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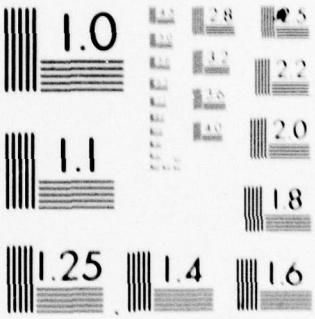


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6 DEMONSTRATION MODEL SYSTEM

VOLUME I

MATHEMATICAL MODELS

by

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## EXECUTIVE SUMMARY

The objective of the project reported in these Volumes is to elucidate the principles of hardware/manpower cost analysis developed in an earlier study, *A Framework for Hardware/Manpower Tradeoff Analysis During the Weapon System Acquisition Process*. In that study, the idea of a linked and graded model system was introduced: a system made up of several models which, while written at widely different levels of complexity, still retain mathematical consistency and differ only in the detail and resulting accuracy of each of its formulations. Unfortunately, that is the kind of idea that can sound good but *be* either good or bad, depending on how it is carried out. Since no set of guidelines, by themselves, could be expected to communicate all the detail necessary, the present demonstration system was undertaken as a way of providing some of that detail by way of an example.

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There are several departures from standard cost analytic practice embodied in the models. The most radical of these is in the formulation of manpower cost equations. The models are written with the use of three billet cost models in mind: the Enlisted Billet Cost Model, the Officer's Billet Cost Model and the Civilian Billet Cost Model. Since the models also incorporate the ability to estimate the costs of contractor operated depots, they not only allow hardware/manpower tradeoffs, they also allow tradeoffs between different forms of manpower. Treating the cost of a man as exogenous, the models concentrate on accurately counting up the number of men of each type required. For some costs, such as compensation, the real number of men is determined on the basis of demand. For others, such as training, the integer number is determined instead. This distinction is important: while wages may only have to be paid to the equivalent of 50 men on 200 ships, 200 men must be trained. The cost elements appropriate to integer quantities can therefore contribute more to life cycle cost than some elements, which, intuitively, seem more costly.

One of the most important departures in the manpower formulations is the recognition of "pools" of labor on board ships. Two pools are distinguished. One consists of men with the appropriate "A" school to staff a system, but not "C" school. They are members of the pool because they are not fully utilized in the job for which they were trained.

These men (or parts of men) represent a resource which can be utilized by a new weapon system if planning is done accurately. The second pool consists of all men aboard, fully occupied with general duties (not requiring advanced training) who could complete the required training for the new system, if required. Our manpower formulations assemble the required staff from members of these two pools, choosing them in the most cost-effective manner.

Spare stockage is another area of departure. It is common practice, in level of repair analysis, to look at each component individually - a better term might be myopically. This leads to errors in the computation of spare stockage costs because the level of confidence in avoidance of a stockout depends on the *system's* characteristics, not just its components' characteristics. In large models it is possible to develop spares estimates for all components simultaneously, working to a system confidence criterion level. We do this, in combination with a marginal return on investment computation, in the most complex of the models presented here. We have also, however, developed a stockage technique, suitable for limited computational power, which gives results that are very close to those achieved with the simultaneous method. This method, however, can be carried out for a single component of the system, making level of repair analysis simple and compact, as well as accurate.

In the cost analysis of support and test equipment, the model pays more detailed attention to software cost than earlier efforts. This attention extends to at least crude formulations for the support cost of software. With the single exception of spares, every cost element is responsive, at both the component and system levels, to the selection of a military or contractor operated depot. All equations are responsive to the general level of repair options: discard at failure, local repair and depot repair.

In this Volume, six models are presented. The Level III Model is the most complex and its exposition is used to set notation and explain the underlying concepts of all the models. It is intended to be implemented on a large, production computer. The Level II Model, less complex, was developed for implementation on a stand-alone micro-computer of the sort one might expect to be available to a design team. There are four Level I Models, all developed for use on a programmable calculator. These include first, a Top-Down Model, intended for use by a system designer, starting with nothing more than system specifications as input data. The next Level I Model is the Lowest Removable Assembly

Model (LRAM), intended for use by a detail designer. Its input data set initially consists of the system designer's guidelines and ultimately of detailed specifications for the new LRA. The third model is the System Aggregation Model (SAM) whose inputs are outputs of the LRAM. Since these are in the same format as the outputs of SAM itself, the latter can also be used as a multi-level aggregation tool. Finally, a System Confidence Model was prepared to provide the system designer with a measurement of support effectiveness achieved by his design.

Of the six models, all but the Level III Model have been programmed as part of this project. The Level II Model has been programmed in BASIC on a 48 K microcomputer. A User's Guide and Program Manual for that model constitute Volumes II and III of this study. All the Level I Models have been programmed on a Texas Instruments TI-59 programmable calculator with printer. The User's Guide and Program Manual for those models are published as Volumes IV and V of the study.

## PREFACE

The cost models presented in this Volume are a distillation of several years' thought about life cycle cost, manpower cost analysis and the use of cost analysis in the design process. The reader will find that the discussion, while unavoidably arid, includes many significant departures from established practice in these areas. While the authors would like to lay exclusive claim to the ideas underlying these formulations, doing so would be a grave misrepresentation.

Our understanding of the design process derives from several years of consulting with design teams in private firms. The most important of these have been Rudy Cazanjian at GTE Sylvania, Stuart Moore and Lyle Whitlock at Honeywell, Bob Lane at CDC and Bruce Whitehead at ITT Gilfillan. In manpower cost analysis, LCDR Lee Mairs and Paul Hogan of OP-212 and Ernie Koehler of NPRCD have helped us to understand the tremendous intricacy of the cost analysis of military billets. The work on spare stockage analysis has benefited by discussions with Marco Fiorello of LMI and John Decker at GTE Sylvania. Finally, several people have helped to enlarge our understanding of the role of cost analysis in the acquisition process. Russ Shorey of OSD/MRA&L has given us much of his wisdom about the realities of DoD acquisition as have Frank Swafford and Mary Snavelly, both of OSN/MRA&L. Among the sponsors of the project, Bob Lehto provided the initial impetus and LCDR Jim Ruland and Lt. Fran York of the HARDMAN office provided encouragement and support.

Notwithstanding these contributions, the authors are, of course, responsible for any errors or omissions remaining.

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## 1.0 INTRODUCTION

A study, carried out during FY 1978 as part of the CNO's Studies and Analysis Program and under the aegis of the HARDMAN project office, sought to identify means by which hardware and manpower resource costs could be minimized through early cost estimation.* The study was sharply critical of several aspects of current Naval cost analysis methods and suggested a number of revisions. In particular, the study developed a mathematical structure by which manning requirements for new equipment could be determined and their cost estimated during the earliest stages of design. Beyond this, the study suggested that single cost models were inappropriate to the way in which new equipment is acquired.

The design process, it was suggested, is one in which the information characterizing a new system is initially very limited and subsequently grows more and more detailed as decisions are made about form, function and technical approach. The cost analysis methods required to aid the decision-making process must recognize this expansion in data and exploit it through gradations of complexity. That is, cost models must be graded - the simplest forms used for the earliest phases of design and the most complex ones for the latest stages.

But grading of cost models is not enough. Since the absolute cost estimates generated by most cost models are largely irrelevant to the costs that ultimately arise over the life of an equipment, their value lies in comparison of alternative approaches at a point in time.

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* See Neches, T. and Robert A. Butler, *Guidelines for Hardware/Manpower Cost Analysis*, AG-PR-A100-2, Volumes I, II and III, The Assessment Group, Santa Monica, 1978.

Thus, the elements of a cost model that cause the estimate to rise or fall must, in fact, be important cost drivers. As the complexity and accuracy of the model increases, therefore, essentially the same cost drivers - and driving mechanisms - must be employed so that decision criteria implicit in each model remain constant. Even though completeness and accuracy of cost estimates rise through increasingly complex models, their relative valuation of alternative design approaches must remain largely constant. This can only be done by ensuring that the various grades of the cost model system are mathematically consistent - linked to each other.

When a model system with these characteristics, called a linked and graded model system, is employed, the economic aspect of the design process resolves itself into an information gathering process, in which each refinement in design yields marginal information on cost. The earliest steps amount to choosing between broad alternatives.

Each alternative can usually be distinguished from its competitors by a few important variables. Once the choice has been made among broad alternatives, detailed design work ensues, generating more complete data on a variety of more finely drawn sub-alternatives. These can only be distinguished, in the cost domain, by a more detailed model. Again, the choice is made and the favored approach carried further. This generates even more detailed data and a new set of alternatives, among which, the most cost effective can only be chosen with the help of an even more detailed cost model.

It was felt that the idea of a linked and graded model system had sufficient merit to be explored in greater detail. This paper reports

the result of that effort. The method chosen was to actually develop a cost model system to demonstrate the principals of manpower cost analysis developed earlier in the context of a linked and graded model system.

The system presented here consists of six different cost models, prepared at four levels of complexity. Two levels of complexity, and four models, have been prepared for use on a desk-top programmable calculator. These are all referred to as Level I Models. Another model was developed for implementation on an inexpensive, self-contained micro-computer. This is the Level II Model. The final model, Level III, was developed for implementation on a large scale production computer. Of the six models, all but the last have been programmed as part of this effort. Thus, in addition to this volume dealing with the mathematical models, there are four other volumes constituting the final report of the project. The others are user's manuals and program manuals for the Level I and II Models, respectively.

The organization of this report is as follows. Chapter 2 is a general description of the model system, focusing on the elements of cost estimated by the models, which elements are not estimated, and the methods chosen for various critical cost areas. Chapter 3 presents the mathematics of the manpower cost computations. The most complex model, Level III, is explained first, establishing general method and notation. The Level II Model is discussed next, with most of the attention focused on means and rationale for simplification of the Level III Model. The Level I Models are then explored in the same fashion.

Chapter 4 has the same format as Chapter 3, but refers to the computation of spare stockage - a particularly weak area in extant Navy models. Chapter 5 discusses all the other cost elements included in the model, the three model levels being explored element by element.

## 2.0 GENERAL MODEL SYSTEM DESCRIPTION

The demonstration model system presented here includes six individual computational equation systems at four levels of detail, intended for three types of computational machinery. Those written specifically for the programmable calculator (a Texas Instruments TI-59 with printer) are the least demanding of input data and the easiest to use. There are four of them, jointly referred to as the Level I Models throughout the paper.

The Level II Model represents a considerable expansion in detail and, as a result, in its demand for input data and complexity of use. It has been programmed in BASIC for use on a 48K Apple II micro-computer. The selection of the class of micro-computers was guided by our belief that models of the Level II type can only receive widespread use if they are disassociated from the production computer environment. Doing so allows the design engineer, not principally concerned with either cost analysis or sophisticated computers, to achieve significantly greater control over the means of computation, eliminating both cost analysts and computer specialists from the set of barriers between his design ideas and information on their cost implications. The low cost of these machines makes their widespread use by design teams feasible, while their tremendous computational power allows considerable detail in the models they drive.

Finally, the Level III Model was developed with a large scale production computer in mind. It's primary purpose is to dot the i's and cross the t's of the cost estimation process well toward the end of the

design phase. Unlike the first two levels, it is not intended for high volume use.

*Cost Domain.* Although we refer to the models presented here as life cycle cost models, that is something of a misnomer. Two elements or phases of life cycle cost have been excluded: research and development and salvage. In both instances, the exclusion stems from two problems. First, there are no really reliable methods for estimating these costs and second, neither is greatly impacted by the design process.

Research and development costs can be estimated in a variety of ways, none of which is particularly reliable. All utilize cost estimating relationships (regression analysis) based on historical data, which may or may not be appropriate to a new effort.* A more compelling reason for excluding research and development from these cost models is that the models are intended for use during the research and development phase. As a consequence, budgets have already been settled for Navy funding and contracts written with design agents. That is, research and development costs are (virtually) historical and do not require estimation. Nor would alterations in design lead to different research and development costs, unless a contractor, for example, proposed to trade some extra design effort for savings down the road. Even if this were the case, the trade-off might use the model system to determine outyear savings, while bottom-up technical estimates (bids) would be used to estimate the increment in design cost.

*See the general argument against the use of CER's in Neches, T. and R. Butler, *op. cit.*, *Guidelines for Hardware/Manpower Cost Analysis*, AG-PR-A100-2, Vol. II, Chapter 4, The Assessment Group, Santa Monica, 1978.

The arguments are similar in the case of salvage: there is no reliable method for predicting the salvage value of a system ten or fifteen years hence, nor will design changes have an appreciable impact on that value: at any rate, not one that can be predicted at the beginning of the life cycle.

Then the cost estimated by these models is really a combination of production and operating and support (O&S) costs. In effect, we pick up the life cycle at the point where design can influence costs and drop it where design no longer has a role to play - or where the role is so speculative as to make little or no difference. By eliding these ends of the life cycle we simplify the resulting models significantly, while losing very little of the total cost - and almost none of the cost influenced by design.

With the estimate of both production and O&S costs, the models have the capability of portraying perhaps the most important and most puzzling of trade-offs in the acquisition process: between expenditures on current accounts and those to which the government commits itself over a period extending ten and even twenty years into the future. The trade-off is often puzzling because the methods by which one compares time streams of cost of different shapes are difficult to understand. Beyond that, even when one understands the economic theory, there are so many problems in political theory and public finance tending to obscure the purely economic issues, that a mutually agreed-upon set of rules has still eluded practitioners. We have chosen a particular way to deal with this issue in the current model system. An explanation and justification for this choice are presented in the next section.

*Intertemporal Cost Comparisons*. If a designer has a bright idea that could save money in the long run, but add to the initial cost of a system, he finds himself in a generally unrewarding - even ticklish - position. His problem is to convince the buyer that he is really concerned about reducing total cost and not merely increasing the size of his company's production contract. This problem is actually absurd - a non-problem if you will - but one that grows out of a variety of political and budgeting realities over which neither the buyer nor the designer have much control. It is usually aggravated by the fact that the buyer lacks a good grasp of intertemporal cost comparisons.*

The models in the demonstration system either include or accommodate a discounting function that allows intertemporal comparisons of different cost streams. We shall attempt to make the rationale for their use understandable in the process of defining the factors themselves.

First, consider the problem of the program management officer (PMO) confronted by the suggestion to spend an additional X dollars in production for a prospective savings of .2X during each of the ten years of the anticipated useful life of the system. His problem breaks down into several parts. Are .2X and X reliable estimates of the relevant values? Is the useful life really going to be ten years? Will his program budget absorb X, whether the change is a good idea or not? Is there some source for additional funding if he doesn't have the budget? And finally, even if all the other answers turn out right, is the change

*Designers, who work for private, profit-maximizing companies, generally *do* have a grasp of the fundamental idea. Their success, or lack of it, is often tied to their ability to make good (profitable) intertemporal comparisons and decisions.

a good idea? Imagine that this is, in fact, the case and all the PMO has to do is figure out where the Navy's best economic interests lie.

The first and most obvious comparison is also the most commonly given and, unfortunately, quite wrong. That is, he could compare X to ten times .2X and find that the net savings to the government is 2X - X or X dollars - to be realized over a period of ten years. What's wrong is that there is something quite different about X dollars today and X dollars ten years from now.* The difference, put simply, is that ten years have gone by in which the government wasn't able to spend X dollars. Until committing to the design change, the government could have done any one of a number of things with that money. Consider one alternative: retirement of public debt.

The government borrows money from citizens through bonds and pays interest on the money. If it buys up a bond, then it no longer has to pay interest. Imagine that X dollars worth of bonds were purchased, all with ten years to go before maturity. Imagine further, that the interest on the bonds cost .1X each year. It should be evident that the savings on investing X in design changes certainly shouldn't be compared to 2X without any adjustment, but instead with what an investment of X would bring elsewhere. There is a useful phrase to define what we mean by elsewhere: in the best alternative use. Put in these terms, the design change can be thought of as a potential investment of X with a return of .2X per year for ten years.

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*And the difference has nothing to do with price inflation. In other words, think of a fixed set of goods or services worth X of today's dollars. We are speaking of the same quantity of goods or services delivered ten years from today.

The return against which it must be compared - that represents the starting place or zero point in a computation of profit - is the discount rate.

So if the discount rate could have been earned elsewhere, the PMO wants to adjust the estimated savings of .2X per year by deleting from that amount the return which would have to be foregone if X is invested in the design change. This is done through a very simple device. To see the sense of it, we must ask how much a stream of .2X a year is worth at the time we make the investment. Let .2X = 1 and the question becomes, how much would have to be invested if its total value is 1 after a year. That is, if $y(1+\rho) = 1$, how much is y? The answer is $1/(1+\rho)$. If a similar amount is paid at the end of a second year, it will have accumulated interest through two periods, so for that payment $1 = y(1+\rho)(1+\rho) = y(1+\rho)^2$. In general, for the $n^{th}$ year and a payment of one, $1 = y(1+\rho)^n$ and $y = 1/(1+\rho)^n$. Now a stream of earnings of one every year is equal to the sum of all these terms:

$$2.1) \quad Y = \sum_{t=1}^{n} \frac{1}{(1+\rho)^t}$$

We can replace the numerator, 1, with any value representing the annual payout of the investment - for example, .2X.

Now we can answer the PMO's question about whether the investment of X is warranted by a stream of savings of .2X a year for ten years. The stream of earnings, if $\rho = .1$, is worth PV:

$$PV = \sum_{t=1}^{n} \frac{.2X}{1.1^t} = .2X \sum_{t=1}^{n} 1.1^{-t} .$$

The sum is equal to 6.14 in this case, so PV = 6.14(.2X) = 1.23X. Since that is greater than X, *after* we've eliminated the alternative rate of return, the investment makes sense. The procedure just followed is called discounting a stream of earnings (or costs) to present value (PV). The discount factor (6.14) tells us that an investment of $6.14 would just be capable of paying out $1.00 a year for ten years at an interest rate of 10%. Therefore, a project which paid less (say $.99) would be rejected, one paying more would be accepted and we would logically be indifferent between $6.14 now and a project that paid exactly $1.00 every year for ten years.

To see how important the discount rate ($\rho$) is, the same sum at 8% would come to $6.70, but at 15% it would only reach $5.00. Therefore, at 15%, PV = 5(.2X) = X and the PMO would be indifferent between the current design and the proposed alternative. By the same token, he could flatly reject it if either the discount rate were greater than 15% or the return were less than .2X.

In the demonstration model system, we have discounted all costs to present value through the use of a discount factor L, defined as:

$$2.2)\quad L = \sum_{t=1}^{LC} \frac{1}{(1+\rho)^t} ,$$

where LC is the number of years in the operating and support phase. All costs are, therefore, discounted to the first day of deployment of the new system. If the acquisition cost of something is A and annual support costs for it can be portrayed as a proportion, m, of A,

the discount factor would be used in the following way:

2.3) $LCC_A = (1+mL)A.$

The equation states that the life cycle cost of the item is equal to its acquisition cost, (A, not discounted) plus a stream of costs in the amount mA, incurred over LC years. The latter are discounted to present value at a discount rate of $\rho$.

*Level of Repair*. Technically, the models presented here are known as single indenture, two-echelon level of repair models. Indentures refer to the hierarchical levels of assembly into which a system can be divided: subsystems, weapon removable assemblies, shop removable assemblies, modules, sub modules and the like. Echelons refer to the various repair facilities, also arranged hierarchically making up the support environment: organization or local repair, intermediate repair activity and depot.

The present model analyses an indenture level, called the lowest removable assembly (LRA), and a hybrid set of echelons including local and depot repair facilities with an intermediate depot for supply purposes. We were led to these choices by modern support and design practice, rather than traditional cost analysis techniques. By choosing a single indenture for analysis, we escape the tremendous complexity of multi-indenture models, while also reflecting the modern design practice of modularization. As this practice becomes more prevalent, that of using an intermediate repair facility also dies away, and we have removed that echelon from the model as a result.

In this regard, however, we have added two things which, to our knowledge, are not usually included in level of repair models: an

option between military and contractor operated repair depots, and a further option between civil servant and military personnel at a military repair depot. We are led to both of these embellishments by our concern with accurate methods of costing both hardware/manpower trade-offs and those between different forms of manpower - military, civil service and contractor.

The determination of level of repair policy, usually seen as the bailiwick of support specialists, is actually an important design decision. As a consequence, level of repair is an integral part of these models. The idea of level of repair analysis is to determine a policy for each element of a system by which that item will be supported. For example, a local repair policy indicates that the item will, as a matter of policy, be repaired aboard ship. The significance of setting such a policy is that several cost-generating consequences accrue. The right men must be trained in the item's repair, appropriate equipment must be purchased in sufficient quantity, spare parts of a certain type must be purchased and located where they will be most useful and so on. Depending upon the characteristics of the equipment and the support environment provided by the Navy, each policy can prove to be the most effective - or least costly. More to the point, however, a designer can alter his approach to a design problem in such a way as to enhance economy of support if he can test different approaches against each feasible policy.

Most designers begin their conceptual work unconsciously biased in the direction of a repair policy (as opposed to discard at failure). This is understandable because the savings accruing to a discard policy

are intuitively less obvious than those associated with repair. Nonetheless, the inherent characteristics of a design approach frequently show discard at failure to be either the least costly policy or close to the nearest repair option (local or depot). A designer equipped with an appropriate cost model may well find that by *planning* for a discard posture, he can save significant amounts of money. The reverse process, while less common, can also occur.

For these reasons, level of repair analysis is emphasized in all of the models in the demonstration system. In concert with production costing, the designer also gets a more accurate picture of the real difference between support policies. In general, the choice of a level of repair amounts to a tradeoff between the purchase of spares and that of repair capability (training and equipment). Therefore, the production costs of the item and the system as a whole are both influenced through the learning curve effect: as more units are produced the average unit cost declines.

The support policies modeled include local repair, contractor operated depot repair, military depot repair and discard at failure. In the equations each of these policies is distinguished by the use of three "switches" or variables, whose value is either zero or one. The switch names and their settings for each policy are shown in table 2-1.

Table 2-1: Switch Settings for Support Policies

$$\begin{array}{l} \\ \text{Local Repair} \\ \text{Depot Repair-Contractor} \\ \text{Depot Repair-Military} \\ \text{Discard at Failure} \end{array} \begin{array}{c} \begin{array}{ccc} r_1 & r_2 & r_3 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

An example of the use of such switches can be given by imagining that the cost accounted for in a category, C, will be L if a local repair policy is followed, D if any depot repair policy is followed, but 2D if a military depot is used and nothing if the item is discarded. The single equation for this group of costs is:

$$C = r_1 L + D(r_2 + 2r_3)$$

The purpose of the posture switches is to turn on or turn off different elements found in a single equation. By being able to write a single equation for all policies, any computer program is simplified: switches are set to one of the policies, a computation made, then reset and the next computations use the same block of code. This reduces the programming at least to a third of what it would be if different blocks of code were used.

*Special Cost Elements: Spares and Manpower*. A large part of this report is concerned with the computation of spare stockage costs. Since the focal point of these models is intended to be manpower, this emphasis may seem stange. Some explanation is in order.

To a large extent, the computation of spare stockage and such associated measurements as availability rates and readiness rates forms the nucleus of most models of operating and support cost. The reason is quite simple: aside from the potential combat capability of a system, there is no way, other than some version of an effectiveness rate, to determine what the Navy gets for a given level of support.

Another way to say this is that there are no technical relationships between the characteristics of a system and how much support cost to undertake. To illustrate with an absurd example, O&S costs could be minimized if the new systems were left in crates, requiring no support at all.

It is, therefore, important to set some quantifiable effectiveness goal and attempt to achieve it at least cost. In spare stockage this is conceptually simple to do. We have chosen as our criterion the probability that a ship will experience no stockout during a deployment. A stockout occurs when a demand for a spare part, generated by a system failure, cannot be satisfied from ship's stores. In the models, this criterion probability is set by the user and the model "buys" the smallest dollar value mix of spare stockage capable of satisfying the criterion.

While we know that other elements of support dealt with in cost models have a similar impact of readiness or some other measure of combat effect, the mathematical apparatus has generally not been developed to sense and measure these relationships. As an example, there is certainly a trade-off between the number of men trained to perform a given task and the probability that one of them will be available to perform it when necessary. There is also a trade-off between the extent of training given and the likelihood that the last subject taught will contribute enough to the firm's effectiveness. In our terms, however, these are difficult questions, while the impact of spares on stockout rates is an easy one. Nor have we solved those hard

questions in this research effort. In the area of manpower cost analysis, however, we have moved ahead to some extent.

A distinguishing feature of the manpower costing formulations in the demonstration model system is their recognition and costing of billets as opposed to manhours. As a consequence of dealing with billets, the formulations have also been made sensitive to the question - where does the manpower come from? Since much of the labor used by a new system is taken from people who are already members of a ship's company, computation of marginal costs demands that we understand how - if at all - this labor is more costly as a consequence of introducing a new system. We have also been concerned with the computation of training cost as a separate issue from compensation or billet cost.

Economic cost is based on the notion that the real cost of doing something is the value of what you could have done otherwise.* In terms pertinent to the model, the cost of using a man as a skilled technician is, for the most part, his value as an unskilled crewman, because that is the service that had to be given up. The training provided him used up other resources which must be valued in the same way. If his compensation rises as a result of training, this is an additional cost in foregone goods and services that must also be counted.

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*For a lucid exposition of the economic interpretation of cost, see Alchian, A, *Economic Forces at Work*, Liberty Press, Indianapolis, 1977, Chapter 12, pp. 301 - 303.

To accurately account for these costs, it should be clear that we must know, not only how many men will be used on the new system, but what their compensation costs and relevant training were before the system was purchased. To do this, we have assumed knowledge of two pools of manpower on the ships to which a new system will be deployed. They are called AN and AG. The first pool consists of skilled manpower not being used full time in their skill area. That area, furthermore, is defined as providing the appropriate "A" school, though not the right "C" school for the new system. For members of the AN pool, total costs will consist of the cost of "C" school *less* the opportunity benefit arising from the fact that they are no longer being overpaid to perform unskilled labor. Costs for people drawn from the AG pool consist of "A" and "C" school costs, the rise in their wages resulting from the new skill and the opportunity cost of the work they were formerly performing. If they are not completely utilized in the new skill, however, and spend part of their time performing duties worth less than their new compensation rate, the difference between their pay and value is an additional cost that must be counted.

In effect, we focus on the AN pool to track "overpayments." Since this pool consists of skilled manpower being used for unskilled labor, the net increase or decrease in its size leads to a net increase or decrease of overpayments. Thus, a system which, while using positive amounts of labor, decreases the size of the AN pool, is credited by that utilization.

A third pool, AS, is conceptually infinite and does not require data collection. In the event that a new system's manpower demands exceed both AN and AS pools, AS must be tapped. It consists of unskilled manpower not already part of the ship's company. As a consequence, the cost of a man from AS exceeds that of one from AG by whatever incremental costs arise from expanding the ship's company.

The models in the demonstration system select people from each of the three pools in a cost minimizing fashion. The algorithms take into account both the actual quantity of labor used (to which compensation costs and certain overhead charges are proportional) and the *integer* number of men who must be trained in each school. As noted elsewhere, billet cost data are used for compensation costs, but per-man "C" school costs are estimated in the model as a function of equipment characteristics and the opportunity cost of trainee's time.

*The Setting*. All of the models in the demonstration system were written for electronics equipment deployed on non-carrier surface ships. They can be used for submarine and land-based "minimally attended" electronic systems as well. There are enough differences between these and the support environment appropriate to land- or carrier-based systems, that estimations for the latter would require extensive revisions. Furthermore, the models, because they use stationary failure mechanisms, are not really appropriate for hardware systems with large elements of mechanical equipment subject to wear-out behavior.

## 3.0 MANPOWER COSTS

Manpower costs are broken down into three elements for purposes of estimation: compensation, training and other costs. Compensation costs include all direct costs, other than training, associated with a particular billet. As such, the output of the Navy's Enlisted Billet Cost Model can be used for the three major "prices" of the manpower section of the model: BG, the cost of a general, relatively unskilled billet; BN, the cost of a specialist in the field required by the new equipment; and BD, the cost of a technician at military operated depot (MOD). If the MOD is staffed by civilian technicians, the Civilian Billet Cost Model (currently under development at the Naval Research and Development Center) can be used for the variable BD. The output of the Officer's Billet Cost Model can be used similarly for BO.

The computation of manpower costs in the present model is based on earlier work to which the reader is referred for a much more detailed discussion.* There are three stages to the computation. First is the estimation of raw demand levels for manpower of different types. Next, these requirements are compared to the pools of manpower available to satisfy the demand, and least costly combinations of manpower are drawn from the pools. Finally, compensation, training and other costs are estimated on the basis of the resulting characteristics of the manpower combination chosen to serve a given weapon system.

*See, Neches, T. M. and Robert A. Butler, *Guidelines for Hardware/Manpower Cost Analysis*, AG-PR-A100-2, Vol. I (for OP-122H), The Assessment Group, Santa Monica, 1978; Chapter 3.0 especially.

The manpower pools mentioned above are AN, AG and AS. The first are underutilized specialists, already assigned to a ship, who have successfully completed the appropriate "A" school training course for either operation or maintenance of the equipment. By underutilized, we mean that some portion of their regular work schedule is taken up by duties other than those they have been trained for. Such underutilization creates a cost to the Navy equivalent to the difference between their billet cost and that of a less well trained or compensated man who would be capable of performing the same unskilled labor. By providing an additional "C" school course to these men, the Navy is able to escape their underutilization as well as the cost of supplying someone else both "A" and "C" schools.

The AG pool consists of those men on board, employed in general duties (i.e. with a relatively low payoff to the Navy) who are capable of successfully completing a course of training leading to the required NEC (Naval Enlisted Code - specialty rating). Their utilization implies a loss to the Navy equivalent to the value of their current services (estimated as BG) together with a gain equivalent to their use as specialists (estimated as BN). Drawing from the AN pool is preferred, under most circumstances, to drawing from the AG pool, since the latter implies the need of "A" school whereas the former does not.

The AS pool is a conceptually limitless pool of people in the Navy with the same characteristics as members of the AG pool, except they are not aboard ship. Drawing from AS, therefore, implies expanding the ship's company in addition to training through "A" and "C"

schools. This pool is the most costly to draw from.

As with other elements of cost, the detail in manpower cost equations is reduced from Level III to II to I. The reader will notice that some of the simplifications are aimed at the complexity of a particular equation where others primarily influence the size and detail required of the input data set.

## 3.1 Manpower Costs in the Level III Model

3.1.1 Demand Relations. Manpower demand is estimated for four personnel groups: officers, system operators, shipboard maintenance technicians and MOD technicians. Two demand estimates, predicted on average and peak demand, are used for shipboard personnel. Average demand is used to compute compensation costs, while peak demand is used to determine the number of people who must be trained to handle the manpower burden under the most stressed conditions. Thus, a total of six demand estimates are computed. The officer requirement, OF, is a direct input, while the other five relations are computed.

The average operator manpower requirement, $M_o$, is a simple function of the average weekly operator man-hour requirement and the available weekly work hour yield for an operator under average steaming conditions:

$$3.1)\quad M_o = Q \cdot AHR \cdot \theta \cdot 7/(D \cdot h \cdot WH_o)$$

where Q is the number of systems per ship, AHR is average operating hours per year, $\theta$ is the number of operators per system, D the number of days in a deployment period, h the number of deployments per year and $WH_o$ the number of available weekly work hours for a watch stander under normal operating conditions. The value of $M_o$ is, therefore, a real number expressing the number of operators required to man the equipment for a single ship. The peak operator manpower requirement

is computed similarly:

$$3.2)\quad M_o' = Q \cdot PK \cdot \theta / WH_o'$$

where PK is the peak system operating hours during a peak operating period and $WH_o'$ is the available work hour yield from an operator under those conditions.*

Maintenance manpower demand includes both the time spent actually removing, replacing and repairing failures and an additional term, OHT, which accounts for the hours necessary (per week) to keep the maintenance technician's skills sharp. While we know that the size of OHT should vary inversely with time spent repairing (the more of the latter, the less study time necessary) we have, as yet, acquired no useful information as to how this relationship might be modelled. The variable OHT is, therefore, an input value. The equation for average demand is:

$$3.3)\quad M_m = \left[ Q \cdot SM + \sum_{i=1}^{n} (\lambda_i \cdot 7/D \cdot h) \cdot (MTRR_i + r_{1,i} MTTR_{i,s}) \right] / (U_m \cdot WH_m - r_{1,i} OHT)$$

The mean time to repair the *system*, $MTRR_i$, by fault isolation, removal and replacement of the $i^{th}$ LRA is distinguished from what is expected to be the longer time requirement to repair an LRA on board

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* For a complete discussion of the variable WH, see ibid. pg 3-31 ff.

ship, $MTTR_{i,s}$. The value $\lambda_i$ gives the number of failures in the $i^{th}$ LRA type per ship per year.* The total number of LRA types is n, and SM is the system weekly scheduled maintenance requirement. Available work hours for repair technicians ($WH_m$) are different than for operators (i.e., watch standers) and in addition they have been reduced by OHT in the event of a local repair policy. Finally, a utilization rate, $U_m$, is applied to hours delivered to the work site, incorporating a ratio of time required for administrative duties arising from repair actions.

Peak maintenance demand is essentially the same thing, except the number of hours of operation (and hence failures) is increased to the peak:

$$3.4)\quad M'_m = M_m \; PHR \cdot h/AHR,$$

where PHR is the peak system operating hours during a deployment period. Since peak hours are measured over a single deployment, they are multiplied by the number of deployments in a year, h, in order to equalize the dimensions of PHR and AHR.

Demand for labor at the depot is essentially the same as on the ship, except that only those items coded depot repair are counted. Furthermore, this is only done for a military depot (i.e., $r_{3,i} = 1$), otherwise the value of the equation for that LRA is zero:

$$3.5)\quad M_d = \sum_{i=1}^{n} r_{3,i} \cdot N \cdot \lambda_i \cdot MTTR_{i,d} \cdot h/(d_r \cdot 52 \cdot U_d \cdot WH_d)$$

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* See Chapter 4 for the computation of $\lambda_i$.

where N is the number of ships on which the system is deployed, $d_r$ the number of MOD's among which the total demand is split, $MTTR_{i,d}$ the mean time to repair the $i^{th}$ LRA at the MOD, $U_d$ the MOD labor utilization rate, and $WH_d$ the weekly depot technician work hour yield. The value 52 convert annual to weekly demand.

Equations 3.1 to 3.5 have given the real number value of men required per ship in the case of operators and maintainers and per depot in the case of depot technicians. As noted above, by substituting a civil service billet cost for the military billet cost in BD, the further option of a civilian operated Navy depot can be estimated.

3.1.2 Allocations from Skill Pools. For shipboard manning requirements, compensation, training and other elements of personnel cost depend, not just on the amount of labor required, but also on the skill level and current cost of the people who will fill the billets. The sizes of underutilized labor pools AN and AG will also condition the actual number of billets necessary.

To find the optimal draw from each pool to fulfull a requirement, M, we draw from the three pools in order of preference, starting with the least expensive, the AN pool. The size of the draw, $A_n(M)$, can take on three values depending on the relative size of M and AN:

$$3.6)\quad A_n(M) = \begin{cases} M & M \le AN \\ AN & M > AN \text{ and } 0 < fp(M) < fp(AN) \\ ip(AN) & \text{Otherwise} \end{cases}$$

These rather odd looking rules can be best explained by an example.* Let AN = 2.5. In the first case, set M = 1.3. Since the demand can be met entirely from the AN pool, we have $A_n(M) = M$. Now suppose M = 3.7. Since M is larger than AN, personnel must be drawn from both the AN and the AG pool. An obvious way to meet the manpower demand would seem to be to use up the AN pool ($A_n(M) = AN = 2.5$) and meet the remaining demand from the AG pool ($A_g(M) = 1.2$). Yet the rules require that only the integer part of the AN pool be utilized ($A_n(M) = ip(AN) = 2$) with the remaining demand met from the AG pool ($A_g(M) = 1.7$), which we assume here to be large enough to meet the requirement. When one examines the manpower cost equations, it will be clear that compensation costs will be the same whether 2 or 2.5 men are drawn from the AN pool. However, when $A_n(M) = 2.5$ a total of five people per ship must receive "C" school training (3 from the AN pool and 2 from the AG pool), whereas when $A_n(M) = 2$ only four need receive "C" training (two from each pool). Thus the second manpower allocation is preferred.

By trying other examples, the reader can check that the rul s for drawing from the AN pool assure that the number of personnel per ship who must receive "C" school training is always the smallest possible.

Having drawn $A_n(M)$ from the AN pool, the number drawn from the AG pool is the residual requirement. If, however, this requirement exceeds the total size of AG, then this pool us used up and the balance drawn from the AS pool:

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* For a complete discussion of the manpower pools, see ibid. Section 3.1.

$$A_g(M) = \begin{cases} M - A_n(M), & M - A_n(M) < AG \\ AG, & \text{otherwise} \end{cases}$$

3.7)

$$A_s(M) = M - A_n(M) - A_g(M)$$

Having determined the real numbers of men drawn from each pool, we now define two collective terms for rounded-up sums:

3.8) $A(M) = \lceil A_g(M) \rceil + \lceil A_s(M) \rceil$

3.9) $C(M) = A(M) + \lceil A_n(M) \rceil = \lceil M \rceil$

where $\lceil x \rceil$ is an operator which rounds x to the next higher integer. The value, A(M), tells how many men must receive "A" school training and the value C(M) tells how many require "C" school.* A(M) also plays an important role in accounting for part of the compensation costs of manpower.

3.1.3 Training Course Costs. We have now derived equations by which the numbers of men required can be estimated. These must be combined with prices to provide manpower costs. Three sets of prices are used: billet costs, training course costs and other costs. Billet costs were discussed at the beginning of the chapter. Other costs, including a variety of personnel costs such as administration, house-

* It is not obvious that $C(M) \equiv \lceil M \rceil$. This will be demonstrated in Section 3.3, below.

keeping and medical personnel costs, security clearance costs, and in some cases, ship alteration costs, are divided into two sets called Z and $Z_s$. The first include all costs associated with a member of a ship's company, working with the equipment being designed. The second include extra costs generated by the augmentation of the ship's company implied by drawing from the AS pool. In this section we discuss the computation of training course costs.

The first training course costs are for operators. There are no support policy implications, nor do individual LRA's play a role, but a distinction is made between men drawn from the AN pool and those from AG or AS pools.

$$3.10 \quad TN_o = TO \cdot \left[ BN_o/WD + \beta + TAD \right] + TX$$

$$3.11 \quad TG_o = TO \cdot \left[ BG/WD + \beta + TAD \right] + TX + TA_o$$

Both equations are the same, except that $TN_o$ uses $BN_o$ to compute the daily course cost while $TG_o$ uses BG. In the equations, the number of school days required to accomplish training, TO, is multiplied by a daily course cost consisting of TAD (temporary assign duty) per diem costs, billet compensation costs and daily school overhead costs, $\beta$. The variable, WD, is the annual number of available work days for enlisted personnel. The next element, TX, is the travel expense incurred going to and from school. Finally, all general

personnel must first attend the appropriate "A" school for the NEC required for system operators. The cost of this course is $TA_o$.

Similar equations may be written for maintenance training. The course length, TO, is replaced by a computation dependent on the support policy for each LRA and $TA_o$ is replaced by the maintenance "A" school course cost, $TA_m$:

$$3.12)\quad TN_m = \left[TS + TFI + \sum^{n} r_{1,i}TR_i\right] \cdot \left[BN_m/WD + \beta + TAD\right] + TX$$

$$3.13)\quad TG_m = \left[TS + TFI + \sum^{n} r_{1,i}TR_i\right] \cdot \left[BG/WD + \beta + TAD\right] + TX + TA_m$$

Again, both equations are similar, except that $TN_m$ uses $BN_m$, while $TG_m$ uses BG. The computation of the number of course hours required to cover the material includes TS hours for system orientation, TFI hours for fault isolation and system repair, and $TR_i$ hours of repair training for the $i^{th}$ LRA type (if it is coded local repair).

The training course cost for depot technicians excludes fault isolation to the LRA and allows the distinction between military and civilian personnel:

$$3.14)\quad TC_d = \left[TS + \sum^{n} r_{3,i}TR_i\right] \cdot \left[BD/WD_d + \beta + TAD_d\right] + TX,$$

where $WD_d$ is annual work days for civilian technicians and $TAD_d = 0$ in the case of a Navy operated civilian depot, otherwise they are set equal to WD and TAD, above. Note that $r_{3,i}$, the switch

for a MOD, is used instead of $r_{1,i}$, which signals local repair.

3.1.4 Manpower Cost Elements. We have now assembled all the ingredients necessary to write out the actual cost equations for each of the three elements of manpowe. For the first, compensation cost, there are two elements applicable to shipboard personnel and another for depot personnel. For shipboard personnel, we rely on the economic notion of opportunity cost: that the cost of using a resource in a particular way is the foregone benefit of its best alternative use. Thus, part of the compensation equation takes the form M·BG, or the product of the number of man years of labor actually used and the *value* of the work previously performed by those now serving the new equipment. The remainder of the equation uses an expression of the form A(M)·(BN - BG) which accounts for the increase in billet cost for those moved from the AS and AG pools into the AN pool or used on the system.* The equation is:

$$3.15)\quad C_1 = N\cdot L\cdot\Big[(M_o + M_m)\cdot BG + A(M_o)\cdot(BN_o - BG) + A(M_m)\cdot(BN_m - BG) + BO\cdot OF\Big] + d_r\cdot L\cdot M_d\cdot BD$$

Notice that the expression in brackets makes the implicit assumption that the value of duty other than that for which he is trained is worth BG for either maintenance or operator personnel. Costs per

* For a detailed discussion of compensation costs, see ibid. pg. 3-15 ff.

ship are multiplied by N and costs per depot are multiplied by the number of repair depots in the military system. Both are raised by the discount factor, L. Note that $M_d$ will be equal to zero if none of the LRA's is coded military depot repair.

In Section 3.1.1 we developed peak and average manpower demand estimates for operators and for maintenance personnel on-board ship at the MOD. In Section 3.1.2 we showed how this demand would be met by drawing from the AN, AG and AS manpower pools. At the same time we determined how many men from each pool would have to receive training: $\lceil A_n(\cdot)\rceil$ , $\lceil A_g(\cdot)\rceil$ and $\lceil A_s(\cdot)\rceil$ , respectively. In Section 3.1.3 we calculated the training course cost associated with each pool: TN for the AN pool and TG for the AG and AS pools. All that remains to do, therefore, is to put the number of personnel and the training course cost together to yield total training costs:

$$3.16)\quad C_2 = \left(\lceil A_n(M_o')\rceil \cdot TN_o + A(M_o')\, TG_o\right)\cdot N\cdot(1 + TOR_o\cdot L) +$$

$$\left(\lceil A_n(M_m')\rceil \cdot TN_m + A(M_m')\cdot TG_m\right)\cdot N\cdot(1 + TOR_m\cdot L) +$$

$$\lceil M_d\rceil \cdot TC_d\cdot d_r\cdot(1 + TOR_d\cdot L)$$

where $A(\cdot)$ is as given in equation 3.8, and $TOR_o$, $TOR_m$ and $TOR_d$ are respectively annual turnover rates for operators, shipboard maintenance technicians and depot technicians at a MOD.* An important point to note is that operator and shipboard personnel requirements

* Note that these are <u>billet</u> turnover rates measuring the proportion of people who either leave the service or go elsewhere in the service.

are based on peak demand and multiplied by N ships, while MOD requirements are based on average demand and multiplied by $d_r$ repair depots. The expression $X(1 + TOR \cdot L)$ indicates that X personnel must receive training initially; $TOR \cdot X$ personnel must be replaced each year of the system life cycle, the cost of which is discounted to present value.

The third element of manpower cost is a collection of various things called other costs. As mentioned earlier, these are collected into two prices or cost factors, Z and $Z_s$. The first applied to all personnel, while $Z_s$ applies only to additions to the ship's company, $\lceil A_s(M) \rceil$. Altogether, there are five different Z values which are exogenous to the model. Their definitions are given as:

$Z_m$: Indirect costs functionally related to the skill group rather than allocated by work shares. If security clearances were required for maintenance men, this would be an appropriate cost. Usually this will be small because clearances are not necessary.

$Z_o$: The same as $Z_m$ except applicable to operators. Security clearances are more likely to be germane here, except in the case of crypto-equipment, where maintenance men must also be cleared.

$Z'$: Indirect costs functionally related to the amount of work done, rather than the number of people. Applicable to both maintenance and operator personnel, it includes most "overhead" types of cost like management, administration, support personnel and so on.

$Z_s$: Ship alteration and administrative costs, as well as travel and permanent change of station costs, associated with an increase in ship's company, applicable only to $A_s$, but for either maintenance or operator personnel.

$Z_d$: All direct costs applicable to the amount of work done, analogous to $Z'$, but appropriate for depot personnel.

The sum of other manpower costs is written out as:

$$3.17)\quad C_3 = \left(C(M_o')\cdot Z_o + \left\lceil A_s(M_o')\right\rceil \cdot Z_s\right)\cdot N\cdot(1 + TOR_o\cdot L) +$$

$$\left(C(M_m')Z_m + \left\lceil A_s(M_m')\right\rceil \cdot Z_s\right)\cdot N\cdot(1 + TOR_m\cdot L) +$$

$$d_r\cdot L\cdot M_d\cdot Z_d + N\cdot L\cdot(M_o + M_m)\cdot Z'$$

The first term in the first line is the "other" costs for all system operators; the second term the cost associated with adding new operator personnel to the ship's company. The second line is the same, except that it refers to maintenance personnel. The last line gives the costs associated with actual manpower equivalents of work done - similar to normal practice in the application of overhead rates to direct labor hours.

## 3.2 Manpower Costs in the Level II Model

The manpower cost elements of the Level II Model, while predicated on the same basic relationships, are considerably simplified. The simplification is accomplished in several ways.

- Cost elements accounting for relatively small additions or subtractions have been dropped.
- Some variables, different for each LRA in the Level III Model, have been changed to system constants.
- Some distinctions between classes of labor have been dropped.

Three objectives are satisfied by these changes. First, the input data set is reduced considerably, making it easier for the user to begin employing the model at a relatively early stage of design development. Second, the computations themselves are less complex, leading to faster machine runs, and making it possible to use the program for extremely large numbers of design alternatives, sensitivity analysis and rapid trade-off analysis. Third, the relatively compact nature of data set and program in concert with fast running properties, makes the Level II Model adaptable to inexpensive computational hardware. The Level II Model, for example, has been programmed in BASIC on a machine whose acquisition cost is under $4,000, including peripheral equipment. The importance of this statistic is that it makes widespread use of the model quite feasible for even the smallest design shops.

The following discussion is organized in the same fashion as section 3.1, with sections covering demand, allocations from skill pools, course cost, and cost element equations. The notation is the

same, except where the model has been changed. This constitutes the focus of the discussion.

3.2.1 Demand Relations. The equations used to estimate manpower demand in the Level II Model are very similar in form to those in Level III. They are:

3.18) $M_o = Q \cdot AHR \cdot \theta \cdot 7/(D \cdot h \cdot WH_o)$

3.19) $M_o' = M_o \cdot PHR \cdot h/AHR$

3.20) $M_{m,i} = \left[Q \cdot SM/n + \lambda_i \cdot 7/D \cdot h(MTRR_i + r_{1,i} MTTR_i)\right]/(U \cdot WH_m)$

$$M_m = \sum_{i=1}^{n} M_{m,i}$$

3.21) $M_{m,i}' = M_{m,i} PHR \cdot h/AHR$

$$M_m' = M_m PHR \cdot h/AHR$$

3.22) $M_{d,i} = r_{3,i} N \cdot \lambda_i \; MTTR_i \cdot h/(d_r \cdot 52 \cdot U \cdot WH_d)$

$$M_d = \sum_{i=1}^{n} M_{d,i}$$

The differences between the Level II and Level III demand equations are: peak operator demand is based on peak operating hours during a deployment period; the concept of OHT is eliminated; and the distinction between $U_m$ and $U_d$ and between $MTTR_{i,s}$ and $MTTR_{i,d}$ has been deleted.

3.2.2 Allocations From Skill Pools. The equations describing allocations from skill pools are identical in the Level II and Level III Models (i.e., equations 3.6 through 3.9 are incorporated in the Level II Model).

Maintenance manpower costs are computed at two aggregation levels in the Level II Model. Manpower demands for individual LRA's are used to compute a manpower cost for that LRA. This cost is used solely as a part of the routine which assigns the level of repair (LOR) posture to the LRA. The posture chosen is the one having the smallest life cycle cost for the LRA among the four LOR options. Once the LOR postures have been chosen, the maintenance manpower requirements for individual LRA's are aggregated into a system level requirement, which is then substituted into the manpower allocation and cost equations.

In computing maintenance manpower costs for individual LRA's, it is necessary to allocate portions of the available skill pools to each LRA. The simplest way, of course, would be to set $AN_{m,i} = AN_m/n$ and use this value in the manpower allocation equations. However, this approach is too rigid. LRA's with higher

requirements (i.e. higher failure rates and/or MTTR's) would be more likely to overrun their alloted pool size, unnecessarily biasing the choice of LOR posture against local repair, which has the highest manpower requirement. Similarly, LRA's with lower manpower requirements would be alloted more manpower than they need. A better approach is to allocate portions of the skill pool to an LRA in rough proportion to its estimated manpower requirement (i.e., LRA's with higher manpower requirements would be allocated a higher percentage of the total available pool), and to keep running track of how much of the pool is actually utilized, reallocating the remaining pool among the remaining LRA's. The size of the AG and AN pools allocated to the $i^{th}$ LRA are given in the following equations:

$$3.23) \qquad AN_{m,i} = \frac{\lambda_i MTRR_i}{\sum_{x=i}^{n} \lambda_x MTRR_x} \left[ AN_m - \sum_{x=1}^{i-1} A_n(M'_{m,x}) \right]$$

$$3.24) \qquad AG_{m,i} = AG_m - \sum_{x=1}^{i-1} A_g(M'_{m,x})$$

where we are using the usual convention for sums that $\sum_{x=1}^{0} \triangleq 0$. Note that we simply allocate all of the AG pool to the first LRA, (possibly) somewhat less to the second, and so on. Since a slightly different AN and AG pool would be assigned to the same LRA if it were coded, say, number three or number ten in the system, it is

conceivable -- but not likely -- that it would be assigned a different LOR posture in the two cases.

The model allows the maintenance manpower demand full access to the general labor pool, AG. Once that demand is met, the operator demand can draw from that pool. That is, the size of the AG pool for operators is the full AG pool less whatever has been used up (if anything) by maintenance demand:

$$3.25)\quad AG_o = AG - A_g(M_m')$$

3.2.3 Training Course Costs. The training course cost calculations in the Level II Models are considerably simplified. The changes involve the use of an exogenous value for the per-day training course cost of attendance instead of the calculations, dependent on BN and BG, made in equations 3.10 to 3.14. As a consequence, the distinctions between TN and TG disappear. The Level II "C" school training cost equations are:

$$3.26)\quad TC_{m,i} = DC\cdot(TS/n + r_{1,i}TR)$$

$$TC_m = \sum_{i=1}^{n} TC_{m,i}$$

$$3.27)\quad TC_{d,i} = r_{3,i}DC\cdot TR$$

$$TC_d = \sum_{i=1}^{n} TC_{d,i}$$

3.28) $TC_o = DC \cdot OTC$

where DC is the daily training course cost, including travel, per student; TS is the system repair and fault isolation training course length (compare to TS + TFI in Level III); TR is the average training days to repair an LRA (compare to $TR_i$ for each LRA in Level III); and OTC is the required days of operator training. The "A" school costs, $TA_o$ and $TA_m$, are still exogenous and are incorporated into the manpower cost equations.

3.2.4 Manpower Cost Equations. The manpower cost equations in the Level II Model are similar in form to the Level III, the main simplification occurring in the equation for "other" costs:

$$3.29)\quad C_1 = N \cdot L \cdot \Big[(MO + MM) \cdot BG + A(MO') \cdot (BN_o - BG) + A(MM')(BN_m - BG) + BO \cdot F\Big] + d_r \cdot L \cdot MD \cdot BD$$

$$3.30)\quad C_2 = N \cdot (1 + TOR_s \cdot L) \cdot \Big[C(MM') \cdot TCM + C(MO')TCO + A(MM') \cdot TA_m + A(MO')TA_o\Big] + d_r \cdot (1 + TOR_d \cdot L) \cdot \lceil MD \rceil \cdot TCD$$

$$3.31)\quad C_3 = N \cdot (1 + TOR_s \cdot L) \cdot \Big[\big[C(MM') + C(MO')\big] Z + \big[\lceil A_s(MM') \rceil + \lceil A_s(MO') \rceil\big] Z_s\Big]$$

where Z is a single value replacing $Z_o$ and $Z_m$ in the Level III Model ($Z'$ and $Z_d$ are eliminated in the Level II Model).

Demand and training course costs are substituted into 3.29 - 3.31 at the LRA and system levels (i.e. $MM = M_{m,i}$, $TCM = TC_{m,i}$ for the $i^{th}$ LRA). At the LRA level $F = MD = MD' = 0$, that is, there are no operator or offices costs.

## 3.3 Manpower Costs In The Level I Models

The Level I Models are generically divided into two model systems: the Top Down Model (TDM), which estimates system costs by assuming uniform characteristics for system subelements, and the Aggregation Model System (AMS), consisting of two models:

1) The Lowest Removable Assembly Model (LRAM) computes costs for individual LRA's;

2) The System Aggregation Model (SAM) configures a system by aggregating LRA's and adding system level costs.

The output of the LRAM, stored on magnetic cards, is used as input to the SAM. The SAM program is written so that SAM output can be used as input to itself. This allows the user the option of breaking down the system into any number of subsystems, and then configuring the subsystems into the system using the SAM. In fact, any number of aggregation levels is possible. This flexibility is accomplished by defining an aggregation factor as follows:

$$3.32)\quad R_{i,j} = \frac{q_j QIPA_{i,j}}{q_i},$$

where $q_j$ is the number of elements of type j in the system, $QIPA_{i,j}$ is the number of elements of type i, the next lower aggregation level in the $j^{th}$ element, and $q_i$ the number of $i^{th}$ elements in the entire system. When the SAM is used at the system level, $q_j = 1$ and $QIPA_{i,j} \equiv q_i$, so

that R = 1. The factor, R turns a liability of slide-rule models - the necessity of aggregating by hand-loading the outputs from the LRAM into the SAM - into an asset: enhanced flexibility of use concomitant to a multi-aggregation level option.

Space and input data restrictions necessitated considerable simplification in the manpower cost formulations used in the Level I Models. However, the basic approach to manpower cost (aggregation of raw manpower requirements, drawing from manpower pools, and relating training costs to equipment designs) is the same as in Levels II and III.

Manpower costs are estimated for maintenance personnel and operators only; officer costs are not computed. In addition, cost calculations are restricted to compensation and training; "other" costs are not estimated. Thus manpower estimates will tend to be lower in the Level I Models than in Levels II and III. For simplicity's sake, the only depot repair option available in the Level I Models is a contractor operated depot. Therefore, manpower costs for depot technicians are not computed but rather are assumed to be included in the COD, the average cost of a repair at a contractor operated depot. The value COD is discussed in detail in Section 5.3.

### 3.3.1 Demand Relations And Training Costs In The Top-Down Model.*

The average maintenance manpower requirement in the Top-Down Model is given by:

$$3.33)\quad M_m = \left[\lambda(MTRR + r_1 MTTR) + Q\cdot SM\right] / (U\cdot WH_m)$$

* It turned out to be most convenient to define demand rates in the

where $\lambda$ is the weekly number of system failures, MTRR and MTTR the mean time to repair the system and an average LRA, $r_1$ the fraction of LRA types in the system coded local repair, SM the weekly scheduled maintenance requirement, U the utilization rate and $WH_m$ the available weekly work hours for maintenance personnel. MTTR and MTRR are defined as the time from failure to restoration of operation. Put away, data recording, and overhead time (OHT) are accounted for by adjusting the value of U. The peak requirement is given by:

$$3.34)\quad M_m' = s \cdot M_m$$

where s is the ratio of peak to averaging operating hours. The operator manpower requirement equation is essentially the same as in the Level II and III Models:

$$3.35)\quad M_o = Q \cdot \theta \cdot AHR \cdot / WH_o$$

$$3.36)\quad M_o' = s \cdot M_o$$

"C" school training costs for maintenance personnel are given by:

$$3.37)\quad TC_m = TS_m + n \cdot r_1 TR,$$

---

* (cont'd) Level I Models on a per week basis. Thus, while variables in the following Sections will have the same meaning as in Sections 3.1 and 3.2, the units will generally be different. Thus $\lambda$ and AHR are given in units of failures and operating hours per week, as opposed to per year in the Level III and Level II Models. This annoying inconsistancy was dictated by space and programming considerations in the Level I computer model.

where $TS_m$ is the system level training course cost, including system orientation and fault isolation and system repair, TR the additional course cost for every LRA being coded local repair, and n the number of LRA types in the system. $TS_m$ and TR are determined exogenously by estimating the required course length and multiplying it by an average daily training course cost factor, which would include amortized values of TX, TAD, and $\beta$, defined in equation 3.10.

The operator "C" school training costs, $TC_o$, and an average "A" school training cost for operators and maintainers, TA, are determined exogenously to the model.

<u>3.3.2 Demand Relations And Training Costs In The Aggregation Model System</u>. The demand and training course equations in the LRAM are given in equations 3.38 - 3.40.

$$3.38)\quad M_{m,i} = \lambda_i(MTRR_i + r_{1,i}MTTR_i)/(U \cdot WH_m)$$

$$3.39)\quad M'_{m,i} = sM_{m,i}$$

$$3.40)\quad TC_{m,i} = TFI_i + r_{1,i}TR_i$$

where the input variables are as defined in Section 3.1.3. Note that there are no operator costs at the LRA level, nor any scheduled maintenance requirement.

The values of M′, M and TC are substituted into the manpower cost equations discussed in the next section. The value of $AN_{m,i}$

used in the equations is assigned to individual LRA's so that

$$\sum_{i=1}^{n} AN_{m,i} = AN_m .$$

The manpower costs computed in the LRAM are used for trade-off analyses at the LRA level. They are *not* passed to the SAM, which instead builds up a new manpower training requirement by aggregating the manpower demand and training course requirements for each LRA appearing in the system. The equations are:

$$3.41)\quad M_{m,j} = Q \cdot SM/U \cdot WH_m + \sum_{i=1}^{n} R_{i,j} M_{m,i} , \quad M'_m = sM_m$$

$$3.42)\quad M_o = Q \cdot \theta \cdot AHR/WH_o , \quad M'_o = sM_o$$

$$3.43)\quad TC_m = TS_m + \sum_{i=1}^{n} R_{i,j} TC_{m,i}$$

$$3.44)\quad TC_o = TS_o + \sum_{i=1}^{n} R_{i,j} TC_{o,i}$$

Recall that if the SAM is used at the system level, then we can drop the j subscript and $R_i \equiv 1$. If LRA's are being aggregated, then $TC_{o,i}$ is set equal to zero.

<u>3.3.3 Manpower Cost Equations In The Level I Models</u>. Once the manpower demand and training course costs have been determined, they are substituted into the manpower cost equations. The formulation of the equations is the same in all three Level I Models.

The number of personnel who must receive "A" school training is given by:

$$3.45)\quad A(M,AN) = \begin{cases} 0 & M \le AN \\ \lceil M - AN \rceil & M > AN \end{cases}$$

where M is the raw manpower requirement and AN the size of the available manpower pool. Note that equation 3.45 can also be expressed as $A = \lceil M - \min(M,AN) \rceil$. The number of men who must receive "C" school training is given by:

$$3.46)\quad C(M) = A(M,AN) + \lceil A_n(M) \rceil,$$

where $A_n(M)$ is given in equation 3.6. However, if we examine the three possible relationships between AN and M in equation 3.6 we note:

Case 1) $M \leq AN$: $A_n(M) = M$, $A(M) = 0$

$$C(M) = 0 + \lceil M \rceil$$
$$= \lceil M \rceil$$

Case 2) $M > AN$ and $0 < fp(M) < fp(AN)$: $A_n(M) = AN$, $A(M) = \lceil M-AN \rceil$

$$C(M) = \lceil M-AN \rceil + \lceil AN \rceil$$
$$= ip(M) - ip(AN) + ip(AN) + 1$$
$$= \lceil M \rceil$$

Case 3) $M > AN$ and $fp(AN) \leq fp(M)$: $A_n(M) = ip(AN)$, $A(M) = \lceil M-AN \rceil$

$$C(M) = \lceil M-AN \rceil + ip(AN)$$
$$= ip(M) - ip(AN) + 1 + ip(AN)$$
$$= \begin{cases} \lceil M \rceil & \text{if } fp(M) \neq 0 \\ \lceil M \rceil + 1 & \text{if } fp(M) = 0 \end{cases}$$

Thus $C(M) \equiv \lceil M \rceil$ except in the extremely unlikely instance that

$M > AN$ and $fp(M) = fp(AN) = 0$.* Therefore, in the Level I Models we simply set:

$$3.47)\quad C(M) = \lceil M \rceil.$$

We define the following two manpower cost functions:

$$3.48)\quad C_1(M,AN,BN) = \lceil M \cdot BG + A(M',AN)(BN - BG) \rceil \cdot L \cdot N$$

$$3.49)\quad C_2(M,AN,TC,TA) = \lceil C(M')TC + A(M',AN)TA \rceil (1 + TOR \cdot L) \cdot N,$$

where $A(M,AN)$ is given in equation 3.45. Then manpower costs in the Level I Models are given by:

$$3.50)\quad C_1 = C_1(M_m, AN_m, BN_m) + C_1(M_o, AN_o, BN_o)$$

$$3.51)\quad C_2 = C_2(M_m, AN_m, TC_m, TA_m) + C_2(M_o, AN_o, TC_o, TA_o).$$

In the LRAM we set $M_o = 0$, and all variables are subscripted $(m,i)$. Note that this manpower formulation requires explicit consideration of the AN pools, but not the AG pool.

---

* Note that if $A(M)$ is defined equal to $\lceil M - \min\{M, A_n(M)\} \rceil$ the difficulty in Case 3 is eliminated and $C(M) = \lceil M \rceil$ in all cases. However, this definition of $A(M)$ is the same as $A(M)$ for the Level III Models, given in equation 3.8. By going through a process exactly like the above for $A_g(M)$ and $A_s(M)$ (equation 3.7) one can easily prove the assertion in equation 3.9 that $C(M) \equiv \lceil M \rceil$.

## 4.0 SPARES AND PRODUCTION COSTS

Spare parts can account for as little as five percent or as much as fifty percent of operating and support cost in a military electronics system. Production costs can run over the same range with respect to total life cycle cost. As a consequence, both are significant elements of cost. Production costs can be estimated in a variety of ways, most of which are more complex than necessary for our purposes. Our central concern in estimating production costs is to properly account for learning curve effects when making the level of repair decisions and to properly portray the intertemporal cost tradeoff between production and ownership costs.

While the cost of spare parts for a military system is significant, its significance is frequently outdistanced by the care analysts take in its estimation for the simple reason that spares estimation incorporates more interesting mathematics than other cost elements. Such interest derives from the tractability of a probabilistic approach to the central driving event in spares computations: the interval between random failures of a system. While we might prefer that the analyst's energies be more appropriately distributed among different cost elements, we are also bound to structure our computations as accurately as possible. For this reason, the reader will find the next sections of this paper more difficult than others.

The logical sequence of the computations detailed below proceeds from estimation of the demand for spare parts through the final estimation of production and spares costs. The intermediate elements are as follows:

- Spares and other elements of life cycle cost are computed for the contribution of a single LRA to total system cost. This is done for each of three fundamental support policies or postures: local repair, depot repair and discard at failure.* The least cost policy is chosen and becomes part of the data set describing the individual LRA.

- Now we start the process over again, considering the system as a whole rather than as individual LRA's. This is done by first assuming no stock in the system and checking to see if the system confidence level criterion has been reached anyway. [This is theoretically possible if failure rates are low enough.]

- The marginal return on investment in a single spare of a given LRA is calculated through one of three formulations. They pertain to local repair, shipboard stock; depot repair, shipboard stock and; depot repair, depot stock. The latter two are both computed in the case of a depot repair item. This is done for every LRA.

- The marginal returns are next compared between all LRA's and the highest return item is "purchased" - i.e. added to the spares bill. One additional marginal return computation is then made, for the second spare of the item chosen. Finally, a new system confidence level is computed and checked against the criterion level. This process is repeated until enough stock has been purchased and properly located to satisfy the system confidence criterion.

- The last step is that the system "buys" all the spares indicated by the procedure just described and adds the bill to production costs.

The reader should note that as the number of spares increases, the unit costs of both the LRA's in question and the system as a whole are affected. This fact is taken into account in the computation of marginal return to further investment.

Sections 4.2 and 4.3, respectively, will discuss the spares formulations in the Level II and I Models. It will become apparent that while considerable similifications are made, the basic approach to sparing remain constant throughout all three model levels.

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* The option of military versus contractor operated depot, while important for other cost elements, has no impact on spares computations.

## 4.1 Spares In The Level III Model

Computation of spare stockage for a system is predicated on three elements: the level of repair, achievement of a system criterion level of confidence against stockout and consideration of the unit cost of an LRA as part of the sparing algorithm. Each of these subjects is discussed below.

*Level of Repair*. There are three basic options in level of repair for any LRA. These are discard, local repair and depot repair. A decision must be made to support each item through one of these policies. In general, the choice rests upon a tradeoff between the amount of stockage (and spares costs) on one side and the costs of training, technical data and support and test equipment on the other. The support environment is capable of returning a failed item to the ship, repaired, in a certain period of time. The ship is capable of repairing the same item far more quickly - if it has invested in the training, data and equipment necessary to do so. The importance of the waiting period (called lead time) derives from the fact that spares must be stockpiled to a level sufficient to cover any demands anticipated during the lead time period. This number, called demands during lead time or, more simply, demand lead time, is the mean of a Poisson arrival distribution for failures generated by an exponential interarrival distribution. A number of spares equal to the mean of the Poisson is called the pipeline. But this amount is only sufficient to provide a confidence of 50% against a stockout: that is, half the time a failure can be expected

to produce an unsatisfied demand for a replacement part on board the ship. To achieve a higher level of confidence that no stockout will occur, an additional quantity of stock called the buffer is required. The number of units needed to achieve any given level of confidence is a function of the standard deviation of the arrival distribution which, in the case of a Poisson, is equal to the square root of the mean.

The effect of reducing the lead time by switching from depot to local repair is, therefore, to reduce the number of demands anticipated during lead time, given that failures are generated at a constant rate over time. The tradeoff consists of comparing the reduction in spares costs to increases in test equipment, training and data costs. Both repair options must also be compared to the discard option. If the policy is to discard at failure, then training, test equipment and data costs are reduced on board the ship and eliminated at the depot level. Stockage must be increased, however, to cover every anticipated failure in the system's operational life.

*System Confidence*. Determining the buffer stock requirement for a particular LRA can either be done independently or by taking the entire system into consideration. The former is most frequently done in life cycle cost models because it is computationally simple. Unfortunately, independent estimates for buffer stock usually result in significant underestimates of stockage requirements. The alternative is to set the stockage level for each item at a level consistent with a system goal. To illustrate, imagine that $\mu$ is the demand lead time

for each of n LRA's comprising a system. If the confidence objective for the system is 95%, many cost models would use the following formulation for pipeline and buffer stock:

$$S = \lceil \mu + 1.645\sqrt{\mu} \rceil$$

The value of 1.645 is the number of standard deviations above the mean of a normal distribution required to cover 95% of the distribution's mass. The brackets indicate that the value found should be rounded up to the next highest integer.

If S were just enough for a 95% confidence level, the probability of a stockout, considering the system as a whole, would be $1-.95^n$. For large values of n, the probability can be very high. For $\mu=10$, the probability of a stockout is about 40% and for $\mu=100$ it is 99.4%. To reach a system confidence level of K*, each of n identical components would have to be stocked to a confidence level of $K^{*1/n}$. For n=10, this requires 2.58 standard deviations and for n=100, it requires 3.28.

The illustration just given provides a method for determining stockage in simple models when simultaneous consideration of all components is not possible. It relies on the assumption that all components are identical. Various modifications can also be made, introducing differences in failure rates that follow certain functional forms. For example, the equation of a straight line can be used, with positive intercept and negative slope, to generate a sequence of failure rates that add up to the system total. Such formulations represent a compromise between computational simplicity and realism under conditions of limited information.

The important point to remember from this discussion is that the appropriate domain over which confidence must be obtained is *system* stockouts: not component stockouts. To do so requires that the sparing of each component be influenced by the system as a whole. When this is computationally difficult to do correctly, a reasonable fall-back position is to estimate component requirements to a confidence level set with the system in mind: $K^{*1/n}$. In the following algorithm we will use both approaches. The independent approach is used to make rough estimates sufficient for the level of repair decision and the simultaneous approach is used to set stockage precisely after the LOR policy is set in each case.

*Unit Cost*. The cost of an LRA enters the sparing computation in two distinct ways. First, there is a gross tradeoff between quantity and unit cost modeled by the application of a learning curve formula to the computation. As the number of units increases, the average unit cost decreases. This produces a "softening" effect on the cost impact of greater numbers of spares. Second, unit cost, however computed, is used as a weighting factor in determining how a limited budget should be distributed among different kinds of LRA's.

Learning curves or progress curves define the way in which average (or marginal) unit cost decreases over the length of a production run.* In general, the unit cost is expected to decrease by a fixed proportion every time the production run doubles. A "90% learning curve," therefore,

*See Alchian, A., *op. cit.*, pp. 335-360. This article is a reprint of one of the earliest contributions to learning curve or "progress curve" theory. A more recent publication with a useful summary of the various mathematical formulations for learning functions is, Boren, H. and H. G. Campbell, *Learning Curve Tables: Volume I.* RM-6191-PR, Rand, Santa Monica, 1970. See especially pp. 1-13.

shows average unit cost decreasing to 90% of the first unit cost by the second unit, 90% of the second unit cost on the fourth unit and so on. The use of learning curves in level of repair analysis tends to favor high stockage slightly more than formulations that don't account for learning. Thus, discard and depot repair options are selected more frequently than they would be if average unit cost were fixed.

The use of unit costs as weights amounts to computing the marginal return to investment in stockage. The idea is to spend the spares budget in the most effective way. The return measured in this process is the increment in system confidence provided by an additional spare of a particular type. The measure of merit is, therefore, the change in system confidence produced by an additional spare divided by its cost, where cost is a function of learning. If this ratio is computed for every LRA type, the largest one is spared first, and the rest are spared in order of decreasing value until the system confidence criterion level has been reached.

4.1.2 Demand Relationships. In order to compute spare stockage in any form, demand relationships must first be derived. These formulae specify the anticipated level of demand faced by a supply or repair facility. Two kinds of demand are required to drive the model presented here: average demand and peak demand.

Based on the number of operating hours anticipated for a given equipment component (we deal with the lowest removable assembly, LRA) throughout the fleet per unit of time, the average demand rate is

used to estimate recurring cost flows. For spare stockage, its use is restricted to the consumption of spares or replenishment purchases over the life of a system. Replenishment is required for two kinds of LRA's: those coded discard at failure must be replaced for every failure while LRA's coded repair must sometimes be condemned and therefore replaced. The conventional device for estimating the latter is to characterize a design by specifying a condemnation rate or proportion of actual failures resulting in condemnation and multiplying this factor by average demand.

The idea of peak demand is quite different from average demand and care must be taken not to confuse the two. Both are based on policy variables to the equipment buyer. While average demand is based on the buyer's real estimate of gross activity to be expected over the life of the system, peak demand is an attempt to specify, indirectly, the capacity required for the support system. The normal method of specifying each of these values is through the value of "operating hours per period of time." Most often, average operating hours are quoted on an annual basis, while peak operating hours (for most land based equipment) are specified for a shorter period such as a month. When equipments are operated in remote locations, out of contact with the support infrastructure for specific periods of time, peak operating hours should be measured over the normal length of such periods. For example, if a ship is normally deployed for 90 days, this should be the period over which peak operating hours are defined.

By specifying peak operating hours, the buyer is stating the

activity level at which he wants the support system to be self-sustaining. In other words, logistics and cost models "buy" sufficient spares and other resources necessary to keep a system operating at the peak level without initiating queues for repair. The resulting costs, unlike those based on average operating hours, are initial or investment costs since the support resources required must be present from the first to the last day of operation.

Average demand is given by:

4.1) $\lambda_i = Qq_i\delta_i AHR/MTBF_i$,

where Q is the number of systems per ship, each system contains $q_i$ of the $i^{th}$ LRA type and operates AHR hours per year an average. The duty cycle of the $i^{th}$ component is $\delta_i$, a ratio of the LRA's operating time per hour of system operating time. $MTBF_i$ is the mean operating hours between failures of the $i^{th}$ LRA. The value of $\lambda_i$ can, therefore, be stated as the exepcted number of failures per ship generated by all appearances of the $i^{th}$ LRA type during a one year period.

Peak demands are given by:

4.2) $\lambda_i' = \lambda_i \; PHR/AHR$,

where PHR is the peak operating hours per deployment. The value of $\lambda_i'$ can be stated as the expected number of failures from a single ship generated by all appearances of the $i^{th}$ LRA during a deployment.

<u>4.1.3 The Sparing Algorithm: Level of Repair Assignment</u>. The first step in determining spare stockage requirements is to develop a tentative estimate for the purpose of choosing the level of repair for

each LRA. The stockage allocation is called tentative because it is replaced later by a more precise estimate. The same method, however, is used in the Level II Model to determine final stockage levels. Once levels of repair have been determined, a more lengthy and computationally complex routine, described in Section 4.1.4, is used in the Level III Model to set the final stockage levels. We will defer a discussion of the tentative (i.e. Level II) sparing algorithms until Section 4.2. At this point we define some basic elements common to the stockage algorithms in both models.

In the case of local repair, lead time is simply local response time, LRT. Lead time for depot repair items is more complex. A failed item is turned into the depot at the end of a deployment period, D, measured in days. Either the depot can issue a new part (sufficient stock) or it can't (stockout at the depot). These two outcomes are assigned probabilities b and 1-b respectively. During b proportion of the time, then, the lead time is only D. During 1-b proportion of the time, the lead time is greater than D by a factor, X. The value of b depends on the amount of stock maintained at the depot. That of X depends on a comparison between depot response time (DRT) and the length of a deployment cycle.

The probability of a stockout at the depot, 1-b, is the probability that demands at the depot will exceed $B_i$, the number of spares of the $i^{th}$ type maintained there. First we compute the demand lead time experienced by each of d depots:

4.3) $\mu_i = r_{2,i}(N\lambda_i DRT)/Dd$

where N is the number of ships; $\lambda_i$ is the consolidated demand from each ship during a deployment period, and DRT is the depot response time. The probability of up to $B_i$ demands at the depot during DRT is given by the Poisson distribution:

$$4.4)\quad b_{i,B} = \sum_{j=0}^{B_i} \frac{\mu_i^j e^{-\mu_i}}{j!}$$

The value of X, the number of deployment periods in a ship's lead time in the event of a stockout at the depot, is given by:

$$4.5)\quad X = \lceil (DRT - P)/(D + P) \rceil + 1.$$

Equation 4.5 can be understood as follows. If there is no stock at the depot and DRT exceeds P (the in-port period) the equipment will have to operate over an additional deployment on its own resources. If DRT stretches past the next departure of the ship, two deployment periods are added to the lead time, and so on. The logic is demonstrated in Figure 4-1.

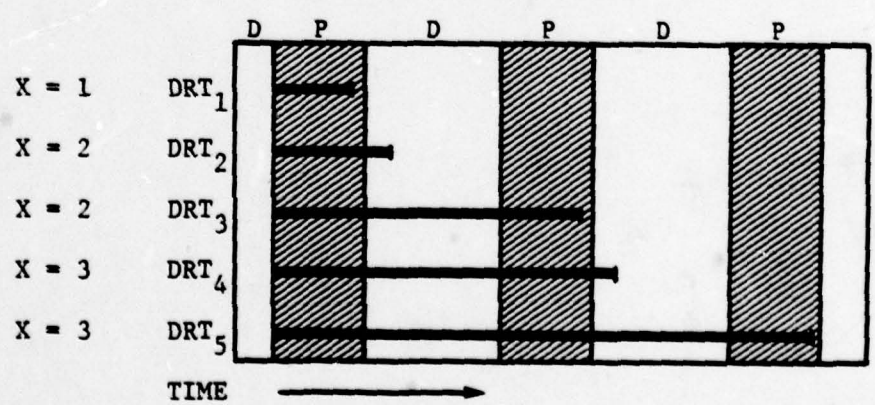


Figure 4-1
Lead time in the Event of a Depot Stockout

The head of the figure is a time line divided into deployment and in-port periods. Failures occur during each D and are turned in at the depot at the end of D. If DRT is less than P, as is $DRT_1$, X is set to 1 by equation 2. The implication of this case is that the repair facility is so fast that no stock need be maintained at the intermediate supply facility and lead time is restricted to the initial operating period, D. $DRT_2$ is a case in which DRT exceeds P, meaning the ship will have to wait an additional D before its part is returned. Notice that once DRT > P, X doesn't change until DRT > 2P + D as with $DRT_4$: $DRT_3$ and $DRT_2$ imply the same waiting time for the ship. The same thing is true of $DRT_4$ and $DRT_5$.

We see, therefore, that the lead time for local and depot repair is given by:

$$4.6)\quad LT_i = \begin{cases} LRT & i^{th} \text{ LRA coded local repair} \\ D & i^{th} \text{ LRA coded depot repair and no stockout at depot } (b_i \text{ proportion of the time}) \\ XD & i^{th} \text{ LRA coded depot repair and stockout at depot } (1 - b_i \text{ proportion of the time}) \end{cases}$$

where $b_i$ is given in equation 4.4 and X is given in equation 4.5. The confidence level achieved by the $i^{th}$ LRA is given by:

$$4.7)\quad K_i = r_{1,i}K(S_i,\lambda_i'LRT) + (1 - r_{1,i})\left[b_iK(S_i,\lambda_i'D) + (1 - b_i)K(S_i,\lambda_i'XD)\right]$$

where the function $K(x,y)$ expresses the confidence level achieved when x spares are stocked for an item with demand lead time y. Thus $b_i = K(B_i, \mu_i)$, where $\mu_i$ is given in 4.3. Note that the confidence levels of the LRA types in the system.

For each LOR posture we must produce estimates for the number of shipboard spares ($S_i$) and the number of depot spares ($B_i$) required. Note that if the LRA is coded local repair ($r_{1,i} = 1$) then $B_i = 0$. If an item is coded discard at failure, the same number of spares is purchased to set up the initial spares support system as in the depot repair posture. The initial support system remains intact throughout the system life cycle. If an LRA is coded discard at failure, each time it fails, a new one is purchased. Repair is attempted for all items coded repair. However, a certain fraction, COND, will have to be condemned and replacements purchased. Annual replenishment spares, then, are given by:

$$4.8)\quad S_i' = \begin{cases} \lambda_i & i^{th} \text{ LRA coded discard at failure} \\ & (r_1 = r_2 = 0) \\ \lambda_i \text{COND} & i^{th} \text{ LRA coded repair} \\ & (r_1 + r_2 = 1) \end{cases}$$

This equation can be compactly expressed by:

$$4.9)\quad S_i' = \lambda_i \left[ 1 - (r_{1,i} + r_{2,i})(1 - \text{COND}) \right]$$

The unit cost of the $i^{th}$ LRA is given by:

$$4.10)\quad f(UC_i) = UC_{\ell,i}\left[\frac{N(Qq + S_i + S_i') + dB_i}{\ell}\right]^{[\log RRATE/\log 2]}$$

where RRATE or reduction rate is the proportion of unit cost to which average unit cost drops after production has doubled, $\ell$ is a lot size for which $UC_{i,\ell}$ is known, Q is the number of systems per ship and $q_i$ is the number of LRA's of type i per system. Notice that $S_i'$, replenishment spares for only 1 year, is used in the lot computation. This is because the model assumes replenishment spares will be purchased on a year-to-year basis and that further gains from learning are unavailable after the first run.

It was noted earlier that the level of repair decision amounts to a tradeoff between the quantity of spares and a variety of other costs associated with actions necessary to reduce lead time and hence the number of spares. The costs mentioned included training, technical data and support and test equipment. The preceding discussion, however, suggests that other elements must be considered. For example, production costs are influenced because larger or smaller spares buys have a direct effect on the average unit cost of the LRA's from which the production systems are assembled. Beyond this, a depot - whether the intermediate supply facility (at which $B_i$ is maintained) or a repair facility - may involve military construction, labor and a variety of other costs. We, therefore, consider all elements of life cycle cost in making the level of repair decision.

Since costs other than spares are discussed in the remainder of this paper, we will not detail their computation here. The trade-off is accomplished by comparing the sum of spares and all other costs computed for each of the three levels of repair and for each LRA. The minimum sum for each LRA indicates the support policy to be followed. With this assignment made, the system can be re-spared using fixed demand lead times for local repair LRA's and variable demand lead times for depot repair LRA's.

4.1.4 Setting Final Spare Stockage Levels. This section details the sequence of computations required to determine spare stockage for all LRA's coded either local or depot repair. If an item is coded discard, the same initial spares purchase at the depot and on-board ship is made as for the depot repair posture. Thus, for the purpose of sparing, discard and depot repair items are considered identical.

Our general objective is to purchase the least cost set of spare LRA's satisfying the system criterion probability, $K^*$, that no stockouts will occur on a ship during deployment. A general expression for achieved system confidence, K, is:

$$4.11)\quad K = \prod_{i=1}^{n} K_i$$

where n is the number of LRA's and $K_i$ is the probability that no stockout will occur for the $i^{th}$ LRA. For the remainder of this chapter we will redefine $\lambda_i'$ as the previous value divided by D.

The new $\lambda_i'$ is now a failure *rate*, which, when multiplied by a lead time, yields the demand lead time.

The actual computation of the probability of no stockout differs between three alternatives: local repair; depot repair, shipboard stock; and depot repair, depot stock. In other words, an additional unit of stock placed at the depot will have a different impact of K than one placed on the ship, and the effect of the latter will be different depending on whether the LRA is coded local or depot repair. The probability of no stockout during deployment for the system as a whole is given by:

$$4.12)\quad K = \prod_{i=1}^{n} \left\{ r_{1,i} \sum_{j=0}^{S_i} \frac{\lambda_i' LRT^{\,j} e^{-\lambda_i' LRT}}{j!} + (1 - r_{1,i}) \sum_{j=0}^{S_i} \left[ b_{i,B} \frac{\lambda_i' D^{\,j} e^{-\lambda_i' D}}{j!} + (1 - b_i) \frac{\lambda_i' XD^{\,j} e^{-\lambda_i' XD}}{j!} \right] \right\}$$

The first expression inside the outer brackets applies to local repair items and the second applies to depot repair or discard items. In the latter, the first ratio gives $K_i$ when there is stock at the depot and the second gives $K_i$ when there is a stockout at the depot. There are three stockage levels and three lead times in the equation. The value $S_i$ is shipboard stockage for local repair items in the first sum and depot repair items in the second. The value $b_i$ implies a depot stockage level, $B_i$, as discussed in the previous section. Its definition is repeated for convenience:

$$4.4)\quad b_i = \sum_{j=0}^{B_i} \frac{\mu_i^{\,j} e^{-\mu_i}}{j!}$$

The lead times are local response time, LRT, for local repair items, D for depot repair items when there is no stockout at the depot and XD when there is a stockout condition. The value of X was also developed earlier and its definition is repeated here for convenience:

4.5) $X = \lceil (DRT-P)/(D+P) \rceil + 1$

The method of determining the spares package is to select one spare at a time on the basis of the maximum contribution to K per dollar expended. If there are $S_i$ units of stock for the $i^{th}$ LRA, already in the system, the effect of adding one more unit is given by:

$$4.13)\quad K_S = K_{S-1}\left[\frac{K_i(S_i)}{K_i(S_i-1)}\right]$$

where the notation K(y) means the probability of no stockout in the $i^{th}$ LRA when y units of stock are available. The ratio of $K_i$'s can be thought of as a quantity $1+\Delta K_i(S_i)$ where $\Delta K_i(S_i) = K_i(S_i)-K_i(S_i-1)$. The 1 can be dropped and the ratio $\Delta K_i(S_i)/f(UC_i)$ used as a measure of the marginal return (in terms of increased system confidence) to investment in a spare of the $i^{th}$ type. The notation $f(UC_i)$ was defined in equation 4.10 above, as the unit cost of the $i^{th}$ LRA adjusted for the new production quantity implied by $S_i$.

The equation for marginal return is different for three classes of spares: on board, local repair; on board, depot repair; and depot stock. These equations are given below:

*Marginal Return for Local Repair Items:*

$$4.14)\quad R_{i,S} = \frac{\lambda_i' LRT^{S_i} e^{-\lambda_i' LRT}}{S_i!}\left[f(UC_i)\sum_{j=0}^{S_i-1}\frac{\lambda_i' LRT^{j} e^{-\lambda_i' LRT}}{j!}\right]^{-1}$$

where the ratio outside the brackets is the probability of exactly $S_i$ failures and the sum inside the brackets is the probability of $S_i - 1$ or fewer failures. The product is, therefore, $P(S_i)/K(S_i - 1) = \Delta K_i(S_i)$. That product is divided by $f(UC_i)$ to produce the measure of marginal return - change in confidence per dollar expended on spare stock.

*Marginal Return for Depot Repair, Ship Stock:*

$$4.15)\quad R_{i,S,B} = \left[ b_{i,B} \frac{\lambda_i' D^{S_i} e^{-\lambda_i' D}}{S_i!} + (1-b_{i,B}) \frac{\lambda_i' XD^{S_i} e^{-\lambda_i' XD}}{S_i!} \right] f(UC_i)^{-1}$$

$$\left[ b_{i,B} \sum_{j=0}^{S_i-1} \frac{\lambda_i' D^j e^{-\lambda_i' D}}{j!} + (1-b_{i,B}) \sum_{j=0}^{S_i-1} \frac{\lambda_i' XD^j e^{-\lambda_i' XD}}{j!} \right]^{-1}$$

Equations 4.14 and 4.15 are substantially the same except that the single Poisson ratio, dependent on LRT in the former, is replaced by a weighted sum of two such ratios dependent on D and XD in the latter. While both of these equations are written down in the form $P(S_i)/K(S_i-1)$, this is inconvenient for depot spares. The reason is that adding a spare at the depot, by reducing lead time (increasing $b_i$), has an impact on every element of the sum of probabilities of 0, 1 ... $S_i$ failures. Therefore, it must be written out as follows.

*Marginal Return for Depot Repair, Depot Stock:*

$$4.16)\quad R_{i,S,B} = \frac{\mu_i^B e^{-\mu_i}}{B!} \sum_{j=0}^{S_i} \left( \frac{\lambda_i' D^j e^{-\lambda_i' D}}{j!} - \frac{\lambda_i' XD^j e^{-\lambda_i' XD}}{j!} \right) f(UC_i)^{-1}$$

$$\cdot \left\{ \sum_{j=0}^{S_i} \left[ b_{i,B-1}\left(\frac{\lambda_i' D^j e^{-\lambda_i' D}}{j!}\right) + (1-b_{i,B-1})\left(\frac{\lambda_i' XD^j e^{-\lambda_i' XD}}{j!}\right)\right]\right\}^{-1} \frac{N}{d}$$

The first term of equation 4.16 gives the decrease in the probability of a depot stockout resulting from addition of the $B^{th}$ unit. More simply, it is the increase in $b_{i,B}$ from the $B^{th}$ spare. The sum is the difference, at $S_i$ units of shipboard stock, between the probability of no local stockout when there is stock at the depot and when there is a stockout at the depot. Essentially, the increment $\Delta b_{i,B}$ is subtracted from the weight $(1-b_{i,B-1})$ and added to the weight $b_{i,B-1}$ as used in equation 4.15. The term $f(UC_i)^{-1}$ plays the same role here as in equations 4.14 and 4.15. The second line of equation 4.16 gives the confidence level achieved on each ship with $S_i$ units of ship stock and B-1 units of depot stock. The last term, N/d, is the number of ships served by each of d depots, all of which benefit by the increase in confidence. This term is not found, of course, in equation 4.14 and 4.15 since they are used for shipboard stock.

The sparing algorithm uses the values calculated for R as a means of identifying which LRA should be spared next. First, the value R is computed for one unit of stock of every LRA, including both ship and depot locations for LRA's coded depot repair. The highest value of R is identified, that LRA spared at the location indicated and a new R computed for the next stock level of that LRA. The list is searched again, this time including the R computed for a second unit of the LRA already spared. After each iteration, K is

computed according to equation 4.12 and compared to a value, K*, representing the system confidence level sought. An alternative is to maximize K given a budget constraint, in which case total spares cost is compared to the budget after each iteration. When the appropriate constraint is satisfied, the least cost spares package has been identified as to both quantities and locations.

## 4.2 Spares in the Level II Model

To begin our discussion of the sparing algorithm in the Level II Model, we note that the confidence level achieved by the $i^{th}$ LRA is given in equation 4.7, which we repeat here for convenience:

$$4.7)\quad K_i = r_{1,i}K(S_i,\lambda_i'LRT) + (1 - r_{1,i})\left[b_iK(S_i,\lambda_i'D) + (1 - b_i)K(S_i,\lambda_i'XD)\right]$$

where $K(S,\lambda t)$ is the confidence level achieved when S spares are purchased for an item which demand $\lambda$ and lead time t. The value of $b_i$ is $K(B_i,\mu_i)$ where $\mu_i = N\lambda_i'DRT/d$.

The job of the sparing algorithm is to find the values of $S_i$ and $B_i$ which minimize the total number of spares purchased, $NS_i + dB_i$, yet still provide for $K_i \geq K_i^*$, where $K_i^*$ is the desired confidence level for the $i^{th}$ LRA type. Since we must have that $\prod_{i=1}^{n} K_i \geq K^*$, the most straightforward approach would seem to be to set $K_i^* = K^*/n$. However, we can do better than that.* First, spares purchases (being integer values) cannot exactly meet desired confidence criteria, but will slightly overshoot them. If each of n LRA types slightly overshoot a desired confidence level of $K^*/n$, the resulting achieved system confidence level could be much higher than was originally desired - more spares than necessary will have been

* The reader will note that this discussion is analogous to the issue of allocating portions of the AN and AG pools to individual LRA's, discussed in Section 3.2.

purchased. Thus, we would like to keep a running account of the achieved confidence level so as to minimize any overshoot. Also, everything else being equal, we would prefer to allocate higher desired confidence levels to LRA's with lower unit costs, since by doing so we can achieve a given confidence level at a lower total spares cost. An allocation equation which accomplishes these two goals is:

$$4.17)\quad K_i^* = \left[\frac{K^*}{\prod_{x=1}^{i-1} K_x}\right]^{\left[UC_i \,/\, \sum_{x=i}^{n} UC_x\right]}$$

where we define $\prod_{x=1}^{0} K_x \triangleq 1$. Note that since the quantity inside the brackets is always less than 1, LRA's with higher unit costs will be assigned lower $K_i^*$'s, as desired.

Now that individual $K_i^*$'s have been assigned, we wish to find the optimal values of $S_i$ and $B_i$ needed to achieve the desired confidence level. If the LRA is coded local repair ($r_{1,i} = 1$), then $B_i = 0$ and $S_i$ is determined by finding the smallest s such that $K(s,\lambda' LRT) \geq K_i^*$. That is, $S_i = S(\lambda' LRT, K_i^*)$, where $S(\lambda t,K)$ is the minimum number of spares needed to achieve a confidence level, K, for an item with demand lead time $\lambda t$.

Sparing becomes much more complicated if the item is coded depot repair. We need a procedure which finds the values of $S_i$ and $B_i$ that minimizes $NS_i + dB_i$ subject to the constraint that $K_i \geq K_i^*$. Now since any spare placed on board is multiplied by the number of

ships, N, which is usually much larger than the number of supply depots, d, the result of such a procedure will usually be to minimize $S_i$. Therefore, we use this as our starting point. Setting $b_i = 1$ in 4.7, we see that the minimum value of $S_i$ is given by $\bar{S}_i = S(\lambda_i' D, K_i^*)$. Once $\bar{S}_i$ has been determined, we can solve for the minimum $b_i$ need to meet $K_i^*$ by setting $K_i = K_i^*$ and $S_i = \bar{S}_i$ in 4.7:

$$4.18) \quad \bar{b}_i = \frac{K_i^* - K(\bar{S}_i, \lambda' XD)}{K(\bar{S}_i, \lambda' D) - K(\bar{S}, \lambda' XD)}$$

The number of spares at the depot needed to meet $\bar{b}_i$ is given by:

$$4.19) \quad \bar{B}_i = S(\mu_i, \bar{b}_i)$$

Therefore, the total number of spares purchased is $N\bar{S}_i + d\bar{B}_i$. We note that if $d\bar{B}_i \leq N$ we know immediately that we can't do any better, because the only way to decrease $B_i$ is to increase $S_i$, which indicates the purchase of at least N additional spares. If, however, $d\bar{B}_i > N$, then the possibility exists that we can decrease the total spares purchase by setting $S_i = \bar{S}_i + 1$ in 4.7 and using 4.18 and 4.19 to determine a new value of $\bar{B}_i$. If the new total, $N\bar{S}_i + d\bar{B}_i$, is smaller than the previous value then we have done better than before. If the new $d\bar{B}_i$ is still greater than N, we try $\bar{S}_i + 2$, $\bar{S}_i + 3$ . . ., until either we reach a value of $\bar{B}_i$ such that $d\bar{B}_i \leq N$ or until we reach a total spares purchase at $\bar{S}_i + x$ which is larger than the purchase for $\bar{S}_i + (x - 1)$. We

could eliminate the second termination criterion and still have an algorithm which always terminates in a finite number of iterations. However, experience has shown that in almost every case for $N >> d$, once going from $S_i$ to $S_i + 1$ causes the total spares purchase to increase, all subsequent total spares purchases are still higher. The second criterion eliminates the necessity of checking many possibilities which are never chosen, saving considerable computer running time.

Once we have decided on $\bar{S}_i$ and $\bar{B}_i$, we can determine the achieved confidence level for the LRA by setting $\bar{b}_i = K(\bar{B}_i, \mu_i)$ and substituting $\bar{b}_i$ and $\bar{S}_i$ into equation 4.7.

Now that we have developed a sparing algorithm, the only problem remaining is to express the functions $K(S,\lambda t)$ and $S(\lambda t,K)$. However, these are easily defined using the Poisson distribution:

$$4.20)\quad K(S,\lambda t) = \sum_{x=0}^{S} \frac{e^{-\lambda t}(\lambda t)^x}{x!}$$

$$4.21)\quad S(\lambda t,K) = \min\left\{ s \geq 0: \sum_{x=0}^{s} \frac{e^{-\lambda t}(\lambda t)^x}{x!} \geq K \right\}$$

Unfortunately, the required number of iterations can make the program running time too lengthy for the Level II program to be practical as a trade-off tool.

We can avoid using the Poisson by taking advantage of a result from renewal theory which states that for an arbitrary renewal counting process, $N_t$, the confidence level (i.e., the probability

that $N_t \leq S$) approaches a normal distribution as $\lambda t$ becomes large:

$$4.22)\quad 1-K = P(N_t > S) \sim Q\left(\frac{S-t/\mu}{\sqrt{t\sigma^2/\mu^3}}\right)$$

where $Q(x)$ is the area of the region to the right of x on a standard normal distribution and $\mu$ and $\sigma^2$ are the mean and variance of the interarrival distribution associated with $N_t$. In this case $N_t$ is the Poisson arrival distribution resulting from exponentially distributed interarrivals with $\mu = 1/\lambda$ and $\sigma^2 = 1/\lambda^2$. Noting that $1 - Q(x) = Q(-x)$ we obtain:

$$4.23)\quad K(S,\lambda t) = \begin{cases} Q\left(\frac{\lambda t-S}{\sqrt{\lambda t}}\right) & S \neq 0 \\ e^{-\lambda t} & S = 0 \end{cases}$$

The term $e^{-\lambda t}$ comes from setting $S = 0$ in 4.20. This is done because the normal approximation to the Poisson distribution breaks down for $S = 0$. To calculate $S(\lambda t,K)$ we use the inverse normal function, $Q^{-1}(K)$, to find the number of standard deviations from the mean of $N_t$ needed to meet a confidence level K, and then buy that number of spares:

$$4.24)\quad S(\lambda t,K) = \begin{cases} 0 & e^{-\lambda t} \geq K \\ \lceil \lambda t + Z\sqrt{\lambda t} \rceil^+ & \text{otherwise} \end{cases}$$

where $Z = Q^{-1}(K)$ and $\lceil x \rceil^+$ is equal to the next higher integer then x provided $x > 0$; if $x \leq 0$ $\lceil x \rceil^+ = 0$. The criterion $e^{-\lambda t} \geq K$ is included

included to avoid incorrectly buying a spare when K could be achieved with S = 0.*

The functions Q(x) and $Q^{-1}(K)$ are computed using polynomial and rational approximations found in any standard mathematical reference.**

The Level II Model user can choose either the Poisson or the normal approximation to the Poisson in the spares calculation subroutine of the model. In the latter case, the user may give up some accuracy in the spares approximations, which is compensated for by considerably decreased program running time.

To complete the discussion of spares we note that $S_i'$ and $f(UC_i)$ are computed in the Level II model in in exactly the same manner in the Level III Model.

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* See Neches, T. and Robert A. Butler, *Guidelines for Hardware/Manpower Cost Analysis*, AG-PR-A100-2, *op cit.*, Volume III, Table A-3, p. A-10 for a comparison of values of $S(\lambda t,K)$ and $K(S,\lambda t)$ using the Poisson and the normal approximation.

** See Abramowitz and Stegun, *Handbook or Mathematical Functions*, National Bureau of Standards, U.S. Government Printing Office, Wash. D.C., 1968, pg. 932.

## 4.3 Spares in the Level I Models

The sparing algorithms developed in Section 4.1 require a great deal of input data and are computationally quite complex. Clearly, such methods are inappropriate to the Level I Models, which are restricted by the computational limitations of small calculators and by the need for minimum data input, easy usage, and quick turn-around time.

Recall that there are three Level I Models, a Top-Down System Model (TDM) and an Aggregation Model System (AMS), consisting of an LRA Model and a System Aggregation Model. Different simplifications and assumptions must be made for each model. These are discussed in Sections 4.3.1 and 4.3.2 below.

4.3.1 Spares In The Top-Down Model. Recall that the system confidence level is given by the product of the confidence level of each LRA in the system. Let $n_1$ be the number of LRA types coded local repair and $n_2$ be the number coded depot repair. Recalling that we have assumed LRA's coded discard to be the equivalent of depot repair, system confidence is given by:

$$4.25)\quad K = \prod_{i=1}^{n_1} K_i \prod_{j=1}^{n_2} K_j,$$

where the i subscript refers to local repair items and the j subscript to depot repair. Explicit formulae for $K_i$ and $K_j$ are given in equation 4.12. The expression for $K_j$ is complicated, but can be much simplified if we assume that $b_i \approx 1$. In this case, equation 4.12

becomes:

$$4.26)\quad K = \exp\left\{-t_1 \sum_{i=1}^{n_1} \lambda_i - t_2 \sum_{j=1}^{n_2} \lambda_j\right\} \prod_{i=1}^{n_1}\left\{\sum_{x=0}^{S_i} \frac{(\lambda_i' t_1)^x}{x!}\right\} \prod_{j=1}^{n_2}\left\{\sum_{x=0}^{S_j} \frac{(\lambda_j' t_2)^x}{x!}\right\},$$

where $t_1$ = LRT and $t_2$ =D.

We can justify setting b = 1 by noting that the number of spares purchased for an LRA coded depot repair is $NS_j+dB_j$, which we desire to be as small as possible. In almost every case this will mean minimizing $S_j$ by choosing a high confidence level at the depot. In many actual cases b will be close to unity. The TDM, therefore, assumes $b \approx 1$ to determine $S_j$. To determine $B_j$, b is set to a high value, normally .95 (if b = 1, then $B_j$ would have to be infinite). This set of assumptions yields values of $S_j$ and $B_j$, which are close and often equal to their optimal values. The conceptual drawbacks of this approach are more than balanced by the simplification in computations which it allows, however. In the AMS, on the other hand, we will be able to relax the assumptions considerably and compute near optimal values of $S_j$ and $B_j$.

Equation 4.26 is still inappropriate for the TDM because it requires as input $\lambda$ for each LRA. This information is not available early in system design. Therefore, we need to make some assumptions about the distribution of the $\lambda$'s in order to obtain a spares formulation which will work in the TDM. One admittedly very arbitrary assumption is to state that failure rates are uniformly distributed over the n LRA types. That is, $\lambda_i' = \lambda'/n$, where $\lambda'$ is the failure

rate of the entire system. This assumption results in considerable simplification in the spares formulation, making the problem a tractable one, even for a limited capacity programmable calculator. However, it is a very strong assumption and should be kept in mind when examining the results of the sparing algorithms.

Using the assumption of uniform failure rates, we can rewrite equation 4.26 as follows:

$$4.27)\quad K = \exp\left[-n_1\nabla_1 - n_2\nabla_2\right] \prod_{i=1}^{n_1} \left\{ \sum_{x=0}^{S_i} \frac{(\nabla_1)^x}{x!} \right\} \prod_{j=1}^{n_2} \left\{ \sum_{x=0}^{S_j} \frac{(\nabla_2)^x}{x!} \right\},$$

where $\nabla_p = \frac{\lambda' t_p}{n}$, $p = 1,2$.

Setting $S_i$ and $S_j$ equal to 0, we get a particularly simple expression for the initial confidence level of the system:

$$4.28)\quad K_0 = \exp\left[-n_1\nabla_1 - n_2\nabla_2\right]$$

If $K_0 \geq K^*$, there is no need to buy any spares at all. This is very unlikely, however, and as before we wish to buy spares in the most economical way possible. We now assume the unit costs of the LRA's are also uniformly distributed (this assumption is somewhat less arbitrary than the equivalent assumption concerning failure rates). This will allow the sparing algorithm to run very quickly, because once we have decided that the next spare to be purchased is, say, a local repair item, we can go ahead and buy up to $n_1$ copies of it. Thus, we can approach $K^*$ in much larger increments than previously.

This is an important consideration. Programmable calculators process much more slowly than full-scale computers. Running time constraints are often critical.

Initially $K = K_0$. If we now purchase one spare for, say, a depot repair item, the new confidence level will be:

$$4.29)\quad K = K_0 \sum_{x=0}^{1} \frac{(\nabla_2)^x}{x!} = K_0(1+\nabla_2),$$

and if we purchase 1 spare for each of h depot repair items,

$$4.30)\quad K = K_0(1+\nabla_2)^h.$$

In general, if we have already purchased $S_1$ spares for each of the $n_1$ local repair and $S_2$ for the $n_2$ depot repair LRA's, we have:

$$4.31)\quad K = K_0 \left\{ \sum_{x=0}^{S_1} \frac{(\nabla_1)^x}{x!} \right\}^{n_1} \left\{ \sum_{x=0}^{S_2} \frac{(\nabla_2)^x}{x!} \right\}^{n_2}$$

The LRA which is purchased next is the one which will cause the largest increase in system confidence level (as before, per unit dollar, but all unit costs are assumed equal here, so UC drops out of the calculation). Increasing $S_p$ by 1 will increase K by a factor of $\Delta_p$, defined as follows:

$$4.32)\quad \Delta_p = \sum_{x=0}^{S_p+1} \frac{(\nabla_p)^x}{x!} \div \sum_{x=0}^{S_p} \frac{(\nabla_p)^x}{x!}$$

If $\Delta_1 > \Delta_2$, then the next spare purchased will be one coded local repair, otherwise it will be a depot repair item. If h spares are purchased, the new system confidence level is given by:

$$4.33)\quad K = K\Delta_p^h$$

Setting K = K*, the desired system confidence level, we can solve for the smallest number of spares needed to achieve K*:

$$4.34)\quad h = \left\lceil \frac{\log(K^*/K)}{\log(\Delta_p)} \right\rceil$$

If $h \leq n_p$, the total number available, h spares are purchased and the sparing algorithm is finished. If $h > n_p$, $n_p$ spares are purchased, $S_p$ is increased by one, new values of $K_p$ and $\nabla_p$ are computed, and the algorithm returns for another round of spare purchases.

Recall that n, $n_1$, and $n_2$ refer to the number of LRA types in the system. Spares are purchased for LRA types; for the purposes of sparing the $i^{th}$ LRA type is considered to be a single LRA with failure rate $q_i\lambda_i$. The unit cost of the spare, however, is not $UC_{sys}/n$, but $UC_{sys}/n'$, where $n'$ is the total number of LRA's in the system (i.e., $n' = \bar{q}n$, where $\bar{q}$ is the average number of appearances of an LRA in the system).

Spares are included in the learning curve calculation of unit cost by dividing the total number of spares purchased by the number of LRA's in the system. Thus, for example, if 20 spares are bought for a system

which has a total of 50 LRA's, for the purpose of learning, it is considered that an additional .4 system has been purchased.

Depot spares at each depot for each LRA type are computed using the formula:

$$4.35)\quad B_j = \lceil \mu + 1.645\sqrt{\mu} \rceil ,$$

where $\mu = (\lambda'/n')\cdot N\cdot DRT$. This approximation to the Poisson distribution was discussed in detail in Section 4.2 The number of depot spares purchased in system equivalents is given by:

$$4.36)\quad B = B_j \cdot d \cdot n_2/n'$$

Replenishment spares are computed as follows:

$$4.37)\quad S' = \lambda\cdot AHR\cdot\left(1 + \frac{n_1+n_2}{n}\,(1-COND)\right)/n'$$

Finally, the sum of the number of systems and spares in system equivalents are used to compute the reduction in unit cost due to learning:

$$4.38)\quad UC = UC_\ell \left[\frac{N(Q+S+S')+B}{\ell}\right]^{\log RRATE/\log 2} .$$

### 4.3.2 Spares In The Aggregation Model System

The Aggregation Model System consists of two cost models. The first (LRAM) computes costs for individual LRA's, the second (SAM) aggregates these costs.

The LRAM computes spares cost based on the number of appearances of the LRA in the system. The SAM aggregates these costs as follows:

$$4.39)\quad S_j = \sum_{i=1}^{n_j} R_{i,j} S_i,$$

Where $R_{i,j}$ is given in equation 3.32. This aggregation approach eliminates the commonality problems which often occur when costs determined on a system-wide basis, like spares, training, documentation, and so on, are allocated at a below-system aggregation level.

The sparing algorithm in the LRAM is a slightly simplified version of the Level II approach. As in the Level II Model, the LRAM sets $\bar{S}_i = K(\lambda_i D, K_i^*)$ and uses equation 4.18 to determine $\bar{b}_i$ (compare this to the TDM, in which b was arbitrarily set to .95) and equation 4.19 to determine $\bar{B}_1$. However, once this is done the LRAM does not compare alternative possibilities for $S_i$ and $B_i$ but rather sets them equal to the values computed above (recall that in most cases these will be the optimal values). A compact expression for the number of spares purchased by the LRAM is given by:

$$4.40)\quad S_i = S\left(\lambda_i'(r_{1,i}LRT + r_{2,i}D), K_i^*\right)$$

4.41) $B_i = r_{2,i} S(\mu_i, b_i)$,

where $b_i$ is given in 4.18. Note that when the LRA is coded local repair ($r_{1,i}=1$ , $r_{2,i} = 0$) then $S_i = S(\lambda_i' LRT, K_i^*)$ and $B_i = 0$. The desired confidence level, $K_i^*$, is a direct input to the LRAM. Its value is computed exogenously to the model, either using equation 4.17 or a preferred technique.

The Poisson definitions of $S(\lambda t, K)$ and $K(S, \lambda t)$ given in 4.20 and 4.21 run far too slowly on a programmable calculator to be practical. Therefore, only the normal approximation to the Poisson (4.23 and 4.24) is used in the LRAM to compute the functions S and K.

The function Q(x) is computed by using a preprogrammed routine available in the plug-in read only memory module which comes with the TI-59 programable calculator on which the LRAM is implemented. The formula used to compute $Q^{-1}(K)$ is:

4.42) $Q^{-1}(K) = h - (2.5+.8h)/(1+1.4h+.19h^2)$,

where $h = \sqrt{\log(1-K)^{-2}}$ . This equation is derived by Chebyshev telescoping the standard formula for approximating $Q^{-1}(K)$.*

To complete the discussion of spares we note that S′ and f(UC) are computed in the LRA model in exactly the same manner as in the Level II Model.

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* See, Stark, Peter, *Introduction to Numerical Methods*, The Macmillian Company, London, 1970

# 5.0 OTHER COST ELEMENTS

Manpower costs were the issue of central importance in the development of the models presented here. Spares computation, on the other hand, is quite complex. As a consequence, each of these topics received rather detailed attention in Chapters 3 and 4. With those two issues explained, however, the balance of the model system is relatively straightforward and its exposition is dealt with entirely in this chapter.

The arrangement of this chapter follows the format previously adopted. For each cost element, the most detailed model is explained first, and then its simplifications for Levels II and I are discussed immediately afterward.

## 5.1 Production And Spares Costs

5.1.1 Production And Spares Costs In The Level III Model. In Chapter 4, equation 4.10 was used to define the adjustment in a unit cost estimate for an LRA predicated on $\ell$ units to the number actually purchased as a result of the sparing solution. This value, called $f(UC_i)$, is the final production cost of the $i^{th}$ LRA. To estimate production costs we must first develop a unit cost for the system as a whole that depends on both the costs of its components and the learning gains involved in assembling them into a whole system.

Conceptually, we model the production of a system by imagining that all of its components (the LRA's) are first produced and represent the material cost input to the system production process. Beyond this a proportion, z, of material cost represents value added to the LRA's through the process of assembly into a system. This amount represents a first unit cost subject to gains from learning in exactly the way that LRA cost was reduced by increases in lot size. The equation for system unit cost is:

$$5.1)\quad f(UC) = (1+z)\left[\sum^{n} q_i f(UC_i)\right](N \cdot Q)^{\log RRATE/\log 2}$$

in which the reduction rate may or may not be different from the one used in equation 4.10, depending on the differences in production processes.

Next, the system and LRA unit costs are used to define initial production and spares costs:

$$5.2)\quad C_4 = f(UC)N\cdot Q + \sum_{i=1}^{n} f(UC_i)(N\cdot(S_i + S_i') + dB_i)$$

Notice that replenishment spares, $S_i'$, are purchased for only the first year of the life cycle. This formulation amounts to the assumption that replenishment is a continuous process in which an annual inventory is drawn down during the course of a year and repurchased at the end of the year. Therefore, the first year's requirement is included in the initial production run, while subsequent purchases are needed for the second to the $LC^{th}$ year where LC is the length of the operating and support period. Replenishment spares are given by:

$$5.3)\quad C_5 = \sum_{t=1}^{LC-1} (1+\rho)^{-t} \sum_{i=1}^{n} f(UC_i)S_i',$$

where $\rho$ is the appropriate discount rate. The first sum is analogous to L, used elsewhere in the cost equations (repeated here for convenience):

$$5.4)\quad L = \sum_{t=1}^{LC} (1+\rho)^{-t},$$

except that it ends at the beginning of the last year, not the end, as does L.

Notice that the unit cost, $f(UC_i)$, depends on the *initial* lot size, $N(Qq_i+S_i+S_i')+dB_i$ and not on the total of replenishment spares to be purchased throughout the life of the system. Effectively, we are stating that learning curve gains are curtailed after the first lot has been produced. Thereafter, the increments of $S_i'$ purchased each

year do not decline in unit cost.

This is a violation of one of the weaker parts of learning curve theory - the assumption that "follow-on" lots, as they are called, simply continue down the same progress or learning curve. It is well known that to assemble a system from individually purchased parts would be far more expensive than to buy a complete system. A number of factors explain this: overhead and administrative costs that are constant with respect to the size of purchase, storage cost of production tooling, costs of maintaining the required technical expertise, tooling set-up and knock-down costs, limited piece-part purchases and limited item shelf life are among the major problems.

Nonetheless, we have no useful information on the mechanisms that tend to raise a unit cost which otherwise tends to decline as a consequence of learning. We therefore compromise in the model formulation by assuming that learning curve gains cease after the initial production run, but don't rise thereafter.

If one accepts this approach to unit costing, then there is a further analytical trade-off that could be incorporated in the model. The trade-off pertains to production scheduling of the LC-1 lots of $S_i'$ spares. The larger the number produced in the initial lot, the lower the unit cost of all units (of all types and time periods). Alternatively, front loading the production of replenishment spares also decreases the degree of discounting applied to outyear purchases. In addition, shelf life problems may cause wastage of front loaded (stockpiled) replenishment spares. By estimating costs reflecting these cost changes for different degrees of front loading, the optimal

production schedule can be determined. This schedule would tend to vary with the size of $S_i'$, and therefore be responsive to both failure rates and support policies.

Notwithstanding the possibility of carrying out the kind of trade-off just described, we have chosen not to do so, even in the Level III Model. The reason is the inadequacy of our understanding of the cost consequences of delayed production runs. As a result, the effects mentioned are, at least in part, no more than a reflection of the simplifying assumptions rather than of reality.

5.1.2 Production and Spares Costs in the Level II Model. Estimation of LRA and system unit costs is accomplished in a simpler and somewhat different manner than in Level III. Rather than a separate equation for replenishment spares in the LC-1 outyears, we have combined this cost with initial spares and production costs to give:

$$5.5)\quad C_{4,i} = f(UC_i)\left[N(Qq_i + S_i + S_i'L) + dB_i\right]$$

where $f(UC_i)$ is given in 4.10 and L is given in 5.4. The net result of this change is to discount replenishment spares costs from the end of each year rather than the beginning - slightly underestimating their cost. The exact difference is given by:

$$5.6)\quad f(UC_i)S_i'\left[1-(1+\rho)^{-LC}\right].$$

For a ten year life cycle and a discount rate of 10%, this amounts to a

total underestimate of less than 40% of the cost of the first year's replenishment cost. It was felt that this constituted a sufficiently small error to warrant deletion of the additional equation. The total system production and spares cost is given by adding a system assembly cost, reduced by learning effects, to the sum of the production and spares costs for all the LRA types appearing in the system:

$$5.7)\quad C_4 = PT_\ell \left[\frac{N \cdot Q}{\ell}\right]^{[\log RRATE/\log 2]} + \sum_{i=1}^{n} C_{4,i}$$

where $PT_\ell$ is the estimated system assembly, or put together, cost at lot size $\ell$.

5.1.3 Spares and Production Costs in the Level I Models. The adjusted unit cost of the system as estimated in the Top Down Model was presented in equation 4.38. Spares and production costs in the TDM are simply the product of this cost and the number of systems and spares.

$$5.8)\quad C_4 = [(Q + S + S'L)N + dB] \cdot UC,$$

where spares are calculated in system equivalents (see equations 4.36 and 4.37) and L, the life cycle discount factor, is given by:

$$5.9)\quad L = \begin{cases} \dfrac{(1+\rho)^{LC} - 1}{\rho \cdot (1+\rho)^{LC}} & \rho \neq 0 \\ LC & \rho = 0 \end{cases}$$

which is equal to $\sum_{t=1}^{LC} (1+\rho)^{-t}$.

Hardware costs are computed in the AMS in exactly the same manner as in the Level II Model.

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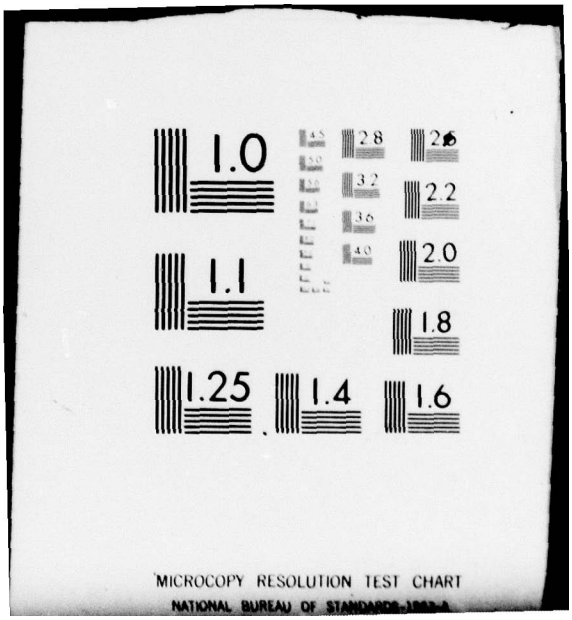


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## 5.2 Support and Test Equipment Costs

5.2.1 Support and Test Equipment Costs in the Level III Model. The Navy distinguishes between external and built-in support and test equipment. The first is abbreviated as S&TE and the second as BITE. The cost of BITE is incorporated in the unit costs of the system and its LRA's while S&TE must be purchased and maintained separately. While logically the support costs of S&TE could be estimated as is the rest of the equipment, this is generally not done. The reason is that it usually generates much more modest costs and these are not worth the same trouble to estimate precisely. Therefore, a "maintenance rate" is used, expressing the expected annual cost of maintenance as a fixed proportion of acquisition cost.

In these models we use the annual maintenance rate, m, for both the hardware and software elements of S&TE. S&TE consists of five different types of hardware and software. These are system level fault isolation hardware, FIH, such as a computer; common software, CS, such as the fault isolation programs necessary to determine which LRA has failed; common repair hardware, CH, used if any of the LRA's are to be repaired locally or at a MOD; specific software, $SS_i$, such as the fault isolation and/or repair subroutine necessary for a specific LRA; and specific hardware, $SH_i$, such as a special plug extender for a particular circuit board LRA.

Each of the variable names introduced above has the value of the installed acquisition cost of the item. To understand the equation for their total cost, the reader must remember that software

includes a one-time cost (development) and a much smaller cost repeated over the number of installations for duplicating, shipping and checking out. Hardware must be purchased for every ship when the repair policy dictates, and common hardware or software must be purchased if *any* LRA is coded repairable. The equation is:

$$5.10)\quad C_6 = \left[ N \cdot FIH + CS + CH\left[ N \cdot SGM \left( \sum_{i=1}^{n} r_{1,i} \right) + d_r SGM \left( \sum_{i=1}^{n} r_{3,i} \right) \right] + \sum_{i=1}^{n} \left[ r_{1,i}(N \cdot SH_i + SS_i) + r_{3,i}(d_r SH_{d,i} + SS_{d,i}) \right] \right] (1 + mL)$$

The logic of the equation is as follows. Fault isolation hardware is purchased for every ship, and a common fault isolation software development cost is incurred. These are the first two terms in the expression. Common repair hardware costs are incurred for each ship if *any* LRA is coded local repair; common hardware costs are incurred at each depot if any LRA is coded repair at a military depot. This is the second term. LRA specific hardware S&TE costs are incurred on each ship for any LRA coded local repair; a repair software development cost is also incurred, but only once. The same is true for LRA's coded repair at a military depot ($r_{3,i}$ =1). The specific hardware and software costs may be different than for local repair, therefore, a subscript, d, is appended to $SH_i$ and $SS_i$. Finally, the annually recurring cost S&TE maintenance, expressed as the fraction, m, of the original purchase cost, is included and discounted to present value through the factor (1 + mL).

5.2.2 Support and Test Equipment in the Level II Model. The Level Level II Model uses an equation for support and test equipment costs which is similar to the Level III Model. The main simplification is that LRA specific hardware and software costs are combined into a single value, $STE_i$, which is the same for both local and depot repair. The equation is:

$$5.11)\quad C_6 = \left[ N \cdot FIH + CS + CH\left[ N \cdot SGM\left(\sum_{i=1}^{n} r_{1,i}\right) + d_r \cdot SGM\left(\sum_{i=1}^{n} r_{3,i}\right)\right] + \sum_{i=1}^{n} STE_i (r_{1,i}\, N + r_{3,i} d_r) \right] (1 + mL)$$

5.2.3 Support and Test Equipment in the Level I Models. The TDM uses a simplified approach to support and test equipment costing which buys system level STE for fault isolation, removal and replacement of failed LRA's and common repair STE if any of the LRA's are coded local repair ($n_1 > 0$):

$$5.12)\quad STE = [STE_{sys} + SGM(n_1) STE_{rpr}]\,(1 + mL)N,$$

where $STE_{rpr}$ can be considered to be the cost of, for example, a test computer, which must be purchased if any LRA is coded local repair. The additional cost of LRA specific STE is not included. The cost of STE at the contractor operator depot is included in COD, defined in the next section.

Support and test equipment for each LRA is computed in the LRAM as:

$$5.13)\quad STE_i = \left[STES_i + r_{1,i}STER_i\right](1 + mL)\cdot N,$$

where $STES_i$ is the addition to system level STE caused by the $i^{th}$ LRA and $STER_i$ is the cost of STE specific to the repair of the $i^{th}$ LRA. Support and test equipment in the SAM is:

$$5.14)\quad STE = STE_{sys}(1 + mL)N + \sum_{i=1}^{n} R_i STE_i.$$

## 5.3 Repair Costs

5.3.1 Repair Costs In The Level III And II Models. While the major costs of repair are accounted for by labor and several items necessary to create a repair capability, there are also material costs of repair. These costs are of two radically different types. First, if the repair is accomplished either on board or at a military depot, the material cost is limited to the parts which must be replaced, RP, where RP is the product of the average price of a repair part and the average number of repair parts required per repair. In the case of a contractor operated depot, the situation is quite different.

The cost of a single repair at a contractor operated depot (COD) may be as much as $500.00, compared to the $5.00 to $50.00, which would be typical values for RP. The reason for this great difference is that the price includes everything associated with repair: labor, training, S&TE, maintenance of parts stockage and even stores of ready-for-issue LRA's, transportation, insurance, technical data and so on. Virtually every equation in the model, other than those dealing with spare stockage, is sensitive to the distinction between a military and civilian depot. Usually, cost element is only accounted for if an LRA is coded military depot repair. Thus, while the costs accounted for in the repair category are very large for contractor operator depots compared to military depots, savings accrue elsewhere in the model. The equation for both level III and Level II Models is:

$$5.15)\quad C_7 = L \cdot N \sum_{i=1}^{n} \lambda_i \left[ (r_{1,i} + r_{3,i}) RP + r_{2,i}(1 - r_{3,i}) COD \right]$$

5.3.2 Repair Costs in the Level I Models. Repair costs in the TDM are computed as follows: The system fails $\lambda$ times per year; $n_1/n$ of the time the LRA is repaired locally, with a repair material cost of RP; $n_2/n$ of the time it is repaired at a contractor operated depot at a cost of COD:

$$5.16)\quad C_7 = \lambda \cdot L \cdot N(r_1 RP + r_2 COD),$$

where $r_1 = n_1/n$, the fraction of LRA types coded local repair and $r_2$ is similarly defined for depot repair.

In the LRAM it is assumed the local repair of the $i^{th}$ LRA is accomplished by removing and replacing one of its components with a repair material cost of $UC_i/C_i$, where $C_i$ is the number of components in the $i^{th}$ LRA:

$$5.17)\quad C_{7,i} = \lambda_i \cdot L \cdot N(r_{1,i} UC_i/C_i + r_{2,i} COD)$$

System repair costs are the sum of the repair costs for each LRA in the system:

$$5.18)\quad C_7 = \sum_{i=1}^{n} R_i C_{7,i}$$

## 5.4 Inventory Entry And Management Costs

**5.4.1 Inventory Entry And Management Costs In The Level III And II Models.** Every time a new part or assembly is entered into Navy stockage inventories a number of cost bearing consequences ensue. Data files must be created or augmented, code numbers assigned, inventory descriptions developed and distributed and so on. These costs are all lumped together in a price called inventory entry cost, or IEC, which applies to every new part included in a new design. While occasionally a new system will introduce new piece parts, the bulk of new entries arise from the system and its stocked LRA's. Thus, IEC costs are modeled as nIEC.

The management of inventory is related to the number of item types stocked and the number of locations at which stockage is maintained. Thus, the recurring management cost per part type per location is multiplied by a lengthy expression that counts up the number of part-location combinations created by the new system. The equation is:

$$5.19)\quad C_8 = nIEC + IMC \cdot L\left[N\sum_{i=1}^{n} SGM(S_i) + d\sum_{i=1}^{n} SGM(B_i) + \sum_{i=1}^{n} PP_i(r_{1,i}N + r_{3,i}d_r)\right]$$

The first expression inside brackets counts up the number of LRA types for which some shipboard stock has been acquired and multiplies by the number of ships. The next term performs the same function for depot stock. The last term sums the product of the number of repair parts

in each LRA ($PP_i$) and the number of ships or repair depots at which that LRA is to be repaired.

Simplifications for the Level II Model are straightforward and include:

- use of an average number of parts per LRA,
- assigning management cost even at locations where no spares are stocked (considering the high confidence levels which are assigned to LRA's, in most cases we will have stockage at every location; thus this is a reasonable simplification).

The resulting equation is:

$$5.20)\quad C_8 = n \cdot IEC + IMC \cdot L \cdot \sum_{i=1}^{n} \left[N + r_{3,i} d_r + PP/n(r_{1,i} N + r_{3,i} d_r)\right]$$

where in this equation PP refers to the number of new piece parts in the entire system.

5.4.2 Inventory Entry and Management Costs in the Level I Models.

Item entry costs are paid for every LRA type in the system; in addition, item management costs are paid for all LRA's and for all new components of LRA's coded either local or depot repair. The TDM formulation is:

$$5.21)\quad C_8 = IECn + IMC \cdot L \cdot n \left[1 + PP(r_1 + r_2)\right]$$

Item entry and management for each LRA is given in the LRAM as:

$$5.22)\quad C_{8,i} = (IEC + IMC \cdot L)\left[1 + PP_i(r_{1,i} + r_{2,i})\right]$$

System item entry and management costs in the SAM is given by:

$$5.23)\quad C_8 = \sum_{i=1}^{n} R_i C_{8,i} \; .$$

## 5.5 Technical Data Costs

### 5.5.1 Technical Data Costs in the Level III and II Models.

Technical data costs are estimated on the basis of the number of pages required for each item of equipment for different repair options. As with software costs, duplication and shipping costs are small and thus included in the Level III Model, but excluded from the Level II Model. With that exception the equations are the same:

$$5.24)\quad C_9 = (1 + \bar{m}L)\left[P + P_f + P_r \cdot \sum_{i=1}^{n} (r_{1,i} + r_{3,i}) \left[TDP + dup_d(r_{1,i}N + r_{3,i}d_r)\right]\right]$$

$$5.25)\quad C_9 = (TDP + ADC \cdot L)\left[P + P_f + P_r \cdot \sum_{i=1}^{n} (r_{1,i} + r_{3,i})\right]$$

The elements of the common part of both equations are P, the number of pages of tehcnical data needed to provide a system overview; $P_f$, the number ot document system repair, that is fault isolation removal and replacement of LDA's, and; $P_r$, the number required to document repair procedures for each LRA. Each of these is weighted by the number of applicable elements. Notice that no documentation costs for LRA repair are accumulated for a contractor operated depot. The value TDP is the cost per page of developing technical data while $dup_d$ is the per page cost of printing. Recurring costs are computed in the Level III Model

using an annual maintenance factor, $\bar{m}$, defined in the same fashion as the value, m, in equation 5.10. The Level II Model uses an annual per page data maintenance cost, ADC.

The reader should also note the dimensions of the page definitions. Technical manuals are frequently used as training manuals and this circumstance best fits the formulation of this equation. If there are distinct training materials these must be incorporated into either P, $P_f$ and $P_r$ of the equations 5.24 and 5.25 or in the training cost per man values developed in Chapter 3. In addition, large quantities of technical data (for example, category E drawings) represent significant costs that must be accounted for one way or another. The most common method is to amortize those costs in the unit production cost estimate dealt with in Chapter 4. Our preference, however, is to include such data costs in the technical data equations where they need not be arbitrarily amortized.

5.5.2 Technical Data Costs in the Level I Models. Documentation in the TDM is the sum of system repair documentation and documentation for the repair of individual LRA's coded local repair.

$$5.26)\quad C_9 = P_s + r_1 n P_r$$

The variable $P_s$ includes $P + P_f$, above. Documentation costs for depot repair items are included in COD.

Documentation costs in the AMS follow the same pattern as STE, RPR and IEMC:

5.27) $C_{9,i} = P_{f,i} + r_{1,i}P_{r,i}$

5.28) $C_9 = P + \sum_{i=1}^{n} R_i C_{9,i}$

## 5.6 Second Destination Transportation Costs

5.6.1 Transportation Costs In The Level III And II Models. Transportation costs can be modeled in the most complex fashion of any element in operating and support cost other than spares and S&TE. However, whereas the payoff to detail in the latter is great, it is all but insignificant for most systems in the transportation account. The greater the detail, the more information required. The kind of detail, however, is the sort that does not tend to be settled until long after a design has been set in concrete. Since all the models in this paper are meant for different parts of the design process, such detail is unlikely to be available and, as a result, not assumed in the equations.

The cost of transportation is modeled by counting up the total pound-miles of cargo cost the system is expected to generate during its life. This amounts to cataloging the total demand by destination. For LRA's it is reasonable to expect that rough estimates of packaged weight, $W_i$, will become available during the later stages of the design process. Transportation costs only arise when the LRA is coded military depot repair (transportation costs for a civilian operated depot are included in the value, COD). The equation is:

$$5.29)\quad C_{10} = L \cdot 2 \cdot DIS \cdot CC \cdot N \sum_{i=1}^{n} r_{3,i} \lambda_i W_i$$

where DIS is the average distance between repair and supply depots and CC is the cargo cost per pound-mile.

For use in the Level II Model, the equation is altered only to the extent of using an average weight for all LRA's, W , instead of the subscripted variable $W_i$.

5.6.2 Transportation Costs in the Level I Models. Transportation costs occur when failed items are shipped to and from the repair depot. For a civilian operated depot, the only type considered in the Level I Models, this cost is included in the value COD. Therefore, there is no explicit transportation cost calculation in any of the Level I Models.