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6 THRESHOLDING USING
THE ISODATA CLUSTERING ALGORITHM.

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ABSTRACT

A recently propcsed iterative thresholding scheme turns out to be essentially the well-known ISODATA clustering algorithm, applied to a one-dimensional feature space (the sole feature of a pixel is its gray level). We prove that in one dimension, ISODATA always converges. We also apply it to requantize images into specified numbers of gray levels.

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1. Introduction

An iterative method for threshold selection has been proposed by Ridler and Calvard in [1] for object-background discrimination. After an initial guess is made, at each iteration we get a new threshold in the following way: given a threshold T_i, the next threshold T_{i+1} is the average of V_{above} and V_{below}, where V_{above} is obtained by integrating all points above T_i and V_{below} by integrating all points below T_i. T_{i+1} is, hopefully, a better threshold than T_i for object-background discrimination. The process terminates as soon as we have $T_{i+1}=T_i$, which usually requires about four iterations. The initial value, T_0, is chosen by selecting a region in the image (its four corners) that is most likely to contain only points of the same class-background. The method is applied to thresholding a low-contrast image containing a handwritten signature.

Except for the choice of the initial threshold, the method described in [1] can be thought of as a one-dimensional application of the ISODATA algorithm (as described in [2]) where we restrict the number of classes to two. (In the ISODATA algorithm what is initially chosen are the means $\hat{\mu}_1, \ldots \hat{\mu}_c$ of the classes.)

In this note we give some other applications of the ISODATA algorithm in which we consider numbers of classes other than two. It is also proved that the algorithm for the one-dimensional, two-class case always converges.

2. <u>The ISODATA algorithm</u>

The basic ASODATA algorithm [2] is a procedure for classifying a set of sample vectors $x = \{\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_m\}$ into c distinct classes.

<u>Algorithm 2.1</u> Basic ISODATA.

1. Choose some initial values for the means $\hat{\mu}_1, \hat{\mu}_2, \ldots, \hat{\mu}_c$;

Loop: 2. Classify the m samples by assigning them to the class having closest mean;

3. Recompute the means as the averages of the samples in each class,

4. If any mean has changed value, go to loop; otherwise stop.

In our case, we have an image in which each point has a "gray level" integer value in the interval $[0,L]$. Each sample is thus a point of the image. The distribution of gray levels is given by a histogram h, where h(0), h(1),...,h(L) are the numbers of points with gray levels 0,1,...L. Let [LO,UP] be the smallest interval containing all non-zero histogram values. In this one-dimensional case, the ISODATA algorithm may be rewritten as:

<u>Algorithm 2.2</u> One-dimensional ISODATA:

1: Choose some initial values for the means $\mu_1, \mu_2, \ldots \mu_c$, such that $LO \leq \mu_1 < \mu_2 < \ldots < \mu_c \leq UP$;

Loop 2. Calculate thresholds $T_1, T_2, \ldots T_{c-1}$ by the formula:

$$T_i = \lfloor (\mu_i + \mu_{i+1})/2 \rfloor, \ 1 \leq i < c;$$

Assign to class i, $1 \leq i \leq c$, all gray levels in the interval $I_i = [T_{i-1}+1, T_i]$; (we define $T_0 = LO-1$ and $T_c = UP$)

3) Recompute the means: for every i make $\hat{\mu}_i$ the nearest gray level to $(\sum_{j \in I_i} j \cdot h(j)) / (\sum_{j \in I_i} h(j))$, $1 \le i \le c$;

4) If any mean has changed value, go to loop; otherwise, stop.

Observation

In step (3) of the algorithm, if it happens that $\sum_{j \in I_i} h(j) = 0$ for some i, in other words, there is no point whose gray level falls in the interval I_i, one should suppress class i and consider just the remaining classes.

For the two-class case we can show that Algorithm 2.2 converges. The proof reveals how Algorithm 2.2 works. First, however, it is necessary to prove two lemmas.

Lemma 2.1

Let μ_1^k and μ_2^k be the means of classes 1 and 2 respectively, and T^k the threshold at the kth iteration. Then we have $LO \le \mu_1^k \le T^k < \mu_2^k \le UP$.

Proof:

For k=0 we have, by Step 1 of the algorithm, that $LO \le \mu_1^0 < \mu_2^0 \le UP$. As $T^0 = \lfloor (\mu_1^0 + \mu_2^0)/2 \rfloor$, then $\mu_1^0 \le T^0 < \mu_2^0$.

Let us suppose that the inequality is true for the (k-1)st iteration. The new values for the means are based on the intervals $I_1^{k-1} = [LO, T^{k-1}]$ and $I_2^{k-1} = [T^{k-1}+1, UP]$. Therefore $\mu_1^k \in I_1^{k-1}$ and $\mu_2^k \in I_2^{k-1}$, which implies that $LO \le \mu_1^k < \mu_2^k \le UP$. Again, as $T^i = \lfloor (\mu_1^k + \mu_2^k)/2 \rfloor$ we have $\mu_1^k \le T^k < \mu_2^k$. //

Lemma 2.2

If $T^i \le T^{i+1}$ then $T^{i+1} \le T^{i+2}$. Similarly, if $T^i \ge T^{i+1}$ then $T^{i+1} \ge T^{i+2}$.

Proof:

We prove only the first part of Lemma 2.2, as the second part is analogous to the first.

If $T^i = T^{i+1}$, then $\mu_1^{i+1} = \mu_1^{i+2}$ and $\mu_2^{i+1} = \mu_2^{i+2}$ and thus $T^{i+1} = T^{i+2}$.

If $T^i < T^{i+1}$, the interval I_1^{i+1} where μ_1^{i+2} will be calculated is the union of the intervals I_1^i and $[T^i+1, T^{i+1}]$. So, besides the points of I_1^i that contributed to μ_1^{i+1}, we have the new gray levels of $[T^i+1, T^{i+1}]$ that are larger than μ_1^{i+1}. Therefore $\mu_1^{i+2} \ge \mu_1^{i+1}$. Similarly, the interval I_2^i where μ_2^{i+1} is calculated is the union of $[T^i+1, T^{i+1}]$ and I_2^{i+1} and, therefore, $\mu_2^{i+2} \ge \mu_2^{i+1}$. Since both means μ_1^{i+2} and μ_2^{i+2} are larger, we have $T^{i+2} \ge T^{i+1}$. //

Lemma 2.2 shows that the threshold, if it moves, moves only in one direction.

Theorem 2.1

Algorithm 2.2 converges in a finite number of steps.

Proof:

By Lemma 2.2, the sequence $T^0, T^1, \ldots$ of thresholds forms either a non-decreasing or non-increasing sequence. By Lemma 2.1 this sequence is bounded and, therefore, there must be a k such that $T^k = T^{k+1}$. If this happens, then $T^k = T^{k+j}$ for all j.

3. Examples

Algorithm 2.2 was tested on three different pictures. Figures 1,3, and 5 show the original pictures together with their histograms. The results for numbers of classes equal to 8,4,3, and 2 for Figures 1 and 3 are shown in Figures 2 and 4 ((a),(b),(c), and (d), respectively). Figure 6 shows the result for Figure 5 when we consider only two classes.

The initial values for the means were chosen so they were evenly spread in the interval [LO,UP]. The number of iterations together with the thresholds are given in Table 1.

Figure	Number of classes	Number of iterations	LO,UP	Thresholds
1	2	5	13,49	36
	3	4		28,37
	4	4		24,31,38
	8	2		17,21,25,29,33,37,41
3	2	3	7,63	37
	3	4		28,44
	4	4		24,34,48
	8	3		15,20,26,32,40,48,55
5	2	3	3,63	37

Table 1. Number of iterations and thresholds for each figure.

Moreover, the maximum number of steps is limited by the size of the interval [LO,UP]. //

Observation:

Although we can prove the convergence of the algorithm, at least for two classes, this does not mean that the algorithm always arrives at the same threshold, irrespective of the initial values for the means.

4. Comments

In this note we showed that the ISODATA algorithm can be used not only for thresholding a picture, as in [1], but also to requantize it into a few gray levels. In the latter application, it seems that one can achieve reasonable data compression without significant distortion of the image, as in [3], However, an advantage over the method proposed in [3] is that one can specify a priori the number of gray levels desired.

References

[1] T. W. Ridler and S. Calvard, Picture thresholding using an iterative selection method, IEEE Trans. on Systems, Man, and Cybernetics 8, 639-632 (1978).

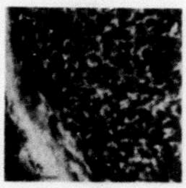
[2] R. O. Duda and P. E. Hart, Pattern Classification and Scene Analysis, John Wiley & Sons, Inc. (1973).

[3] S. Peleg, Iterative histogram modification 2, IEEE Trans. on Systems, Man, and Cybernetics 8, 555 (1978).

(a) (b)

Figure 1. Original FLIR image (a) and histogram (b).

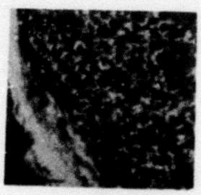
Figure 2. Results for 8 classes (a), 4 classes (b), 3 classes (c), 2 classes (d).



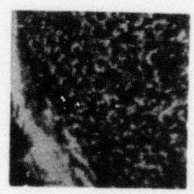
(a)

(b)

Figure 3. Satellite image (clouds) (a) and histogram (b).

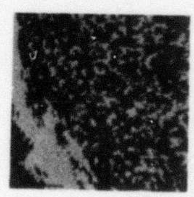


(a)



(b)

(c)



(d)

Figure 4. Results for 8 classes (a), 4 classes (d), 3 classes (c), 2 classes (d).

(a) (b)

Figure 5. Signature (a) and histogram (b).

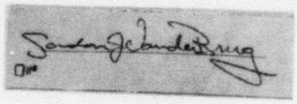


Figure 6. Result of thresholding (two classes).

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A recently proposed iterative thresholding schemes turns out to be essentially the well-knwon ISODATA clustering algorithm, applied to a one-dimensional feature space (the sole feature of a pixel is its gray level). We prove that in one dimension, ISODATA always converges. We also apply it to requantize images into specified numbers of gray levels.

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