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Radar Backscatter From a Vegetation
Terrain

A Discrete Scattering Approach

F. H. Lang

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Errata Sheet for Final Report ETL-1059

1. page 32, Eq. (4-12)

change $\cos \alpha$ to $\cos \theta$

2. page 34

ϵ_r = relative dielectric constant

$\epsilon_r = 66. -j36.$

3. page 35

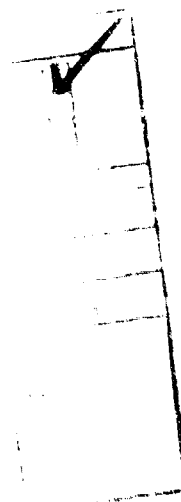
$\epsilon_r = 60. -j51.$

4. page 51

delete last line in first paragraph, "The validity..."

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dielectric constant for the vegetation is obtained by the Foldy technique, single scattering is employed to evaluate the back-scattering cross section. The resulting expression is found to compare favorably with experimental data. In addition to the three dimensional work, a one dimensional problem is analyzed. This analysis is compared with the Foldy approximation in the high density limit.



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TABLE OF CONTENTS

	Page
Abstract	
I. Introduction	i
II. Isotropic Scattering By a Half-Space of Randomly Distributed Spheres	4
III. Multiple Scattering of a Horizontally Polarized Wave By a Half-Space of Uniformly Distributed Discrete Scatterers	18
IV. Numerical Results and Comparison With an Empirical Model	29
V. Discrete One Dimensional Random Slab	36
VI. Conclusions and Recommendations	50
VII. References	52
Appendix A - Induced Current on the Dielectric Spheres - Rayleigh Scattering	54
Appendix B - Calculation of the Backscattering Coefficient	56

I. INTRODUCTION

This report studies microwave backscattering from vegetated terrain. Discrete random media theory is employed in an effort to relate the backscattering cross section to characteristic objects in the vegetation. The resulting theory is then compared with results obtained by more empirical methods.

The study is motivated by the need to relate radar return to the characteristics of the scattering objects. This situation is particularly complex in the case of vegetated terrain which consists of an ensemble of many highly irregular objects placed in a more or less random fashion. Using the scattered return from a vegetated terrain one would like to obtain such information as height, density and moisture content of vegetated regions in addition to obtaining the characteristic shapes of scatterers, so that the vegetation can be classified. This type of information would be important for military analysis of the terrain. In addition, if the terrain could be determined adequately from an electromagnetic point of view, the backscatterer information could be used to predict returns at other angles and frequencies not measured. This would be of great aid to the development of manageable data bases for radar simulations.

The above applications have served as a motivation for the development of electromagnetic models for the vegetated terrain. These models have been constructed by replacing the vegetation with a slab of random medium whose statistical

characteristics are related to the physical quantities in the medium. The models can be divided into two categories: continuous and discrete. In the continuous case, the vegetation is modeled by assuming its permittivity $\epsilon(\underline{r})$, is a random process whose moments are known. The average backscattering cross-section is then calculated from a knowledge of the statistics of $\epsilon(\underline{r})$. Usually just the mean and correlation of the permittivity are required. Following this, some quasi quantitative techniques are used to relate the medium's statistics to the actual vegetation under consideration. General development of these techniques are attributed to Keller⁽¹⁾ and Tatarskii and Gertsenshtein⁽²⁾, however, particular applications to vegetated terrain have been made by Lang⁽³⁾, Hevenor⁽⁴⁾ and Fung⁽⁵⁾.

When modeling is done by the discrete technique, on the other hand, the individual objects in the medium, such as leaves, are identified by their deterministic cross section, and then each object is placed randomly and its position is given by means of a probability density function. The average backscattered cross section is computed in such a way that all multiple interaction between objects is accounted for. The systematic development of these techniques has been made by Foldy⁽⁶⁾, Lax⁽⁷⁾ and Twersky⁽⁸⁾. In applying the discrete technique to vegetation, there is no need to relate the correlation function of the medium to the scattering object, as in the continuous case, since this has already been included.

As mentioned earlier, we will employ discrete scattering techniques to model the vegetation. We assume a lossy half space of vegetation is present, i.e., one in which the ground is not visible. We then consider the vegetation to be made up of small water droplets which are randomly distributed. In Section II, the average backscattering cross section is found by assuming the electromagnetic field can be treated as a scalar field. The Foldy technique is employed to calculate the field. This technique requires that the scatterers be in the Rayleigh regime, i.e., small compared to the wavelength. The use of the scalar assumption introduces difficulties in explicitly evaluating the cross section. These difficulties are removed in Section III where the full electromagnetic problem is treated for horizontal polarization. In Section IV numerical results are presented and compared with data from other sources. Certain restrictions are placed on the use of Foldy's technique in Section V. There a one dimensional model is analyzed exactly and examined in the thermodynamic limit. A comparison is made with the approximate results of the Foldy method.

II. ISOTROPIC SCATTERING BY A HALF-SPACE OF RANDOMLY DISTRIBUTED SPHERES

In this chapter the problem of multiple scattering of scalar waves by a half-space of randomly distributed dielectric spheres is considered. The spheres are assumed to be of the same size and their radius is taken much smaller than the wavelength of the incident wave. In addition, we also assume that the spheres are independent and uniformly distributed.

An approximate expression for the mean field is derived by employing Foldy's method⁽⁶⁾. This approximate mean field is then used along with the Born approximation to calculate the backscattered field. The scattering coefficient is then obtained from the backscattered field.

2-1 SINGLE SCATTERING

Consider a half-space ($z > 0$) of randomly distributed spheres (see Fig. 2-1). A scalar wave $\psi_0(\underline{r}) = \exp(-j\mathbf{k}_0 \cdot \underline{r})$ with harmonic time dependence $\exp(j\omega t)$ is incident upon the scatterers. The angle of incidence is θ . In free space $\psi_0(\underline{r})$ satisfies the wave equation.

$$\nabla^2 \psi_0(\underline{r}) + k_0^2 \psi_0(\underline{r}) = 0 \quad (2-1)$$

where $k_0 = \omega \sqrt{\mu_0 \epsilon_0}$ and μ_0 and ϵ_0 are the free space permeability and permittivity respectively.

The wave scattered by a single sphere which is small compared to wavelength is given by

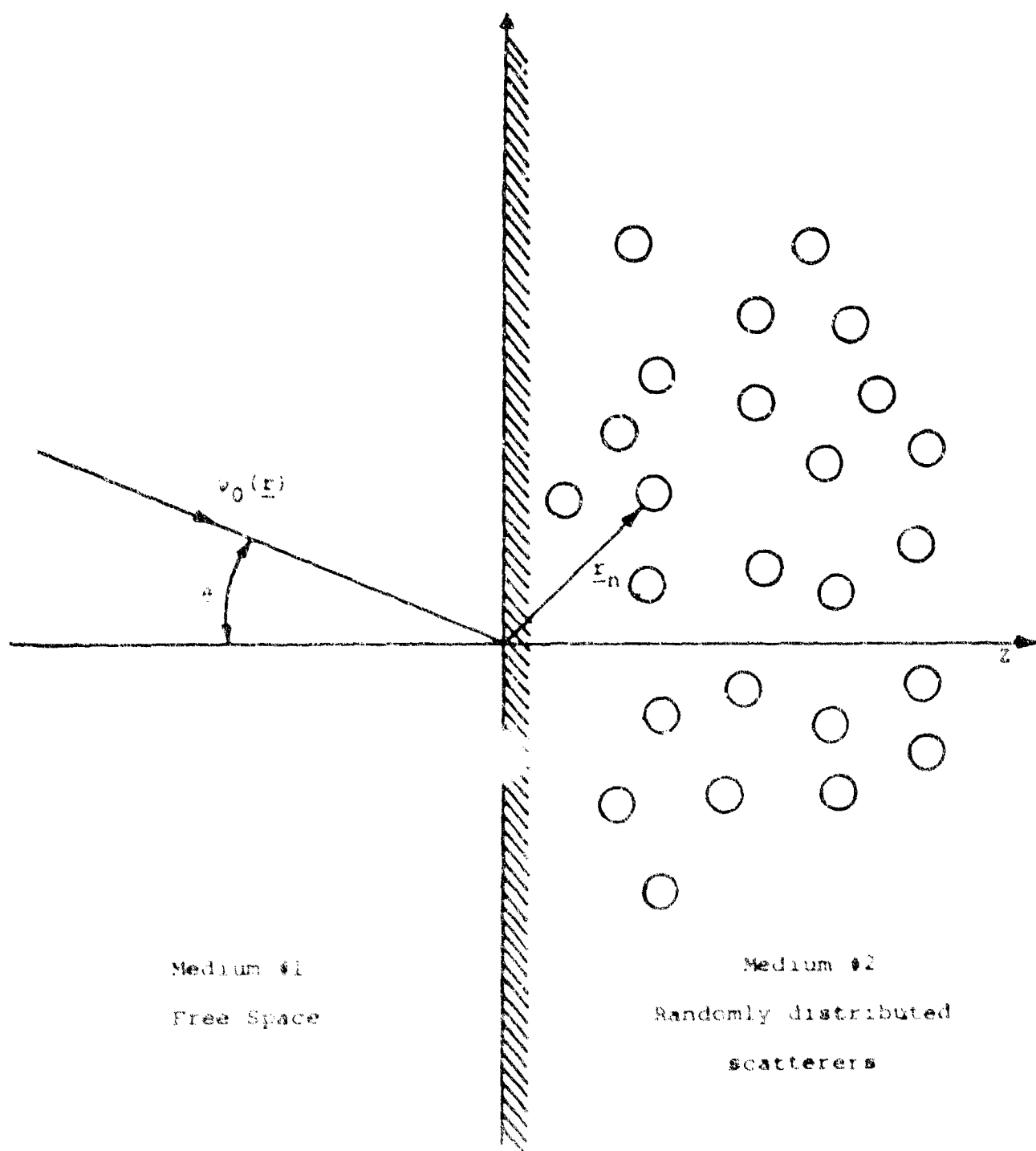


Fig. 2 1 Scalar Plane Wave Obliquely Incident Upon a Half-Space of Randomly Distributed Scatterers.

$$v_s^{(n)}(\underline{r}) = A_n g(\underline{r} - \underline{r}_n) \quad (2-2)$$

Here

$$g_0(\underline{r}) = \exp(-jk_0 \underline{r}) / 4\pi \underline{r} \quad (2-2a)$$

is the free space scalar Green's function which satisfies the wave equation:

$$\nabla^2 g_0(\underline{r} - \underline{r}_n) + k_0^2 g_0(\underline{r} - \underline{r}_n) = -i(\underline{r} - \underline{r}_n) \quad (2-3)$$

The scattering properties of the spheres are characterized by the relationship

$$A_n = v_s^{(n)}(\underline{r}_n) \quad (2-4)$$

which makes the strength of the scattered wave from the n^{th} sphere proportional to the external field $v_s^{(n)}(\underline{r}_n)$ acting on it. The proportionality constant v is called the scattering coefficient and is the same for all spheres. The simple spherical wave behavior of the scattered field results from the assumption that the fields are approximately constant in the vicinity of the sphere. As a result, each sphere can be replaced by a point source located at its origin (Rayleigh assumption).

2-2 MEAN FIELD

Now consider scattering by N spheres which are all small compared to wavelength and distributed randomly in a volume V . The ensemble of configurations is described by the joint probability function $p(\underline{r}_1, \dots, \underline{r}_N)$ where $\underline{r}_i$, $i=1, \dots, N$ is the position for the i^{th} sphere. We shall be interested only in probability functions in which the probability that a

particular scatterer is located in the volume V is independent of the locations of the other scatterers, thus we can write

$$P(\underline{r}_1, \underline{r}_2, \dots, \underline{r}_N) = P(\underline{r}_1)P(\underline{r}_2) \dots P(\underline{r}_N) \quad (2-5)$$

$$\text{where } p(\underline{r}_n) = \frac{1}{V}. \quad (2-6)$$

The fundamental equations of multiple scattering will now be formulated. Consider a particular configuration of the spheres. The wave function at the point $\underline{r}$ is given by

$$\psi(\underline{r}) = \psi_0(\underline{r}) + \sum_{n=1}^N \gamma \psi^{(n)}(\underline{r}_n) g_0(\underline{r} - \underline{r}_n) \quad (2-7)$$

The equation represents the field as the sum of the incident wave and spherical waves diverging from each of the spheres. The external field acting on the n^{th} scatterer is then

$$\psi^{(n)}(\underline{r}_n) = \psi_0(\underline{r}_n) + \sum_{i \neq n}^N \gamma \psi^{(i)}(\underline{r}_i) g_0(\underline{r}_n - \underline{r}_i) \quad (2-8)$$

The equations (2-7) and (2-8) represent the fundamental equations of multiple scattering.

The direct method of solving the problem would then consist of solving the set of simultaneous linear algebraic Eqs. (2-8) for the $\psi^{(n)}(\underline{r}_n)$ and substituting these in Eq. (2-7), thus giving $\psi(\underline{r})$ as a function of the positions and scattering parameters of the spheres. Taking the mean value of this quantity would then give us the desired results.

Unfortunately, it does not seem possible to carry out this procedure because of its complexity when N is large and it is necessary to resort to another procedure. This alternative method consists of attempting to find an approximate equation satisfied by $\langle \psi(\underline{r}) \rangle$ and then solving this equation for the $\langle \psi(\underline{r}) \rangle$.

We now proceed to find an approximate equation for $\langle \psi(\underline{r}) \rangle$ by taking the mean value of both sides of Eq. (2-7). We have

$$\langle \psi(\underline{r}) \rangle = \psi_0(\underline{r}) + \sum_{n=1}^N \langle \psi^{(n)}(\underline{r}_n) q_0(\underline{r}-\underline{r}_n) \rangle \quad (2-9)$$

By using the definition of the mean value we have for the bracketed part of the right hand side

$$\langle \psi^{(n)}(\underline{r}_n) q_0(\underline{r}-\underline{r}_n) \rangle = \int \dots \int d\underline{r}_1 \dots d\underline{r}_N P(\underline{r}_1, \dots, \underline{r}_N) \psi^{(n)}(\underline{r}_n) q_0(\underline{r}-\underline{r}_n) \quad (2-10)$$

From probability theory we know that (10)

$$P(\underline{r}_1, \underline{r}_2, \dots, \underline{r}_N) = P(\underline{r}_1, \dots, \underline{r}_{n-1}, \underline{r}_{n+1}, \dots, \underline{r}_N | \underline{r}_n) P(\underline{r}_n) \quad (2-11)$$

Note that $\psi^{(n)}(\underline{r}_n)$, as we can see from Eq. (2-8), depends on the position of all the other spheres but $q_0(\underline{r}-\underline{r}_n)$ depends only on the position of the n th sphere. Incorporating Eq. (2-11) into Eq. (2-10) we obtain

$$\begin{aligned} \langle \psi^{(n)}(\underline{r}_n) q_0(\underline{r}-\underline{r}_n) \rangle &= \int d\underline{r}_n q_0(\underline{r}-\underline{r}_n) P(\underline{r}_n) \int \dots \int P(\underline{r}_1, \dots, \underline{r}_{n-1}, \underline{r}_{n+1}, \dots, \\ &\quad \underline{r}_N | \underline{r}_n) \psi^{(n)}(\underline{r}_n) d\underline{r}_1 \dots d\underline{r}_{n-1} d\underline{r}_{n+1} \dots d\underline{r}_N \end{aligned} \quad (2-12)$$

Denote

$$\langle \psi(\underline{r}_n | \underline{r}_n) \rangle = \int \dots \int d\underline{r}_1 \dots d\underline{r}_{n-1} d\underline{r}_{n+1} \dots d\underline{r}_N \psi^{(n)}(\underline{r}_n) d\underline{r}_1 \dots, \\ d\underline{r}_{n-1} d\underline{r}_{n+1} \dots d\underline{r}_N \quad (2-13)$$

The quantity $\langle \psi(\underline{r}_n | \underline{r}_n) \rangle$ represents the external field acting on the n^{th} sphere averaged over all possible configurations of all the other spheres. For large N we approximate the external field on the n^{th} sphere by the mean field which would exist at the position of the n^{th} sphere when the sphere is not present. Thus we can write

$$\langle \psi(\underline{r}_n | \underline{r}_n) \rangle \approx \langle \psi(\underline{r}_n) \rangle \quad (2-14)$$

By using Eqs. (2-14), (2-13), Eq. (2-12) becomes

$$\langle \psi^{(n)}(\underline{r}_n) \epsilon_0(\underline{r} - \underline{r}_n) \rangle = \int d\underline{r}_n \epsilon_0(\underline{r} - \underline{r}_n) \langle \psi(\underline{r}_n) \rangle p(\underline{r}_n) \quad (2-15)$$

The quantity $\langle \psi^{(n)}(\underline{r}_n) \epsilon_0(\underline{r} - \underline{r}_n) \rangle$ is the same for all the spheres. Substituting Eq. (2-15) into Eq. (2-9) we obtain

$$\langle \psi(\underline{r}) \rangle = \psi_0(\underline{r}) + N \int d\underline{r}_n \epsilon_0(\underline{r} - \underline{r}_n) \langle \psi(\underline{r}_n) \rangle p(\underline{r}_n) \quad (2-16)$$

But from Eq. (2-6) we have $p(\underline{r}_n) = 1/V$ thus Eq. (2-16) becomes

$$\langle \psi(\underline{r}) \rangle = \psi_0(\underline{r}) + \frac{N}{V} \int d\underline{r}_n \epsilon_0(\underline{r} - \underline{r}_n) \langle \psi(\underline{r}_n) \rangle \quad (2-17)$$

Taking the limit of both sides as $V \rightarrow \infty$ and $N \rightarrow \infty$ we have

$$\langle \psi(\underline{r}) \rangle = \psi_0(\underline{r}) + \rho \int d\underline{r}_n \epsilon_0(\underline{r} - \underline{r}_n) \langle \psi(\underline{r}_n) \rangle \quad (2-18)$$

where

$$\rho = \lim_{V \rightarrow \infty} \frac{N}{V}$$

To see the physical significance of the Eq. (2-18), let us apply the operator, $\nabla^2 + k_0^2$ to both sides of Eq. (2-18)

$$\begin{aligned} \nabla^2 \langle \psi(\underline{r}) \rangle + k_0^2 \langle \psi(\underline{r}) \rangle &= (\nabla^2 + k_0^2) \psi_0(\underline{r}) + \\ &+ \gamma \rho \int d\underline{r}_n (\nabla^2 + k_0^2) g_0(\underline{r} - \underline{r}_n) \langle \psi(\underline{r}_n) \rangle \end{aligned} \quad (2-19)$$

By using Eqs. (2-1) and (2-3), Eq. (2-19) becomes

$$\nabla^2 \langle \psi(\underline{r}) \rangle + k_{eq}^2 \langle \psi(\underline{r}) \rangle = 0 \quad (2-20)$$

where

$$k_{eq}^2 = k_0^2 + \rho \gamma \quad (2-21)$$

We see that $\langle \psi(\underline{r}) \rangle$ in the region $z > 0$ satisfies the wave equation in a "continuous medium" in which the wave number depends upon the scattering coefficient and density distribution of the spheres.

The problem of finding the average value of the wave function has been essentially reduced to solving a boundary value problem for the wave equation and determining the value of γ . Carrying out this calculation, we find that the mean wave in the equivalent or effective medium is

$$\langle \psi(\underline{r}) \rangle = T e^{j(k_x x - k_z' z)}, \quad z > 0 \quad (2-22)$$

where

$$T = \frac{2k_z^i}{k_z^i + k_z^t} \quad (2-23)$$

with

$$k_z = \sqrt{k_0^2 - k_x^2} \quad , \quad \text{Im} k_z < 0 \quad (2-24)$$

$$k_z' = \sqrt{k_{eq}^2 - k_x^2} \quad , \quad \text{Im} k_z' < 0 \quad (2-25)$$

and

$$k_x = k_0 \sin \theta \quad (2-26)$$

2-3 CORRELATION FUNCTION OF THE SCATTERED FIELD

We now obtain an integral representation for the total scattered intensity. The second term of the right hand side of Eq. (2-7) gives the total scattered wave.

$$\psi_s(\underline{r}) = \sum_{n=1}^N \gamma \psi^{(n)}(\underline{r}_n) g(\underline{r} - \underline{r}_n) \quad (2-27)$$

Since the scattered field radiates into the equivalent medium, k_{eq} , we have used the Green's function $g(\underline{r})$ instead of $g_0(\underline{r})$ where

$$g(\underline{r}) = \frac{e^{-jk_{eq}|\underline{r}|}}{4\pi|\underline{r}|} \quad (2-28)$$

To obtain the mean value of $|\psi_s(\underline{r})|^2$ we follow an analogous procedure to that used for $\langle \psi(\underline{r}) \rangle$, but the analysis now becomes somewhat more involved. To begin we first multiply the expression for $\psi_s(\underline{r})$ as given by Eq. (2-27) by the corresponding expression for $\psi_s^*(\underline{r})$; thus we have

$$\langle \psi_s(\underline{r}_1) \psi_s^*(\underline{r}_2) \rangle = |\gamma|^2 \left\langle \sum_{n=1}^N \sum_{m=1}^N g(\underline{r}_1 - \underline{r}_n) g^*(\underline{r}_2 - \underline{r}_m) \psi^{(n)}(\underline{r}_n) \psi^{*(m)}(\underline{r}_m) \right\rangle \quad (2-29)$$

We next assume that the external field at the n^{th} sphere can be replaced by the mean field; thus we have⁽⁶⁾

$$\langle \psi_s(\underline{r}_1) \psi_s^*(\underline{r}_2) \rangle = |\gamma|^2 \left\langle \sum_{n=1}^N \sum_{m=1}^N g(\underline{r}_1 - \underline{r}_n) g^*(\underline{r}_2 - \underline{r}_m) \langle \psi(\underline{r}_n) \rangle \langle \psi^*(\underline{r}_m) \rangle \right\rangle \quad (2-30)$$

If we let $h(\underline{r}_n, \underline{r}_m) = g(\underline{r}_1 - \underline{r}_n) g^*(\underline{r}_2 - \underline{r}_m) \langle \psi(\underline{r}_n) \rangle \langle \psi^*(\underline{r}_m) \rangle$, Eq. (2-31) is written as

$$\langle \psi_s(\underline{r}_1) \psi_s^*(\underline{r}_2) \rangle = |\gamma|^2 \left\langle \sum_{n=1}^N \sum_{m=1}^N h(\underline{r}_n, \underline{r}_m) \right\rangle \quad (2-32)$$

Using Eqs. (2-5) and (2-6) we obtain for the bracketed term at the right hand side

$$\begin{aligned} \left\langle \sum_{n=1}^N \sum_{m=1}^N h(\underline{r}_n, \underline{r}_m) \right\rangle &= \left(\frac{1}{V} \right)^N \int \left[\sum_{n=1}^N \sum_{m=1}^N h(\underline{r}_n, \underline{r}_m) \right] d\underline{r}_1 d\underline{r}_2 \dots d\underline{r}_N \\ &= \left(\frac{1}{V} \right)^N \left[(N^2 - N) \int h(\underline{r}_n, \underline{r}_m) d\underline{r}_n d\underline{r}_m \int \dots \int_{\substack{i=1 \\ i \neq n, m}}^N d\underline{r}_i \right. \\ &\quad \left. + N \int h(\underline{r}_n, \underline{r}_n) d\underline{r}_n \int \dots \int_{\substack{i=1 \\ i \neq n}}^N d\underline{r}_i \right] \end{aligned} \quad (2-33)$$

The first integral consists of the (N^2-N) terms where $n \neq m$, while the second integral consists of N terms where $n=m$.

Since

$$\int \dots \int_{\substack{i=1 \\ i \neq n, m}}^N d\underline{r}_i = V^{N-2} \quad (2-34)$$

and

$$\int \dots \int_{i=1}^N d\underline{r}_i = V^{N-1} \quad (2-35)$$

Therefore

$$\left\langle \sum_{n=1}^N \sum_{m=1}^N h(\underline{r}_n, \underline{r}_m) \right\rangle = \rho^2 \iint h(\underline{r}', \underline{r}'') d\underline{r}' d\underline{r}'' + \rho \int_V h(\underline{r}', \underline{r}') d\underline{r}' \quad (2-36)$$

where for large N and V

$$\frac{N^2-N}{V^2} \rightarrow \frac{N^2}{V^2} = \rho^2 \quad (2-37)$$

Substituting Eq. (2-36) into Eq. (2-32) we have

$$\langle \psi_{\mathbf{g}}(\underline{r}_1) \psi_{\mathbf{g}}^*(\underline{r}_2) \rangle = |\gamma|^2 \rho^2 \iint h(\underline{r}', \underline{r}'') d\underline{r}' d\underline{r}'' + |\gamma|^2 \rho \int_V h(\underline{r}', \underline{r}') d\underline{r}' \quad (2-38)$$

or

$$\begin{aligned} \langle \psi_{\mathbf{g}}(\underline{r}_1) \psi_{\mathbf{g}}^*(\underline{r}_2) \rangle &= |\gamma|^2 \rho^2 \iint g(\underline{r}_1 - \underline{r}') g^*(\underline{r}_2 - \underline{r}'') \langle \psi(\underline{r}') \rangle \langle \psi^*(\underline{r}'') \rangle d\underline{r}' d\underline{r}'' \\ &+ |\gamma|^2 \rho \int g(\underline{r}_1 - \underline{r}') g^*(\underline{r}_2 - \underline{r}') \langle \psi(\underline{r}') \rangle \langle \psi^*(\underline{r}') \rangle d\underline{r}' \end{aligned} \quad (2-39)$$

2-4 EVALUATION OF THE TRANSVERSE SPECTRAL DENSITY AND BACK-SCATTERING COEFFICIENT

In this section we evaluate the transverse power spectral density of the fluctuating portion of the scattered field. From this spectral density a direct calculation yields the backscattering coefficient as we show in Appendix B.

We start by defining the fluctuating portion of the scattered field by

$$\psi_f(\underline{r}) = \psi_s(\underline{r}) - \langle \psi_s(\underline{r}) \rangle \quad (2-40)$$

where we note that $\langle \psi_f(\underline{r}) \rangle = 0$. The spectral density can be obtained by Fourier transforming the correlation function of $\psi_f(\underline{r})$. We proceed by calculating the correlation of $\psi_f(\underline{r})$:

$$\begin{aligned} \langle \psi_f(\underline{r}_1) \psi_f^*(\underline{r}_2) \rangle &= \langle (\psi_s(\underline{r}_1) - \langle \psi_s(\underline{r}_1) \rangle) (\psi_s^*(\underline{r}_2) - \langle \psi_s^*(\underline{r}_2) \rangle) \rangle \\ &= \langle \psi_s(\underline{r}_1) \psi_s^*(\underline{r}_2) \rangle - \langle \psi_s(\underline{r}_1) \rangle \langle \psi_s^*(\underline{r}_2) \rangle \end{aligned} \quad (2-41)$$

Equation (2-41) shows that at $\underline{r}_1 = \underline{r}_2$ the fluctuating intensity is the total intensity minus the coherent intensity. By using Eqs. (2-39) and (2-18) in Eq. (2-41), we find

$$\langle \psi_f(\underline{r}_1) \psi_f^*(\underline{r}_2) \rangle = |\gamma|^2 \int g(\underline{r}_1 - \underline{r}') g^*(\underline{r}_2 - \underline{r}') |\psi(\underline{r}')|^2 d\underline{r}' \quad (2-42)$$

Now proceed by taking the transverse Fourier transform of Eq. (2-42) with respect to $\underline{r}_{t1}$ and $\underline{r}_{t2}$ where

$$\underline{r}_{t_i} = x_i \underline{a}_x + y_i \underline{a}_y, \quad i=1,2$$

with $\underline{a}_x$ and $\underline{a}_y$ being unit vectors in the x and y directions respectively.

We have

$$\begin{aligned} \langle \tilde{\psi}_f(\underline{k}_{t_1}, z_1) \tilde{\psi}_f^*(\underline{k}_{t_2}, z_2) \rangle &= \frac{|Y|^2 \phi}{(2\pi)^4} \int d\underline{r}_{t_1} d\underline{r}_{t_2} d\underline{r}' \\ &\quad g(\underline{r}_1 - \underline{r}') g^*(\underline{r}_2 - \underline{r}') |\langle \psi(\underline{r}') \rangle|^2 e^{+j(\underline{k}_{t_1} \cdot \underline{r}_1 - \underline{k}_{t_2} \cdot \underline{r}_2)} \end{aligned} \quad (2-43)$$

Here we have defined $\tilde{\psi}_f$ by

$$\tilde{\psi}_f(\underline{k}_t, z) = \int d\underline{r}_t \psi_f(\underline{r}) e^{+j\underline{k}_t \cdot \underline{r}} \quad (2-44)$$

and $\underline{k}_t = k_x \underline{a}_x + k_y \underline{a}_y$.

We now set $z_1 = z_2 = 0$ since it is the spectral density at the interface that is related to the backscattering coefficient. To simplify Eq. (2-42) represent the Green's functions by their transverse Fourier representation, i.e.,

$$g(\underline{r}) = \frac{1}{(2\pi)^2} \int d\underline{k}_t \frac{e^{-j(\kappa_z |z| + \underline{k}_t \cdot \underline{r})}}{-2j\kappa_z} \quad (2-45)$$

with $\kappa_z = \sqrt{k_{eq}^2 - |\underline{k}_t|^2}$, and use the explicit expression given in Eq. (2-22) for the mean field. Substituting these in Eq. (2-43) and simplifying, we obtain

$$\langle \tilde{\psi}_f(\underline{k}_{t_1}, 0) \tilde{\psi}_f^*(\underline{k}_{t_2}, 0) \rangle = S(\underline{k}_{t_1}) \delta(\underline{k}_{t_1} - \underline{k}_{t_2}) \quad (2-46)$$

where

$$S(\underline{k}_{t_1}) = |\gamma T|^2 \int_0^\infty \frac{e^{+\text{Im}(\kappa'_{z_1} + k'_z)z'}}{|\kappa'_{z_1}|^2} dz' \quad (2-47)$$

with $\kappa'_{z_1} = \sqrt{k_{eq}^2 - |\underline{k}_{t_1}|^2}$, $\text{Im}\kappa'_{z_1} > 0$ and k'_z being given in Eq. (2-25). The quantity $S(\underline{k}_{t_1})$ is the transverse power spectral density at the interface. The expression for it given in Eq. (2-47) can be further simplified by performing the integration. We have

$$S(\underline{k}_{t_1}) = \frac{|\gamma T|^2 \rho}{|\kappa'_{z_1}|^2 \text{Im}(\kappa'_{z_1} + k'_z)} \quad (2-48)$$

Now applying the results of Appendix B relating σ^* , the backscattering coefficient directly to $S(\underline{k}_{t_1})$, we have

$$\sigma^* = \frac{k_0^2 \cos^2 \theta}{4\pi} S(\underline{k}_{t_0}) \quad (2-49)$$

when $\underline{k}_{t_0} = k_0 \sin \theta \underline{\hat{x}}$. Noting that when $\underline{k}_{t_1} = \underline{k}_{t_0}$ we have $\kappa'_{z_1} = k'_z$ and thus

$$\sigma^* = \frac{\rho |\gamma T|^2 k_0^2 \cos^2 \theta}{8\pi |k'_z|^2 \text{Im}k'_z} \quad (2-50)$$

where $k'_z = \sqrt{k_{eq}^2 - k_0^2 \sin^2 \theta}$.

This is the final expression for the backscattering coefficient. Because of the scalar treatment of the problem the constant γ can not be evaluated. The situation will be remedied in the next section where the same techniques as used here are applied directly to the more complex vector case.

III. MULTIPLE SCATTERING OF A HORIZONTALLY POLARIZED WAVE BY A HALF-SPACE OF UNIFORMLY DISTRIBUTED DISCRETE SCATTERERS

The problem of multiple scattering of a horizontally polarized plane wave is considered in this chapter. The wave is assumed to have unit amplitude and a harmonic time dependence of $\exp(j\omega t)$. It is incident at angle θ on the half-space occupied by the scatterers. We model the scatterers by dielectric spheres. As in Chapter Two we assume that the spheres are identical and randomly distributed. The size of the spheres is again taken much smaller than the wavelength of the incident wave (Rayleigh scattering).

The main quantity of interest in this chapter, as in Chapter Two, is the backscattering coefficient, σ° . In this chapter, in contrast with Chapter Two, the scattering coefficient is evaluated directly in terms of the geometric and electrical properties of the spheres. This allows us to find an explicit expression for σ° in terms of known quantities.

In determining the expression for σ° we follow basically the same steps as in Chapter Two.

3-1 PROBLEM FORMULATION AND FUNDAMENTAL EQUATION

A horizontally polarized plane wave is incident upon the half space of random spheres at an angle θ . The incident electric field $E_0(\underline{r})$ is given as

$$E_0(\underline{r}) = e^{jk_0(x \sin \theta - z \cos \theta)} \underline{a}_y \quad (3-1)$$

The incident field is scattered by the dielectric spheres, and thus gives rise to a scattered field, $\underline{E}_s(\underline{r})$. The geometry of the problem is basically the same as the scalar problem shown in Fig. 2-1.

We now derive the fundamental equations for the multiple scattering of an electromagnetic wave in the Rayleigh limit. As we have shown in Appendix A, the spheres can be represented by an equivalent dipole current. From electromagnetic theory we know that a current distribution $\underline{J}(\underline{r}')$ produces an electric field given by⁽⁹⁾

$$\underline{E}_s(\underline{r}) = -j\omega\mu_0 \int_V \underline{G}_0(\underline{r}-\underline{r}') \cdot \underline{J}_0(\underline{r}') d\underline{r}' \quad (3-2)$$

where $\underline{G}_0(\underline{r}-\underline{r}')$ is the free space dyadic Green's function, and $\underline{J}_0(\underline{r}')$ the total current distribution in volume V .

The free space dyadic Green's function satisfies the following equations

$$(\nabla \times \nabla \times - k_0^2) \underline{G}_0(\underline{r}-\underline{r}') = \underline{I} \delta(\underline{r}-\underline{r}') \quad (3-3)$$

$$\underline{G}_0(\underline{r}-\underline{r}') = \left(\underline{I} + \frac{\nabla \nabla}{k_0^2} \right) g_0(\underline{r}-\underline{r}') \quad (3-4)$$

where $\underline{I} = \underline{a}_x \underline{a}_x + \underline{a}_y \underline{a}_y + \underline{a}_z \underline{a}_z$ is the unit dyadic and $g_0(\underline{r})$ is the free space Green's given by Eq. (2-2a).

The total current $\underline{J}_0(\underline{r}')$ is given by

$$\underline{J}_0(\underline{r}') = \sum_{n=1}^N \underline{J}_{eq}^{(n)}(\underline{r}') \quad (3-5)$$

where N is the number of dielectric spheres in the volume V and $\underline{J}_{eq}^{(n)}(\underline{r}')$ is the dipole current induced in the n^{th} sphere. By using Eq. (A-9) in Eq. (3-5) becomes

$$\underline{J}_{eq}(\underline{r}') = j\omega 4\pi\epsilon_0 K a^3 \sum_{n=1}^N \underline{E}^{(n)}(\underline{r}_n) \delta(\underline{r}' - \underline{r}_n) \underline{a}_y \quad (3-6)$$

where $\underline{E}^{(n)}(\underline{r}_n)$ is the external field acting on the n^{th} sphere, and $K = (\epsilon_r - 1)/(\epsilon_r + 1)$ with ϵ_r being the relative dielectric constant of the spheres. By putting Eq. (3-6) into Eq. (3-2) we obtain

$$\underline{E}_s(\underline{r}) = \sum_{n=1}^N \underline{G}_0(\underline{r} - \underline{r}_n) \cdot \underline{E}^{(n)}(\underline{r}_n) \underline{a}_y \quad (3-7)$$

where

$$\underline{r} = 4\pi K a^3 \epsilon_0 \quad (3-8)$$

The total electric field at the position $\underline{r}$ is the sum of the incident field and the total scattered electric field, thus we can write

$$\underline{E}(\underline{r}) = \underline{E}_0(\underline{r}) + \sum_{n=1}^N \underline{G}_0(\underline{r} - \underline{r}_n) \cdot \underline{E}^{(n)}(\underline{r}_n) \underline{a}_y \quad (3-9)$$

The external field acting on the n^{th} sphere is the sum of the incident field and the scattered field by all the other spheres at the position $\underline{r}_n$ with the n^{th} sphere removed. We obtain

$$\underline{E}^{(n)}(\underline{r}_n) = \underline{E}_0(\underline{r}_n) + \rho \sum_{\substack{l=1 \\ l \neq n}}^N \underline{G}_0(\underline{r}_n - \underline{r}_l) \underline{E}^{(l)}(\underline{r}_l) \underline{a}_y \quad (3-10)$$

Eqs. (3-9) and (3-10) are the fundamental equations of multiple scattering of an electromagnetic wave from a random collection of dielectric spheres which are small compared to wavelength.

3-2 EVALUATION OF THE MEAN FIELD OF $\underline{E}(\underline{r})$

We now proceed to find an approximate equation for $\langle \underline{E}(\underline{r}) \rangle$. Following the same steps as in the derivation of Eq. (2-18) in Chapter Two we have

$$\langle \underline{E}(\underline{r}) \rangle = \underline{E}_0(\underline{r}) + \rho \int d\underline{r}_n \underline{G}_0(\underline{r} - \underline{r}_n) \cdot \langle \underline{E}(\underline{r}_n) \rangle \quad (3-11)$$

where ρ is the density of the scatterers.

Operating on both sides of Eq. (3-11) by $\nabla \times \nabla \times$ we have

$$\nabla \times \nabla \times \langle \underline{E}(\underline{r}) \rangle = \nabla \times \nabla \times \underline{E}_0(\underline{r}) + \rho \int d\underline{r}_n \nabla \times \nabla \times \underline{G}_0(\underline{r} - \underline{r}_n) \cdot \langle \underline{E}(\underline{r}_n) \rangle \quad (3-12)$$

Note that the incident wave $\underline{E}_0(\underline{r})$ satisfies the wave equation in free space

$$(\nabla \times \nabla \times - k_0^2) \underline{E}_0(\underline{r}) = 0 \quad (3-13)$$

By using Eqs. (3-13) and (3-3), Eq. (3-12) becomes

$$\begin{aligned} \nabla \times \nabla \times \langle \underline{E}(\underline{r}) \rangle &= k_0^2 \underline{E}_0(\underline{r}) + \rho \int d\underline{r}_n \left[\nabla \times \nabla \times \underline{G}_0(\underline{r} - \underline{r}_n) - k_0^2 \underline{G}_0(\underline{r} - \underline{r}_n) \right] \cdot \langle \underline{E}(\underline{r}_n) \rangle \\ &= k_0^2 \underline{E}_0(\underline{r}) + \rho \langle \underline{E}(\underline{r}) \rangle + \rho k_0^2 \int d\underline{r}_n \underline{G}_0(\underline{r} - \underline{r}_n) \cdot \langle \underline{E}(\underline{r}_n) \rangle \end{aligned} \quad (3-14)$$

By using Eq. (3-11) we have

$$\nabla \times \nabla \times \langle \underline{E}(\underline{r}) \rangle = \rho \delta \langle \underline{E}(\underline{r}) \rangle + k_0^2 \langle \underline{E}(\underline{r}) \rangle \quad (3-15)$$

We can write this as

$$\nabla \times \nabla \times \langle \underline{E}(\underline{r}) \rangle = k_{eq}^2 \langle \underline{E}(\underline{r}) \rangle \quad (3-16)$$

where

$$k_{eq}^2 = k_0^2 + \rho \delta \quad (3-17)$$

We see that $\langle \underline{E}(\underline{r}) \rangle$ satisfies the wave equation in a "continuous medium" in which the propagation constant depends upon the scattering coefficients and density of the scatterers. Thus the problem of finding the average value of the electric field has been essentially reduced to finding the transmitted field, in a half-space ($z > 0$) of "continuous medium".

Carrying out this calculation, we find that the mean wave in the equivalent medium is

$$\langle \underline{E}(\underline{r}) \rangle = T e^{j(k_x x - k_z' z)} \underline{a}_y, \quad z > 0 \quad (3-18)$$

where

$$T = \frac{2k_z'}{k_z' + k_z} \quad (3-19)$$

with

$$k_z' = \sqrt{k_{eq}^2 - k_x^2}, \quad \text{Im} k_z' \leq 0 \quad (3-20)$$

and k_z and k_x are given by Eq. (2-24) and (2-26) respectively. For parameters values typical of vegetation, we will find that $\beta \ll 1$ which implies the Tel.

3-4 EVALUATION OF THE TRANSVERSE SPECTRAL DENSITY AND BACKSCATTERING COEFFICIENT

In this section we evaluate the transverse power spectral density. The procedure differs slightly from the previous section. Here the Fourier transform of the scattered field is computed first and then the spectral density is obtained. Previously we computed the correlation of the scattered field and then transformed it.

The single scattered field in the equivalent medium is obtained by modifying Eq. (3-2). We have

$$\underline{E}_s(\underline{r}') = -j\omega\epsilon_0 \int \underline{G}(\underline{r}-\underline{r}') \cdot \underline{J}(\underline{r}') d\underline{r}' \quad (3-21)$$

where $\underline{G}$ and $\underline{J}$ have replaced $\underline{G}_0$ and $\underline{J}_0$. Here $\underline{G}$ is the dyadic Green's function in the equivalent medium. It is given by

$$\underline{G}(\underline{r}) = \left(\underline{I} + \frac{\underline{r}\underline{r}}{r^3} \right) g(\underline{r}) \quad (3-22)$$

ϵ_{eq}

and

$$g(\underline{r}) = \frac{e^{-jk_{eq}|\underline{r}|}}{4\pi|\underline{r}|} \quad (3-23)$$

with k_{eq} being given in Eq. (3-17). The induced current $\underline{J}(\underline{r})$ is the same as $\underline{J}_0(\underline{r})$ with $E^{(N)}(\underline{r})$ replaced by the average field in the equivalent medium, i.e., $\langle E^{(N)}(\underline{r}) \rangle$. We have

$$\underline{J}(\underline{r}') = \frac{\beta^2}{-j\omega\mu_0} \sum_{n=1}^N \langle \underline{E}(\underline{r}_n) \rangle \delta(\underline{r}' - \underline{r}_n) \underline{a}_y \quad (3-24)$$

Next we take the transverse Fourier transform of the scattered field (the transform was defined in Eq. (2-44)). Noting the fact that Eq. (3-21) is a convolution, we have

$$\underline{E}_s(\underline{k}_t, z) = -j\omega\mu_0 \int \underline{G}(\underline{k}_t, z - z') \cdot \underline{J}(\underline{k}_t, z') dz' \quad (3-25)$$

where $\underline{E}_s(\underline{k}_t, z)$ is the Fourier transform of $\underline{E}_s(\underline{r})$,

$$\begin{aligned} \underline{J}(\underline{k}_t, z) &= \int d\underline{r}_t \underline{J}(\underline{r}) e^{j\underline{k}_t \cdot \underline{r}} \\ &= \frac{\beta^2}{-j\omega\mu_0} \sum_{n=1}^N \langle \underline{E}(\underline{r}_n) \rangle \cdot e^{j\underline{k}_t \cdot \underline{r}_n} \underline{a}_y \delta(z - z_n) \end{aligned} \quad (3-26)$$

and

$$\begin{aligned} \underline{G}(\underline{k}_t, z) &= \int d\underline{r}_t \underline{G}(\underline{r}) e^{j\underline{k}_t \cdot \underline{r}} \\ &= \int d\underline{r}_t \left(1 + \frac{z}{k_{eq}}\right) \underline{g}(\underline{r}) e^{j\underline{k}_t \cdot \underline{r}} \end{aligned} \quad (3-27)$$

$$= \underline{g}(\underline{k}_t, z) + \frac{1}{k_{eq}} \int d\underline{r}_t \underline{r} \underline{g}(\underline{r}) e^{j\underline{k}_t \cdot \underline{r}} \quad (3-28)$$

with

$$\underline{g}(\underline{k}_t, z) = \int d\underline{r}_t \underline{g}(\underline{r}) e^{j\underline{k}_t \cdot \underline{r}} = \frac{e^{-j|z|/2}}{-2j^2 z} \quad (3-29)$$

and

$$k_z = \sqrt{k_{eq}^2 - |\underline{k}_t|^2}, \quad \text{Im} K_z \leq 0 \quad (3-30)$$

If we now integrate the second term in Eq. (3-28) by parts to remove the derivative with respect to x and y , we obtain

$$\tilde{G}(\underline{k}_t, z) = L(\underline{k}_t, z) \tilde{g}(\underline{k}_t, z) \quad (3-31)$$

where

$$L(\underline{k}_t, z) = I + \frac{\gamma \gamma}{k_{eq}^2} \quad (3-32)$$

and

$$\tilde{\gamma} = -j\underline{k}_t + \frac{3}{jz} \underline{a}_z \quad (3-33)$$

The Fourier transform of the scattered field can now be computed explicitly. By using Eqs. (3-26) and (3-31) in Eq. (3-25), we obtain

$$\tilde{E}_s(\underline{k}_t, z) = \sum_{n=1}^N \tilde{E}_s^{(n)}(\underline{k}_t, z; \underline{r}_n) \quad (3-34)$$

where

$$\tilde{E}_s^{(n)}(\underline{k}_t, z; \underline{r}_n) = 3L(\underline{k}_t) \cdot \underline{a}_y \tilde{g}(\underline{k}_t, z - z_n) \langle E(\underline{r}_n) \rangle e^{-j\underline{k}_t \cdot \underline{r}_n} \quad (3-35)$$

Next we remove the specular field from consideration by defining the fluctuating field as follows:

$$\tilde{E}_f(\underline{k}_t, z) = \tilde{E}_s(\underline{k}_t, z) - \tilde{E}_s(\underline{k}_t, z) \quad (3-36)$$

Now we form the dot product of $\tilde{\underline{E}}_f$ with its conjugate and average. Letting $V, N \rightarrow \infty$ such that $N/V = \rho$, we find

$$\langle \tilde{\underline{E}}_f(\underline{k}_{t_1}, z_1) \tilde{\underline{E}}_f^*(\underline{k}_{t_2}, z_2) \rangle = \rho \int \tilde{\underline{E}}_s^{(n)}(\underline{k}_{t_1}, z_1; \underline{r}') \cdot \tilde{\underline{E}}_s^{(n)*}(\underline{k}_{t_2}, z_2; \underline{r}') d\underline{r}' \quad (3-37)$$

Substituting Eq. (3-35) into Eq. (3-37), it follows that

$$\begin{aligned} \langle \tilde{\underline{E}}_f(\underline{k}_{t_1}, z_1) \tilde{\underline{E}}_f^*(\underline{k}_{t_2}, z_2) \rangle &= (2\pi)^2 |\underline{ST}|^2 \rho \int_0^\infty dz' [L(\underline{k}_{t_1}, z_1) \cdot \\ &\quad \underline{a}_y \tilde{g}(\underline{k}_{t_1}, z_1 - z')] \cdot [L^*(\underline{k}_{t_2}, z_2) \underline{a}_y \tilde{g}^*(\underline{k}_{t_2}, z_2 - z')] e^{+2i \underline{k}'_z z'} \\ &\quad S(\underline{k}_{t_1} - \underline{k}_{t_2}) \end{aligned} \quad (3-38)$$

where the mean field from Eq. (3-18) has been used.

Since we require the spectral density in the $z=0$ plane we first use the fact that

$$L(\underline{k}_{t_1}, z_1) \tilde{g}(\underline{k}_{t_1}, z_1 - z') = L(\underline{k}_{t_1}, -z') \tilde{g}(\underline{k}_{t_1}, z_1 - z'), \quad i=1,2 \quad (3-39)$$

in Eq. (3-38) and then set $z_1 = z_2 = 0$. We obtain

$$\langle \tilde{\underline{E}}_f(\underline{k}_{t_1}, 0) \tilde{\underline{E}}_f^*(\underline{k}_{t_2}, 0) \rangle = S(\underline{k}_{t_1}) S(\underline{k}_{t_1} - \underline{k}_{t_2})$$

where

$$S(\underline{k}_{t_1}) = (2\pi)^2 |\underline{ST}|^2 \rho \int_0^\infty dz' [L(\underline{k}_{t_1}, -z') \underline{a}_y \tilde{g}(\underline{k}_{t_1}, z')]^2 e^{+i \text{Im} \underline{k}'_z z'} \quad (3-40)$$

From appendix B, we have

$$\sigma^0 = \frac{k_0^2 \cos^2 \theta}{4\pi^3} S(\underline{k}_{t_0}) , \quad \underline{k}_{t_0} = k_0 \sin \theta \underline{a}_x \quad (3-41)$$

Since there is no k_y component of the incident wave, we have

$$\underline{L}(\underline{k}_{t_0}, -z') \cdot \underline{a}_y = \underline{a}_y \quad (3-42)$$

Now putting Eq. (3-40) into Eq. (3-41), we obtain

$$\sigma^0 = \frac{\partial k_0^2}{\pi} \frac{\partial T|^2 \cos^2 \theta}{\pi} \int_0^\infty dz' |\tilde{g}(\underline{k}_{t_0}, z')|^2 e^{+2\text{Im}k'_z z'} \quad (3-43)$$

By using Eq. (3-29) in Eq. (3-43) and evaluating the integral, we find

$$\sigma^0 = \frac{\partial k_0^2}{16\pi} \frac{\partial T|^2 \cos^2 \theta}{|k'_z|^2 \text{Im}k'_z} \quad (3-44)$$

with

$$k'_z = \sqrt{k_0^2 \cos^2 \theta + \partial^2} \quad (3-45)$$

In the study of vegetation such as forest canopies, we find that $\partial \ll 1$. Then Eq. (3-45) can be written approximately as

$$k'_z = k_0 \cos \theta + \frac{\partial^2}{2k_0 \cos \theta} + O(\partial^4) , \quad (3-46)$$

Using this in Eq. (3-19), we see $T \approx 1$, i.e., there is very little reflection at the interface. Employing this result

and the small $\rho\delta$ approximation given in Eq. (3-46), the formula for σ° given in Eq. (3-44) can be substantially simplified. We find

$$\sigma^\circ = \frac{|K|^2 (k_0 a)^3 \cos\theta}{2 |\text{Im}K|} \quad (3-47)$$

This is our final result. We see that it is independent of the density ρ to first order in $\rho\delta$. The angular variation is a cosine. This angular dependence corresponds to the third empirical model proposed by Clapp^(10,11), however, Clapp did not determine the multiplicative constant as we have for the case of small spheres.

Various limitations on the above formula should be pointed out: first, the Foldy closure assumption which allowed us to obtain an equation for the mean wave is most likely only good when $\beta\rho \ll 1$; second, the backscatter angle must be bounded away from grazing or the approximation in Eq. (3-46) will not be valid; and third, the spheres must have sufficient loss so that the mean wave does not penetrate too far into the medium. This insures the applicability of the Born approximation to compute the backscatter.

IV. EMPIRICAL TECHNIQUES AND NUMERICAL RESULTS

In this chapter we compare our results to an empirical model developed by Attema and Ulaby⁽¹²⁾. The comparison aids in the physical interpretation of the results obtained in Chapter III. Following this, we evaluate the backscattering cross-section obtained in Chapter III for various parameters. Plots of σ^0 versus the angle of incidence, θ , are presented for different values of frequency and sphere radii a .

4-1 EMPIRICAL MODELING

We have studied the scattering properties of vegetated terrain by treating the target as a collection of lossy dielectric spheres and deriving the backscattering cross-section directly from Maxwell's equations and the statistics of the medium. Attema and Ulaby treated the same problem by a more empirical approach which we will describe below.

They assumed the vegetation could be modeled by a layer of water droplets having thickness h . The basic geometry of the model is shown in Fig. 4-1. To keep the model as simple as possible, the following assumptions were made: first, the water droplet cloud representing the vegetation consisted of identical water particles, uniformly distributed throughout the layer; second, only "single scattering" was considered. Here single scattering was taken to mean single scattering by the effective or average wave in the medium.

The reflectivity factor or radar cross-section per unit volume and the power attenuation coefficient per unit length

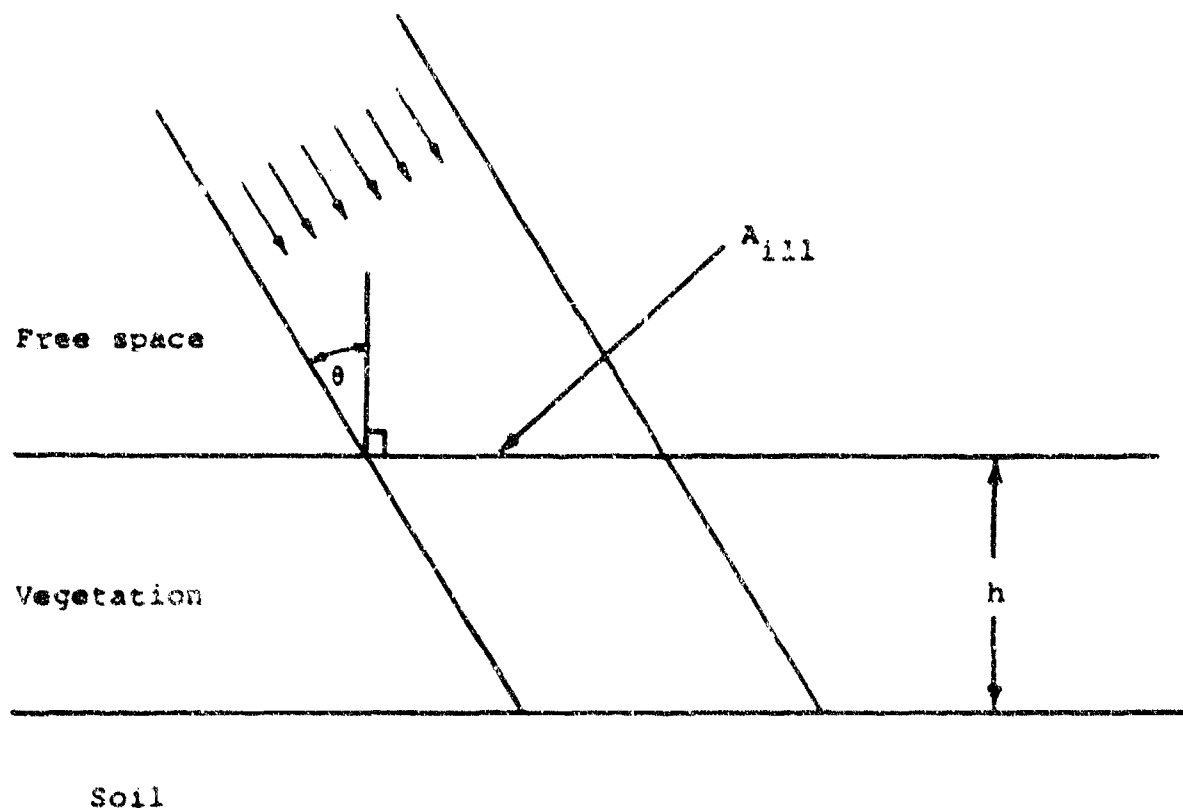


Fig. 4 - 1 Beam Incident Upon Slab of Vegetation

were expressed as

$$\eta = \rho \sigma \quad (4-1)$$

$$\alpha = \rho Q \quad (4-2)$$

where ρ is the number of water particles per unit volume, σ is the radar backscattering cross-section of a single droplet and Q is the total attenuation cross-section for a single particle.

To calculate the average backscattered power due to the incident radar beam, the contributions by all particles in the beam are summed, taking into account the two-way attenuation by the vegetation layer between the scattering particle and

the vegetation surface. Referring to Figure 4-1, the incident wave is assumed to be a plane wave with power density S confined to an ideal cylindrical beam illuminating the surface at an angle of incidence θ . Let the illuminated area, A_{ill} , be defined as the intersection of the beam with the horizontal plane at the top of the vegetation layer. The incident power P_i will then be given by:

$$P_i = SA_{ill} \cos \theta \quad (4-3)$$

and the average backscattered power $\langle P_r \rangle$ is then found to be

$$\langle P_r \rangle = SA_{ill} \cos \theta \int_0^{h/\cos \theta} \exp(-2xz) dz \quad (4-4)$$

Consequently,

$$\sigma^0 = \frac{\langle P_r \rangle}{SA_{ill}} = \left(\frac{\rho g}{2\alpha} \right) [1 - \exp(-2\alpha h / \cos \theta)] \cos \theta \quad (4-5)$$

This is the final formula they used to relate σ^0 to the medium cross sections and density.

For our purposes the formula can be simplified by first assuming $\alpha h \gg 1$; thus Eq. (4-5) becomes

$$\sigma^0 = \frac{\rho g}{2\alpha} \cos \theta \quad (4-6)$$

Now we specialize considerations to Rayleigh scattering, i.e., we assume the wavelength is large compared to the droplet size. If we further assume the droplets are spheres of radius a , then the backscattering cross-section of an individual sphere

is given approximately by⁽¹³⁾

$$\sigma \approx 4\pi k_0^4 |K|^2 a^6 \quad (4-7)$$

where K has been defined in Appendix B. Next, we write Q as a sum of an absorption cross section, Q_a , and scattering cross-section, Q_s .

$$Q = Q_a + Q_s \quad (4-8)$$

In the Rayleigh limit for spherical scatterers, we have⁽¹³⁾

$$Q_a \approx 4\pi k_0 \operatorname{Im} K a^3 \quad (4-9)$$

and

$$Q_s \approx \frac{8\pi}{3} k_0^4 |K|^2 a^6 \quad (4-10)$$

For the X band region of the spectrum under consideration, the attenuation cross-section is much larger than the scattering cross-section; i.e., $Q_s \ll Q_a$. Thus

$$Q \approx 4\pi k_0 \operatorname{Im} K a^3 \quad (4-11)$$

Now using Eqs. (4-2), (4-7) and (4-11) in Eq. (4-6), we obtain

$$\sigma^* = \frac{|K|^2 (k_0 a)^3 \cos \theta}{2 \operatorname{Im} K} \quad (4-12)$$

This is exactly the expression obtained for σ^* in Chapter III. Thus we see in the Rayleigh limit for spheres we can be satisfied that the empirical technique and the analytic vector

method give the same results.

4-2 NUMERICAL EVALUATION OF σ^0

In this section the backscattering cross-section as given in Eq. (4-12) is evaluated for several sphere sizes and frequencies as a function of incidence angle, θ . We assume that the spheres are droplets of water. The complex permittivity of water as given in Peake⁽¹⁶⁾ is

$$\epsilon_r = 5 + \frac{75}{1 + j11.85/\lambda}$$

where λ is the wavelength given in centimeters.

The plots are shown in Figs. 4-2 and 4-3. The frequencies used were 9 and 15 GHz respectively. An examination of the plots shows that the general behavior of the plots is in agreement with experimental data⁽¹²⁾ if spheres in the order of a few millimeters are chosen. To obtain more definitive information the model in Chapter III would have to be generalized to consider discs and cylinders. This work is in progress.

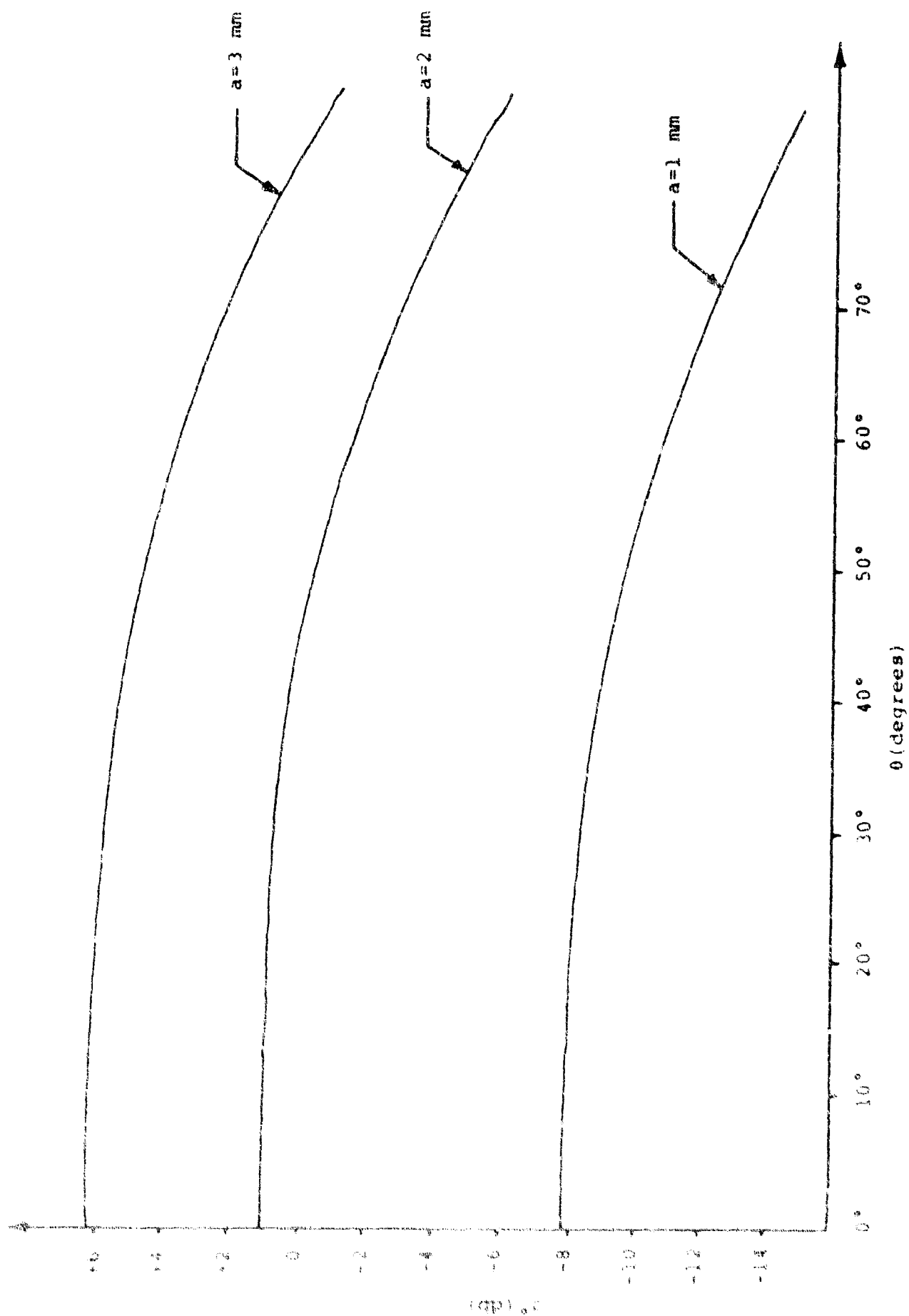


Fig. 4 - 2 Backscattering Coefficient versus Angle of Incident at 9 GHz

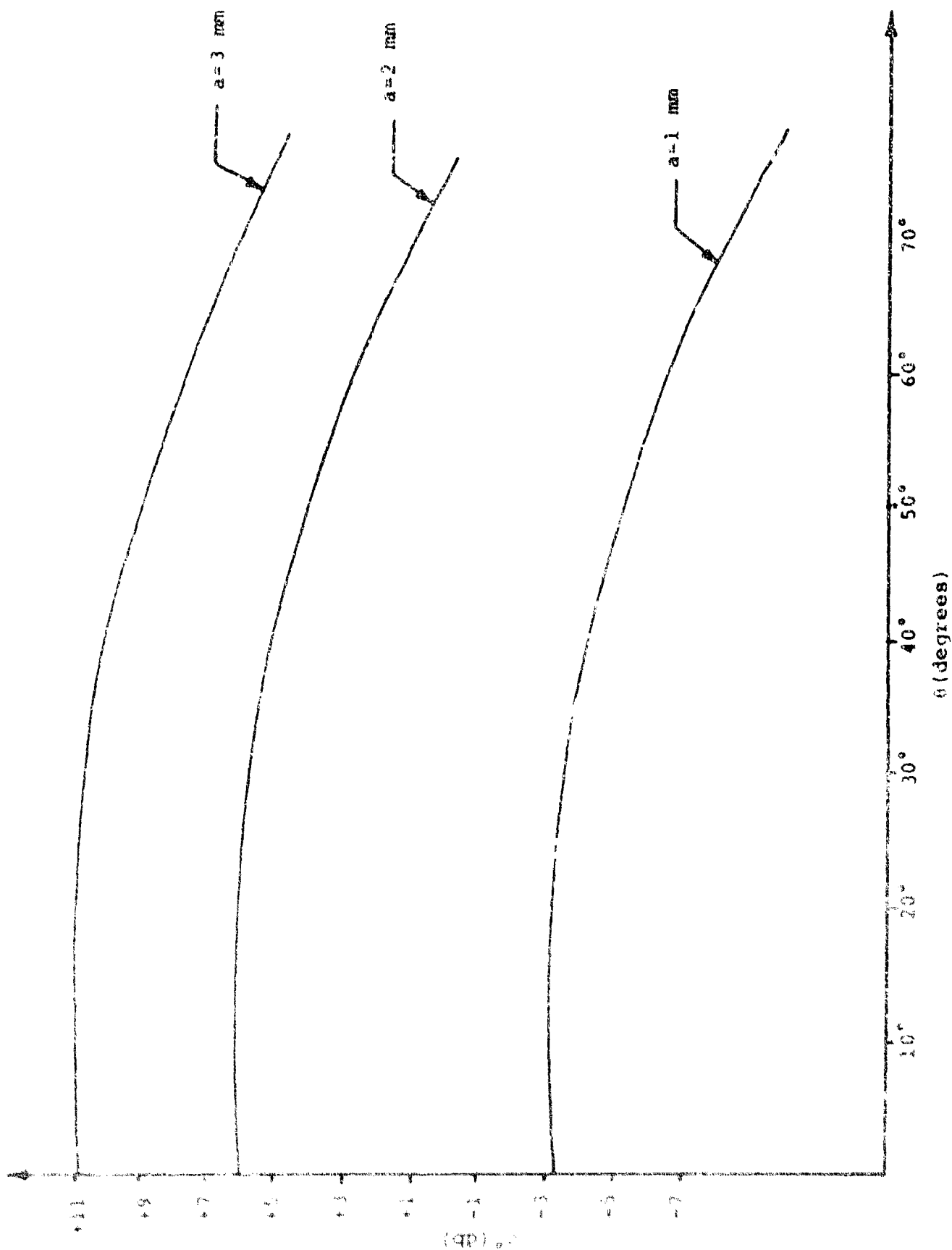


Fig. 4 - Backscattering Coefficient versus Angle of Incident at 15 GHz

V. DISCRETE ONE DIMENSIONAL RANDOM SLAB

The preceding two chapters have been concerned with backscattering from three dimensional scatterers that were randomly distributed. The calculation of the backscattering coefficient was performed by employing Foldy's approximate method to calculate the mean field. In this chapter we will compare the Foldy approximation for the average dielectric constant to the expression obtained in the thermodynamic limit, that is, in the limit when particle size becomes small and particle density large. To accomplish this, we will restrict our attention to one dimensional discrete models. Here the medium is composed of identical dielectric slabs which are distributed probabilistically. The results obtained should carry over to the three dimension case, however, this work has not been completed at this time.

The methods used in this chapter differ substantially from those employed previously. First, identical slabs of width " a " are chosen so that they obey a renewal process which will be described in more detail in the next section. This process has the important property that overlapping of slabs never occurs. If one chooses the slab position to be distributed in a Poisson manner, then overlapping will occur.

Using a process without overlapping is desirable since this corresponds to more physical situations of interest.

Following the choice of the renewal process, differential equations are found for the probability of occurrence and nonoccurrence of a slab. The solutions to these equations enable one to calculate the average dielectric constant of the random medium exactly.

In the remainder of the chapter, the renewal process is used to drive a nonlinear system of ordinary differential equations having solution $u(t)$. A diffusion equation is derived for the probability density of $u(t)$ and the solution to this equation is obtained in the thermodynamic limit. Finally, these general results are applied to the one dimensional slab and compared to Foldy's results.

5-1 SLAB RENEWAL PROCESS

In this section we will describe the random process that we will use to model the one dimensional random medium. The medium will be composed of dielectric slabs of width "a" which are placed in a random manner to be specified. The dielectric variation, $\epsilon(z)$ of the medium is related to the random process $\epsilon(z)$ as follows:

$$\epsilon(z) = \epsilon_0 \epsilon_r(z) \quad (5-1)$$

where ϵ_0 is the free space permittivity and c is a constant determining the strength of the fluctuations. The process $r(z)$ takes on values 0 or 1 if z is outside a slab or inside a slab respectively.

The probabilistic character of the process $r(z)$ will now be specified. Associate the random variable Z_i with the trailing edge of the i^{th} slab. Rather than specify the Z_i directly, we form the difference

$$W_i = Z_i - Z_{i-1}, \quad i = 1, 2, 3, \dots \quad (5-2)$$

where $Z_0 = 0$. The W_i are just the distance between trailing edges of adjacent slabs. We assume that the W_i are independent identically distributed random variables. This definition of the W_i forces the process $r(z)$ to be a renewal process⁽¹⁴⁾. We now choose the distribution of the W_i to be

$$P_{W_i}(W) = \lambda e^{-\lambda(W-a)} u(W-a) \quad (5-3)$$

where $u(W)$ is the step function. For small " a ", the parameter λ can be interpreted as the number of pulses per unit length or the pulse density.

This assumed distribution has two important properties. First, W cannot get smaller than " a " and thus slabs cannot overlap. This is a critical physical assumption since if the slabs overlapped the dielectric constant would double in the overlap region. This would lead to an unphysical result. A

second property of Eq. (5-3), is that for small "a" the distance between slabs is approximately exponentially distributed. This implies that the slabs are randomly distributed which is physically appealing. If the z_i had been chosen to be uniformly distributed at the outset, then the process would have been Poisson. This is a usual assumption made, however, it leads to an overlap problem.

Since the process $r(z)$ has now been defined, some of its properties can be calculated from its probability density $P_r(z)$ where

$$P_r(z) = \begin{cases} p_0(z) & , \quad r=0 \\ p_1(z) & , \quad r=1 \end{cases} \quad (5-4)$$

Here $p_0(z)$ is the probability that the point z is outside a slab while $p_1(z)$ is the probability that z is located inside a slab. We see that

$$\langle \epsilon(z) \rangle = \epsilon_0 \langle (1 + \alpha r(z)) \rangle \quad (5-5)$$

and

$$\begin{aligned} \langle r(z) \rangle &= 0 \cdot p_0(z) + 1 \cdot p_1(z) \\ &= p_1(z) \end{aligned} \quad (5-6)$$

Thus a knowledge of $p_1(z)$ will allow us to calculate the average dielectric constant. Using the method of conservation of probability⁽¹⁵⁾ one can show that $p_0(z)$ and $p_1(z)$ satisfy the following equations:

$$\frac{dp_0}{dz} = -p_0(z) + p_0(z-a)u(z-a) \quad (5-7)$$

$$p_1(z) = 1 - p_0(z) \quad (5-8)$$

Eq. (5-7) can be solved by employing Laplace transform techniques. Denoting $\phi_0(s)$ as the Laplace transform of $p_0(z)$, i.e.

$$\phi_0(s) = \int_0^\infty p_0(z) e^{-sz} dz \quad (5-9)$$

the solution to Eq. (7) becomes

$$\phi_0(s) = \frac{1}{s+1 - (e^{-as})} \quad (5-10)$$

Now employing the inverse Laplace transform $p_0(z)$ becomes

$$p_0(z) = \frac{1}{2\pi i} \int_{\delta-i\infty}^{\delta+i\infty} \frac{e^{sz}}{s+1 - (e^{-as})} ds, \quad \delta > 0 \quad (5-11)$$

Now using Eqs. (5-5) (5-6), we find

$$\langle \epsilon(z) \rangle = \epsilon_0 \left\{ 1 + \left(1 - \frac{1}{2\pi i} \int_{\delta-i\infty}^{\delta+i\infty} \frac{e^{sz}}{s+1 - (e^{-as})} ds \right) \right\} \quad (5-12)$$

In general the mean will be a function of z and thus the process is not stationary; however, an examination of the mean for large z shows it approaches a constant value. By using the final value theorem, we find

$$\lim_{z \rightarrow \infty} p_0(z) = \lim_{s \rightarrow 0} s \phi_0(s) = \frac{1}{1+1a} \quad (5-13)$$

A more detailed calculation shows that this constant mean is approached within a few pulse distances. Thus the nonstationary character of the mean is just due to the boundary effects

at $z=0$.

Before concluding this section, we will discuss the thermodynamic limit. In this limit we let the pulse width $a \rightarrow 0$ as the pulse density $\lambda \rightarrow \infty$. We do this in such a way that $\lambda a = \text{constant} = \alpha$. In this case, the medium approaches a non-trivial limit which we call the macroscopic or thermodynamic limit. Performing this limiting operation on Eq. (5-12), we find

$$\langle \epsilon(z) \rangle = \epsilon_0 \left\{ 1 + \alpha \left(1 - \frac{1}{1-\alpha} \right) \right\}, \quad z > 0 \quad (5-14)$$

Thus we see the mean has a constant value for all z greater than zero. We shall return to this behavior at a later point.

5-2 NONLINEAR SYSTEM DRIVEN BY RENEWAL PROCESS

We will now divert our attention from the specific study of the one dimensional problem and obtain a result that will be needed in the next section. Consider an N^{th} order system of nonlinear ordinary differential equations driven by a renewal process $r(z)$ i.e.

$$\frac{d\mathbf{U}(z)}{dz} = \mathbf{F}(\mathbf{U}, r(z), z), \quad z \geq 0 \quad (5-15)$$

where

$$\mathbf{U}(0) = \mathbf{u}_0, \quad \text{w.p. } 1$$

Here $\mathbf{U}$, $\mathbf{u}_0$ and $\mathbf{f}$ are N dimensional vectors. Now by employing the conservation of probability argument⁽¹⁵⁾, a diffusion equation can be derived for the joint probability density

$p_0(\underline{u}, z)$ where

$$p_i(\underline{u}, z) d\underline{u} = \text{Prob} \left\{ (\underline{u} < \underline{U}(z) \leq \underline{u} + d\underline{u}) (r(z) = i) \right\}, \quad (5-16)$$

with $d\underline{u} = \sum_{i=1}^N du_i$. The diffusion equation is

$$\begin{aligned} \frac{\partial p_0(\underline{u}, z)}{\partial z} = & - \sum_{i=1}^N \frac{\partial}{\partial u_i} (f_i(\underline{u}, 0, z) p_0(\underline{u}, z) - \lambda p_0(\underline{u}, z) \\ & + J(z-a, \underline{u}, z) p_0(\underline{u}(z-a; \underline{u}, z), z-a) \underline{u}(z-a)) \quad (5-17) \\ p_0(\underline{u}, 0) = & \delta(\underline{u} - \underline{u}_0) \end{aligned}$$

where $\delta(\underline{u} - \underline{u}_0)$ is an N dimensional delta function. Here $\underline{u}(\xi; \underline{u}, z)$ is the solution to the final value problem

$$\begin{aligned} \frac{\partial \underline{u}}{\partial \xi} &= \underline{f}(\underline{u}, 1, \xi), \quad \xi \leq z \\ \underline{u}(z; \underline{u}, z) &= \underline{u} \end{aligned} \quad (5-18)$$

and $J(\xi, \underline{u}, z)$ is the Jacobian of the transformation from $\underline{u}$ to $\underline{u}$, i.e.

$$J(\xi, \underline{u}, z) = \det \left(\frac{\partial \underline{u}_i}{\partial u_j} \right) \quad (5-19)$$

Although the equation is complicated, one must realize it is an exact equation with no approximations for $p_0(\underline{u}, z)$ and the equation is deterministic.

Again employing conservation of probability arguments, an additional equation can be obtained that relates $p_i(\underline{u}, z)$ to $p_0(\underline{u}, z)$. It is

$$p_1(\underline{u}, z) = \lambda \int_{(z-a)_+}^z p_0(\underline{u}(\xi; \underline{u}, z), \xi) J(\xi, \underline{u}, z) d\xi \quad (5-20)$$

where p_1 is defined in Eq. (5-16) and $(z)_+$ is z if $z > 0$ and zero if $z \leq 0$.

We would now like to see how these equations reduce in the thermodynamic limit, i.e., as $a \rightarrow 0$ and $\lambda \rightarrow \infty$ such that $\lambda a = 1$. Expanding Eqs. (5-17) and (5-20) in a power series in " a ", replacing λ by $1/a$ and keeping only dominant terms, we obtain

$$\begin{aligned} \frac{\partial p_0(\underline{u}, z)}{\partial z} &= - \frac{1}{1+\epsilon} \sum_{i=1}^N \frac{\partial}{\partial u_i} \left(f_i(\underline{u}, 0, z) p_0 \right) \\ &\quad - \frac{a}{1+\epsilon} \sum_{i=1}^N \frac{\partial}{\partial u_i} \left(f_i(\underline{u}, 1, z) p_0 \right) \end{aligned} \quad (5-21)$$

$$p_0(\underline{u}, 0) = \frac{1}{1+\epsilon} \delta(\underline{u} - \underline{u}_0)$$

$$p_1(\underline{u}, z) = a p_0(\underline{u}, z) \quad (5-22)$$

The probability density with respect to just $\underline{u}$ can be obtained by summing out the r variable. We have

$$p(\underline{u}, z) = p_0(\underline{u}, z) + p_1(\underline{u}, z) \quad (5-23)$$

By using Eqs. (5-21) and (5-22) an equation for $p(\underline{u}, z)$

$$\frac{\partial p}{\partial z} = - \sum_{i=1}^N \frac{\partial}{\partial u_i} \left(\langle f_i(\underline{u}, r, z) \rangle p \right) \quad (5-24)$$

$$p(\underline{u}, 0) = \delta(\underline{u} - \underline{u}_0)$$

If we now make the important assumption that f depends linearly upon the renewal process r then we can write

$$\langle f_i(\underline{u}, r, z) \rangle = f_i(\underline{u}, \langle r \rangle, z) \quad (5-25)$$

Using this result in Eq. (24) one has

$$\frac{\partial p}{\partial z} = \sum_{i=1}^N \frac{\partial}{\partial u_i} \left(f_i(\underline{u}, \langle r \rangle, z) p \right) \quad (5-26)$$

$$p(\underline{u}, 0) = \delta(\underline{u} - \underline{u}_0)$$

By direct substitution, it can be shown that the solution to Eq. (5-26) is

$$p(\underline{u}, z) = \delta(\underline{u} - \hat{\underline{u}}(z)) \quad (5-27)$$

where

$$\frac{d\hat{\underline{u}}}{dz} = \underline{f}(\hat{\underline{u}}, \langle r \rangle, z) \quad (5-28)$$

$$\hat{\underline{u}}(0) = \underline{u}_0$$

Thus we see the probability density in the thermodynamic limit is a delta function, i.e., at each point $z, \underline{u}$ only takes on one value $\hat{\underline{u}}(z)$. Therefore the solution is deterministic. Note that this deterministic solution, $\hat{\underline{u}}(z)$, obeys the same equation as the random solution, $\underline{u}(z)$, Eq. (5-15) with r replaced by $\langle r \rangle$. Thus in this limit the process acts in a deterministic way. There are so many particles per unit length that the system only sees the average effect of the particles. This result will be used in the next section.

5-3 REFLECTION FROM A DISCRETE ONE DIMENSIONAL SLAB

In this section we will conclude the discussion of one dimensional problems by employing the results of the last two sections in the treatment of a slab of one dimensional discrete

random medium. We shall calculate the average reflected wave and compare these results with those obtained by the Foldy's technique.

Consider a scalar problem having field component u which obeys the reduced wave equation:

$$\frac{d^2}{dz^2} + k^2 \epsilon(z) u = 0, \quad -\infty < z < \infty \quad (5-29)$$

when k is the free space wave number of the medium and $\epsilon(z)$ is the dielectric permittivity. The domain of the variable permittivity is confined to $0 < z < L$. Outside this region $\epsilon(z)$ takes on a constant value one. Inside the slab region the dielectric fluctuations are modelled by a renewal process as follows:

$$\epsilon(z) = \begin{cases} 1 & , \quad z > L \\ 1 + r(z) & , \quad 0 < z < L \\ 1 & , \quad z < 0 \end{cases} \quad (5-30)$$

where $r(z)$ has been defined earlier. We assume a plane wave is normally incident upon the slab from the right and a reflected and transmitted wave is generated in the homogeneous region to the right and left of the slab respectively. This is shown in FIG. 5-1.

The field in the homogeneous regions can be written as

$$u(z) = \begin{cases} e^{-ik(z-L)} + r(L) e^{ik(z-L)} & , \quad z > L \\ T(L) e^{-ikz} & , \quad z < 0 \end{cases} \quad (5-31)$$

where $r(L)$ is the reflection coefficient of the slab and $T(L)$ is the transmission coefficient. We have explicitly indicated

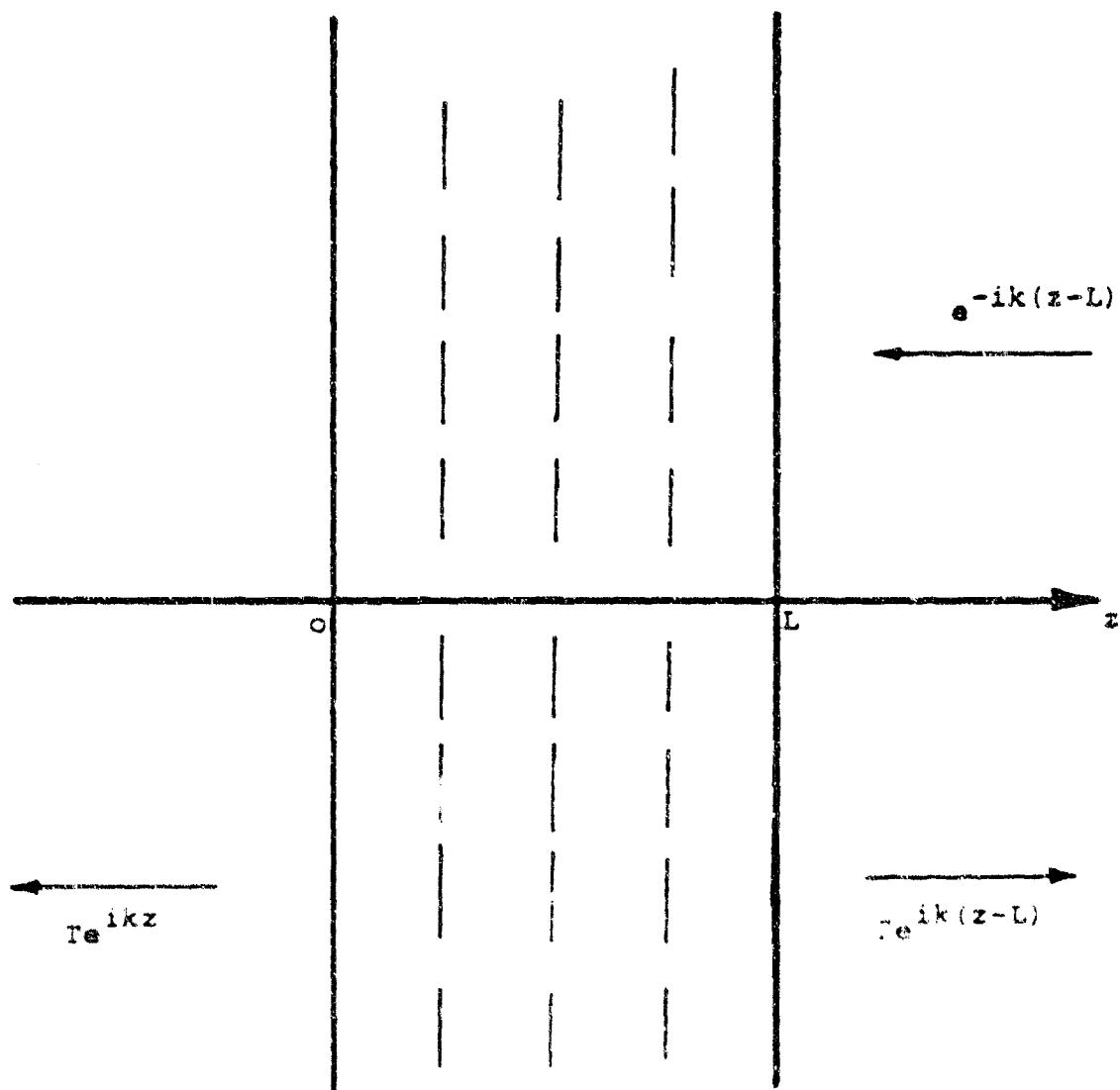


Fig. 5 - 1 One-Dimensional Random Slab

that both r and T are functions of the slab thickness.

Since we want to draw upon the results of the last section, the boundary value problem of Eq. (5-29) will have to be converted to an initial value problem. This can be accomplished by looking at the reflection coefficient $r(L)$ rather than the field $u(z)$. One can derive a Ricatti equation for this reflection coefficient. It is

$$\frac{dr}{dL} = ikr + \frac{ikr(L)}{2} (1+r(L))^2 \quad (5-32)$$

$$r(0) = 0 \quad (5-33)$$

We see it is a first order nonlinear ordinary differential equation. The problem is now an initial value problem since the condition in Eq. (5-33) is only given at one point.

The problem has almost been cast in the form of the last section except for the fact that Eq. (5-32) is complex. To transform Eq. (5-30) into a real form, we let

$$r = \rho e^{i\phi} \quad (5-34)$$

where ρ is the amplitude of r and ϕ is its phase. Plugging this into Eq. (5-32) and equating real and imaginary parts, we find

$$\frac{d\rho}{dL} = \frac{k}{2} \rho r(L) (1-\rho^2) \sin\phi \quad (5-35)$$

$$\frac{d\phi}{dL} = 2k + \frac{k\rho r(L)}{2} (2 + (1-\rho^2)^{-1}) \cos\phi$$

$$\rho(0) = 0 \quad \phi(0) = \phi_0$$

This is the same as the system given in Eq. (5-15) with z

replaced by L and with

$$\underline{U} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \underline{U}_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

In addition, since the system is linear in $r(L)$, the results derived for the thermodynamic limit hold for Eq. (5-35). Basically this says that when $a \rightarrow 0$ and $\lambda \rightarrow \infty$ such that $a\lambda = a$ the solution for ϵ and ρ is deterministic and can be obtained by replacing $r(L)$ by $\langle r(L) \rangle$. Physically this says that in the thermodynamic limit the solution to the problem can be obtained by replacing the statistical medium $r(z)$ by its average $\langle r(z) \rangle$. Thus the medium can be replaced by one having an average dielectric constant of

$$\begin{aligned} \langle \epsilon(z) \rangle &= \epsilon_0 (1 + \langle r(z) \rangle) \\ \langle r(z) \rangle &= p_1(z) = \frac{1}{1+i} \end{aligned} \tag{5-36}$$

An analysis of the one dimensional problem by the Foldy technique leads to the following result for the average dielectric constant:

$$\langle \epsilon(z) \rangle_F = \epsilon_0 (1 + a) \tag{5-37}$$

Thus we see that the results from the Foldy technique overlap with the thermodynamic limit results when $a \rightarrow 1$. It also indicates that the Foldy method is not correct when $a \rightarrow \infty$, $a \rightarrow 0$ and a is moderate or large.

Although the analysis at present has not resolved the nature of the differences between the "analytic theory" and the "transport theory", the existence of an exact equation for the

probability density should prove to be a useful tool in the future.

IV. CONCLUSION AND RECOMMENDATIONS

We have developed and analyzed a model for a half space of vegetation. This model views the vegetation as a collection of lossy dielectric spheres. Within the constraints of this model the backscattering cross section has been related to sphere size and dielectric constant.

In Section II the scalar problem was analyzed by the Foldy technique. This section served the purpose of illustrating the technique, however, because of the scalar nature of the problem a complete relationship between the medium parameters and backscattering cross section could not be obtained. In Section III this defect was remedied by applying the Foldy technique to the complete electromagnetic problem. The case of horizontally polarized waves was treated and an expression was obtained relating the radar cross section to the medium parameters. This expression was found in agreement with the angular dependence of Clapp's third model. An interesting feature of this expression is that it was independent of density.

In Section IV the results of Section III were compared with the data presented by Attenu and Ubiy. The agreement was found to be quite good for water droplets of 1 to 5 mm in the X band region of the spectrum. In addition a connection was established between the more empirical analysis of Attenu and Ubiy and the systematic procedure employed in Section III. The connection showed that the method we employed has, in effect, assumed that the scattering cross section of an

individual particle is small compared to the absorption cross section. The validity of this assumption in the frequency range of interest is shown in Figure 4-4.

Finally in Section V, we explored some fundamental limitations of the Foldy technique. There we saw by employing a one dimensional mode, that the Foldy approximation only gives correct results when the product of slab width and density are small.

In view of the success of the method we used in relating physical parameters of the medium to backscattering cross section, we make the following recommendations:

1. Analyze a medium represented by discs having a random location and a probabilistic angular orientation. Since the discs would represent leaves, the wavelength would be of the order of the scattering object. Twersky's technique (a generalization of Foldy's method which removes the Rayleigh assumption) would be employed. The results would be compared to those of Du⁽¹⁶⁾ who calculated backscattering from leaves by a different, more heuristic, technique.
2. By using the technique employed in recommendation 1., analyze the effect of ground lying under the vegetation. This requires the analysis of the slab problem.
3. Investigate the effects of depolarization. See how they compare to the results of continuous random medium models.

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Appendix A

Induced Current on a Dielectric Sphere

When the size of a dielectric sphere is much smaller than the wavelength of the radiation incident upon it, the electric field within and near the sphere behaves like a constant electrostatic field. From elementary electrostatics, we know the scattered field can be represented by a dipole located at the center of the sphere. In this appendix we will calculate the equivalent current $\underline{J}_{eq}(\underline{r})$. This will be derived from the induced dipole moment as given by Stratton.

Let us assume there is an electric field $E(\underline{r})\underline{a}_y$ incident upon a sphere of radius a whose center is located at $\underline{r}=\underline{r}'$. If the sphere's radius is small compared to wavelength then the induced dipole moment density $\underline{P}(\underline{r})$ is given by⁽⁹⁾

$$\underline{P}(\underline{r}) = P(\underline{r})\underline{a}_y = 4\pi\epsilon_0 ka^3 E(\underline{r}') \delta(\underline{r}-\underline{r}')\underline{a}_y \quad (A-1)$$

where $K=(\epsilon_r-1)/(\epsilon_r+2)$ and ϵ_r is the relative dielectric constant of the dielectric sphere.

The equivalent charge density $\rho_{eq}(\underline{r})$ used to represent the scattered field is then

$$\rho_{eq}(\underline{r}) = -\nabla \cdot \underline{P}(\underline{r}) \quad (A-2)$$

$$= -4\pi\epsilon_0 ka^3 E(\underline{r}') \delta'(x-x') \delta'(y-y') \delta'(z-z') \quad (A-3)$$

when $\delta'(z)$ is the derivative of $\delta(z)$ with respect to z . From the continuity equation

$$\nabla \cdot \underline{J}_{eq}(\underline{r}) = -j\omega\rho_{eq}(\underline{r}) \quad (A-4)$$

we find

$$\underline{J}_{eq}(\underline{r}) = J_0 \delta(\underline{r}-\underline{r}') \underline{a}_y \quad (A-5)$$

where

$$J_0 = j\omega 4\pi\epsilon_0 ka^3 E(\underline{r}') \quad (A-6)$$

Thus we can replace each sphere whose radius is small compared to wavelength by equivalent dipole current as given in Eqs. (A-5) and (A-6).

Appendix B

Relationship Between Backscattering Coefficient and Transverse Spectral Density

In both Chapters II and III we have used the relationship between transverse power spectral density $S(\underline{k}_t)$ and σ^0 . In this appendix we will derive that relationship.

Assume that a vector component of the scattered field, say $\phi_s(\underline{x}_t, 0)$ is known on the boundary ($z=0$ plane) between the random medium ($z>0$) and free space ($z<0$). Then the far field in the free space region due to a region A on the plane $z=0$ is given by the Kirchhoff diffraction formula⁽¹⁷⁾. It is

$$\phi_s(\underline{r}) = \frac{jk_0 \cos \theta}{2\pi r} e^{-jk_0 r} \int_A \phi_s(\underline{x}'_t, 0) e^{jk_0 \underline{x}'_t \sin \theta} d\underline{x}'_t \quad (B-1)$$

where θ is measured with respect to the normal to the $z=0$ plane as is shown in Fig. 2-1 and $r = |\underline{r}| = \sqrt{x^2 + y^2 + z^2}$.

The backscattering coefficient σ^0 is defined as

$$\sigma^0 = \lim_{A \rightarrow \infty} \frac{4\pi r^2 I_r}{A I_i} \quad (B-2)$$

where I_r is the average intensity at the receiver of the fluctuating scattered field, i.e., $I_r = \langle \phi_f(\underline{r}) \phi_f^*(\underline{r}) \rangle$ where $\phi_f = \phi_s - \langle \phi_s \rangle$, and I_i is the field intensity incident upon the illuminated area A. Assuming an incident field of the form

$$\phi_i(\underline{r}) = e^{jk_0(x \sin \theta - z \cos \theta)} \quad (B-3)$$

then we have $I_i = |\phi_i(\underline{r})|^2 = 1$.

The average at the receiver from Eq. (B-1) is given by

$$I_r = \langle \phi_f(\underline{r}) \phi_f^*(\underline{r}) \rangle$$

$$= \frac{k_0^2 \cos^2 \theta}{(2\pi r)^2} \int_A d\underline{r}'_t \int_A d\underline{r}''_t \langle \phi_f(\underline{r}'_t, 0) \phi_f^*(\underline{r}''_t, 0) \rangle e^{jk_0(\underline{x}' - \underline{x}'') \sin \theta} \quad (B-4)$$

where $\phi_f(\underline{r}'_t, 0) = \phi_s(\underline{r}'_t, 0) - \langle \phi_f(\underline{r}'_t, 0) \rangle$. Using Eqs. (B-3) and (B-4) in Eq. (B-2), we have

$$\sigma^0 = \lim_{A \rightarrow \infty} \frac{k_0^2 \cos^2 \theta}{\pi A} \int_A d\underline{r}'_t \int_A d\underline{r}''_t \langle \phi_f(\underline{r}'_t, 0) \phi_f^*(\underline{r}''_t, 0) \rangle e^{jk_0(\underline{x}' - \underline{x}'') \sin \theta} \quad (B-5)$$

We will now use the transverse Fourier transform of $\phi_f(\underline{r}_t, 0)$ and its conjugate. Denoting the transform of $\phi_f(\underline{r}_t, 0)$ by $\tilde{\phi}_f(\underline{k}_t, 0)$, we have

$$\phi_f(\underline{r}_t, 0) = \frac{1}{(2\pi)^2} \int \tilde{\phi}_f(\underline{k}_t, 0) e^{j\underline{k}_t \cdot \underline{r}_t} d\underline{k}_t \quad (B-6)$$

Using this in Eq. (B-5),

$$\sigma^0 = \lim_{A \rightarrow \infty} \frac{k_0^2 \cos^2 \theta}{(2\pi)^4 \pi A} \int d\underline{k}'_t d\underline{k}''_t \int_A d\underline{r}'_t \int_A d\underline{r}''_t \langle \tilde{\phi}_f(\underline{k}'_t, 0) \tilde{\phi}_f^*(\underline{k}''_t, 0) \rangle e^{j(\underline{k}' - \underline{k}'') \cdot \underline{r}'_t} e^{-j(\underline{k}' - \underline{k}'') \cdot \underline{r}''_t}$$

Now we made use of the fact that the $\phi_f(\underline{r}_t, 0)$ is a wide sense stationary process, i.e., $\langle \phi_f(\underline{r}_t, 0) \phi_f^*(\underline{r}''_t, 0) \rangle = \langle \phi_f(\underline{r}_t, 0) \phi_f^*(\underline{r}_t, 0) \rangle$ and

$$\langle \bar{\phi}_f(\underline{k}_t', 0) \bar{\phi}_f^*(\underline{k}_t'', 0) \rangle = S(\underline{k}_t') \delta(\underline{k}_t' - \underline{k}_t'') \quad (B-8)$$

where $S(\underline{k}_t')$ is the transverse power spectral density of the interface field. Putting Eq. (B-8) into Eq. (B-7) and making the change of variables

$$\begin{aligned} \underline{\bar{r}} &= \underline{r}' - \underline{r}'' \\ \underline{r}'' &= \underline{r}'' \end{aligned} \quad (B-9)$$

we find

$$\sigma^* = \lim_{A \rightarrow \infty} \frac{k_0^2 \cos^2 \theta}{2^4 \pi^5} \int d\underline{k}_t' \int_A d\underline{\bar{r}} S(\underline{k}_t') e^{-j(\underline{k}_t' \cdot \underline{\bar{r}} - k_0 \bar{r} \sin \theta)} \quad (B-10)$$

Illuminating the whole plane ($A \rightarrow \infty$), the final result is obtained. It is

$$\sigma^* = \frac{k_0^2 \cos^2 \theta}{4\pi^3} S(\underline{k}_t) \quad (B-11)$$

where $\underline{k}_t = k_0 \sin \theta \underline{a}_y$

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