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PERTURBATIONS IN PERIGEE AND ECCENTRICITY AT THE CRITICAL INCLINATION

by
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Strategic Systems Department

DECEMBER 1978

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A cluster of three satellites showed large, unpredicted orbital perturbations shortly after launch. There was immediate concern for the success of its mission which depends on the stability of the cluster geometry.

The Naval Surface Weapons Center found the zonal harmonics of degree five and seven to cause most of the observed perturbations. Subsequently, the three orbits were computed for the time frame June 1976 to July 1984. It was also shown that the cluster configuration remains unaffected by the large orbital perturbations, and the mission appears safe.

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FOREWORD

The work described in this report was performed in the Astronautics and Geodesy Division, Warfare Analysis Department.

Dr. T. J. Sherrill of the Lockheed Missile and Space Company also studied the behavior of this satellite system. The principal features of the motion were obtained for 20 years by analytical techniques. Conclusions and numerical results of both investigations agree very well over the eight years covered by our numerical integration.

Released by:

R. A. NIEMANN, Head
Warfare Analysis Department

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INTRODUCTION

The Doppler Beacon 14 (DB-14), a Navy satellite system, was launched into orbit in April 1976. The orbits for this cluster of three satellites were designed such that the relative configuration and motion shown in Figure 1 could be maintained for a number of years. In a coordinate frame centered at Satellite No. 182, No. 181 remains stationary along the tangential axis, leading No. 182 by about 150 km. In the orbital plane, No. 183 describes the roughly elliptical path shown in the side view. The period of this relative motion is identical to the orbital periods of the three satellites, namely 107.5 min.

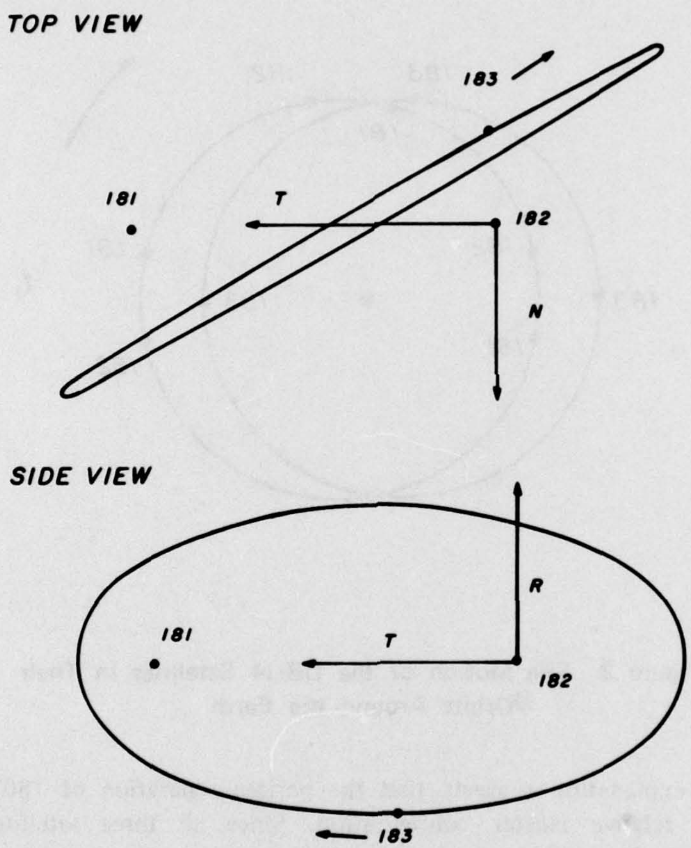

Figure 1. The Relative Motion of Satellite No. 183 with Respect to No. 181 and 182. R, T, and N are the Radial, Tangential, and Normal Components Defined by the Position, Velocity, and Angular Momentum Vectors of No. 182

The geometry depicted in Figure 1 is achieved as follows. All three satellites are placed in orbits with eccentricities of about 0.003. However, the perigee of No. 183 is 180° away from the perigee position for No. 181 while the other orbital elements are approximately identical. Of course, the true anomaly of No. 183 also differs by about 180°. Using Figure 2 and recalling that the angular motion varies from a minimum at apogee to a maximum at perigee, the reader will be able to verify the side view in Figure 1. The relative geometry shown in the top view is achieved by a small separation in the longitudes of the node.

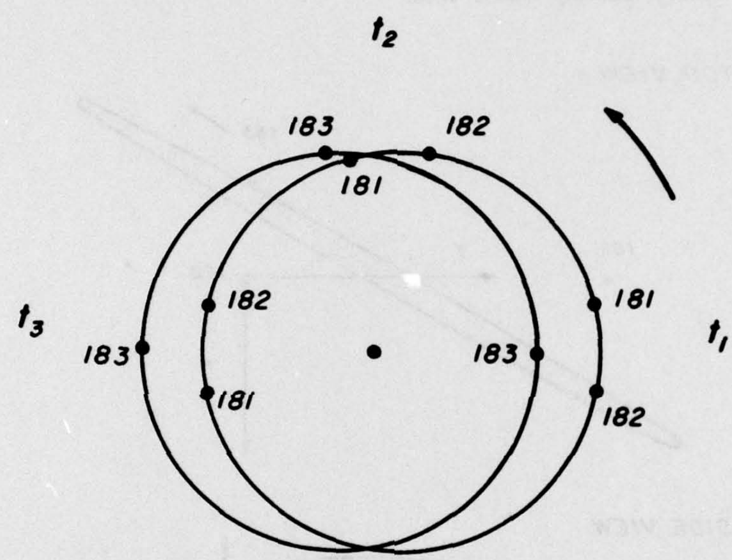

Figure 2. The Motion of the DB-14 Satellites in Their Orbits Around the Earth

The above explanation suggests that the perigee separation of 180° is crucial in maintaining the relative cluster configuration. Since all three satellites are at the critical inclination, I = 63°.43, for which the secular perigee motion is known to vanish, the perigees were expected to remain stationary.

However, within a few months after launch, significant drifts of both perigees away from their initial values were noticed. The argument of perigee of No. 183

appeared to be increasing and the other decreasing in a seemingly secular fashion at roughly 50° per year. This behavior was unexpected and raised the question whether the relative cluster configuration, necessary to the mission objective, would eventually decay.

At this point, NSWC was requested to investigate the phenomenon. The questions to be answered were as follows:

1. What causes the perturbations observed in the arguments of perigee?

2. How will the perigees and eccentricities evolve over a period of eight years from launch?

3. Will the relative cluster geometry remain stable?

ORBIT COMPUTATION

ANALYTICAL THEORIES

NSWC briefly surveyed the literature for applicable analytical solutions, and none were found. Existing theories for the critical inclination do not allow eccentricities smaller than 0.1. They are also restricted to the first few zonal harmonics.

NUMERICAL INTEGRATION

Consequently, orbits were computed by numerical integration. A fast, suitable program was available. Originally designed for the integration of five planetary orbits around the sun (Reference 1), the program was later successfully used for the NRL solar radiation (SOLRAD) orbits. The routine will, in fact, be called SOLRAD Program in the remainder of this report. Programming details may be found in References 2 and 3 while the integration routine is described in Reference 4. Basically, it is a Cowell algorithm of order 12 using backward accelerations rather than their differences.

The original program was developed for five-point masses orbiting a sixth. They represent sun, moon, and three artificial satellites with the earth at the center. This model was more than adequate for the high-altitude SOLRAD satellites. The DB-14 system, however, is at a low altitude, and gravitational perturbations must be added.

ZONAL HARMONICS

The contribution of the zonal harmonics are initialized by computing

$$U_{2,x} = \frac{3}{2}\mu \frac{R^2}{r^5}\left(5\frac{z^2}{r^2} - 1\right)x$$

$$U_{2,y} = \frac{3}{2}\mu \frac{R^2}{r^5}\left(5\frac{z^2}{r^2} - 1\right)y$$

$$U_{2,z} = \frac{3}{2}\mu \frac{R^2}{r^5}\left(5\frac{z^2}{r^2} - 3\right)z$$

$$U_{3,x} = \frac{1}{2}\mu \frac{R^3}{r^6}\left(35\frac{z^3}{r} - 15\frac{z}{r}\right)x$$

$$U_{3,y} = \frac{1}{2}\mu \frac{R^3}{r^6}\left(35\frac{z^3}{r^3} - 15\frac{z}{r}\right)y$$

$$U_{3,z} = \frac{1}{2}\mu \frac{R^3}{r^6}\left(35\frac{z^3}{r^3} - 30\frac{z}{r} + 3\frac{r}{z}\right)z$$

where we use R = 6378.15 km and = 3.98602×10^5 km^3/sec^2 throughout the project. Additional terms are obtained by the recurrence relations

$$U_{n,x} = \frac{2n+1}{n}\,\frac{R}{r}\,\frac{z}{r}\,U_{n-1,x} - \frac{n+1}{n}\,\frac{R^2}{r^2}\,U_{n-2,x}$$

$$U_{n,y} = \frac{2n+1}{n}\,\frac{R}{r}\,\frac{z}{r}\,U_{n-1,y} - \frac{n+1}{n}\,\frac{R^2}{r^2}\,U_{n-2,y}$$

$$U_{n,z} = \frac{2n+1}{n}\,\frac{R}{r}\,\frac{z}{r}\,U_{n-1,z} - \frac{n}{n-1}\,\frac{R^2}{r^2}\,U_{n-2,z}$$

Finally, the additions to the equations of motion are

$$\Delta\ddot{\mathbf{r}} = \sum_{n=2} J_n \begin{pmatrix} U_{n,x} \\ U_{n,y} \\ U_{n,z} \end{pmatrix}$$

ZONAL HARMONIC COEFFICIENTS

In selecting a set of numerical coefficients, J_n, we examined solutions made by Kozai and others, which are based on observations spanning long periods of time. They were inadquate. The NSWC set listed in Table 1 was found to match early DB-14 observations quite well. It was therefore used in all subsequent computations.

Table 1. Zonal Harmonic Coefficients

n	J_n
2	+1082.634 E-06
3	-0.2536 E-05
4	-0.1664 E-05
5	-0.2195 E-06
6	+0.6355 E-06
7	-0.3720 E-06
8	-0.3508 E-06
9	-0.8733 E-07
10	-0.5730 E-07
11	+0.1686 E-06
12	-0.3809 E-06

TESSERAL HARMONICS

Addition of the above zonal harmonics to the SOLRAD program produced satisfactory agreement between observed and computed orbits. Hence, no tesseral harmonics or any other forces were examined. It is believed that tesserals do not produce any perturbations that would be of significance in this study.

RELATIVE COORDINATES

One minor program addition was made in order to compute and plot the relative cluster geometry shown later in this report. First, calculate

$$\Delta\bar{r}_i = \bar{r}_i - \bar{r}_{182}, \quad i = 181, 183$$

Let

$$\hat{r} = \frac{\bar{r}_{182}}{|\bar{r}_{182}|}$$

$$\hat{v} = \frac{\dot{\bar{r}}_{182}}{|\dot{\bar{r}}_{182}|}$$

$$\hat{h} = \hat{r} \times \hat{v}$$

Then the relative tangential, radial, and normal components are given by

$$T_i = \Delta\bar{r}_i \cdot \hat{v}$$

$$R_i = \Delta\bar{r}_i \cdot \hat{r}$$

$$N_i = \Delta\bar{r}_i \cdot \hat{h}, \quad i = 181, 183$$

ORBIT ERRORS

As will be seen below, the DB-14 orbits were numerically integrated over eight years. At a step size of 60 sec, this represents over 4,200,000 integration steps. It is difficult to obtain a realistic estimate of the total accumulated errors. Nevertheless, it is known that the principal error in such an integration results from the truncation of the integration formula, but such truncation errors are mostly along-track and affect neighboring orbits in a similar fashion. Hence, our three satellites could be off by several degrees in orbital longitude, but they would have been displaced by almost identical amounts. However, our best estimate for the total truncation error is on the order of 250 m.

Roundoff errors are believed to be much smaller, even after 4 million steps, since the crucial calculations are done in double precision on our CDC 6700. In this mode, the binary mantissa has 96 bits. It is estimated that the total roundoff error is on the order of 50 cm along-track.

INITIAL CONDITIONS

The state vectors for sun and moon were taken from an earlier NSWC solution. Great accuracy is not required for these bodies. The coordinates are listed in Table 2.

Table 2. Initial Conditions

	$\bar{r}_0$	$\dot{\bar{r}}_0$
SUN	2.93 049 446 E+07	-2.87 322 846 E+01
	1.36 729 435 E+08	5.36 204 583 E+00
	5.92 876 673 E+07	2.32 655 692 E+00
MOON	-2.37 734 502 E+05	8.14 086 131 E-01
	-2.54 045 685 E+05	-6.86 699 432 E-01
	-1.05 515 879 E+05	-1.91 766 484 E-01
No. 181	-0.763 215 752 E+02	-0.497 922 233 E+01
	0.496 713 487 E+04	0.393 500 353 E+01
	-0.562 991 695 E+04	0.355 628 009 E+01
No. 182	0.183 933 509 E+02	-0.497 972 796 E+01
	0.489 143 089 E+04	0.402 290 333 E+01
	-0.569 670 929 E+04	0.345 488 903 E+01
No. 183	-0.393 440 660 E+02	-0.497 281 590 E+01
	0.494 473 620 E+04	0.400 936 925 E+01
	-0.560 147 416 E+04	0.355 475 489 E+01

The masses for sun and moon were already in the SOLRAD program from previous users. Given to many more significant figures than necessary, the following values were left unchanged

$$m_S = 332{,}945.5619\ 2544\ m_E$$

$$m_M = 0.0122\ 999\ 7171\ m_E$$

Initial conditions for the three satellites were based on a Naval Space Surveillance (NAVSPASUR) solution done with an NSWC CELEST program using two days of Doppler data. An independent four-day solution performed by NSWC for No. 182 differed by only a few units in the sixth significant figures. The coordinates listed in Table 2 are for 1976 June $0^h.0$ UT(Day 162).

DIFFERENTIAL CORRECTIONS

A test run using above coordinates showed that the satellites diverged at a rapid rate which would result in several thousand kilometers after a year. Obviously, initial conditions from fits over two days are inadequate to predict for hundreds of days. Fortunately, the errors proved to be almost entirely tangential, so that a simple adjustment in the mean motion, n, was sufficient. The increment, Δn, was translated into corrections to the state vector by

$$\Delta\bar{r} = -\frac{2}{3}\frac{\Delta n}{n}\bar{r}$$

$$\Delta\dot{\bar{r}} = \frac{1}{3}\frac{\Delta n}{n}\dot{\bar{r}}$$

This simple device reduced the errors by three orders of magnitude. After applying such corrections to Nos. 181 and 183, the improved coordinates are listed in Table 3. Coordinates for the other bodies were taken from Table 2.

Table 3. Corrected Initial Conditions

	$\bar{r}_0$	$\dot{\bar{r}}_0$
No. 181	-0.763 212 351 668 73 E+02	-0.497 923 342 188 51 E+01
	0.496 711 274 007 93 E+04	0.393 501 229 574 80 E+01
	-0.562 989 186 720 54 E+04	0.355 628 801 209 00 E+01
No. 183	-0.393 447 165 106 69 E+02	-0.497 277 478 998 94 E+01
	0.494 481 795 575 11 E+04	0.400 933 610 475 27 E+01
	-0.560 156 677 418 88 E+04	0.355 472 550 302 60 E+01

Examination of the relative cluster plots, given in a later part of this report, show the initial conditions in Table 3 to be good enough to permit a forward integration for five years. At that time, 1981, the relative ellipse of No. 183 is seen to have shifted somewhat to the left, indicating a mean motion that is slightly too large. It was decided to perform a second differential correction. This adjustment is different from the previous one since not only the mean motion is to be corrected but also, in order to center the ellipse again, the mean anomaly ℓ_0. The state vector corrections become

$$\Delta r = \frac{\partial \bar{r}}{\partial n} \Delta n + \frac{\partial \bar{r}}{\partial \ell} \Delta \ell$$

$$\Delta \dot{\bar{r}} = \frac{\partial \dot{\bar{r}}}{\partial n} \Delta n + \frac{\partial \dot{\bar{r}}}{\partial \ell} \Delta \ell$$

However, determination of n must take into account the short-periodic effect on n due to changing ℓ. Hence,

$$\Delta n(\ell_0 + \Delta \ell) = \Delta n(\ell_0) + \left(\frac{\partial n}{\partial \ell}\right)_{\ell_0} \Delta \ell .$$

In practice, the state vector correction was obtained by an alternate but less desirable procedure which will not be recorded.

THE COMPUTER RUNS

As mentioned earlier, the behavior of the satellite cluster was studied over a period of eight years, from 1976 to 1984. To this end, all three orbits were obtained by numerical integration through a series of computer runs which are listed in Table 4. The long runs, one to three years, serve to study the evolution of the orbital elements. The short 170-min runs were used to obtain the relative cluster configuration around one revolution.

Table 4. Computer Runs

Run No.	t_0	t_{end}	Initial Conditions
18	1976 Jun 10.0000	$t_0 + 170^m$	From Table 3
11	1976 Jun 10.0000	1978 Jun 20.8125	Same as No. 18
13	1978 Jun 20.8125	$t_0 + 170^m$	End point No. 11
16	1978 Jun 20.8125	1981 Jun 24.3875	Same as No. 13
17	1981 Jun 24.3875	$t_0 + 170^m$	End point No. 16
26	1981 Jun 24.3875	$t_0 + 170^m$	From differential correction
27	1981 Jun 24.3875	1982 Jun 24.4500	Same as No. 26
28	1982 Jun 24.4500	$t_0 + 170^m$	End point No. 27
29	1982 Jun 24.4500	1984 Jul 4.2625	Same as No. 28
30	1984 Jul 4.2625	$t_0 + 170^m$	End point No. 29

RESULTS

CAUSE OF PERTURBATIONS

An analytical first-order expression for the secular rate of perigee shows that $\bar{\dot{\omega}} = f(a, e, I)$. Hence, all three of these parameters must be consistent in order for $\bar{\dot{\omega}}$ to remain zero. Accordingly, it occurred to us to examine the deviations of a, e, and I from their nominal values to see if any of these might be sufficiently large to cause a nonzero $\bar{\dot{\omega}}$. All three effects were found to be orders of magnitude too small to yield the observed motion in ω.

It was clear that the cause for the unexpected large perturbations had to be found elsewhere. By that time we had reason to believe that some zonal harmonics were involved. Consequently, a series of test runs were made designed to isolate the term or terms in question. The first of these runs included J_2, J_3, and J_4 only. For each subsequent run one more J_n was added, through J_9. Finally, J_{10} through J_{12} were added simultaneously in the last run.

Figure 3 shows the outcome in graphical form. Obviously, J_5 and J_7 are the principal contributors to the unexpected perigee motion. They account for about 87% of observed perturbations.

Points plotted in Figure 3 are averages of 400 osculating values. This large number of points was necessitated by the 15° short-periodic noise in the argument of perigee.

Table 5 shows the numerical data used in Figure 3 as well as the results of this experiment.

Once the zonals through J_{12} were included, our computed ω and e matched the observed values quite well. Any remaining differences can be reduced by improving the numerical values of J_5 and J_7.

EVOLUTION OF PERIGEES AND ECCENTRICITIES

The development of the arguments of perigee and the eccentricities as function of time are shown in Figures 4 to 6. All points are averages of eight osculating values.

Table 5. Zonal Harmonic Contributions to Perigee Motion

Coefficient	$\bar{\omega}_{182}$ t = 3^d	t = 5^d	t = 28^d	$\Delta\omega$ (25^d)	$\Delta\omega$ over previous	motion	$\Delta\omega$ (deg/day)
J_4	83.99	83.91	83.79	-0°.20	0°.20	4.7%	0.008
J_5	83.85	83.07	82.20	-1.65	1.45	34.0	0.058
J_6	83.86	83.13	82.31	-1.55	-0.10	-2.3	-0.004
J_7	83.64	81.81	79.81	-3.83	2.28	53.4	0.091
J_8	83.63	81.79	79.77	-3.86	0.03	0.7	0.001
J_9	83.62	81.71	79.63	-3.99	0.13	3.0	0.005
J_{10} - J_{12}	83.58	–	79.31	-4.27	0.28	6.6	0.011
						100.1	0.170

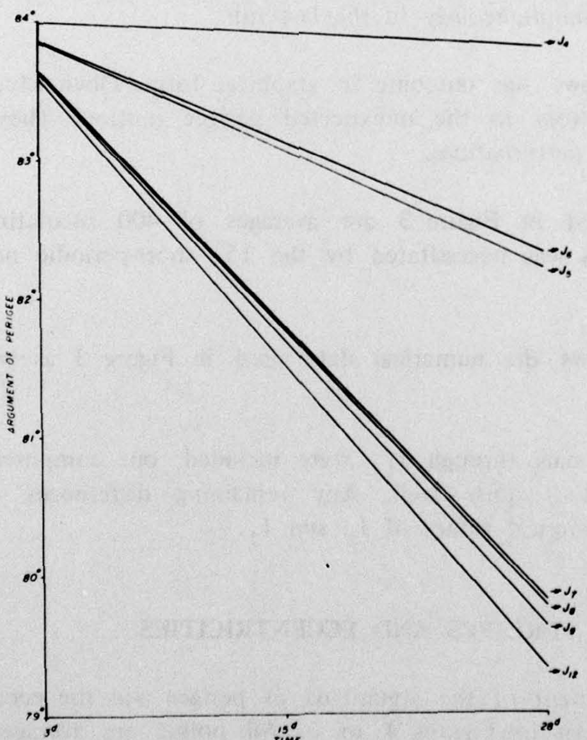

Figure 3. Effect of Adding Zonal Harmonics on the Perigee of No. 182. J_5 and J_7 Are Seen to Contribute Most to the Unpredicted Perigee Motion

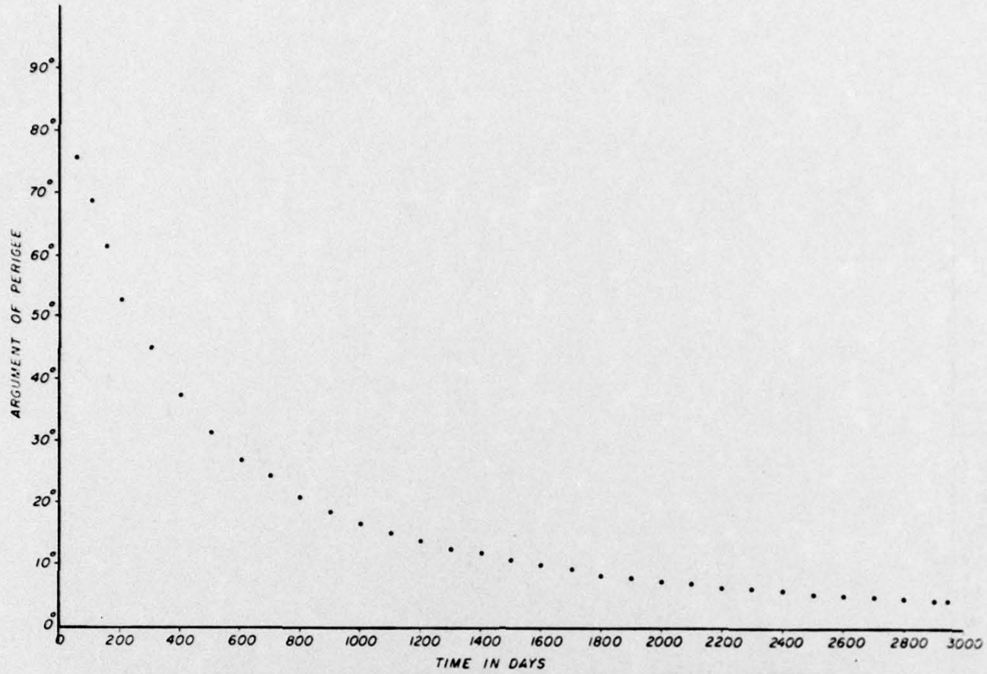

Figure 4. Arguments of Perigee for Nos. 181 and 182 from 1976 June 10 to 1984 July 4

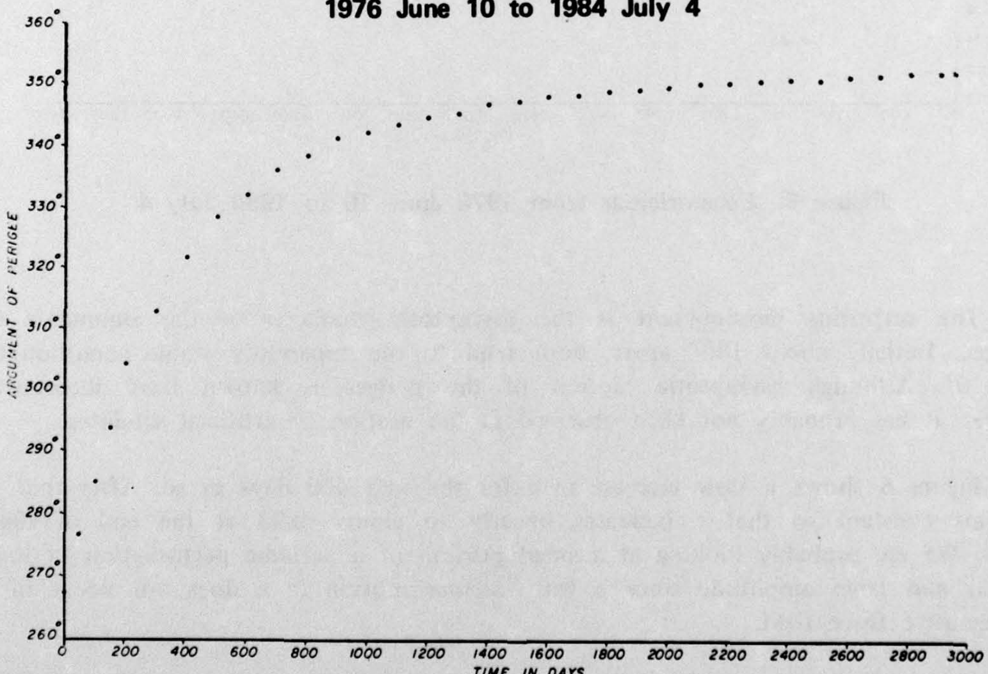

Figure 5. Argument of Perigee for No. 183 from 1976 June 10 to 1984 July 4

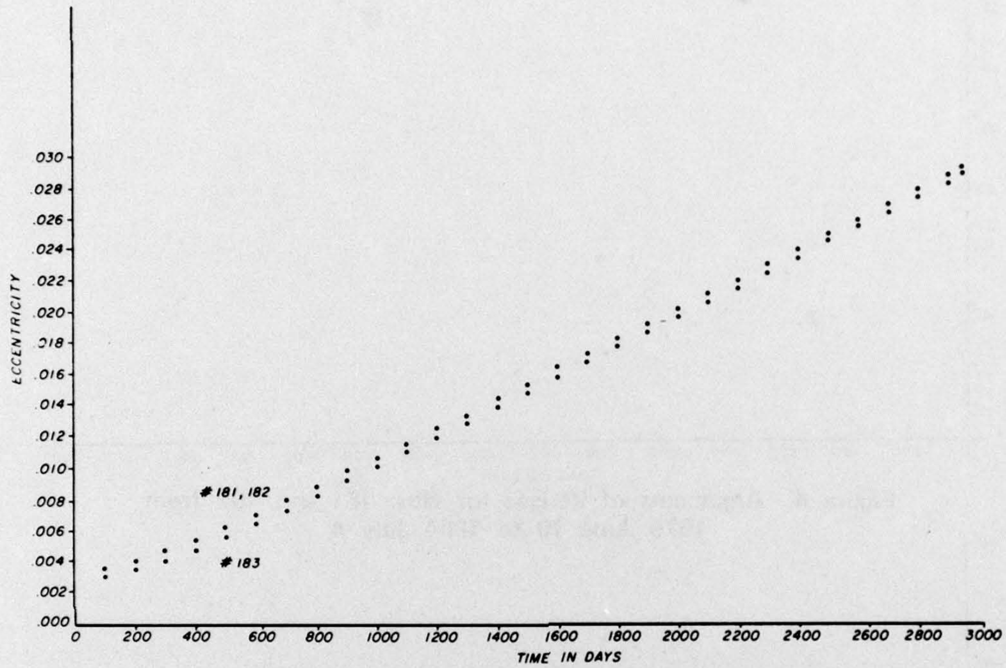

Figure 6. Eccentricities from 1976 June 10 to 1984 July 4

The surprising development is the asymptotic character of the arguments of perigee. Initially about 180° apart, both tend to the apparently stable condition of $\omega \approx 0°$. Although asymptotic motion of the perigee is known from theoretical studies, it has probably not been observed in the motion of artificial satellites.

Figure 6 shows a slow increase in $\dot{e}$ for the first 400 days or so. After that, $\dot{e}$ remains constant so that e increases linearly to almost 0.03 at the end of eight years. We are probably looking at a small portion of a periodic perturbation of long period and large amplitude since a truly secular motion in e does not occur in a conservative force field.

We are presently trying to capture the principal features of these perturbations by analytical means. Results should appear in print in the near future (Reference 5).

CLUSTER GEOMETRY

The coordinates of Nos. 181 and 183 relative to No. 182 for the eight-year period are shown in Figures 7 to 14. The side view at the beginning of the numerical integration is depicted in Figure 7. Subsequent diagrams show the relative configuration to remain absolutely stable. The slow drift of the No. 183 ellipse to the left is due to a minute error in the initial mean motion. In the real world, any such along-track deviations are corrected by a small thrust along the velocity vector. In our computer simulation, this can be done a bit easier by an instantaneous adjustment of the mean motion. As explained in an earlier chapter, this correction was made on 1981 June 24, between Figures 9 and 10.

The remaining side views show that the cluster preserves its geometry throughout the eight-year period. The mean motion correction applied in 1981 appears to have been too small as another, but smaller, drift of the ellipse can be seen.

Throughout this period, the relative path of No. 181 seems to grow into a circle of 8 km diameter. This is of no consequence to our study. A proper adjustment of the No. 181 state vector at epoch would eliminate this blemish.

Finally, Figures 13 and 14 illustate that the desired spatial separation in the plane perpendicular to the radius vector is also preserved. Since only plots for 1976 and 1984 are given, it should be added that the pattern was carefully monitored throughout the integration.

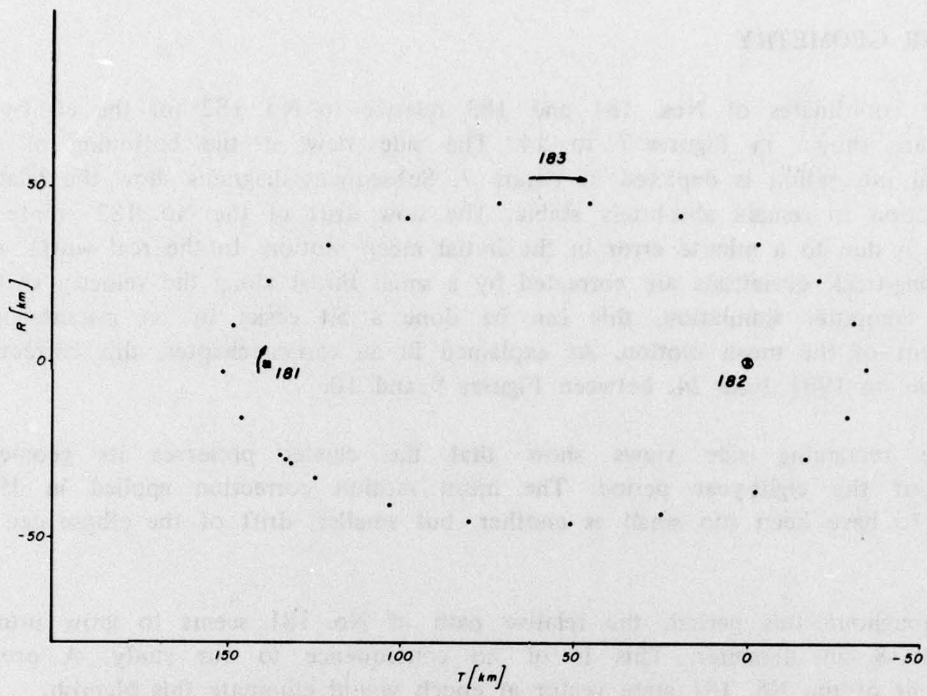

Figure 7. Relative Positions on 1976 June 10. R = Radial, T = Tangential Component

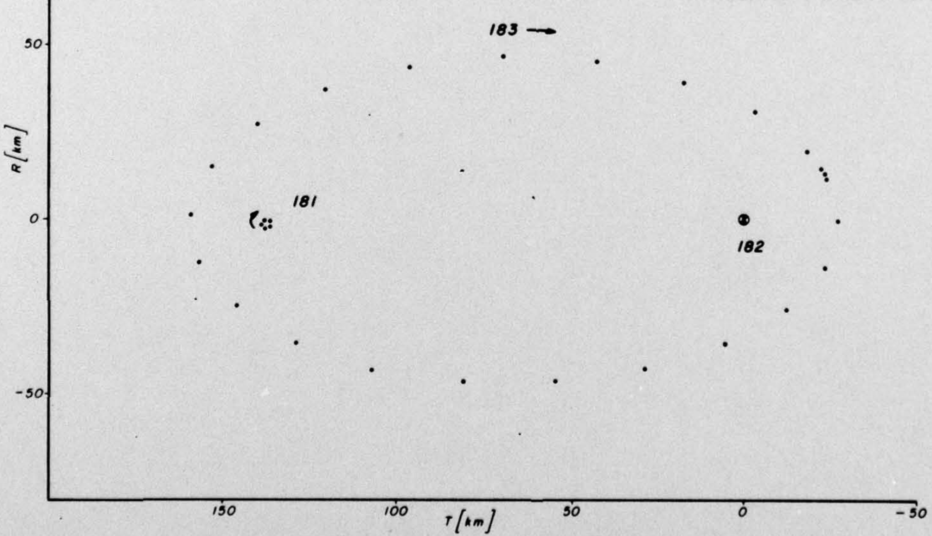

Figure 8. Relative Positions on 1978 June 20

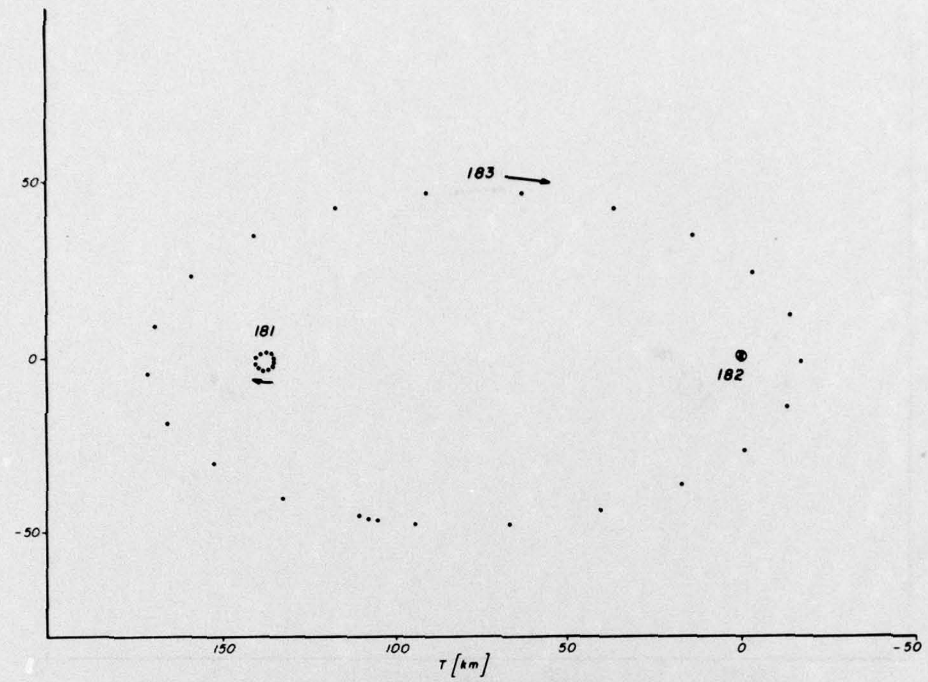

Figure 9. Relative Positions on 1981 June 24 Before Differential Correction

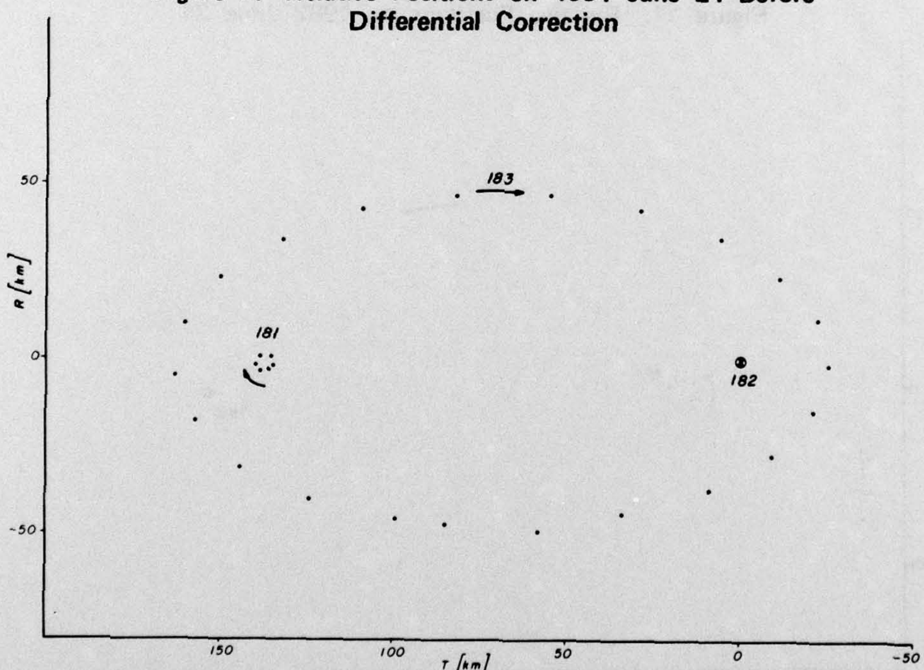

Figure 10. Relative Positions on 1981 June 24 After Differential Correction

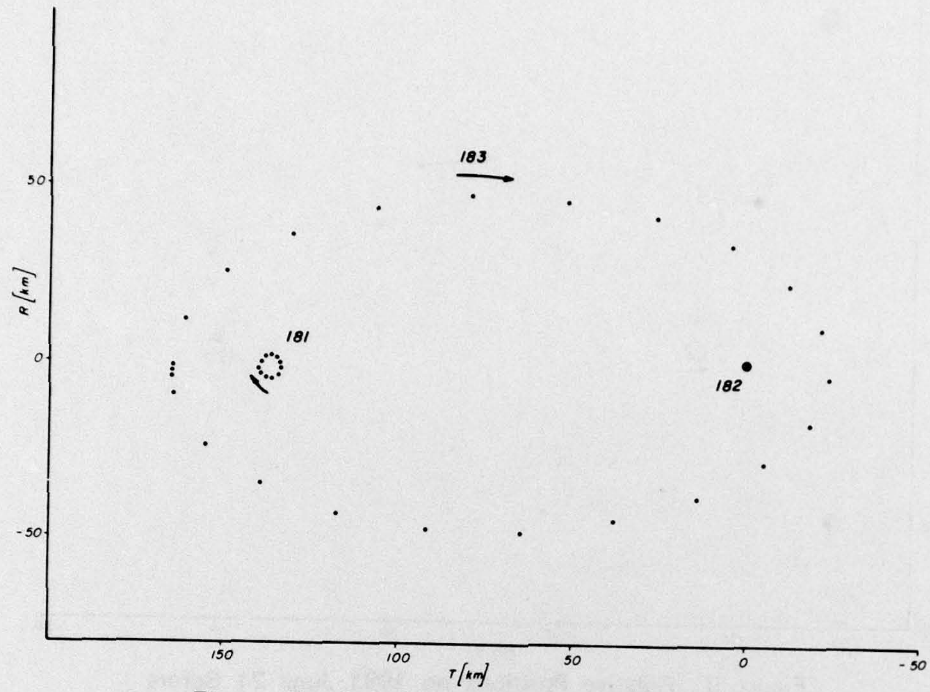

Figure 11. Relative Positions on 1982 June 24

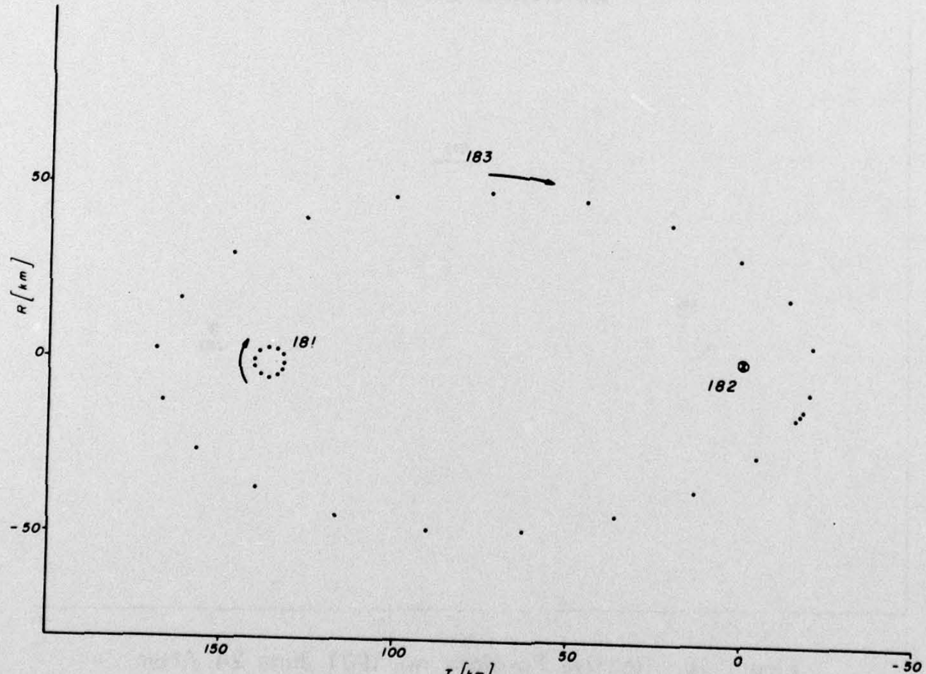

Figure 12. Relative Positions on 1984 July 4

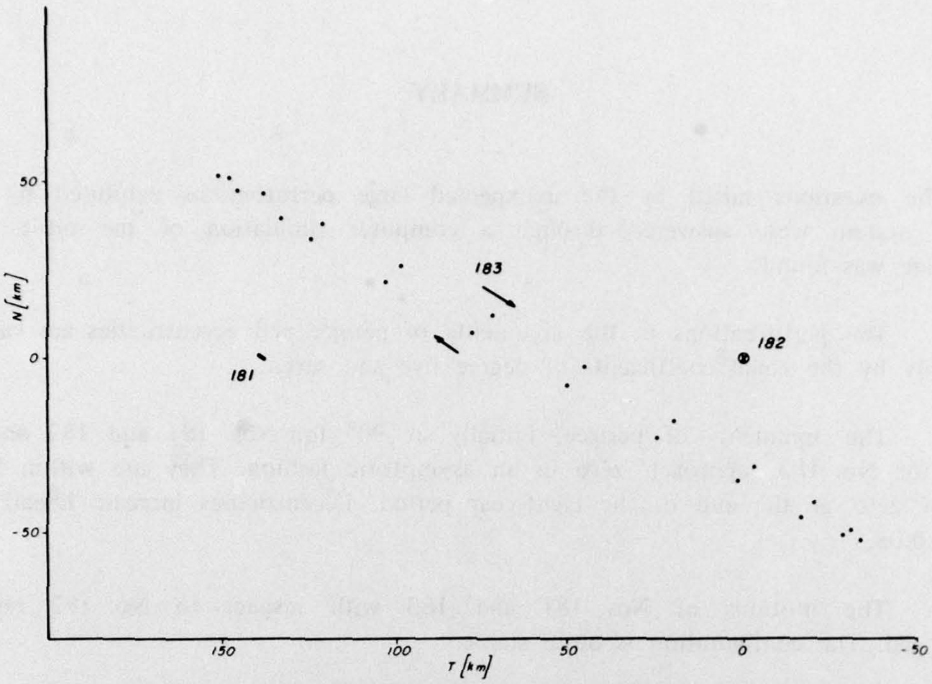

Figure 13. Relative Positions on 1976 June 10 (Top View, N = Normal Component)

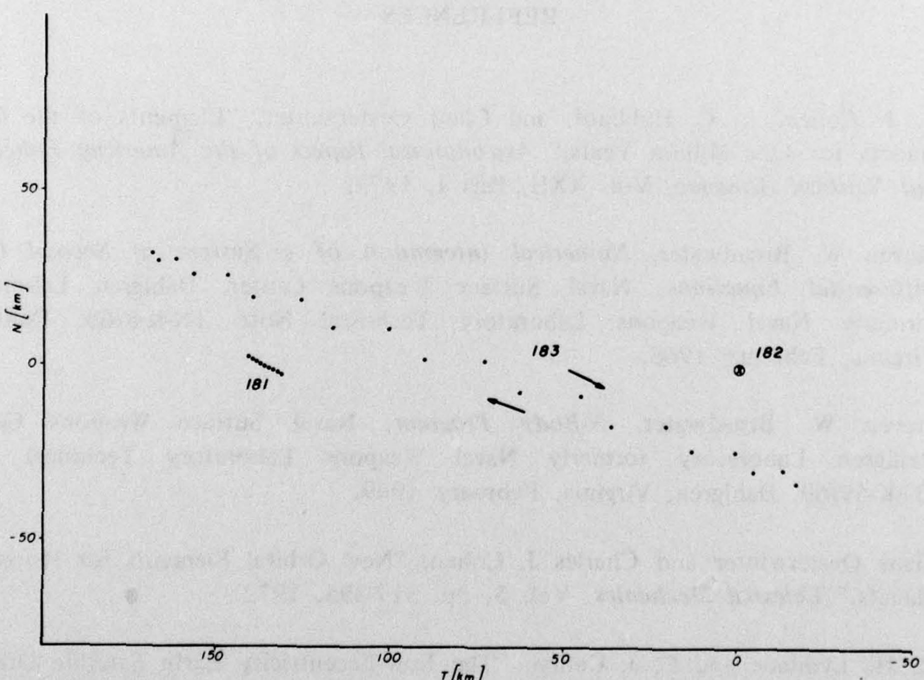

Figure 14. Relative Positions on 1984 July 4 (Top View)

SUMMARY

The questions raised by the unexpected large perturbations exhibited by the DB-14 system were answered through a computer simulation of the orbit. The following was found:

1. The perturbations in the arguments of perigee and eccentricities are caused primarily by the zonal coefficients of degree five and seven.

2. The arguments of perigee, initially at 90° for Nos. 181 and 182 and at 270° for No. 183, approach zero in an asymptotic fashion. They are within 5 to 10° of zero at the end of the eight-year period. Eccentricities increase linearly to about 0.03.

3. The motions of Nos. 181 and 183 with respect to No. 182 remain unchanged. The configuration is quite stable.

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