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MIXED BOUNDARY VALUE PROBLEMS OF ELASTICITY.(U)
AUG 79 R P SRIVASTAV, Y M CHEN DAAG29-76-G-0197
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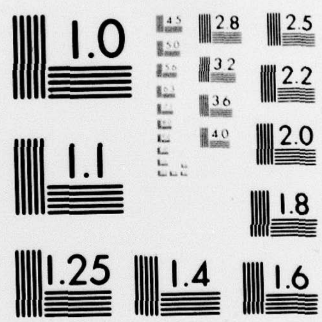
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18. SUPPLEMENTARY NOTES

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19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

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brittle elastic solids
anisotropic media
boundary value problems
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The central theme of the project was the theoretical (as opposed to experimental) study of the fracture of brittle elastic solids. The purely numerical approach based upon the direct solution of the partial differential equations using the adaptations of the 3-d Hemp Code led to the initiation of a multipurposecode for the simulation of wave propagation in anisotropic media. The boundary value problems for which, in principle, a solution with a desired degree of accuracy can be calculated or computed were considered. The results of these studies are summarized in this report.

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Final Report

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II. Grant Number: DAAG 29 -76-G-0197

III. Title of the Proposal :

Mixed Boundary Value Problems of Elasticity

IV. Grant Period : April 15, 1976 - June 14, 1979

V. Name of the institution:

State University of New York at Stony Brook

VI. Scientific Personnel Supported By The Grant:

1. Ram P. Srivastav , Principal Investigator and Project Director
2. Yung Ming Chen , Principal Investigator and Co- Project Director
3. Tai-Yih Chang , Graduate Research Assistant
4. Wei-Min Cheng , "
5. A. Gerasoulis , "
6. Robert Hatcher , "
7. Kelvin Lee , "
8. J. Quattrochi , "
9. Emanuel Tsimis , "

VII. Advanced Degrees Earned By The Participating Scientific Personnel:

1. Tai-Yih Chang was awarded Ph.D.The title of his thesis was *Stress Distribution in the vicinity of a crack in non-homogeneous wedges* (August 1976).
2. Emanuel Tsimis was awarded Ph.D. The title of his thesis was "*On theinverse problem by means of the integral equations of the first kind*". (March 1977)
3. Apostolos Gerasoulis was awarded Ph.D. The title of his thesis was *The numerical solution of singular integral equations and applications to mixed boundary value problems of fracture mechanics* (March 1979).
4. Wei-Min Cheng has successfully defended his Ph.D. thesis.The title of his Ph.D. thesis was *Numerical Methods for solving partial differential equations with inadequate data* (June 1979).
5. Kelvin Lee was awarded M.S. in Applied Mathematics (May 1977). He has made substantial progress on his Ph.D. thesis.
6. J.Quattrochi was awarded M.S. in Applied Mathematics (December 1978).

VIII. Publications :

1. Application of a novel finite difference method to dynamic crack problems - Y.M.Chen and M.L.Wilkins: Advances in Engineering Sciences, Vol 1,(1976) pp. 227 - 238.
2. Numerical methods for constructing optimal solutions of differential equations - Y.M.Chen, Proceedings 14th Annual Meeting,Society of Engineering Science, Lehigh University (1977)pp. 955 - 965.
3. Numerical Solutions of three dimensional crack problems and simulation of dynamic fracture phenomena by a non-standard finite-difference method - Y.M. Chen, Engineering Fracture Mechanics,Volume 10 (1978) pp. 699 -
4. A numerical study on effects of fiber-orientation on a unidirectional elastic solid under impact - Y.M.Chen and Kelvin J.Lee, Journal of Composite Materials, 13(1979) pp. 117 - 125.
5. A note on dual integral equations with trigonometric kernels - R.P.Srivastav, Quarterly of Applied Mathematics,Vol. 35 (1978)pp. 524 - 526.
6. A Griffith crack problem for a non-homogeneous medium - A. Gerasoulis and R.P. Srivastav, International Journal of Engineering Science (to appear).
7. A Neumann Series reduction of simultaneous triple integral equations of elasticity to equations of Cauchy type and an application to a crack problem - R.P. Srivastav and A. Gerasoulis, International Journal of Engineering Science (to appear). (Invited paper for the Society of Engineering Science 16th Annual Meeting Northwestern University September 5 - 7, 1979.)
8. A method for the numerical solution of singular integral equations with a principal value integral - A. Gerasoulis and R.P. Srivastav Presented to 25th Conference of Army Mathematicians, Baltimore June 1979; submitted to Quarterly of Applied Mathematics.
9. Order of singularity in mixed boundary value problems of elasticity - R.P. Srivastav (Presented at the Eighth Congress of Applied Mechanics, University of California at Los Angeles,June 1978 under a different title) to be submitted for publication.

IX. A statement of the problem studied and a summary of the most important results:

The central theme of the project was the theoretical (as opposed to experimental) study of the fracture of brittle elastic solids. Professor Chen had the primary responsiblity for the purely numerical approach based upon the direct solution of the partial differential equations using the adaptations of the 3-d Hemp Code. Professor Chen's work has led to the initiation of a multipurposecode for the simulation of wave propagation in anisotropic media. Professor Srivastav looked at the boundary value problems for which , in principle, a solution with a desired degree of accuracy can be calculated or computed. The results of these studies are summarized below.

The first paper on our list reports the time-dependent stress intensity factors for cracks in thick cylindrical pipes. The HEMP 3-D computer program was modified to analyze the stress fields around the cracks in a solid of finite dimension subject to dynamic loads. It was found that the surface cracks in a plane normal to the axis of the cylinder experience much less stress than the surface cracks in a plane parallel to the axis of the cylinder. These effects were found to be more pronounced for longer cracks than for shorter cracks.

The stress intensity factors for the axial semi-elliptical surface cracks of different sizes on the interior surface of a section of cylindrical pipes subject to Mode I load are computed in item 3 of the list of publications. This paper also simulates rupturing of a solid cylinder indented by a rigid cylindrical punch under the assumption that a crack will appear and propagate in any region when a critical stress has been reached, and that the propagation will be in the direction perpendicular to the maximum principal stress.

Both the finite difference method and the finite element method for solving the crack problems of elastic solids depend to a great extent on a prior knowledge of the precise nature of the singularity in the vicinity of the crack boundary. The bounds for error in a typical field quantity have the form constant.$h^a\ U^{(b)}(P)$, where a and b are positive integers and h is the 'grid size'. P is an unknown point in the computational grid. In the neighborhood of a singular point the gradient is steep and/or a violent oscillation, with the result that $U^{(b)}(P)$ is extremely large.Hence, for the grid size, which is practical, the error estimates are often meaningless. Moreover, reducing the grid size fails to improve the accuracy of the approximate solution to any appreciable extent in the critical region. In item 2, Professor Y.M. Chen has initiated the development of a projection-like method and a collocation like method with better and sharper criteria. These methods have been tested for the ordinary differential equations, and found superior to either the finite-difference method or the finite-element method for certain types of problems.

The paper by Chen and Lee contains a numerical study of the effects of the fiber orientation on the wave propagation in a unidirectional linear elastic solid not orthogonal to the direction of impulsive loading. A finite difference method is incorporated inthe AWMP CODE. A two-dimensional case of the penetration of a rigid punch into a uni-directional linear-elastic solid is indicative that the fiber-orientation plays an important role in causing asymmetric wave patterns and generating asymmetric surface waves under symmetric loads. These asymmetric waves resemble those in isotropic solids caused by asymmetric loads.

Dual integral euations arise when the integral transforms are used to solve mixed boundary value problems of elasticity. The literature dealing with the questions of existence and the uniqueness of the solution is somewhat thinner than the formal solutions, and the analysis tends to be abstruse. Item 5 contributes a necessary and sufficient condition for the dual integral equations with trigonometric kernels, usually encountered in the two-dimensional theories of elasticity , but also related to the axisymmetric problems leading to the dual equations with Bessel functions.

Gerasoulis and Srivastav have developed a solution for the boundary value problem of two bonded nonhomogenous similar planes containing a crack at the interface.The main feature of this paper is the adaptation of the Plevako solution leading eventually to a weakly singular integral equation. This equation is solved using a trapezoidal approximation and the stress intensity factors and the crack energy for certain known variations of the shear modulus have been calculated. The success of the piecewise linear approximation for this problem lead us to consider the singular integral equations of the Cauchy type using similar approximation.

The method reported in item 8 exploits the knowledge of the singular behavior of the unknown function (the index theorems of Noether). The singularity is built explicitly into the solution by multiplying a piecewise linear function with the appropriate weight function. The Cauchy principal value integral of the dominant term can be evaluated in the closed form and we found that the method gives as good a result as the more sophisticated procedures based on Gauss-Chebyshev and Lobatto-Chebyshev quadrature formulae for the problems we considered. We shall like to describe one feature of the numerical solution of integral equations. In such solutions the value of the free function can be used only at a discrete set of points. If this function is a constant or varies slowly and smoothly, the distance between the collocation points does not matter. On the other hand , if the free function changes rapidly,many more collocation points will be needed in the neighborhood of rapid change. Using a Chebyshev type formula a very high degree of polynomial will be needed.The reason for this is that the largest distance between the two zeros of a Chebyshev polynomial of degree $2n-1$ is $\pi/2n$. If the distance between the collocation points necessary to capture the salient features of the function is ,say, .01 , then a polynomial of degree three hundred would be needed; and , if the function changes significantly over an interval of length .001, then a polynomial of degree 3,000 would be required! The difficulty is that to choose two sample points, which are close, all the meshes have to be made small.

The numerical solution of singular integral equations in this way is further utilized to study the stress field in the vicinity of a Griffith crack situated at the interface of two bonded dissimilar half-planes,which is opened by a rigid inclusion determining the crack profile over a specified region.

Professor Srivastav has also written a preliminary version of an expository monograph dealing with the dual integral equations and singular integral equations of the Cauchy type. The manuscript is being typed and as usual a copy would be furnished to the Army Research Office. Professor Srivastav has also reduced the problem of two charged parallel plates to a singular integral equation. It is expected that this formulation will perhaps give acceptable numerical results at low cost when the plates are close. The problem was recently discussed by Professor J.B.Rosser at the Baltimore Meeting of Army Mathematicians using conformal mapping and Fast Fourier Transforms.