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Final Report. on
Contract N00014-78-C-0833.

by

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Merl L. Kardatzke
Leo R. Katzenstein
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ACADEMY FOR INTERSCIENCE METHODOLOGY

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Contents

Overview

This report summarizes work completed between 1 October 1978 and 30 September 1979 under contract N00014-78-C-0033 for the U.S. Navy (OP604) by the Academy for Interscience Methodology.

Three technical developments are discussed in this report.

Chapter 1 describes soft target methodology in the Force Mix model. The results of research into the possibilities for increasing model efficiency are furnished.

Chapter 2 documents an addition to the Cruise Missile routing capability of the RPM strategic analysis model. This addition constrains development of routes so that barriers are not crossed and so that routes will avoid specific sites.

The development of a method for constructing DGZ's employing multiple yield weapons with avoidance constraints is discussed in Chapter 3. This work was supported by both the U.S. Navy and the Joint Strategic Target Planning Staff (JP). The method has been incorporated into the RPM model.

Chapter 4 contains short descriptions of several support efforts that received attention during the last year.

The span of interesting and relevant tasks that the AIM technical staff was asked to work on during this year is much appreciated. Mr. Paul Garvin has fostered a challenging and technically rewarding atmosphere for our work.

Chapter 1. The Force Mix Model*

A. Organizing Concepts

Long range planning for new strategic weapon systems is founded on the imperative that certain basic requirements be met within constraints of budget and other resources. These requirements do not exist for a single time period. They must be satisfied in forseeable time periods and they may involve several types of targets included in separate countries. Many different weapon systems must be considered. These weapon systems may come into the force or may exist but need modification to extend useful contributions to the force. Alternative force configurations are evaluated with respect to this total environment.

Following the lead of some manual methods that had been used to deal with specific problems, it was decided that a computer implemented linear programming approach would provide a suitable evaluation framework. Damage to general target types that are known as soft targets had been approximated by exponential functions. Damage requirements can be transformed to equivalent warheads by the inverse of the exponential function. Different weapon types have different effectiveness coefficients, and these coefficients multipled by the number of weapons allocated to each target type form the linear equations that are the basis of this model.

Force effectiveness against hard targets requires greater modeling complexity and a completely different formulation. The probability of damage for each weapon type against a single target site is calculated. Hard target deployment has the characteristic of having little interaction between sites. Damage from different weapon types or by a second weapon of the same type are considered. This formulation contains more structural complexity in the LP model, but it gives an adequate representation of damage for up to two warheads of any weapon type on each hard target.

One set of hard targets in each of three countries is included in the model. There are also two soft target types within each country. Required levels of damage can be set for each of these country-target types. In each time era considered, different requirements may be input. Up to nine time periods

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* See References 1 and 2 for further discussion of the Force Mix Model.

are available although three or four have been adequate to show the consequences of decisions that can be made at the present time. The number of weapon systems is open ended so that alternate existing systems can be considered and so that competitive configurations of a proposed system can be compared over the time frames studied.

An important part of this model is the SYSTEM TREE network which relates total weapon systems to components. It ties the availability of components or technology bought in one time period to the weapon system availability in a later time period.

Alternative weapon systems may be able to use the same components such as submarines or benefit from the same developmental effort such as guidance systems. They may compete for the use of the same production facilities such as shipyards. The component subsystem networks that support each weapon system's concept are interrelated with each other and also over time as components are converted from one kind of weapon system to another. There are costs associated with conversion for example as when submarine launch tubes are changed. This network provides a more adequate way to include costs rather than using system costs which depend on which other systems are developed and supported. Budgetary constraints, production rates, conversion rates, and buy-in costs can all be represented in this part of the model. Buy-in cost gives rise to binary variables that require mixed integer programming solutions.

In running a particular problem the linear programming model may be used to minimize cost, minimize a uniform budget, or to maximize damage to some difficult target type while holding the budget as a fixed requirement for each time period. The technology of LP models allows efficient solution of problems much larger than those generated by this application which is called the Force Mix Model. The LINMIX program takes the specification of the model and the problem to be solved and generates the linear programming problem in a standard format. The APEX-III Mixed Integer Programming optimization system is used to solve each problem.

B. Damage to Soft Target Types

In the Force Mix Model the aggregate of all targets of a soft target type is approximated by an exponential damage function. This exponential represents the concentrated value damaged in a few areas and the smaller incremental value available at scattered locations. An exponential damage function is fitted to each soft target type. Within each target type the weapon types differ in effectiveness. Specific weapon effectiveness coefficients are required for each target type.

The form of the damage function is

$$d(W_i) = A + B \cdot \exp(-(\gamma_i \cdot W_i)^F)$$

where W_i is number of weapons and $d(W_i)$ is the damage for weapon type i. γ_i is the effectiveness of weapon type i against the target type. If the number of weapons is zero, then $d(0) = 0$. For $d(0)$ to be equal to 0, $A = -B$. As W_i becomes large, the $\exp(-(\gamma_i \cdot W_i)^F)$ approaches zero and $d(W_i)$ is asymptotic to A. If $A = -B = 1.00$ is used, the damage function is asymptotic to 100%. After A and B are fixed, the exponent F, which is characteristic of the data base, is fitted in a non-linear regression while the effectiveness coefficients γ_i are fitted by linear regression within each weapon type.

If several weapons are used on the same target type it is not assumed that the damage may be added. The effectiveness for each weapon is added.

The total of the effective weapons is $T = \sum_{i=1}^{ILM} \gamma_i W_i$, where ILM is the number of weapon types considered.

The combined damage is $D(T) = A + B \cdot \exp(-T^F)$. If the required level of damage, P, to the target type is substituted for D(T), the function may be inverted to give $(-\ln((P-A)/B))^{1/F} = T$. This transforms the level of damage P into effective number of warheads T. This limit, called ZMIN, is the requirement for the sum $\sum_{i=1}^{ILM} \gamma_i W_i$ which corresponds to one row of the linear programming problem.

The data to be fit will include a series of observations for each weapon type. Each observation consists of the number of weapons W_i and the damage Q_i done by that many weapons. The damage is the fraction of total value destroyed for the whole target type. Here target type may be an aggregate of several facility lists in a data base for a given time period. For example, categories used may be soft military targets and industrial - economic targets.

The damage for a series of numbers of weapons must be computed separately for each weapon type. General models such as RPM or SIRNEM can compute this damage with any appropriate values of yield, CEP, reliability, survivability, or special restrictions on altitude, range, or avoidance. However, there are often several possibilities of reliability and survivability that may be studied and it is more practical to set reliability to 1.0 and compute perfect weapon curves. The data analysis in the Force Mix Model includes a special iterative process to approximate imperfect weapon data from perfect weapon functions.

C. Using Perfect Weapon Functions for Imperfect Weapons

There is an exponential function for damage by perfect weapons similar to the exponential function for imperfect weapons.

$$PK(X_i) = a + b \cdot \exp(-(G_i \cdot X_i)^H)$$

where X_i is the number of weapons and $PK(X_i)$ is the damage for weapon type i. G_i is the effectiveness and a, b and H are parameters characteristic of the target type. Since G_i is mainly a function of yield of the weapon, it is possible to fit the set of G_i for weapons over several yields. A quadratic function in log yield ($\ln Y_i$) is used. It fits well for the range of yields used. The coefficients of the quadratic are stored as parameters of each target type and the individual G_i for perfect weapons are not saved. When these functions are used they may be interpolated easily for a new yield or employed to recalculate a yield previously considered. It is therefore useful to have a representative set of yields, covering the range of interest, when the perfect weapon data is fitted, but it is not necessary to include all yields or weapons which will be used.

Suppose that certain survivability and reliability factors for installations which are not retargetable, combine to give some probability, P_i. It is assumed that these factors are sufficiently well known to allow multiple targeting of the most valuable parts of the data base. Even though we do not know which weapons will fail, we will double up on the highest value target areas because the probable value achieved is greater than the value achieved by attacking the next most valuable target.

Assume that weapons are assigned up to some level Z_1, then the incremental value $V(Z_1)$ for one weapon at Z_1 is the minimum value required for any weapon being used. Therefore, a second layer of weapons is put on the richest targets up to the point Z_2 where the incremental value for the second weapon $(1-P_i)\cdot V(Z_2)$ is equal to $V(Z_1)$. Similarly a third and more layers may be considered until the cutoff Z_j is less than one weapon. The damage for each layer is accumulated to give the proportion damaged, Q. The total of $Z_1 + Z_2 + \ldots Z_j = W$ is the number of weapons used. After several points are generated for each weapon type, the points may be used as imperfect weapon data and fitted as described above.

D. Model Efficiency Research

The Force Mix Model is a general linear programming model with mixed integer options. It is supported by a program designated LINMIX which generates the LP problem in a standard format. This standard format could be processed by any available mixed integer programming optimization system. But in fact these problems have been solved only by APEX III which is available for a 70% surcharge on Control Data machines. Advances in network analysis offered promise of impressive savings. There were some rather long APEX running times due to extensive integer programming submodeling in 1978 during a series of production runs. If these runs were excessively long due to the requirements of the problem, the expense could be justified, but if an alternative is available that could solve the same problem it should be explored. Besides possibly significant computer costs, the actual production time for a study might be held back by the time for processing a long running problem in a series of runs necessary to explore the study background.

First of all it should be clarified that although parts of LINMIX may be formulated as a network, it is not a network problem. Demonstrations of network analysis efficiency can be compared with APEX. Network problems, even general networks are a special case of linear programming. A general LP solver like APEX can solve network problems, but programs which are specifically designed to efficiently solve network problems cannot solve general linear programming problems. Efficient network solvers such as PNET use a modified simplex method that exploits the special characteristics of a network and can result in solving large problems quickly. Even the more general networks with attenuation factors on the arcs may result in reduction to 1/15 the running time for the same problem under APEX. Sometimes it is possible to reformulate a problem to allow solution by another model system. It was found that to do so in this case would enlarge the problem.

Remember the problem is not a network. It could be seen as a network with matching flow requirements. It is possible to do this matching when each missile is treated as a node. If a new model were written and if it gave verified equivalent results, it would be so large for production problems that any savings would probably disappear. All the problems of verification not only for accuracy but for validity would be required because the problem solved would not be exactly the same problem, but the problem recast as a network flow. It was decided that this was not worth pursuing for LINMIX. However, there are other problems which are well suited to a network approach which are currently solved with PNET.

A bigger and faster new APEX-like mixed integer solver has not yet become available. LINMIX problems are produced in the standard MPS (Mathematical Programming Standard) format to be compatible with any standard system. In view of the 70% CDC surcharge, even a slightly less efficient system could handle Force Mix Model problems. While use of APEX continues, it is possible to save by using it in a more integrated way and only call APEX functions as needed. This would allow streamlining the input directly to a matrix for APEX instead of to a file to be read by APEX. It could mean selective and summarized output formated as OP604 requires. It would also imply that reduction of the problem by elimination of redundant rows and column variables would be taken over by LINMIX. Many summary rows would only be created on request, completely outside of the APEX. These summary rows would be computed from the solution matrix. The research conclusions are that a decision can be made to go in the direction of further development of LINMIX to incorporate APEX functions or to wait for a new system to replace APEX-III at a savings. If developmental effort is invested to develop LINMIX to call APEX functions the model would be more efficient but it would also be more dependent upon the CDC owned royalty software. (Perhaps if the Navy could negotiate a reduced royalty it might satisfy users and owners in the long run.)

Recent experience has shown that the problems run with LINMIX have not continued to grow in running time. Current problems have gone in the direction of more time periods (moving from 3 to 4) and fewer weapon types, usually 8. The running times have been reasonable and storage requirements are less than 150 K. The main block to production schedules is not running time but the time to structure the problem through several time periods. The documentation of the inputs could be better, especially where the same input has different interpretation in separate phases of the program. This refers specifically to limits on weapon and targets whether in probability or number of weapons as used to control the data analysis with the iterative process of data generation or the constraints and requirements used to generate the LP problem.

In addition, experience with using LINMIX has led to the following suggestions that should make the solution to problems faster.

1. There are extraneous equations which could be omitted or included by request.

2. Linear combination variables are created at execution time by APEX. It is not necessary to have APEX do this, but it also makes it difficult or impossible to delete unnecessary component parts that are in these combinations.

3. The System Component Tree can be reduced by deleting weapons from time periods for which they will not be available.

These suggestions will not result in important computer savings but they will reduce the unnecessary information in the output. It was also suggested that the number of printouts of the TEMP data base be reduced. This has been done in the new LINMIX now maintained under UPDATE. At the same time the latest probability of damage calculations were incorporated.

Experience shows that any replacement for APEX must have a mixed integer capability and that just an LP solver will not be sufficient.

Chapter 2. Barriers to Cruise Missile Routes

A. Introduction

Cruise missiles have the capability to fly around obstacles, so it is desirable to be able to generate routes which reflect this capability. For example it might be desirable, for geographic reasons, to insure that no cruise missile flies over a particular area, or for operational reasons, that no cruise missile flies too close to a SAM site.

The RPM strategic nuclear exchange model has the capability to develop cruise missile routes from launch points through a set of IP points.* Under contract N00014-78-C-0033 a subroutine has been developed which provides this avoidance capability. If requested, cruise missile routes will be generated which avoid all of the obstacles contained in a designated facility. This is accomplished by designating as unacceptable any potential route leg which may cross an obstacle.

B. Statement of the Problem

Although avoiding a SAM site, and flying around a particular area are both problems of avoidance, their formulation and solution are significantly different. The first problem may be described by giving the location of the SAM site, and requiring that cruise missile not come within a distance R of the SAM site. The second problem may be described by saying that the cruise missile must not cross a given border.

Through the remainder of this chapter it shall be assumed that as the cruise missile flies from IP point to

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* See References 1 and 3 for discussion of cruise missile routing using the RPM model.

IP point its path is the minor arc of the great circle connecting these two points. Also we will assume the area borders are represented by a sequence of great circle arcs, each arc less than 180°, joined at the end points. Points on the surface of the earth will be represented by vectors in 3-space, normalized to have length 1. The same letter will be used to represent a point and the corresponding vector, but it will be clear from context which notion is intended. Note that the center of the earth corresponds to the zero vector which will be denoted 0.

C. Mathematical Solution

Consider first the case of the SAM. Suppose that the proposed segment of the cruise missile route joins the points P and Q, that the SAM site is located at the point B and it has a radius R. Let Θ denote the angle subtended at the center of the earth by a distance of R along the surface. We will assume $\Theta < 90°$.

Step 1:

Determine whether or not B is within a distance R of the great circle determined by P and Q.

Method:

Check to see if the angle between P x Q and B is greater than or equal to $90° - \Theta$. If we let $PQN = PxQ/|PxQ|$, and note that taking the cosine of both sides reverses the above inequality then the condition reads

$$|B \cdot PQN| \leq \cos(90° - \Theta) \qquad (1)$$

If we replace $\cos(90° - \Theta)$ by $\sin \Theta$, square both sides, and replace $\sin^2 \Theta$ by $1 - \cos^2 \Theta$, then condition (1) becomes

$$(B \cdot PQN)^2 + \cos^2\theta \leq 1. \tag{2}$$

If this condition is not met then no point on the great circle determined by P and Q is within a distance R of B so we do not have an intersection, and further testing is unnecessary.

Step 2:

If the condition in Step 1 is satisfied we next determine whether or not B is between P and Q. If this is the case then some point on the arc PQ is within a distance R of B.

Method:

B is between P and Q if and only if B and Q are on the same side of the plane determined by P and PQN, and B and P are on the same side of the plane determined by Q and PQN. Using vector algebra these conditions reduce to

$$\mathrm{Sign}(B \cdot P \times PQN) = \mathrm{Sign}(Q \cdot P \times PQN) \quad \text{and} \tag{3}$$

$$\mathrm{Sign}(B \cdot Q \times PQN) = \mathrm{Sign}(P \cdot Q \times PQN)\ . \tag{4}$$

Step 3:

Even if these conditions are not met it still might be the case that there are some points on arc PQ within a distance R of B. But since B is not between P and Q, if there are points on arc PQ within a distance R of B, then one of P or Q must be within R of B. Noting that taking cosines reverses the direction of an inequality, P is within R of B if

$$B \cdot P \geq \cos(\theta) \tag{5}$$

and Q is within R of B if

$$B \cdot Q \geq \cos(\Theta) \tag{6}$$

If either of these conditions is met we know that some point on the arc is within a distance R of B. Otherwise no point on the arc PQ is within a distance R of B.

Consider, now, the case of the arc crossing a barrier. Because the barrier is represented by a sequence of great circle arcs the problem reduces to a determination of whether or not two great circle arcs intersect.

As before, suppose the cruise missile route joins the points P and Q and suppose the barrier arc is the minor arc of the great circle determined by the points A and B. There are two cases to consider. Either the arcs lie in the same plane or they do not lie in the same plane.

Consider the first case that the arcs do not lie in the same plane. If the arcs cross then clearly the points A and B must be on opposite sides of the plane POQ. Similarly P and Q must be on opposite sides of the plane AOB. Using vector algebra these conditions become

$$\text{Sign } (A \cdot P \times Q) \neq \text{Sign } (B \cdot P \times Q) \quad \text{and} \tag{7}$$

$$\text{Sign } (P \cdot A \times B) \neq \text{Sign } (Q \cdot A \times B) \ . \tag{8}$$

They are not sufficient to insure an intersection. We need to guarantee that the arcs are not "antipodal". That is the case where there is a point X such that X is on arc PQ and -X is on arc AB (see Figure 1).

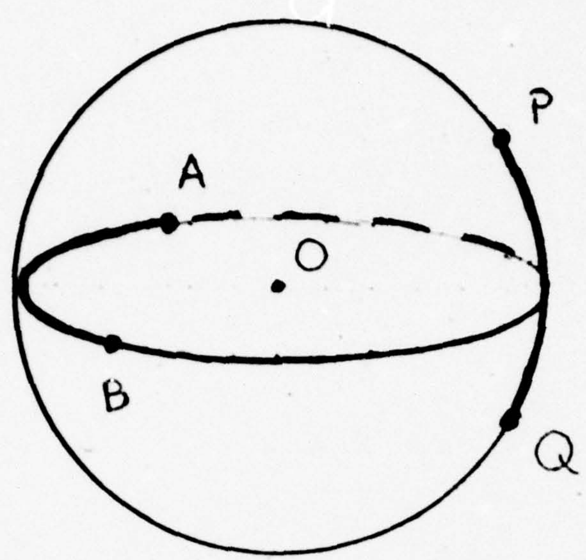

Figure 1. Antipodal Arcs.

Using vector algebra (see appendix), in the presence of conditions (7) and (8) the condition that the arcs not be antipodal is

$$\text{Sign}\ ((A-B)\cdot P \times Q)) \neq \text{Sign}\ ((P-Q)\cdot A \times B)). \qquad (9)$$

Conditions (7), (8), and (9) are both necessary and sufficient for the two arcs to intersect.

The only case left to be dealt with is when both arcs lie in the same plane. In this case the above method breaks down. Because the situation is symmetric with respect to the two arcs, we can suppose without loss of generality that arc PQ is longer than arc AB. Then, if the arcs intersect at least one of A or B must be contained in arc PQ, and the methods from Step 2 of the SAM site carry over directly to this situation. Thus the condition for intersection reads

$$\mathrm{Sign}\ (A \cdot P x PQ) = \mathrm{Sign}\ (Q \cdot P x PQ) \text{ and}$$
$$\mathrm{Sign}\ (A \cdot Q x PQ) = \mathrm{Sign}\ (P \cdot Q x PQ)$$

or

$$\mathrm{Sign}\ (B \cdot P x PQ) = \mathrm{Sign}\ (Q \cdot P x PQ) \text{ and}$$
$$\mathrm{Sign}\ (B \cdot Q x PQ) = \mathrm{Sign}\ (P \cdot Q x PQ)\ .$$

D. Computational Considerations

Because the computer has finite precision we must examine the effects of truncation error. Consider first the case of a SAM site. The very small errors in the positions of P, Q, and B, and the size of the radius that truncation error may introduce will only make a difference if arc PQ is tangent to the circle. And in this case, considering the situation being modeled, both accepting the segment or rejecting the segment are reasonable and justifiable decisions. So special consideration of truncation effects is not necessary in this case.

We turn now to the effects of truncation on the problem of determining whether or not two arcs cross. Truncation error causes two significant problems. Mathematically it is easy to distinguish the case when the arcs lie in a single plane, this happens if and only if

$$A \cdot P x Q = B \cdot P x Q = P \cdot A x B = Q \cdot A x B = 0.$$

But considering the special nature of the floating point zero, in the presence of truncation error it is very unlikely that all four of these quantities will be zero. Yet it is necessary to distinguish those cases which should be treated as though the arcs did lie in a plane.

We also must be able to determine when the endpoint of one arc is in the plane determined by the other arc. Again, mathematically this is no problem, but due to the special nature of the floating point 0, truncation error can give rise to visibly incorrect answers.

E. Numerical Defense

Thus what is needed are criteria for deciding whether or not two arcs lie in a plane and for deciding whether or not an endpoint of one arc lies in the plane of the other arc.

There are two principal effects of truncation error. The most obvious is that computations are slightly in error. The second effect is that site locations are slightly inaccurate. The effect of truncation error may be adequately represented by supposing that the four points P, Q, A, and B are displaced by an amount no greater than some small number E. On CDC $E = 2^{-24}$ which corresponds to 1.25 feet or an angular displacement of at most 0.0123" of arc.

Any test used to determine whether or not to treat a particular case as though the arcs lay in a single plane should have the following two desirable properties. Firstly, if the case could be obtained by starting with the two arcs in the same plane and displacing each of the four points by less than E, then we will treat the case as though the arcs did lie in a plane. Secondly, for those cases which are treated as though the arcs lie in a single plane, but could not be produced by starting with the arcs in a single plane and displacing the four endpoints by a distance of E or less, we require that the answer so obtained be reasonable and justifiable.

The following test has the above attributes. We proceed in steps.

Step 1:

If at least one of $|P \cdot AxB|$ and $|Q \cdot AxB|$ is greater than or equal to $3 \cdot E$ and at least one of $|A \cdot PxQ|$ and $|B \cdot PxQ|$ is greater than or equal to $3 \cdot E$ then the arcs do not lie in the same plane.

Step 2:

Select two of the four vectors whose included angle is closest to 90°. The reason for this selection is that the plane that these two vectors determine is least affected by small changes in the positions of the two determining points.

Step 3:

Compute the distances from the two remaining points to this plane. If these distances are both less than $3 \cdot E$ we will treat the case as though the arcs lie in a plane, otherwise not.

Suppose that we have any three points, say P, Q, and A, lying in a plane and of all possible pairs the angle between P and Q is closest to 90°. Then if we arbitrarily displace these points, to P', Q', and A', by a distance of no more than E, it can be shown by means of vector algebra that the distance from A' to the plane determined by P' and Q' is less than $3 \cdot E$.

We also have the problem of determining when the end-point of one arc is in the plane of the other, as this may change the determination of whether or not the arcs intersect. However this test is only of interest when one of the conditions (7) or (8) is met and the other fails; if both conditions are met then this test makes no difference, and if both fail the arcs don't intersect and the test is unnecessary. Suppose, for example condition (7) is met but condition (8) fails. Then, in turn for the point P and point Q, we perform the following test in three steps. Let X denote the point currently under consideration.

Step 1

If $|X \cdot A \times B|$ is greater than 1.5 E the test fails, otherwise proceed to Step 2.

Step 2

Is the point X close to the plane determined by arc AB? Specifically is

$$|X \cdot A \times B| / |A \times B| < 1.5 \text{ E ?}$$

If no, the test fails, otherwise proceed to Step 3.

Step 3

If X is between A and B the test succeeds. The method from Step 2 of the SAM site carries over directly.

If the test succeeds the point is presumed to be in the plane and condition (9) is checked. Note that if both P and Q could pass this test then the case would be treated as though both arcs lay in the same plane.

F. Overview of Subroutine BAR

Input:

A proposed cruise missile route segment PQ and a facility of BARriers. Sites with a group number of 4095 are treated as circular defense sites, consecutive sites with the same group number different from 4095 are treated as defining barrier arcs. N consecutive sites $S_1, \ldots, S_N$ with the same group number different from 4095 define the N-1 barrier arcs $S_1S_2, S_2S_3, \ldots, S_{N-1}S_N$. Note that it is permissible to have $S_N = S_1$ in which case the arcs form a closed loop, i.e, an unacceptable region for cruise missile overflight.

Output:

A 0 or 1, stored in the variable INTER. 0 if the proposed path does not cross any of the barriers. 1 otherwise.

What follows is an outline of Subroutine BAR.

1. Declarations and Initializations.

   Let VA be a one dimensional array of vectors.

   E = 2 ** (-24)   E3 = 3.*E   E15 = 1.5*E

   CX(I,J,K) is the Ith component of VA(J) x VA(K).

   DET(I,J,K) = VA(I) · VA(J)xVA(K).

   INTER = 1 (presume an intersection).

2. Main Loop. This loop will step through sites in the barrier facility.

3. B = SITE(IT).

Get the location and group number of the ITth site in the barrier facility. If the group number is 4095 go to 9. If the group number of the current site is different from the group number of the previous site, known as site A, go to 8.

4. If Sign(A·PxQ) = Sign(B·PxQ) and both |A·PxQ| and |B·PxQ| are sufficiently large that neither of the tests described in section E can succeed then go to 8.

5. If the arcs don't lie in a plane go to 7. This is the first test described in section E.

6. If the arcs, viewed as lying in a plane, intersect RETURN, otherwise go to 8. This test is described in section C.

7. If appropriate, check to see if the endpoint of one arc lies in the plane of the other. If the arcs intersect RETURN. This test is described in section C.

8. Site A = Site B. Go to 10.

9. Does the arc cross the circular site? If yes, RETURN.

10. End of Main Loop: Set the group number associated with Site A equal to the group number associated with Site B.

11. INTER = 0
RETURN

12. END

Chapter 3. DGZ's for Multiple Weapon Systems

A call which will develop desired ground zeros (DGZ's) for multiple weapon systems has been added to the RPM program.* Until this call was constructed, DGZ's were developed for a single weapon system, data base values were modified to reflect damage and then the process was repeated for the next weapon system. The DGZ call in RPM has been used when DGZ's which are constrained to avoid categories of sites are to be developed for single weapon systems. The DGZ call required perfect weapons, e.g., a CEP of zero and a probability of arrival of one. The WGZ call in RPM has been used to develop aimpoints for single weapon systems for imperfect weapons which are not constrained to avoid. The new call, WHIZ, will develop DGZ's for multiple types of imperfect weapons with avoidance.

The WHIZ call considers all of the weapon systems as each DGZ is developed. Weapon systems are characterized by yield, CEP, PA, height of burst, minimum range, maximum range, fission fraction, basing reference, and a weighting factor. A series of criteria determines which weapon systems are eligible to be considered for the next DGZ to be assigned. The assignment is given to the weapon system which meets all of the criteria and gives the most reliable, weighted objective damage.

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* See Reference 3.

Consider a data base containing objective installations, other installations which are not objectives, and sites to be avoided. The data base is "grouped". Grouping separates the data base into sets of sites which can be considered independently for DGZ construction. Given a particular group, the objective with the most remaining damage requirement is selected to form the kernel of the DGZ that will be constructed next. Each weapon system is considered. Several possibilities exist.

1. If ranging is being checked, the weapon system may be out of range of the group. In this case the weapon system is not considred for this DGZ development.
2. The weapon system may have no capability against the objective kernel. For example the weapon system may be assigned a fixed height of burst at which no weapon radius can be developed for the VN of the objective. In this case the weapon system is not considered.
3. The weapon system may have little capability against the objective. This condition exists when the ratio of CEP to weapon radius is greater than 1. In this case, only the center of the objective is considered as the position of the potential DGZ for that weapon system.
4. The weapon system may have a good capability against the objective, but may not be able to attain the required damage level with one weapon. In this case

only the center of the objective is considered as the position of the potential DGZ for the weapon system.

5. There may be an area within which the weapon system can meet the objective damage requirement. In this case either two or three points are considered as potential DGZ's for the weapon system, depending on whether avoidance is a requirement. These points are the center of the objective, the point of maximum return and a point constructed to accommodate the avoidance requirement.

If avoidance is not to be considered, the DGZ assigned will be that location, weapon system combination which gives the most reliable weighted damage. If avoidance is required, then the DGZ selected must also meet the avoidance criterion which is input as a ratio of avoidance damage to objective damage.

In RPM the "category" of a site is an integer between 0 and 63. The category is used to distinguish types of sites. Objective site types count up from category zero. The lowest category of sites to be avoided is input to the WHIZ call. All sites whose categories are between this input value up through 59 are to be avoided. Sites with categories between the highest objective category code and the lowest avoidance category code are installations which can be the source of by-product damage. If the option to include by-product damage is selected then when the objective kernel is in Case 5 above, the process for locating the point of maximum return considers both other objective and by-product sites.

Categories 60 through 63 are reserved for the WHIZ call. If equivalent area targets are encountered in the data base, they are put in category 63 and are ignored.

Separate damage criteria are input for each objective category code. Each damage criteria has an integer and a fractional part. The integer part is surviving value; the fraction is probability of survival (PS). The damage requirement against an objective sites is satisfied when either the value is reduced to the integer surviving value or the probability of survival of the site is reduced to the fractional PS.

Range may be taken into account in the WHIZ call. Each weapon system has a minimum and a maximum range, and a base facility containing a set of base sites. In WHIZ, range is checked against the centroid of each group. If at least one base site for a weapon system exists which is within the range limits of the centroid of the group, then that weapon system is considered within range of the entire group.

Figure 2 illustrates DGZ development for Case 5. In this illustration, Q_0 is the objective in the group with the most remaining damage requirement. Then Q_0 is the kernel of the next DGZ to be developed.

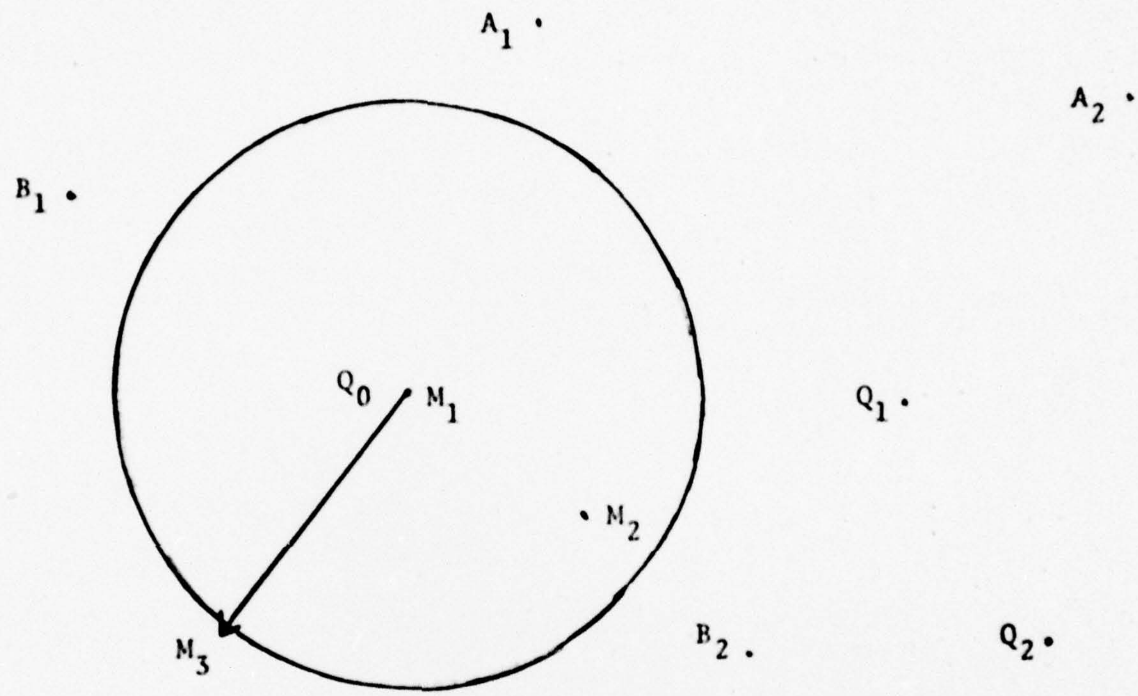

Figure 2. DGZ Development for Case 5.

An example to illustrate the meaning of "most remaining damage requirement" may be helpful. Suppose there are two objectives, Q_0 with initial value 1000 in category 0, and Q_1 with initial value 500 in category 1. Suppose the category 0 damage requirement parameter is 0.5. This means, allocate to each site in category 0 until the value remaining at the site is 0 or the site's probability of survival is .5. Suppose the category 1 damage requirement parameter is 200.3. This means allocate to each site in category 1 until the value remaining at the site is 200 or the site's probability of survival is .3. If neither Q_0 nor Q_1 have been previously damaged, then at least 500 points must be damaged for Q_0 and at least 300 points must be damaged for Q_1. In this case, Q_0 will be considered as an objective kernel before Q_1 is considered. On the other hand, suppose Q_0 has been partially damaged by a previously developed DGZ. Suppose 300 points have already been killed. Then damage of 200 points more is necessary to fulfill the damage requirement. In this case, Q_1 will be considered as an objective kernel before Q_0 is considered.

Once the objective kernel is selected and the remaining required damage is known, each weapon system is examined. For a weapon system to be in Case 5, there must be a radius within which the damage requirement to the objective kernel can be met. This radius may be seen in Figure 2. A maximum of one weapon radius is placed on this damage requirement radius. If there is no avoidance, a procedure similar to the WGZ procedure for developing the point of maximum return is used. This point of maximum return, M_2, will be the next candidate DGZ for the weapon system. The location of M_2 may depend on other objectives such as Q_1 and Q_2 and on by-product sites such as B_1 and B_2. In the WHIZ call the procedure for developing the point of maximum return first considers all possible neighbors to the objective kernel for the particular weapon system. To determine if a site is a neighbor of an objective kernel, find the weapon radius for the site at the weapon height of burst determined by the kernel. If this circle overlaps the damage requirement circle for the kernel then the site is a neighbor. Initially the assumption is made that all of these points may lie within one weapon radius of the DGZ that will be developed. The objective probability of survival function is used as input to develop the optimum aimpoint. If it is true that all of the original neighbors are within one weapon radius of the developed DGZ then the process is complete. If all neighbors are not within one weapon radius of the DGZ, then the one neighbor which is more than one weapon radius from the DGZ and which contributes the least value to the DGZ is eliminated. Then the position of the DGZ is recomputed. This procedure is repeated until the set of neighbors is reduced to one in which each neighbor is within one weapon radius of the DGZ. This method of eliminating neighbors is different from the method in the WGZ procedure. In the WGZ procedure, all 'out of range' values are weighted to one-half value before the second DGZ location is computed. If on the second pass some of the original neighbors are still 'out of range', then their value to the DGZ is set to zero. Neighbors come into and go out of the WGZ as the location shifts and settles.

If avoidance is considered, then the objective return and the avoidance damage by a DGZ at the center of the kernel are computed. At the same time, a vector weighted by avoidance damage pointing away from the sites to be avoided is constructed. The avoidance damage at the point M_3, where this vector intersects the damage requirement circle is estimated. Figure 3 illustrates how this construction is made.

To determine the location of point M_3,

1. Compute the avoidance value killed, VK_1, at M_1.
2. Compute VK_i at M_i.
   The vector for each A_i, V_i, points away from A_i.
   Its length is $(VK_1 - VK_i)$.
   M_3 is along the vector sum of V_i's and positioned at the limit of the requirement circle.

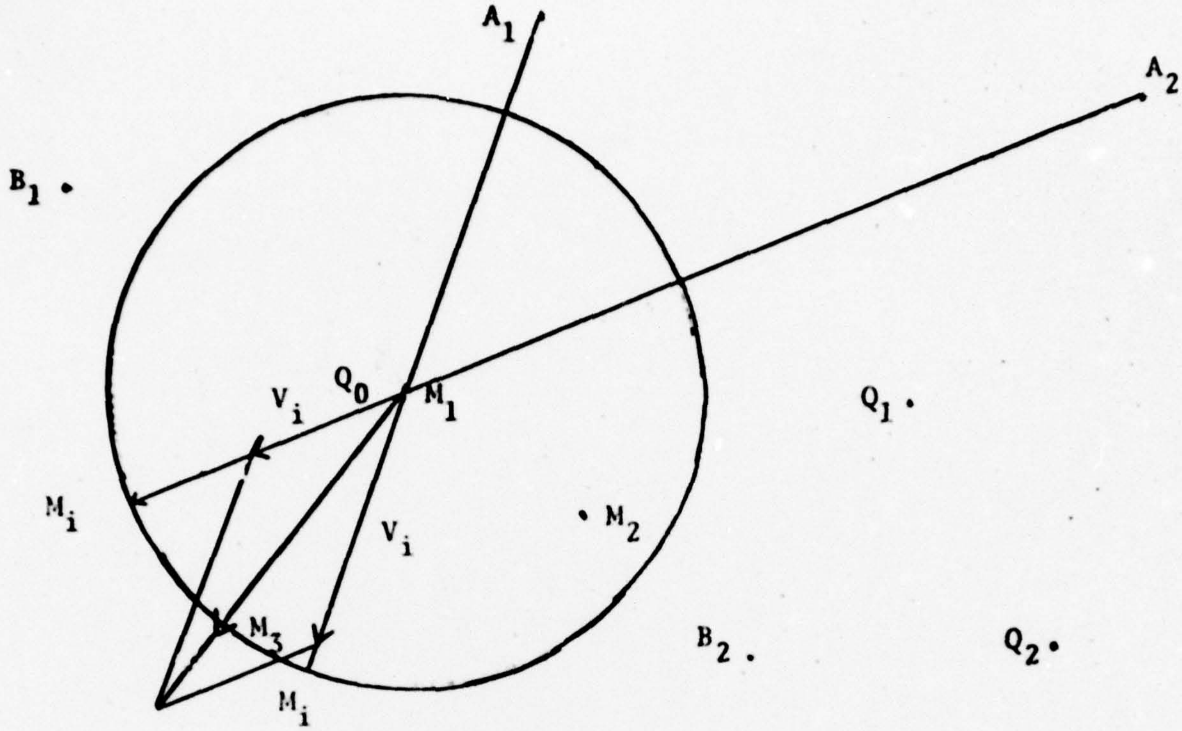

Figure 3. Construction of Vector to M_3.

The avoidance damage at M_3 is the damage at M_1 reduced by the product of the amount saved per nmi along the vector sum and the radius (nmi) of the requirement circle.

Using the avoidance damage at M_1 and at M_3, an estimate of the avoidance damage at M_2 is made by projecting the point M_2 onto the line through M_3 and M_1. By linearly interpolating the avoidance damage at the point of intersection, an approximation to the avoidance damage at M_2 is made. If the ratio of avoidance damage to objective damage plus by-product at M_2 is within the input avoidance ratio then M_2 becomes the candidate DGZ for the weapon system. The avoidance estimating procedure implies that if the perpendicular from M_2 to the line connecting M_1 and M_3 lies between M_1 and M_3 then the avoidance damage at M_2 is lower than at M_1. Therefore the ratio of avoidance damage to objective damage will be smaller at M_2 than at M_1. If the perpendicular from M_2 does not lie between M_1 and M_3 and if M_2 is not within the criterion, then the ratio of avoidance damage to objective and by-product damage at M_1 is considered. Only if this ratio too does not meet the avoidance criterion is M_3 considered. At M_3, the objective return from only Q_0 itself is compared to the avoidance site damage. If this ratio is within the criteria then M_3 becomes the candidate DGZ for the weapon system.

The estimation procedure just described seems to work well in most cases for the purpose of developing DGZ's. These estimates are made in the interest of efficient computing. As each weapon system/DGZ choice is made, then the PD for each site is explicitly updated to include damage from that DGZ. The intermediate damage calculations at M_1, M_2 and M_3 may be summarized as follows:

1. M_1. The damage to the objective is explicitly calculated. The damages to other objectives and to by-product sites are explicitly calculated if CEP divided by weapon

radius is less than 1, otherwise damage is estimated to be 0. The damage to avoidance sites is explicitly calculated.

2. M_2. The damage to the objective, other objectives and to by-products sites are well estimated using the same maximizing procedure as is used in the WGZ call. The damage to sites to be avoided is based on linear interpolation.

3. M_3. Only damage to the objective kernel is considered at M_3. This damage is explicitly known by the definition of the requirement radius. Avoidance damage at M_3 is estimated as the weighted vectors from M_1 away from sites to be avoided are summed.

Note that as damage requirements become partially met, the area around an objective within which the DGZ may lie increases is size. By this means, if the damage requirement for an objective has not been met by a single weapon, then the objective remains a candidate for a potential DGZ for a second weapon of the same or a different type.

The methods used to locate the point of maximum return in the WHIZ call and the WGZ call differ in that while the WHIZ call will locate the point within an area determined by the requirement radius, the WGZ call will locate the point within one weapon radius of the objective kernel.

In the WHIZ call, area targets are not busted. If busting is desired, it must be done prior to WHIZ. The CIRCLE call in RPM may be used to accomplish area busting.

When the WGZ and DGZ procedures are used to develop aim points one weapon system at a time, the list of aim points is ordered on value returned and chopped to match inventory before the data base is updated to reflect damage. This is possible since all data base groups are processed for a specific system before the next system is considered. In the WHIZ procedure, decisions among weapon systems are made within a group of sites, before the next group is processed. Some problems require consideration of specific inventory limits. In order to give the analyst some means of influencing the weapon system selection with respect to inventory, a weighting factor is associated with each weapon system. Efficient use of this function to match DGZs to inventory remains to be measured against a span of real cases.

Chapter 4. Navy Study Support

Throughout the contract year Navy study requirements have received technical attention. Standard programs have been updated, special programs have been developed, data processing procedures have been supplied, prototype decks have been constructed and analyses has been supported. Some of this support is described in the paragraphs which follow.

A. One Navy study required development of DGZs constraining damage to urban areas. The data base for this study was large enough to require processing with blocked ATLAS, GSPLIT and GMERGE calls which were recently added to RPM. A prototype deck demonstrating use of these calls was constructed. In this study it was of interest to know the total damage to sites which made up each urban area. To compute this damage a modification was made to the merging procedure in RPM which reduces a group to a single site. The modification computed the probability of kill for the single merged site based on the damage to each site in the group.

B. The cruise missile routing path matrix editor in RPM was reprogrammed so that larger cases could be printed. Provision was made to select intervals of launch point sites for the simultaneous mode. More control over print options was provided to the analyst. Binary atlases of coverage circles with route distances in the value fields are now written to file.

C. Other RPM related support included

1. Development of a procedure to convert a multiple allocation warhead list to a list with an entry for each DGZ by repeated splits on the zone bits.
2. Modification of the STRIKE, DGZ, WGZ and TARGET calls to process Z type vulnerabilities.
3. An updated RPM program was prepared for use at Dahlgren Naval Proving Grounds.

D. Data processing programs to collate and reformat data tapes were supplied.

E. A set of LINMIX demonstration cases was developed.

F. Study of printing procedures and programs from Dahlgren which incorporate the FOZ program was initiated. A tape containing the FOZ program and decks for a small test case were prepared for CCTC.

G. Development of procedures using COPE to maintain standard control card sequences and prototype input decks continued.

H. A program, ARC, was furnished in support of a research project to develop a footprinting system which respects target value. Part of the footprinting procedure is geographic, but given the geographic constraints there still remains an optimization problem. This optimization problem can be formulated as a network flow problem. Program ARC, using the geographic information, constructs a file which can be used as input data to PNET.

I. A program, ARPMCON, was furnished in support of a research project to develop a procedure by which avoidance constraints against sites or groups of sites from a set of DGZ's can be studied. This is in contrast to procedures in RPM which consider avoidance constraints on a single site, single DGZ basis. The program ARPMCON uses information from an RPM warhead list to construct a file which can be used as input data to APEX-III.

Appendix

1. Suppose we have two pairs of unit vectors P and Q, and A and B, and we want to determine whether arc AB intersects arc PQ. Here and hereafter, if X and Y are unit vectors then arc XY denotes the minor arc of the great circle determined by X and Y. Also, sector XOY will mean the infinite sector bounded by ray OX and ray OY and containing arc XY.

Proposition 1:

Suppose the points A, B, P, Q, and O are not coplanar. Then arc AB intersects arc PQ if and only if line AB intersects sector POQ and line PQ intersects sector AOB.

Proof:

Suppose $X \in$ arc PQ $\cap$ arc AB. Then ray OX is contained in both sector POQ and sector AOB. But line AB intersects ray OX. Therefore line AB intersects sector POQ. Similarly, line PQ intersects sector AOB.

Now suppose line AB intersects sector POQ and line PQ intersects sector AOB. Let X be the point of intersection of line PQ and sector AOB. Then, by construction ray OX is contained in the intersection of the plane POQ and sector AOB. Similarly if Y is the point of intersection of line AB and sector POQ then ray OY is contained in the intersection of plane AOB and sector POQ. Thus ray OX and ray OY are colinear, and since both intersect line PQ, ray OX = ray OY. Therefore arc AB intersects PQ.

Any plane in 3-space may be described as the zero set of some suitable affine function.

(a) $F(X) = X \cdot V + k$

where X and V are vectors. Any line in 3-space may be parameterized

(b) $L(\tau) = \tau V + P$

where V and P are vectors.

Finally note that two points X and Y are on the same side of the plane defined by F if Sign(F(X)) = Sign(F(Y)), which can be restated as

(c) $F(X) \cdot F(Y) \geq 0$

Note that (c) implies that if one of X or Y is actually on the plane we will always say that the two points are on the same side.

Suppose that the hypothesis of proposition 1 holds. Consider the problem of determining, for example, whether line AB intersects sector POQ. Let Z by the point of intersection of line AB and plane POQ. It is possible to use the method of Chapter 2, section C to determine if Z is between P and Q, but a more efficient method is available. Choose a plane Rp which is parallel to line AB and whose intersection with plane POQ is line OP. Similarly, choose a plane Rq, parallel to line AB and whose intersection with plan POQ is line OQ. Then Z is in sector POQ if and only if Z is on the same side of Rp as is Q and Z is on the same side of Rq as P. A function Fp defining Rp, as in (a) is given by

(d) $Fp(X) = X \cdot (B-A) \times P$

and a function Fq defining Rq is given by

(e) $Fq(X) = X \cdot (B-A) \times Q$.

Observe that both Fp and Fq are constant on line AB. So $Fp(Z) = Fp(A)$ and also $Fq(Z) = Fq(A)$. Therefore the conditions for line AB to intersect sector POQ are

(f) $[A \cdot (B-A) \times P] * [Q \cdot (B-A) \times P] \geq 0$ and

(g) $[A \cdot (B-A) \times Q] * [P \cdot (B-A) \times Q] \geq 0$.

Using the fact that cyclic permutations leave the triple products unchanged, and basic properties of the dot and cross products, (f) and (g) may be reduced to:

(f') $[P \cdot A \times B] * [(A-B) \cdot P \times Q] \leq 0$ and

(g') $[Q \cdot A \times B] * [(A-B) \cdot P \times Q] \geq 0$.

Because the situation is symmetric in the two arcs, if we exchange P and A, and Q and B the conditions that line PQ intersect sector AOB are

(h) $[A \cdot P x Q] * [(P-Q) \cdot A x B] \leq 0$ and

(i) $[B \cdot P x Q] * [(P-Q) \cdot A x B] \geq 0$.

Suppose that conditions (7) and (8) in Chapter 2, section C are met. If we have condition (9) as well then conditions (f'), (g'), (h), and (i) follow at once and the arcs intersect. To see this consider the case where $(P-Q) \cdot A x B > 0$. Then $P \cdot A x B > Q \cdot A x B$ and since these two terms have opposite signs (8), $P \cdot A x B \geq 0 \geq Q \cdot A x B$. Conditions (f') and (g') are immediate and all the other cases are similar. Conversly if the arcs intersect, the inequalities (f'), (g'), (h), and (i) are satisfied. Conditions (7) and (8) are immediate consequences, and if we subtract (g') from (f') (or (i) from (h)) we get condition (9). This proves that when the arcs are not coplanar conditions (7), (8), and (9) are both necessary and sufficient for the two arcs to intersect.

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