would vary depending on the number, quality, and distribution of observations.) Now suppose that the total information content of any analysis, I_τ, decays with time, the information carried forward in time to the next analysis being RI_τ where $R < 1$. Thus, for an analysis sequence commencing at time τ:

$$\begin{aligned} I_\tau &= I, \\ I_{\tau+1} &= I + RI_\tau, \\ I_{\tau+2} &= I + RI_{\tau+1}, \text{ and so on.} \end{aligned} \tag{2}$$

The effect of carrying, and accruing, information along the time axis in accordance with Eq. (2) is shown in Fig. 3 using I = 6 "units" of information and R = 0.5 (i.e., half the information contained in one analysis is still relevant at the next analysis time).

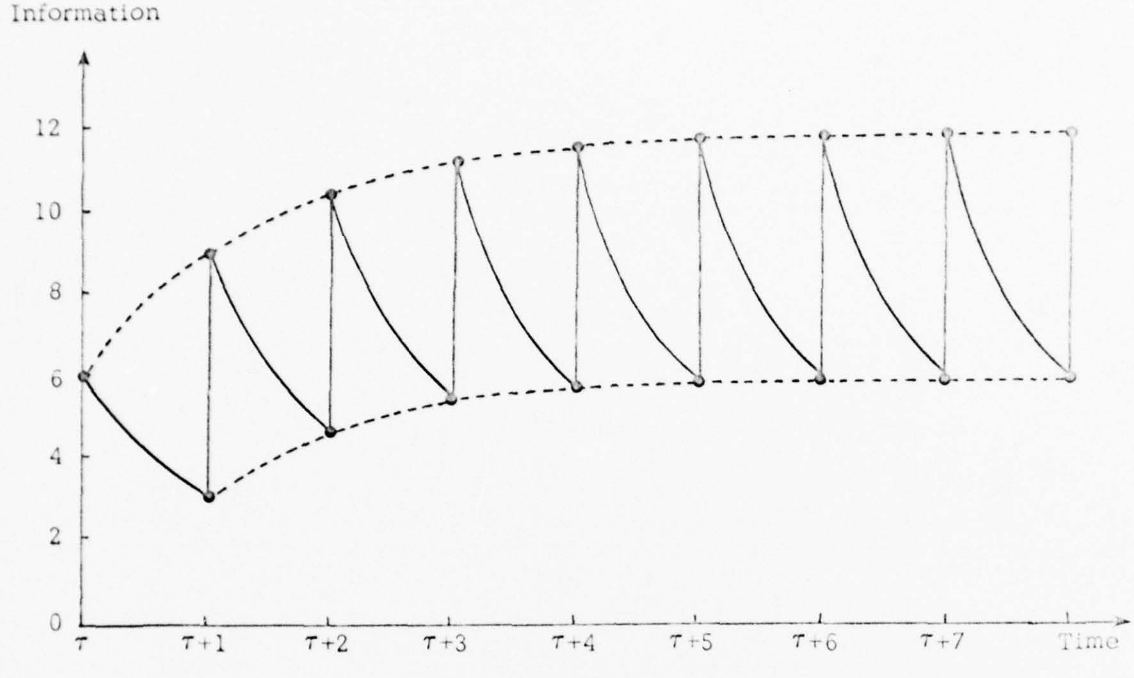

Figure 3 The effect of carrying information along the time axis. The total information available at any time is shown by the solid line. Information decays with time (R = 0.5) but for each new analysis a further 6 "units" of information are provided by the concurrent observations. The upper dashed line shows the total information attained by successive analyses. The lower dashed line shows the accruing information available from previous analyses in the sequence.

(This schematic is based on a simple concept--decay in the worth of persistence. The application of appropriate prediction capabilities would slow the decay rate and result in higher plateaus of information yield. Prediction capabilities represent information contributions.

Well formulated comprehensive models which interrelate the evolution of a relatively closed physical system of parameters can produce information growth for a period following each analysis input of concurrent observations, to the point where net decay again sets in. The information yield which can be achieved in such analysis/prediction systems by the "dynamic compounding" can be spectacular.)

The effect of carrying and accruing information along the time axis may be generalized as follows. Let I be the total concurrent information available for each analysis. As noted earlier I will vary depending on the number, quality and distribution of observations. As discussed later, I may also contain information derived from sources other than direct observation of the object parameter. However we now identify I with the mean total information contribution to each analysis from all sources. If R is the fraction of information still relevant at the next-in-sequence analysis, then

$$I_{\tau+n} = I \sum_{n=0}^{n} R^n \tag{3}$$

where $I_{\tau+n}$ is the total information in the $(\tau+n)$ analysis, and the analysis sequence commenced at $n = 0$ with no earlier information available.

If $n = N$ where N is large, then

$$I_{\tau+N} \rightarrow \frac{1}{1-R} I \quad . \tag{4}$$

Thus for the example illustrated in Fig. 3, $I_{\tau+N} = 2\,I$, -- in other words after an appropriate number of analyses (4 or 5 in this case) the information content of the analysis is about double that available solely from concurrent information. If $R = 0.75$ then $I_{\tau+N} = 4\,I$. It can be seen that previous observations can provide a very significant information contribution to an analysis; no analysis scheme should ignore this potential contribution. Clearly the simple and subjective analysis method--drawing isopleths--cannot take full advantage of previous information. Objective methods for carrying information along the time axis form an integral part of the FIB analysis methodology and are described later. However at this point it may be noted that the relevance of $I_{\tau+n}$ to the next analysis is determined not only by the time-rate of decay of information but also by the frequency of analysis. For any environmental parameter the analysis frequency may be chosen to maximize the yield available from information carried along the time axis. The optimum frequency of analysis[1] will vary from parameter to parameter.

Returning to Fig. 2, each analysis has been shown as encompassing observations made over a relatively brief period of time $\Delta\tau$. This illustrates the fact that observations are not truly synoptic and neither are they instantaneous. For example not all observations for, say, 00Z are made precisely at that time--the time of observation depends to some extent on the observer and each observation requires a finite time to make. In addition some environmental parameters are deliberately recorded as a mean. Observations of surface wind are a good example; a wind averaged over a number of minutes (usually about 10) is reported as the "observed" wind in order to eliminate non-representative gusts. Sea and swell waves are another example--the associated parameters (height and period) are observed and reported in terms of "significant" values, defined to be the mean of the one-third highest values of each parameter observed over a period of time. The problems of non-representative values generally are not recognized for observations of other parameters.

In some types of analyses the observations of P contributing to a single analysis may range over a period of time $\Delta\tau$ ranging from a few

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[1] The optimum frequency depends on various factors including the frequency of synoptic collections of data, the naturally-occurring time variability of a particular parameter and the availability of computer resources. To maximize the yield from information carried along the time axis analyses should be carried out as frequently as possible. However, in a real-time operational context, the optimum frequency has to be determined based on all relevant factors.

hours to many days. For example where there are insufficient observations available to provide a realistic analysis we may choose to utilize all observations made over a period of 24 hours in order to increase data density. However resolution in time is decreased--the ability to determine changes in time (based on actual observations) within the 24-hour period is lost. In some circumstances the full resolution in time available from observed data may not be required. Suppose we require the mean sea-surface temperature for a particular calendar month. Although better methods are available this could be produced by plotting all observations for that month and carrying out an analysis using the simple subjective technique outlined above (i.e., by drawing isopleths).

In general it can be seen that all analyses, even those termed synoptic, actually encompass information from observations made over a period of time $\Delta\tau$ which may vary from perhaps a few minutes in the case of a synoptic analysis to many days or even months for other analyses. In addition each analysis may contain an information component provided by non-synoptic information carried forward and accrued along the time axis.

So far discussion of what an analysis actually shows has been avoided. Using the subjective analysis method previously described for purposes of illustration, what do the isopleths represent?

Suppose the object parameter of the analysis is sea-surface temperature, the available concurrent observations are synoptic in the sense that they were made within ± 10 minutes of the time to which the analysis refers, and that each observation provides a value of sea-surface temperature which is appropriate in local space (say 100 meters x 100 meters) and time (say a 1-minute time span).[1] If there is a high density of

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[1]If there are no observer or instrument errors these assumptions are reasonably realistic with regard to data from ships available for a synoptic analysis of sea-surface temperature. The requirement to assimilate satellite-derived data introduces other considerations which are discussed later.

data coverage over the whole area to be analyzed (say the North Atlantic) then, because there are no data-sparse or data void regions, the scale of resolution of the analysis in space (i.e., the detail which can be represented) is determined largely by the scale of the chart upon which the isopleths are drawn. (Objective analysis methods use an analysis grid, the upper limit of analysis resolution being determined by the grid spacing. However analysis resolution generally is limited by data density.) Now assume that in a reasonably small region (say 50 kms x 50 kms) there are a large number of ships all reporting temperatures between 19°C and 21°C but normally distributed around 20.0°C. (For the moment we are assuming there are no errors in any observation.) Clearly the representative temperature for this small region is 20.0°C, the difference between reported values and the representative value being due to the effect of features which are sub-scale to the object scale-of-resolution. The 20°C isotherm could be drawn through this small region with considerable confidence. <u>Note that the isotherm is drawn to conform to the representative value, not to a particular observation which may differ considerably from the representative value</u>. By estimating representative values over the whole area a family of isotherms could be drawn, thus providing the required analysis. Because representative values have been used to construct the isotherms, <u>the analysis shows significant variabilities in the object scale-of-resolution</u>[1] for the two-dimensional sea-surface temperature field. Because of the (assumed) high data density, thus allowing representative values to be accurately determined from the estimates provided by actual observations, our confidence in the analysis would be high.

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[1]This is the purpose of an analysis. It is a misconception--unfortunately very prevalent--that the sole purpose of an analysis is to fit <u>observed</u> data rather than to produce <u>representative</u> values. This point is very significant and will arise again.

However now suppose that in some regions there are very few observations. Although each observation provides an estimate of the associated representative value, this value cannot be determined with any certainty from a small number of observations. If, in a small region, only one observation were available then this provides our "best estimate" of the representative value (ignoring for the moment any information carried along the time axis from the previous analysis). Although isopleths could be drawn to "fit" this isolated observation no great confidence should be placed in the analysis in that region because the observation is known to contain information pertinent to features which are sub-scale to the desired representative value. If no observations are available in a region then isopleths have to be interpolated from surrounding information; analysis confidence is even less. In general the greater the density of data the more certain is the analysis with regard to the representation of variabilities which are significant in the object scale-of-resolution. It can be seen that measures of data reliability and measures of the reliability of the resultant analysis at all points in the analyzed field are factors which should be considered as part of the analysis process.

Information which can contribute to an analysis may be obtained from sources other than concurrent observations. One source already outlined is the contribution provided by information carried along the time axis. Other sources are available--for example, analyses of other environmental parameters which provide information concerning the object parameter by way of diagnostic relationships. These sources, and the manner in which their information contribution can be assimilated into analyses of the object parameter, are discussed in subsequent sections.

Putting aside considerations of reliability it can be seen that the purpose of the simple two-dimensional analysis described above is to determine the representative value of the object parameter at any point in the analyzed field where the representative value is appropriate to the object scale-of-resolution of the analysis. Drawing isopleths is only a convenience for (subjectively) estimating representative values. An appropriate interpolation scheme may be used to determine the representative value at any point in the field--in other words the field is assumed to be more or less continuous. In addition the isopleths provide an easily-interpreted visual representation of significant features in the field.

The concept of two-dimensional synoptic analysis may be extended to three-dimensional space. In subjective analyses this is usually achieved by analyzing a "stacked set" of horizontal (or quasi-horizontal[1]) fields for a common time. Note that analyzing each horizontal field independently of others in the set does not provide a three-dimensional analysis. Information must be exchanged (or "spread") vertically to provide three-dimensional continuity. For example significant features in a sea-surface temperature analysis should be evident as sub-surface features even if not directly supported by concurrent sub-surface observations. The vertical extent of horizontal features must be determined in a realistic manner by the analysis process itself. Subjective analysis methods concentrate on two-dimensional variabilities; spreading information in three-dimensional space is a difficult process depending very much on the skill and insight of the analyzer. In practice hand-drawn analyses pay little attention to significant variabilities in three dimensions. Nevertheless, the purpose of a three-dimensional analysis in space is the same--to determine representative values at any point in space where the representative value is appropriate to the object scale-of-resolution of the analysis.

Extension of the analysis process to encompass significant variabilities in space and time now may be considered. Suppose that observations of an environmental parameter P are made every n hours

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[1] For example isobaric surfaces are quasi-horizontal.

both horizontally and vertically in space. P is the object parameter of the analysis process.[1] As is usual with conventional environmental data, the observations are distributed throughout space but are "clustered" along the time axis. The process of analysis is illustrated in Fig. 4.

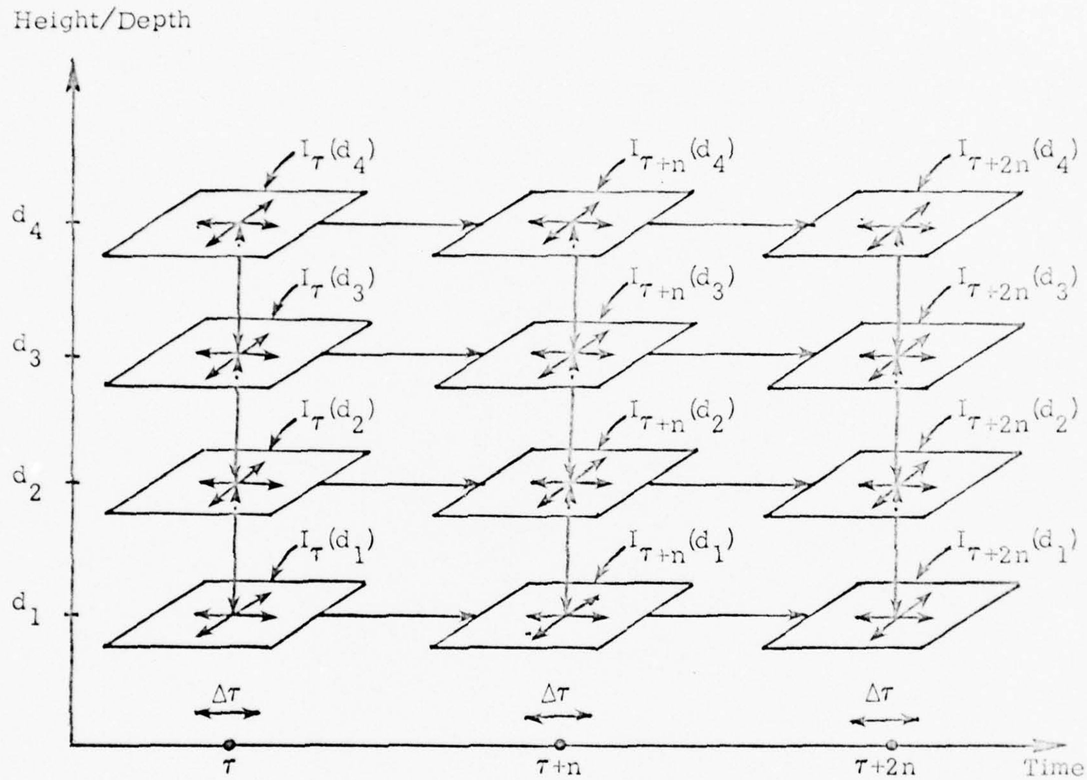

Figure 4 An analysis in space and time. See text for explanation.

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[1] P has deliberately not been specified. To use any particular environmental parameter as an example for analysis in space and time would involve considerations unique to that parameter. Such considerations are inappropriate to this general discussion.

At analysis time τ+n, concurrent information is available as input to each of the levels to enter into the analysis. (Four are shown, d_1 through d_4. In practice the number of levels, and their separation, is chosen to provide an appropriate degree of vertical resolution.) As previously noted (page 22), concurrent information may be obtained from sources other than direct observations of the object parameter, P, itself. The allowable time-span for direct observations is shown by $\Delta\tau$. In addition to concurrent information from a variety of sources, information also is available from the analysis at time τ carried forward along the time axis to time τ+n by an appropriate method.[1] At each level all information relevant to that level is combined and blended horizontally, thus providing a set of two-dimensional analyses. Information from these horizontally analyzed fields is then spread vertically through <u>all</u> levels, the information arriving from above and below being used to adjust each horizontal analysis so that it now represents a field which is an optimum combination of the <u>total</u> information available. The set of fields provides, in effect, a three-dimensional analysis for τ+n with continuity in time.

The τ+n analysis is then carried forward in time to τ+2n, thus continuing to accrue information along the time axis, and the analysis procedure repeated. The complete process can provide representative values of the object parameter P, in the object scale of the analysis resolution, at any point in space and time.

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[1] In certain circumstances--for example the analysis of historical sequences--information may be carried in both directions along the time axis, thus further enhancing the analysis for any particular synoptic time. This capability is discussed later (Section 7.4).

The overall procedure described is, of course, an outline of a 4-dimensional analysis method. In 4-D objective analysis systems (such as FIB), each of the dimensions is generally discretized by finite increments--in effect there is a grid array in both space and time. For example, referring to Fig. 4, there is a grid array for the 2-D (quasi-) horizontal surfaces. The third component of the grid array is provided by the vertical separation between those surfaces, and the fourth component is provided by the time-separation between analyses. The continuum between "points" in 4-dimensional space and time is defined by appropriate interpolation schemes. In many cases variations in the vertical dimension, in the atmosphere and oceans, can be better represented by a set of parameters which correspond to a modelling of significant degrees of profile variability rather than by a set of levels. If so then parameters which are physically independent of one another are to be preferred. (The full significance of this paragraph will become apparent when the FIB system for the 4-D analysis of ocean thermal-structure parameters is discussed--Section 5.) Clearly however, the sophisticated information-processing capabilities implied by this outline of a 4-D analysis system lie far beyond any subjective analysis skills.

The Fields by Information Blending analysis methodology is a comprehensive information-processing methodology for general application to the analysis of scalar and vector fields. It encompasses all the capabilities required to provide analyses of meteorological and oceanographic parameters on any desired scale of resolution in space and time. The analyzed fields which result provide the maximum knowledge of distributions of the object parameter of the analysis which is to be gained from the total relevant information available. FIB encompasses a capability for providing the reliability of analyzed fields at every grid point used in the analysis, and a capability for assigning to every observation utilized in an analysis a measure of the estimated reliability of that observation.[1] A qualitative and quantitative description of FIB capabilities, with specific application to the analysis of ocean thermal-structure parameters, occupies the remainder of this publication.

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[1] Based on the foregoing discussion it can be seen that the reliability of an observation depends on the object scale of analysis resolution. For example an observation used in a synoptic analysis may well represent the current synoptic situation. Using the same observation in a monthly-mean climatology would show, say, a lower reliability if the synoptic situation (for which the observation has a high reliability) were highly anomalous. An example of this will be shown in Section 4.13 using two different degrees of spatial resolution.

3. FUNDAMENTAL CONCEPTS AND FORMULATIONS OF THE FIELDS BY INFORMATION BLENDING ANALYSIS METHODOLOGY

3.1 The Reliability of Observations as Information

Assume that a number N of independent, unbiased and well-distributed observations of an environmental parameter P have been made in a certain "module" in space and time. Each observation of P provides an independent estimate of the representative value, P_R, for the module. If N is large a plot of the frequency distribution of the observed values probably would be a close approximation to a normal distribution. Figure 5 shows an assumed distribution[1] for the observations where, for convenience, P has been identified as sea-surface temperature, T.

Our best estimate of the representative temperature, T_R, is provided by the mean, $\overline{T}_N$, of all available data:

$$T_R \approx \overline{T}_N = \frac{1}{N} \sum_{n=1}^{N} T_n \quad . \qquad (5)$$

If the sample size (N) is large then we may assume that $T_R = \overline{T}_N$.

The variance of the total population may be computed from

$$\sigma_R^2 = \frac{1}{N} \sum_{n=1}^{N} \left(\overline{T}_N - T_n\right)^2 \quad . \qquad (6)$$

(For the distribution shown in Fig. 5, $\overline{T}_N = 20.0^\circ C$, σ_R (the standard deviation) = 0.61 and σ_R^2 (the variance) = 0.38.)

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[1] Although Fig. 5 shows a normal distribution, this is not a <u>necessary</u> assumption.

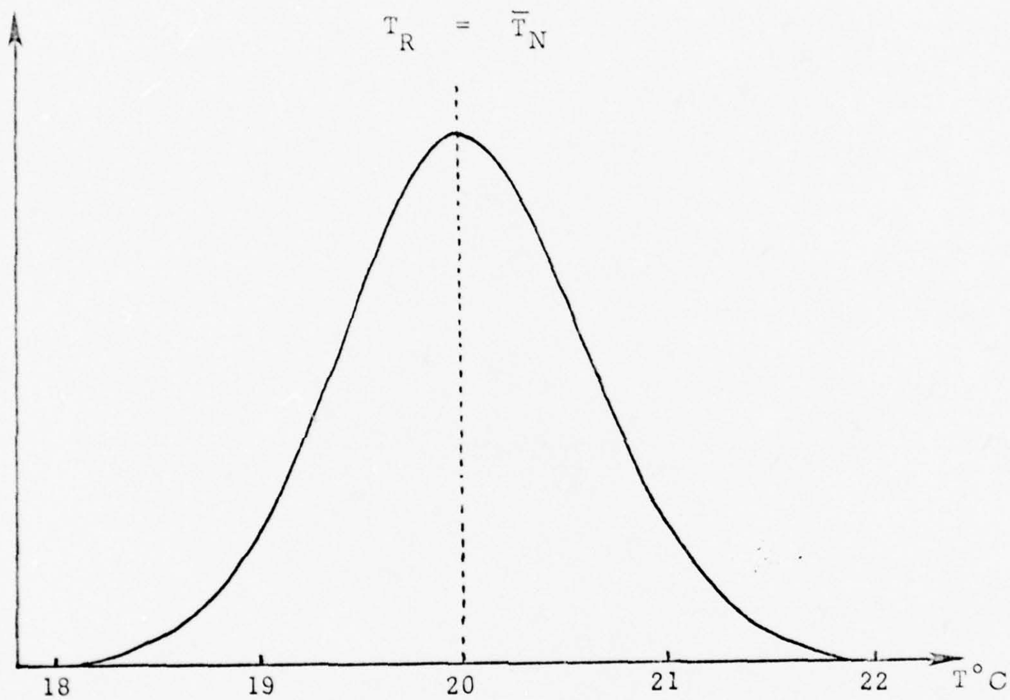


Figure 5 An assumed distribution of observations of sea-surface temperature. ($\sigma = 0.61$, $\sigma^2 = 0.38$)

Now suppose that a number of random samples, each of size M where M << N , are picked from the total population used to compute T_R and σ_R^2. The arithmetic mean, $\overline{T}_M$, of each set of M observations provides an estimate of the representative temperature. The variance of the means as estimates of T_R is given by:

$$\sigma_M^2 = \frac{\sigma_R^2}{M} \quad . \tag{7}$$

Consider the effect of increasing the sample size M :

If M = 1 , $\frac{1}{\sigma_M^2} = \frac{1}{\sigma_R^2}$ i.e., $\sigma_M^2 = \sigma_R^2$

M = 2 , $\frac{1}{\sigma_M^2} = \frac{1}{\sigma_R^2} + \frac{1}{\sigma_R^2}$ i.e., $\sigma_M^2 = \frac{\sigma_R^2}{2}$

M = 3 , $\frac{1}{\sigma_M^2} = \frac{1}{\sigma_R^2} + \frac{1}{\sigma_R^2} + \frac{1}{\sigma_R^2}$ i.e., $\sigma_M^2 = \frac{\sigma_R^2}{3}$.

It can be seen that for any M , σ_M^2 may be calculated by taking the inverse of the sum of M terms of $(\sigma_R^2)^{-1}$. Clearly the larger the number of observations used to compute $\overline{T}_M$, the less is the variance associated with the sample mean and the more confident we therefore can be that any particular value of $\overline{T}_M$ provides a close approximation to T_R computed from the total population N .

This leads to the concept of the reliability (or weight) to be associated with any estimate of T_R provided by the mean of M observations of T . We define the weight, A , of a single observation by

$$A = \frac{1}{\sigma_R^2} \quad . \tag{8}$$

Thus, for M observations, the weight to be associated with $\overline{T}_M$ (the best estimate of T_R provided by the M actual observations) is MA.

Considering Fig. 5 as an example, $A = \frac{1}{0.38} \approx 2.7$. Thus if any single observation were picked at random from all those available, its associated reliability (as an estimate of T_R) is 2.7. If 2 observations were selected at random the mean of these two observations would have an associated reliability of 5.4.

The constraint that M be a subset of N is not necessary. The evaluation of $(\sigma_R^2)^{-1}$ based on a large sample provides a "class weight" for, in this example, sea-surface temperature observations. Suppose that, for another but similar module in space and time, a single observation of T is made. The reliability of this observation as an estimate of the local representative temperature is $A\ (\equiv 1/\sigma_R^2)$. If 5 observations are available then the associated reliability of the mean temperature as an estimate of the unknown representative temperature is 5A.

One of the fundamental concepts upon which FIB is based may now be stated. Any piece of information is incomplete without an associated reliability. For an independent piece of information the reliability, or "report weight", is defined as the inverse of the error variance inherent in the observation and/or associated with the class of observation.

The FIB methodology is based on fundamental rules for adding uncorrelated variance contributions and for adding independent information.

FIB has the ability to accept and blend together, in a precisely controlled and formulated manner, all information of relevance to the distribution of the object parameter in the scale of the analysis resolution.

Before formulating methods for combining information and variances some of the factors which contribute to the lack of reliability of observed data should be considered. These factors include for example:

a. Observer error.

b. Instrument error including calibration bias.

c. Errors due to the method of measurement--for example, for sea-surface temperature reports, engine-intake temperatures refer to the temperature of sea-water a few feet below the surface whereas a sea-bucket measures the temperature of a water sample obtained from the top foot or so.

d. Errors in recording and transmitting data.

e. Sub-scale "noise" due to the fact that observations in general contain an information component which is sub-scale to the desired representative value.

All of these factors contribute to the uncertainty of an observation as an estimate of the representative temperature in the object scale of resolution. Statistically however (apart from one factor discussed below) all errors are random and uncorrelated; the data sample for a particular analysis generally consists of a reasonably large number of observations all made by different observers all using different and independently calibrated instruments. The normally-occurring dispersion[1] associated

[1] In any large data sample there generally is a small number of reports which are patently in "gross error"--for example, for a particular location and time, a report of sea-level pressure may be received which differs by many millibars (say 20 or more) from other local information. In a FIB analysis a "gross error check" is carried out to eliminate such reports.

with any class of observations may be taken into account by the FIB concept of report weight as defined above. As will be seen, this concept and associated formulations means that FIB is able to handle low-quality data and from it extract (or distill) the information component which is relevant to the object parameter in the object scale of resolution. FIB should be regarded as an information-processing methodology.

The factor which may introduce errors which are correlated is concerned with calibration bias of a whole class of instruments. In general this does not occur--for example comparison of a large sample of barometers with a standard barometer would not reveal a consistent bias. The calibration of individual instruments may be incorrect but, as a class, it is reasonable to assume that calibration errors are normally distributed. Even for observations which are subjective estimates (such as observations of wave height) it is not likely that observers, as a whole, report consistently high or low. However a calibration bias problem is encountered with satellite-derived data and not only because the measurements are made by a common instrument. The transmission functions on which is based the interpretation of the measured radiances omit a variety of secondary attenuation factors. Their variance contributions can be expected to correlate spatially to some degree. Using a FIB capability known as "Alternating Parallel Analysis" (APA), the calibration bias of satellite-derived data may be circumvented. (See Section 4.12.)

3.2 Basic FIB Rules, Formulations and Definitions

The Fields by Information Blending methodology is based on a small number of fundamental rules and associated formulations for adding (or subtracting) uncorrelated variance contributions and for adding (or subtracting) independent information. These rules and formulations are presented in this Section. As will be seen by the numerical example given in Section 3.2.6, although derivation of the fundamental FIB rules is a matter of some complexity, these rules, once derived, are relatively easy to apply.

3.2.1 Definition of the Arbitrary ℓ,m Module

Consider a 2-dimensional orthogonal grid system of dimensions LxM grid points covering the area for which an analysis of the object parameter is to be performed. Within limits imposed by the computer system L and M may be chosen to provide any desired degree of resolution in L,M space. (Whether or not the desired degree of resolution can be meaningfully achieved depends on the information available for analysis.) An arbitrary grid point may be designated by the integers ℓ,m. We define the <u>module</u> of the analysis area to be associated with ℓ,m as the corner point ℓ,m, two sides, and the interior area; see Fig. 6.

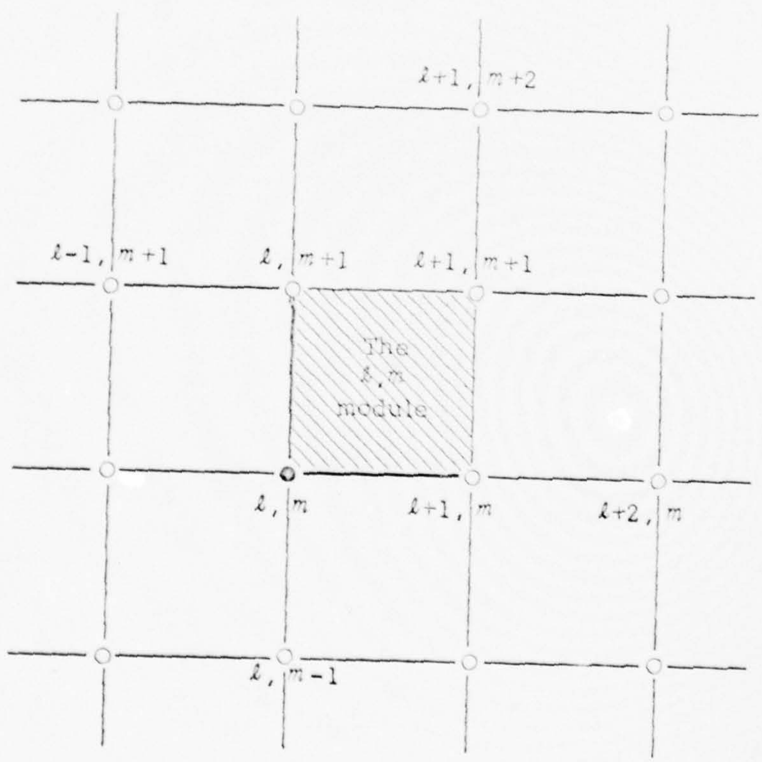


Figure 6 The arbitrary ℓ,m area module, consisting of the corner point, two sides and the interior area.

3.2.2 Weight or Reliability

Let P be any parameter. Let P_R be the representative value of P for a small region in space and time where P_R is determined from a large number N of independent observations--see Eq. (5). Let the variance associated with these observations be σ_R^2--see Eq. (6). Let P_n be an independent measurement of P. If P_n is made in the same region as the N observations used to compute P_R, then P_n is an estimate of P_R. As discussed in Section 3.1, the reliability of P_n as an estimate of P_R is given by $1/\sigma_R^2$. If P_n is made in another locality (but with similar variabilities) then P_n is an estimate of the (unknown) value of P_R for that locality. However the reliability to be associated with P_n as an estimate of the unknown value of P_R is still given by $1/\sigma_R^2$--σ_R^2 is the "class variance" to be associated with observations of P, and the "class weight" is defined as $1/\sigma_R^2$.

However the concept of class weight may be extended. By way of illustration suppose that we measure SST by two distinct classes of instrument--say a sea-bucket and the surface reading from an expendable bathythermograph (XBT). For reasons given in Section 3.1 the dispersion for the two types of instrument may be different. Clearly, therefore, there is a class variance associated with one type (or class) of instrument which differs from the class variance associated with the other type of instrument. How do we combine an observation from one class with an observation from another class, and what is the reliability to be associated with the resultant of the combination? This question is answered in the following Section.

3.2.3 Addition of Information

Suppose we measure values of an object parameter P by two classes of instrument. Let the class variance (based on a large number of independent observations) for one class of instrument be σ_1^2 and for the second class be

σ_2^2. Then

$$\sigma_1^2 = \frac{1}{N} \sum_{n=1}^{N} \left(P_R - P_{1,n}\right)^2 \qquad (9)$$

and

$$\sigma_2^2 = \frac{1}{N} \sum_{n=1}^{N} \left(P_R - P_{2,n}\right)^2 \qquad (10)$$

where N is large, $P_{1,n}$ and $P_{2,n}$ are observations in a given module made by the two classes of instrument, and P_R is the representative value of P for that module.

Let P^* be the optimum resultant to be obtained from combining an observation, P_1, from one class with an observation, P_2, from the second class. We assume the form

$$P^* = \frac{W_1 P_1 + W_2 P_2}{W_1 + W_2} \qquad (11)$$

where W_1 and W_2 are arithmetic weighting factors to be chosen in such a way that P^* is indeed the optimum resultant.

We may re-write Eq. (11) as follows:

$$P^* = \frac{P_1 + \alpha P_2}{1 + \alpha} \qquad (12)$$

where $\alpha = W_1/W_2$.

Based on a large number N of such pairs, the variance to be associated with $P^{\star}$ is given by

$$\sigma_{\star}^{2} = \frac{1}{N} \sum_{n=1}^{N} \left(P_R - \frac{P_{1,n} + \alpha P_{2,n}}{1 + \alpha} \right)^2$$

$$= \frac{1}{(1+\alpha)^2} \left(\frac{1}{N} \sum_{n=1}^{N} \left(P_R - P_{1,n} \right)^2 + \frac{1}{N} \alpha^2 \sum_{n=1}^{N} \left(P_R - P_{2,n} \right)^2 \right.$$

$$\left. + \frac{1}{N} 2\alpha \sum_{n=1}^{N} \left(P_R - P_{1,n} \right)\left(P_R - P_{2,n} \right) \right) \quad . \qquad (13)$$

Since the differences $(P_R - P_{1,n})$ and $(P_R - P_{2,n})$ are uncorrelated and unbiased

$$\frac{1}{N} \sum_{n=1}^{N} \left(P_R - P_{1,n} \right)\left(P_R - P_{2,n} \right) = 0 \quad . \qquad (14)$$

Thus from Eqs. (9), (10) and (14), Eq. (13) becomes

$$\sigma_{\star}^{2} = \frac{\sigma_1^2 + \alpha^2 \sigma_2^2}{(1+\alpha)^2} \quad . \qquad (15)$$

To determine that value of α which minimizes σ_*^2 we set

$$\frac{\partial \sigma_*^2}{\partial \alpha} = 0 \quad .$$

Thus from Eq. (15)

$$2 \alpha \sigma_2^2 \left(1 + \alpha\right)^2 - 2\left(1 + \alpha\right)\left(\sigma_1^2 + \alpha^2 \sigma_2^2\right) = 0 \quad .$$

Rearrangement provides

$$\alpha = \sigma_1^2 / \sigma_2^2 \quad . \tag{16}$$

Substituting for α in Eq. (12) and dividing throughout by σ_1^2 provides

$$P^* = \left(\frac{1}{\sigma_1^2} \cdot P_1 + \frac{1}{\sigma_2^2} \cdot P_2\right) \bigg/ \left(\frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}\right) \quad . \tag{17}$$

In accordance with Eq. (8) we now define the class weight A_1 to be associated with the first class of instrument by $A_1 = 1/\sigma_1^2$ and the class weight A_2 to be associated with the second class of instrument by $A_2 = 1/\sigma_2^2$. From Eq. (17)

$$P^* = \frac{A_1 P_1 + A_2 P_2}{A_1 + A_2} \quad . \tag{18}$$

What weight, A^*, is to be associated with P^*?

From Eqs. (15) and (16)

$$\sigma_*^2 = \left(\sigma_1^2 + \frac{\sigma_1^4}{\sigma_2^2}\right) \Bigg/ \left(1 + \frac{\sigma_1^2}{\sigma_2^2}\right)^2 .$$

Thus

$$\frac{1}{\sigma_*^2} = \frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2} . \tag{19}$$

Defining the weight $A^* \equiv 1/\sigma_*^2$ we see that

$$A^* = A_1 + A_2 . \tag{20}$$

To sum up, if we have two classes of observations whose class weights (defined as the reciprocal of the respective class variances determined from large samples) are A_1 and A_2, then the best estimate P^* of the representative temperature P_R provided by an observation P_1 from the first class and an observation P_2 from the second class is given by Eq. (18), and the reliability of P^* as an estimate of P_R is given by Eq. (20).

Suppose now we wish to combine a third observation P_3 of class weight A_3, with the resultant P^* of weight A^*. Denoting the new resultant by P'^* and its associated weight by A'^*, then

$$P'^{*} = \frac{A^{*} P^{*} + A_3 P_3}{A^{*} + A_3}$$

$$= \frac{A_1 P_1 + A_2 P_2 + A_3 P_3}{A_1 + A_2 + A_3} \qquad (21)$$

and

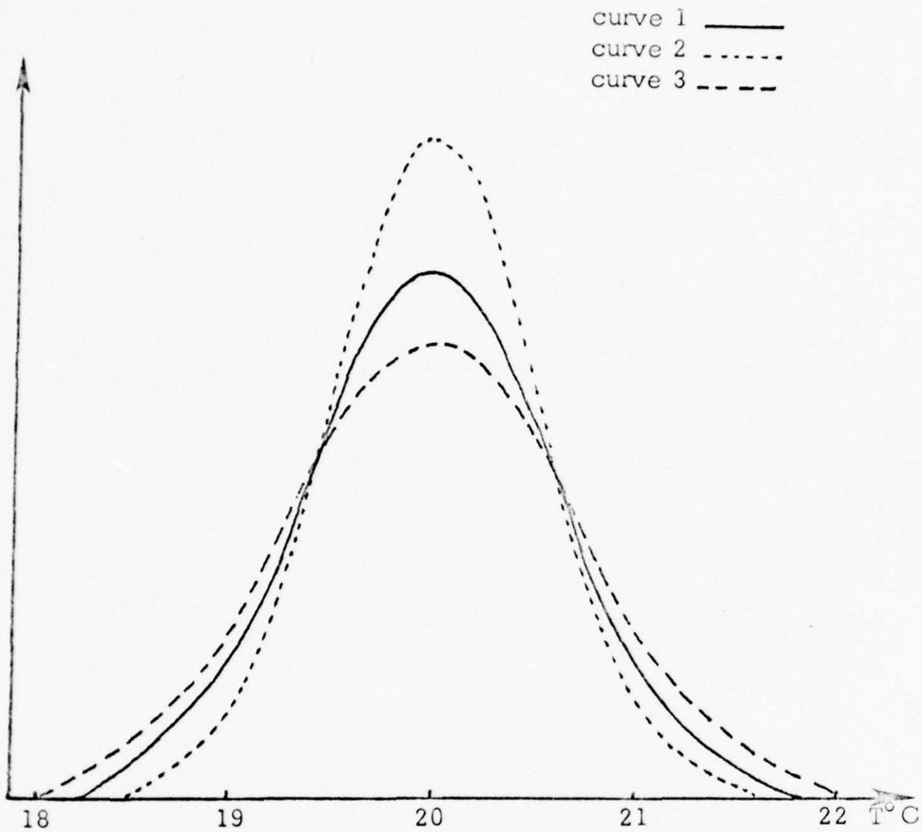
$$A'^{*} = A^{*} + A_3 \quad ,$$

$$= A_1 + A_2 + A_3 \quad . \qquad (22)$$

Note that the resultant is independent of the order in which the observations are assembled.

Equations (21) and (22) generalize the discussion of the reliability of observations given in Section 3.1 where we assembled M observations of equal weight A--Eq. (21) provides $P^{*} = \overline{P}$ and Eq. (22) gives $A^{*} = MA$. A question which might be posed at this point is "Why introduce class weights? Why not assemble all observations on an equal basis using a single weight which is a measure of the expected variance to be associated with observations of P in general?" The answer to this is best explained by a numerical example. (This example also serves to illustrate utilization of the FIB rule for adding information.)

Suppose that in a certain module in space and time an observation of sea-surface temperature is made using both a sea-bucket and an XBT. Let the two observed temperatures be 12.6° C and 14.2° C respectively. Based on previously determined distributions it is estimated that the associated class weights are $A_1 = 4.0$ and $A_2 = 2.0$--see curves 2 and 3 in Fig. 7. (The reason for curve 1 will become apparent shortly.) From Eq. (18)



curve 1	curve 2	curve 3
σ = 0.61	σ = 0.50	σ = 0.71
σ^2 = 0.38	σ^2 = 0.25	σ^2 = 0.5
A = 2.7	A = 4.0	A = 2.0

Figure 7 Curve 1 shows the total distribution of sea-surface temperature observations for a given module in space and time. Curves 2 and 3 show the distributions for each of the two classes of instrument contributing to the total distribution. (Curve 1 is the same as Fig. 5.)

or Eq. (21)

$$T^{*} = \frac{4.0 \times 12.6 + 2.0 \times 14.2}{4.0 + 2.0} = 13.1^{\circ}C$$

where T^{*} is the best estimate of the local representative temperature available from the two observations. From Eq. (20) or Eq. (22) the weight A^{*} to be associated with T^{*} is 6.0. (T^{*} and A^{*} have been obtained using the fundamental FIB rule for adding information.)

To answer the question originally posed, we now need to determine what T^{*} and A^{*} would be if all observations were treated as a single class. Since A_1 ($\equiv 1/\sigma_1^2$) and A_2 ($\equiv 1/\sigma_2^2$) both have been determined from a large number of observations then the variance σ_c^2 of the composite data set is given by

$$\sigma_c^2 = \frac{\sigma_1^2 + \sigma_2^2}{2} \quad .$$

Thus, using the numerical values given above, $\sigma_c^2 = 0.38$ and $A_c = 2.7$--see curve 1 in Fig. 7.

Using observed values of $12.6^{\circ}C$ and $14.2^{\circ}C$ as before, but this time with the variance appropriate to a single combined class, $T_c^{*} = 13.4^{\circ}C$ and $A_c^{*} = 5.4$. Note that by failing to utilize separate classes we have lost resolution--i.e., information.

In general it can be seen that if the reports available for analysis are of diverse qualities--falling into distinguishable classes based on associated variances--then they must be treated as separate classes in order to extract the maximum amount of available information.

3.2.4 Removal of Information

The contribution of a piece of information, P_n of weight A_n, which already has been assembled into a resultant, P^* of weight A^*, may be removed by subtraction:

$$P_B^* = \frac{A^* P^* - A_n P_n}{A^* - A_n} \tag{23}$$

$$A_B^* = A^* - A_n \tag{24}$$

where P_B^* of weight A_B^* may be regarded as the "background information" remaining at the location after removal of the information P_n of weight A_n.

3.2.5 Addition of Contributing Variances

Let σ_P^2 be the variance associated with P_n, an estimate of quantity P_R:

$$\sigma_P^2 = \frac{1}{N} \sum_{n=1}^{N} \left(P_R - P_n\right)^2$$

where N is large.

Let σ_b^2 be the variance associated with b_n, an estimate of quantity b_R:

$$\sigma_b^2 = \frac{1}{N} \sum_{n=1}^{N} \left(b_R - b_n\right)^2$$

where N is large.

What is the variance to be associated with the combination $P_n \pm b_n$?

By straightforward analysis, again where N is large,

$$\sigma^2_{P\pm b} = \frac{1}{N} \sum_{n=1}^{N} \left[\left(P_R \pm b_R\right) - \left(P_n \pm b_n\right) \right]^2$$

$$= \frac{1}{N} \sum_{n=1}^{N} \left[\left(P_R - P_n\right) \pm \left(b_R - b_n\right) \right]^2$$

$$= \frac{1}{N} \sum_{n=1}^{N} \left[\left(P_R - P_n\right)^2 + \left(b_R - b_n\right)^2 \pm 2\left(P_R - P_n\right)\left(b_R - b_n\right) \right] .$$

As before (see Eq. (14)), since the differences $(P_R - P_n)$ and $(b_R - b_n)$ are unbiased and uncorrelated,

$$\frac{2}{N} \sum_{n=1}^{N} \left(P_R - P_n\right)\left(b_R - b_n\right) = 0$$

and hence

$$\sigma^2_{P\pm b} = \sigma^2_P + \sigma^2_b \quad . \tag{25}$$

Equation (25) is the fundamental FIB rule for the addition of contributing variances. At this point, to aid understanding, we should now consider some practical interpretations of this rule.

Suppose at grid point m , parameter P has a representative value of $P_{R,m}$ and at grid point $m+1$ has a representative value of $P_{R,m+1}$. Now suppose we have an independent estimate (i.e., measurement) b_m of the difference in P between m and $m+1$. Thus b_m is an estimate of $P_{R,m+1} - P_{R,m}$. This will be written

$$b_m \Rightarrow P_{R,m+1} - P_{R,m} \quad .$$

The reliability of this estimate may be defined by the class weight $B \equiv 1/\sigma_b^2$.

Now assume that at the grid points m and $m+1$ we do not have the representative values but actual observations P_m and P_{m+1}. (Note $P_m \Rightarrow P_{R,m}$ and $P_{m+1} \Rightarrow P_{R,m+1}$.) Thus at $m+1$ we have two estimates of $P_{R,m+1}$--P_{m+1} and $P_m + b_m$. The weight associated with P_{m+1} is say A. But what weight should be associated with the linear combination $P_m + b_m$? According to Eq. (25) the associated variance is given by $1/A + 1/B$. In general for a linear combination of such independent estimates

$$P = P_0 \pm b_1 \pm b_2 \pm \dots \tag{26}$$

the associated variance is given by

$$\sigma^2_{P\pm b} \equiv A^{-1}_{P\pm b} = A_0^{-1} + B_1^{-1} + B_2^{-1} + \dots \quad . \tag{27}$$

(To give a numerical example, if the SST at m is observed as 20.0° C with a reliability of 4.0 and the SST-difference between m and $m+1$ is observed as 3.0°C ($m+1$ warmer) with a reliability of 2.0, then the best estimate of $P_{R,m+1}$ provided by these two pieces of information is 23.0°C with an associated reliability of 1.33.)

One interesting point is worth noting--our best available estimate of b may be zero but we still have to take its associated variance into account. For example suppose at grid point ℓ, m an SST observation of, say, 18.0°C was made two hours ago with a reliability of 4.0. If we have no other information available then our best estimate of the current value of SST at this grid point is still 18.0°C (i.e., b = 0). However it is clear that the reliability of this observation, when carried along the time axis, is less than it was two hours ago. If the variance associated with a time-increment of two hours (based on a large number of observations) is, say, 0.25, then a reliability of 2.0 is to be associated with 18.0°C as an estimate of the current representative value of SST at grid point m.

As a final point in this Section we now consider the reliability of the product kP_n where k is a specified constant. If σ_R^2 is the variance associated with observations of P, then

$$\sigma_R^2 = \frac{1}{N} \sum_{n=1}^{N} \left(P_R - P_n\right)^2 .$$

The variance associated with kP_n is

$$\frac{1}{N} \sum_{n=1}^{N} \left(kP_R - kP_n\right)^2 = k^2 \frac{1}{N} \sum_{n=1}^{N} \left(P_R - P_n\right)^2$$

$$= k^2 \sigma_R^2 . \tag{28}$$

It will be noted that the fundamental formulations of the FIB methodology are not based on the requirement of normal distributions.

3.2.6 A Numerical Example

m • T_m : 14.2, 2.0; 14.6, 3.0; 16.8, 6.0

b_m $(\Rightarrow T_{m+1} - T_m)$: 3.3, 2.0; 3.5, 3.0; 3.8, 4.0

$m+1$ • T_{m+1} : 17.0, 2.0; 18.4, 4.0; 19.5, 5.0

b_{m+1} $(\Rightarrow T_{m+2} - T_{m+1})$: 1.1, 4.0; 0.8, 2.0

$m+2$ • T_{m+2} : 20.0, 2.0; 19.6, 3.0; 21.8, 5.0

The above diagram shows 3 locations--m , $m+1$, $m+2$. At each location 3 observations of temperature are available of known weight (say by class weight).[1] In addition estimates of b_m and b_{m+1} are available from independent sources. These estimates and associated weights are shown in a similar manner. Based on the FIB rules and definitions given previously, what is the best estimate of temperature (and associated reliability) at the points m , $m+1$, $m+2$?

First, Eqs. (21) and (22) may be used to assemble the information. For example at m ,

$$T_m = \frac{14.2 \times 2.0 + 14.6 \times 3.0 + 16.8 \times 6.0}{2.0 + 3.0 + 6.0} = 15.7^\circ C$$

and $A_m = 11.0$.

---

[1] Parameter magnitude P and associated weight A are shown as P,A. Thus, for the first observation at location m , the parameter value is $14.2^\circ C$ and the associated weight is 2.0.

Similarly

$$T_{m+1} = 18.6^\circ C,\ 11.0$$

$$T_{m+2} = 20.8^\circ C,\ 10.0 \quad .$$

The estimates for b_m and b_{m+1} also may be assembled:

$$b_m = 3.6,\ 9.0$$

and $$b_{m+1} = 1.0,\ 6.0 \quad .$$

The effects of assembly may be illustrated thus:

$$\begin{array}{llcl}
m & \bullet\ T_m & = & 15.7,\ 11.0 \\
 & \quad b_m & = & 3.6,\ \ 9.0 \\
m+1 & \bullet\ T_{m+1} & = & 18.6,\ 11.0 \\
 & \quad b_{m+1} & = & 1.0,\ \ 6.0 \\
m+2 & \bullet\ T_{m+2} & = & 20.8,\ 10.0
\end{array}$$

Note for example that b_m ($\Rightarrow T_{m+1} - T_m$) is 3.6° C whereas $T_{m+1} - T_m$ from direct observation is 2.9° C. The information sources conflict and the best compromise, based on reliability, must be effected.

Let the resultant temperatures based on <u>all</u> available information be T^*_m, T^*_{m+1} and T^*_{m+2} at m, $m+1$ and $m+2$ respectively, the corresponding weights being A^*_m, A^*_{m+1} and A^*_{m+2}. To find T^*_{m+2}, T_m must be combined with b_m, the resultant assembled with T_{m+1}, the resultant combined with b_{m+1}, and the resultant assembled with T_{m+2}. At all stages the appropriate weights must be taken into account.

a. $T_m + b_m \Rightarrow T_{m+1} = 19.3^\circ C$ (Eq. (26))

Associated weight of the estimate is given by

$$\frac{1}{A} = \frac{1}{A_m} + \frac{1}{B_m} \qquad \text{(Eq. (27))}$$

i.e., $A = 5.0$.

b. Resultant at $m+1$ is given by

$$\frac{5.0 \times 19.3 + 18.6 \times 11.0}{5.0 + 11.0} \qquad \text{(Eq. (21))}$$

$$= 18.8^\circ C = T^1_{m+1} \quad \text{say .}$$

Associated weight is 16.0 . (Eq. (22))

c. $T^1_{m+1} + b_{m+1} \Rightarrow T_{m+2} = 19.8$ (Eq. (26))

Associated weight of estimate is 4.4 . (Eq. (27))

d. $$T^*_{m+2} = \frac{19.8 \times 4.4 + 20.8 \times 10.0}{4.4 + 10.0} = 20.5^\circ C \qquad \text{(Eq. (21))}$$

and $A^*_{m+2} = 14.4$. (Eq. (22))

Similar calculations provide T^*_{m+1}, A^*_{m+1} and T^*_m, A^*_m . The best estimate of temperature and associated reliability at each location, based on a blending of all available information, is as follows:

$m \bullet T^{\star}_{m} = 15.6^{\circ}C, \quad A^{\star}_{m} = 16.6$

$m+1 \bullet T^{\star}_{m+1} = 19.0^{\circ}C, \quad A^{\star}_{m+1} = 19.7$

$m+2 \bullet T^{\star}_{m+2} = 20.5^{\circ}C, \quad A^{\star}_{m+2} = 14.4$

Five points may be noted:

a. The above example is a very simple FIB analysis--all available information has been blended together to provide the best estimate of temperature at m, $m+1$ and $m+2$.

b. If desired $b^{\star}$ and $B^{\star}$ values could be calculated.

c. The expected variance of analysis is given by $(A^{\star})^{-1}$--i.e., 0.06, 0.05, and 0.07 at m, $m+1$ and $m+2$ respectively. (The corresponding standard deviations are $0.25^{\circ}C$, $0.22^{\circ}C$ and $0.26^{\circ}C$.)

d. The analysis has used class weights. The analysis may be refined by "reevaluating" the reliability of reports in the context of all other independent (i.e., "background") information. How this is done is described later. However the statement in point a., that we have computed the best estimate of temperature, may not be quite correct.

e. A simple subjective analysis might compute the mean temperature at each location ($15.2^{\circ}C$, $18.3^{\circ}C$ and $20.5^{\circ}C$ for m, $m+1$ and $m+2$ respectively). How the gradient information could be realistically assimilated is not clear. Neither is such a scheme capable of determining the reliability of the analysis.

This simple example also may be used to demonstrate how information may be spread to a point where no direct observations are available.

Suppose that no observations were available at the $m+1$ location. After assembly we would have:

m • $T_m = 15.7,\ 11.0$

$b_m = 3.6,\ 9.0$

$m+1$ •

$b_{m+1} = 1.0,\ 6.0$

$m+2$ • $T_{m+2} = 20.8,\ 10.0$

This information provides

m • $T^{*}_{m} = 15.8^{\circ}C$, $A^{*}_{m} = 13.6$ ($\sigma^2 = 0.07$, $\sigma = 0.27$)

$m+1$ • $T^{*}_{m+1} = 19.5^{\circ}C$, $A^{*}_{m+1} = 8.7$ ($\sigma^2 = 0.11$, $\sigma = 0.34$)

$m+2$ • $T^{*}_{m+2} = 20.7^{\circ}C$, $A^{*}_{m+2} = 12.7$ ($\sigma^2 = 0.08$, $\sigma = 0.28$)

Note how the decrease in information has affected not only the final values of the object parameter but also the reliability of these values. As would be expected the greatest change in reliability occurs at the point where no data is available--the weight has been reduced from 19.7 to 8.7 (i.e., the standard deviation has increased from $0.22^{\circ}C$ to $0.34^{\circ}C$).

As shown later, information may be spread in more than one dimension. However it can be seen that information-spreading (in this simple example) is achieved by way of the gradient estimates (b,B). In general such estimates are not provided by direct observation. Sources of spreading information are discussed under "Parameter Initialization Fields" (Section 4.4).

3.3 Explicit Blending

The numerical example given in the previous Section is an example of explicit blending. A general solution for such a problem is discussed in this Section.

Suppose we have a one-dimensional array of independent information concerning parameter P .

Level	Values	Weights
1	P_1	A_1
	$b_1 \Rightarrow P_2 - P_1$	B_1
2	P_2	A_2
	$b_2 \Rightarrow P_3 - P_2$	B_2
3	P_3	A_3
	$b_3 \Rightarrow P_4 - P_3$	B_3
4	P_4	A_4

P_1 through P_4 denote the (assembled) values P at levels 1 through 4 respectively. Associated weights are A_1 through A_4 . The estimates b of weight B are <u>independent</u> estimates of the gradient of P , expressed in finite-difference form, along the axis of the array.

What is the resultant value P^* of weight A^*, say at level 4, that is provided by the available information?

At first sight it would seem that four estimates are available for P_4:

Value	Weight
$P_1 + b_1 + b_2 + b_3$	$\left(A_1^{-1} + B_1^{-1} + B_2^{-1} + B_3^{-1}\right)^{-1}$
$P_2 + b_2 + b_3$	$\left(A_2^{-1} + B_2^{-1} + B_3^{-1}\right)^{-1}$
$P_3 + b_3$	$\left(A_3^{-1} + B_3^{-1}\right)^{-1}$
P_4	A_4

However it would be wrong to combine these by the FIB rule for adding independent estimates; the estimates are not independent. The errors are correlated. There are common contributions to the variances of the first three estimates listed.

There is a stepwise procedure for combining the available information which satisfies the condition of independence. The information which propagates toward level 4 can be accrued step by step. (This stepwise procedure was used in the numerical example.) At level 2 the values

$$P_1 + b_1 \quad \text{of weight} \quad \left(A_1^{-1} + B_1^{-1}\right)^{-1}$$

$$\text{and} \quad P_2 \quad \text{of weight} \quad A_2$$

can be combined by the rule for the addition of independent estimates:

$$P_{2(1+2)} = \frac{\left(A_1^{-1} + B_1^{-1}\right)^{-1}\left(P_1 + b_1\right) + A_2 P_2}{\left(A_1^{-1} + B_1^{-1}\right)^{-1} + A_2} \tag{29}$$

$$A_{2(1+2)} = \left(A_1^{-1} + B_1^{-1}\right) + A_2 \tag{30}$$

where the subscript parentheses have been added to show the sequence of combination.

Next, at level 3, the values

$$P_{2(1+2)} + b_2 \quad \text{of weight} \quad \left(A_{2(1+2)}^{-1} + B_2^{-1}\right)^{-1}$$

$$\text{and} \quad P_3 \qquad\qquad \text{of weight} \quad A_3$$

can be combined to form

$$P_{3(1+2+3)} \quad \text{of weight} \quad A_{3(1+2+3)} \quad .$$

Finally, the procedure is repeated to form

$$P_{4(1+2+3+4)} \quad \text{of weight} \quad A_{3(1+2+3+4)}$$

which are the resultants P_4^* of weight A_4^*.

Note that the information <u>arriving</u> at level 4 from external sources is

$$P_{3(1+2+3)} + b_3 \quad \text{of weight} \quad \left(A_{3(1+2+3)}^{-1} + B_3^{-1}\right)^{-1} \quad .$$

The weight is upper-bounded by $A_{(1+2+3)}$ and, more significantly, by B_3.

For an intermediate point, say level 2, the ambient information arrives from two directions. The available independent estimates are:

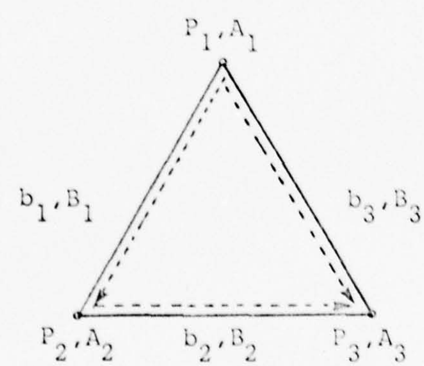
$$P_1 + b_1 \qquad \text{of weight} \quad \left(A_1^{-1} + B_1^{-1}\right)^{-1}$$

$$P_{3(4+3)} - b_2 \qquad \text{of weight} \quad \left(A_{3(4+3)} + B_2^{-1}\right)^{-1}$$

$$P_2 \qquad \text{of weight} \quad A_2 \quad .$$

This stepwise procedure is termed "explicit blending" and provides resultant values and resultant weights. Explicit blending can be applied to any array in which each point is connected to any other point by a <u>single</u> path.

The reason why stepwise (or explicit) blending may only be used for linear arrays is clear from the following diagram which shows the simplest possible two-dimensional array:



At any specified grid point, information from either of the two other grid points can arrive by two paths--either directly or indirectly. (The dotted lines in the diagram show the two information paths from location 1 to location 3). The total information arriving at the specified grid point is not independent. Explicit blending, although useful in certain applications, cannot provide an analysis of two-dimensional arrays of information.

To analyze such two-dimensional arrays is a more difficult task involving "implicit blending". In the following Sections the concepts of implicit blending are introduced and illustrated by using a simple linear array. These concepts are then generalized and extended to encompass an array such as that shown in Fig. 6.

3.4 Simple Implicit Blending in One Dimension

3.4.1 The Error Functional

Consider the following linear array:

$$
\begin{array}{rcl}
m-2 & \bullet & P_{m-2},\ A_{m-2} \\
 & - & b_{m-2},\ B_{m-2} \\
m-1 & \bullet & P_{m-1},\ A_{m-1} \\
 & - & b_{m-1},\ B_{m-1} \\
m & \bullet & P_{m},\ A_{m} \\
 & - & b_{m},\ B_{m} \\
m+1 & \bullet & P_{m+1},\ A_{m+1} \\
 & - & b_{m+1},\ B_{m+1} \\
m+2 & \bullet & P_{m+2},\ A_{m+2}
\end{array}
$$

Figure 8 A one-dimensional array of independent information.

This shows five locations with the parameter value P and weight A at each location and b shows the finite first-difference values of weight B. For any m, $b_m \Rightarrow P_{m+1} - P_m$. All information is independent.

At any location the difference between the assembled values and the resultant values may be regarded as disparity. To produce the optimum resultant <u>over all m</u> the sum of these disparities must be minimized in an appropriate manner.

To carry out this process for the one-dimensional array shown here, we <u>define</u> an "error functional" E as follows:

$$E = \sum_{m=m-2}^{m+2} \left\{ A_m \left(P_m^* - P_m\right)^2 + B_m \left(P_{m+1}^* - P_m^* - b_m\right)^2 \right\} \quad . \tag{31}$$

(E is the sum over all m of the weighted squares of the contributing disparities.)

For any resultant P^*, the value required is that which minimizes its contribution to E. For any location m this can be achieved by setting

$$\frac{\partial E}{\partial P_m^*} = 0 \quad . \tag{32}$$

To do this, terms in Eq. (31) involving P_m^* are separated:

$$E = \ldots + A_m\left(P_m^* - P_m\right)^2 + B_m\left(P_{m+1}^* - P_m^* - b_m\right)^2$$
$$+ B_{m-1}\left(P_m^* - P_{m-1}^* - b_{m-1}\right)^2 + \ldots \quad .$$

Setting $\frac{\partial E}{\partial P^*_m} = 0$ and rearranging provides P^*_m :

$$\left(A_m + B_m + B_{m-1}\right) \cdot P^*_m = A_m \cdot P_m + B_m \cdot \left(P^*_{m+1} - b_m\right)$$

$$+ B_{m-1} \cdot \left(P^*_{m-1} + b_{m-1}\right) \quad . \tag{33}$$

There is one such equation for every m .

It will be noted that Eq. (33) does not directly provide the reliability, A^*_m , associated with P^*_m . Rewriting Eq. (23) for a particular grid point m gives:

$$P^*_{B(m)} = \frac{A^*_m P^*_m - A_m P_m}{A^*_m - A_m} \tag{34}$$

where $P^*_{B(m)}$ is the resultant <u>background</u> information available at m if information contributed by P_m , A_m is removed. Clearly however, the same resultant, $P^*_{B(m)}$, would be obtained if the analysis were carried out without including P_m , A_m in the first place. Rearranging Eq. (34) gives

$$A^*_m = A_m \cdot \frac{P_m - P^*_{B(m)}}{P^*_m - P^*_{B(m)}} \quad . \tag{35}$$

To solve this equation, Eq. (33) is used to determine P^*_m (of course including the information P_m , A_m). Then Eq. (33) is used again, this time with $A_m = 0$, thus providing $P^*_{B(m)}$. Substituting these values and the known value of A_m in Eq. (35) provides a simple method for determining A^*_m .

3.4.2 A Numerical Example of Implicit Blending in One Dimension

Considering the same example as used in Section 3.2.6, assembly provided the following information:

m	•	T_m , A_m	= 15.7, 11.0
	–	b_m , B_m	= 3.6, 9.0
$m+1$	•	T_{m+1} , A_{m+1}	= 18.6, 11.0
	–	b_{m+1} , B_{m+1}	= 1.0, 6.0
$m+2$	•	T_{m+2} , A_{m+2}	= 20.8, 10.0

By setting $m = m$, $m = m+1$ and $m = m+2$ in Eq. (33) and inserting the appropriate parameter values, three equations with terms in T^*_m , T^*_{m+1} and T^*_{m+2} are produced. For example, the equation for location $m+1$ simplifies to

$$T^*_{m+1} = 0.23\, T^*_{m+2} + 0.35\, T^*_m + 8.88 \quad . \tag{36}$$

Solving these three equations simultaneously provides 15.6°C, 19.0°C and 20.5°C for T^*_m, T^*_{m+1} and T^*_{m+2} respectively--i.e., the same values as provided by explicit blending (see Section 3.2.6).

To find say A^*_{m+1} , the analysis must be carried out again but without the data at $m+1$ (18.6°C, 11.0). Once again three equations are produced. For example this time the equation for location $m+1$ simplifies to

$$T^*_{B(m+1)} = 0.40\, T^*_{m+2} + 0.60\, T^*_m + 1.76 \quad . \tag{37}$$

(Compare Eqs. (36) and (37), noting the effect of setting omitting $m+1$ data.) Solving these equations gives $T^{*}_{B(m+1)} = 19.5^{\circ}C$. This is the same result as produced by explicit blending--see page 51. Substituting in Eq. (35) provides $A^{*}_{m+1} = 19.7$--again the same result as produced by explicit blending.

To find A^{*}_{m} the analysis must be repeated without the data at location m, thus providing $T^{*}_{B(m)}$. Substitution in Eq. (35) provides A^{*}_{m}. A similar procedure provides A^{*}_{m+2}.

These examples demonstrate the equivalence of explicit blending and implicit blending (least-squares computations) in the case of one-dimensional arrays.

3.5 Implicit Blending in Two Dimensions

3.5.1 Weighted Information Fields

Blending so far has been discussed in terms of a linear array. An example of a linear array involving an object parameter P of weight A and independent estimates of first-differences of P, b of weight B, is given in Fig. 8. A linear array may be considered as a one-dimensional field. It can be seen that Fig. 8 may be regarded as two pairs of fields, one pair being P,A and the other being b,B. These fields may be shown separately, thus:

	Grid Point	Object Parameter		First Difference	
		Value[1]	Weight	Value	Weight
$m-2$	•	P_{m-2}	A_{m-2}	b_{m-2}	B_{m-2}
$m-1$	•	P_{m-1}	A_{m-1}	b_{m-1}	B_{m-1}
m	•	P_m	A_m	b_m	B_m
$m+1$	•	P_{m+1}	A_{m+1}	b_{m+1}	B_{m+1}
$m+2$	•	P_{m+2}	A_{m+2}	b_{m+2}	B_{m+2}
	etc.				

(Note that for any grid point m, $b_m \Rightarrow P_{m+1} - P_m$.)

The meaning of "Fields by Information Blending" now is apparent. In the context of the simple examples so far discussed, the techniques of both explicit and implicit blending outlined under Sections 3.3 and 3.4 have blended together two "weighted information fields" to produce a resultant field-pair P^*, A^* which provides the best knowledge of the "true" representative value P_R (in the object scale of resolution) which is provided by the total <u>available</u> information. The <u>reliability</u> of the resultant at any point m is given by A^*_m . If more information were available then, of course, P^* would be (slightly) modified and the increased reliability of the analysis as an estimate of P_R would be reflected in the new value of A^* .

The concept of weighted information fields may be applied to fields other than those associated only with the object parameter and first-difference values. For example, suppose that <u>independent</u> estimates of

[1] The term "value" always is used in the sense of numerical magnitude. Weight (or reliability) is used for value in the sense of worth.

$P_{m+2} - P_m$ were available. We term such estimates "double differences".[1] A weighted information field for double-difference estimates could be blended with the P,A and b,B fields, thus assimilating the additional information. However, rather than develop the error functional needed to blend double-difference fields with the P,A and b,B fields for a linear array, we shall consider the definition and blending of eight weighted information fields for a two-dimensional array.

Figure 6 showed an arbitrary grid point ℓ,m of a two-dimensional orthogonal array of size LxM grid points. Setting aside for the moment any boundary considerations we may define the following elements of information at the ℓ,m grid point:

P_m of weight A_m

b_m of weight B_m where $b_m \Rightarrow P_{\ell,m+1} - P_{\ell,m}$ (38)

c_m of weight C_m where $c_m \Rightarrow P_{\ell+1,m} - P_{\ell,m}$ (39)

d_m of weight D_m where $d_m \Rightarrow P_{\ell+1,m+1} - P_{\ell,m}$ (40)

e_m of weight E_m where $e_m \Rightarrow P_{\ell+1,m-1} - P_{\ell,m}$ (41)

f_m of weight F_m where $f_m \Rightarrow P_{\ell,m+2} - P_{\ell,m}$ (42)

g_m of weight G_m where $g_m \Rightarrow P_{\ell+2,m} - P_{\ell,m}$ (43)

q_m of weight Q_m where $q_m \Rightarrow P_{\ell,m+1} + P_{\ell+1,m} + P_{\ell,m-1} + P_{\ell-1,m} - 4P_{\ell,m}$. (44)

---

[1] The term "double-difference" is used rather than "second difference" which is more appropriate to a finite-difference expression for the second derivative of P with respect to distance. It may be noted that values of b are both single-differences and first-differences; either term applies to a finite-difference expression for the gradient given by $(P_{m+1} - P_m)$ per grid-length. Henceforth, for consistency, b will be referred to as a single-difference estimate.

These eight elements of information are shown in Fig. 9. The six difference estimates (4 single-differences and two double-differences) are indicated by arrows joining the grid points involved in Eqs. (38) through (43). $P_{\ell,m}$, $A_{\ell,m}$ represents "direct" information pertinent to the object parameter. $q_{\ell,m}$ is an estimate of the Laplacian $\nabla^2 P^*_{\ell,m}$ expressed in finite-difference form.

Figure 9 shows the eight elements of information for the arbitrary grid point ℓ,m. These eight elements over all ℓ (i.e., where ℓ assumes the values 1 through L which may be written $\ell = 1 \rightarrow L$) and over all m ($m = 1 \rightarrow M$) provide eight two-dimensional arrays, or fields, of parameter values, each with an associated weight field.

(Weighted information fields other than the eight given above also could be defined, a simple example being a "double-difference" Laplacian. However current applications of FIB to the analysis of two-dimensional distributions of an object parameter P are based on these eight.)

Depending on the object parameter of the analysis there is a variety of sources capable of contributing to the required eight weighted information fields. These sources are described in Section 4. However, given that such fields can be provided, the concept of what these fields represent should be appreciated.

The field of P shows the magnitude of the object parameter at each grid point, $P_{\ell,m}$, together with its associated weight $A_{\ell,m}$. The fields for parameters b, c, d, e, f, g and q are not directly concerned with the magnitude of the object parameter; they are measures (or estimates) of the shape of the field of the object parameter. Thus $b_{\ell,m}$, $c_{\ell,m}$, etc. are measures of the shape of the object parameter field in the vicinity of the grid point ℓ,m. The local "shape" is expressed in terms of single-differences (b, c, d, e), double-differences (f, g) and the Laplacian (q) defined in accordance with Eqs. (38)-(44) and shown on Fig. 9.

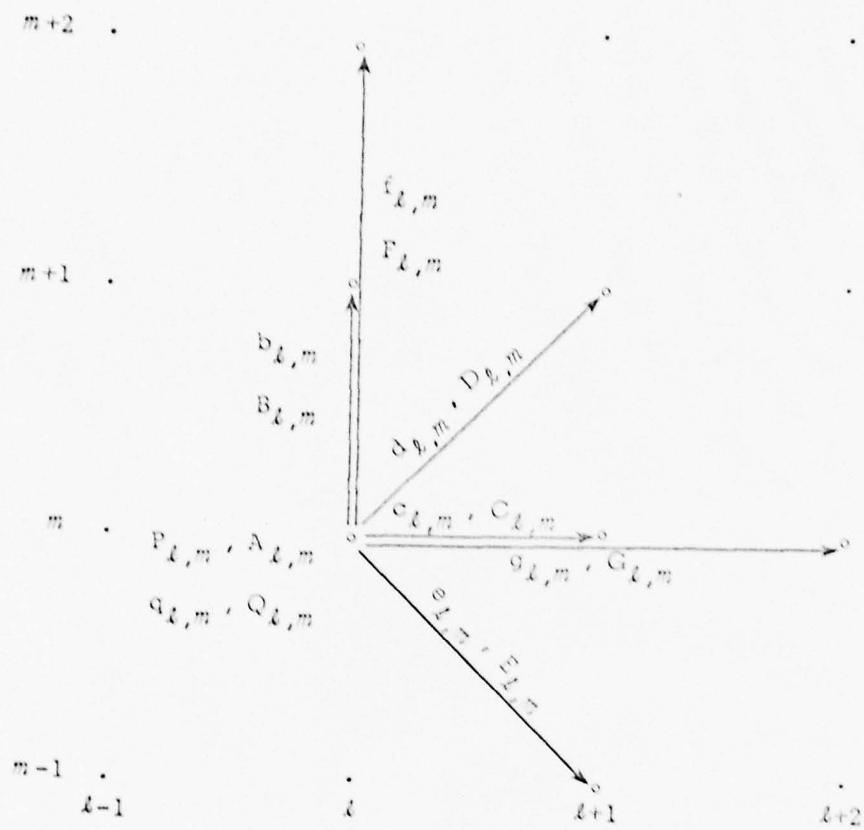


Figure 9 Symbols and subscripts for the eight information-elements referred to the arbitrary grid point ℓ, m .

Estimates of the shape parameters are derived from a different source (or sources) than estimates of the magnitude of the object parameter; in other words shape parameter estimates are <u>independent</u> of estimates of the magnitude of the object parameter. The concept of separating the shape of the object parameter field from its magnitude is a fundamental and essential component of the FIB analysis methodology.

Since shape and magnitude are derived from different sources there will be conflict between the available information. For example in the case of a simple SST analysis using a one-dimensional array (see Section 3.2.6) we saw that independent gradient estimates did not agree with the difference between the magnitudes at two successive grid points. FIB takes all weighted information fields and blends them together so as to produce the best possible compromise, in a least-squares sense, between the total available information. The blended resultant is the field of P^* with an associated resultant-reliability field A^*. What does the P^* field show? In the object scale of analysis resolution the value of P^* at any grid point is the best estimate of the "true" representative value of the object parameter at that grid point which is provided by the total information contributing to the analysis. A^* at the same grid point is a direct measure of the reliability we can place in P^* as an estimate of the true value. If A^* is large the error variance is small. A^* will be large where there is a considerable amount of information. Conversely, in regions where only a small amount of information is available the confidence or reliability we can place on the local values of P^* is low and will be reflected in a low value of A^*.

The following Section shows how the contributing weighted information fields may be blended to produce P^* and A^*.

3.5.2 The Blending Process in Two Dimensions

(NOTE: The blending process given below is for a specified object parameter P for which the blended resultant P^*, A^* can be produced using the eight weighted information fields defined in the previous Section. Analyses of other object parameters may utilize different sets of weighted information fields. Nevertheless, for all object parameters, the underlying blending methodology is essentially similar in concept. The mathematics which follows is an extension of the simple one-dimensional implicit blending process given in Section 3.4.)

The objective of the blending process is always to produce that field of an object parameter which gives a best fit in a least-squares sense to the ensemble of weighted information fields. Holl [1] has established that this best fit, consistent with the concept of explicitly combining independent estimates, is given by the solution which minimizes an appropriate error functional E . For the specified object parameter P (see note above) this best fit field is denoted by P^* of weight A^* and the error functional is defined by:

$$\begin{aligned}
E \equiv \sum_{\ell,m} \Big\{ & A_{\ell,m}\left(P^*_{\ell,m} - P_{\ell,m}\right)^2 \\
& + B_{\ell,m}\left(P^*_{\ell,m+1} - P^*_{\ell,m} - b_{\ell,m}\right)^2 \\
& + C_{\ell,m}\left(P^*_{\ell+1,m} - P^*_{\ell,m} - c_{\ell,m}\right)^2 \\
& + D_{\ell,m}\left(P^*_{\ell+1,m+1} - P^*_{\ell,m} - d_{\ell,m}\right)^2 \\
& + E_{\ell,m}\left(P^*_{\ell+1,m-1} - P^*_{\ell,m} - e_{\ell,m}\right)^2 \\
& + F_{\ell,m}\left(P^*_{\ell,m+2} - P^*_{\ell,m} - f_{\ell,m}\right)^2 \\
& + G_{\ell,m}\left(P^*_{\ell+2,m} - P^*_{\ell,m} - g_{\ell,m}\right)^2 \\
& + Q_{\ell,m}\left(P^*_{\ell,m+1} + P^*_{\ell+1,m} + P^*_{\ell,m-1} + P^*_{\ell-1,m} - 4\,P^*_{\ell,m} - q_{\ell,m}\right)^2 \Big\} \,. \qquad (45)
\end{aligned}$$

(This expression for E may be compared with Eq. (31). The essential differences are that an additional six weighted information fields have been included, and that the error functional has been extended into two dimensions.)

The minimum value of E occurs when

$$\frac{\partial E}{\partial P^*_{\ell,m}} = 0 \tag{46}$$

simultaneously, for every element $P^*_{\ell,m}$ of the solution field. Equation (46) yields <u>one equation per grid point</u> thus producing a simultaneous system of liner equations. The equation for the point ℓ,m is given by Eq. (47).

The Blending Equation:

$$
\begin{aligned}
S_{\ell,m} \cdot P^{*}_{\ell,m} \;=\; & A_{\ell,m} \cdot P_{\ell,m} \\
& + B_{\ell,m} \cdot \left(P^{*}_{\ell,m+1} - b_{\ell,m}\right) + B_{\ell,m-1} \cdot \left(P^{*}_{\ell,m-1} + b_{\ell,m-1}\right) \\
& + C_{\ell,m} \cdot \left(P^{*}_{\ell+1,m} - c_{\ell,m}\right) + C_{\ell-1,m} \cdot \left(P^{*}_{\ell-1,m} - c_{\ell-1,m}\right) \\
& + D_{\ell,m} \cdot \left(P^{*}_{\ell+1,m+1} - d_{\ell,m}\right) + D_{\ell-1,m-1} \cdot \left(P^{*}_{\ell-1,m-1} + d_{\ell-1,m-1}\right) \\
& + E_{\ell,m} \cdot \left(P^{*}_{\ell+1,m-1} - e_{\ell,m}\right) + E_{\ell-1,m+1} \cdot \left(P^{*}_{\ell-1,m+1} + e_{\ell-1,m+1}\right) \\
& + F_{\ell,m} \cdot \left(P^{*}_{\ell,m+2} - f_{\ell,m}\right) + F_{\ell,m-2} \cdot \left(P^{*}_{\ell,m-2} - f_{\ell,m-2}\right) \\
& + G_{\ell,m} \cdot \left(P^{*}_{\ell+2,m} - g_{\ell,m}\right) + G_{\ell-2,m} \cdot \left(P^{*}_{\ell-2,m} + g_{\ell-2,m}\right) \\
& + 16\,Q_{\ell,m} \cdot \tfrac{1}{4}\left(P^{*}_{\ell,m+1} + P^{*}_{\ell+1,m} + P^{*}_{\ell,m-1} + P^{*}_{\ell-1,m} - q_{\ell,m}\right) \\
& + Q_{\ell-1,m} \cdot \left(4\,P^{*}_{\ell-1,m} - P^{*}_{\ell-1,m+1} - P^{*}_{\ell-1,m-1} - P^{*}_{\ell-2,m} + q_{\ell-1,m}\right) \\
& + Q_{\ell,m-1} \cdot \left(4\,P^{*}_{\ell,m-1} - P^{*}_{\ell+1,m-1} - P^{*}_{\ell,m-2} - P^{*}_{\ell-1,m-1} + q_{\ell,m-1}\right) \\
& + Q_{\ell+1,m} \cdot \left(4\,P^{*}_{\ell+1,m} - P^{*}_{\ell+1,m+1} - P^{*}_{\ell+2,m} - P^{*}_{\ell+1,m-1} + q_{\ell+1,m}\right) \\
& + Q_{\ell,m+1} \cdot \left(4\,P^{*}_{\ell,m+1} - P^{*}_{\ell,m+2} - P^{*}_{\ell+1,m+1} - P^{*}_{\ell-1,m+1} + q_{\ell,m+1}\right). \qquad (47)
\end{aligned}
$$

In Eq. (47), $S_{\ell,m}$ denotes the sum of all weights appearing as coefficients (preceding the dots) on the right-hand side of the equation. Thus:

$$S_{\ell,m} \equiv A_{\ell,m} + B_{\ell,m} + B_{\ell,m+1} + \ldots\ldots + Q_{\ell,m+1} \, . \tag{48}$$

The terms after the dots show the required solution P^* together with estimates provided by the weighted information fields.

We now come to the problem of solving this set of linear equations. The system of blending equations may be expressed in matrix notation:

$$\underset{\approx}{M}\,\underset{\sim}{P}^* = \underset{\sim}{F} \tag{49}$$

where Eq. (47) represents an arbitrary row of Eq. (49). The formal solution is

$$\underset{\sim}{P}^* = \underset{\approx}{M}^{-1}\,\underset{\sim}{F} \quad . \tag{50}$$

If only a small number of grid points is involved explicit inversion of the matrix is possible. However typical FIB analyses utilize grids of 63x63 grid points, 89x89 grid points, or 125x125 grid points. In the latter case (used for hemispheric analyses of SST) a simultaneous solution to 15625 linear equations is required. Even for a 63x63 grid the number of equations involved is 3969. In general the matrix is far too large for routine explicit inversion and solution by an iterative technique is necessary.

At one time FIB utilized only Successive Over Relaxation (SOR) techniques to arrive at a solution for P^*. The most advanced schemes were found to converge very slowly and, consequently, were expensive in terms of computer resources. An additional drawback is that SOR requires a first-guess to the final solution.

This problem was overcome when, in 1976, Holl [6] reported the development of a technique, which he termed Blending by Weighted Spreading (BWS), for producing an effective solution to the system of blending equations which converged far more rapidly than SOR schemes alone. Of particular note is the fact that no first-guess to the solution is involved. In practice it has been found that the most effective technique for solving the system of blending equations is to commence with BWS then transfer to SOR at a late stage of convergence. This procedure is described in the following Section.

The resultant reliability weight, $A^*_{\ell,m}$, which is a measure of the firmness of the solution element $P^*_{\ell,m}$, is basic to the FIB methodology. The A^* elements appear, inverted, as the diagonal elements of the inverse matrix given by Eq. (50). A possible approach for determining A^* is suggested by Eqs. (34) and (35). However this requires repeating the analysis with $A_{\ell,m} = 0$ to obtain $P^*_{B(\ell,m)}$. Clearly this is not a realistic approach as it would require LxM separate FIB analyses. In practice a good approximation to A^* is sufficient. The Blending by Weighted Spreading technique incorporates a formulation which provides a reasonable approximation to A^*.

3.5.3 Blending by Weighted Spreading

Blending by Weighted Spreading is a technique of general applicability to the blending component of all applications of the FIB methodology.[1] In general such applications lead to a system of linear

---

[1] Although Blending by Weighted Spreading was conceived in the context of the FIB methodology (Holl [6]) the scheme is applicable to the solution of any system of linear equations for which the matrix is not only symmetric and positive definite but also has some degree of diagonally dominant rows/columns. In the FIB context the diagonal dominance occurs wherever $A_{\ell,m} > 0$.

equations defined by

$$\underset{\approx}{M}\,\underset{\sim}{P}^{*} = \underset{\sim}{F}$$

where P is any object parameter and P^* is the resultant of the analysis process. The coefficient matrix $\underset{\approx}{M}$ is formed only of parameter weights; all parameter values are grouped in the elements of the forcing vector $\underset{\sim}{F}$. The formal solution to this system of equations is

$$\underset{\sim}{P}^{*} = \underset{\approx}{M}^{-1}\,\underset{\sim}{F} \quad .$$

As indicated in the previous Section, the system of equations normally is too big to permit explicit matrix inversion. Until the development of Blending by Weighted Spreading, the sole recourse to solution was to Successive Over-Relaxation (SOR) techniques. These require a first-guess solution and also converge at an undesirably slow rate. The Blending by Weighted Spreading technique does not require a first-guess solution, and converges much more effectively in the initial stages.

The technique of Blending by Weighted Spreading may be demonstrated by application to the object parameter P used in previous Sections. The blending equation for the arbitrary grid point ℓ,m is given by Eq. (47). For reasons which will become apparent later in this Section, initially we shall omit the Laplacian terms--in effect $Q \equiv 0$. Eq. (47) then becomes

$$
\begin{aligned}
S_{\ell,m} \cdot P^*_{\ell,m} = {} & A_{\ell,m} \cdot P_{\ell,m} \\
& + B_{\ell,m} \cdot \left(P^*_{\ell,m+1} - b_{\ell,m}\right) + B_{\ell,m-1} \cdot \left(P^*_{\ell,m-1} + b_{\ell,m-1}\right) \\
& + C_{\ell,m} \cdot \left(P^*_{\ell+1,m} - c_{\ell,m}\right) + C_{\ell-1,m} \cdot \left(P^*_{\ell-1,m} + c_{\ell-1,m}\right) \\
& + D_{\ell,m} \cdot \left(P^*_{\ell+1,m+1} - d_{\ell,m}\right) + D_{\ell-1,m-1} \cdot \left(P^*_{\ell-1,m-1} + d_{\ell-1,m-1}\right) \\
& + E_{\ell,m} \cdot \left(P^*_{\ell+1,m-1} - e_{\ell,m}\right) + E_{\ell-1,m+1} \cdot \left(P^*_{\ell-1,m+1} + e_{\ell-1,m+1}\right) \\
& + F_{\ell,m} \cdot \left(P^*_{\ell,m+2} - f_{\ell,m}\right) + F_{\ell,m-2} \cdot \left(P^*_{\ell,m-2} + f_{\ell,m-2}\right) \\
& + G_{\ell,m} \cdot \left(P^*_{\ell+2,m} - g_{\ell,m}\right) + G_{\ell-2,m} \cdot \left(P^*_{\ell-2,m} + g_{\ell-2,m}\right). \qquad (51)
\end{aligned}
$$

Equation (51) states that the <u>total</u> weight due to "arrive" at the arbitrary grid point ℓ, m during the blending procedure (performed by successive approximations) is given by $S_{\ell,m}$. At any stage during the blending let (R) be the number of successive approximations made so far, and let $\alpha^{(R)}_{\ell,m}$ represent the corresponding portion of $S_{\ell,m}$ that has arrived by stage (R). Clearly $\alpha^{(R)}_{\ell,m}$ tends toward $S_{\ell,m}$ as (R) increases. A "progress factor", $\beta^{(R)}_{\ell,m}$ may be defined

$$
\beta^{(R)}_{\ell,m} = \alpha^{(R)}_{\ell,m} / S_{\ell,m} \qquad (52)
$$

where $\beta^{(R)}_{\ell,m}$ begins at zero and grows monotonically with each pass to asymptotically approach 1.

The process of Blending by Weighted Spreading for object parameter P is defined by:

$$\alpha^{(R+1)}_{\ell,m} \cdot P^{(R+1)}_{\ell,m} = A_{\ell,m} \cdot P_{\ell,m}$$

$$+ B_{\ell,m} \frac{\alpha^{(R)}_{\ell,m+1}}{S_{\ell,m+1}} \cdot \left(P^{(R)}_{\ell,m+1} - b_{\ell,m}\right) + B_{\ell,m-1} \frac{\alpha^{(R)}_{\ell,m-1}}{S_{\ell,m-1}} \cdot \left(P^{(R)}_{\ell,m-1} + b_{\ell,m-1}\right)$$

$$+ C_{\ell,m} \frac{\alpha^{(R)}_{\ell+1,m}}{S_{\ell+1,m}} \cdot \left(P^{(R)}_{\ell+1,m} - c_{\ell,m}\right) + C_{\ell-1,m} \frac{\alpha^{(R)}_{\ell-1,m}}{S_{\ell-1,m}} \cdot \left(P^{(R)}_{\ell-1,m} + c_{\ell-1,m}\right)$$

$$+ D_{\ell,m} \frac{\alpha^{(R)}_{\ell+1,m+1}}{S_{\ell+1,m+1}} \cdot \left(P^{(R)}_{\ell+1,m+1} - d_{\ell,m}\right) + D_{\ell-1,m-1} \frac{\alpha^{(R)}_{\ell-1,m-1}}{S_{\ell-1,m-1}} \cdot \left(P^{(R)}_{\ell-1,m-1} + d_{\ell-1,m-1}\right)$$

$$+ E_{\ell,m} \frac{\alpha^{(R)}_{\ell+1,m-1}}{S_{\ell+1,m-1}} \cdot \left(P^{(R)}_{\ell+1,m-1} - e_{\ell,m}\right) + E_{\ell-1,m+1} \frac{\alpha^{(R)}_{\ell-1,m+1}}{S_{\ell-1,m+1}} \cdot \left(P^{(R)}_{\ell-1,m+1} + e_{\ell-1,m+1}\right)$$

$$+ F_{\ell,m} \frac{\alpha^{(R)}_{\ell,m+2}}{S_{\ell,m+2}} \cdot \left(P^{(R)}_{\ell,m+2} - f_{\ell,m}\right) + F_{\ell,m-2} \frac{\alpha^{(R)}_{\ell,m-2}}{S_{\ell,m-2}} \cdot \left(P^{(R)}_{\ell,m-2} + f_{\ell,m-2}\right)$$

$$+ G_{\ell,m} \frac{\alpha^{(R)}_{\ell+2,m}}{S_{\ell+2,m}} \cdot \left(P^{(R)}_{\ell+2,m} - g_{\ell,m}\right) + G_{\ell-2,m} \frac{\alpha^{(R)}_{\ell-2,m}}{S_{\ell-2,m}} \cdot \left(P^{(R)}_{\ell-2,m} + g_{\ell-2,m}\right) \tag{53}$$

According to the "convention of dots" between coefficient weights and estimates, Eq. (53) defines a second equation for the coefficients:

$$\alpha_{\ell,m}^{(R+1)} = A_{\ell,m} + B_{\ell,m}\frac{\alpha_{\ell,m+1}^{(R)}}{S_{\ell,m+1}} + B_{\ell,m-1}\frac{\alpha_{\ell,m-1}^{(R)}}{S_{\ell,m-1}}$$

$$+ \ldots + G_{\ell-2,m}\frac{\alpha_{\ell-2,m}^{(R)}}{S_{\ell-2,m}} \quad . \qquad (54)$$

The process of Blending by Weighted Spreading consists of proceeding from grid point to grid point, in any preferred ordering, computing new estimates of $\alpha_{\ell,m}^{(R+1)}$ and $P_{\ell,m}^{(R+1)}$ by explicit solution of Eqs. (54) and (53) <u>in that order</u>. Each pass at all the grid points advances (R) by 1.

The successive approximations are initiated by setting

$$\alpha_{\ell,m}^{(0)} \equiv 0$$

and $$P_{\ell,m}^{(0)} \equiv 0 \quad .$$

No extraneous first-guess field is introduced.

As stated above, any ordering of grid points is permitted. The preferred ordering is successive in ℓ, increasing or decreasing, within a successive ordering of m, increasing or decreasing. A complete set of four such passes consists of:

Pass 1:	ℓ increasing	,	m increasing
Pass 2:	ℓ decreasing	,	m decreasing
Pass 3:	ℓ decreasing	,	m increasing
Pass 4:	ℓ increasing	,	m decreasing

The progress factor, $\beta^{(R)}_{\ell,m}$, is carried along with each successive approximation for $P^{(R)}_{\ell,m}$ and is an absolute measure of the proportion of the total due influence that has arrived at the ℓ,m location. However it is <u>not</u> a measure of the firmness of the value $P^{(R)}_{\ell,m}$. Further due influence may subsequently arrive to produce a pronounced change in the value.

Pronounced changes are very unlikely to occur after only a few multi-directional successive iterations have been effected. A set of four such passes spreads all influences to all locations--although not immediately to their full measure. Subsequent changes will generally be gradual and minor, as β asymptotically approaches unity everywhere.

The Laplacian element in the error functional, Eq. (45), gives rise to five terms in the blending equation, Eq. (47). These five compound estimates can be included in the formulations of weighted spreading. Inclusion requires the imposition of a suitable progress factor in the coefficient of each estimate.

The Laplacian terms have been omitted from the blending by weighted spreading process for several reasons:

a. Inclusion of the five Laplacian terms almost doubles the computational work per pass.

b. The Laplacian terms contribute absolute resolution at a slower pace than do the single-difference and double-difference terms.

c. The Laplacian information can be included in a final supplemental blending in which all progress factors are set to unity.

d. Exclusion of the Laplacian terms enables formulation and use of an effective approximation to the resultant reliability weight field, $A^*_{\ell,m}$, in just one computational pass.

The final blending operation for including Laplacian information (item c. above) resembles an SOR scheme used previously in certain FIB analyses such as sea-level pressure. Its use here, however, is only for adding "finishing touches" to the blending solution obtained by weighted spreading; it is an economical method for adding the Laplacian information. No regional biases remain; none are introduced by extraneous first guesses. The Laplacian information is introduced gradually. Each full pass at the array is made in terms of five subsets to produce an accelerated form of simultaneous displacement. In the case of sea-level pressure, the over-relaxation factor, ω, is set <u>less</u> than 1 in the initial passes, rising in the sequence $\omega = 0.3, 0.6, 1.0, 1.2$, and continuing at 1.5 with the fifth pass.

Basic to the FIB methodology is the field of A^*--an absolute measure of the analysis variance. The combined weight, $S_{\ell,m}$, of the total information due to arrive at grid point ℓ,m does not represent information which is completely independent. Only the contribution provided by $A_{\ell,m}$ is <u>known</u> to be independent; the additional weight, $S_{\ell,m} - A_{\ell,m}$, represents, at least to some degree, a feedback of the $A_{\ell,m}$ contribution. It may be concluded that

$$A_{\ell,m} \leq A^*_{\ell,m} < S_{\ell,m} \quad . \tag{55}$$

In developing a simplified approximation to A^*, it is important to keep two considerations in mind:

a. The approximation should be representative of the gathering of independent information that has actually been gathered by the weighted spreading, in the specified total of R passes, giving the solution

$$P^*_{\ell,m} \equiv P^{(R)}_{\ell,m} \quad .$$

b. An overestimation of A^* is preferred for use in the reevaluation of reports. Underestimation can result in rejection of good reports.[1]

The first of these two considerations permits a further narrowing of the limits expressed by Eq. (55). According to the second consideration this narrowing is valuable because it lowers the upper bound on $A^*_{\ell,m}$:

$$A_{\ell,m} \leq A^*_{\ell,m} < \alpha^{(R)}_{\ell,m} < S_{\ell,m} \quad . \tag{56}$$

The upper bound on $A^*_{\ell,m}$ can be brought down even lower by removal of the direct feedback of information through the first-difference estimates. This reduced upper bound provides a simple, adequate approximation to $A^*_{\ell,m}$. The formula is given by Eq. (57).

[1]Reevaluation and rejection of reports is described in Section 4.

$$A^*_{\ell,m}\ (\text{estimate}) = A_{\ell,m}$$

$$+\frac{B_{\ell,m}}{S_{\ell,m+1}}\left(\alpha^{(R)}_{\ell,m+1} - \frac{B_{\ell,m}}{S_{\ell,m}}\,\alpha^{(R-1)}_{\ell,m}\right) + \frac{B_{\ell,m-1}}{S_{\ell,m-1}}\left(\alpha^{(R)}_{\ell,m-1} - \frac{B_{\ell,m-1}}{S_{\ell,m}}\,\alpha^{(R-1)}_{\ell,m}\right)$$

$$+\frac{C_{\ell,m}}{S_{\ell+1,m}}\left(\alpha^{(R)}_{\ell+1,m} - \frac{C_{\ell,m}}{S_{\ell,m}}\,\alpha^{(R-1)}_{\ell,m}\right) + \frac{C_{\ell-1,m}}{S_{\ell-1,m}}\left(\alpha^{(R)}_{\ell-1,m} - \frac{C_{\ell-1,m}}{S_{\ell,m}}\,\alpha^{(R-1)}_{\ell,m}\right)$$

$$+\frac{D_{\ell,m}}{S_{\ell+1,m+1}}\left(\alpha^{(R)}_{\ell+1,m+1} - \frac{D_{\ell,m}}{S_{\ell,m}}\,\alpha^{(R-1)}_{\ell,m}\right) + \frac{D_{\ell-1,m-1}}{S_{\ell-1,m-1}}\left(\alpha^{(R)}_{\ell-1,m-1} - \frac{D_{\ell-1,m-1}}{S_{\ell,m}}\,\alpha^{(R-1)}_{\ell,m}\right)$$

$$+\frac{E_{\ell,m}}{S_{\ell+1,m-1}}\left(\alpha^{(R)}_{\ell+1,m-1} - \frac{E_{\ell,m}}{S_{\ell,m}}\,\alpha^{(R-1)}_{\ell,m}\right) + \frac{E_{\ell-1,m+1}}{S_{\ell-1,m+1}}\left(\alpha^{(R)}_{\ell-1,m+1} - \frac{E_{\ell-1,m+1}}{S_{\ell,m}}\,\alpha^{(R-1)}_{\ell,m}\right)$$

$$+\frac{F_{\ell,m}}{S_{\ell,m+2}}\left(\alpha^{(R)}_{\ell,m+2} - \frac{F_{\ell,m}}{S_{\ell,m}}\,\alpha^{(R-1)}_{\ell,m}\right) + \frac{F_{\ell,m-2}}{S_{\ell,m-2}}\left(\alpha^{(R)}_{\ell,m-2} - \frac{F_{\ell,m-2}}{S_{\ell,m}}\,\alpha^{(R-1)}_{\ell,m}\right)$$

$$+\frac{G_{\ell,m}}{S_{\ell+2,m}}\left(\alpha^{(R)}_{\ell+2,m} - \frac{G_{\ell,m}}{S_{\ell,m}}\,\alpha^{(R-1)}_{\ell,m}\right) + \frac{G_{\ell-2,m}}{S_{\ell-2,m}}\left(\alpha^{(R)}_{\ell-2,m} - \frac{G_{\ell-2,m}}{S_{\ell,m}}\,\alpha^{(R-1)}_{\ell,m}\right)$$

$$\cong A^*_{\ell,m}\ (\text{true}) \tag{57}$$

The application of Eq. (57) requires saving the fields of $\alpha^{(R-1)}$ and $\alpha^{(R)}$ from the blending by weighted spreading. As an expedient the program saves only $\alpha^{(R)}$ which it also uses in place of $\alpha^{(R-1)}$. If the number of passes, R, is adequate, there should be little difference.

Additional information concerning BWS:

As pointed out earlier the Laplacian terms can be included in the Blending-by-Weighted-Spreading (BWS) process. The formulation is not difficult. Inclusion, however, makes every row/column of the matrix less diagonally dominant and this slows the BWS process. We have designed the solution process to use the features of BWS and SOR to best advantage: A few sets of successive alternating-direction (i.e., alternating ordering) passes of BWS are followed by a few passes of SOR. A selected ordering and an over-relaxation factor which changes from pass to pass are used in the SOR stage. Second-and-higher-order terms (e.g., the Laplacian) are omitted in the BWS stage. All terms are included in the SOR stage.

The effect of exaggerating $\beta^{(R)}$ in each iteration was investigated in the context of Ref. [2], in the application of the FIB methodology to the *processing of satellite multi-channel scanning-radiometer radiances for* the diagnosis of clear-column radiance components. The incremental growth of $\beta^{(R)}$, toward +1, can be simply increased, for example, by replacing β with $1.1\,\beta^{(R)} \leq 1$, each time β is recalculated. This refinement was found to increase the effectiveness of the BWS process. Another refinement which was introduced in Ref. [2] is the use of passes in which either the BWS or the SOR algorithm is used at a grid-point location, depending on the value of β at that location. Once $\beta^{(R)}$ has attained a prescribed value, $\beta^{(R)}$ is jumped to +1, and the algorithm of use switches from BWS to SOR for that grid-point location in all succeeding passes. The use of these two refinements in the present context, however, would defeat the useful, expedient approximation for A^* that is afforded by Eq. (57). The "reevaluation" component, discussed in Section 4.8, depends on a good approximation of A^* at every grid-point location.

A general method can be made available for measuring the resolution weight corresponding to each element of each parameter in the blended resultant. That is, not only A^* but also B^*, C^* ... Q^* can be estimated at each grid-point location. We remind the reader that the starred weights are not measures of independent worth; they are measures of the robustness of the blended solution for each corresponding element. We have termed this general method the "Stiffness Method".

The Stiffness Method is quite simple to describe. Suppose for example that we wish to estimate $B^*_{\ell,m}$ corresponding to the resultant single-difference element $b^*_{\ell,m}$. In theory the method involves calculating a complete new solution subject only to a finite change, $\delta b_{\ell,m}$, in the forcing value, $b_{\ell,m}$, of that element. The corresponding weight, $B_{\ell,m}$, is not changed. Let the change in the new solution for the element be expressed by $\delta b^*_{\ell,m}$.

In accordance with the FIB methodology, $B^*_{\ell,m}$ is given by

$$B^*_{\ell,m} = \frac{\delta b_{\ell,m}}{\delta b^*_{\ell,m}} B_{\ell,m} \quad .$$

Note the s[illegible]larity in form with Eq. (35).

It would seem that the Stiffness Method should only be applied to a very accurate solution of the blended system of equations. However this requirement can easily be circumvented. The robustness of the solution depends only on the input weights of all contributions of assembled information. The robustness is independent of all forcing values. An exact blended solution as the basis for application of the Stiffness Method is therefore always at hand: By setting all forcing values equal to zero, the blended solution is given exactly by a zero field. As a further simplification, the finite change imposed on an element, say on $b_{\ell,m} = 0$, is chosen to make $\delta b_{\ell,m} B_{\ell,m} = 1$. [If the value of $B_{\ell,m}$ happens to be zero then it is simply changed to a finite value, e.g., one. This value must then be subtracted from the value of $B^*_{\ell,m}$ that results.] In practice the zero solution is revised as a result of the element change only in the ambience of the element. The extent of this ambient region depends on the desired degree of accuracy and on the ambient density of information in parameter assemblies.

The Stiffness Method is not utilized in routine applications because it is very expensive in terms of computations. It is to be reserved for special applications in which resultant weights are especially relevant--such as in evaluations of measurements obtained from satellite sensor systems.

3.5.4 Boundary Treatment

Figure 6 shows the area module associated with an arbitrary grid point ℓ,m of a two-dimensional orthogonal array of size LxM grid points. Figure 9 shows the symbols and subscripts for the eight information-elements referred to the ℓ,m grid point. The shape parameters are defined by Eqs. (38) to (44). The boundary of the L,M grid may be accommodated in a very simple manner--at any arbitrary grid point the weight of any information-element which extends outside the grid, or which is undefined, is set to zero.

However internal boundaries may also exist within the analysis region. For example in the analysis of oceanographic parameters such as SST, internal boundaries are provided by coastlines. The representation and effects on the analysis of internal boundaries is discussed in the following Section.

3.6 Spatial Covariance Dissociation

In the context of environmental analyses which utilize a grid, spatial covariance may be used to express how strongly a change in the object-parameter value at one grid point affects the value at another grid point.

A FIB analysis uses estimates of the object parameter together with first-difference estimates to propagate the effect of an observation in all directions, the strength of the propagation depending on the weights associated with the first-difference estimates. It can be seen that these weights, in effect, control the spatial covariance between corresponding grid points (see Fig. 9). In analyses of oceanographic and marine parameters it is realistic to restrict the propagation of information across land barriers. This is particularly important in the case of peninsulas and isthmi which separate different water masses. For example, in the vicinity of Panama, the Atlantic SST analysis should not propagate information into the Pacific and vice versa; the water masses are independent and so therefore should be the parameters which are measures of their physical state.

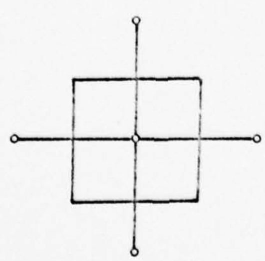
In essence the propagation of information across land/sea interfaces may be prevented by setting gradient weights equal to zero where the geographical dispositions of the finite-difference expressions for the gradient magnitudes at any grid point involve a land/sea boundary. This process is termed Spatial Covariance Dissocation (SCD).

Consider an analysis region covered by an analysis grid of size LxM. Using a high-resolution land/sea table an "SCD field" may be constructed which shows, for any arbitrary grid point ℓ,m, whether or not there is significant land interruption between ℓ,m and $\ell,m+1$ and/or between ℓ,m and $\ell+1,m$ (see Fig. 9). The SCD parameter at each grid point consists of two bits. The first bit indicates land (1) or no land (0) between ℓ,m and $\ell,m+1$; the second bit indicates land (1) or no land (0) between ℓ,m and $\ell+1,m$. Figure 10a shows part of an analysis grid within which is located an island (shown by shading). Figure 10b shows an associated SCD field. The solid lines in Fig. 10b show the closed (dissociated) connections caused by the island. Figure 10c shows the same island located within an analysis grid whose grid-point spacing is half that shown in Fig. 10a. Figure 10d shows an SCD field associated with Fig. 10c.

The method for arriving at an SCD field described above is a basic method only--in practice the method is more sophisticated. The land/sea information is used to derive a matrix consisting of ten rows of ten bits for each module of the analysis grid. Each bit is set to 1 (land) or 0 (water). The bits then are examined to determine the SCD parameter appropriate to any area module. The objective rules used to classify modules are as follows:

1. FOUR-WAY DISSOCIATION

If a modular area centered on a grid point is entirely land then the grid point is dissociated (closed) in all four links:



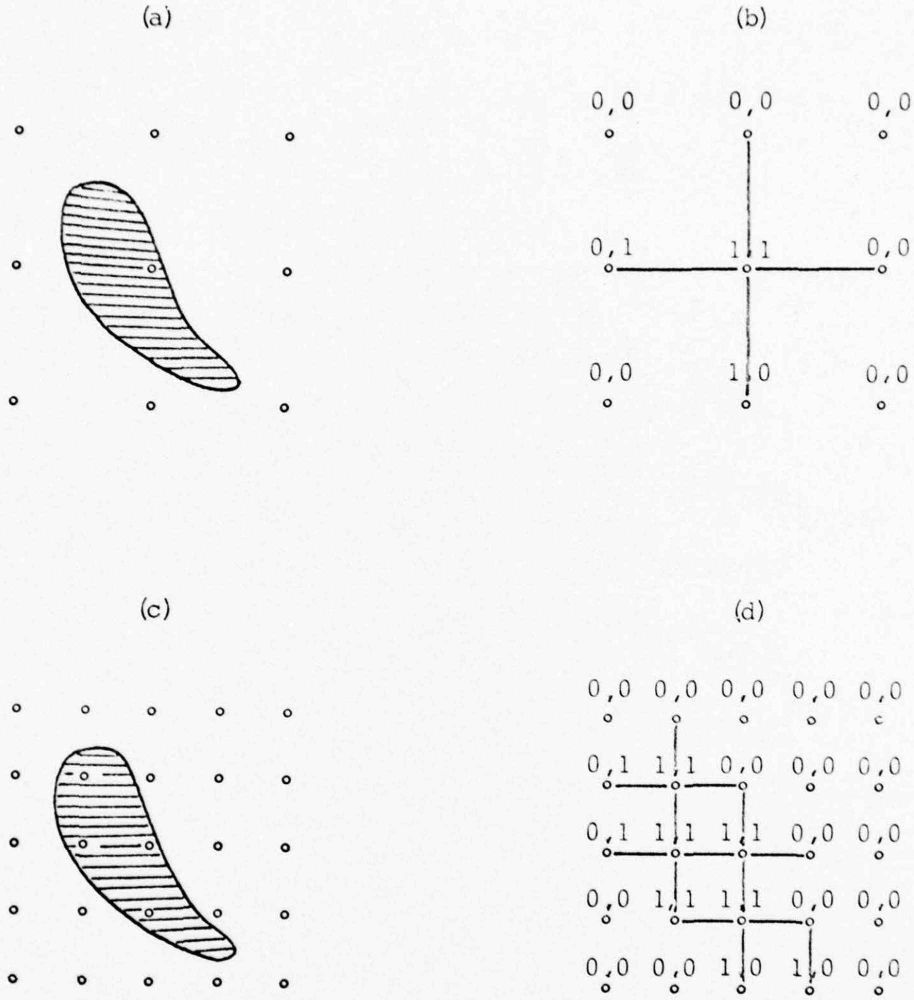


Figure 10 (a) and (c) show the same island located within two analysis grids, one having half the grid spacing of the other. (b) and (d) show the SCD fields for (a) and (c) respectively. Solid lines show the connections which, due to the island, are <u>closed</u> to the flow of information. The flow is limited to the open links around the island.

2. OPEN

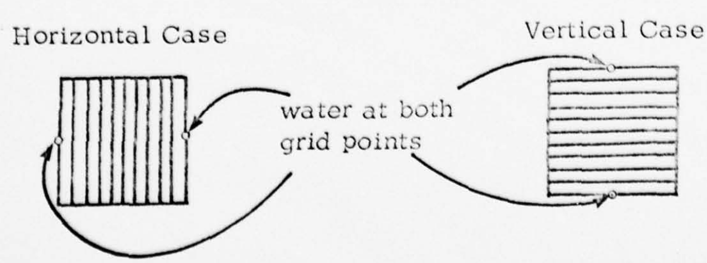
If a modular area centered midway between two grid points has any of the 5 center lines (i.e., rows or columns of bits) open, the connection between the two points should be associated (open):

Based on this rule, the center grid point of Fig. 10a should have two links closed and two open--the upward link and the link to the right.

3. CLOSED

a. If a modular area centered midway between two grid points has the 5 center lines all closed, then the two center points should be dissociated:

b. If any <u>interior</u> line is completely closed and both module points are sea then center points should be dissociated:



4. UNRESOLVED

All grid-point connections not resolved by rules 1-3 are unresolved.

Using these objective rules and subjective decisions where appropriate, the SCD field corresponding to Fig. 10c may be as follows:

0,0	0,0	0,0	0,0	0,0
0,0	1,1	0,0	0,0	0,0
0,0	0,1	0,0	0,0	0,0
0,0	0,0	1,1	0,0	0,0
0,0	0,0	0,0	1,0	0,0

The solid lines connect grid points which are <u>dissociated</u>.

The generation of SCD fields is a semi-automated procedure forming part of the overall capabilities of appropriate analysis systems based on the FIB methodology. Based on the rules given above the program (OBJSCD) automatically classifies grid-point connections as closed, open or unresolved. Output consists of the objective SCD field and visual information in plot and print form. Using this information SUBSCD is called to update the SCD field for subjective decisions. The User may specify unresolved connections as open or closed and, in addition, may modify any of the objectively-determined connections. A capability also exists for specifying all unresolved connections as either open or closed.

It will have been noted that the SCD parameter only directly specifies whether or not the first-difference gradient weights B and C shown in Fig. 9 should be set to zero. Simple rules are applied to local SCD parameter values to determine whether or not the remaining information-spreading weights should be set to zero.

In some applications it is desirable to reduce spreading weights rather than to set them to zero--information flow is restricted rather than stopped. For example, in the case of sea-level pressure analyses, the effects of terrain may be realistically represented in the analysis by restricting the flow of information between valleys separated by mountain ranges.

With regard to oceanographic parameters where SCD is utilized to prevent information flow if the ocean continuum is interrupted by land, a "depth dependent" SCD field should be used. For example where two deep ocean basins are separated by a ridge, the flow of information from one basin to another can be prevented. Near the top of the ridge information flow could be restricted rather than prevented, thus taking account of spillover from one basin to the other.[1]

The analysis algorithm recognizes the presence of land in terms of the SCD parameter--if two neighboring grid points are dissociated then land lies between them; no information is directly passed between two such grid points. The analysis of oceanographic parameters is invalid within such discontinuity regions.

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[1] As yet, depth dependent SCD fields (based on depth-variable land/sea tables) are not used by the Expanded Ocean Thermal-Structure analysis system. It is hoped to incorporate this capability in the near future.

For any analysis, the number and extent of invalid regions depends on the resolution afforded by the analysis grid and by appropriate objective and subjective SCD decisions. Figure 11 shows the invalid regions for the northern hemisphere on a 63x63 analysis grid, polar stereographic projection, using a specified SCD field. (The analysis grid, with the land outlined, is shown in Fig. 12 on page 93.)

Figure 11 Invalid areas for analysis using SCD where SCD extends over water--Northern Hemisphere 63x63, polar stereographic projection.

4. A SYSTEM FOR THE ANALYSIS OF SEA-SURFACE TEMPERATURE DISTRIBUTIONS

4.1 Introduction

Section 2 discussed, in a largely qualitative manner, some of the considerations to be taken into account by an effective analysis system. The Section had two primary objectives--to introduce certain FIB concepts and associated terminology in an appropriate context, and to introduce some of the problems of analyzing distributions of environmental parameters which can be solved by the objective FIB analysis methodology but which are largely ignored by subjective analysis methods (and some objective methods) either because the problems are too intractable or due to a lack of awareness on the part of the analyst.

Section 3 provided, in a largely quantitative manner, other FIB concepts and associated formulations. In a sense FIB is based on one fundamental premise--that no piece of information is complete without an associated reliability. This premise, coupled with an appreciation of the characteristics of observed data and an appreciation of the purpose of an analysis and its resultant, permits the derivation of all formulations given in Section 3.

Sections 2 and 3 do not in themselves provide sufficient information to show how FIB analyses of a particular object parameter may be produced. In addition, many other FIB concepts and formulations remain to be introduced. The most convenient way to satisfy both of these objectives is in the context of the FIB analysis system for a particular environmental parameter.

When designing a system for the analysis of an object parameter, the particular characteristics of that parameter must be taken into consideration. For example, even though based on the same underlying information-processing concepts, a system for producing analyses of

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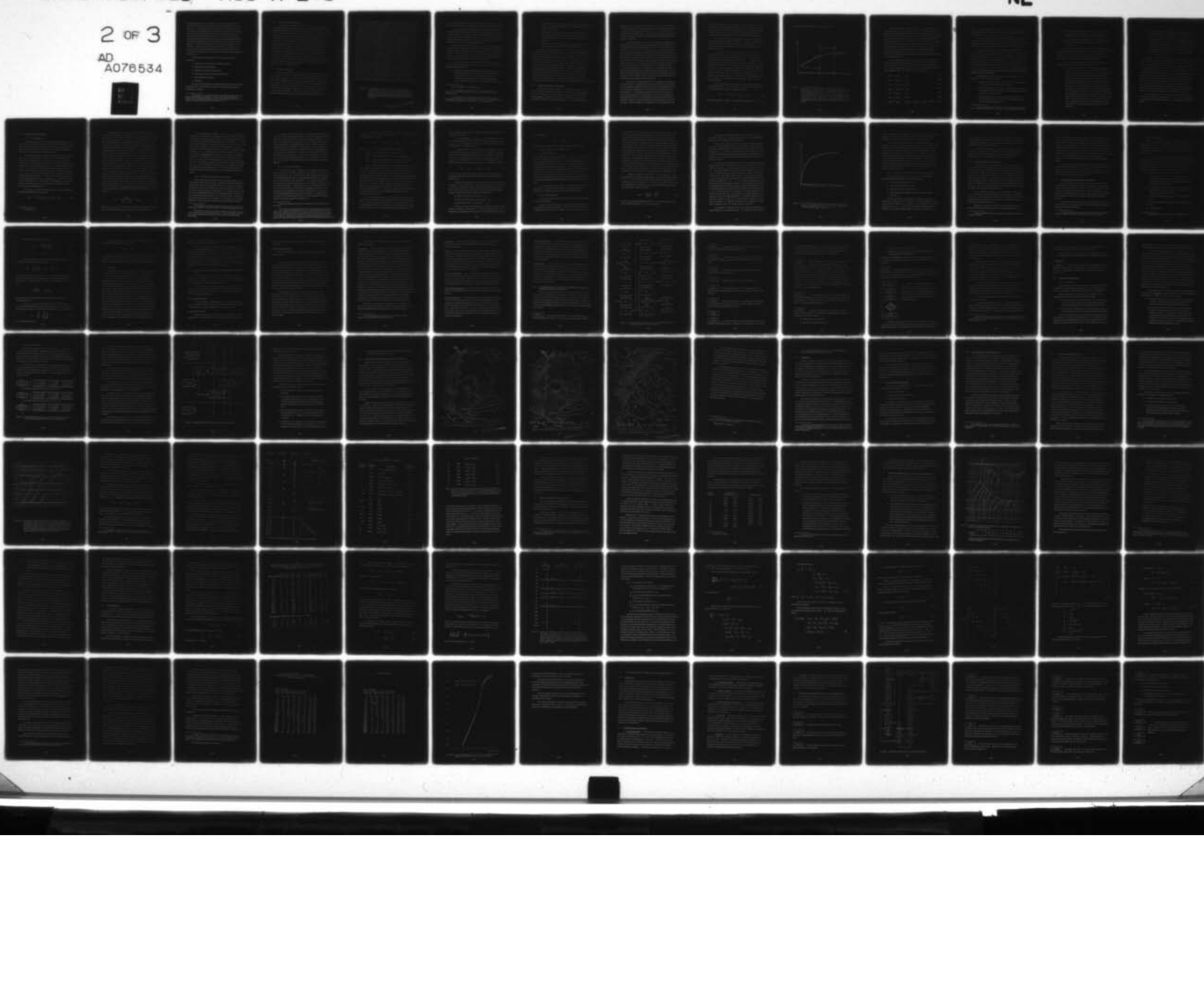

sea-level pressure is not converted to a system for the analysis of significant wave-height distributions merely by substituting one data set for another. A FIB analysis system therefore must be described in terms of a particular object parameter. Sea-surface temperature is an appropriate choice--a FIB system for producing sea-surface temperature analyses is relatively uncomplicated and, in addition, SST is one of the parameters analyzed by the Expanded Ocean Thermal-Structure (EOTS) analysis system. This Section therefore describes an SST analysis system, which, in later Sections, is extended to encompass all parameters analyzed by EOTS. However, although specifically oriented toward SST analyses, some information also is provided indicating how the concepts presented can be modified for application to the analysis of other environmental parameter distributions.[1]

A FIB analysis of SST consists of the following sequence of operations:

1. Preparation of the Parameter Initialization Fields.
2. Assembly of new information.
3. Blending for sea-surface temperature.
4. Computing the reliability field for blended SST.
5. Reevaluation and rejection.
6. Recycling.

Each step in this sequence is described below following a preliminary discussion of the object scale of resolution and available sources of information for analysis.

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[1]For example, the FIB analysis system developed for horizontal-wind field analysis differs considerably. There are two wind components per module, and the spreading parameters include divergence and vorticity, key elements in dynamic compounding (see caption Fig. 3).

4.2 The Object Scale of Resolution

As discussed in Section 2, the purpose of an analysis is to produce representative values of (in this case) sea-surface temperature which show significant variabilities in the object scale of resolution. It will be assumed that, for the purposes for which the SST analyses are required, these significant variabilities may be revealed by a 63x63 analysis grid, polar stereographic projection, covering the whole of the northern hemisphere (see Fig. 12), with an analysis frequency of once every 24 hours. This defines the object scale of resolution. Note that no matter how much SST data is available for analysis, the analysis resolution cannot exceed that appropriate to the object scale of resolution defined by the analysis grid and analysis frequency. In practice we will not achieve the object scale of resolution over the whole analysis area because of paucity of data in some regions. Given a sufficient data density, significant variabilities on a finer scale of resolution may be obtained by decreasing the grid-spacing and/or increasing the analysis frequency. The limit to resolution is determined by the available data.

4.3 Sources of Information

An elementary system for the analysis of a particular object parameter would utilize only direct synoptic observations of that parameter. However a far superior analysis is obtained by utilizing information available from all sources of relevance to the object parameter distribution. Such sources vary according to the particular object parameter to be analyzed but, in general terms, encompass observations of the object parameter, estimates obtained by way of diagnostic relationships from estimates and/or analyses of other relevant parameters, previous analyses using prediction or extrapolation techniques to carry and accrue information along the time

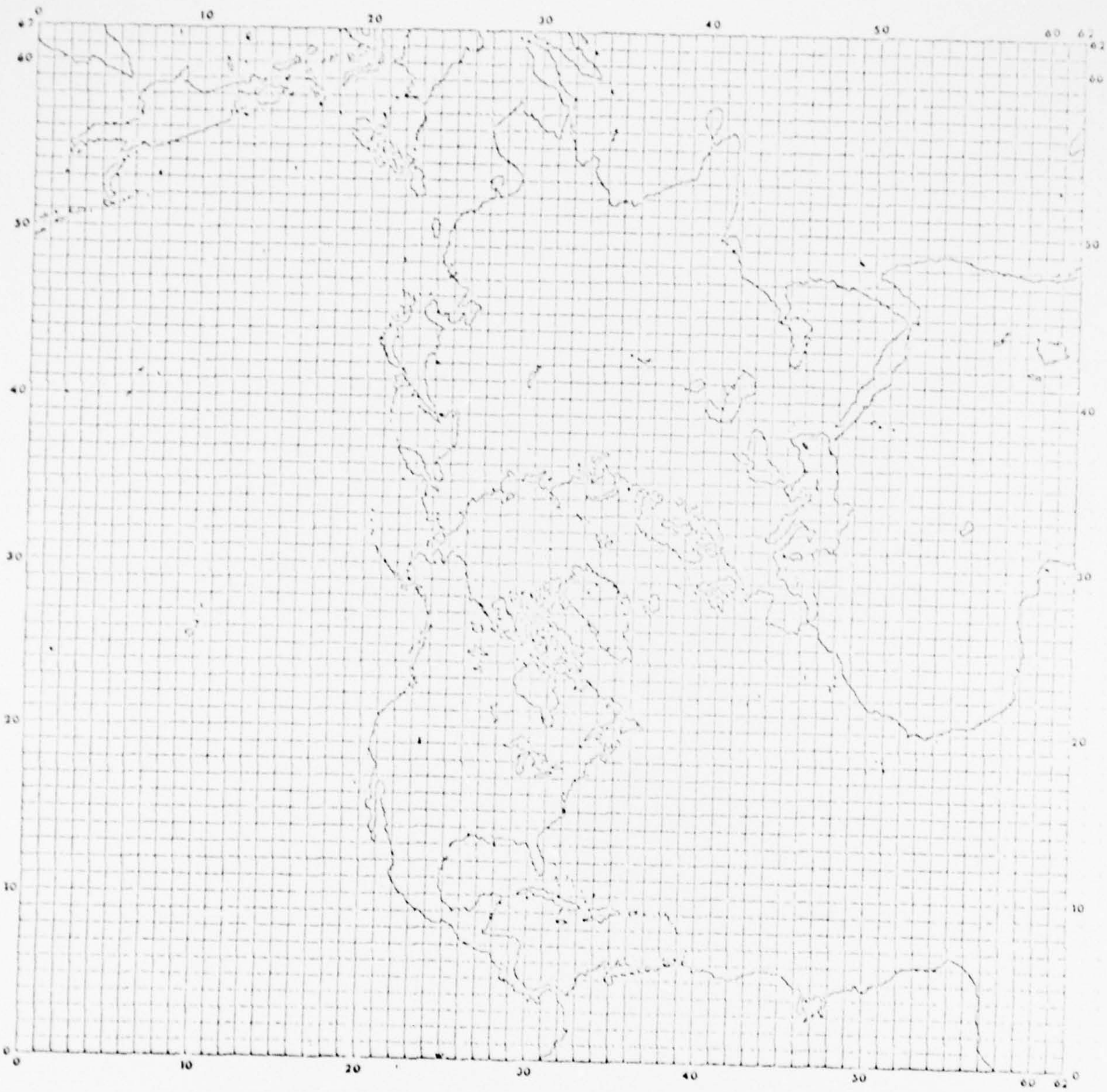

Figure 12 A 63x63 Northern Hemisphere polar stereographic grid. The north pole is at 31,31. Grid spacing = 381 km (206 n.m.) at 60° N. An arbitrary module of this (or any other) grid is shown in Fig. 6. Figure 9 shows the symbols and subscripts for the eight information-elements referred to an arbitrary grid point.

axis[1] (thus also enhancing continuity in time), and climatology. These sources can all contribute information to an analysis of an object parameter.

For any particular object parameter, not all possible sources of information (even if available) are necessarily appropriate. In the case of sea-level pressure for example, even though climatologies are available the naturally-occurring variabilities in space and time are such that climatology can make no significant information contribution in the context of normal (e.g., six-hourly) synoptic analyses.[2]

It is not only estimates of the object parameter itself which can provide significant information; the information contributing to an analysis may be in other forms including "shape" parameters such as gradients,[3] curvature, circulation, divergence, Laplacian and others. As explained in Section 3, this information may be assimilated (i.e., blended) into an analysis of the object parameter in finite-difference form.

Sources of information for an analysis may be grouped in terms of the age of observations contributing to each source. In the case of SST analyses these sources are:

a. "Current" information provided by synoptic or near-synoptic observations. "In-situ" observations are provided by ships and instrumented buoys. The large majority of in-situ observations are made as part of a ship's routine weather report--usually every 6 hours. A lesser number of observations

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[1] See Fig. 3 and associated discussion.

[2] The contribution of climatology, or any other applications of statistics, would tend to reduce (i.e., smooth) features in the object range of scale.

[3] For example, by way of an appropriate diagnostic relationship, surface wind observations can provide estimates of pressure gradient which can contribute information to the analysis of sea-level pressure.

are provided by the surface reading of XBTs, and by instrumented buoys. Current information also is available from remotely-sensed satellite data.

b. "Recent-past" information provided by observations of SST (including both in-situ and remotely-sensed data) which have been assimilated into previous analyses in the synoptic sequence.

c. "Distant-past" information provided by observations which have been used in the compilation of SST climatology fields.

Current SST data may be referred to as "synoptic" or "near-synoptic". If synoptic data collections are available at 6-hourly intervals (usually 00Z, 06Z, 12Z and 18Z) we could produce, say, 00Z analyses every 24 hours by utilizing only the synoptic 00Z observations. However, in order to increase data density and hence the information content of the analysis, we could use 00Z synoptic data plus all observations made during the previous 24 hours which were not utilized in the previous analysis. These additional observations may be termed "near-synoptic". In fact the near-synoptic period could be greater than 24 hours. For example if it is accepted that an SST observation still has relevance even if made, say, 30 hours previously then it could be used to contribute information to an analysis as long as it has not been used in any previous analysis; no observation should be directly assimilated more than once in a well-designed analysis system.

4.4 Parameter Initialization Fields (PIFS)

PIFS are an essential component of all FIB analysis systems. Each analysis system requires a set of PIFS, one of which is the object-parameter PIF. This is defined as the best estimate of what the forthcoming analysis will be before any information contribution from current observations is considered. In effect, for an object parameter P, the object-parameter PIF, denoted by $P_0^\star$, is a forecast of the resultant field, $P_\tau^\star$, for analysis time τ. $P_0^\star$ is (usually) derived by an appropriate method for carrying information in the $P_{\tau-1}^\star$ field along the time axis to time τ. (A PIF should not be regarded as a first-guess field. In FIB the process of carrying information along the time axis is a carefully controlled and formulated procedure; no "guess" is required.)

Methods for producing a PIF of the object parameter depend on the object parameter itself, and other considerations such as the availability of suitable forecast methods (including models). For sea-level pressure for example, a PIF for time τ may be derived from the $\tau-1$ analysis by kinematic extrapolation using an appropriate steering field based on large-scale features of the circulation. Alternatively a numerical forecast model could be used. In fact the best estimate field may be produced by combining any number of other estimates, each weighted according to its assessed degree of reliability. The weighting need not be a constant over the whole of the contributing field. For example suppose the object-parameter PIF for sea-level pressure is to be produced by a weighted combination of kinematic extrapolation and the forecast produced by a PE model. If it is known that the PE forecast verifies better in high latitudes than in tropical regions, then the weighting (which controls the relative contribution of information) may be made a function of latitude.

In general, an analysis system which is designed for continuous-in-time resolution of a geophysical system of variability must be coupled with an appropriate prediction model for carrying the analyzed information from one synoptic analysis along the time axis for assimilation into the next analysis. An appropriate prediction model also is required for projecting the evolution of the geophysical system as an environmental forecast-service for use in other applications; such an initial-value time-integrated prediction usually extends over several analysis cycles into the future. The same model is not necessarily the most suitable for both purposes; demands differ. In the analysis context the emphasis is on maximizing the information yield in terms of specific analysis parameters for the relatively short period between analyses. In the forecast-service context emphasis is on the prediction of operationally significant variabilities with maximum skill over the whole of the extended range.

Now consider an appropriate method for producing a PIF where the object parameter is sea-surface temperature T. The previous analysis at time $\tau-1$ provided the resultant field $T^{\star}_{\tau-1}$ of weight $A^{\star}_{\tau-1}$. Let the object-parameter PIF be denoted by $T^{\star}_{0}$, $A^{\star}_{0}$ and the resultant of the τ analysis by $T^{\star}_{\tau}$, $A^{\star}_{\tau}$.

In the object scale of resolution specified in Section 4.2, representative values of sea-surface temperature will not (usually) show a marked change in a 24-hour period (the analysis interval). Significant features present in one analysis will be apparent in e next analysis, albeit slightly modified with regard to absolute value and shape. Thus the information content of the $\tau-1$ analysis still has considerable relevance at time τ. For this parameter, persistence is apparently a simple but effective method for carrying information along the time axis from one analysis to the next. This can be achieved by setting $T^{\star}_{0} = T^{\star}_{\tau-1}$. (It is important to realize, however, that we may control the reliability of $T^{\star}_{0}$ as an estimate of $T^{\star}_{\tau}$ by appropriate specification of $A^{\star}_{0}$. This is discussed below.)

Although persistence ($T^{\star}_{0} = T^{\star}_{\tau-1}$) could be used to provide the object-parameter PIF, $T^{\star}_{0}$ as an estimate of $T^{\star}_{\tau}$ may be enhanced by an adjustment toward the predicted climatological value:

$$T^{\star}_{0} = \left(1 - \epsilon\right)\left(T^{\star}_{\tau-1} - T_{c,\tau-1}\right) + T_{c,\tau} \tag{58}$$

where the subscript c refers to the climatological value and $\epsilon < 1$. The following figure shows the effect of this formulation:

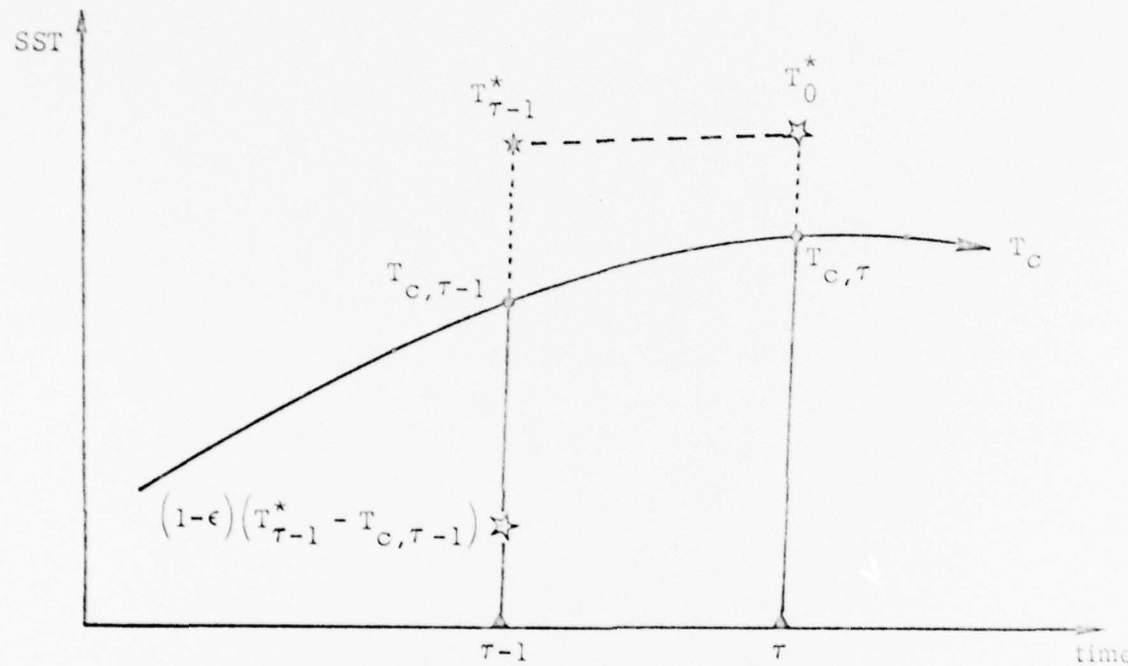

Figure 13 T_c shows SST climatology as a function of time at a particular location (i.e., a particular ℓ, m grid-point of the two-dimensional array shown in Fig. 10). $T^{\star}_{\tau-1}$ is the analyzed value of SST at the same location for time $\tau-1$. $T^{\star}_{\tau-1} - T_{c,\tau-1}$ represents the local analysis anomaly from climatology. This anomaly is reduced by the factor $(1-\epsilon)$. Adding $T_{c,\tau}$ to the adjusted anomaly provides $T^{\star}_{0}$. This is done for each grid point. Values of T_c for a particular analysis time may be interpolated from monthly-mean values.

Suppose an anomalous feature--say a cold gyre--is established by observed data in the τ-1 analysis. If, in the following analyses, no observations are available in the vicinity of the gyre, then setting $T_0^* = T_{\tau-1}^*$ would cause this feature to persist until such time as its existence were denied by observed values of SST. This is not reasonable--anomalous features decay with time. Using Eq. (58) with a high value of ϵ will cause a rapid return to climatologically normal conditions; a low value of ϵ will allow the anomaly to decay over a longer period of time. (Conversely, if the gyre were climatologically normal then it would persist until its existence were denied by observed data. If no more data then were forthcoming the gyre would gradually be reestablished in following analyses at a rate determined by ϵ.)

Having established the object-parameter PIF--in this case the T_0^* field--the inherent information is then partioned in terms of component information fields to produce a set of Parameter Initialization Fields, each with its own associated weight field. Seven PIFS are derived from the object-parameter PIF, T_0^*, using Eqs. (59) through (65).

$$b_{\ell,m} = T_{\ell,m+1} - T_{\ell,m} \tag{59}$$

$$c_{\ell,m} = T_{\ell+1,m} - T_{\ell,m} \tag{60}$$

$$d_{\ell,m} = T_{\ell+1,m+1} - T_{\ell,m} \tag{61}$$

$$e_{\ell,m} = T_{\ell+1,m-1} - T_{\ell,m} \tag{62}$$

$$f_{\ell,m} = T_{\ell,m+2} - T_{\ell,m} \tag{63}$$

$$g_{\ell,m} = T_{\ell+2,m} - T_{\ell,m} \tag{64}$$

$$q_{\ell,m} = T_{\ell,m+1} + T_{\ell+1,m} + T_{\ell,m-1} + T_{\ell-1,m} - 4T_{\ell,m} \tag{65}$$

where, to avoid a confusion of superscripts and subscripts, T at any arbitrary grid point ℓ,m has been used for the value provided by the T_0^* field at that grid point--e.g., $T_{\ell,m}$ is given by $T_{0,\ell,m}^*$.

It will be noted that the above seven PIFS have been derived from a single PIF of the object parameter; in these circumstances the complete set of eight PIFS will be mutually consistent. Such consistency is not necessary; individual PIFS may be derived from different sources of information.

In several previous Sections, in particular Sections 3.5.1 and 3.5.2, the FIB blending process was described and formulated in terms of eight weighted information fields; discussion of the source of these information fields was postponed. Thus in the formulation for E (Eq. (45) and the blending equation (Eq. (47)) it was assumed that values for these parameters were available. The eight weighted information fields are comprised of one field for the magnitude of the object parameter and seven fields for the shape parameters--see Fig. 9 and associated discussion. The shape parameters used in the blending equation are provided, at least in part, by the PIFS defined in Eqs. (59) through (65).

In simple terms we now can envisage a FIB analysis of object parameter P as the following procedure:

a. Produce an object parameter PIF, P_0^*, which is a best estimate of P^* before considering current information.

b. Partition P_0^* into eight PIFS, one of which is P_0^* itself and the other seven are estimates of shape. Each PIF has an associated weight field, discussed below.

c. Modify the P_0^*, A_0^* field by assembling new information (i.e., current observations) into this field.[1] This results in a new field which we may denote by P, A.

[1] As will be seen in Section 4.5.1 a somewhat different procedure is used to assemble information for the analysis of oceanographic parameters.

d. Blend these eight fields--seven shape PIFS and the object parameter PIF--using Blending by Weighted Spreading (Section 3.5.3). This produces the P^*, A^* field for time τ.

e. Carry the information provided by P^*, A^* along the time axis to $\tau+1$, and repeat the whole procedure to provide P^*, A^* for time $\tau+1$.

(This simple procedure for producing an analysis of object parameter P has omitted many important stages in the FIB analysis process. However it serves to provide an easily-comprehended framework which may be expanded into a more effective procedure. In particular it illustrates the source and function of PIFS in the FIB analysis methodology.)

Before returning to developing an analysis system for SST, two general points with regard to PIFS should be appreciated:

a. The seven shape PIFS are not necessarily based <u>entirely</u> on the P_0^* field. As pointed out above, individual PIFS may be derived from difference sources of information. In these circumstances two (or more) PIFS may be available for particular shape parameters, one derived from the P_0^* field and the other from some independent source or sources. These PIFS could be combined using the standard FIB rules for adding information (see Section 3.2.3). In addition current information may be assembled into appropriate PIFS other than the object parameter PIF. For example, in the case of a sea-level pressure analysis, current wind observations may be diagnosed in terms of pressure gradient and assembled into the shape PIFS. In the case of SST analyses, satellite-derived data could be expressed either in terms of absolute values of SST for assembly into the object parameter PIF, or in terms of gradient for assembly into the shape PIFS.

b. As demonstrated by the numerical examples given in Section 3, and as expressed in Eqs. (45) and (47), the shape parameters do more than contribute an estimate of the shape of the forthcoming analysis. They control the spreading of information provided by current observations. For this reason they may be termed shape and spreading parameters. The degree of spreading is controlled by the weight fields associated with the seven spreading parameters. (To illustrate this the simple numerical example given in Section 3.2.6 may be re-run with reduced weights for b_m and b_{m+1}.)

Returning to SST analyses we have seen how to compute PIFS for the object parameter and the seven spreading parameters. However the associated weight fields have not been defined; such weight fields must be available as input to the blending process. Weight fields for the spreading parameters may be specified by the User to provide any appropriate degree of spreading. This specification, usually in terms of a uniform weight field for each spreading parameter, must be consistent with the object scale of resolution. Spreading weights determine the amount of information contributed to an analysis by the shape parameters and consequently, particularly in data-sparse areas, determine to what extent isolated observations are fitted. (It should be borne in mind that an analysis should provide the best estimate of local representative temperature available from all sources of information. This best estimate is not (necessarily) that provided by a limited number of current observations. See footnote on page 21.)

In SST analyses (and all other ocean thermal-structure parameters encompassed by EOTS) a special procedure is used for weights associated with estimates of the object parameter. This is discussed in the following Section.

4.5 Assembly of New Information

4.5.1 The Assembly Field

In many applications of FIB the new information provided by current observations of an object parameter P is assembled into the P_0^*, A_0^* fields. For SST however (and other EOTS parameters) we use a separate assembly field. This field is used to accumulate the information provided by all recent-past[1] observations of the object parameter T.

Assume that the analysis sequence has been running for some time and that the assembly field for time $\tau-1$, after assembly of the $\tau-1$ observations, is given by $T_{N,\tau-1}$ of weight $A_{N,\tau-1}$. At some grid points--those on shipping lanes for example--the information represents the accumulation of many observations and hence the associated reliability will be high. At other grid points the information may represent only a small number of observations; the reliability will be low. We must now carry $T_{N,\tau-1}$, $A_{N,\tau-1}$ along the time axis to time τ to provide an assembly field for the current observations. Denote this field by T_0 of weight A_0. (It will be appreciated that $T_{N,\tau-1}$ of weight $A_{N,\tau-1}$, and T_0 of weight A_0 are <u>not</u> analyzed fields. They contain an accumulation of information provided by observations of the object parameter. The utility of these fields will shortly become apparent.)

To carry $T_{N,\tau-1}$ along the time axis to time τ, thus providing T_0, a formulation similar to Eq. (58) is used:

$$T_0 = \left(1 - \epsilon'\right)\left(T_{N,\tau-1} - T_{c,\tau-1}\right) + T_{c,\tau} \quad . \qquad (66)$$

[1]See Section 4.3.

On completion of the $\tau-1$ analysis the best information we have concerning the reliability of the analyzed field $T^*_{\tau-1}$ is given by $A^*_{\tau-1}$. Suppose that at a particular grid point the analysis resultant for $\tau-1$ was $T^*_{\ell,m}$, $A^*_{\ell,m}$ where $T^*_{\ell,m} = 19.0°C$ and $A^*_{\ell,m}$ is low because not much information was available for analysis in the vicinity of ℓ,m. Four observations then arrive, all made by independent ships very close to the grid point and all reporting about 19°C for time $\tau-1$. The analysis is re-run incorporating these reports. After re-analysis $T^*_{\ell,m}$ would still be 19.0°C but $A^*_{\ell,m}$ would be considerably higher, reflecting the greater confidence we have in $T^*_{\ell,m}$ as an estimate of the (true) representative temperature in the object scale of resolution. Then another 36 independent observations arrive, all reporting about 19°C near ℓ,m for time $\tau-1$. The analysis is again re-run giving $T^*_{\ell,m} = 19.0°C$ with a very high value for $A^*_{\ell,m}$. Now consider the same grid point say 3 days later. Assuming that no additional information is available the best estimate of the temperature at ℓ,m would be 19°C enhanced by a controlled adjustment toward the predicted climatological value in accordance with Eq. (58). However is the reliability we can place in this estimate made significantly greater by being based on 40 reports 3 days old rather than 4 of the same vintage? Clearly there would not be much difference between the two reliabilities to be associated with the SST estimate. There is an absolute limit on the resolution provided by old data imposed by the growth of variance with lapsed time. This must be taken into account in determining the reliability of the assembly field for time τ:

$$A_0 = \frac{\sigma^2_{*,\tau-1}}{\sigma^2_{*,\tau-1} + \sigma^2_{\Delta t}} \cdot A_{N,\tau-1} \qquad (67)$$

where $\sigma^2_{*,\tau-1} = (A^*_{N,\tau-1})^{-1}$ and $\sigma^2_{\Delta t}$ is the growth in variance with lapsed time. This formula is an expedient treatment of a complex process.

The assembly field for analysis time τ (i.e., T_0, A_0) contains the accrued (but time-decayed and trend-adjusted) information of relevance to the analysis provided by <u>all</u> observations of the object parameter (SST) that have been assimilated into previous analyses in the sequence.[1] At some grid points, especially if the analysis sequence to time τ represents only a small number of analyses, there may be no information--i.e., no past observations have been assembled at that grid point. Before each analysis therefore, T_0, A_0 is combined with the object-parameter PIF T_0^*, A_0^* using a very low <u>assigned</u> weight for A_0^*. Grid points in the assembly field where $A_0 >> A_0^*$ will be virtually unaffected. However in regions where A_0 is very small, the combination of T_0 and T_0^* will tend toward T_0^* which, in data-sparse regions will tend toward the local climatological value T_c. This is reasonable--if no information is available from recent-past observations the best estimate of the representative temperature is provided by the distant-past observations used to derive climatological values.

4.5.2 The Data List and Removal of Duplicates

Assume that the SST analyses, carried out every 24 hours, are for 00Z and that analyses are run at 07Z. As explained previously (Section 4.3) observations other than those made at 00Z may be utilized in an analysis. We shall assume that all observations of SST made between 00Z +5 hours and 00Z -17 hours are used in the 00Z analysis. In this way the possibility of using an observation more than once is avoided although, of course, we may lose some data--for example, an observation made at 00Z but not received until 06Z will not be included in any analysis if this simple procedure is used to determine current observations for a 00Z analysis.[2]

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[1] The benefits of accruing information along the time axis were described in Section 2; in particular see Fig. 3 and associated discussion.

[2] In practice a rather more involved procedure is followed to ensure, as far as possible, that all data is utilized. This is particularly important for BT data because of the relatively small number of sub-surface observations.

All current data is compiled into a data list containing separate sections for in-situ observations made by ships as part of a normal weather report, SST observations from BT reports, and satellite-derived data. Each section is ordered by time of observation and by latitude. This ordering makes the detection of duplicates a simple matter. Each observation is compared with the one following it in the data list. If time of observation, location (lat/long) and SST value are identical then a duplicate exists and is eliminated from the data list. (In practice the BT and satellite-derived components of the current data list do not contain duplicates. BT data is quality controlled during data collection before compilation of the data list, and the nature of satellite-derived information precludes duplicate reports.)

4.5.3 The Assembly Process

The simplest method is to take each SST observation in the current data list, carry it to the nearest grid point, and assemble it into the T_0, A_0 field (see Section 4.5.1). To carry an SST report to the nearest grid point, the object parameter PIF (T_0^*) is interpolated (non-linearly) to provide a value at the location of the observation. The difference between the interpolated value and the station value is then added to the value of T_0^* at the nearest grid point. The resulting value is an estimate of SST <u>at</u> the grid point provided by an observation made <u>near</u> the grid point.[1] This value may then be assembled at the same grid point in the T_0, A_0 field using the standard FIB rules for combining information. To assemble such an estimate the associated reliability is required; initially this may be provided by the class weight[2]--see Section 3.1.

---

[1] Special provisions are made for interpolating in the presence of an SCD field.

[2] This weight may be modified during the reevaluation process (Section 4.8). Also, in some FIB applications (though not SST and EOTS) the assembly weight is reduced as a function of the magnitude of the gradient in the vicinity of the report, thus reflecting the additional uncertainty caused by extrapolation in such regions. Other factors (such as land-station elevation for pressure reports) also may be taken into consideration when determining assembly weights.

Let i,j be the exact coordinates of the report in the L,M grid. The nearest grid point ℓ,m may be found by rounding i,j to the nearest integers. The extrapolation formula is:

$$T_{\ell,m} = \left(T_0^*\right)_{\ell,m} + \left(T - T_0^*\right)_{i,j} \tag{68}$$

where $T_{\ell,m}$ is the extrapolated value at grid point ℓ,m

$\left(T_0^*\right)_{\ell,m}$ is the object parameter PIF value at ℓ,m

$\left(T\right)_{i,j}$ is the reported value of SST at station location i,j

$\left(T_0^*\right)_{i,j}$ is the PIF value interpolated at the station location using a 12-point operator and the SCD field.

However before assembling $T_{\ell,m}$ into the assembly field a "Gross Error Check" is performed by comparing the observed value with the PIF value at the i,j location. If the difference between the two values--i.e., $\left(T - T_0^*\right)_{i,j}$--exceeds a certain specified large limit then the report is considered a gross error and is rejected. The maximum allowable difference depends on latitude--SST variabilities are greater in higher latitudes so a greater difference must be allowed before a report may be considered as a gross error. For SST, up to 10°C difference is allowed poleward of 60°, and 5°C equatorward of 20°; the allowable difference varies linearly between 20° and 60° latitude. Only reports passing this Gross Error Check are assembled into T_0, A_0 at the appropriate grid point for further evaluations and error checks.

As each report is assembled, the information provided by current observations is accumulated in the assembly field. After all observations of SST have been assembled the resultant field is $T_{N,\tau}$ of weight $A_{N,\tau}$. This field contains all information provided by current observations of the object parameter, and all information provided by observations of the object parameter used in previous analyses carried and accrued along the time axis to time τ.

(For the next analysis, $T_{N,\tau}$, $A_{N,\tau}$ is carried along the time axis to provide the assembly field for time $\tau+1$ in exactly the same manner as $T_{N,\tau-1}$, $A_{N,\tau-1}$ was carried along the time axis to provide the assembly field for time τ. See Section 4.5.1.)

To clarify the purpose of this assembled field which, in general, we may denote by T_N, A_N, it provides one of the weighted information fields to be blended with other weighted information fields. The blending equation, Eq. (47), is given for the arbitrary grid point ℓ,m. The value of T_N, A_N at ℓ,m provides the first term on the right-hand side of the blending equation; i.e., for the object parameter sea-surface temperature

$$A_{\ell,m} \cdot P_{\ell,m} = A_{N,\ell,m} \cdot T_{N,\ell,m} \quad . \tag{69}$$

The other terms in the equation are provided by the PIFS derived in accordance with Eqs. (59) through (65) together with their associated weight fields.

As noted at the beginning of this Section the method of assembly just described is the simplest method. All observations have been assembled using their appropriate class weights where class weight is determined by the inherent variance in values of SST provided by the various observing systems. Suppose we have three classes of observation:

SST reports from ships, class weight $= A_1$.

BT reports (surface reading), class weight $= A_2$.

Satellite-derived reports, class weight $= A_3$.

In practice, rather than using these class weights as initial assembly weights, we define a "base weight". Initial assembly weights are then determined by multiplying this base weight by an assigned class multiplier.

Thus, in general,

$$A_n = f_n A_R \tag{70}$$

where A_n is the initial assembly weight for observations in class n,

f_n is the assigned class multiplier, and

A_R is the base weight.

(In the EOTS analysis system for example, $A_R = 1.0$ for the SST parameter. The class multipliers currently are $f_1 = 0.5$, $f_2 = 1.0$ and $f_3 = 0.1$, thus giving initial assembly weights of 0.5, 1.0 and 0.1 respectively. Note the low reliability associated with satellite data, reflecting the uncertainty of indirect measurements of the object parameter. The EOTS system provides 4-D analyses of ocean thermal-structure parameters. The surface readings from BT reports are weighted more highly than SST reports from ships because, in the context of EOTS, they are of greater worth. However, if we were concerned solely with an SST analysis, then this would not be so and equal initial assembly weights could be used for the two classes of data.)

The reason why we define initial assembly weights in terms of a base weight (Eq. (70)), rather than directly using class weights, is necessary to the reevaluation process described in Section 4.8.

4.5.4 Staged Assembly

This is a more sophisticated assembly process than that described in the previous Section.

Suppose that a very large number of satellite-derived reports are assembled at a particular grid-point. Clearly, since report weights are combined linearly (Eq. (22)), the weight of that grid point due to these satellite reports may be very high. Now suppose that a relatively small number of ship reports also are available at the same grid point. Even though the assembly weight for satellite-derived data is lower than for ship data, the combined satellite reports can overwhelm the ship data--in other words most of the information at the grid point would be based on satellite data. Does this matter? If all reports were true random samples of the representative temperature then it would not. However the satellite-derived data may contain a bias. This bias is not necessarily a fixed calibration error. The bias could vary with time (calibration drift) and could vary in a non-random manner over the field; such biases, being non-random, are not always represented in the class weight for satellite-derived data. Thus no matter how many satellite-derived observations of SST are assembled at a grid point, the combined weight of these observations in general is not a true reflection of the reliability of the object-parameter value. Some method is required for limiting the combined weight of a particular class of observations at a given grid point to a maximum value.

Consider the class of observations S of class weight A_S. If N_S observations in this class are assembled at a grid point the resulting weight is $N_S A_S$. Let V_S be the maximum assembled weight allowed due to assembly of class S observations at the grid point. (V_S^{-1} is the minimum allowed variance.) The weight at the grid point resulting from assembly of N_S observations may be limited thus:

$$A_{SN} = \left(\frac{1}{N_S A_S} + \frac{1}{V_S}\right)^{-1} \tag{71}$$

where A_{SN} is the limited combined weight. (It can be seen that Eq. (71) adds a "limiting variance", V_S^{-1}, at the grid point.)

The effect of this formulation is shown in Fig. 14 where V_S has been taken as 10.0 (i.e., the limiting variance is 0.1, corresponding to a limiting standard deviation of about 0.32° C).

Although the concept of a limiting variance has been explained in terms of satellite-derived data, it is applicable to any data class. For example observations of SST may be grouped into 3 classes. Where appropriate (e.g., in EOTS) limiting variances could be added to any class (or classes) to prevent more numerous observations from overwhelming less numerous reports which have a greater significance in the context of the analysis objective.

To make use of the concept of a class-limited variance, we utilize two assembly fields. The first one is T_0, A_0 (see Section 4.5.1). The second one, say T_S, A_S, is initialized by setting T_S, $A_S = 0,0$ over all ℓ,m. Individual reports of the same class are assembled into T_S, A_S using the assembly process previously described. When all reports for this class have been assembled an appropriate limiting variance is added to each grid point and the resulting fields (object-parameter value and weight) are added into T_0, A_0. T_S, A_S is again initialized and used for the assembly of the next class of reports. When all reports for this class have been assembled a limiting variance is added and the resulting fields of object-parameter value and weight are combined with the contents of T_0, A_0. Repeating this process constitutes a "staged assembly". (The resultant field provides T_N, A_N for entry into the blending process.) By using this method the "sub-assemblies" of individual reports in their respective classes, as well as the full assembly of classes, can be made in any order. In addition an added limiting variance will effect only its specific class, not classes already assembled.

(The process of staged assembly facilitates the withholding of all reports belonging to a particular class. The withheld reports remain in

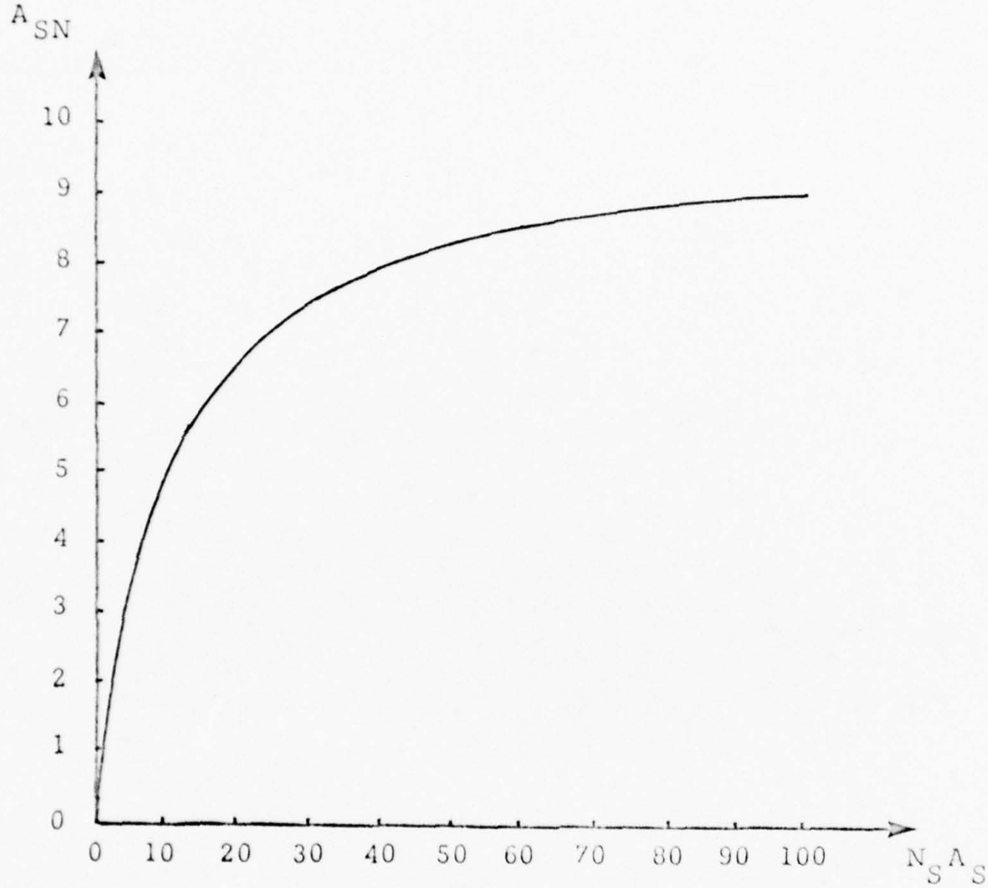

Figure 14 The effect of adding a limiting variance (0.1) to the combined weight of N_S observations each of weight A_S. A_{SN} is the limited combined weight. See text.

the data list and can be "reevaluated" along with the reports from assembled classes. In this way an objective assessment can be made of the usefulness and accuracy of a particular class (or type) of data.)

In the previous Section initial assembly weights were used to emphasize the value of surface readings from BTs in the context of EOTS analyses. Staged assembly provides a superior method. Nevertheless the problems of satellite-calibration bias, although reduced by staged assembly, still remain. By imposing a limiting variance on the large number of SST reports provided by satellite sensors we are sacrificing some of the information contribution potentially available. FIB is capable of circumventing calibration problems--in fact can detect and determine calibration bias and its variability in space and time--but an explanation of how this can be achieved must be postponed until a FIB system for SST analysis has been described.

4.5.5 Assembly of Satellite-Derived SST Gradient Information

There are four methods which could be used to assimilate satellite-derived SST information into an SST analysis:

a. Direct assembly (Section 4.5.3).

b. Staged assembly (Section 4.5.4).

c. Gradient assembly (discussed here).

d. Alternating Parallel Analysis (Section 4.12)--the superior approach.

In staged assembly we utilize an ancilliary field T_S, A_S for assembling a particular class of information. Following assembly a limiting class variance is added to T_S, A_S which is then combined with the main assembly field T_0, A_0. In the case of assembled satellite-derived information, T_S could be used to provide estimates of gradient (i.e., the first-differences b, c, d, e, f and g) and these estimates assembled into the appropriate shape PIFS. This procedure would avoid the effects of calibration bias and drift as long as, at any particular time, the bias did not obscure the local shape information.

(At this time EOTS utilizes the direct assembly method.[1] Gradient assembly is utilized in a bogus capability for assimilating significant SST gradients subjectively interpreted from satellite imagery or any other appropriate source. See Appendix C of the Users Manual [3].)

4.6 Blending for Sea-Surface Temperature

To summarize the analysis procedure so far, the resultant of the previous analysis has been carried along the time axis to provide T_0^*, A_0^*. This field has been partitioned in terms of component information fields to provide seven "shape" PIFS, each with an associated weight field. The assembly field of the previous analysis also has been carried forward along the time axis to provide T_0, A_0 and combined with T_0^*, A_0^* using a low assigned weight for A_0^*. Following a Gross Error Check new observations have been assembled into this field to provide the new assembly field T_N, A_N. We thus have eight weighted information fields--T_N, A_N and the seven weighted shape (or spreading) PIFS.

For any arbitrary grid point ℓ,m the corresponding values from these eight weighted information fields are entered into the Blending Equation (Eq. (47)), thus yielding one equation per grid point. The system of linear equations is solved using the technique of Blending by Weighted Spreading described in Section 3.5.3.

Blending is the process of compounding <u>all</u> information into an optimum determination of the distribution of the object parameter. This

[1]Staged assembly exists as a capability but as yet (June 1979) has not been activated.

process is achieved by determining the best fit, in a least squares sense, to the ensemble of weighted information fields. During blending, grid-point information is spread to surrounding grid points through the seven gradient and Laplacian fields. The degree of spreading increases with the reliability at the grid points and reliability of the surrounding gradients. After blending each grid point will have a new value of temperature, reflecting surrounding information as well as information at the grid point itself. Each grid point will have a new weight value, reflecting the reliability of surrounding information contributing to the new temperature value.

At first sight it would seem that the resultant of the blending process provides the required analysis of the object parameter distribution. Indeed, some less sophisticated analysis techniques are unable to refine an analysis beyond a single least-squares-fit process. However in FIB, this analysis is considered to be preliminary only.[1]

4.7 Computing the Reliability Field for Blended SST

After the blending process each new grid-point value will have a different weight or reliability than it did before blending. To illustrate, suppose that before blending a certain grid point had nothing but PIF information, but nearby grid points had the help of many reports. After blending the deserted grid point would have been provided with information from its data-rich neighbors and thereby would have an increased reliability.

The new weight field, computed in accordance with the formulations given in Section 3.5.3, provides the basis for the reevaluation procedure described in the following Section.

[1] In general a FIB analysis consists of three cycles--the blending equation is applied three times to determine the optimum resultant.

4.8 Reevaluation and Rejection

Knowledge of the blended temperature at each grid point and the resulting reliability enables the FIB analysis system to reevaluate, individually, each piece of information that entered the analysis. The reevaluation provides a quality measure for each contributing observation. This reevaluation capability is one of the most valuable features of the FIB methodology.

To reevaluate sea-surface temperature reports, each report and its associated reliability is removed from its grid-point resultant[1] and compared with the local independent weighted information--i.e., the "background information". A parameter, λ_n^2, is computed for each report; λ_n is the difference between the reported temperature and the "background temperature" expressed in units of the expected difference.

Parameters entering into report evaluation include:

T_n the magnitude of the report in $^\circ C$ after it has been extrapolated to the nearest grid point by the shape of the PIF in cycle 1 and by the shape of the preceding analysis in cycles 2 and 3 (Eq. (68)).

A_n the assembly weight with which T_n has been previously assembled at the nearest grid point. (Note that both T_n and A_n change with each assembly.)

T^* the resultant (blended) analysis value at the nearest grid point.

A^* the resultant (blended) analysis weight at the nearest grid point.

A_R the base weight.

---

[1]The removal of information is described in Section 3.2.5.

The background information is given by

$$T_B = \frac{A^* T^* - A_n T_n}{A^* - A_n} \tag{72}$$

$$A_B = A^* - A_n \quad . \tag{73}$$

The difference between T_n and T_B may be expressed in scaled units (λ) of the expected difference thus:

$$\lambda_n^2 = \frac{A_R A_B}{A_R + A_B} \cdot \left(T_n - T_B\right)^2 \cdot K \tag{74}$$

where the K factor reduces λ_n^2 in the first reevaluation to account for the larger variance in the report population before more refined evaluation has taken effect. (K = 0.25 for cycle 1, and unity for cycles 2 and 3). Note that

$$\frac{A_R A_B}{A_R + A_B} = \frac{1}{\sigma_R^2 + \sigma_B^2} \quad .$$

Additional variance contributions can be included in the denominator wherever appropriate.

Let λ^2_{max} be a specified upper limit for λ^2; if λ^2 reaches λ^2_{max} then the report is withheld from the subsequent analysis (i.e., the report is tentatively rejected). The weight reduction parameter, F_n, is defined by

$$F_n = \left(1 - \frac{\lambda_n^2}{\lambda^2_{max}}\right) \geqq 0 \tag{75}$$

(F_n = 0 indicates rejection.)

Reevaluation consists of replacing the assembly weight, A_n, by a reevaluated assembly weight, A_{nR}, where

$$A_{nR} = F_n A_R \quad . \tag{76}$$

(Note that λ_n^2 is defined in terms of A_R. If defined in terms of A_n then reevaluation is less stringent for reports of low reliability because the expected variance is higher. Using A_R as the basis for reevaluation ensures that all reports are judged in the context of a fixed expectation).

4.9 Recycling

The procedures described in Sections 4.5 to 4.8 constitute the first analysis cycle. The sequence of operations is now repeated by returning to the assembly stage (Section 4.5). Cycle 2 commences with the reassembly of reports with their reevaluated weights as determined in Cycle 1. The program then proceeds just the same as in Cycle 1 (apart from setting K in Eq. (74) equal to 1.0). The reevaluation in Cycle 2 will be more precise since doubtful reports were downgraded (with respect to weight) in the first cycle and had less or no effect in the second cycle. The reevaluation now is more meaningful because it is based on checked, re-weighted reports. Of course, in spite of the fact that K was set to 0.25, it is possible that some good reports were thrown out in the first cycle. This usually occurs when only one good report is found at an isolated grid point along with a bad one. The reevaluation has insufficient information from surrounding areas to determine which report is actually good. The result is that both are rejected. In the second reevaluation the good report matches the analysis fairly well and may be allowed to enter the analysis again in the third and final cycle. (In practice it is found that the discrimination capabilities provided by the Cycle 1 evaluation of report reliability are such that reports are rarely reinstated.) The third cycle gains further accuracy because it is based on the second, more accurate reevaluation.

Reports are not usually reevaluated during the third cycle as doing so does not contribute information to the final analysis. However, if a measure of report reliability is required, then this measure may be provided by Cycle 3 reevaluation. Note that report reliability depends to some extent on the object scale of analysis resolution. A report which is judged to be a poor estimate of the local representative temperature on one scale of resolution may prove to be a new good estimate of the local representative temperature on another scale of resolution (see page 27).

4.10 The Design, Modular Structure and Implementation of a FIB Analysis System

Sections 4.4 through 4.9 describe the sequence of operations utilized in a FIB analysis of SST. This sequence was defined in Section 4.1. At this point it is convenient to discuss, in general terms, how a new FIB analysis capability is established and implemented.

All FIB analysis systems are designed as an ensemble of modules where each module has a specific function to perform in the overall analysis process. A FIB system for the analysis of a particular object parameter utilizes three types of modules:

General-Purpose Modules

These are modules of general applicability to all relevant analysis systems, requiring only the setting of various adjustable constants and tuning parameters in order to employ them in a specific system.

Multi-Purpose Modules

These are modules which can be employed in a variety of similar applications. In general the use of a multi-purpose module in a new FIB system requires some varying degree of program modification to tailor it to its new role.

Specific-Purpose Modules

These are modules which are employed only by a specific FIB analysis system.

The ability to employ general and multi-purpose modules in the design of a new FIB analysis system is made possible because all FIB analysis systems are based on the same underlying concepts of information processing--the FIB analysis methodology. If a new application of FIB requires the programming of a new module, it is a deliberate policy to make this module as flexible as possible so that, in future, it is available for other appropriate applications. For example the concepts of staged assembly (Section 4.5.4), although originally developed and programmed specifically for treating satellite-derived data as a separate class, can be applied to any class of data in any FIB application; the source code for staged assembly thus constitutes a general-purpose module.

The utilization of general and multi-purpose modules in FIB methodology has many advantages. Improvements made in a module, perhaps because of the development of a new concept or because of the specific requirements of a new application, automatically are incorporated in all analysis systems which utilize that module. The ability to utilize pre-existing program components which have been tried and tested in previous applications reduces development costs and times for a new application. The use of generalized components permits common formats and procedures to be utilized, and avoids unnecessary duplication on the system libraries.

To establish a new FIB analysis capability the following stages generally are required:

Design Study   Methods for achieving the required analysis capability are considered. There are many factors to be taken into account--the reliability, frequency and density of observations of the object parameter;[1] other possible sources of relevant information and methods for extracting the information component of relevance to the object parameter; how to represent or parameterize the available information so as to best satisfy the objectives of the analysis; methods for combining the diverse sources of information; methods for carrying and accruing information along the time axis to provide the necessary PIFS; available data bases and climatologies (if appropriate); the object scale of analysis resolution in space and time; the required output; operational scheduling including the availability of required fields produced by other analysis and/or forecast capabilities; and so on. Based on these considerations all necessary formulations are developed. The modular structure of the new analysis capability may then be defined. As much use as possible is made of program components which already are used in other FIB applications; necessary modifications to multi-purpose modules are noted, thus increasing their range of applicability to encompass the new requirements. The functions of new modules are defined, including the main driving programs which set adjustable specifications in general and multi-purpose modules to take account of variations required in the general procedures by specific applications.

Production of an effective and efficient system design is an essential first step in establishing a new FIB analysis capability. The time and effort expended during the design phase is very cost-effective.

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[1] The object parameter of the analysis need not be a directly observed (or measured) environmental parameter.

Programming Based on the design study, the main driver is programmed together with all other new modules and necessary modifications to pre-existing modules.

Testing A series of test runs is made to ensure that all program components are functioning in accordance with expectations. The overall FIB methodology encompasses a number of program components which provide field verification statistics and a variety of measures of system performance. These are used to set tuning parameters to provide optimum output. All other FIB analysis systems making use of modules which have been modified in establishing the new capability are checked to ensure that these modifications do not produce unexpected results in other analyses.

Operational Evaluation The new FIB analysis capability is then run in real time using data obtained during normal operations. The products are evaluated against the design objectives on a continuing basis. If necessary, improved formulations are developed and the appropriate modifications made to program components. The values of adjustable constants are refined.

Operational Implementation The new analysis capability now may be placed in the operational jobstream and the product utilized for its intended purpose.

Program Maintenance FIB analysis systems in general are very flexible. There is normally a wide variety of user options with regard to adjustable parameters, analysis grid dimensions, spacing, location and orientation, analysis frequency, the ability to force in and force out real or invented data, output formats, and so on. On occasion problems arise which were not uncovered during the testing and operational evaluation phases. Such problems are usually minor in nature and their rectification is covered by "program maintenance".

Additional Capabilities The User may require to extend the capabilities of a FIB analysis system beyond the original design specifications. For example several capabilities have been added to EOTS including the production of temperature:depth cross-sections along any specified line and the ability to assimilate zones of strong SST gradient as determined subjectively from satellite imagery. FIB systems are sufficiently flexible to allow such additional capabilities to be provided; they then form part of the overall system.

Documentation Using the EOTS analysis system as an example, program maintenance information is provided by comments in the source code. A comprehensive Users Manual [3] is available which enables the system to be used without requiring an in-depth understanding of the underlying principles of the FIB analysis methodology. The Users Manual is designed to facilitate documentation of system modifications and additions. The third necessary component of EOTS documentation is provided by this Scientific/Technical Report.

4.11 The Modular Structure of a Simplified FIB System for the Analysis of Sea-Surface Temperature Distributions

Figure 15 shows the information flow and processing in a simplified FIB system for the analysis of sea-surface temperature. The schematic shown is a subset of the EOTS analysis system; a more complete information flow and processing chart is given and discussed in Section 6. However the simplified chart serves to illustrate a practical application of the formulations and discussions presented in previous Sections. The function of each module is as follows:

INITSCD Tests to determine whether or not SCD field generation is required (includes User option). If not, then control is passed to OTSBT.

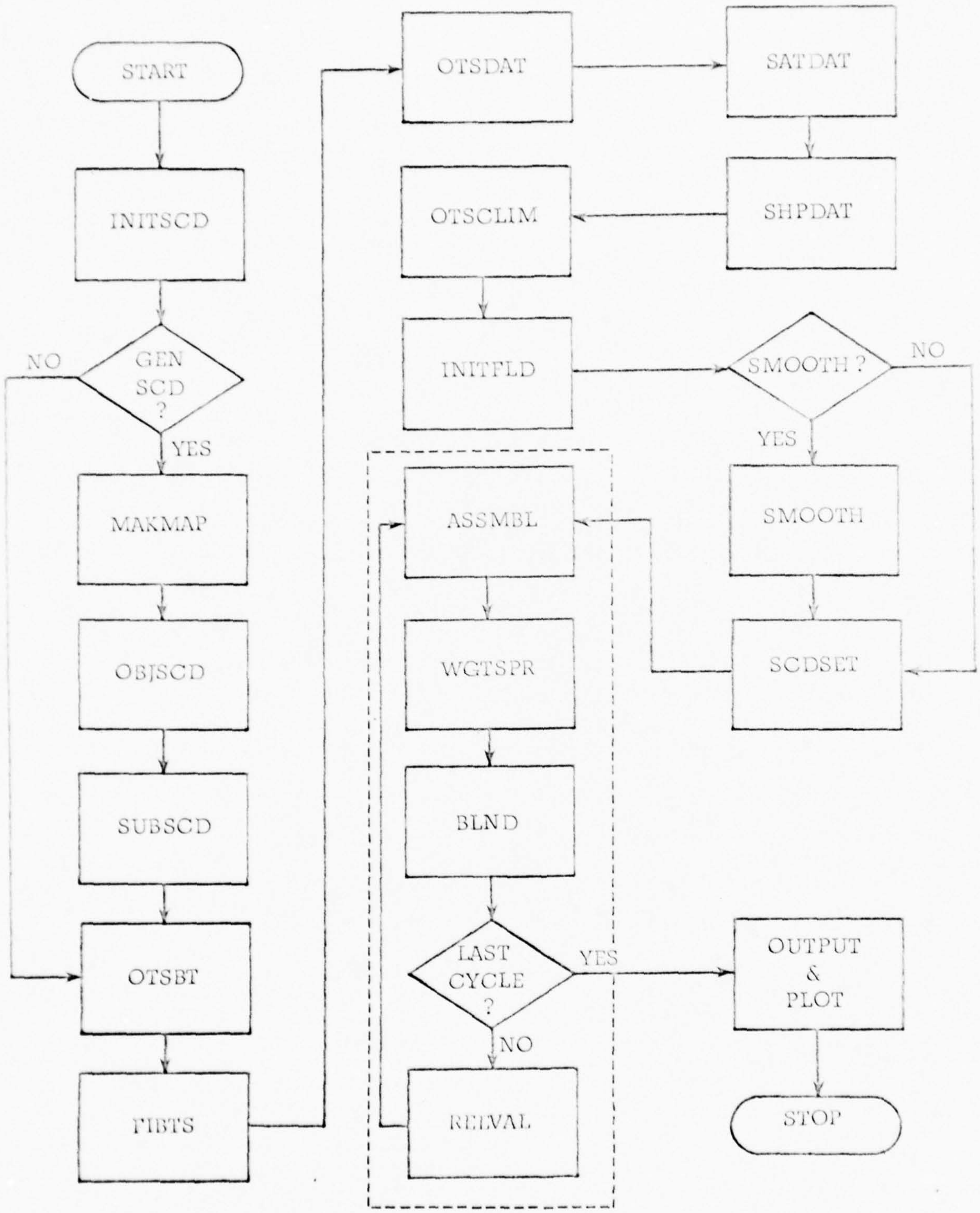


Figure 15 Information flow and processing in a simplified FIB system for the analysis of sea-surface temperature.

MAKMAP

Computes land/sea distribution in a new area (or for a previously analyzed area on a new analysis grid).

OBJSCD

Determines the objective SCD field from the land/sea distribution. Provides output needed to allow subjective decisions to be made.

SUBSCD

Updates OBJSCD using subjective decisions. Saves final SCD field for use by SCDSET.

OTSBT

Prepares data list to be used in the analysis.

FIBTS

Drives the SST analysis. This routine reads all field identification information and calls all modules necessary to carry out an analysis.

OTSDAT

Selects and reformats all observational data pertinent to the parameter to be analyzed--in this case SST reports from ships, satellites and the surface reading from BTs.

SATDAT

↓

SHPDAT

For reasons previously discussed (Section 4.5.3), in-situ observations provided by ships, including both the SST element of BT reports and SST observations available from synoptic marine reports, should not be directly assembled with equal weights with satellite-derived SST data. In these two modules, called by OTSDAT, SST data are grouped together in the data list according to their source for later assembly.

OTSCLIM

The previous analysis at time $\tau-1$ produced the blended resultant $T^{*}_{N,\tau-1}$, $A^{*}_{N,\tau-1}$ and the assembly field $T_{N,\tau-1}$, $A_{N,\tau-1}$. OTSCLIM carries information along the time axis to provide object-parameter PIFS T^{*}_{0} and T_{0} for the current analysis (see Sections 4.4 and 4.5.1 respectively). A_{0}, the weight field to be associated with the climatologically-enhanced recent-past data fields used in the current analysis, is computed during the course of the previous analysis run in accordance with Eq. (67). This avoids the necessity to save the field of $A^{*}_{N,\tau-1}$. The variance contribution $\sigma^{2}_{\Delta t}$, a User-controlled tuning parameter, is set to take account of the growth in variance of information during the time between analyses.

INITFLD

The object-parameter PIF, T^{*}_{0}, is used to produce the shaping and spreading PIFS--six first-differences (b, c, d, e, f and g) and the Laplacian (q). INITFLD computes and stores each PIF derived from T^{*}_{0} with its associated (assigned) weight field.

SMOOTH

To improve information control, optional smoothing processes may be applied to the shaping and spreading PIFS. If smoothing is selected, then options include

a. Smoothing with or without SCD;

b. Smoothing in coastal regions only;

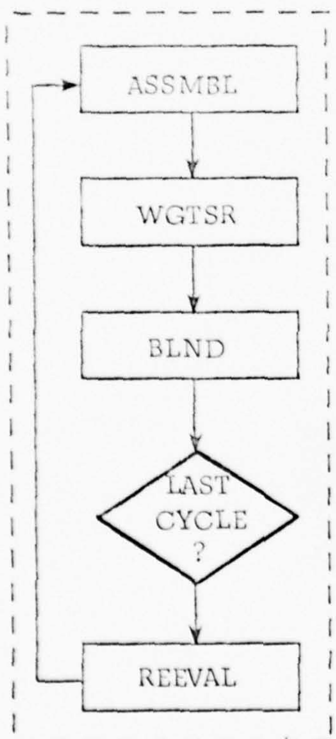
c. Smoothing at flagged grid points only;

d. Smoothing at only those grid points having a specified sign for the Laplacian of the parameter.

(An example of the application of these controls is the use of options c. and d. in adjusting gradients in the wake of major reported retreats in sea-ice coverage.)

SCDSET

Sets the appropriate spreading weights to zero in order to "decouple" the direct flow of information across land barriers. The zeroes for B and C are directly specified by the SCD field; simple SCD rules are used to set the zeroes of the other spreading weights.



Program components ASSMBL through REEVAL (enclosed by a dotted line on Fig. 15) form the heart of any FIB analysis system; all other components essentially are concerned with "setting up" the observational data and other relevant information, and for displaying the analysis product.

In ASSMBL all observations are carried to the nearest grid point by an adjustment obtained from the shape of the appropriate PIF. A Gross Error Check is carried out. All information passing this check is assembled into the assembly fields T_0, A_0 (thus providing T_N, A_N) by applying the standard FIB rules for adding uncorrelated variance contributions and independent information. If direct assembly[1] is used (Section 4.5.3), initial assembly weights are determined by multiplying an assigned class multiplier by a base weight which is determined by the inherent variance in the parameter.

Program components WGTSR and BLND determine the best fit to the ensemble of weighted information field. Eight pairs of input fields are involved--the object-parameter field into which current information has been assembled (T_N), six first-difference fields (b, c, d, e, f, g), the Laplacian (q), and the eight associated weight fields (A_N, B, C, D, E, F, G, and Q respectively). The resultant of the blending process is the field of the object parameter (T^*) and the field estimating the resolved or blended weight (A^*) used in reevaluation.

Program component REEVAL carries out the reevaluation process, adjusting downward the weight of those reports which lie outside their expected range when compared with all other available information. The process provides a quality measure in the object scale of resolution for each observation contributing information to the analysis.

The sequence of operations is repeated by returning to ASSMBL. Three cycles are used.

The final analysis consists of a field of the object parameter in which every piece of contributing information has been assessed twice for reliability. Associated with the analyzed field of the object parameter (T^*) is a blended weight field (A^*). The reciprocals of these weights provide

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[1] Superior methods for assembling data from diverse reporting systems are available--see Staged Assembly (Section 4.5.4) and Alternating Parallel Analysis (Section 4.12).

an estimate of the unresolved (i.e., residual) analysis variance in the object scale of resolution at every grid point in the field.

On completion of the final assembly a new assembly weight field, A_0, is produced for use in the next analysis--see OTSCLIM and INITFLD.

OUTPUT & PLOT

A wide variety of plotted and printed information may be output for such purposes as tuning evaluation, quality control, and further research and development. Examples of SST analyses are given in Section 4.13.

4.12 Alternating Parallel Analysis

4.12.1 Introduction

Remote sensing satellite systems provide measurements of certain atmospheric and oceanographic parameters which must be combined with in-situ measurements to provide optimum analyses of these parameters.

Major complications, to the point of being obstacles to the operational exploitation of remotely sensed data, include:

a. Cloud contamination of the primary data from those systems which must see cloud-free views for accurate measurement of objective atmospheric and oceanographic variabilities; and

b. The difficulty of realizing and maintaining absolute calibration of the measures relative to an object component of variability.

Both of these problems must be resolved by any analysis technique which purports to utilize remotely-sensed data in an effective manner. However the first complication is largely shunned, primarily because of difficulty in coping with measures of the object parameter which are of uncertain (and variable) quality. Analysis systems based on FIB methodology are inherently capable of coping with such complications.

The second complication generally is approached by a direct method--essentially that of determining a mean difference between in-situ observations and satellite-derived data and applying this mean difference as a correction. This direct approach has met with limited success because the error may vary over the object parameter field and with time--in effect there is a "bias field" and the remote sensing system measures a combination of the bias field and the object-parameter field. Assuming that the remote sensing systems are well designed--that in each case the bias field is larger in scale and smaller in variance than the resolution of the satellite system--then the FIB methodology can provide a capability for completely divorcing the analysis from the calibration of the remotely sensed data. The absolute resolution of the analyzed distribution may be derived entirely from in-situ measurements--measurements made by many instruments all independently calibrated. A valuable by-product of each analysis is the associated bias field.

The FIB capability to utilize remotely sensed data in an analysis in a manner which circumvents calibration problems is termed "Alternating Parallel Analysis" (APA). Two points should be noted:

a. Although presented only in terms of analyses of sea-surface temperature distributions, the concepts are applicable to a variety of remotely-sensed environmental parameters.

b. As yet APA has not been fully formulated, programmed and utilized in a specific application such as sea-surface temperature analyses. A description of APA is appropriate in this publication primarily because, in due course, it is intended to utilize APA in the overall EOTS analysis system. In addition an outline of APA further demonstrates the inherent information processing capabilities of FIB.

4.12.2 Outline of APA

Consider three analyses, at time $\tau-1$, τ, and $\tau+1$, which form part of an analysis sequence. Depending on the object parameter of the analysis (in this case SST), analysis times may be separated by periods ranging from hours to days (or even longer for the production of climatological fields). As previously discussed, information is carried and accrued along the time axis from one analysis to the next.

For SST analyses, each analysis is "cycled" three times as part of the reevaluation process. As will be seen, the APA concept introduces a requirement for three analyses at <u>each</u> analysis time, each of these three analyses involving three analysis cycles. This is made clear by the following figure:

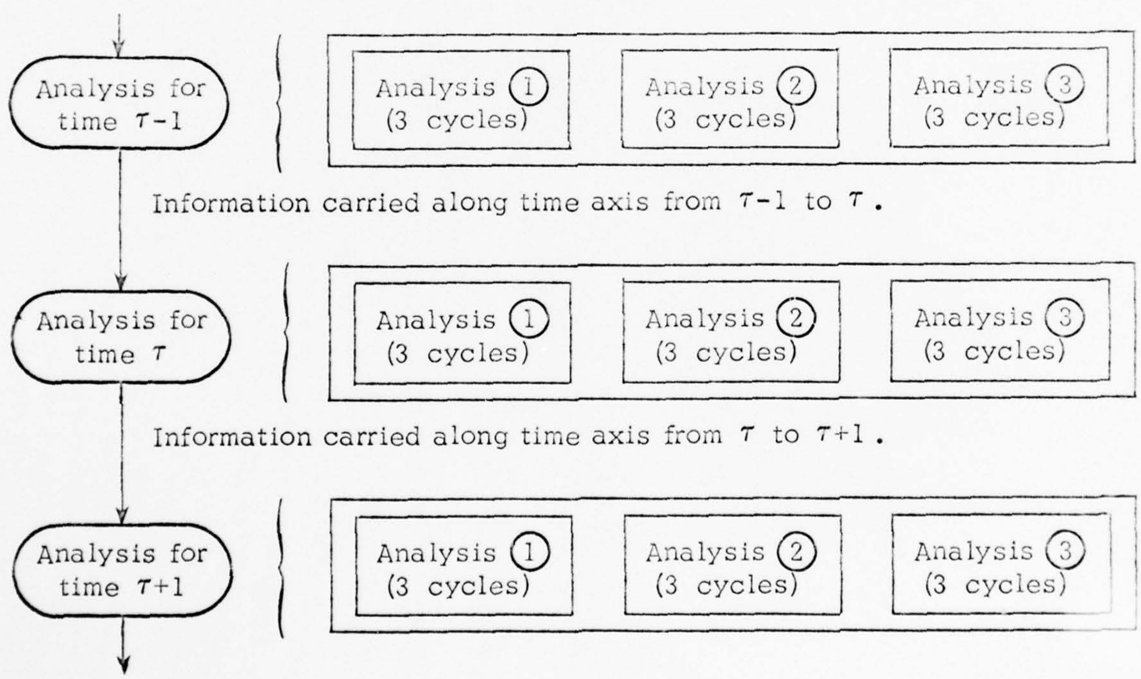

Figure 16 For each analysis time (e.g., time τ) the APA concept requires three analyses <u>each</u> of which is a 3-cycle FIB analysis.

One other point should be recalled--that FIB methodology can represent the shape of a field of the object parameter independently of absolute object parameter values. These shape fields are provided by fields of gradient and curvature. The concept of transferring shape between analyses 1, 2 and 3 for a given analysis time (such as analysis time τ) is fundamental to the APA capability--essentially it is this feature which permits calibration problems associated with remotely sensed data to be circumvented.

The APA concept is shown schematically in Fig. 17. The three vertically aligned flow lines, marked (a), (b) and (c), carry information along the time axis from one analysis time to the next--e.g., from $\tau-1 \rightarrow \tau$. For a given analysis time (e.g., time τ), line (b) also passes the analysis-resultant shape field from analysis 1 to 2 and from analysis 2 to 3.

Line (a) carries a field of assembled in-situ estimates and the associated time-decayed weight field from analysis 3 at $\tau-1$ to analyses 1 and 3 at τ. Line (c) carries similar information for remotely sensed estimates from analysis 2 at $\tau-1$ to analysis 2 at τ.

For analysis time τ, the resultant shape field from analysis 1 is passed by line (b) as input to analysis 2. The enhanced resultant shape field from analysis 2 is passed by line (b) as input to analysis 3. It can be seen that, in effect, analyses 1 and 3 are "calibrated" to the in-situ reports whereas analysis 2 is "calibrated" to the remotely sensed estimates. Clearly this APA design exploits all relevant information inherent in the estimates derived from remotely sensed data while completely circumventing the calibration problems commonly associated with such estimates.

The configuration given in Fig. 17 is designed for exploitation of remotely sensed estimates which are all derived from one sensor system, thus sharing bias characteristics. The scheme can be extended to

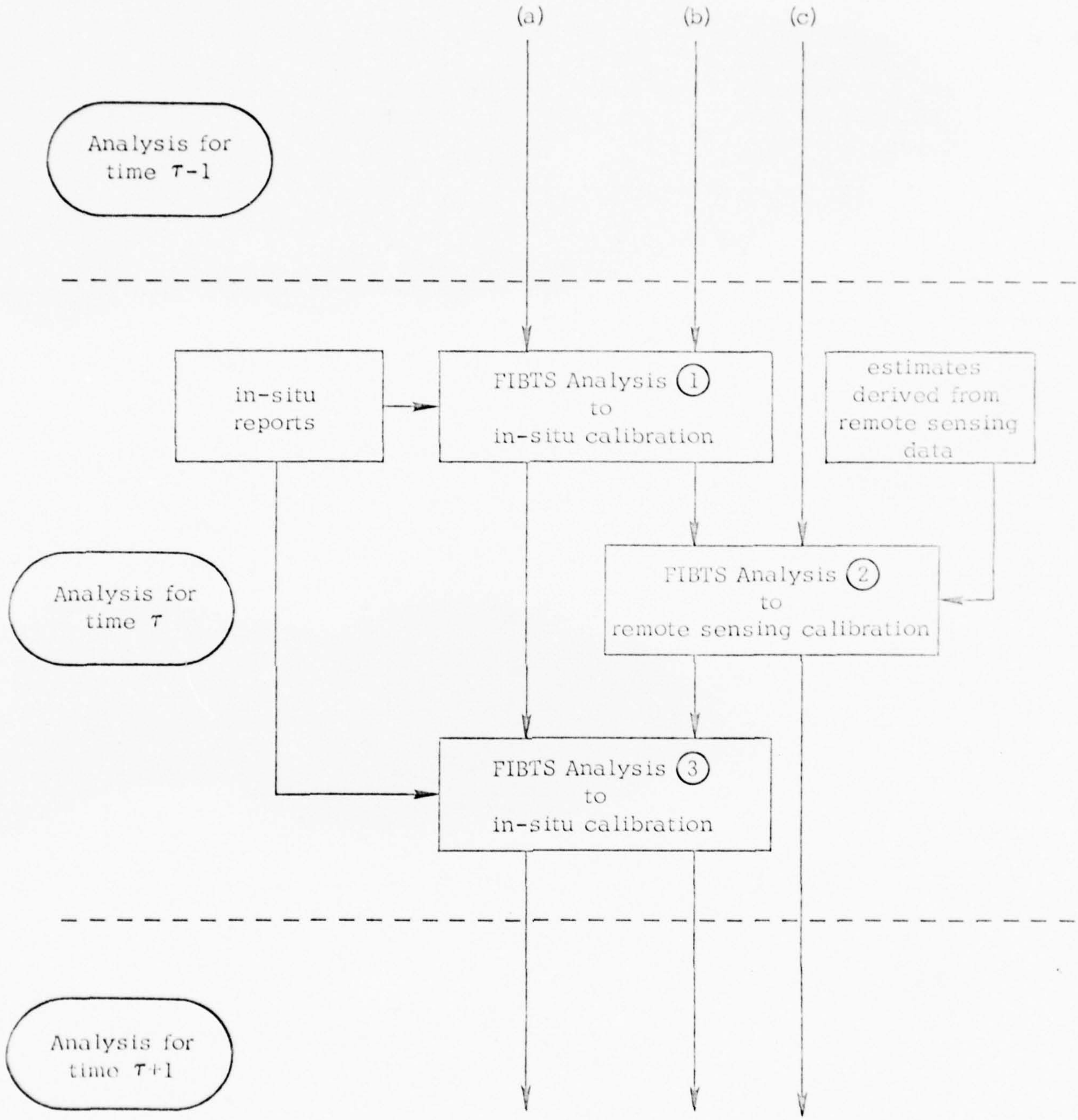

Figure 17 Schematic of the Alternating-Parallel-Analysis concept.

exploitation of sets of remote sensing estimates--each set being derived from one distinct sensor system.

Although presented in terms of SST analyses, in principle APA can be applied to analyses of any object parameter. The generalized concepts of APA, when developed to the point of operational utility, will provide a technique permitting remotely sensed data to be used in analyses of atmospheric and oceanographic parameters without any of the problems associated with sensor-calibration bias and drift. The only two requirements are that the sensor systems are well designed (i.e., that the bias field, including drift, is of a larger scale and smaller variance than the resolution of the satellite system), and that in-situ measurements of the object parameter also are available. The satellite data should not have been corrupted by ineffectual calibration adjustments.

Sea-surface temperature is particularly suitable for developing an APA capability for the following reasons:

a. Satellite-derived and in-situ observations are readily available.

b. SST analyses form part of the input to comprehensive atmospheric/oceanographic prediction models and are thus needed on an operational basis by all numerical weather analysis and forecasting organizations such as FNWC and NMC.

c. SST is the keystone to analysis of ocean thermal-structure distributions which are of great significance theoretically, commercially (e.g., fishing), and to military planning and operations.

d. A knowledge of SST distributions is essential to understanding air/sea interaction processes and development of air/sea interaction algorithms (such as the planetary boundary layer).

e. SST distributions play an important role in determining weather developments and climate variability.

4.13 Examples of Analyzed Fields of Sea-Surface Temperature

Figures 18 and 19 show two synoptic analyses for the Northern Hemisphere on a 63x63 analysis grid, polar stereographic projection. The times are 00Z on 29 May and 30 May 1979 respectively, and the contour interval is 2° C. A total of 2689 reports (ship and satellite) were available for the 29 May analysis and 2983 were available for the 30 May analysis. (The number of current reports varies but usually is well in excess of 2500. For example 9458 reports were available for the 27 May analysis.) The latest ice boundary is read by the system and shown by a heavy line. This can be seen between Iceland and Greenland (and elsewhere). Implications of the ice extent are assimilated in the analysis.

Figures 18 and 19 show variabilities in the SST field which are significant in the object scale of resolution--i.e., an analysis grid of 63x63 covering the Northern Hemisphere. The California Current and Gulf Stream are readily apparent as larger-scale features of the hemispheric SST field.

FNWC also produces fine-mesh synoptic SST analyses for a number of sub-regions. Figure 20 shows an analysis for the Gulf Stream (produced every 24 hours) at 12Z, 29 May 1979, again on a 63x63 analysis grid, polar stereographic projection, but with a mesh size equal to one-eighth of that used in the Northern Hemisphere analyses. The contour interval in Fig. 20 is 1° C. The number of reports used in the analysis was 294. For fine-mesh analyses such as that shown in Fig. 20, both ship and satellite reports are plotted--value in $^{\circ}$C at the appropriate location. Reports plotted as white-on-black have been rejected by the analysis system.

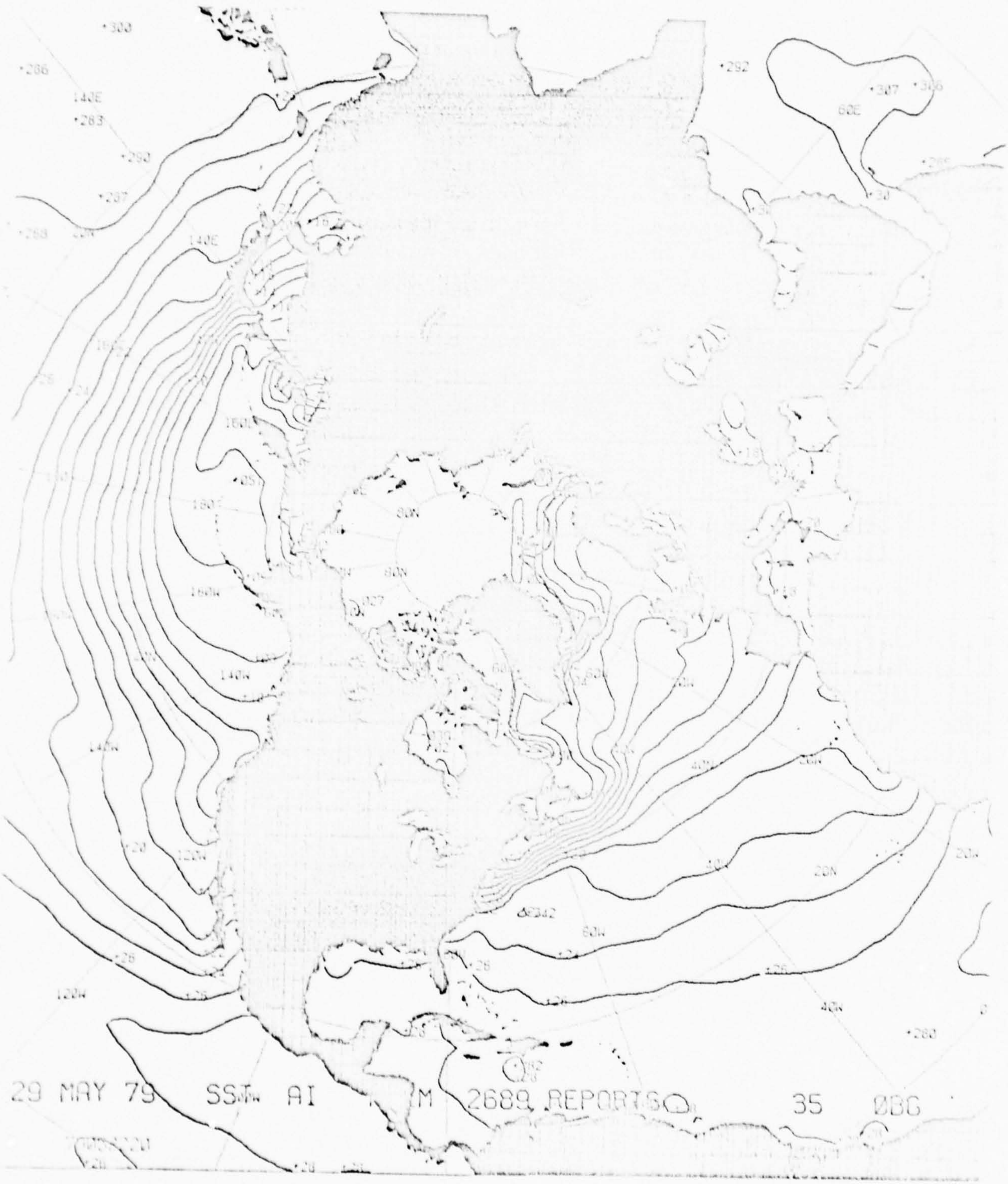

Figure 18 Northern Hemisphere SST analysis for 00Z, 29 May 1979.

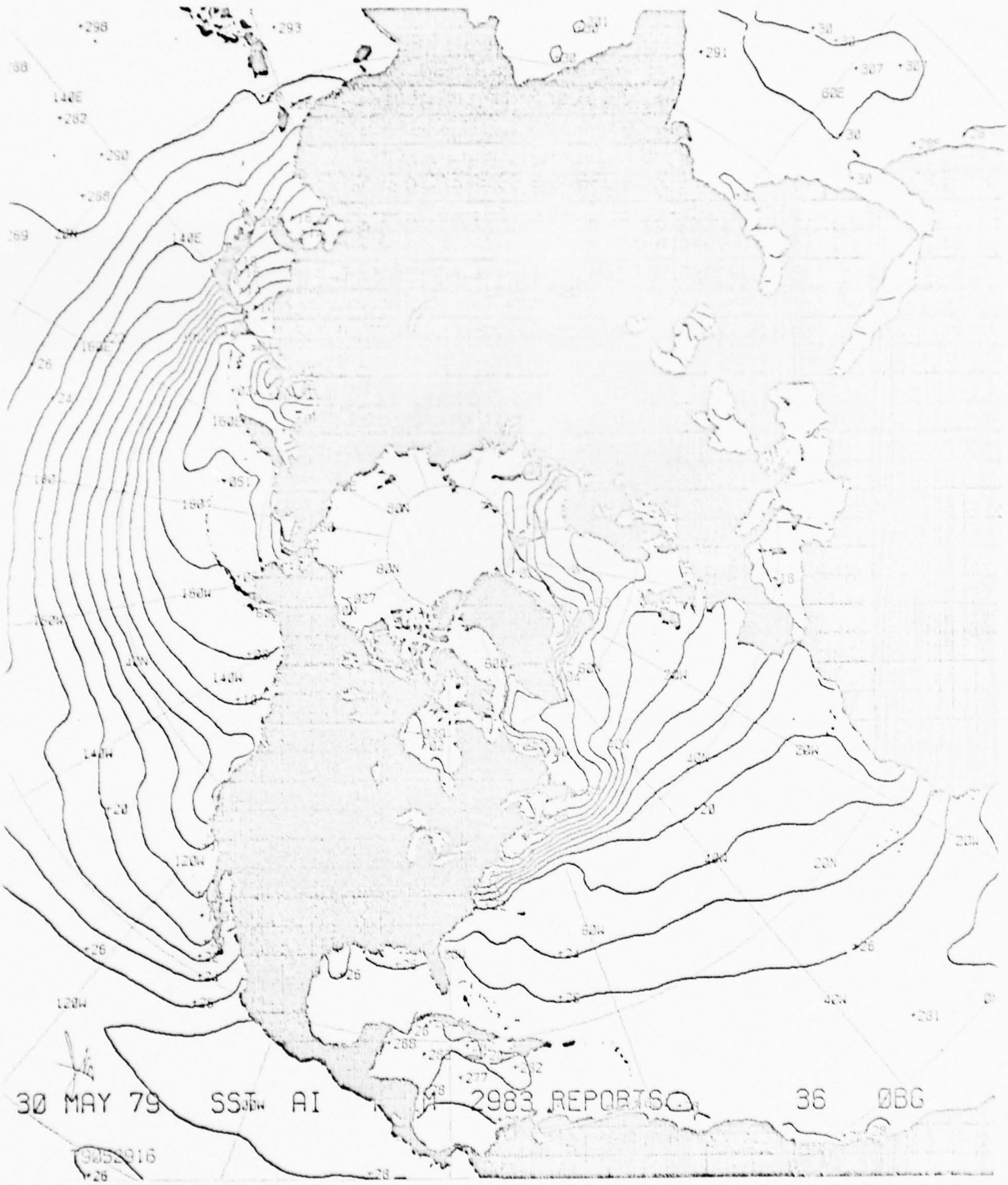

Figure 19 Northern Hemisphere SST analysis for 00Z, 30 May 1979.

Figure 20 Gulf Stream SST analysis for 12Z, 29 May 1979.

Figure 20 again shows variabilities in the SST field which are significant in the object scale of resolution. However the object scale of resolution for Fig. 20 is 64 times greater than that for Figs. 18 and 19. As a consequence the fine-mesh Gulf Stream analysis shows far greater detail than the analysis for the same region provided by the hemispheric analyses. Note that this greater detail is due to the change in object scale of resolution.[1]

These analyses graphically illustrate many of the points raised in Section 2 (and elsewhere) concerning the worth (or reliability) of a measured value of an environmental parameter as an estimate of the representative value in the object scale of resolution. Clearly observations which enable a significant feature to be resolved in Fig. 20--say the cold gyre evident in the vicinity of 41°N;68°W--are less valuable estimates of SST in the context of a coarse-scale hemispheric analysis. In fact, because the gyre is not recognized as a significant feature in Figs. 18 and 19, these observations would be considered as (partially) in error and their weight appropriately reduced during the reevaluation process.[2] Both analyses are "correct" in the sense that they produce the best SST field from the total information available but what is "best" depends on the object scale of analysis resolution.

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[1] Current reports used in the Gulf Stream analysis also contributed to one or other of the hemispheric analyses.

[2] See footnote 1 on page 21.

5. THE DEFINITION OF OCEAN THERMAL-STRUCTURE PARAMETERS AND THEIR ANALYSIS IN SPACE AND TIME

5.1 Introduction

Section 4 describes the analysis of SST distributions based on the FIB analysis methodology. Clearly the concepts and formulations developed are not only applicable to the temperature at the sea surface; they could be applied to the analysis of temperature at any depth. This depth need not even be constant over the analysis region. For example temperature analysis could be carried out on a surface defined locally, say, by the depth of the seasonal thermocline.

In order to encompass the added dimension of depth the information available from temperature profiles provided by BT reports and other sources must be utilized appropriately. However, as explained in Section 5.4, a simple analysis of ocean thermal-structure based entirely on independent horizontal (or quasi-horizontal) temperature analyses at various depths is entirely inadequate for realistically determining representative values of ocean temperature in space and time.

The procedure adopted is to parameterize the vertical profile in terms of a set of "ocean thermal-structure parameters". The parameters used for this purpose, defined in Section 5.4, have been chosen to capture significant temperature variations in the vertical structure while having sufficient continuity to be amenable to horizontal (and independent) analysis. The analysis system described in Section 4, with adjustable constants set to suit the characteristics of individual parameters, is used to analyze each of these thermal-structure parameters.[1] Having completed analysis of each thermal-structure parameter (which includes continuity along the time

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[1] The analysis system described in Section 4 is, in fact, a general-purpose system capable of application to a wide variety of ocean parameters. For example, appropriately modified, it is used to analyze significant wave-height distributions.

axis), vertical blending, described in Section 5.7, completes the synoptic analysis process. The overall resultant is the best estimate of representative temperature in the space-and-time object scale of resolution that is available from the total available information.

The FIB concepts and formulations presented in this Section are oriented primarily toward the production of EOTS analyses in an operational time-frame. However the EOTS system is able to cope with any quantity of input data (subject to computer resource limitations) and can be used to produce climatologies and historical sequences from appropriate data bases (see Section 7).

5.2 Current Information for Analysis

In general, for any analysis time τ, raw data for the analysis is available from the following sources:

a. BT reports of all types.

b. SST reports from ships and buoys.

c. SST data from satellites.

To encompass the vertical dimension the information available from temperature profiles provided by BT reports must be utilized appropriately. However, as will be seen, the vertical blending process propagates information from the surface to sub-surface regions. In other words BT reports are not the sole source of information contributing to a knowledge of sub-surface thermal structure.

In the real-time production of ocean thermal-structure analyses certain types of data--such as Nansen casts--are not available; these may be included in the production of historical sequences and climatologies.

5.3 Representation of a Temperature Profile

A BT observation provides a continuous trace of temperature vs. depth from the sea-surface to the maximum depth achieved by the sounding. In the case of XBTs this depth is generally about 400-450 meters. Due to the response time of the instrument as it drops the trace obtained actually shows a smoothed representation of the true vertical temperature variability. However it may be assumed that these fine-structure variabilities are not significant in the context of sound propagation conditions. An additional loss of variance is caused by the translation of a BT observation to a BT report. The observer subjectively interprets the trace in terms of a variable number of paired values of temperature and depth commencing with the temperature at zero depth--i.e., the SST. These paired values provide the BT report. The number of paired values used to represent the BT observation depends on the complexity of the trace, the skill of the observer in recognizing significant features, and the effort he is willing to expend in encoding these features.[1]

A module of the EOTS system accepts each BT report available for an analysis. The reports are first checked and corrected for duplicate levels and obvious single-level reporting errors. The BT is then converted to meters and °C and the temperature linearly interpolated at every 2.5 meters from the surface to 450 meters (or to the greatest depth reported if the BT did not reach this depth). Thus, in the EOTS analysis system, each BT report is represented by up to 181 values of temperature separated in depth by increments of 2.5 meters.

[1] An increasing number of BT observations are being interpreted automatically. In such cases over 100 depth:temperature values are not uncommon.

5.4 The Thermal-Structure Parameters

The essentials of a 4-dimensional analysis scheme are illustrated in Fig. 4. An elementary approach to the problem of analyzing temperature distributions in the ocean is to define an appropriate number of fixed depths--say about twenty (four such levels are shown in Fig. 4). At analysis time $\tau+n$ the temperatures at these fixed levels would be selected from the interpolated BT reports and assembled at the nearest grid point. At the surface additional information is available--the SST reports from ships and satellites. Horizontal analyses for each of these fixed levels may then be carried out using the methods and formulations presented in Section 4. Once all horizontal analyses have been completed a temperature profile at any grid point of the analysis grid could be constructed from the corresponding values extracted from the horizontally analyzed fields. Appropriate interpolation schemes would define the space-time continuum.

This, of course, is not a realistic approach. The horizontal levels have been analyzed independently of one another; no information has been exchanged vertically. For example suppose that the available SST information revealed an anomalous cold gyre in a certain region but that no BT observations had been made in this region for some time. The cold gyre would not be shown by sub-surface analyses. If the first sub-surface level were, say, 25 meters, then the reconstructed profile would show a completely fictitious positive temperature gradient between the surface and 25 meters. Clearly the 25-meter analysis (and analyses at greater depths) should be adjusted to reflect the cold gyre revealed by SST information. To do this an appropriate method for the vertical exchange of information is required.

Before defining such a method other drawbacks to the elementary approach should be considered. An important reason for performing ocean thermal-structure analyses is to determine sound propagation conditions.

In this context sound speed gradients (directly related to temperature gradients) are more significant than absolute values of sound speed (or temperature). In other words it is the shape of the profile which is the over-riding consideration. This immediately suggests that the analysis of gradient fields (i.e., the vertical temperature difference between two levels), in addition to temperature analyses, is a necessary feature of a realistic ocean thermal-structure analysis system.

The elementary approach suggested a specified number of fixed levels for the horizontal analysis of temperature. This can have serious consequences in the realistic representation of vertical temperature gradients. Figure 21c shows the smoothed profile that results from using fixed levels which, in terms of vertical separation, fail to resolve the significant variabilities of the reported profile. This failure is particularly important in regions of strong negative curvature such as the Primary or Principal Layer Depth (PLD).[1]

In summary, a simple approach based on horizontal temperature analyses at fixed levels is inadequate because:

a. Information is not spread between levels.

b. The shape of the profile is not considered.

c. Unless a very large number of fixed levels are utilized (precluded by currently available computer resources), features of the profile which are particularly significant to sound propagation are unrealistically modified.

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[1] PLD is a depth at which the rate of change of temperature with depth becomes markedly more negative; a more precise definition is given later in this Section. The term "curvature" is used to denote a second difference of the temperature profile with depth rather than the radius of curvature.

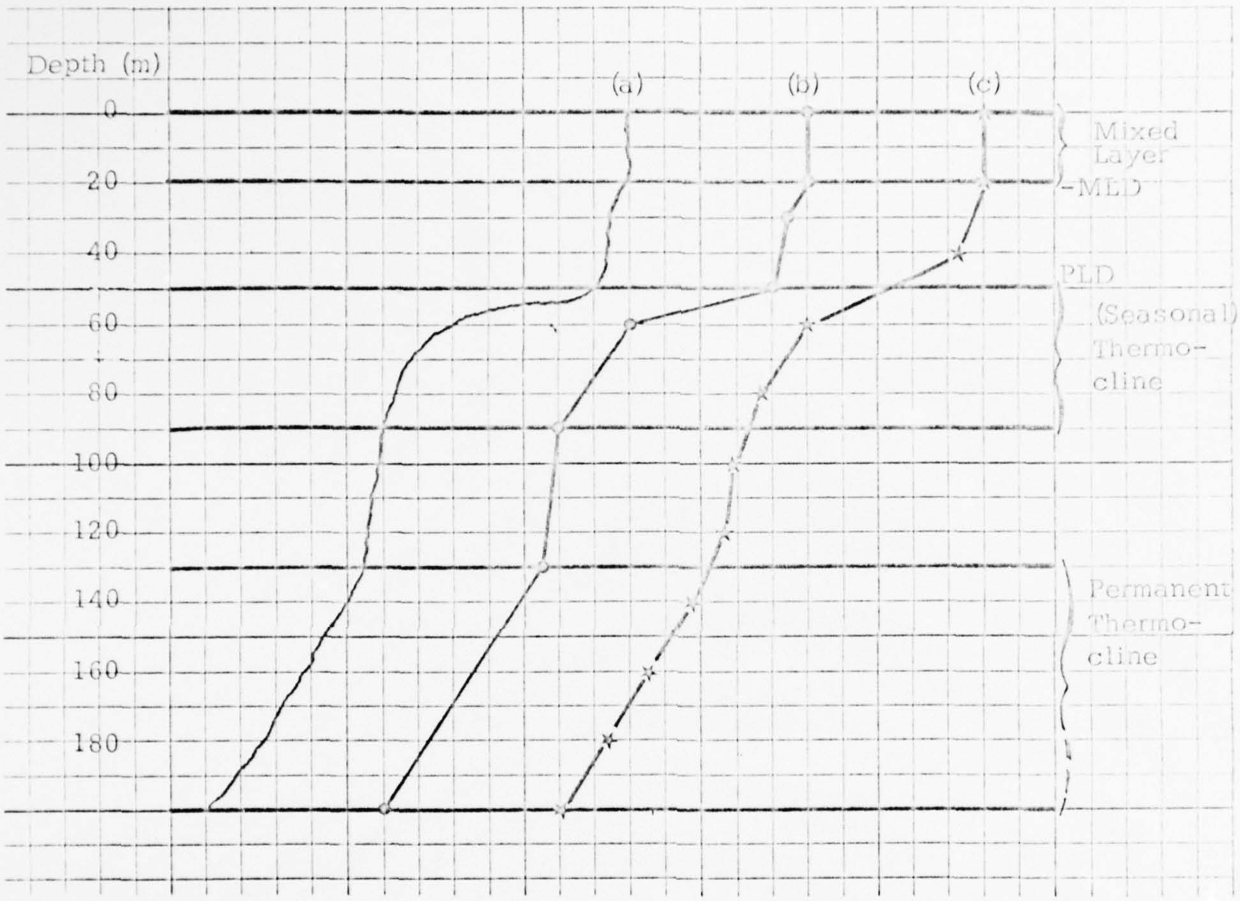

Figure 21 Features of a typical BT are named to the right. (a) shows a BT observation. (b) shows a BT report based on this observation. (c) shows the profile that would result from selecting temperatures at 20-meter intervals from (b). Comparison of (a) or (b) with (c) shows the smoothing effect that arises from the use of fixed levels--temperature differences are spread over a greater depth range and hence gradient magnitudes are modified.

On page 26 it is stated that "In many cases variations in the vertical dimension...can be better represented by a set of parameters which correspond to a modelling of significant degrees of profile variability rather than by a set of levels." This concept, combined with the capabilities provided by FIB, enables items (a) and (b) above to be overcome and item (c) to be minimized.

The EOTS system uses up to 26 "thermal-structure parameters" to represent a BT profile from the surface to 400 meters, the same number of parameters being used for all profiles in a given analysis. These parameters are measures of profile temperature, "gradient" (i.e., first difference) and "curvature" (i.e., second difference).

For sound propagation purposes the most significant feature of the profile is the Primary Layer Depth. Consider a reported profile such as that shown in Fig. 21b. Associated with this profile there is a list (i.e., an array) of linearly-interpolated temperatures at intervals of 2.5 meters (Section 5.3). For every reported depth, the nearest-in-depth 2.5-meter temperature, T_d, is consulted and the following quantity Φ calculated:

$$\Phi_d = 2T_d - \left(T_{d-25} + T_{d+25}\right) \tag{77}$$

where T_{d-25} and T_{d+25} refer to the interpolated temperature 25 meters above and below the depth d. If T_{d-25} extends above the surface then T_{d-25} is set equal to T_0, the sea-surface temperature.

Using all reported depths the maximum value of Φ is found; this provides a "PLD candidate". The procedure is repeated to find the next largest value of Φ, and so on. Each profile, with certain exceptions which are discussed later, provides up to nine PLD candidates.[1] Associated

[1]The search of a profile for PLD candidates ceases when either the number of candidates, arranged in order of decreasing Φ reaches nine <u>or</u> $\Phi < 0.2°C$.

with each candidate, eight thermal-structure parameters are obtained based on the 2.5 meter interpolated temperature array--the PLD itself, the temperature at this depth, four measures of temperature difference over prescribed depth ranges relative to the PLD, and two measures of curvature. These measures provide fine-resolution of the ocean thermal-structure above and below each PLD candidate. At a later stage, in a manner to be described, one of the nine candidates is selected for inclusion in the analysis. The chosen PLD varies from profile to profile. Since the set of eight parameters is defined relative to the PLD it can be seen that we have a set of parameters which "float"--i.e., they are not at a fixed depth relative to the sea surface.

The remainder of the profile from the surface to 400 meters is represented by up to 18 "fixed-level" parameters.

Figure 22 shows the scheme used to parameterize a given profile in terms of the twenty-six thermal-structure parameters. These parameters are defined in Table 1; the class standard deviations associated with each parameter also are given.

As can be seen from Fig. 22, the fixed level parameters (9 → 26) provide 8 measures of temperature at pre-defined depths and 10 measures of first-differences of temperature over pre-defined finite depth intervals. Note that fixed-level resolution increases with decreasing depth in order to capture the greater variability of temperature in near-surface waters.

As previously noted, the majority of BT reports do not extend below 400 meters or so and 400 meters is therefore the greatest depth analyzed in a synoptic time-frame. At a later stage in the analysis process, climatological fields of temperature for 600m → 5000m are used to provide deep-level parameters shown on Fig. 22--9 values of temperature, 8 direct first-differences of temperature, and 7 direct second-differences of temperature, all at fixed levels. Note that these deep-level parameters

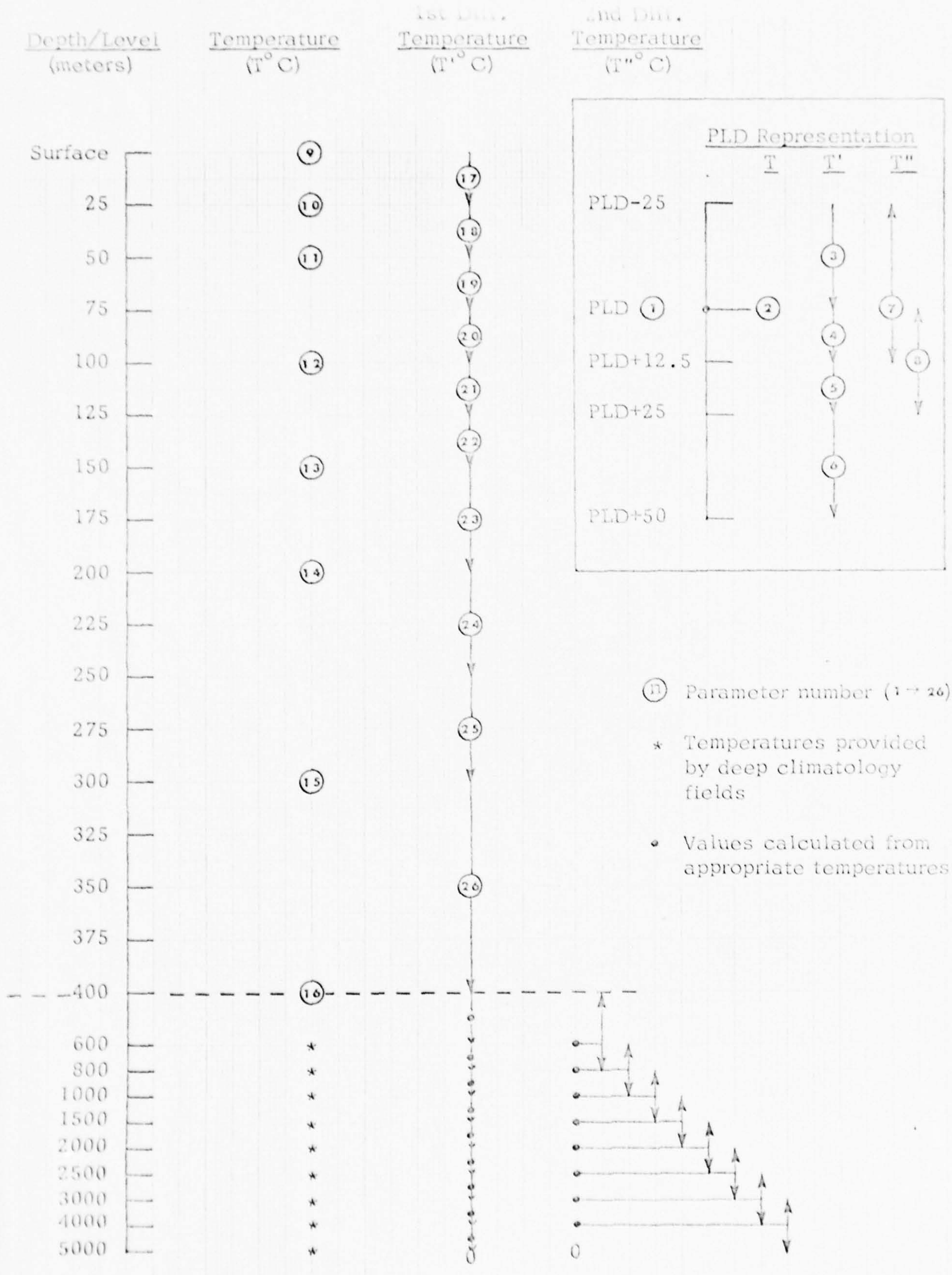

Figure 22 Parameterization of profiles--see text for description.

Table 1
The ocean thermal-structure parameters

Parameter Number	Parameter Name	Description	Class Std. Dev.
1	PLD	Primary Layer Depth (m)	15.0
2	TV00	T_{PLD} (°C)	0.7
3	GVM1	$T_{PLD} - T_{PLD-25m}$	0.3
4	GV00	$T_{PLD+12.5m} - T_{PLD}$	0.8
5	GVP1	$T_{PLD+25m} - T_{PLD+12.5m}$	0.5
6	GVP2	$T_{PLD+50m} - T_{PLD+25m}$	0.4
7 (c)	CV00	$T_{PLD+12.5m} - 2T_{PLD} + T_{PLD-25m}$	0.5
8 (c)	CVP1	$T_{PLD+25m} - 2T_{PLD+12.5m} + T_{PLD}$	0.5
9	SST	T_{0m}	1.0
10 (c)	T025	T_{25m}	1.0
11	T050	T_{50m}	1.0
12 (c)	T100	T_{100m}	0.8
13	T150	T_{150m}	0.7
14 (c)	T200	T_{200m}	0.6
15	T300	T_{300m}	0.6
16 (c)	T400	T_{400m}	0.5
17	G000	$T_{25m} - T_{0m}$	0.4
18	G025	$T_{50m} - T_{25m}$	0.8
19	G050	$T_{75m} - T_{50m}$	0.7

Table 1 (continued)

20	G075	$T_{100m} - T_{75m}$	0.6
21	G100	$T_{125m} - T_{100m}$	0.3
22	G125	$T_{150m} - T_{125m}$	0.3
23	G150	$T_{200m} - T_{150m}$	0.4
24	G200	$T_{250m} - T_{200m}$	0.4
25	G250	$T_{300m} - T_{250m}$	0.3
26	G300	$T_{400m} - T_{300m}$	0.3

NOTE: All units correspond to those of the object data. For example, the G300 value of 0.3 is in units of $^{\circ}C$ per 100m (300-400m), whereas GVM1 is in units of $^{\circ}C/25m$. Certain parameters normally are not used in routine synoptic analyses; these are shown by (c) against the parameter number.

do not remain constant; they are modified in each synoptic analysis by the vertical blending process which spreads information from upper levels of the ocean to depths below 400 meters. If the vertical blending process did not extend beyond 400 meters (i.e., if the deep climatological values merely were "added on" to the analyzed upper levels) there would be a discontinuity in the profile when passing the 400-meter level. Clearly, climatological values near 400 meters will be modified to a greater extent than values at greater depths. Using synoptically-modified climatological values below 400 meters is a reasonable and necessary procedure since, at great depths, significant changes are not likely to occur in a synoptic time-frame and neither is synoptic data available to reveal any such changes that might occur.

To summarize, each BT report may be represented by twenty-six thermal structure parameters of which eight (the floating parameters) are devoted to a fine-resolution representation of a PLD selected from a list of up to nine possible candidates. The remaining parameters are at fixed levels. Since acoustic propagation depends on gradients of temperature rather than absolute values, the parameterization process is designed to highlight vertical shaping characteristics--gradients and curvatures.

In the production of synoptic EOTS analyses in an operational context the full set of twenty-six parameters generally is not used, the parameters indicated by (c) against the parameter number in Table 1 being omitted. The full set normally is used only for the production of ocean thermal-structure climatologies. However, for the time being, it is convenient to describe profile parameterization and analysis in terms of the full set.

5.5 Analysis of the Fixed-Level Parameters[1]

As can be seen the profile parameterization process provides the temperature at eight fixed levels (parameter nos. 9 → 16). Parameter no. 9 is the sea-surface temperature, the analysis of which was discussed in some detail in Section 4.

Current data available for the analysis of parameter no. 9 is provided by ship and satellite observations in addition to the surface reading from BTs. The EOTS analysis of this parameter initially is carried out exactly as described in Section 4 with one significant exception. The object-parameter PIF, T_0^*, was produced in accordance with Eq. (58).

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[1]The EOTS system actually analyzes the floating levels first. However it is more convenient to commence with a discussion of the analysis of fixed-level parameters.

The inherent information in T_0^* then was partitioned in terms of component information fields to provide the seven shape PIFS defined in Eqs. (59) through (65). Note that T_0^* (and hence the shape PIFS) contains only information derived from surface observations. In general however, an EOTS analysis for a given time exchanges information vertically as well as horizontally--a double-blending process is involved.

Using the thermal-structure parameter names given in Table 1 (e.g., SST for parameter no. 9 rather than T as before) the previous EOTS analysis at time τ-1 provides the doubly-blended field $SST_{\tau-1}^{**}$. Using Eq. (58), this is carried forward along the time axis to time τ to provide the PIF SST_0^{**}. The information is partitioned to provide the shape PIFS--Eqs. (59) through (65). Eq. (66) provides the assembly field SST_0 which is combined with the object-parameter PIF in the same manner (and for the same reasons) as presented in Section 4.5.1. Following assembly of current SST data, a horizontal-only analysis provides SST_τ^*. (SST_τ^* will be modified during vertical blending to provide SST_τ^{**}; this is discussed later.)

Using current data provided by BT reports, a similar procedure may be followed to provide the horizontally-analyzed fields for parameter nos. 10 through 16--i.e., $T025_\tau^*$, $T050_\tau^*$, ... $T400_\tau^*$. Various adjustable constants and tuning parameters have to be appropriately set to reflect the characteristics of each thermal-structure parameter, but the underlying principles and procedures are the same.

Parameter nos. 17 through 26 are estimates of temperature profile gradient. Thus from Fig. 22 and Table 1 it can be seen that parameter no. 17 is the difference in temperature between 25 meters and the surface and therefore represents the mean temperature gradient over the first 25 meters of the profile. By convention gradient names are referred to the upper of the two levels; thus parameter no. 17 is denoted by G000 (Table 1). Similarly parameter no. 23 is the mean gradient between 150 and 200 meters and is denoted by G150.

Analysis of these 10 gradient parameters is no different in principle to the analysis of temperature. For any particular gradient parameter the previous analysis provides PIFS for the shape fields and the assembly field. New information for an analysis is provided by gradient estimates extracted from the 2.5 meter interpolated temperature array for each available BT. This information is assembled using the methods given in Section 4. A three-cycle FIB analysis then provides the analyzed fields for each gradient parameter.

On completion of the FIB analyses for the fixed-level parameters (nos. 9 through 26) it can be seen that we end up with 18 horizontally-analyzed fields of which 8 are temperature fields and 10 are gradient fields. Clearly at every grid point we have a linear array as follows:

Depth (meters)	Object Parameter (Temperature, °C)		First Difference	
	Value	Weight[1]	Value	Weight[1]
0	SST*	ASST	G000*	B000
25	T025*	A025	G025*	B025
50	T050*	A050	G050*	B050
75	--	--	G075*	B075
100	T100*	A100	G100*	B100
125	--	--	G125*	B125
150	T150*	A150	G150*	B150
200	T200*	A200	G200*	B200
250	--	--	G250*	B250
300	T300*	A300	G300*	B300
400	T400*	A400		

[1] See note b. below.

The similarity between these two pairs of weighted information fields and those given early in Section 3.5.1 is immediately apparent. Using an error functional similar to Eq. (31) vertical blending at every grid point of the horizontal array could be carried out. This would provide a 3-dimensional analysis; a sequence of such 3-D analyses constitutes a 4-dimensional analysis. However the scheme outlined above has encompassed only the fixed-level parameters; the high-resolution representation of the PLD has yet to be analyzed and assimilated into the analysis of ocean thermal structure. This is described in the following Sections.

With regard to fixed-level analysis the following points may be noted:

a. The values of the object parameter and gradient fields shown in the above array--e.g., T025* or G050*--are the resultant of horizontal analysis and thus only have a single "*".

b. The weights shown (e.g., A025 or B050) are not to be confused with the starred resultants of the horizontal analyses of the associated parameters. They represent a partitioning of information. The horizontally analyzed parameters do not represent fully independent information; they are derived from a common set of BT soundings. The information partitioning weights are assigned[1] in proportion to the adjudged significance of the parameters in the profile blending context. The sea-surface temperature value, generally having the greatest information content, is given the largest weight. The vertical blending process gradually propagates this information downward.

[1] A weight-specification table for vertical blending is given and discussed in Section 5.7.

Note also the evolving complexity of the EOTS analysis system--as so far described we require a 3-cycle FIB analysis similar to that discussed in Section 4 to be carried out 18 times, resetting adjustable constants and tuning parameters for each thermal-structure parameter.

5.6 PLD Selection and Analysis of the Floating-Level Parameters

As described in Section 5.4, each BT profile provides up to nine PLD candidates. Associated with each candidate there is a set of eight thermal-structure parameters--see Fig. 22 and Table 1. The purpose of these eight parameters is to provide fine-resolution of that feature of the profile which, in the upper levels of the ocean, is most significant to sound propagation. The appropriate PLD candidate must be selected. The following points may be noted:

a. The PLD candidates which are not chosen for fine-resolution representation are not disregarded--they will be resolved to the extent provided by the fixed-level parameters;

b. The candidates from each profile are chosen in descending order of Φ where Φ, defined in Eq. (77), is the difference between the mean gradient 25 meters above and 25 meters below the reported level being considered. This procedure for selecting the PLD candidates from each profile ensures that a balance is struck between strong temperature gradients which extend over only a limited depth range and weaker gradients which extend over a greater depth range.

Why select nine candidates? Why not choose that value of PLD for which Φ is greatest? Figure 23 shows a series of profiles ($^\circ$C vs. depth in meters). The abscissa also is used to represent either distance (in which case the series is for a particular analysis time with the separation between profiles being, say, one or two grid-lengths) or time (in which case the series

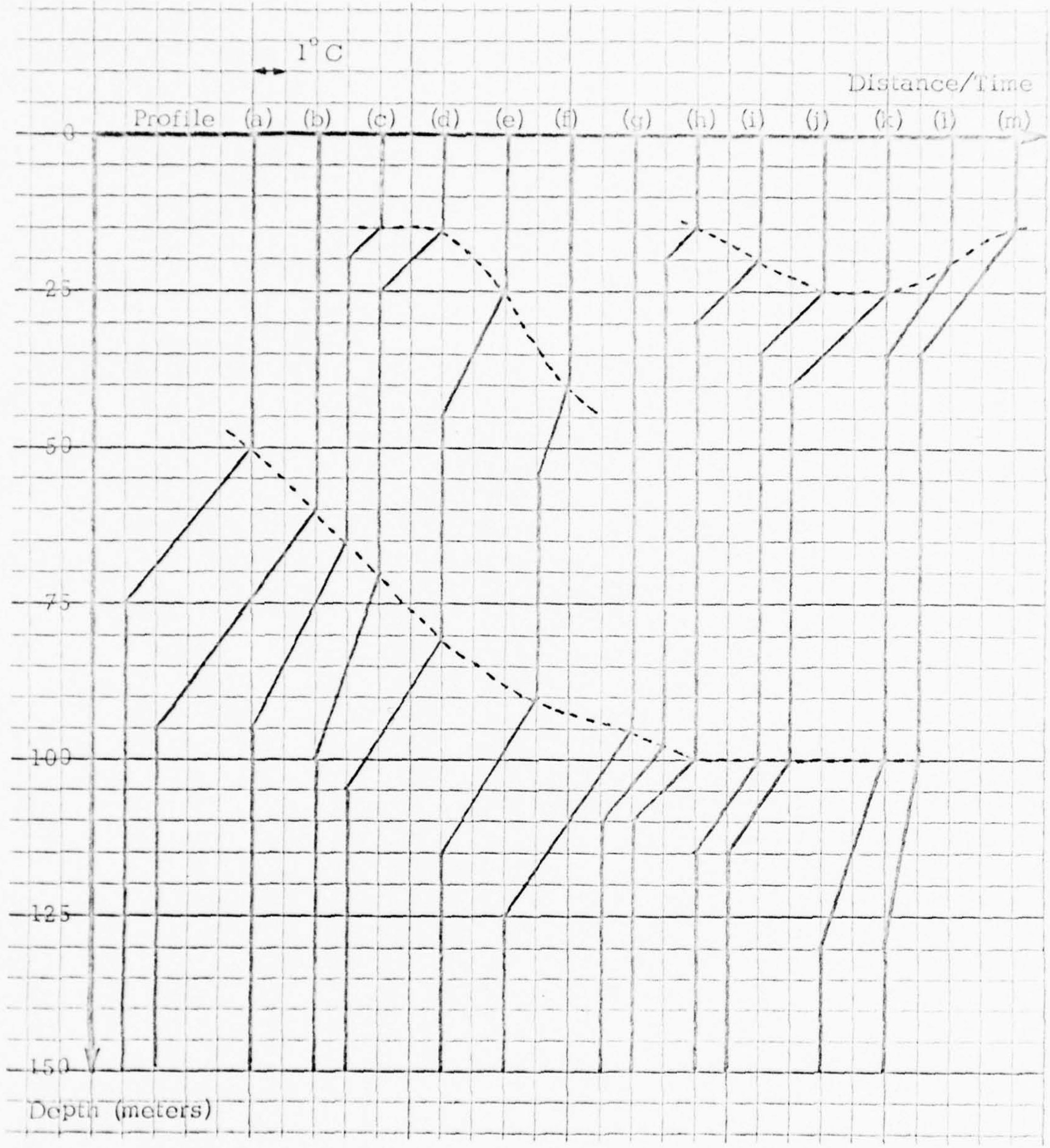

Figure 23 A series of BT reports for distance or time. Values of Φ in accordance with Eq. (77) are as follows:

	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)	(k)	(l)	(m)
Φ (upper)	-	-	1.0	2.0	2.0	1.0	-	1.0	2.0	2.0	3.0	2.0	3.0
Φ (lower)	4.0	3.6	2.5	1.7	3.0	3.0	3.3	2.0	2.0	2.0	2.0	1.7	0.8

(Internal waves and other sub-scale variances are not included in this idealized schematic.)

is for a particular grid point in space with the separation between profiles being equal to the analysis increment). Values of $\bar{\Phi}$ also are shown. To simplify matters each profile provides only one or two PLD candidates. The problem is to define an algorithm for choosing from each profile that PLD candidate which it is most appropriate to represent by the fine-resolution floating-level parameters.

For profiles (a) and (b) the choice is obvious. For profiles (c) through (f) the choice is between an upper PLD and a lower PLD where, apart from profile (d), the lower PLD is stronger (in terms of $\bar{\Phi}$) than the upper. If the choice is based purely on the magnitude of $\bar{\Phi}$ then, depending on whether the abscissa represents time or distance, the selected PLD would jump from 65 meters to 15 meters then to 80 meters over two grid-spacings or over two analysis increments respectively. Such variabilities in space and time are not realistic. For example mid-way between the two points in space or time with PLDs of 15 and 80 meters respectively, an interpolation scheme would indicate a PLD of about 48 meters; none of the profiles indicate that this is a representative value. Clearly the choice of PLD cannot be based on the magnitude of $\bar{\Phi}$. Space-and-time continuity in the object scale of resolution must be provided.

Considering now profiles (g) through (m), the profiles are intended to show an initially weak but intensifying thermocline overlying an initially strong but eroding thermocline. Such occurrences are common in many areas with the onset of spring. The problem is obvious--when (or where) to transfer the fine-resolution set of parameters available for representing the chosen PLD from the lower PLD to the upper PLD.

The algorithm for PLD selection encompasses the following steps for analysis time τ:

a. All profiles observed during a specified number of days prior to analysis time[1] are inspected. If the maximum depth reached by any profile does not exceed the local value of PLD resulting from the analysis at time $\tau-1$ by at least 10 meters then that profile does not enter into PLD selection for analysis time τ.

b. All PLD candidates (up to 9 per profile) from the remaining profiles are used in a preliminary 3-cycle FIB analysis (see Section 4). It will be appreciated that this is an analysis of a depth field rather than an analysis of a temperature or temperature-gradient field as for parameter numbers 9 through 26. However, although adjustable constants and tuning parameters must be set to reflect the characteristics of the PLD parameter, the concepts underlying the analysis process are the same. The PIFS for this preliminary analysis are derived from the PLD analysis-resultant at time $\tau-1$ in the usual manner including an adjustment toward the predicted climatological value for time τ.

c. Having completed this preliminary PLD analysis, all PLD candidates from the profiles to be used in the analysis for time τ are inspected and one candidate set from each profile is selected. The set chosen from each profile is that whose PLD value is closest to the value provided by the preliminary analysis at the same location. Selection is independent of the magnitude of Φ.

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[1] Normally 5 days of data are used. For some applications the data period may be varied, depending on the expected degree of PLD persistence. If the User does not specify the maximum age of data to be used the analysis program defaults to 5 days.

d. Using only these selected values--i.e., one per profile--a second 3-cycle FIB analysis of PLD is carried out. The PIFS for the analysis are the same as those used in (b) above. This second analysis provides the analyzed field of PLD, parameter no. 1.

This algorithm ensures reasonable continuity in space and time but also allows transfer of the set of parameters used for fine-resolution of thermal-structure in the vicinity of the PLD from one "PLD regime" to another. This transfer occurs when (and/or where) the accumulating information in the preliminary PLD analysis forces a more-appropriate choice of PLD from the candidates provided by each profile.

Determining the profiles which provide PLD candidates for the analysis at time τ ((c) and (d) above) requires some explanation. Clearly the period covered by the preliminary analysis (typically 5 days) may be greater than, equal to, or less than the interval between synoptic EOTS analyses. If the period used in the preliminary analysis for time τ is greater than or equal to the period since the previous synoptic analysis for $\tau-1$, then profiles from which single PLD candidates are chosen for the second analysis for time τ are those profiles received since the previous analysis whose times of observation do not precede that of the earliest possible profile used in the preliminary analysis. This ensures that all profiles used in the preliminary analysis also are used in the final analysis for time τ unless they have been used in the final analysis for $\tau-1$. (Profiles should not be used directly in more than one final analysis.) If the period used in the preliminary analysis is less than the period since the previous synoptic analysis, then profiles from which single PLD values are chosen for the final synoptic analysis at time τ are those profiles received during the period covered by the preliminary PLD analysis, irrespective of their times of observation. In all cases the maximum depth reached by the BT must exceed the local value of PLD resulting from the

τ-1 synoptic analysis by at least 10 meters; BTs not satisfying this requirement are not utilized by the τ analysis although, of course, they may enter into the τ+1 analysis if the local PLD resultant field for time τ decreases sufficiently in depth.

Having completed the second PLD analysis, parameter no. 2, the temperature at the PLD, is analyzed. The temperatures used in the analysis are those corresponding to the single PLD candidate selected from each BT profile. Analysis of the four gradient-fields, parameter nos. 3 through 6 and the two curvature fields, parameter nos. 7 and 8 (see Fig. 22 and Table 1) then follows. As in the case of the PLD temperature, these floating parameter values are drawn from that BT-diagnosed set corresponding to the selected PLD candidate--in effect the floating structure is moved to the analyzed local PLD level, thus removing sub-scale internal waves to some degree. Note that parameters 4 and 5 are measures of the mean gradient over two 12.5-meter intervals immediately below the PLD; these parameters provide extra resolution of the shape of the profile in a region of particular significance to sound propagation.

5.7 Vertical Blending

To summarize the analysis procedure so far, we have produced analyzed fields of the twenty-six ocean thermal-structure parameters for analysis time τ. Of these, eighteen are fixed-level parameters and eight are floating-level parameters which provide fine-resolution of the PLD.

Consider any arbitrary grid-point ℓ, m, τ. Values for all twenty-six parameters at this location in space and time are provided by the analyzed fields. Essentially the process of vertical blending recombines these thermal-structure parameters to provide the best available estimate of the representative profile in the object-scale of resolution at ℓ, m, τ; vertical blending completes the analysis process.

As noted previously, certain thermal-structure parameters are not analyzed in the normal course of operational synoptic analysis sequences.

The "synoptic set" is made up of those parameters not marked by (c) in Table 1. Vertical blending will be described in terms of the synoptic set. The procedure for the full climatological set is similar.

Table 2 shows the weighted information fields entering into synoptic vertical blending. Depths (meters) are shown in the left-hand column. Across the top of the table, T , T' and T" refer to temperature, gradient and curvature respectively. For each type of parameter, three columns are given. The first, headed "P. No.", gives the parameter number from Fig. 2 . Values of T and T' provided directly by Analysis are denoted by "A" in the second column. Assigned weights (discussed later) are given in the third column.

The upper section of Table 2 refers to the fixed-level parameters. The first level, 0 meters, is parameter no. 9, the sea-surface temperature. The value of SST is provided by analysis (as shown by "A" in the center column under T) and the assigned weight is 10.0. The gradient over the first 25 meters, parameter no. 17, also is provided by analysis. Note that gradients are entered opposite the upper level encompassed--thus parameter no. 17, an estimate of $T_{25m} - T_{0m}$, is shown against level 0. In Table 2 a gradient entered at any level d refers to $T_{d+1} - T_d$ where d+1 is the next deeper level shown in the left-hand column.

At some levels such as 75 meters there is no analyzed field of T--see Fig. 22. Where indicated, formulation F is used to compute T . In general

$$F = T_{d-1} + T'_{d-1} \quad . \tag{78}$$

At 400 meters formulation F3 is used to form T'_{400}:

$$F3 = T'_{300} + 0.5\, C'_{600} \quad . \tag{79}$$

The definition of C' follows.

Table 2
The weighted information fields entering into synoptic vertical blending. See text for description. A similar table is used for the climatological set of thermal-structure parameters.

	T			T'			T"		
Fixed Levels	P. No.	A/F/C	Wt.	P. No.	A/F/C	Wt.	P. No.	A/F/C	Wt.
0	9	A	10.0	17	A	3.0	--	--	--
25	--	--	--	18	A	2.0	--	--	--
50	11	A	0.01	19	A	1.0	--	--	--
75	--	F	0.01	20	A	1.0	--	--	--
100	--	F	0.01	21	A	1.0	--	--	--
125	--	--	--	22	A	1.0	--	--	--
150	13	A	0.1	23	A	1.0	--	--	--
200	--	F	0.1	24	A	1.0	--	--	--
250	--	F	0.1	25	A	1.0	--	--	--
300	15	A	0.2	26	A	1.0	--	--	--
400	--	F	0.1	--	F3	0.5	--	--	--
600	--	C	0.005	--	C'	0.10	--	C" = 0	0.3
800	--	C	0.01	--	C'	0.15	--	C"	0.2
1000	--	C	0.05	--	C'	0.20	--	C"	0.2
1500	--	C	0.10	--	C'	0.25	--	C"	0.1
2000	--	C	0.20	--	C'	0.30	--	C"	0.1
2500	--	C	0.30	--	C'	0.35	--	C"	0.1
3000	--	C	0.40	--	C'	0.40	--	C"	0.1
4000	--	C	0.50	--	C'	0.45	--	C"	0.1
5000	--	C	1.0	--	--	--	--	--	--
Floating Levels (Relative to Parameter No. 1, the PLD)									
-25	--	--	--	3	A	2.0	--	--	--
0	2	A	0.01	4	A	2.0	--	F1	1.0
+ 12.5	--	--	--	5	A	1.5	--	F2	0.5
+ 25	--	--	--	6	A	1.0	--	--	--
+ 50	--	--	--	--	--	--	--	--	--

At 600 meters and below, values of T are provided by the stored fields of climatology. Values of T' are computed by formulation C':

$$C'_d = T_{d+1} - T_d \quad , \quad d \geq 600 \text{ meters} \quad . \tag{80}$$

Values of C" are given by

$$C''_d = C'_d - C'_{d-1} \quad , \quad (d \geq 800 \text{ meters}) \quad . \tag{81}$$

At 600 meters the appropriate expression would be

$$C''_{600} = C'_{600} - F3 \quad . \tag{82}$$

However, to exercise control of the curvature when passing from analyzed levels to deep levels whose parameter-values are provided by climatology, C''_{600} is set equal to zero with an associated weight of 0.3. In so-doing we are providing a contrived piece of weighted information to the effect that there is little change in gradient between (400 → 600m) and (600 → 800m). This reduces any "kinking" when combining the upper and lower levels. Since C''_{600} is set equal to zero (shown in Table 2), there is no need, in practice, to apply Eq. (82).

The second part of Table 2 refers to the floating levels. Analyzed values of parameter numbers 2 through 6 are available. Values of T" are computed as follows:

$$F1 = T'_0 - T'_{-25} \tag{83}$$

$$F2 = T'_{12.5} - T'_0 \tag{84}$$

where the subscripts refer to depths relative to the analyzed value of PLD, parameter no. 1.

The next stage is to merge the fixed and floating levels. This is achieved by replacing the fixed-level parameter values by the <u>finer-resolution</u> floating-level parameter values over the depth range covered by the floating levels (i.e., from PLD-25 to PLD+50 meters). The procedure is most easily explained by example.

Figure 24 shows the top 150 meters of the ocean at grid point ℓ , m . Column (a) shows the temperature and gradient values provided by the fixed levels. Analyzed values are represented by "○" and computed values by "O"--see Table 2. Similarly Col. (b) shows values of T , T' and T" provided by the floating levels using an assumed PLD of 55 meters. Analyzed values are represented by "▲" and computed values by "△". Column (c) shows the effect of merging the fixed and floating levels. Note that all fixed-level parameters falling within the range PLD-25m to PLD+50m are replaced by the floating levels. Values of temperature difference must be computed for the ranges 25 → 30 meters and 105 → 125 meters. A simple linear relationship is used. For example

$$T'_{25\,(\text{new})} = \frac{(\text{PLD} - 25) - 25}{50 - 25} \times T'_{25}$$

where T'_{25} is the fixed-level gradient value at 25 meters. This provides a new value for T'_{25} which is a measure of the temperature difference between 30 meters and 25 meters. The associated weight is given by

$$\left(\frac{T'_{25}}{T'_{25\,(\text{new})}}\right)^2 \times \left(\text{weight of } T'_{25} \text{ given in Table 2}\right) \quad .$$

[This is a direct application of Eq. (28).]

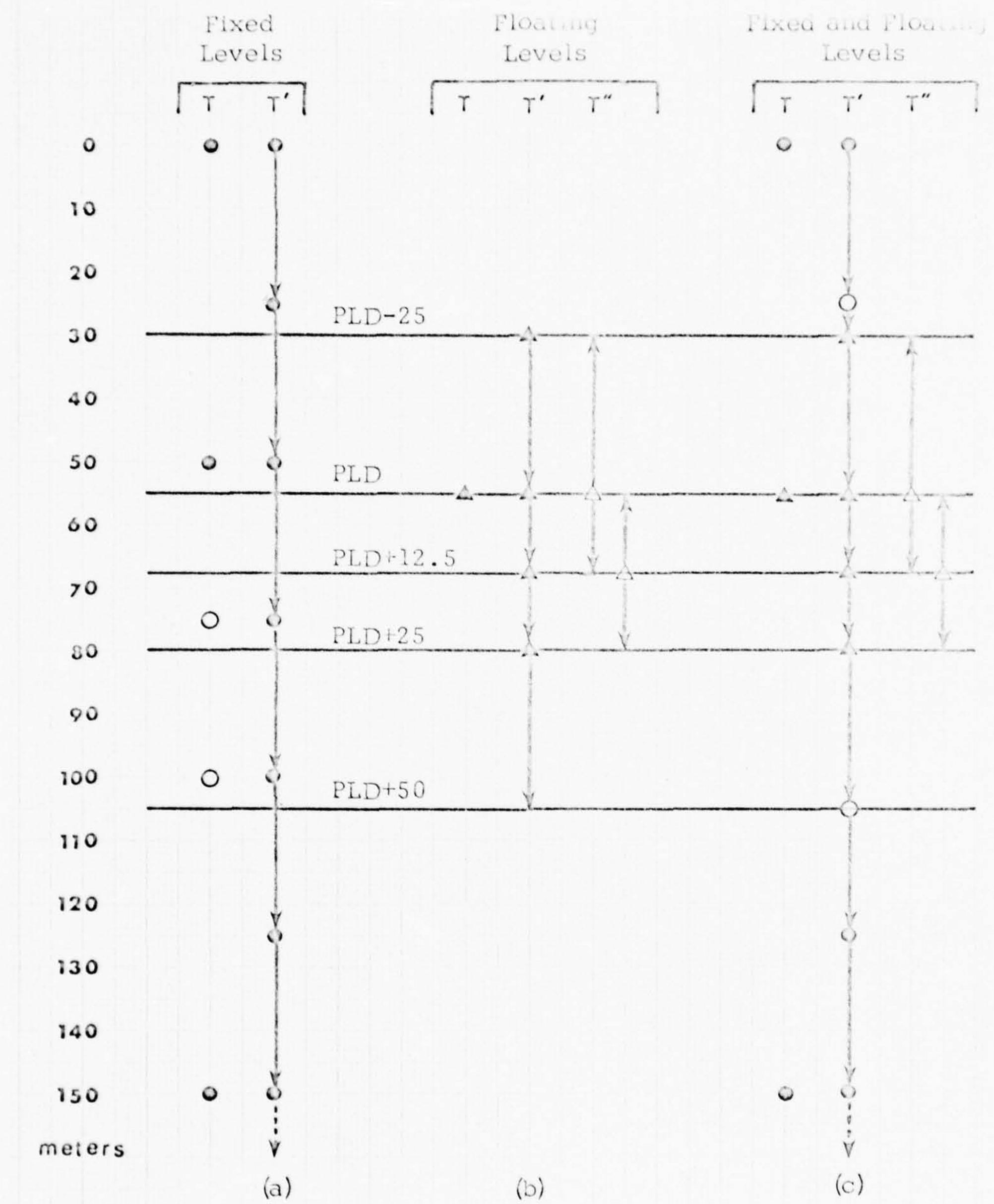

Figure 24 Fixed-level parameters are shown in (a). Floating-level parameters are shown in (b) for an assumed PLD of 55 meters. The effect of combining the two parameter-sets is given in (c). ● and ▲ represent analyzed values; ○ and △ represent computed values. Note that, following combination of the two parameter sets, gradient-component values must be computed for 25 → 30m and for 105 → 125m.

A similar calculation provides the new temperature difference between 125 meters and 105 meters and the associated weight. In general, no matter where the PLD falls in relation to the fixed levels, no more than two such calculations are required to adjust the "interrupted" fixed-level gradient values and associated weights. If PLD < 25 meters then T'_0 is multiplied by PLD/25.

Let d be any level in the ocean;

T_d be the temperature at that level. (T_d is provided either by analysis of the ℓ, m field or by formulation F);

A_d be the weight associated with T_d;

b_d be the first-difference temperature referred to level d where $b_d = T_{d+1} - T_d$;

B_d be the weight associated with b_d;

c_d be the second-difference temperature referred to level d where $c_d = T_{d+1} + T_{d-1} - 2T_d$; and

C_d be the weight associated with c_d.

During the vertical blending process, information from all levels is to be spread and assimilated, and the final result will be the "best estimate" of each parameter that can be provided by the total information available. This best estimate will be denoted by a double-star superscript since estimates of the thermal-structure parameters for vertical blending are derived from fields which have been analyzed horizontally (and include information accrued along the time axis). Thus T^{**}_d denotes the final blended resultant of the temperature at level d of associated weight A^{**}_d, and similarly for the other parameters and their associated weights. We regard the difference between any parameter-value before and after vertical blending as a disparity, and to produce the "best-estimate" profile we must minimize the total weighted squared disparities over all parameters (i.e., T, b, c) for the whole profile (i.e., for all values of d).

To do this we set up the appropriate error functional E where

$$E = \sum_d \left\{ A_d\left(T_d^{**} - T_d\right)^2 + B_d\left(T_{d+1}^{**} - T_d^{**} - b_d\right)^2 + C_d\left(T_{d+1}^{**} + T_{d-1}^{**} - 2T_d^{**} - c_d\right)^2 \right\} . \qquad (85)$$

To minimize we set

$$\frac{\partial E}{\partial T_d^{**}} = 0 .$$

Noting from Eq. (85) that there are terms involving T_d^{**} associated with the levels d-1, d, and d+1,

$$\begin{aligned}
\frac{\partial E}{\partial T_d^{**}} &= A_d\left(T_d^{**} - T_d\right) \\
&\quad + B_{d-1}\left(T_d^{**} - T_{d-1}^{**} - b_{d-1}\right) \\
&\quad - B_d\left(T_{d+1}^{**} - T_d^{**} - b_d\right) \\
&\quad + C_{d-1}\left(T_d^{**} + T_{d-2}^{**} - 2T_{d-1}^{**} - c_{d-1}\right) \\
&\quad - 2C_d\left(T_{d+1}^{**} + T_{d-1}^{**} - 2T_d^{**} - c_d\right) \\
&\quad + 2C_{d+1}\left(T_{d+2}^{**} + T_d^{**} - 2T_{d+1}^{**} - c_{d+1}\right) \\
&= 0
\end{aligned} \qquad (86)$$

Rearrangement gives:

$$
\begin{aligned}
S_d \cdot T_d^{**} = \; & A_d \cdot T_d \\
& + B_d \cdot \left(T_{d+1}^{**} - b_d\right) \\
& + B_{d-1} \cdot \left(T_{d-1}^{**} + b_{d-1}\right) \\
& + 4C_d \cdot \frac{1}{2}\left(T_{d+1}^{**} + T_{d-1}^{**} - c_d\right) \\
& + C_{d-1} \cdot \left(2T_{d-1}^{**} - T_{d-2}^{**} + c_{d-1}\right) \\
& + C_{d+1} \cdot \left(2T_{d+1}^{**} - T_{d+2}^{**} + c_{d+1}\right) \qquad (87)
\end{aligned}
$$

where $S_d \equiv \left(A_d + B_d + B_{d-1} + 4C_d + C_{d-1} + C_{d+1}\right)$.

There is one such equation for every d , forming a system of simultaneous equations.

The set of blending equations may be expressed in matrix notation. To deduce matrix elements for the level d , Eq. (87) may be rearranged as follows:

$$
\begin{aligned}
\left(C_{d-1}\right)T_{d-2}^{**} & + \left(-B_{d-1} - 2C_d - 2C_{d-1}\right)T_{d-1}^{**} + \left(S_d\right)T_d^{**} \\
& + \left(-B_d - 2C_d - 2C_{d+1}\right)T_{d+1}^{**} + \left(C_{d+1}\right)T_{d+2}^{**} \\
& = A_d T_d - B_d b_d + B_{d-1}\, b_{d-1} - 2C_d c_d \\
& + C_{d-1}\, c_{d-1} + C_{d+1}\, c_{d+1} \quad . \qquad (88)
\end{aligned}
$$

In matrix notation the full set of equations is given by

$$\underset{\approx}{M}\,\underset{\sim}{T}^{**} \;=\; \underset{\sim}{F} \tag{89}$$

where $\underset{\approx}{M}$ is pentadiagonal, symmetric and positive definite.

This set of equations is solved most expediently by the procedure known in the literature as "forward elimination, backward substitution".

Since $\underset{\approx}{M}$ is symmetric and positive definite it can be decomposed uniquely into $\underset{\approx}{L}\,\underset{\approx}{U}$ where $\underset{\approx}{L} = \underset{\approx}{U}^T$ is a lower triangular matrix with positive diagonal elements. Equation (89) becomes

$$\underset{\approx}{L}\,\underset{\approx}{U}\,\underset{\sim}{T}^{**} \;=\; \underset{\sim}{F} \quad . \tag{90}$$

The solution for $\underset{\sim}{T}^{**}$ can then be obtained by first solving

$$\underset{\approx}{L}\,\underset{\sim}{Z} \;=\; \underset{\sim}{F} \tag{91}$$

for $\underset{\sim}{Z}$ and then solving

$$\underset{\approx}{U}\,\underset{\sim}{T}^{**} \;=\; \underset{\sim}{Z} \tag{92}$$

for $\underset{\sim}{T}^{**}$. The elements of $\underset{\sim}{Z}$ are easily obtained from Eq. (91) in the order $Z_1, Z_2, Z_3 \ldots$ since the first equation involves only Z_1, the second equation involves Z_1 and Z_2, and so on. Then the elements of $\underset{\sim}{T}^{**}$ are obtained similarly from Eq. (92) in the reverse order, i.e., $\ldots T_3^{**}$, T_2^{**}, T_1^{**}. These two steps--obtaining the intermediate solution, $\underset{\sim}{Z}$, from Eq. (91) and obtaining the *final solution*, T^{**}, from Eq. (92) are referred to as forward elimination and backward substitution respectively.

To explain the procedure in more detail, the relationship $\underset{\approx}{M} = \underset{\approx}{L}\,\underset{\approx}{U}$ may be expanded as follows:

idealized schematic.) ... are not included in this

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$$
\underset{\approx}{M} = \begin{bmatrix}
a_1 & b_2 & c_3 & & & & & \\
b_2 & a_2 & b_3 & c_4 & & & & \\
c_3 & b_3 & a_3 & b_4 & c_5 & & & \\
 & c_4 & b_4 & a_4 & b_5 & c_6 & & \\
 & & c_5 & b_5 & a_5 & b_6 & c_7 & \\
 & & & \cdot & \cdot & \cdot & \cdot & \cdot \\
 & & & & \cdot & \cdot & \cdot & \cdot \\
 & & & & & \cdot & \cdot & \cdot
\end{bmatrix}
$$

$$
= \underset{\approx}{L}\,\underset{\approx}{U} = \begin{bmatrix}
d_1 & & & & & & & \\
e_2 & d_2 & & & & & & \\
f_3 & e_3 & d_3 & & & & & \\
 & f_4 & e_4 & d_4 & & & & \\
 & & f_5 & e_5 & d_5 & & & \\
 & & & \cdot & \cdot & \cdot & & \\
 & & & & \cdot & \cdot & \cdot & \\
 & & & & & \cdot & \cdot & \cdot
\end{bmatrix}
\begin{bmatrix}
d_1 & e_2 & f_3 & & & & & \\
 & d_2 & e_3 & f_4 & & & & \\
 & & d_3 & e_4 & f_5 & & & \\
 & & & d_4 & e_5 & f_6 & & \\
 & & & & d_5 & e_6 & f_7 & \\
 & & & & & \cdot & \cdot & \cdot \\
 & & & & & & \cdot & \cdot \\
 & & & & & & & \cdot
\end{bmatrix}
$$

$$
= \begin{bmatrix}
d_1^2 & d_1e_2 & d_1f_3 & & & & & \\
e_2d_1 & e_2^2+d_2^2 & e_2f_3+d_2e_3 & d_2f_4 & & & & \\
f_3d_1 & f_3e_2+e_3d_2 & f_3^2+e_3^2+d_3^2 & e_3f_4+d_3e_4 & d_3f_5 & & & \\
 & f_4d_2 & f_4e_3+e_4d_3 & f_4^2+e_4^2+d_4^2 & e_4f_5+d_4e_5 & d_4f_5 & & \\
 & & \cdot & \cdot & \cdot & \cdot & \cdot & \\
 & & & \cdot & \cdot & \cdot & \cdot & \cdot \\
 & & & & \cdot & \cdot & \cdot & \cdot \\
 & & & & & \cdot & \cdot & \cdot
\end{bmatrix} \tag{93}
$$

Equating the individual elements of $\underset{\approx}{M}$ to the corresponding elements of the product $\underset{\approx}{L}\,\underset{\approx}{U}$ and solving for d, e and f the following relationships result:

$$
\begin{aligned}
d_1 &= (a_1)^{1/2} \\
e_2 &= b_2/d_1 \\
f_3 &= c_3/d_1 \\
d_2 &= (a_2 - e_2^2)^{1/2} \\
e_3 &= (b_3 - f_3e_2)/d_2 \\
f_4 &= c_4/d_2 \\
d_3 &= (a_3 - e_3^2 - f_3^2)^{1/2} \\
&\quad \text{etc.}
\end{aligned}
$$

Having determined the elements of $\underset{\approx}{L}$ and $\underset{\approx}{U}$ we can employ forward elimination to solve for $\underset{\sim}{Z}$ in Eq. (91):

Initialize-- $Z_1 = F_1/d_1$

$$Z_2 = \left(F_2 - e_2 Z_1\right)/d_2$$

Iterate-- $n = 3 \rightarrow N$

$$Z_n = \left(F_n - e_n Z_{n-1} - f_n Z_{n-2}\right)/d_n \quad . \qquad (93)$$

Backward substitution then follows to solve Eq. (92) for $\underset{\sim}{T}^{**}$:

Initialize-- $T^{**}_N = Z_N/d_N$

$$T^{**}_{N-1} = \left(Z_{N-1} - e_{N-1} Z_N\right)/d_{N-1}$$

Iterate-- $n = \left(N-2\right) \rightarrow 1$

$$T^{**}_n = \left(Z_n - e_n Z_{n+1} - f_n Z_{n+2}\right)/d_n \quad . \qquad (94)$$

Vertical blending is carried out at every grid point of the horizontal analysis array. The initial values of thermal-structure parameters contained in the horizontally-analyzed fields are replaced by the corresponding values resulting from vertical blending. These re-written fields represent the final analyzed product and are used to derive PIFS for the next analysis in the manner previously described. A series of such analyses provides a 4-dimensional analysis of ocean thermal structure.

The information-partitioning weights utilized in vertical blending have to be specified. By appropriate assignment of these weights the relative information contribution provided by any one or any sub-set of the thermal-structure parameters can be controlled. For example if gradients are emphasized then conflict between, say, an analyzed temperature at a certain level and an estimate of the temperature at the same level provided by information carried to that level by way of gradient estimates will be resolved by a compromise which more closely agrees with the ambient gradient information. The shape of the profile will tend to be preserved rather than the actual temperature. Conversely, by a different specification of weights, the relative significance of temperature may be emphasized although, of course, the shape will be modified. (Note that after assembly, the example given in Sections 3.2.6 and 3.4.2 is a simple exercise in vertical blending. Some appreciation of the complex interplay between the weighted contributions of temperature and first-differences of temperature may be gained by reworking the example with different assigned sets of weights for T_m and b_m.)

The assigned weights for EOTS vertical blending must be specified so that the resultant profile provides the best estimate of the local representative profile; this best estimate must take into account those features of the profile which are of greatest significance to the objective of the overall analysis process. In EOTS these features are those which determine sound propagation[1]--gradients and curvatures. The set of weights specified in Table 2 is the result of much careful thought and experimentation.

At a depth of zero meters the profile at any point effectively is "anchored" to the SST analysis by the assigned weight of 10.0. At great depths the profile is anchored to climatological values. Between these levels the significance of gradient information generally is emphasized.

SST information is propagated downwards by way of the first-difference weights. In near-surface waters the SST analysis contributes

---

[1] Examples of vertical and horizontal cross-sections of sound speed produced by the EOTS analysis system are given in Section 7.

significantly to the vertically-blended resultant temperature. At greater depths the influence of SST becomes less marked especially in transiting the floating structure. Thus if the SST analysis showed, for the first time, the presence of a warm gyre which was not detected (resolved) by the initially-available sub-surface information then, following vertical blending, this feature would be spread downwards. The gyre would be evident as a sub-surface feature even though no direct sub-surface data was available to confirm its sub-surface existence. In other words the spreading process infers, realistically, that the feature revealed solely by the SST analysis actually has vertical extent to a degree controlled by the vertical spreading weights. The surface and near-surface levels now contain information showing the existence of a 3-dimensional gyre and this information is carried along the time axis by way of the PIFS to the next analysis.

With regard to the PLD, the temperature at this level is assigned a low weight compared with the associated gradients and curvature. The shape of the profile in the vicinity of the PLD is emphasized.

As the transition from analyzed to climatological values is approached the T' weight tapers to 0.5 for T'_{400} (Eq. (79)) then drops to 0.10 for T'_{600}. At the same time T''_{600} is set to zero with a weight of 0.3. Clearly the upper part of the climatologically-determined profile is synoptically modified to provide a smooth and realistic transition from upper to lower levels. Note that the climatological values are modified by the downward passage of information; the upper synoptically-analyzed levels are not significantly changed by the upward passage of information because of the assigned weight distribution with depth. The synoptic influence dies away with increasing depth. At great depths the profile will conform to climatology--the best information available. The upper climatological levels now contain synoptic information and therefore are a better estimate of the representative profile.

Table 3 shows vertical blending at a point in the deep ocean (34.3° N; 66.0° W) in the vicinity of the Gulf Stream. The first table (NHEM) is based on a 63x63 analysis grid, polar stereographic projection, covering the whole of the Northern Hemisphere. The second table (GLFS) is provided by the Gulf Stream regional analysis which utilizes a grid-spacing of one-eighth that of the hemispheric analysis. (The SST analyses given in Section 4.13 utilized the same analysis grids.) Because of operational scheduling there is a time difference of 12 hours between the two analyses--the NHEM analysis is for 00Z, 6 June 1979, the GLFS analysis being 12 hours earlier. The time difference is not significant to this discussion.

The second line of each table heading provides the location of the profile in analysis-grid coordinates, and the corresponding latitude and longitude. Locations are designated "open ocean" (as in the tables), "coastal" or "land" as appropriate. If coastal, the number of associated links is given.

The third line of the heading provides depth information--PLD and bottom depth.

For each depth (Z meters), columns A, B and C show the weights associated with T, M(≡ T') and O(≡ T") respectively[1]--see Table 2. Column R shows the blended resultant temperature. The upper 1000 meters of each profile have been plotted (by hand)[2] in Fig. 25. The climatological values of T and M (i.e., for depths ≥ 600 meters) are provided from common climatological fields, the GLFS climatologies being "zoomed" from

---

[1]The weight of 266.6 at 73 meters for GLFS may seem unlikely at first sight. This is realistic and due to the fact that a gradient over only 1.5 meters is required to couple (PLD+50) meters to the fixed level at 75 meters. (See calculation of associated weight given on page 164.)

[2]A machine-plot capability is available.

Table 3
Vertical blending at a point in the deep ocean
for two different scales of horizontal analysis resolution.
See text for description.

NHEM 79060600
I=35 J=15 34.3N 66.0W OPEN OCEAN
PLD=31.1M BOTTOM DEPTH=5013.4M

Z	A	B	C	T	M	O	R
0.	10.0	50.6	0.0	24.5	-.1	0.0	24.5
6.	0.0	2.0	0.0	0.0	-.3	0.0	24.5
31.	.0	2.0	1.0	24.2	-1.1	-.8	24.2
44.	0.0	1.5	.5	0.0	-.8	.3	23.1
56.	0.0	1.0	0.0	0.0	-1.2	0.0	22.3
81.	0.0	1.7	0.0	0.0	-.4	0.0	21.1
100.	.0	1.0	0.0	20.8	-.5	0.0	20.7
125.	0.0	1.0	0.0	0.0	-.4	0.0	20.2
150.	.1	1.0	0.0	19.8	-.7	0.0	19.7
200.	.1	1.0	0.0	19.1	-.4	0.0	19.1
250.	.1	1.0	0.0	18.7	-.5	0.0	18.6
300.	.2	1.0	0.0	18.2	-.8	0.0	18.1
400.	.1	.5	0.0	17.4	-2.7	0.0	17.4
600.	.0	.1	.3	15.4	-3.9	0.0	14.4
800.	.0	.2	.2	11.5	-3.9	0.0	11.1
1000.	.1	.2	.2	7.6	-3.3	0.0	7.7
1500.	.1	.3	.1	4.3	-.6	0.0	4.8
2000.	.2	.3	.1	3.7	-.4	0.0	3.7
2500.	.3	.4	.1	3.3	-.4	0.0	3.3
3000.	.4	.4	.1	2.9	-.6	0.0	2.8
4000.	.[illegible]	.5	.1	2.3	-.1	0.0	2.3
5000.	1.0	0.0	0.0	2.2	0.0	0.0	2.2

Table 3 (continued)

GLFS 79060512
I=31 J=31 34.3N 66.0W OPEN OCEAN
PLD=23.5M BOTTOM DEPTH=5013.4M

Z	A	B	C	T	M	O	R
0.	10.0	2.3	0.0	23.7	-.2	0.0	23.7
23.	.0	2.0	1.0	23.4	-.9	-.7	23.5
36.	0.0	1.5	.5	0.0	-.7	.2	22.5
48.	0.0	1.0	0.0	0.0	-1.0	0.0	21.8
73.	0.0	266.6	0.0	0.0	-.0	0.0	20.8
75.	.0	1.0	0.0	20.7	-.7	0.0	20.8
100.	.0	1.0	0.0	20.1	-.4	0.0	20.1
125.	0.0	1.0	0.0	0.0	-.4	0.0	19.7
150.	.1	1.0	0.0	19.2	-.7	0.0	19.2
200.	.1	1.0	0.0	18.6	-.4	0.0	18.6
250.	.1	1.0	0.0	18.2	-.3	0.0	18.2
300.	.2	1.0	0.0	17.8	-.7	0.0	17.8
400.	.1	.5	0.0	17.1	-2.7	0.0	17.1
600.	.0	.1	.3	15.4	-3.9	0.0	14.2
800.	.0	.2	.2	11.5	-3.9	0.0	10.9
1000.	.1	.2	.2	7.6	-3.3	0.0	7.6
1500.	.1	.3	.1	4.3	-.6	0.0	4.7
2000.	.2	.3	.1	3.7	-.4	0.0	3.7
2500.	.3	.4	.1	3.3	-.4	0.0	3.3
3000.	.4	.4	.1	2.9	-.6	0.0	2.8
4000.	.5	.5	.1	2.3	-.1	0.0	2.3
5000.	1.0	0.0	0.0	2.2	0.0	0.0	2.2

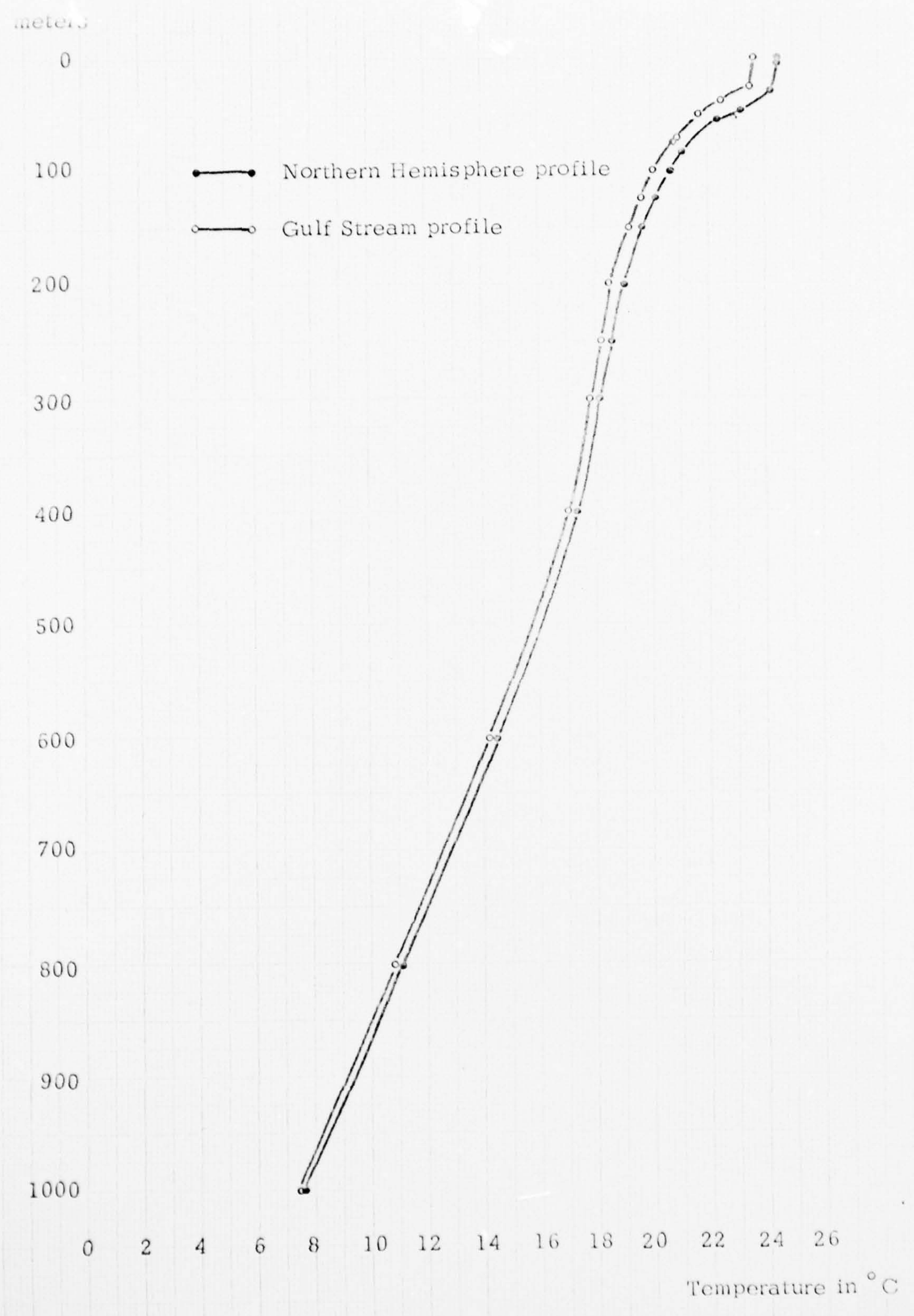

Figure 25 The resultant profiles at a point in the deep ocean for two different scales of horizontal analysis resolution.

the available NHEM climatologies. Regional climatologies should be used but, as yet, these have not been generated.

The effect on the resultant profile of the two different scales of horizontal analysis resolution may be noted--both resultants are best estimates of the representative profile provided by the available information, the differences being due to the different object scales of analysis resolution.

Note how synoptic information is propagated downward to modify the upper climatological levels, and how the shape of the profile is preserved in these regions.

(As currently configured, the vertical blending process extends to 5000 meters irrespective of bottom depth. In due course the EOTS analysis system is to be modified to take account of local bathymetry.)

6. THE EXPANDED OCEAN THERMAL-STRUCTURE ANALYSIS SYSTEM

6.1 Introduction

Previous Sections have described how the concepts and formulations of the FIB analysis methodology may be applied to produce horizontally-analyzed distributions of the ocean thermal-structure parameters and how these independently-analyzed parameters then may be blended vertically to produce the resultant temperature at any point in space and time. This Section, based very largely on the more detailed account contained in the EOTS Users Manual, provides a basic outline of the EOTS analysis system which carries out the overall analysis process.

The EOTS system exists in two configurations--the "off-line" and the operational (or "on-line") mode. The on-line mode is used for the production of synoptic sequences in a real-time operational context. The off-line mode permits non-routine tasks to be performed without having to modify or provide an input to the on-line mode. The off-line version is used for R&D; tuning and evaluation; setting up new areas for subsequent transfer to the operational configuration; generating climatological fields; and for providing a rapid response to operational requests for products not available on a routine basis.

6.2 Summary of System Capabilities

a. Vertical Resolution The EOTS analysis system uses up to 26 parameters to represent the temperature profile at any point in the ocean from the surface to 400 meters (Fig. 22). Eight of these parameters are devoted to a high-resolution representation of the PLD. Below 400 meters (to a depth of 5000 meters), climatological values are used which are synoptically modified during the course of vertical blending. Any number of parameters may be specified for analysis. Thus specifying parameter no. 9 would result in an SST analysis. Normally either one, twenty (the synoptic set) or twenty-six (the climatological set) are specified.

b. Horizontal Resolution EOTS utilizes an analysis grid, polar stereographic projection, which can be specified with regard to location, orientation, size (LxM grid points) and grid-point spacing.

c. Temporal Resolution Any analysis frequency may be specified. In practice the analysis interval is lower bounded by the frequency (and quantity) of synoptic observations. For example, Northern Hemisphere EOTS analyses on a 63x63 analysis grid are carried out every 24 hours.

d. Analyses produced by a given Run Given the necessary inputs and appropriate specifications, the EOTS system automatically will perform analyses for any number of regions in a single run. The analysis-grid dimensions may be specified for each individual region, as may be the parameters to be analyzed.

e. Data Bases All necessary fields and data bases are referenced, supported and/or kept current by the EOTS system. The latest sea-ice information is included automatically in non-climatological analyses. No User intervention is necessary (apart from providing initial analysis specifications). For example, given a new area for analysis, the EOTS system will automatically access the land/sea table, compute the appropriate SCD field, and store this for future use as required.

f. Bogussing A bogus capability is available for subjective influence. Actual or contrived single-level or full-profile reports may be forced in or out. In addition, SST gradients subjectively interpreted from satellite imagery may be imposed on the analysis. (This capability could be applied to the assimilation of weighted-gradient estimates into the analysis of other thermal-structure parameters.)

g. Output A wide variety of plotted and printed information can be output including, for example, plots of any analyzed thermal-structure parameter; the land background is included in such plots. Cross-sections of temperature and sound speed may also be specified and plotted. (Samples of EOTS products in this publication include Table 3 and Figs. 18, 19, 20, 28, 29, 30, 31, 32, 33, 34 and 35.)

6.3 Information Flow and Processing in the EOTS Analysis System

Figure 26 shows information flow through the EOTS analysis system. This may be compared with Fig. 15. A brief description of the function of each module is given below; the concepts underlying these functions and the formulations which enable the functions to be realized are contained in Sections 2 through 5.

INITSCD

Tests to see whether or not SCD field generation is required (includes User option). If not, then control passes to OTSRATS.

MAKMAP

Computes land/sea distribution for a new area.

OBJSCD

Determines the objective SCD field from the land/sea distribution. Provides output needed to allow subjective decisions to be made.

SUBSCD

Updates OBJSCD using subjective decisions. Saves final SCD field for use by SCDSET.

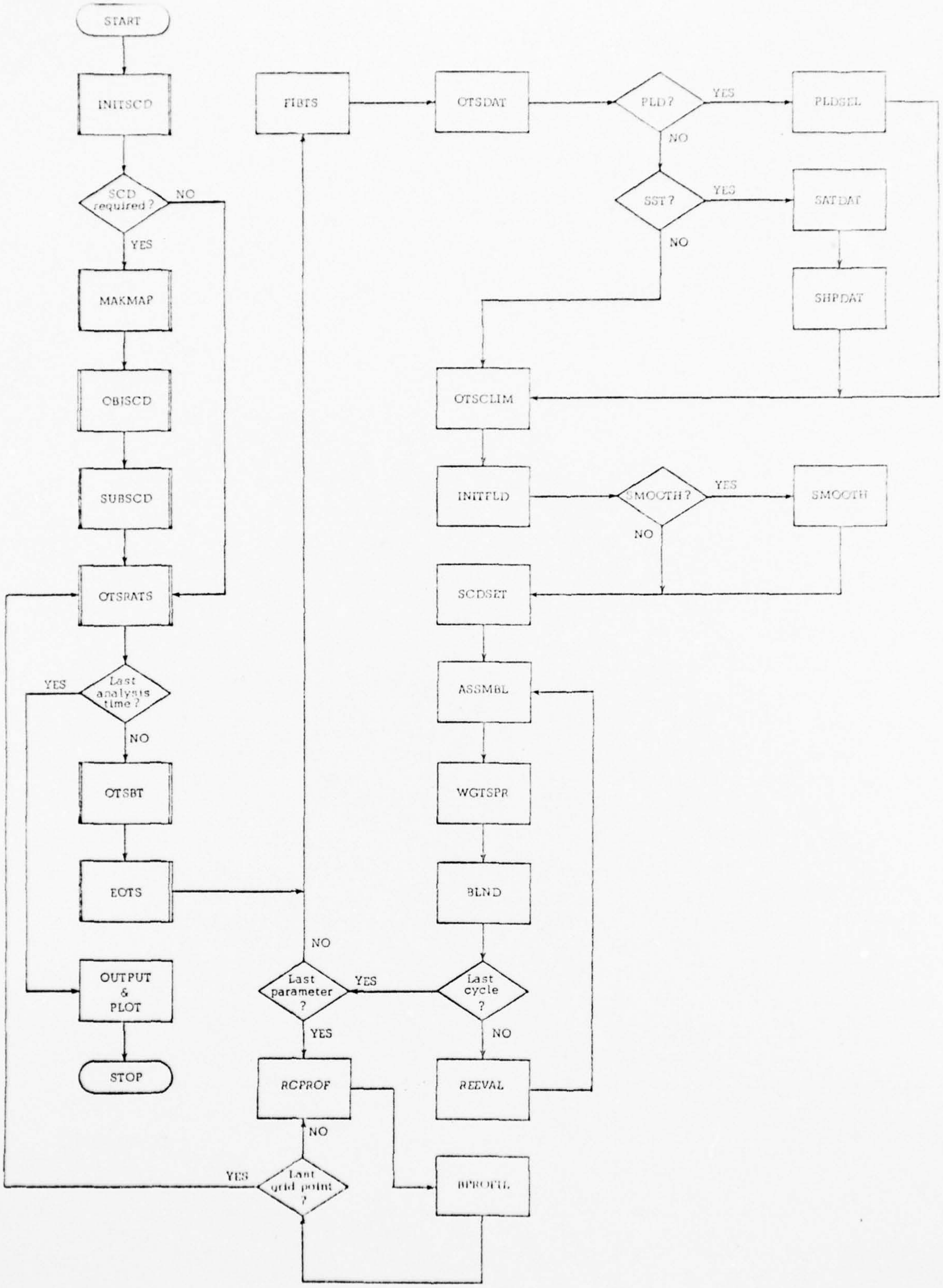

Figure 26 Information flow through the EOTS analysis system.

OTSRATS

In any operational run of EOTS, analyses for several areas may be required and, in general, these requirements vary from run to run. OTSRATS is a scheduler which determines and controls the sequence of analyses produced in a given run. If any necessary PIFS are missing, OTSRATS computes the appropriate tape number and invites the operator to restore the missing fields. A "restart from climatology" option is available.

OTSBT

BT reports are checked and corrected for duplicate levels and obvious single-level reporting errors. The BT then is converted to units of meters and $^{\circ}$C and the temperature linearly interpolated at every 2.5 meters from the surface to 450 meters (or to the bottom of the profile if the BT did not reach this depth). All fixed and floating level ocean thermal-structure parameters provided by each profile (including up to nine PLD candidates together with their associated parameters) are computed and stored in a variable length format. SST reports from ships and satellites are saved in much shorter formats.

EOTS

EOTS drives the analysis of thermal-structure parameters required for each area using relevant observational data, previously analyzed fields and climatological information. As can be seen from Fig. 26, the analysis is performed, one parameter at a time, by entering an information-processing loop which commences with FIBTS.

FIBTS

This module drives the analysis of individual thermal-structure parameters, one at a time. (FIBTS and its associated modules form the subject of Section 4.)

OTSDAT

Only part of the information parameterized by OTSBT is needed for the analysis of a particular thermal-structure parameter. OTSDAT selects the relevant information provided by current observations. Two parameters--PLD and SST--are treated as special cases.

PLDSEL

This module carries out the selection of one PLD candidate from each profile in the manner described in Section 5.6. (This process, involving a FIB 3-cycle analysis, involves all modules from OTSCLIM to REEVAL.)

SATDAT

↓

SHPDAT

For reasons previously explained (Section 4.5.3), satellite-derived and synoptic marine SST data are not assembled with equal weights. These two modules convert the SST data to an internal format which includes a class number indicating source. The SST data are grouped together in the data list according to their source for later assembly.

OTSCLIM

Adjusts the assembly field and the analyzed resultant field from the previous analysis to provide P_0 and P_0^{**}. Having no prediction model in concurrent operation, the present EOTS system bases the adjustment on climatological trend. See Eqs. (58) and (66).

INITFLD

Computes, from P_0^{**}, the shaping and spreading PIFS and stores them together with their associated weight field.

SMOOTH

To improve information control, optional smoothing processes may be applied to the shaping and spreading PIFS. If smoothing is selected, then options include

a. Smoothing with or without SCD;

b. Smoothing in coastal regions only;

c. Smoothing at flagged grid points only;

d. Smoothing at only those grid points having a specified sign for the Laplacian of the parameter.

(An example of the application of these controls is the use of options c. and d. in adjusting gradients in the wake of major reported retreats in sea-ice coverage.)

SCDSET

Sets the appropriate spreading weights to zero in order to "decouple" the flow of information across land barriers.

ASSMBL → **WGTSPR** → **BLND** → **REEVAL**

These modules carry out a 3-cycle FIB analysis of each ocean thermal-structure parameter, including the gross error check and reevaluation. On completion of the final assembly, A_0 is produced for use in the next analysis.

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THE EXPANDED OCEAN THERMAL-STRUCTURE ANALYSIS SYSTEM: A DEVELOP--ETC(U)
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RCPROF

As can be seen from Fig. 26, the information-processing loop entered by EOTS is exited when all analysis cycles have been completed for all requested parameters. As the horizontal analysis of each thermal-structure parameter is completed it is stored. The function of RCPROF is to reconstruct a vertical profile at each grid point using values from the stored set of horizontally analyzed fields. Floating levels are merged with the fixed levels as described in Section 5.7. RCPROF also acquires the deep climatological parameters (see Table 2) from storage.

BPROFIL

This module carries out the vertical blending process, grid point by grid point. (The corresponding values from the horizontally-analyzed fields are replaced by the resultant from BPROFIL to provide the final analyzed products. This is carried out by RCPROF.)

Having completed the analysis sequence for a given area, the program returns to the scheduler OTSRATS to commence the next thermal-structure analysis (if any).

6.4 Tunable Parameters

The EOTS analysis system encompasses a number of tunable parameters, some of which can be readily accessed and modified by the User and others which, deliberately, are less accessible.[1] Tunable

[1] For example the weights used in the vertical blending process (Table 2) are not under direct control of the operational User.

parameters under User-control include the class standard deviation of the object data; parameters concerned with the Gross Error Check; the number of smoothing passes and the smoothing coefficient used to smooth shape fields (usually only the Laplacian); reevaluation parameters; a limiter on the total accrued weight during assembly; the weight assigned to P_0^{**}; the weights assigned to the shape PIFS derived from P_0^{**}; the rates of adjustment toward predicted climatology to produce P_0 and P_0^{**}; and the variance growth (weight reduction) with time A_0, the weight to be associated with P_0.

The EOTS Users Manual contains a qualitative description of the effects to be expected as a result of adjusting the tunable parameters listed above. Adjustments should be based upon a reasoned appreciation of the role played by that particular parameter in the overall analysis process. The quantitative effects of adjustment may be assessed from study of the relevant formulations and associated discussions given in previous Sections of this Report. It may also be noted that the net effects of tuning are not necessarily immediately apparent (i.e., in the next analysis run). For many tuning parameters the effects are cumulative. In order to use and tune the EOTS system effectively, the User requires knowledge and understanding of the scientific principles and information processing concepts upon which the EOTS analysis system is based. The necessary principles and concepts are encompassed by the Fields by Information Blending analysis methodology and are contained in this Report.

7. ADDITIONAL EOTS-RELATED TOPICS

7.1 Introduction

This Section presents a number of topics forming part of the FIB analysis methodology and of relevance to the EOTS analysis system and its capabilities and applications.

7.2 The Production of Climatologies

By way of example, suppose it is required to produce a hemispheric climatology for June (all years) of sea-surface temperature. We shall assume that 30 years of data are available. Consider these four approaches:

a. Divide the hemisphere into a number of sub-regions, partition the data accordingly, and compute the arithmetic mean for each sub-region. This is the "Marsden square approach".

b. Use all the observations made in all Junes and carry out an analysis of this data. (A simple method would be to carry out this analysis in isolation; a superior technique is to impose forward-and-backward continuity in time[1] using the analyses from preceding and succeeding months.)

c. Use all the observations for a given June in an EOTS analysis of the SST parameter, repeat this process for all Junes, then derive an all-Junes climatological field by taking the grid-point means of the 30 contributing analyses.

d. Divide each June into six 5-day periods, produce analyses for each period, then derive an all-Junes climatological field by taking the grid point means of the 180 contributing analyses.

Which of these approaches is the best?

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[1] See Section 7.4.

The Marsden square approach, though often used, destroys much of the inherent information, has no significant merit when evaluated against the capabilities of an effective analysis system, and therefore will not be discussed further.

If the observations used for producing the climatology are (reasonably) uniformly distributed in space and time then no significant difference would be apparent between the three products based on analysis. However this is an unjustified (and unnecessary) assumption. In general SST observations are not uniformly distributed. In these circumstances method b. is the least satisfactory and method d. is the best.

To illustrate this point, admittedly by exaggerating the likely circumstances, assume that the only observations in a particular area of the analysis come from a ship which normally makes one measurement of SST every 5 days. Thus there will be 6 observations normally available for each June. However, assume that for one particular 5-day period a low-temperature anomaly affects the immediate vicinity and that, in order to obtain data regarding this unique phenomenon, the ship makes measurements of SST every hour. Thus there will be 120 observations of anomalously low SST values for this 5-day period.

Now consider method b. A total of 294 observations (29 x 6 + 120) are available. However over 40% (120/294 x 100%) of the information contributing to this product is derived from one particular 5-day period. This is clearly unrealistic.

Now consider method c. For the June encompassing the 5-day period during which more frequent measurements were made, there are 125 observations available. 95% (120/125 x 100) of the information contributing to the analysis for this particular June comes from one 5-day period. If now a mean value for all Junes is derived, giving equal weight to each June, then 3.2% of the contributing information (1/30 x 95%) comes from the uniquely cold 5-day period. This again is unrealistic because only 5 days of anomalously low temperature out of a possible 900 days (30 days x 30 years) were observed--i.e., 0.56%.

Now consider method d. Each 5-day period is equally weighted in deriving the mean. There are 180 5-day periods (6 x 30). For one of these 100% of the information reflects the cold anomaly--and correctly so since it has been postulated that the anomaly persisted throughout the 5 days. If now a mean value for all Junes is derived giving equal weight to each 5-day period, then the contribution of information from any 5-day period is equal to 0.56%.

(This is not an unduly contrived example. Oceanographic cruises and Naval exercises notoriously bias observations in space and time.)

It is clear that to avoid unrealistic weighting of information, equal weight should be given to equal time periods irrespective of the number of observations in each time period. The analyses should be performed as frequently as can be justified by the available data, the object range of scale, and by computer resources.

7.3 Commencing an Analysis Sequence

In previous discussions of the analysis process it generally has been assumed that the analysis for time τ forms part of an analysis sequence--in other words the τ-1 analysis is available to provide the necessary PIFS[1] for time τ. Now consider the first analysis in a synoptic sequence. Our best estimate of the object-parameter PIF is provided by climatology. (If

[1]As defined in Section 4.4, the object-parameter PIF (from which the shaping and spreading fields are derived) is the best estimate of what the forthcoming analysis will be before any information contribution from current observations is considered.

even climatology is not available then we must commence with a field of T_0^* = constant or some other appropriate field--say a monotonic decrease of temperature with increasing latitude.)

Assuming that climatology is available then the first analysis in the sequence may have only a small number of SST reports as synoptic input. Following assembly (using PIFS derived from climatology) the blending operation then spreads the assembled reports and produces the optimum analysis resultant T_τ^* and associated resolution weight A_τ^*. The field T_τ^* will closely resemble the climatological field in much of the area because only a relatively small number of reports were available to "close the gap" between climatology and the actual current situation.

The assembly and blending procedure is repeated for the next analysis. However the PIFS for the second analysis contain not only climatological information but also synoptic information incorporated into the first analysis.

Clearly, as the sequence of analyses is continued, information is accrued along the time axis and the resultant analysis approaches the actual current synoptic situation. (See Fig. 3 and Eq. (3).) It is a gradual process, being dependent upon the number (and distribution) of synoptic reports. For some areas--the Norwegian Sea in winter being a noteworthy case--the process may take considerable time. This, of course, is not a fault of the analysis system; no system can produce a meaningful analysis from insufficient information. However, even in these circumstances, the EOTS analysis system produces the best estimate of the representative value of the object parameter in the object scale of resolution that is to be obtained from the total available information; analysis resolution will be limited by the density of observations in space and time, not by the analysis grid. The weight A^* provides a measure of the uncertainty of the analysis (i.e., the residual analysis variance).

The process of starting a new analysis sequence is termed "restarting from climatology". As mentioned in Section 6.1, the off-line mode of the EOTS system may be used to set up a new analysis sequence. A file of (currently) 14 days of synoptic data is maintained and an operational run, starting from climatology, can be simulated. This capability--given sufficient data--can be used to overcome the problem of converging on the current synoptic situation in a real-time operational mode.

Although the need to restart from climatology can arise because the necessary PIFS have been "lost", in general the procedure is needed to satisfy U. S. Navy operational requirements for fine-mesh EOTS analyses of a new area. In these circumstances the fine-mesh climatology for that specific area usually does not exist. Rather than start from a "flat field", interpolated Northern Hemisphere climatology is used. (A better procedure would be to start from the interpolated Northern Hemisphere analysis; if this were done the larger-scale synoptic features would already be present. This change can easily be made.)

7.4 The Production of Historical Analysis Sequences

In terms of SST analyses, suppose that we produce an analysis sequence by progressing in the normal manner from time $\tau-1$ to time τ. Information is carried and accrued along the time axis. Now consider that a hitherto data-void region is shown by observation to have, say, a markedly higher SST than shown in the analyses up to the time at which this information was assimilated. Clearly it is unrealistic to assume that such a jump in SST value has indeed occurred between one analysis and the next. It is much more likely that the previous analysis was partially in error because of the lack of observations. A better estimate of the representative temperature may be obtained by carrying information along the time axis in both directions. Figure 27 illustrates how this is achieved.

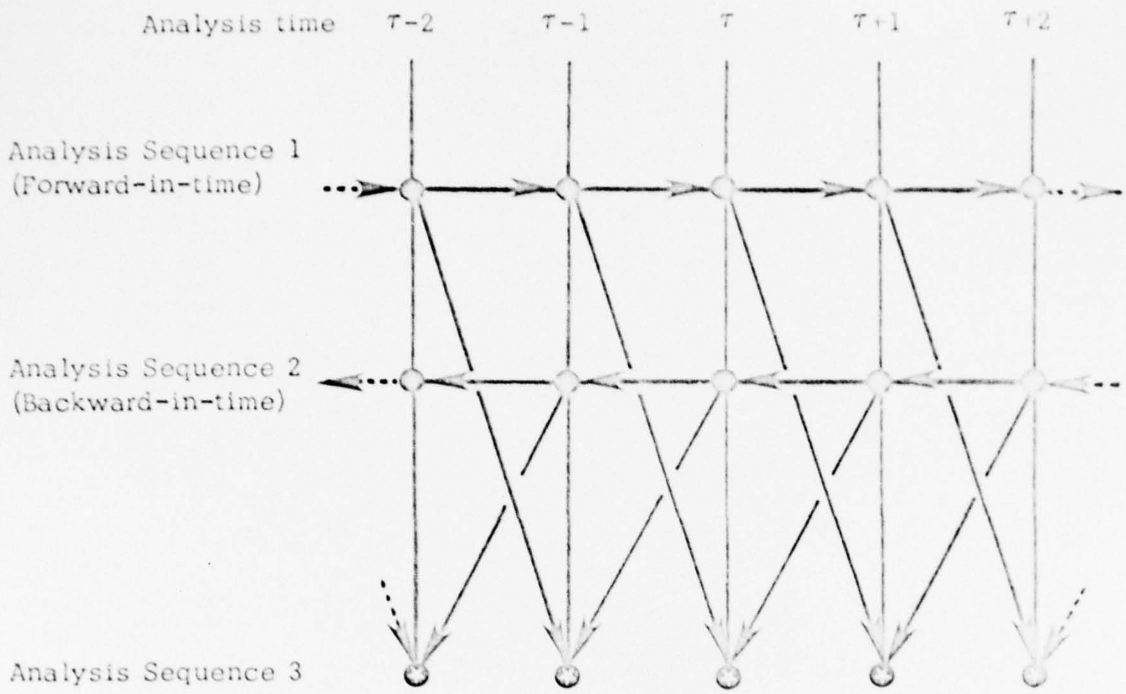

Figure 27 The production of a historical analysis sequence utilizing information carried and accrued along the time axis in both directions. Analyses are indicated thus "○" or "◎". Arrows indicate PIFS; the tail of the arrow is the source analysis and the head is the analysis in which the PIFS are used.

Analysis Sequence 1 shows 5 analyses with information being carried forward in time by way of the PIFS. For analysis time τ, the $\tau-1$ analysis provides the assembly field P_0^+, A_0^+ and the object-parameter PIF, T_0^{*+}, where the "+" indicates the forward-in-time accrual of information.

Analysis Sequence 2 is carried out backward-in-time. For analysis time τ, the $\tau+1$ analysis provides the assembly field P_0^-, A_0^- and the object-parameter PIF, T_0^{*-} where the "-" indicates the backward-in-time accrual of information.

For analysis time τ the assembly fields P_0^+, A_0^+ and P_0^-, A_0^- are combined using the standard FIB rules, thus providing the assembly field P_0, A_0. (Synoptic data for τ are assembled in the usual way to provide

P_N, A_N.) The object-parameter PIF (used to provide the shape and spreading PIFS) is given by $T_0^{\star} = (T_0^{\star +} + T_0^{\star -})/2$. Thus Analysis Sequence 3 has utilized information carried in both directions along the time axis. (Note that the third analysis run does not have to be produced in any particular order.)

For the production of historical sequences this procedure provides superior space-and-time continuity, preventing unrealistic changes between analyses due to poor data distribution such as can occur when information is carried in only one direction along the time axis.[1]

7.5 The Continuity of Ocean Thermal-Structure Parameters

Within an oceanic region delineated by the SCD field, all thermal-structure parameters are analyzed as continuous fields--every analyzed parameter has a value at every point in the field and the space-and-time interpolations assume continuity between grid points.

However, in an operational context, the PLD parameter may be discontinuous. For example, in some regions (such as the Northeast Atlantic in winter), the ocean may be isothermal to 400 meters--there is no PLD of any operational significance. Parameter number 4 (GV00 in Table 1) is a measure of the gradient in the first 12.5 meters of the thermocline. Where the thermocline is weak and ill-defined this gradient (a continuous field) will be small. In such regions the PLD field will show large variations in space and time--the variability of the PLD field increases as the strength of the thermocline, measured by GV00, decreases. To

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[1] A modified procedure which utilizes only two analysis sequences to carry information in both directions along the time axis has been developed and applied to the production of a 30-year history of northern hemisphere analyzed sea-level pressure fields at 6-hourly intervals--a total of some 44000 analyses; diagnosed wind fields also have been produced for the same times [7].

determine whether or not analyzed PLD values in a particular area have operational significance to the User the GV00 field also must be studied.

(By requiring that GV00 reaches a certain minimum value before PLD values were plotted it would be possible to blank off appropriate areas of the PLD field. This would provide a discontinuous field. However at what value of GV00 does the PLD cease to have significance? Thermoclines which are not significant in one operational context may be so in another.)

7.6 An Example showing the Effect of Two Different Object Scales of Analysis Resolution on Vertical Cross-Sections of Temperature

Figure 28 shows a vertical cross-section of temperature through the Gulf Stream for 00Z on 23 August 1978. This cross-section was obtained from the 63x63 Northern Hemisphere EOTS analysis.[1] The dotted line shows the PLD. The left-hand part of the cross-section is affected by land and should be ignored. Blanking of below-bottom areas is a refinement yet to be incorporated.

Figure 29 shows the same cross-section obtained from a fine-mesh EOTS analysis for the Gulf Stream sub-region. The analysis grid used had a grid-point spacing equal to 1/8 that of the Northern Hemisphere analysis. Due to the scheduling of operational runs, the cross-section shown in Fig. 28 was obtained from EOTS fields at a synoptic time 12 hours earlier than those used in Fig. 27; the difference is not significant to this discussion.

The two cross-sections are based on EOTS fields using a common vertical resolution (see Fig. 22). However one horizontal analysis-module (see Fig. 6) for Fig. 28 encompasses 64 analysis modules for Fig. 29.

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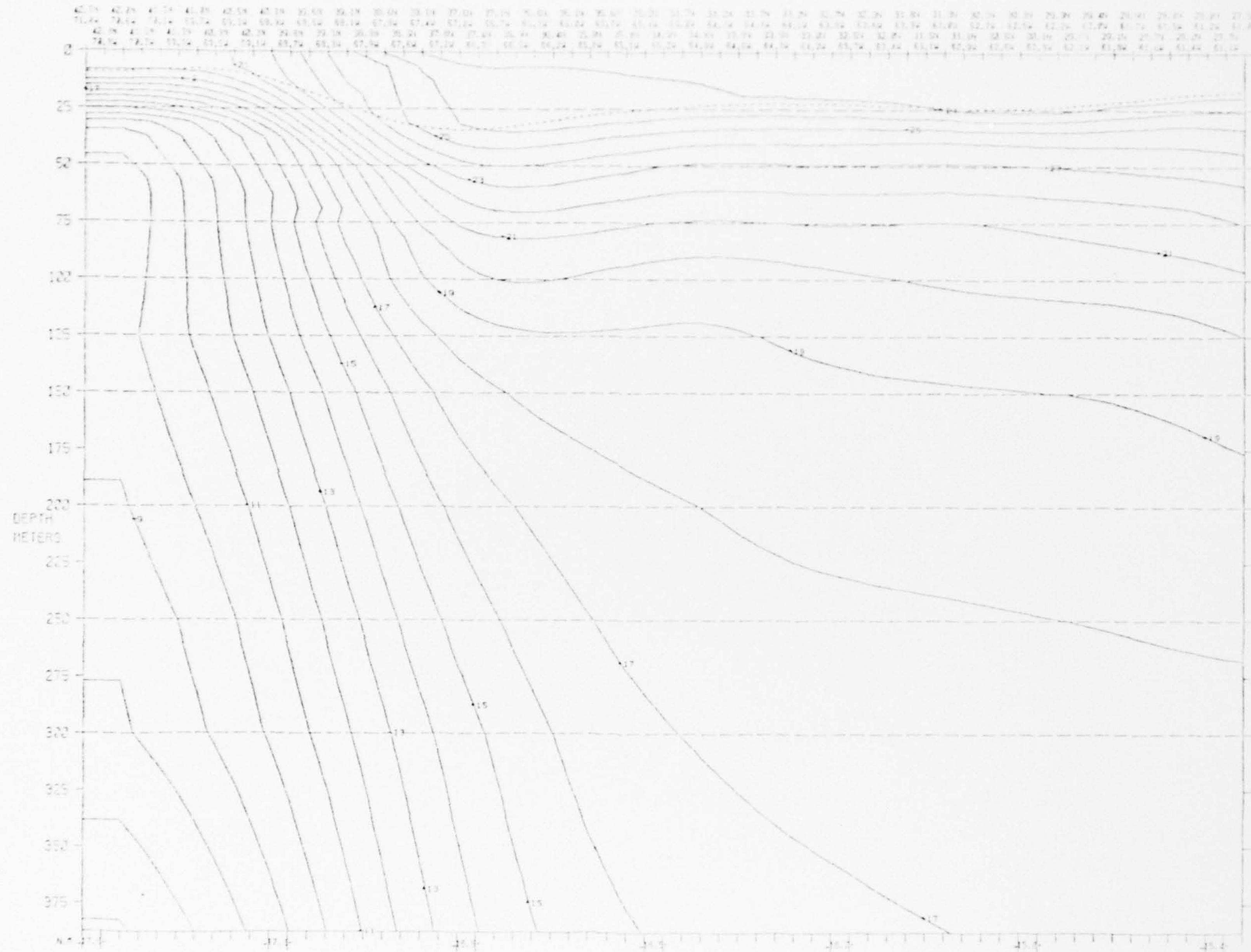
[1] The capability of producing such cross-sections forms part of the overall EOTS system. Details are given in Appendix A of the Users Manual. Note that this plot routine utilizes linear interpolation of a grid having a 6.25 meter vertical spacing. This introduces "kinks" in the isopleths and limits matching of the analyzed PLD.



78282300Z NHEM SEA TEMPERATURE CROSS SECTION FROM 42.5N 71.0W TO 27.5N 61.0W

Figure 28

73082212Z GLFS SEA TEMPERATURE CROSS SECTION FROM 42.5N 71.0W TO 27.5N 61.0W

Figure 29

Although the same data contributed to both cross-sections (ignoring the time difference of 12 hours) there are obvious differences between the two figures. Both cross-sections are equally "correct" in the sense that all available information was used to provide the best estimate of a representative cross-section. The differences are due to the fact that the representative cross-section on one scale of resolution is not the same as that on another scale of resolution.[1]

7.7 An Example of Temperature Fields at Three Different Depths

Figures 30, 31 and 32 show temperature fields at the surface (i.e., the SST), 150 meters and 300 meters on 12Z, 30 May 1979 for the Western Pacific in the vicinity of Taiwan. The analysis grid is 63x63, polar stereographic projection, with a mesh size equal to one-eighth that of the "standard" 63x63 hemispheric grid and a time "grid spacing" of 24 hours. These fields, produced by the EOTS analysis system, in effect are 2-D horizontal sections through the 4-D EOTS analysis for this region.

Figure 30, the SST, shows a fairly flat field to the east of Taiwan, with temperature decreasing fairly rapidly to the north. Note also, apart from the cold water near the China coast, that temperatures in the South China Sea are similar to those in the Western Pacific. For example, along 20°N, Pacific temperatures are about 28°C and South China Sea temperatures are about 28 or 29°C.

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[1] See the examples of SST analyses and associated discussion given in Section 4.13, and the blended profiles given in Table 3 and Fig. 25.

Figure 30 Fine-mesh SST analysis for 12Z, 30 May 1979 in the vicinity of Taiwan.

Figure 31 Fine-mesh temperature analysis at 150 meters for 12Z, 30 May 1979 in the vicinity of Taiwan.

Figure 32 Fine-mesh temperature analysis at 300 meters for 12Z, 30 May 1979 in the vicinity of Taiwan.

Figure 32 Fine-mesh temperature analysis at 300 meters for 12Z, 30 May 1979 in the vicinity of Taiwan.

Figure 31 shows the temperature at 150 meters and, as is readily apparent, there is a very different regime at this depth. Over the Western Pacific the temperature field is still flat--as it is over the South China Sea. However there is a marked east-west temperature gradient through the Luzon Straits. In broad terms, along 20°N the Pacific has decreased in temperature by about 6°C between the surface and 150 meters whereas the South China Sea has decreased by 12°C. By 300 meters (see Fig. 32) these decreases are about 11°C and 17°C respectively. Figures 30 to 32 are relevant to studies of the structure and distribution of the Kuroshio current.

Temperature fields at other depths could be provided by the EOTS analysis system, as could temperature vs. depth cross-sections along any line (straight or curved). However Figs. 30 to 32 are not given to illustrate applications of EOTS to oceanographic research. They have been provided to demonstrate that the EOTS analysis system easily can cope with marked changes in temperature regime with depth--features in the SST analysis have not been imposed on temperature fields at greater depths in defiance of other available information.

7.8 Sound Speed in the Oceans

Given temperature, salinity and depth (or pressure) it is a relatively straightforward matter to compute sound speed using, for example, Wilson's or Leroy's formulation.

The EOTS analysis system produces 4-D analyses of ocean thermal structure. As has been demonstrated a capability exists for extracting both vertical cross-sections (e.g., Figs. 28 and 29) and horizontal cross-sections[1] (e.g., Figs. 30, 31, 32).

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[1] These can be quasi-horizontal sections--for example the temperature at 25 meters above the PLD.

Although EOTS, appropriately modified, could be used to analyze salinity distributions, this is not useful in a synoptic time-frame because of the lack of synoptic data. Currently, the best-available salinity information is provided by climatological fields.[1]

Thus, given vertical and/or horizontal temperature cross-sections (from the EOTS system), corresponding salinity cross-sections (from climatology) and the depth, then cross-sections of sound speed may be computed. (Further details are given in the EOTS Users Manual.)

Figure 33 shows a horizontal field of sound speed at a depth of (PLD-25) meters--i.e., 25 meters above the PLD--based on the fine-mesh EOTS analysis for the Gulf Stream at 12Z, 24 JUN 1979. The heavy line drawn on Fig. 33 corresponds to the vertical cross-section of sound speed shown in Fig. 34. Figure 35 shows another vertical cross-section of sound speed, based on the Northern Hemisphere analysis of 00Z, 08 JUN 1979, and running along longitude 20°W from the equator to Iceland.

Knowledge of vertical and horizontal sound speeds in the ocean is of considerable importance to surface and sub-surface naval operations.

7.9 Quality Control, Evaluation, and Verification

7.9.1 Output Statistics and Diagnostics

As an integral part of any FIB analysis system, a wide variety of output statistics and diagnostics are generated during the analysis process to assist with quality control. Some of these measures are derived from the observed data contributing to the analysis, others provide the values of adjustable constants used by the analysis, and others are based on the

[1] The available fields for salinity are compiled monthly climatologies--they have not been analyzed by a FIB analysis system.

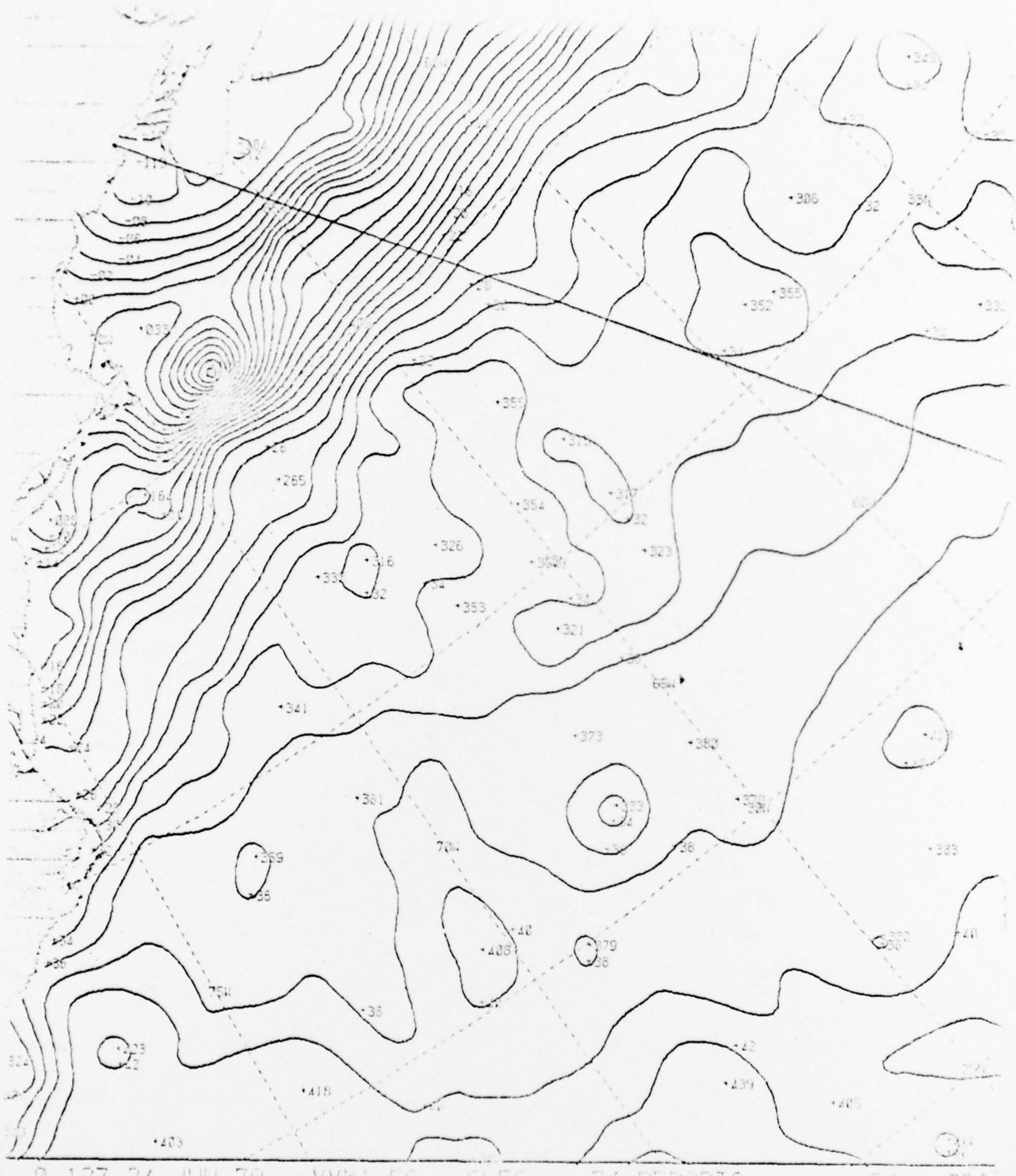

Figure 33 Sound speed at 25 meters above the PLD for the Gulf Stream for 12Z, 24 June 1979.

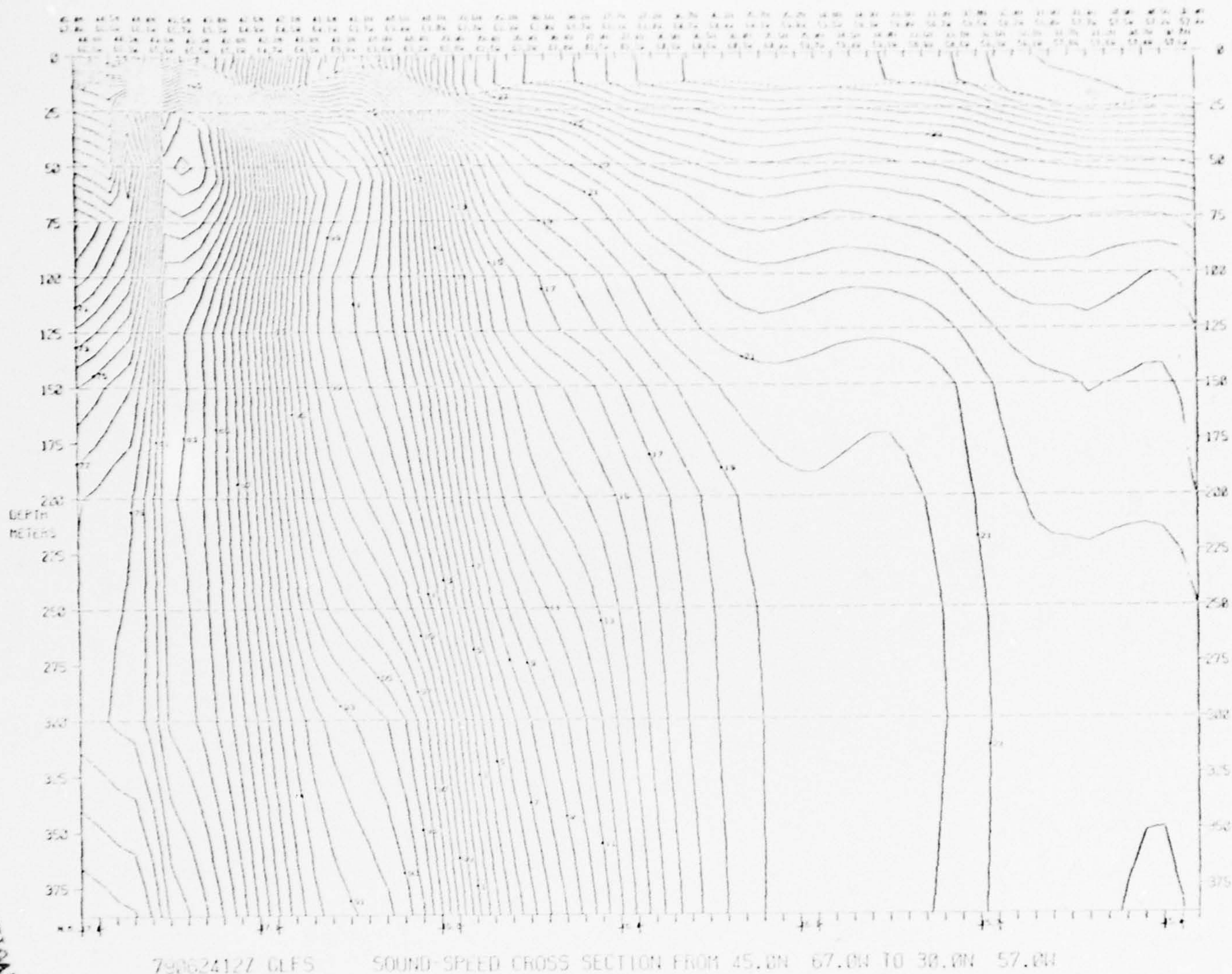

Figure 34. Vertical cross-section (to 400 meters) of sound speed for the Gulf Stream for 12Z, 24 June 1979. This section corresponds to the straight line shown on Fig. 33. The dotted line on this figure shows the PLD.

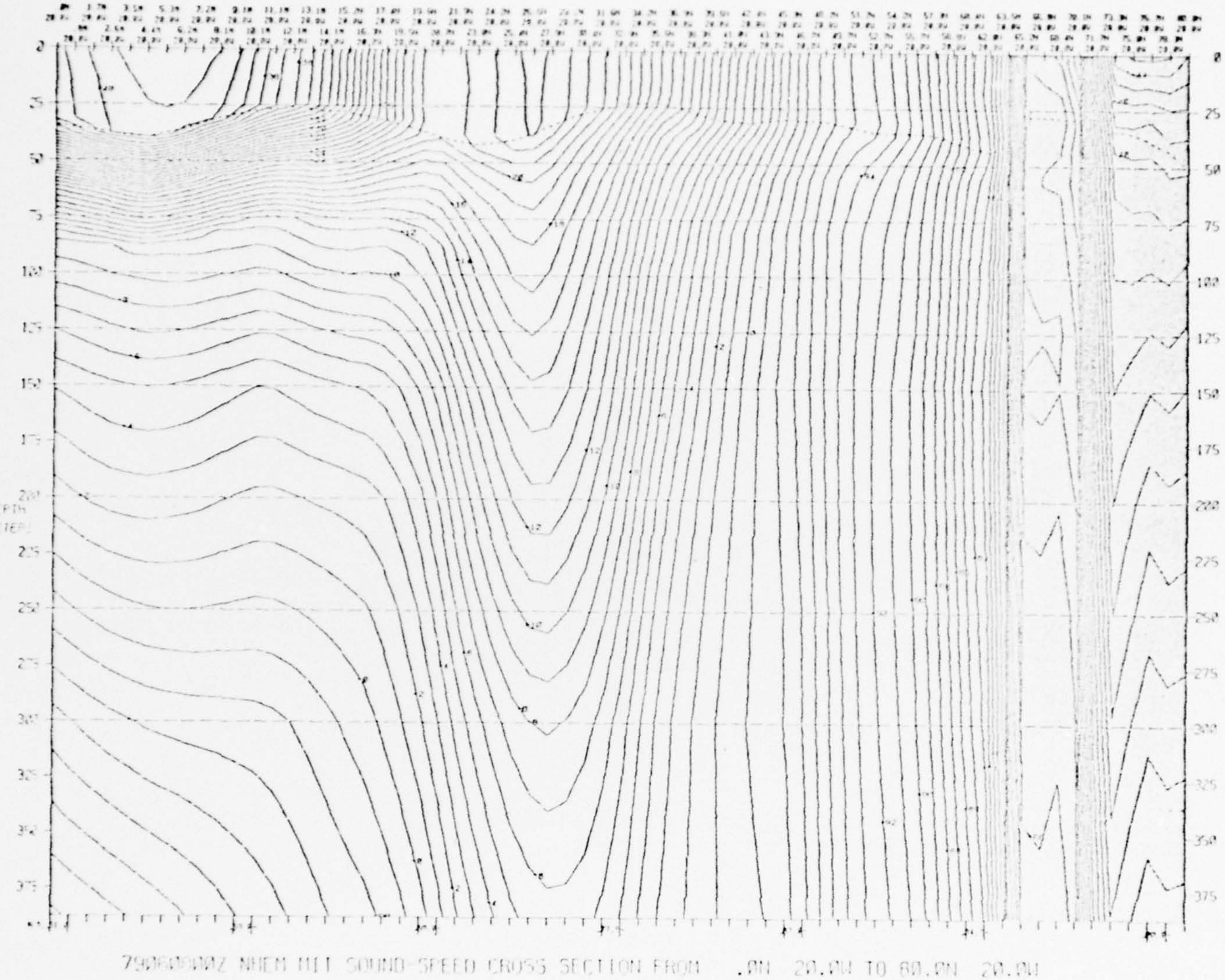

Figure 35 Vertical cross-section (to 400 meters) of sound speed along 20° W from the equator to 80° N for 00Z, 8 June 1979. The right-hand part of the section (shown shaded) is affected by land and ice. The dotted line shows the PLD.

analysis product. The following list, grouped by type, shows a selection of the statistics and diagnostics which may be output during any EOTS analysis run:

a. Data

List of bogusses.

Number of satellite reports.

Number of ship reports.

Number of days of BT data used in the analysis.

Number of BT observations available for each hour.

Total number of available BT reports.

Number of BT reports for each type of BT.

Number of BT reports entering the analysis.

Maximum number of levels reported by any BT.

Percentage of BTs corrected for single-level reporting errors.

Frequency distribution of maximum depth reached by BTs.

Frequency distribution of the number of PLD candidates provided by BTs.

Number of BTs not reaching a depth sufficient for them to be included in the preliminary PLD analysis.

Mean value of parameter nos. 1 through 26 in 15° latitude bands.

b. Analysis System

Number of hours since last analysis run for the selected area.

Values of all adjustable parameters including area definition.

Gross error tolerances for all object parameters.

Climatological summary including measures of the mean adjustment used to derive P_0 and P_0^{**}.

Assembly weight table.

c. Analysis Product

Mean report value for each object parameter.

Mean value of the parameter when carried to the nearest grid point.

Mean background value.

Mean analyzed value.

Mean difference between analyzed and assembled values.

Total field value for A^*, A_N and A_0 (for the next analysis).

Number of grid points in ice.

Data list considered for analysis of each parameter including ship name, DTG, latitude and longitude of the report, grid coordinates of the assembly point, initial report weight, report weight after reevaluation, background weight, resultant analysis weight, λ^2, and whether or not the report was rejected or withheld.

Reject summary showing number of reports accepted, rejected, and withheld.

Statistical measures for checking consistency of a priori estimates of class variance.

7.9.2 EOTS Evaluation

Evaluation may be considered as the process of studying the analysis product to determine whether or not it provides a realistic representation of significant variabilities in the object scale of resolution. (We reserve the term "verification" for the process of determining statistical measures--such as bias and variance--associated with the field and/or the observations. Verification statistics are discussed in Section 4.9.3.)

For evaluating EOTS analyses a variety of products are available. Horizontal fields of any parameter may be output--normally a Varian plot is used. A number of examples of temperature analyses have been presented in this Report. Other horizontally-analyzed parameters--such as gradient or PLD--also may be output. Any desired vertical cross-section of temperature may be plotted--see Figs. 28 and 29. In addition there are capabilities for plotting analyzed and climatological profiles at any point, for plotting observed profiles, and for providing output of the type shown in Table 3.

Based on evaluation, adjustable constants may be appropriately modified to enhance the product. To give a simple example, at one time the same climatic anomaly control factor was used in Eqs. (58) and (66). Over a period of time, evaluation of EOTS products revealed that shape features needed to be decayed at a greater rate than the anomaly of the object parameter. The EOTS analysis system was modified to include a second climatic anomaly control factor set to an appropriate value. Many adjustments and refinements have been made to the EOTS analysis system as the result of product evaluation.

7.9.3 Field Verification

Suppose we have an analysis of parameter P. Let P_f^* be the analysis resultant at an arbitrary location in the field. Now assume that at the same location (space and time) we have two observations P_a and P_b, where P_a was used in the analysis but P_b was not. P_f^*, P_a and P_b are all estimates of the same (but unknown) locally representative value P_R. How can we verify P_f^*, P_a and/or P_b without knowing P_R? Clearly this cannot be done in any simple manner--a large number of associated triplets P_R, P_f^* and P_b form a trivariate distribution. In the case of triplets P_R, P_f^* and P_a an added complication is that P_f^* and P_a are not independent because P_a has been assimilated into the analysis which produced P_f^*.

An approach sometimes adopted is to assume that observations are "true" (i.e., equal to P_R), and that any difference between P_b and P_f^* is due to "analysis error". An alternative approach is to argue that P_f^* should be a better estimate of P_R than P_b because P_f^* is based on more information; thus any difference between P_b and P_f^* is due to "observational error". Of course neither of these approaches is correct. Both P_f^* and P_b (or P_a) have associated variances relative to P_R, and to assume that either variance is zero produces misleading "results".

The subject and problems of field verification have been discussed in more detail by Holl [8]. Based on this study a statistical verification package for general use has been developed.

Figure 36 shows an example of the type of output generated by an early version of this verification package. It is based on a comparison of over 500 ship reports (B) with analyzed values (F) at the same location. Paired values of F and B are divided into classes of $\pm 1.5^\circ C$ centered on multiples of $3^\circ C$. Thus there were 10 cases where an observed value of $12^\circ C$ (actually 10.5 to $13.5^\circ C$) corresponded to an analyzed value of $15^\circ C$, 16 cases where an analyzed value of $12^\circ C$ corresponded to an observed value of $15^\circ C$,

```
VERIFICATION: SEA-SURFACE TEMPERATURE
  DTG: 79062100 ANALYSIS
    GRID: NH 63X63       LAT.BAND:  -90.0 ---  90.0
      INPUT:  FIELDS,  TYPE: DIAGNOSED    SOURCE:              PARAMETER NAMES: T  SW
                  527   DEPENDENT SHIP REPORTS,  (  30 DIFFS >  5.0)        15 OUT-OF-RANGE

                                  (1) FREQUENCY DISTRIBUTION,     UNITS: DEGREES C
      --FIELD--
 (7)   (5)   (3)
 B-F    N   CLASS
 MEAN
                   F
                   .
  -.5   88    30 .                                             1    2    1   13   49
                   .
  -.6  100    27 .                          1         2             1    9   72   15
                   .
  -.2   50    24 .           1                             1        9   27   10    2
                   .
   .2   56    21 .                                            10   36    5    2    1
                   .
  -.2   47    18 .                     1             9       29    8
                   .
  -.2   56    15 .                          2   10   33      11             .
                   .
  -.1   65    12 .       3                  9   35   16       2
                   .
  -.1   49     9 .       1     1    11     24   12
                   .
  -.2   50     6 .            12    31      6    1
                   .
  -.7    7     3 .       1     5     1
                   .
    I    0     0 .       *
                   .
    I    0    -3 .   *

                   * . . . . . . . . . . . . . . . . . . . . . . . . . . . . . ...B

       (2) CLASS      -3     0     3     6     9    12    15    18    21    24    27    30
--OBS--
            (4) N      0     5    19    44    42    58    60    55    50    44    98    67

    (6) F-B MEAN       I   8.3   2.7    .8    .8   -.2    .1    .1    .3    .2    .1   -.9

(8) OVERALL MEAN DIFFERENCE:  (F-B)=   .0     ABS(F-B)= 1.1       NO. DIFFERENCES: 527

(9) OVERALL VARIANCE:    2.56    NO.DIFFERENCES:      532   PERCENTAGE EXCLUDED:     2.

(10) SLOPE OF BEST-FIT STRAIGHTLINE (F/B) THROUGH ORIGIN:  1.094
```

Figure 36 Verification statistics for observed (B) and corresponding field (F) values of SST using dependent observations. The analysis system used is not the EOTS system. Note that gross errors in observations have not been eliminated--see text.

and 35 cases where both analyzed and observed values were 12° C. Two points may be noted:

a. Figure 36 is not based on the EOTS system but on an earlier (circa 1970) FIB-based SST analysis capability.

b. All observed data falling within the scatter diagram limits are shown, including obvious gross errors. For example there is an analyzed value of 24° C where there is a reported value of 3° C, an obvious gross reporting error (perhaps of 20° C?). (To remove gross errors the differences $|F - B|$ may be sorted in descending order and a suitable number (or percentage) eliminated.)

Based on paired values of F and B a variety of statistics are calculated. These are shown in Fig. 36. The verification statistics must be interpreted with care--conclusions which may seem "intuitively obvious" are often invalid.

8. CONCLUDING COMMENTS

As will be appreciated by study of the cited reports, the Fields by Information Blending analysis methodology has been developed over many years. This Report has traced the concepts and formulations of FIB from first principles to their application in a comprehensive and flexible system for the analysis of ocean thermal-structure--the EOTS analysis system. The primary objective has been to provide knowledge and understanding of the EOTS system and its capabilities, including some yet to be realized. A secondary objective has been to generate an appreciation of the general applicability of the FIB methodology to the analysis of a wide range of environmental parameters, many of which have been mentioned in the text.

Any analysis system based on FIB methodology is "open ended"--refinements can be made as the need (or the ability to do so) arises. Refinements can range from provision of new capabilities to special treatment of geophysical phenomena. A refinement currently (June 1979) being made is intended to remove analysis inconsistencies caused at certain times of the year by the discontinuous advance and retreat of the reported polar ice boundary. A refinement recently completed was concerned with "coastal smoothing" where the spreading of information may be unduly limited by the SCD field. The capabilities for producing vertical temperature cross-sections such as shown in Figs. 28 and 29, and for producing vertical and horizontal sound speed fields such as shown in Figs. 33, 34 and 35, represent additions to the evolving EOTS analysis system--they were not part of the original system.

Refinements yet to be made include the utilization of bathymetry in horizontal analyses (i.e., a depth-variable SCD field) and in vertical blending, and the production of PIFS based on an effective prediction model when one becomes available. It can be seen that the EOTS analysis system, because of its inherent flexibility, is capable of continuous development.

With regard to analysis resolution, any desired degree of horizontal resolution may be specified although, of course, this should be warranted by the information available for analysis. Vertical resolution currently is determined by the fixed and floating levels described in Section 5. Setting aside any problems due to computer resources the vertical resolution could be increased to accommodate, for example, not only the primary layer depth but also secondary layer depths, one of which may be the MLD.[1] Clearly secondary layer depths may not be sufficiently resolvable (in the context of the object scale of analysis resolution in space and time) to warrant the added effort--it remains to be shown that sufficient information is available for establishing the space-and-time continuum of secondary layer depths. Providing finer spacing, or other thermal-structure parameters, must be justified on the basis of increasing the 4-D analysis resolution yield.

In this Report little mention has been made of applications of the EOTS analysis system. Its primary intended purpose is to enhance the surface and sub-surface capabilities of the operational U. S. Navy by providing a real-time knowledge of ocean thermal-structure (and hence sound propagation) conditions. However there are many other potential applications including commerce, air-sea interaction studies, climate-trend studies, case studies, and so on. Very comprehensive global data bases of environmental observations (including SST and ocean soundings of all types) are available. The authors would be pleased to discuss any application of the EOTS analysis system and its products, as well as any other FIB-based analysis system, either realized or potential.

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[1] It will be appreciated that the PLD may or may not be the MLD. For example, in the morning the two may be the same. However the development of an "afternoon effect" may provide a shallow mixed layer (typically 20 feet or so in mid-latitudes). No great distance away, cloudy weather or high winds (say 20 knots or more) may preclude an afternoon effect. Overnight such a "transient thermocline" often dissipates. It can be seen that analyzing such a feature would provide a very "noisy" field with no significant continuity in space and time.