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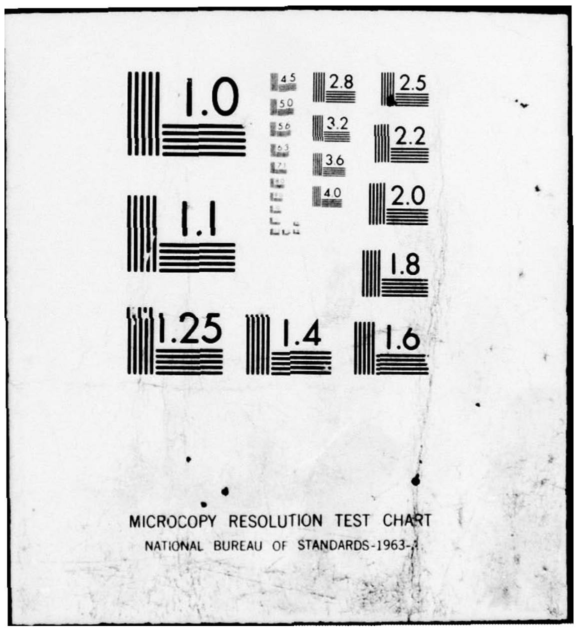
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THESIS

STIMULATED EMISSION BY RELATIVISTIC
ELECTRONS TRAVERSING
A PERIODIC MAGNETIC FIELD

by

Robert Boyce Ellis

June 1979

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Thesis Advisor: F. R. Buskirk

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A historical survey of scientific developments predating the free electron laser (FEL) concept is presented. A general theory of operation of the FEL is discussed. The use of electromagnets to generate a static periodic magnetic field of alternating polarity (as a pump wave source) is proposed. Preliminary measurements with respect to the proposed electromagnet design are analyzed.

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Stimulated Emission by Relativistic Electrons

Traversing a Periodic Magnetic Field

by

Robert Boyce Ellis
Lieutenant, United States Navy
B.S., University of South Carolina, 1971

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requirements for the degree of

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ABSTRACT

A historical survey of scientific developments predating the free electron laser (FEL) concept is presented. A general theory of operation of the FEL is discussed. The use of electromagnets to generate a static periodic magnetic field of alternating polarity (as a pump wave source) is proposed. Preliminary measurements with respect to the proposed electromagnet design are analyzed.

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I. INTRODUCTION

Stimulated emission from intense relativistic electron beams has received a great deal of attention during the last decade. Of particular note are the success at Stanford University in first demonstrating (1975) the feasibility of the free electron laser (FEL) in what is termed the Compton regime and the follow-on success at Columbia University (1977) where the FEL concept was demonstrated in the Raman regime. The basic underlying physical processes for both these devices are the relativistic Doppler shift and stimulated emission of backscattered radiation from an extremely relativistic electron beam. The emitted radiation considered may be upshifted in frequency and is characteristic of the electron beam energy. Hence, devices of the above type based on the complementary effect of these basic phenomena may prove to be a source of high-frequency coherent radiation tunable over a wide range.

II. HISTORICAL SURVEY

A. Einstein (circa 1916) introduced the concept of induced emission and absorption of radiation for a system in equilibrium with electromagnetic radiation by an application of Planck's quantum hypothesis within the framework of Blackbody radiation [Ref. 1]. The salient points of this phenomenon are the phase coherence of stimulated radiation with respect to the stimulating radiation and the equilibrium transition rate for absorption of radiation which is equal to the sum of the transition rate for spontaneous and induced emission of radiation. The extension of this analysis to non-equilibrium conditions has formed the basis for devices which are known as lasers and masers. With respect to the emission of radiation in a scattering process, Kramers and Heisenberg (circa 1925) theorized that the spectrum of electromagnetic radiation scattered from a moving particle would contain other frequencies in addition to frequencies characteristic of the incident radiation [Ref. 2]. The Raman effect [Ref. 3] predicts the appearance of a frequency difference (Stokes or anti-Stokes lines appearing) in scattered spectra which is characteristic of the scattering medium. The frequency shift is attributable to an exchange of energy between the struck system and incident radiation.

In 1933, P. Kapitza and P. Dirac [Ref. 4] proposed an experiment to observe stimulated scattered radiation through the stimulated Compton effect. They considered an electron beam interacting with an intense standing wave light beam (standing waves produced by reflection of a collimated beam back along its incident path). The standing waves being considered as two parallel beams of the same frequency, a photon absorbed from one beam by the electron ensemble would be stimulated to emit by the second beam. The re-emitted photon would be of the same frequency as the stimulating radiation and the electron would suffer a recoil which was to be measured in the lab frame.

Their analysis indicated the scattered electrons would behave according to Bragg's law with a lattice spacing one half the wavelength of the incident light. They showed that with ordinary continuous sources of light the intensity of the photon flux would be insufficient to produce an observable number of refracted electrons.

In 1951, H. Motz [Ref. 5] considered radiation from fast electrons in a beam which passed through a succession of static electric or magnetic fields of alternating polarity. In a decidedly classical device, he later developed a narrow band source of synchrotron radiation by passing an electron beam through an "undulated" magnetic field. Coherent amplification was achieved through harmonic coupling of the spatial period of the transverse magnetic field and a "bunched" beam.

Motz' classical analysis encompasses many of the equations which one encounters in the semi-classical study of stimulated radiation wherein a transverse magnetic field is used to perturb the motion of the electrons. In setting the stage for future development he acknowledged that in the limit of his theoretical assumptions the radiation reaction per electron is no longer negligible with respect to other forces and the effect should then be considered for analysis. Phillips and Ury developed several devices of the type considered by Motz. These devices were called Ubitrons (Undulated beam interaction) [Ref. 6].

In 1959, J. Schneider [Ref. 7] described a cyclotron maser through application of relativistic considerations to the harmonic oscillator solution of the Schrodinger wave equation for an electron moving perpendicular to a magnetic field. In this device relativistic electrons traveling parallel to a longitudinal magnetic field are made to oscillate in helical orbits (hence transverse motion to the magnetic field) under the influence of an alternating current electric field to produce stimulated emission amplified through cyclotron resonance.

By 1963 the development of the laser had come into its own and many "new" ideas related to its use were being explored. R. Milburn [Ref. 8] and F. Arutyunian and V. Tumanian [Ref. 9] in separate papers proposed that the high-energy photon flux produced by lasers incident on a counter-streaming extreme relativistic pulsed electron beam could result in collimated high-energy scattered photons with polarization strongly approaching that of the incident photon beam.

Noting the results of R. Milburn with respect to his analysis of the results of Feenberg and Primakoff [Ref. 10] (Compton scattering as a mechanism of energy degradation of high-energy electrons in intergalactic space) it can be shown (Appendix A) that the possible maximum and minimum scattered photon energies are, respectively,

$$h\nu_2 = 4\gamma^2 h\nu_1 \tag{1}$$

$$h\nu_2 = \gamma mc^2 \tag{2}$$

where subscripts one and two are before and after collision in the laboratory frame respectively and the right-hand side of equation (2) is the electron energy with $\gamma = (1 - v^2/c^2)^{-1/2}$. It was assumed that the scattered photon travels in the same direction as the electron after the collision.

Other scattering experiments using lasers and electron beams were carried out or proposed in the sixties (Refs. 11, 12, and 13]. In 1965, L. Bartell et al. [Ref. 11] performed an experiment which recorded the first observation of stimulated Compton scattering in confirmation of the prediction of Kapitza and Dirac. Shortly thereafter, Eberly [Ref. 12], in an analogous vein, outlined an experiment of the Kapitza and Dirac type. In closing, he noted that other photon electron interactions might compete with observations of the Compton scattering. They are included here as an interesting consideration. He considered three photon (p) interactions,

$$e^- + p \rightarrow e^- + p' + p'$$

$$e^- + p + \gamma \rightarrow e^- + p'$$

noting that these are non-negligible in very intense photon beams but kinematically forbidden for an experiment using non-relativistic electrons. He observed that the most favorable interaction for the Kapitza and Dirac type of experiment was

$$e^- + p + p \rightarrow e^- + p' + p'$$

where one might consider the Bragg refraction law as Kapitza and Dirac did.

In 1968, R. Pantell et al. [Ref. 14] first introduced the concept of stimulated Compton scattering as a laser mechanism. He noted that previous considerations of Compton scattering involved spontaneous emission with large energy exchange (Milburn, Arutyunian and Tumanian) or stimulated emission with no energy exchange (Kapitza and

Dirac, Bartell, Eberly). Most notably, Pantell observed that with a relativistic electron beam the frequency shift is given to a close approximation by equation (1) and it might therefore be possible to construct a high-energy laser based on stimulated Compton scattering. He also noted that it could be tunable through the beam energy parameter gamma.

In 1971, J. Madey, at Stanford, [Ref. 15] addressed this concept. He noted that the high Q cavity for microwave emission proposed by Pantell *et al.* to produce the photon flux could be replaced by a strong direct current magnetic or electric field to obtain virtual photons of large flux (Appendix B). During this same time frame, work was being done at the Naval Research Laboratory, Washington, D.C., toward developing new mechanisms for the production of radio frequency radiation using relativistic electron beams. In 1972, M. Friedman and M. Herndon [Refs. 16 and 17] reported the generation of microwave radiation by an intense relativistic electron beam propagating in a spatially modulated magnetic field. R. Palmer [Ref. 18], in a paper independent of Madey's earlier proposal with respect to use of a magnetic field, outlined a scheme to produce a circularly polarized magnetic field using a bifilar helix.

The bifilar helix produces a magnetic field having the same effect on an electron as circularly polarized electromagnetic radiation. The concept had been considered earlier by Madey in the context of a permanent magnet arrangement. The use of helical windings in radiation devices involving fast electrons had been studied earlier [Ref. 19] and were also investigated with respect to electron cyclotron maser applications [Ref. 20].

In 1973, M. Friedman and M. Herndon [Ref. 21], in a modification to their earlier experiment, reported the generation of infrared radiation which was unexpected. In attempting to understand this phenomenon, they considered the possibility of stimulated Compton scattering as proposed by Pantell et al. In all likelihood and with a generous application of hindsight, the radiation observed was probably the first recorded demonstration of the FEL concept in the Raman regime. This observation has its basis in the reported density of the electron beam which was several kiloamperes. Hence a collective (Raman type) effect was observed vice a Compton (single electron-photon) interaction. In explanation of this occurrence, it is noted that they had earlier reported a device capable of producing intense microwave radiation. The modification to the experiment included the attachment of a cavity downstream. Hence, microwave reflections from the end of this cavity would be counterstreaming toward the electron beam and this could conceivably account for the infrared radiation observed. The conclusions reached here are also presented in a similar vein by V. L. Granatstein et al. [Ref. 22] with comments on other possibilities.

During the period 1973-1975, several papers were published relative to the subject of beam scattering which are of interest with respect to theoretical limitations to gain in Compton scattering [Ref. 23], electron beam stability [Ref. 24], scattering in the presence of an electromagnetic wave [Ref. 25], transverse beam energy and emission of microwave radiation [Refs. 26, 27 and 28] and the momentum modulation of an electron beam through scattering in a medium vice vacuum environment [Ref. 29].

In 1975, P. Sprangle et al. [Ref. 30] addressed the extension of the analysis of Pantell et al. to the Raman regime. The experiment outlined proposed the use of a microwave "pump" (photon-flux source) counterstreaming to a dense relativistic electron beam to produce sub-millimeter high-power radiation.

Stanford's success was published in 1976 [Ref. 31]. In the experiment an electron beam (70 milliamperes) was passed through a circular polarized magnetic field. The generation of the magnetic field is noteworthy. A superconducting bifilar helical wire was used to sustain the high current used to produce a kilogauss magnetic field [Ref. 32]. The first experiment was operated in an amplifier mode using a CO_2 laser. The following year, D. Deacon et al. [Ref. 33] reported the operation of the Stanford FEL in an oscillator mode.

Also in 1977, P. Efthimion and S. Schlesinger [Ref. 34] reported success at Columbia in producing stimulated Raman scattering. The device used involved a drift tube (hence cut-off frequency consideration was required) with a coaxial undulator. The undulator (alternating brass and iron rings) was used to create a rippled magnetic field through interaction with an axial magnetic guide field. The annular electron beam then encountered an electromagnetic pump wave in passing down the length of the cavity.

Following these successes, numerous theoretical considerations have been independently addressed. W. Colson [Ref. 35] analyzed the stimulated Compton effect and developed equations of motion for a single electron using the Lorentz force equations. F. Hopf et al. [Refs. 36 and 37] considered the electron dynamics with consideration given the electron distribution through the coupled Maxwell-Boltzman equation. N. Kroll and W. McMullin [Ref. 38], in a comprehensive manner, analyzed stimulated scattering and concluded that the growth of backscattered radiation is due to the bunching of electrons in both the Raman and Compton regime. Their analysis may be considered as the fulfillment of the classical analysis begun by H. Motz. Other contributions to the theory by these authors and others have been put forth which place the FEL concept description in the classical analysis arena vice the quantum-mechanical description normally considered in laser analysis.

III. GENERAL THEORY OF OPERATION

One may approach an understanding of the FEL concept by considering the space-time form of the covariant force equation [Ref. 39] for a particle moving in external fields which may be rewritten (Appendix C) as given by Colson [Ref. 40] in the following form (cgs units).

$$\frac{d\gamma\bar{\beta}}{dt} = \frac{e}{mc}(\bar{E} + \bar{\beta} \times \bar{B}) \tag{3}$$

$$\frac{d\gamma}{dt} = \frac{e}{mc}(\bar{\beta} \cdot \bar{E}) \tag{4}$$

In the above equations $\bar{\beta}c$, e, and m are, respectively, the electron velocity, charge and mass; t is the time; $\bar{E}$ and $\bar{B}$ are the total fields in the laboratory frame influencing the electron and γ is as given before.

A qualitative description is considered here which yields some quantitative results. Following the initial proposal of Pantell et al. [Ref. 14], the pump wave may be considered as microwave radiation directed oppositely and collinearly to the electron traveling in the +Z direction. Further, the pump wave may be considered as linearly polarized plane electromagnetic waves.

With respect to the last assumption, equation (3), using the form in Appendix C, may be used to describe the momentum perturbation due to the influence of the pump wave acting alone.

$$\frac{d\delta\overline{P}(z,t)}{dt} = e\ \overline{E}_1\ (1 + \beta) \tag{5}$$

The magnitudes of the components of the plane electromagnetic linearly polarized pump wave have been equated ($E_{10} = B_{10}$) and the field above is given as follows.

$$\overline{E}_1 = -E_{10} \cos\ (k_1 Z + \omega_1 t)\ \hat{y} \tag{6}$$

This result, equation (5), is easily obtained by considering the velocity vector of the electron and the components of electromagnetic wave.

Radiation is then emitted by the electrons and a transverse electron velocity component is now seen to exist which remains in phase with the electric field component of the backscattered radiation since the radiation pulse travels at speed c and the electron travels at $\beta c \simeq c$. Thus the electric field of a wave traveling with the electron where the polarization of scattered radiation is determined by the polarization of the incident wave may be given as follows.

$$\overline{E}_2 = E_{20} \cos\ (\omega_2 t - k_2 Z)\ \hat{y} \tag{7}$$

The pump wave may be considered as determining the trajectory of the electron through the argument that the electric and magnetic forces generated by the radiation wave approximately cancel each other. The radiation fields traveling with the electron would of themselves have no significant effect on the electron path or energy.

Noting that the pump wave determines the electron trajectory and neglecting such effects as electron recoil, the momentum perturbation equation after one complete oscillation is constrained to satisfy the following equation.

$$k_1 Z + \omega_1 t_1 = \pm 2\pi \tag{8}$$

where

$$t_1 = \frac{Z}{\beta c} \tag{9}$$

from the electron velocity. The distance Z may be defined as

$$Z = \lambda_e = \frac{2\pi}{K_e} \tag{10}$$

and

$$\omega_1 = v_1 k_1 \tag{11}$$

such that

$$k_1 \left(1 + \frac{v_1}{\beta c}\right) = \pm k = k_e \tag{12}$$

where the negative sign has been dropped to agree with the physical description of the direction of propagation of the pump wave, but k is seen as a propagation constant associated with the electron for the defined radiation wavelength. Similarly, as the electron advances in phase with the radiation wave,

$$k_2 Z - \omega_2 t = \pm\, 2\pi \quad . \tag{13}$$

Then after some manipulation using the above relationships,

$$k_2\ (1 - \frac{v_2}{\beta c}) = \pm\ k = -k_e \ . \tag{14}$$

The plus sign has been dropped to agree with the physical description of the direction of propagation of the radiation and k is as described above. Then by equating these two relations, equations (12) and (14), the following relationship is obtained.

$$k_1\ (1 + \frac{v_1}{\beta c}) = k_2\ (\frac{v_2}{\beta c} - 1) \tag{15}$$

or

$$\nu_1\ (1 + \frac{v_1}{\beta c}) = \nu_2\ (\frac{v_2}{\beta c} - 1) \tag{16}$$

Noting that v_1 and v_2 in these latter equations are, respectively, the phase velocity of the pump wave and the scattered radiation which are equal to c, an expression for the frequency shift of scattered radiation by a counterstreaming pump wave is obtained.

$$\nu_2 = \nu_1 (1 + \beta) / (1 - \beta) \tag{17}$$

or using

$$\gamma = (1 - \beta)^{-1/2} \tag{18}$$

$$\nu_2 = \nu_1 \gamma^2 (1 + \beta)^2 \tag{17a}$$

for $\beta \simeq 1$

$$\nu_2 \simeq 4\gamma^2 \nu_1 \quad . \tag{17b}$$

Similarly for the wavelength

$$\lambda_2 = \lambda_1 / (1 + \beta)^2 \gamma^2 \tag{19}$$

and for $\beta \simeq 1$

$$\lambda_2 \simeq \lambda_1 / 4\gamma^2 \quad . \tag{19a}$$

The frequency shift of scattered radiation by a static periodic magnetic field is then easily arrived at by noting that a spatial wavelength (λ_o) is defined for this zero frequency pump wave. Then considering equation (12) where the phase velocity of the pump wave is zero, one obtains

$$k_o = k_2(1 - \beta)/\beta \qquad (20)$$

where v_2, the phase velocity of the scattered radiation is equal to c and

$$k_1 = k_o = 2\pi/\lambda_o \qquad (21)$$

such that the frequency shift is

$$\omega_2 = k_o c\beta/(1 - \beta) \qquad (22)$$

and

$$\lambda_2 = \lambda_o/\beta(1 + \beta)\gamma^2 \qquad (22a)$$

or for $\beta \simeq 1$.

$$\omega_2 \simeq 2k_o c\gamma^2 \qquad (23)$$

and

$$\lambda_2 \simeq \lambda_o / 2\gamma^2 \quad . \tag{23a}$$

It is noted that the wavelength shift for a linearly polarized plane electromagnetic pump wave with $\beta \simeq 1$ (equation 19a) contains a factor of four whereas equation (23a) above contains a factor of two for the static periodic magnetic pump wave.

The utility of the static periodic magnetic pump was first considered by J. Madey [Ref. 15] as noted earlier. The sample calculation in Appendix B serves to illustrate this point. With this in mind, the pump wave considered for the remainder of this paper will be limited to the static periodic magnet field case.

Equation (23) may be looked upon as a frequency condition required for energy exchange between the pump field and the radiation field with the electron performing in a role similar to that of a catalyst. Then equation (4) for the model now under consideration is seen as describing an intermediate step involving the electron, i.e.,

$$\frac{d\gamma}{dt} = \frac{e}{mc} (\overline{\beta} \cdot E_r) \tag{24}$$

where the subscript denotes the radiation electric field. Colson [Ref. 40] observed that the combined effect of the radiation field and the pump magnetic field guide the electron along a trajectory such that the radiation electric field does work proportional to the product of the transverse component of electron velocity and the radiation electric field. Examination of the arguments and assumptions which result in equation (19) for the frequency shift of the backscattered radiation reveals that a resonant condition has been obtained. By comparison of the frequency for classical Compton backscattered radiation (Appendix A) for the initial conditions described, the resonant frequency of the first case is observed to be valid, i.e., the forced frequency is equal to the natural frequency of the system; hence a resonance is defined. By inference then it is concluded that

$$\omega_2 = k_0 c\gamma^2(1 + \beta)\beta \tag{25}$$

is also descriptive of a valid resonance.

For the resonant condition to exist over many of the defined radiation wavelengths of the electron the work done must be consistent with respect to direction. From this it is concluded [Ref. 40] that the phase relationship of the transverse velocity component relative to the radiation electric field is maintained. Then by examination of equation (24), the time rate of change of energy, one may conclude that if the electron loses energy the radiation field grows; i.e., stimulated emission occurs [Ref. 40].

Madey discusses [Ref. 15] a quantitative estimate of the effect of pump field strength on the emitted radiation using a classical argument. With reference to equation (13) and the foregoing it may be argued that the choice of the minus sign in equation (14) is predicated by the description of the phase relationship between the electron in traveling one radiation length and the emitted radiation; i.e., to paraphrase Madey [Ref. 15], the electron, after one magnet period, lags behind the radiation emitted at a corresponding point in the preceding period by a distance which corresponds to the wavelength of the radiation. Assuming an abrupt transverse magnetic field and relating the electron's path to the magnet spatial wavelength, the change in the wavelength of emitted radiation due to a strong magnetic field may be calculated subject to the phase constraint of equation (13) (Appendix D).

The shift of emitted radiation wavelength for a spatially periodic transverse magnetic pump field is then given below.

$$\lambda_2 = \frac{\lambda_o}{2\gamma^2}\left[\frac{2}{\beta(1+\beta)} + \frac{1}{3}\left(\frac{\lambda_o}{4}\right)^2\left(\frac{eB}{mc^2}\right)^2\frac{1}{\beta^3}\right] \qquad (26)$$

Madey's result [Ref. 15] for $\beta \simeq 1$ then follows.

In analogy to other lasers the net power generated within a unit volume is proportional to the transition rates between stimulated emission and absorption. For conventional laser theory the intensity of radiation is seen to vary exponentially with the length of the medium by a factor termed the gain constant. The gain constant is the product of the population inversion of excited states, the induced transition rate and lineshape factor (frequency bandwidth) of the transition. The net gain for the FEL concept may be defined as the difference between the gain due to stimulated emission and the gain (loss) due to absorption. At this point the analogy between the FEL and conventional lasers diverges. The novel idea of a population inversion is not applicable.

The electron may be described as occupying some initial plane wave state with transitions to some final plane wave state. In its given initial state, the electron may absorb radiation or undergo stimulated emission. However, it is noted [Ref. 43] that a kinematic correction separates these two equally strong resonances slightly in frequency so that the emission process can be made dominant with respect to the absorption process. This condition is outlined in the following discussion.

A logical progression to the description of the electron states from the classical description considered earlier may be realized by rewriting the covariant Lorentz Force equation in terms of the Hamiltonian for a relativistic charged particle followed by a transformation to the Dirac Hamiltonian. Scattering of radiation may then be treated as a perturbation with the transition rate given by the Fermi "Golden Rule" [Ref. 41].

$$R = \frac{2\pi}{\hbar} \;|\langle f \,|H'|\, i\rangle|^2 \,\rho(E_f) \qquad (27)$$

where $\rho(E_f)$ is the density of final states and H' is the interaction Hamiltonian [Ref. 22] defined as follows.

$$H' = e\bar{\alpha} \cdot \bar{A}(\bar{r},t) \qquad (28)$$

where $\bar{A}(\bar{r},t)$ is the radiation field vector potential first appearing in the classical Hamiltonian description alluded to above and $\bar{\alpha}$ is a matrix resulting from Dirac's linearization of the classical Hamiltonian. The quantum theory of radiation admits the description of energy eigenstates, $|n\rangle$, called photon number states and introduces the annihilation (absorption) and creation (emission) operators, a and a*, respectively; these operators change a state with n photons to one with n-1 or n+1 photons respectively [Ref. 42] and thus describe absorption and emission of radiation, respectively. A complete quantum-electrodynamics treatment of scattering theory is beyond the scope of this paper. The foregoing is presented for completeness with respect to the following discussion of gain.

Madey *et al*. [Ref. 43] notes that the possibility of stimulated emission and absorption is due to the presence of the creation (a^*) and annihilation (a) operators which arise from the vector potential (perturbation energy) in the interaction Hamiltonian. By definition [Ref. 42],

$$a|n\rangle = \sqrt{n}\,|n - 1\rangle \tag{29}$$

and

$$a^*|n\rangle = \sqrt{n + 1}\,|n + 1\rangle \tag{30}$$

which are seen [Ref. 43] to introduce a multiplicative factor $\sqrt{n + 1}$ ($\sqrt{n}$) in the matrix element and $n + 1$ ($n - 1$) in the transition rate for stimulated emission (absorption) where n is the number of photons. Madey *et al*. [Ref. 43] then note that for $n \gg 1$ the transition rates for absorption and emission become nearly equal. This analysis does not of itself admit to a gain mechanism.

That gain is possible at all is a consequence of a small frequency shift between the emission and absorption lines arising from the kinematics for absorption and emission of photons in the stimulated processes considered. The existence of intermediate states in the transition probability is duscussed by Heitler [Ref. 44] for free electron scattering and Madey *et al*. [Ref. 43] note that it can be shown that absorption takes place at a slightly higher frequency than stimulated emission. Heitler [Ref. 44] notes that for the intermediate states the momentum is conserved (but not energy). Hence it is seen here that a mechanism exists which may account for the shift of the absorption line up in frequency relative to the emission line center.

The transition rate equation (Fermi "Golden Rule") as given by V. Sukhatme and P. Wolff [Ref. 23] reflects this consideration.

$$R = \frac{2\pi}{\hbar} \left| \sum_i \frac{\langle F | H' | i \rangle \langle i | H' | 0 \rangle}{E_o - E_i} \right|^2 \delta(E_F - E_o) \quad (31)$$

where $|i\rangle$ represents some intermediate state and $|0\rangle$ and $\langle F|$ are the initial and final states, respectively. The frequency shift between absorption and emission lines is stated quantitatively by Madey et al. [Ref. 43] as a ratio of the radiation energy and electron energy.

$$\text{Fractional Frequency Shift} = \frac{h\nu}{\gamma mc^2} \quad (32)$$

Recalling equation (26) for the shift of the radiation wavelength, it may be rewritten to include the factor for the shift between emission and absorption.

$$\lambda_2 = \frac{\lambda_o}{2\gamma^2}\left[\frac{2}{\beta(1+\beta)} + \frac{1}{3}\left(\frac{\lambda_o}{4}\right)^2 \left(\frac{eB}{mc^2}\right)^2 \frac{1}{\beta^3}\right]\left[1 \pm \frac{h\nu_2}{\gamma mc^2}\right] \qquad (33)$$

where the plus and minus is applicable for emission and absorption, respectively.

Madey _et_ _al_. [Ref. 43] derive a net gain formula in decibel per meter based on the assumption that the electron energy distribution and power spectral density of the equivalent plane wave states of the electron are Gaussian (Appendix E). Noting (a) that the emission and absorption lineshapes are the same through like dependence upon the electron beam energy linewidth ($\Delta\gamma$) and line broadening ($\Delta\nu$) of the equivalent plane wave states of the electron and (b) the small shift between the lineshapes as given by equation (32), it is observed [Ref. 43] that the maximum value for the net gain occurs on the low-frequency side of the emission lineshape and is given as

$$G_m = \frac{36.8\left(\frac{r_o^2}{\hbar c}\right)(\lambda_2\lambda_o B)^2 \rho_e \left(\frac{h\nu_2}{\gamma mc^2}\right) FF}{\left[\left(\frac{\Delta\gamma}{\gamma}\right)^2 + \left(\frac{\Delta\nu}{2\nu}\right)^2\right]} \qquad (34)$$

where (cgs units)

ρ_e = electron density

r_o = classical electron radius

λ_o = magnet spatial period

λ_2 = radiation wavelength

ν_2 = radiation frequency

$\frac{\Delta\gamma}{\gamma}$ = electron beam homogeniety

$\frac{\Delta\nu}{\nu}$ = spectral purity of equivalent plane wave states of the electron

and FF is a filling factor (ratio of beam cross-section to radiation cross-section) required to define the coupling of the electron beam to the stimulating radiation.

The derivation of the gain equation is based on the description of the interaction as a perturbation. Perturbation theory is in general applicable to weak field interactions. The gain equation is observed to be dependent on the square of the magnetic field amplitude which implies that increased gain is realized for stronger fields. The validity of the expression for gain is seen [Ref. 43] to hold true for large fields, however, by considering that the magnitude of the displacement of the absorption line relative to the emission line governs the useful gain. The controlling factor becomes the beam energy variable (γ) which operates quadratically in the wavelength shift equation to maintain the emission and absorption lineshape separation but operates linearly in the gain equation. The amplitude of the magnetic field is quadratic in both instances.

Madey et al. [Ref. 43] note that Planck's constant may be canceled out of the maximum net gain equation and that the absence of the constant then raises the question of a classical interpretation of the interaction to arrive at some net gain. This argument further buttresses the validity discussion above concerning the use of perturbation theory to describe the gain.

With this early (1971) speculation concerning a possible classical interpretation of the FEL concept, a number of authors have put forth both semi-classical (some quantum considerations) and classical derivations. N. Kroll and W. McMullin [Ref. 38] have developed a classical description based on a circularly polarized pump wave. Their analysis encompasses the interaction of individual electrons, the Compton regime, and plasma oscillations which is seen to include both the middleground between the Compton and Raman regimes, and the Raman regime. They note that the growth rate for backscattered radiation is due to longitudinal bunching of the electrons. The factor which delineates the different regimes is then the electron beam density. For a dense electron beam, plasma oscillations (collective response to charge fluctuations) are dominant and this is the Raman regime. For a less dense beam the interaction of individual electrons comes to the fore resulting in the Compton regime. As noted earlier the electrons interacting with the total electromagnetic field acquire a small oscillation velocity transverse to the beam direction due to the pump wave component. Then the velocity, through the Lorentz force equations, interacts with the electromagnetic field to provide a mechanism which couples the electrons, pump wave, and scattered radiation to provide growth of the backscattered radiation.

The collision-less form of the relativistic Maxwell-Boltzman (Vlasov) equation is seen as providing an approach capable of encompassing high-frequency oscillations (plasma) and low-frequency oscillations (e.g., pump wave effect in the Compton regime). This analysis considers, in the formulation, the effects of charge separation and displacement current. Kroll and McMullin's [Ref. 38] approach to the analysis is outlined here to illustrate the classical approach to interpretation of the FEL concept.

They note [Ref. 38] that the distribution function of momentum, position and time of the electrons satisfies the one-dimensional Vlasov equation as follows.

$$\frac{\partial f}{\partial t}(p,z,t) + \overline{V}(z,t) \cdot \hat{z}\,\frac{\partial f}{\partial z}(p,z,t) + e\,[\overline{E}(z,t) + \overline{V}(z,t) \times \overline{B}(z,t) - \nabla\phi(z,t)] \cdot \hat{z}\,\frac{\partial f}{\partial p}(p,z,t) = 0 \qquad (35)$$

where $\phi(z,t)$ satisfies Poisson's equation which relates the charge distribution to the potential field.

$$\nabla_z{}^2\phi = 4\pi ne - 4\pi ne \int dpf(p,z,t) \qquad (36)$$

The velocity given above satisfies the relativistic Lorentz force equation which is arrived at by combining equations (3) and (4) (note $\bar{V} = \bar{\beta}$) and is then as given by Kroll and McMullin [Ref. 38]:

$$\frac{d\bar{V}}{dt} = \frac{e}{\gamma mc} [\bar{E} + \bar{V} \times \bar{B} - \bar{V}(\bar{V} \cdot \bar{E})] \tag{37}$$

The above equations are solved iteratively by expanding the velocity, distribution and potential in powers of their amplitude. After arduous manipulation, a dispersion relationship is obtained for the emitted field. Their analysis of the dispersion relationship leads to gain equations through consideration of limiting cases characteristic of each regime. The limiting cases summarize salient characteristics which have been noted earlier.

The electron density is low for Compton scattering (on the order of milliamperes when expressed in terms of the total current). Hence the interaction between the bunched electrons is not important and the single-interaction analysis noted earlier will suffice. For the low-density and high-density (Raman) limit, the interaction between bunched electrons takes on significance. The beam density for the Raman effect is on the order of kiloamperes in terms of the total current.

In closing, it is noted that the analysis of N. Kroll and McMullin is based entirely on a classical interpretation of a beam interaction with a total electromagnetic wave which encompasses the pump wave and the scattered radiation wave.

The results of this interaction are directly related to the electron density of the beam. Thus the characterization of the FEL concept as either a Compton or Raman phenomenon in the classical definition of these phenomena may not be truly descriptive of the FEL since a single unified analysis has been developed.

IV. PROPOSED EXPERIMENT

Operation of devices based on the FEL concept have been reported by other groups [Refs. 45 and 46] following the initial successes described earlier. To date no device has been constructed based on an early proposal by J. Madey [Ref. 15] in which the operation of an FEL was proposed using permanent magnets to generate the required pump wave.

The generation of a static periodic transverse magnetic pump "wave" through the use of electromagnets is considered. Two electromagnets are envisioned at present. Each electromagnet is designed with a wire-wound base plate from which numerous pole pieces may be extended with each pole piece forming what may be considered a "C" magnet. By interlocking the pole pieces (figure 1) of these two electromagnets and supplying current to one coil oppositely directed to the current in the other coil, a transverse magnetic field of alternating polarity is generated with respect to axial motion (parallel to the plane of the base plates) down the line of pole faces. To investigate the feasibility of such a device for use in conjunction with the Naval Postgraduate School's LINAC facility, two prototype electromagnets with three pole pieces each have been constructed.

V. THEORETICAL CONSIDERATIONS FOR MAGNET DESIGN

The prototype electromagnets were constructed without benefit of a detailed analysis to determine the characteristics of the fields generated. A rudimentary analysis based on the extension of the first approximation to the magnetic field in the gap of a "C" magnet was of merit to gain insight for design purposes. The following well known equations are applicable [Ref. 47] in any case.

$$\nabla \cdot \bar{B} = 0 \qquad \Rightarrow \qquad \oint_S \bar{B} \cdot \bar{n} dS = 0 \tag{38}$$

$$\nabla \times \bar{H} = \frac{\bar{I}}{\varepsilon_o c^2} \qquad \Rightarrow \qquad \oint \bar{H} \cdot d\bar{\ell} = \frac{NI}{\varepsilon_o c^2} \tag{39}$$

An order of magnitude magnetic induction field value across each pole face for each electromagnet may be easily calculated by neglecting edge effects, air gaps in the construction of adjoining pieces and assuming that the magnetic induction flux through any cross-section is constant with the proviso that the distribution of flux from the base plate is equally distributed to each pole piece.

Consideration of these equations in a qualitative vein serves to reinforce considerations for small air gaps between pole faces, current requirements, number of wire turns and field losses between the adjacent poles for the interlocked electromagnet apparatus.

With respect to the general theory of the FEL the wavelength shift in a device such as this is given by equation (33) which reduces to the approximation,

$$\lambda_2 \approx \lambda_o/\beta(1 + \beta)\gamma^2 \tag{40}$$

where λ_o is the spatial wavelength of the interlocked electromagnets and βc is the electron velocity. The gain can be calculated from equation (34).

VI. EXPERIMENTAL PROCEDURE

The prototype electromagnets are constructed from a single piece of steel plate which was readily available. The availability of inexpensive materials is considered of paramount importance, i.e., the production of a stable magnetic field without the use of expensive superconducting materials is desirable. No foreknowledge of the magnetic qualities or past history with respect to magnetization of the steel plate was known at the time of selection.

Each magnet (figure 1) consists of a base plate with six "L" shaped pole pieces attached. The base plate of each magnet has one hundred turns of enamel-coated number eight copper wire. Heat generation by the coils is not considered a significant factor in the design of the prototypes, but would limit the field since saturation of the iron is not approached.

The fields produced across the pole faces are measured by a Hall Effect probe. Current to the coils is supplied by a direct current power supply. An ammeter in series with the magnet(s) is used to monitor the current. The Hall Effect probe is mounted on a stand to ensure that the vertical position of the probe within the pole faces is constant. Horizontal positioning of the probe within the pole faces for field measurements is done by hand. Measurements are taken relative to the intersecting centerlines of the pole face at positions halfway across the air gap.

Fields measured by the probe compare favorably (figure 2) with those measured by a rotating coil Gaussmeter in calibration tests using an electromagnet manufactured by Atomic Laboratories, Inc. The sensing element of the probe is sealed in an opaque material. The centroid, location of strongest field detection, of the Hall Effect element was located to within one sixteenth of one inch in the probe by using pole face pieces which are beveled at forty-five degrees to produce field lines directly across a gap of one eighth of an inch. This air gap for calibration purposes was predicated by the probe thickness.

VII. ELECTROMAGNET ANALYSIS

Following construction each electromagnet was tested with respect to field generation as a function of applied current. The results are plotted in figure 3 and figure 4 for each electromagnetic designated as Coil A and Coil B, respectively. Permanent magnetism on the order of forty Gauss measured in the gap was induced in each electromagnet.

Hysteresis typical of the material used to construct the prototype electromagnets is depicted in figure 5. The magnetic induction field strength is plotted for increasing and decreasing values of the current for Coil A. Figure 5 by comparison with figure 3 also graphically indicates the effect of a reduced air gap on the magnetic induction field strength between the pole faces. The decreased air gap for the fields measured in figure 5 was obtained by adding pole face pieces (same material) that are of the dimensions of the pole faces and one eighth of an inch thick. The air gap was reduced from three fourths of one inch to one half of one inch.

The variation of field strength with respect to the direction parallel to the plane of the base plate is shown in figure 6 and figure 7. The centerline pole piece generates a slightly higher field value in each case. Note that these figures have two different currents applied. For figure 7, Coil B was operated at ten amperes to highlight the disparity between the fields generated for the end pole pieces. This effect for Coil B was not readily apparent when the measurements were conducted at eight amperes.

A graph of typical variation of magnetic induction field strength across a pole face in the direction normal to the plane of the base plate is depicted in figure 8. The heavy black lines on figure 8 indicate the outer edges of the pole piece face with respect to the centerline. Note that the distribution fall-off is more abrupt for higher field values but shifted further out from the centerline of the pole face. Note also that for the rectangular face plate design used, the distribution is shifted slightly to the left with respect to the centerline. The base plate coils and arm of the pole piece are physically located to the right for the graph of figure 8.

The last measurements taken are those of the interlocked pole pieces of the coupled electromagnets. The data for these measurements are graphically displayed in figure 9. The curve of figure 9 depicts the alternating polarity of the magnetic field induction in the direction parallel to the base plates along the centerlines of the pole faces. Note the reversal of the effect mentioned with respect to figure 6 and figure 7. The outside pole pieces with respect to the axis of symmetry for the coupled electromagnets now generate the highest field values. The air gap for adjoining side faces of the pole pieces is one half inch in comparison to three quarters of an inch for the air gap across the pole piece face. Also the side faces present a larger area cross-section with respect to the direction of magnetic flux than the pole piece faces. These characteristics explain the reduced field strength of the interior pole pieces with respect to the end pole pieces.

The field values for each pole piece are individually labeled in figure 9. The effect noted earlier for pole piece B3 in figure 7 is evident in figure 9. The symmetry of the coupled magnets is slightly skewed due to the weak pole of Coil B. However, the symmetry of Coil A is noteworthy with respect to the symmetry displayed in figure 6 and figure 9.

VIII. CONCLUSION

The results of the analysis and measurements taken on the prototype electromagnets are positive with respect to the principle considered. Better materials for construction and some redesign of the pole pieces would enhance the magnetic field distribution. Cooling coils intertwined with the coil windings would be necessary to offset heat generation for an increased number of wire windings and higher operating currents. Weight consideration for a multipole electromagnet could preclude the use of two longer electromagnets. A combination of several longer magnets would allow decreased weight for each individual component.

With respect to operation of a FEL device in conjunction with the Naval Postgraduate School's LINAC facility, some further investigation is warranted. The overall length required to achieve significant gain is seen to be of the order of meters. This produces a dimensional conflict with respect to the existing housing structure. A shorter overall length might be considered in an oscillator device where multiple passes of the radiation emitted could enhance the gain. An oscillator device, however, requires electron beam deflection and focusing magnets to guide it into and out of the resonator cavity.

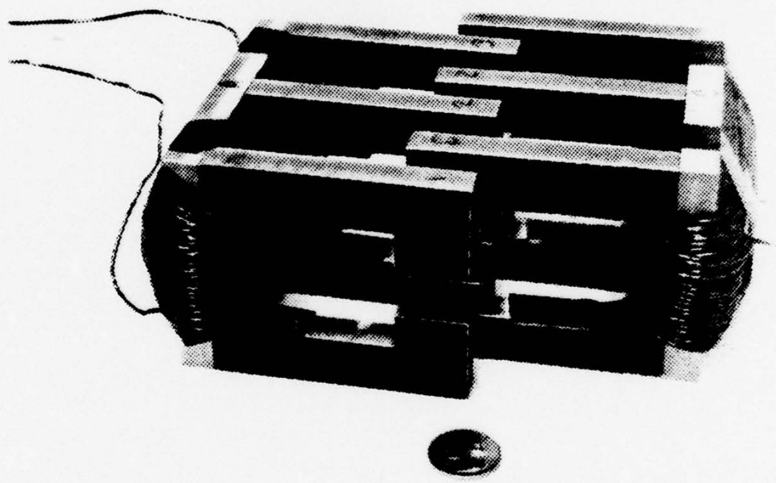


Figure 1

Prototype Electromagnets

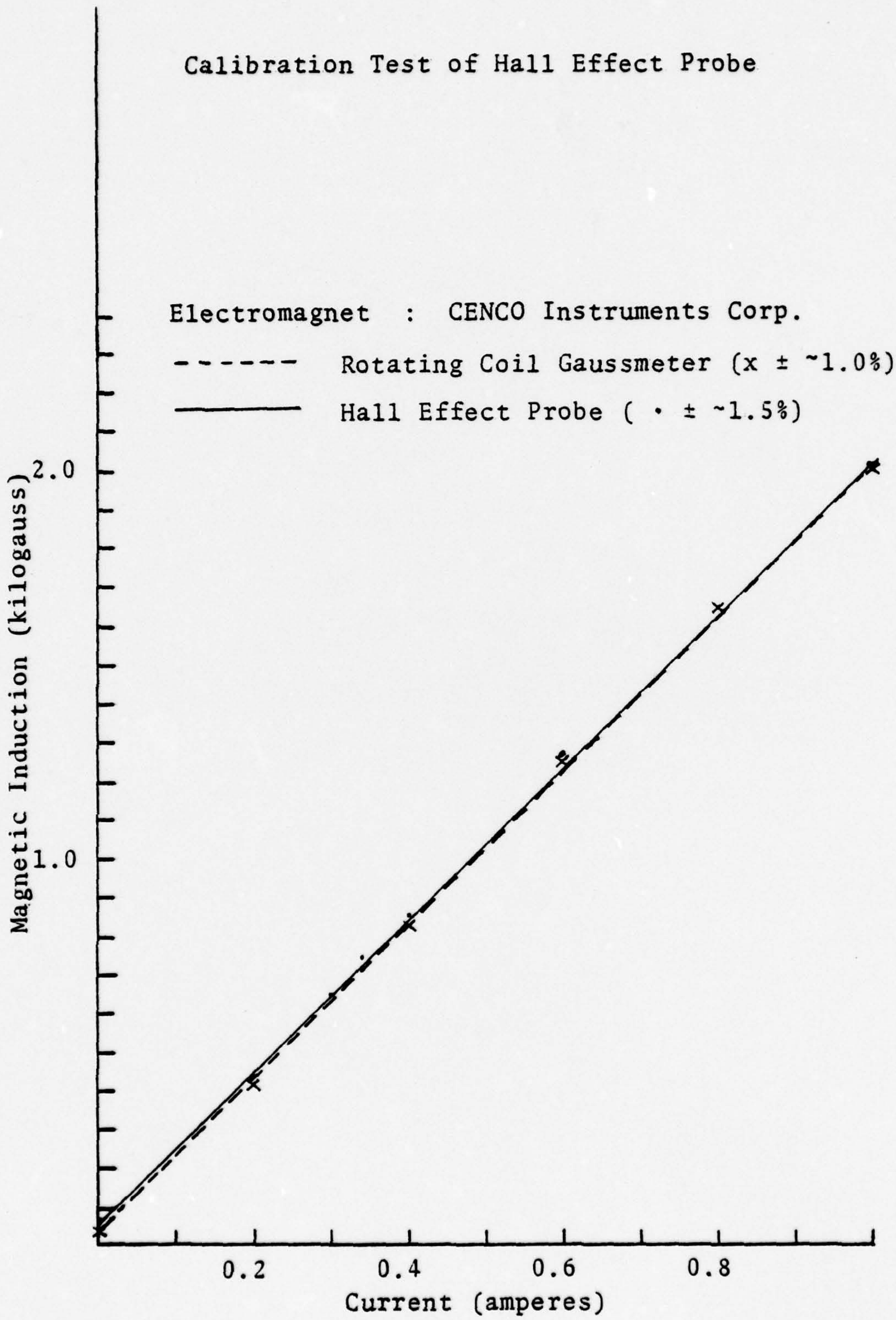

Calibration Test of Hall Effect Probe
Electromagnet : CENCO Instruments Corp.
Rotating Coil Gaussmeter (x ± ~1.0%)
Hall Effect Probe (• ± ~1.5%)
Magnetic Induction (kilogauss)
2.0
1.0
0.2
0.4
0.6
0.8
Current (amperes)

Figure 2

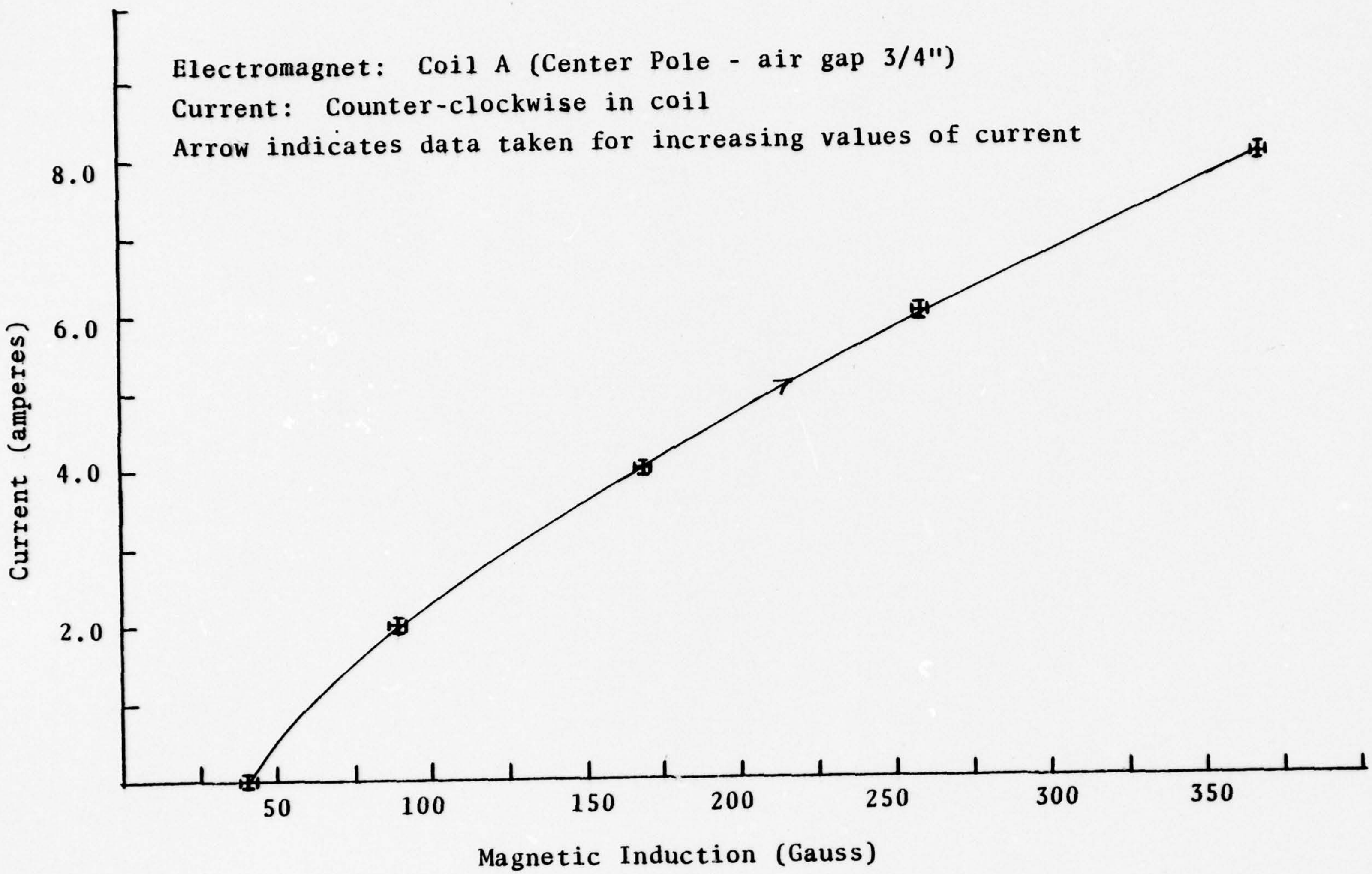

Electromagnet: Coil A (Center Pole - air gap 3/4")
Current: Counter-clockwise in coil
Arrow indicates data taken for increasing values of current
Current (amperes)
8.0
6.0
4.0
2.0
50
100
150
200
250
300
350
Magnetic Induction (Gauss)

Figure 3

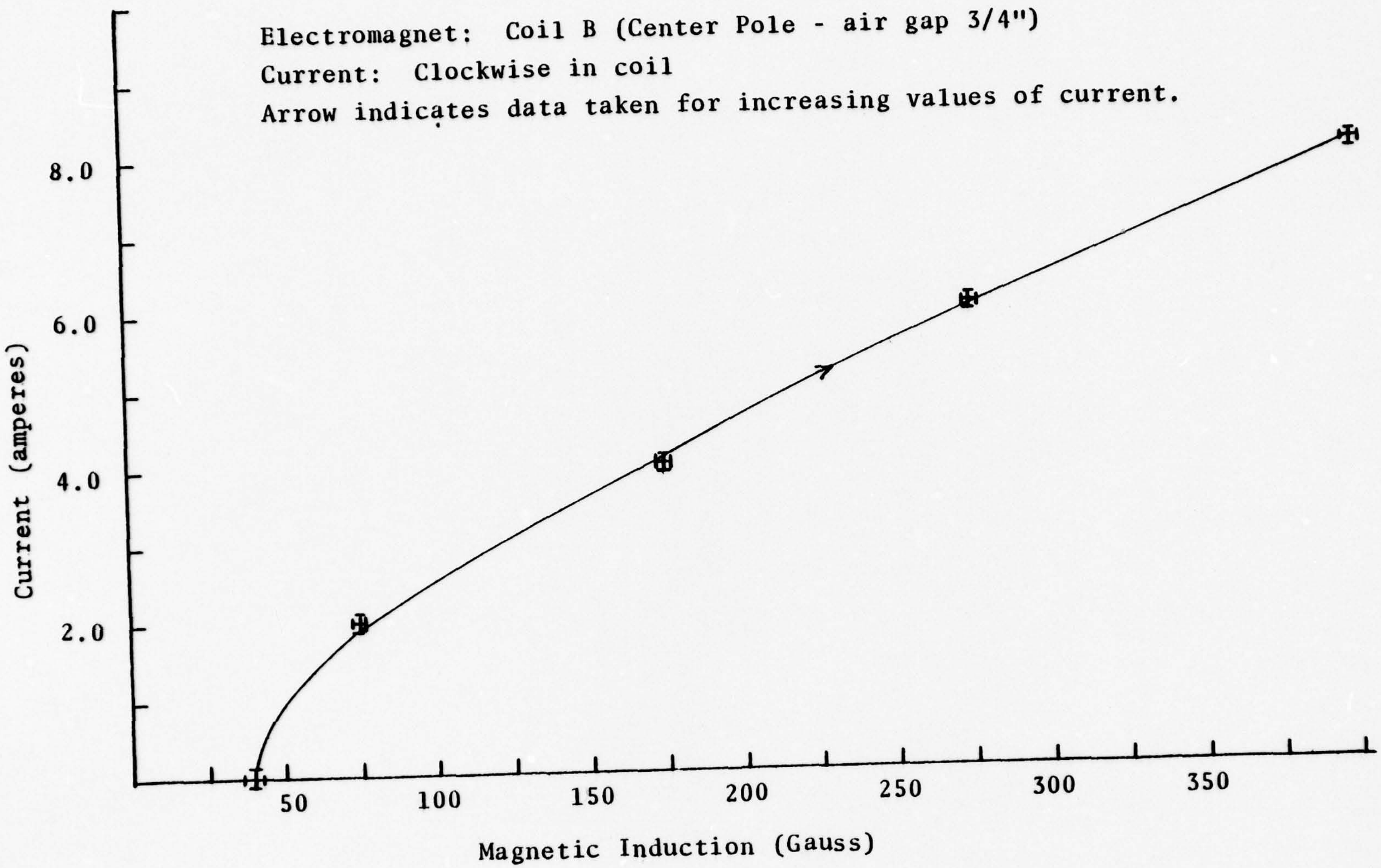

Electromagnet: Coil B (Center Pole - air gap 3/4")
Current: Clockwise in coil
Arrow indicates data taken for increasing values of current.
8.0
6.0
4.0
2.0
Current (amperes)
50
100
150
200
250
300
350
Magnetic Induction (Gauss)

Figure 4

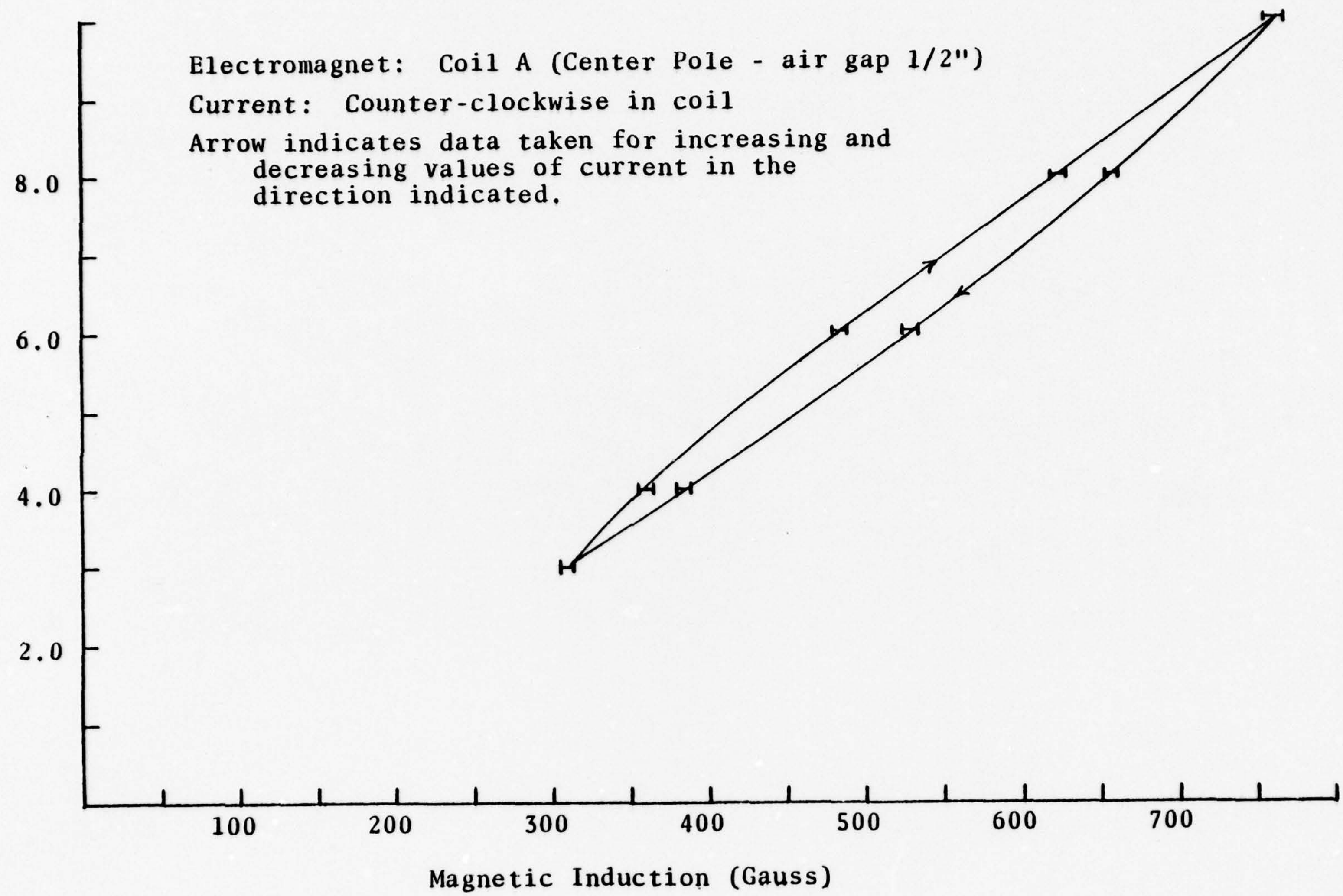

Electromagnet: Coil A (Center Pole - air gap 1/2")
Current: Counter-clockwise in coil
Arrow indicates data taken for increasing and decreasing values of current in the direction indicated.
8.0
6.0
4.0
2.0
100
200
300
400
500
600
700
Magnetic Induction (Gauss)

Figure 5

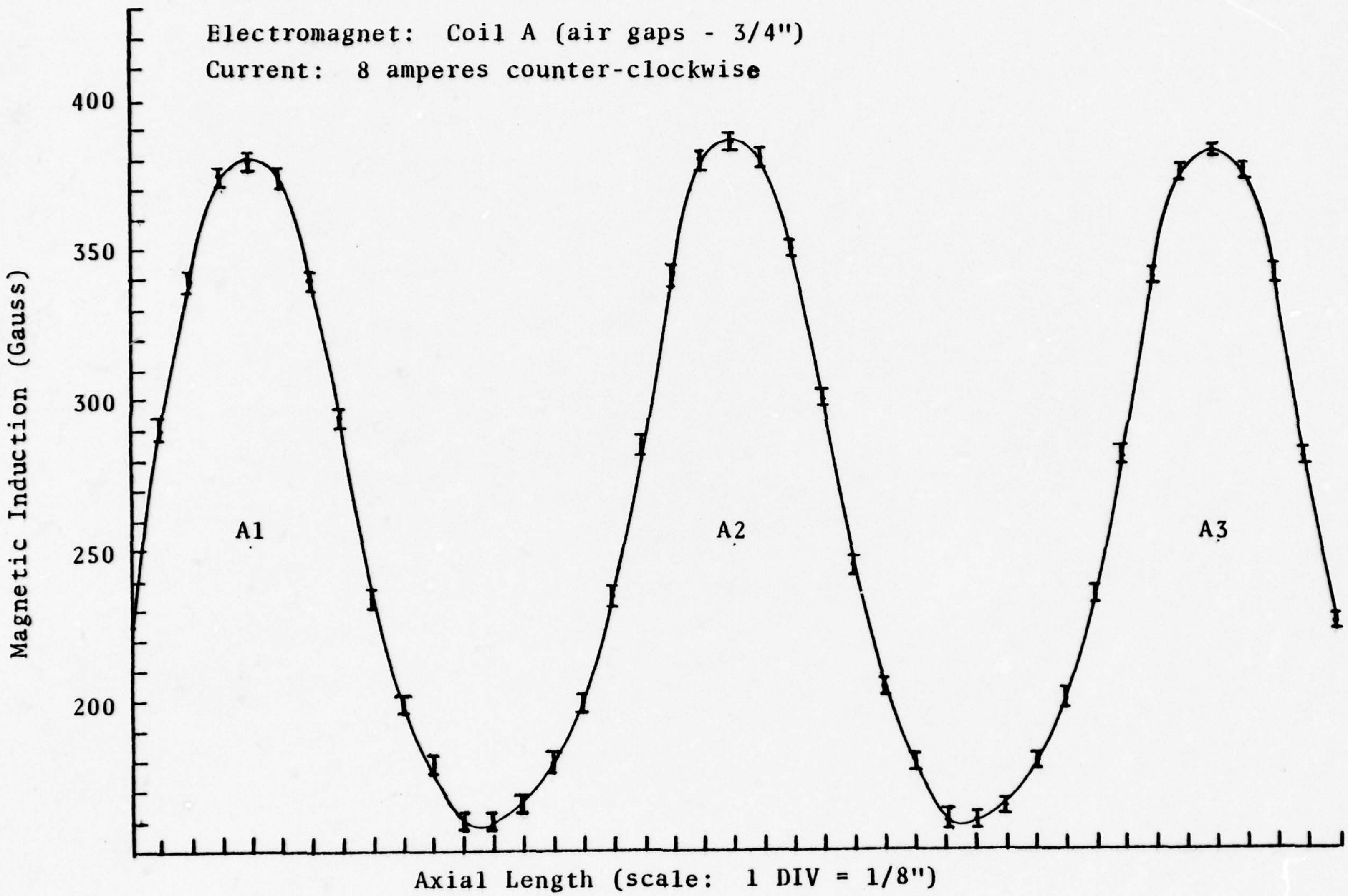

Electromagnet: Coil A (air gaps - 3/4")
Current: 8 amperes counter-clockwise
400
350
300
250
200
Magnetic Induction (Gauss)
A1
A2
A3
Axial Length (scale: 1 DIV = 1/8")

Figure 6

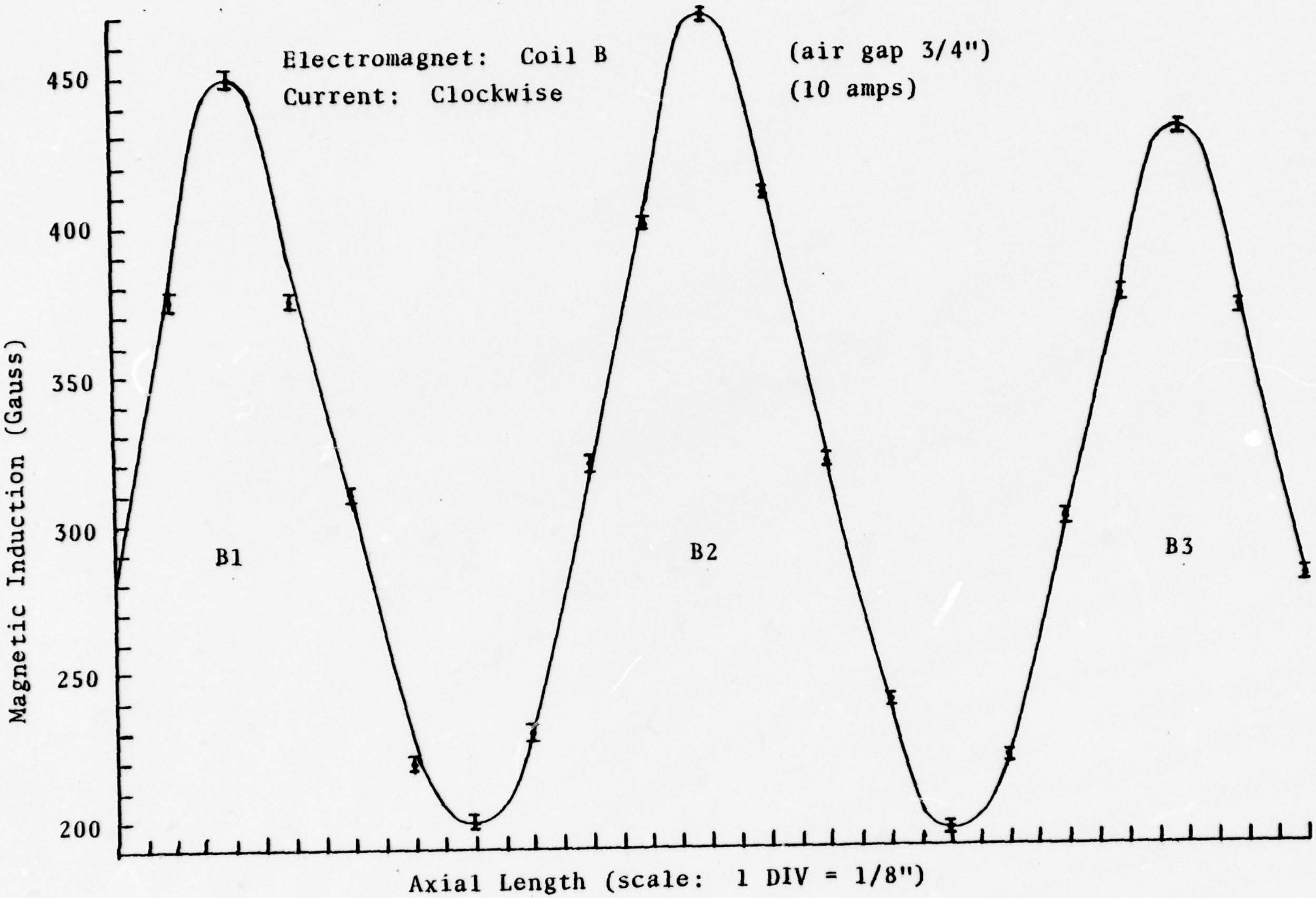

Electromagnet: Coil B
Current: Clockwise
(air gap 3/4")
(10 amps)
B1
B2
B3
450
400
350
300
250
200
Magnetic Induction (Gauss)
Axial Length (scale: 1 DIV = 1/8")

Figure 7

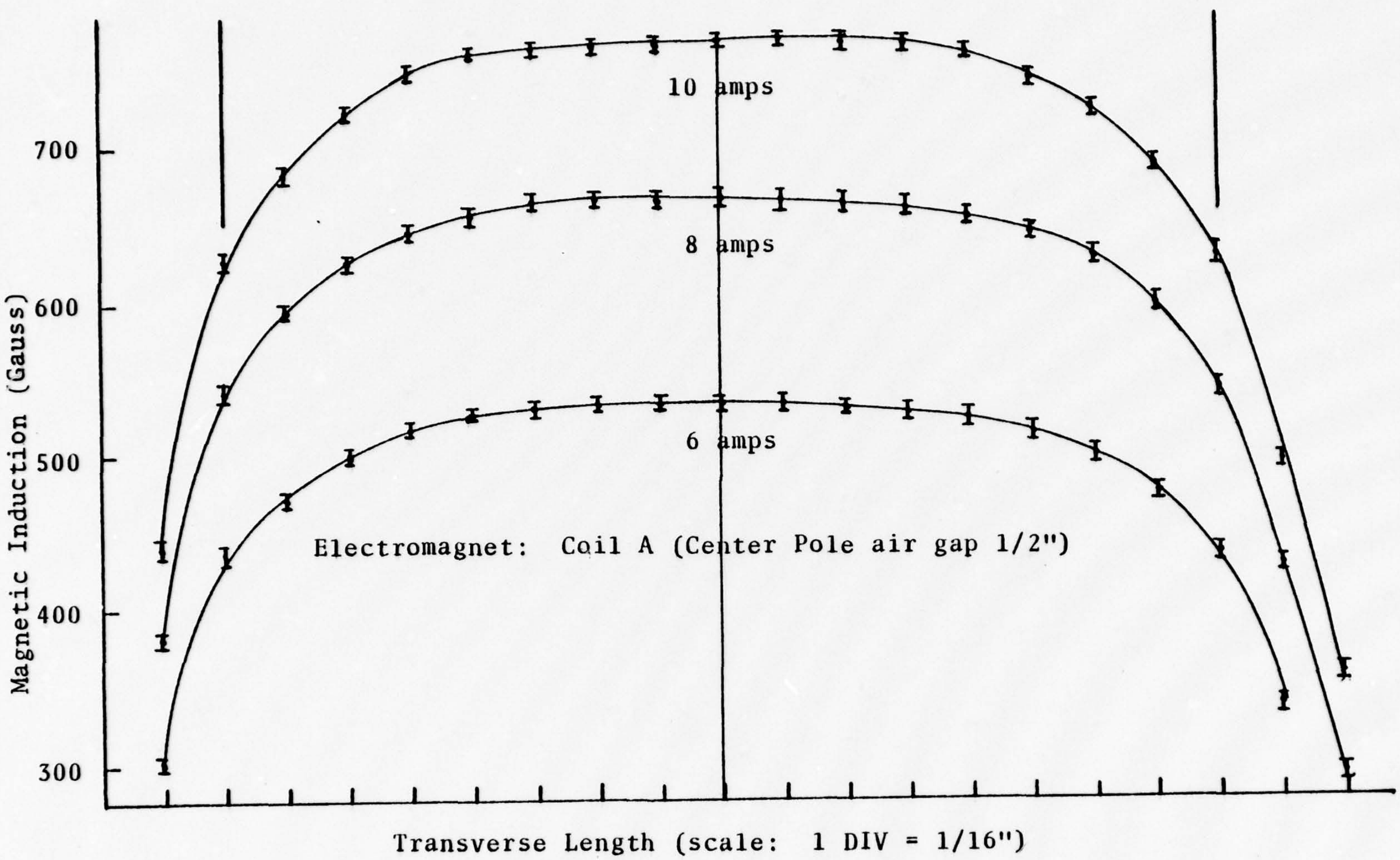

10 amps
8 amps
6 amps
Electromagnet: Coil A (Center Pole air gap 1/2")
700
600
500
400
300
Magnetic Induction (Gauss)
Transverse Length (scale: 1 DIV = 1/16")

Figure 8

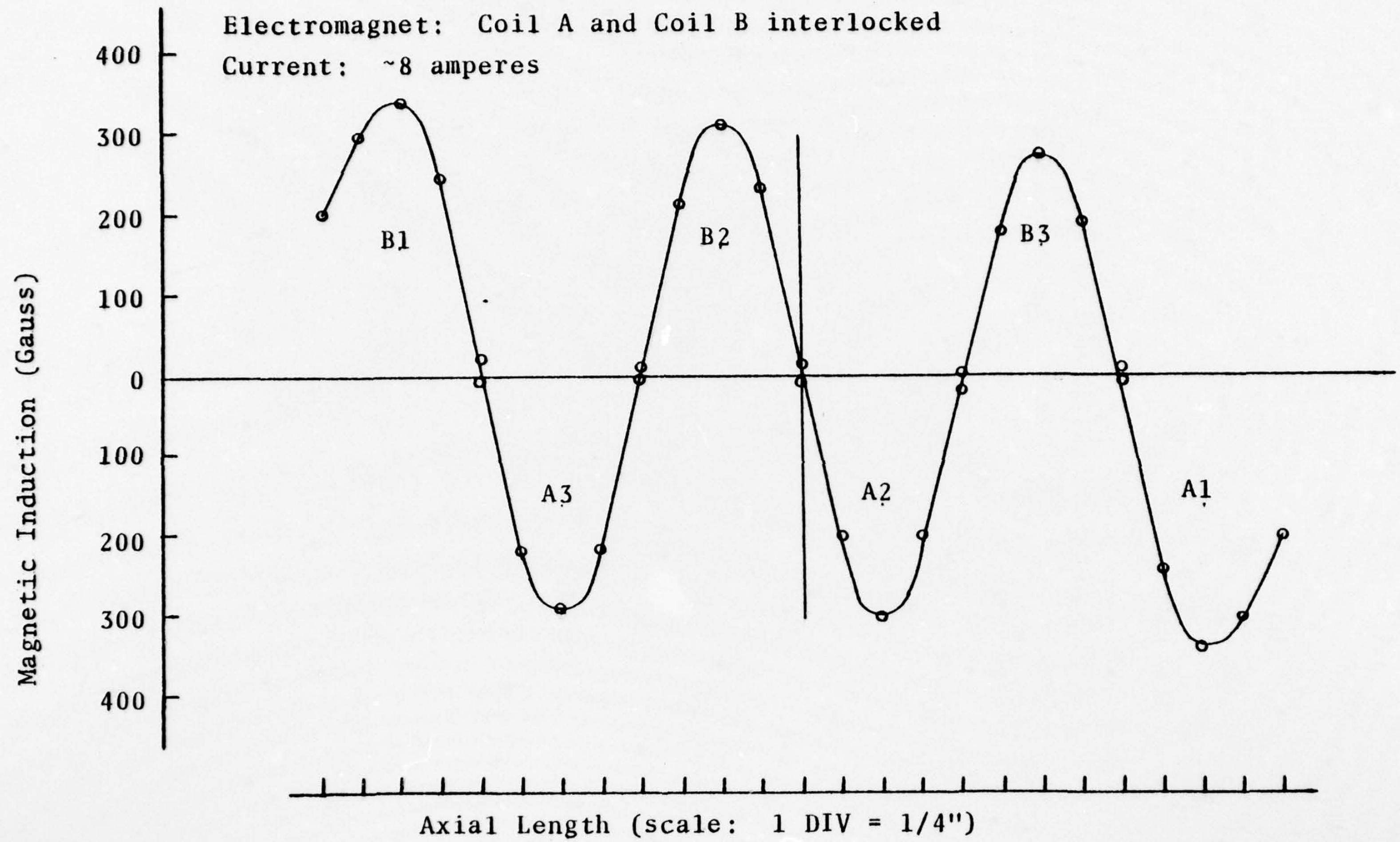

Electromagnet: Coil A and Coil B interlocked
Current: ~8 amperes
400
300
200
100
0
100
200
300
400
Magnetic Induction (Gauss)
B1
B2
B3
A3
A2
A1
Axial Length (scale: 1 DIV = 1/4")

Figure 9

APPENDIX A

Compton Scattering in the Laboratory Frame

Arutyunian and Tumanian (1963) noted that γ-ray beams of high energy might be possible by considering Compton scattering (laboratory frame) in their analogous but independent treatment of the idea first proposed by Milburn. At this point, it is worthwhile to perform a few calculations. Using the usual Compton effect result for the electron rest frame,

$$h\nu_2 = \frac{h\nu_1}{1 + \frac{h\nu_1}{mc^2}(1 - \cos\phi)}$$

where $h\nu_2$ is the energy of the scattered photon, $h\nu_1$ is the energy of the incident photon, mc^2 is the rest energy of the electron and ϕ is the scattering angle of the emitted photon with respect to the direction of the incident photon. The transformation to the laboratory frame (see Primakoff and Feenberg) results in

$$h\nu_2 = \frac{h\nu_1\ (1 - \beta\cos\theta_1)}{(1 - \beta\cos\theta_2) + \frac{h\nu_1}{E}(1 - \cos\phi)}$$

where the subscripts are applicable as above but all terms are considered in the laboratory frame. The energy of the electron is given by E and $\beta = v/c$. Considering an electron-photon interaction for oppositely directed photons and electrons with the photon scattering angle equal to π, then

$$h\nu_2 = \frac{h\nu_1(1 + \beta)}{(1 - \beta) + (2\, h\nu_1/E)}$$

and using

$$\gamma = (1 - \beta^2)^{-1/2}$$

$$E = \gamma mc^2$$

$$\beta \simeq 1$$

the result may be written as

$$h\nu_2 = \frac{2h\nu_1}{\frac{1}{2}\left(\frac{1}{\gamma}\right)^2 + \frac{2h\nu_1}{E}} \quad .$$

Then

$$h\nu_2 = \frac{4\gamma^2 h\nu_1}{1 + (4\gamma h\nu_1/mc^2)} \quad .$$

Consider

$$(4\gamma h\nu_1/mc^2) << 1$$

$$h\nu_2 \simeq 4\gamma^2 h\nu_1 \quad .$$

Note also

$$\lambda_1 >> 4\gamma\lambda_c$$

$$\lambda_c = \frac{h}{mc^2} \qquad \text{(Compton wavelength)}$$

Now considering

$$(4\gamma h\nu_1/mc^2) >> 1$$

$$h\nu_2 \simeq \gamma mc^2 \quad .$$

APPENDIX B

Pump Wave Power Density

A power density comparison for pump wave sources serves to illustrate the utility of the magnetic pump field versus electromagnetic wave sources. With respect to photon flux, it is observed that power is proportional to the intensity which is in turn proportional to the photon flux. Assume a one-kilogauss magnetic field.

$$S = \frac{c}{\mu} B_o^{\ 2}$$

$$S = \frac{3 \times 10^8}{4\pi \times 10^{-7}} \left(.1 \ \frac{\text{Weber}}{m^2}\right)^2$$

$$S = 2.4 \times 10^{12} \ \text{Watts/m}^2$$

The high-power density above is approximately that required for an equivalent continuous wave source. The generation of a one-kilogauss magnetic field or more is obviously the easiest to obtain and most economical by comparison.

APPENDIX C

Covarient Lorentz Force Transformation

The covariant Lorentz force transformation to space-time form is given [Ref. 39]

$$\frac{dP_\mu}{d\tau} = \frac{e}{m} F_{\mu\nu} P_\nu \quad .$$

Then

$$\text{(i)} \quad \frac{d\bar{P}}{dt} = e(\bar{E} \times \frac{\bar{V} \times \bar{B}}{c})$$

$$\text{(ii)} \quad \frac{dE}{dt} = e(\bar{V} \cdot \bar{E})$$

and

$$\bar{\beta} = \bar{V}/c$$

$$\bar{P} = \gamma m \bar{V}$$

$$\gamma = 1/(1 - \frac{V^2}{c^2})^{1/2}$$

$$E = \gamma m c^2 \quad .$$

Then (i)

$$\frac{d}{dt}\left(\frac{\gamma m\overline{V}}{c}\right) = \frac{e}{c}\left(\overline{E} + \overline{\beta} \times \overline{B}\right)$$

$$\frac{d\gamma\overline{\beta}}{dt} = \frac{e}{mc}\left(\overline{E} + \overline{\beta} \times \overline{B}\right)$$

and (ii)

$$\frac{1}{mc^2}\,\frac{dE}{dt} = \frac{e}{mc^2}\,\overline{V} \cdot \overline{E}$$

$$\frac{d\gamma}{dt} = \frac{e}{mc}\left(\overline{\beta} \cdot \overline{E}\right) .$$

Note

$$\gamma = 1/\left(1 - \frac{\overline{V} \cdot \overline{V}}{c^2}\right)^{1/2}$$

$$\gamma^2 = 1/(1 - \overline{\beta} \cdot \overline{\beta})$$

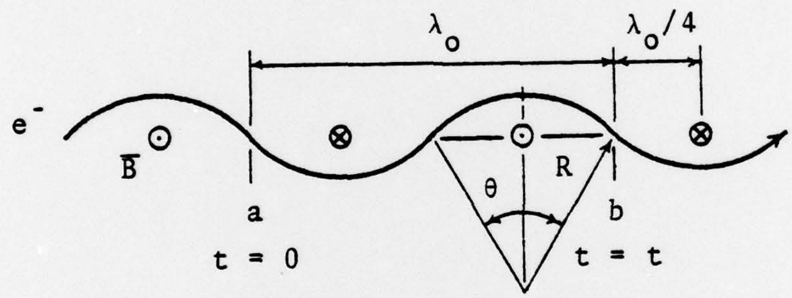
$$\gamma^{-2} = 1 - \overline{\beta} \cdot \overline{\beta} \quad .$$

APPENDIX D

Effect of Pump Wave Field Strength

(Compton Regime)

After one period the electron lags the radiation emitted previously at a corresponding periodic point by a distance corresponding to the wavelength of the emitted radiation. An abrupt magnetic field of alternating polarity is assumed transverse to the plane of electron motion.



Electron velocity = βc

Radiation : $E = E_o \sin (k_2 z - \omega_2 t)$

Point b $\left\{ \begin{array}{l} z = \lambda_o \\ k_2 z - \omega_2 t + 2\pi = 0 \end{array} \right.$

$k_2 = \frac{2\pi}{\lambda_2}$ $\qquad \omega_2 = \frac{2\pi c}{\lambda_2}$

Electron path : $\overline{ab} = \ell$

$$t = \ell/\beta c$$

Then

$$\frac{2\pi}{\lambda_2} \lambda_o - \frac{2\pi c}{\lambda_2} \frac{\ell}{\beta c} = -2\pi \quad .$$

Considering momentum:

$$\frac{\Delta P}{\Delta t} = \frac{evB}{c} \qquad\qquad \Delta P = P\Delta\theta$$

$$\Delta\theta = \frac{\Delta\ell}{R}$$

$$\frac{P}{R}\frac{\Delta\ell}{\Delta t} = \frac{evB}{c}$$

$$\frac{P}{R} = \frac{eB}{c}$$

$$\frac{\gamma mc\beta}{R} = \frac{eB}{c} \quad .$$

From geometry:

$$R \sin\frac{\theta}{2} = \frac{\lambda_o}{4}$$

$$\frac{\ell}{2} = R\theta$$

$$\ell = 2\left(\frac{\lambda_o}{4}\right) \frac{\theta}{\sin(\theta/2)}$$

$$\theta << 1$$

$$\ell \simeq \lambda_o\left[1 + \frac{1}{6}\left(\frac{\theta}{2}\right)^2\right]$$

$$\text{Sin}\,\frac{\theta}{2} = \left(\frac{\lambda_o}{4}\right) \frac{eB}{\gamma mc^2\beta}$$

$$\frac{\theta}{2} \simeq \frac{\lambda_o}{4} \frac{eB}{\gamma mc^2\beta}$$

Consider now the phase relationship.

$$\frac{2\pi}{\lambda_2}\lambda_o - \frac{2\pi c}{\lambda_2}\frac{\ell}{\beta c} = -2\pi$$

$$\lambda_2 = \lambda_o\left[\frac{\ell}{\beta\lambda_o} - 1\right]$$

$$\lambda_2 = \lambda_o\left[\frac{1}{\beta} + \frac{1}{6\beta}\left(\frac{\lambda_o}{4}\right)^2\left(\frac{eB}{\gamma mc^2\beta}\right)^2 - 1\right]$$

$$\lambda_2 = \lambda_o\left[\frac{(1-\beta)(1+\beta)}{\beta(1+\beta)} + \frac{1}{6\beta}\left(\frac{\lambda_o}{4}\right)^2\left(\frac{eB}{\gamma mc^2\beta}\right)^2\right]$$

$$\lambda_2 = \lambda_o\left[\frac{1}{\gamma^2\beta(1+\beta)} + \frac{1}{6\beta}\left(\frac{\lambda_o}{4}\right)^2\left(\frac{eB}{\gamma mc^2\beta}\right)^2\right]$$

Then assuming $\beta \approx 1$, Madey's result [Ref. 15] is

$$\lambda_2 \approx \frac{\lambda_o}{2\gamma^2} [1 + \frac{1}{3} (\frac{\lambda_o}{4c})^2 (\frac{eB}{mc})^2] \quad .$$

This equation defines a shift in the emitted radiation wavelength due to the field strength of a transverse magnetic pump "wave."

APPENDIX E

Net Gain Equation

Compton Regime

Net gain is the difference between the gain for stimulated emission and the gain (loss) due to absorption. The net gain (dB per meter) assuming the electron energy distribution and power spectral density of the equivalent plane wave state are Gaussian is given [Ref. 43].

$$G = 15.8 \left(\frac{r_o}{\hbar c}\right)^2 \frac{(\lambda_2 \lambda_o B)^2 \rho_e \,(FF)}{\left[\left(\frac{\Delta\gamma}{\gamma}\right)^2 + \left(\frac{\Delta\nu}{2\nu}\right)^2\right]^{1/2}} \left[\exp\,(EF) - \exp\,(AF)\right]$$

where

r_o = classical electron radius

ρ_e = electron density

λ_o = magnet spatial wavelength

λ_2 = radiation wavelength

ν_2 = radiation frequency

B = magnetic induction field

FF = filling factor

$$EF = - \frac{\left(\frac{\delta\nu}{2\nu_2}\right)^2}{\left(\frac{\Delta\gamma}{\gamma}\right)^2 + \left(\frac{\Delta\nu}{2\nu}\right)^2} \qquad \text{(emission factor)}$$

$$AF = - \frac{\left(\frac{\delta\nu}{2\nu_2} - \frac{h\nu_2}{\gamma mc^2}\right)^2}{\left(\frac{\Delta\gamma}{\gamma}\right)^2 + \left(\frac{\Delta\nu}{2\nu}\right)^2} \qquad \text{(absorption factor)}$$

$\delta\nu$ = difference between the radiation frequency and the emission line center

$\Delta\gamma$ = electron beam energy linewidth

$\Delta\nu$ = line broadening of the equivalent plane wave states of the electron.

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