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A Study of Elastomeric Materials for Tank Track Pads

by

Costas G. Gogos
and
Rodney D. Andrews

June 1978

Prepared for
United States Army Tank-Automotive
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$$T = f(\rho, C_p, K, E, \mu, \Delta E, \omega, \varepsilon_o, h, b)$$

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20. Abstract

The first six quantities are material variables which depend on the type and structure of the elastomers used. The next three are operation variables and the last, geometric.

We are currently investigating experimentally each of the material variables for a given number of "model" elastomers and are conducting heat generation experiments with these elastomers during cyclic shear deformations. The experimental results will be used to improve our simulation model for heat generation.

A second problem which we feel is important in connection with track pad life and performance is the structural uniformity of the pads achieved during manufacturing. We are embarking on an investigation of the problem of the effects of material, process and geometric variables during injection mold curing. Our initial efforts are numerical having as a goal the prediction of crosslink density as a function of space and time during the molding process.

STEVENS INSTITUTE OF TECHNOLOGY
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Department of
Chemistry and Chemical Engineering

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REPORT SIT-DL-78-9-2084

Department of
Chemistry and Chemical Engineering

A. HEAT GENERATION AND TRANSFER IN CYCLICALLY DEFORMED LINEAR VISCOLEASTIC BODIES

(A Simulation of the Heat-up Problem in
Elastomeric Tank Track Pads)

B. MOLECULAR CONSIDERATION OF THE HEAT GENERATION PROBLEM

I. DEPENDENCE OF ELASTOMER PROPERTIES ON MOLECULAR PARAMETERS

II. HEAT GENERATION - MOLECULAR PHENOMENA

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C. MATHEMATICAL FITTING OF NON-LINEAR STRESS-STRAIN CURVES

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Final Report - Prepared Under Contract
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ABSTRACT

One of the major problems in track pad performance and expected life is heat generation brought about by the periodic loading of the pads and their viscoelastic (partly dissipative) nature. This problem has been simulated numerically making use of a number of simplifying assumptions. The results are included in this final report. Conceptually, the temperature increase can be expressed as

$$T = f(\rho, C_p, K, E, \mu, \Delta E, \omega, \varepsilon_o, h, b)$$

The first six quantities are material variables which depend on the type and structure of the elastomers used. The next three are operation variables and the last, geometric.

We are currently investigating experimentally each of the material variables for a given number of "model" elastomers and are conducting heat generation experiments with these elastomers during cyclic shear deformations. The experimental results will be used to improve our simulation model for heat generation.

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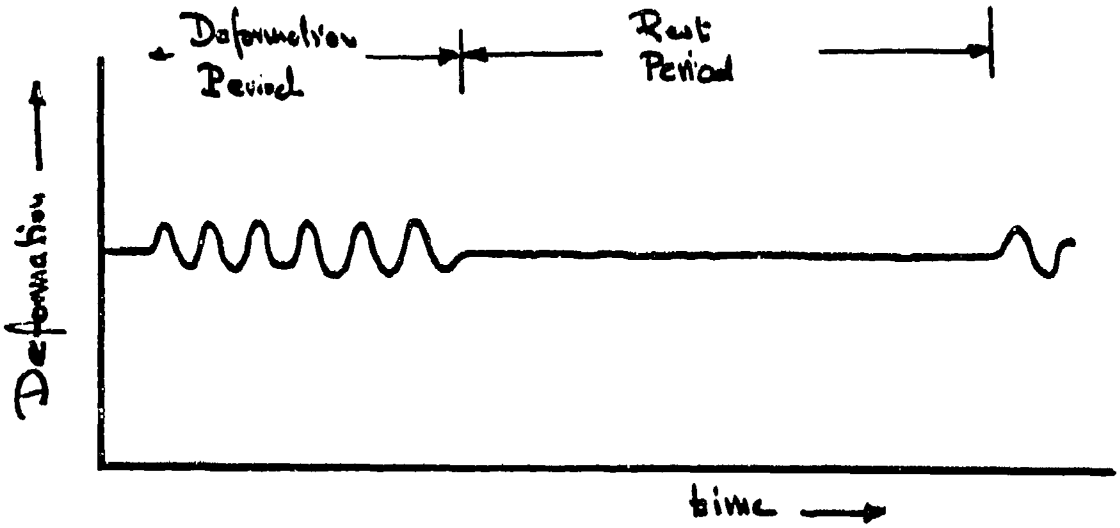
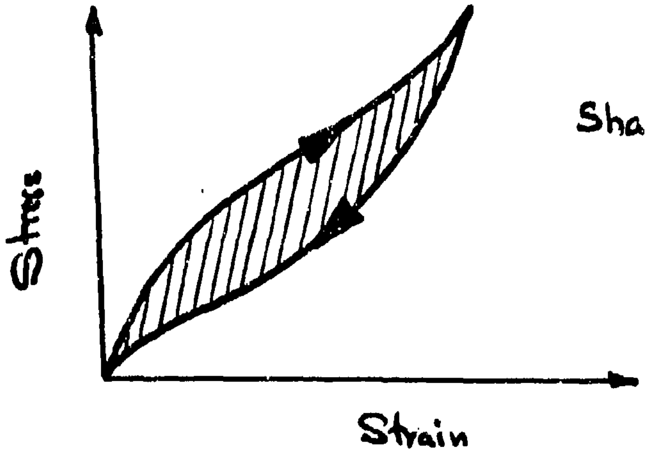
A. HEAT GENERATION

I. Statement of the actual problem

When a tank is in motion, the rubber track pads undergo shear and compressive deformations as they contact the ground surface (which is either smooth or rough) and as they pass the region directly beneath the six wheels of the vehicle. The compressive stresses applied to the track pads could be schematically represented as a function of time as follows:



During the "deformation period" a certain amount of the mechanical work is dissipated into heat resulting in the heat of the rubber pads. This is because the rubber pads are viscoelastic substances which during a cyclic deformation would exhibit hysteresis shown below



Shaded area: Work lost, converted to heat.

Assuming that the rubbery track pads are "ideal rubbers", then the internal energy is not a function of the state of strain and the First Law after the end of each cycle becomes

$$\Delta Q = \Delta W$$

since $dE = 0$.

The thermal energy generated increases the average temperature of the track pads. The temperature distribution depends on the heat transfer conditions around the pad and on the uniformity of the generated heat rate by viscous dissipation, (ΔQ) above. A temperature field $T(x_i,t)$ will result in variations of the rubber pad properties with space and time, and may well cause mechanical deterioration and aging because of

(a) Oxidative degradation which is accelerated at high temperatures
(b) Thermal degradation
(c) Promotion of unfinished cross-linking reactions, which when they occur, may render the pads brittle and susceptible to failure
(d) The increased probability of reactions with soil ingredients
(e) The temperature dependence, in the absence of any reaction, of the mechanical properties of the rubber pads.

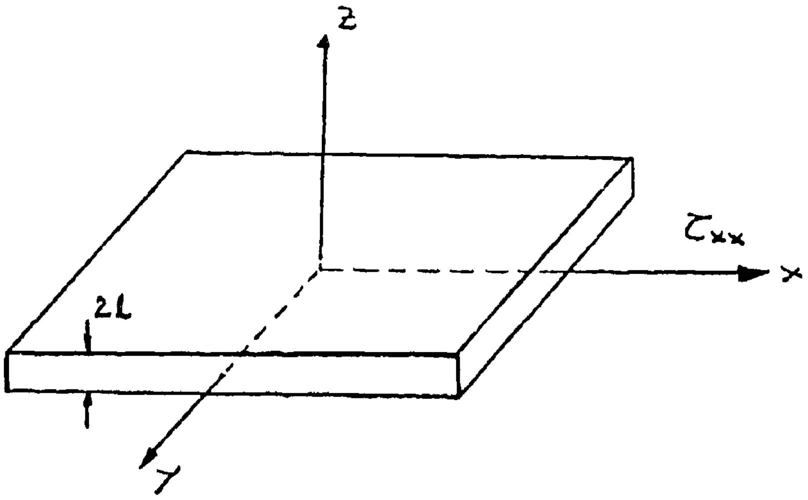
During the "rest period" the rubber pads have an opportunity to transfer the heat generated in the previous period. Heat transfer is by conduction to the initial belt of the track and by forced convection past their top surface.

In the following we report on our <u>simulation</u> work of the above problem. We simplified the physical situation so that we end up with a tractable mathematical problem the solution of which gives useful guides on the effects of material and operation variables on the heat generation and heat transfer in rubber track pads.

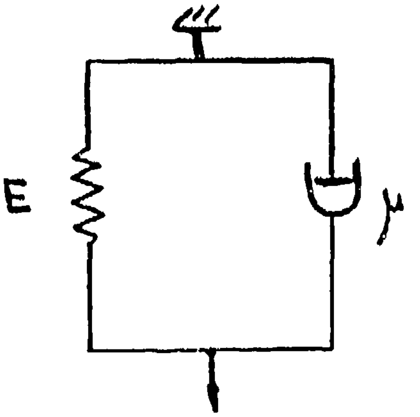
II. <u>Problem simulation</u>

The following assumptions were made about the physical conditions and material response during the actual track pad deformation history:

a. A geometrically simple flat rectangular body undergoing cyclic tensile strains of amplitude ε_o and frequency ω is considered



b. The mechanical response of the track pads is assumed to be adequately described by a Voigt-Kelvin linear viscoelastic mechanical model. Such a model would be appropriate for small deformations



Material parameters : μ, E or μ, $\lambda = \mu/E$.

c. Only the viscosity of the material (and that used in the Voigt model) is assumed to be temperature dependent. All the other relevant properties, ρ , C_p , k and the heat transfer coefficient h are taken to be temperature independent. In reality they are linear functions of temperature, a mild functional dependence compared to the exponential temperature dependence of viscosity.

d. The initial and boundary conditions for the simulation of the heat transfer problems are:

i. $\partial T/\partial z = 0$ at $z = 0$ - symmetric temperature profile

ii. $K \left.\frac{\partial T}{\partial z}\right|_{z = \pm \ell} = h(T\big|_{z=\pm\ell} - T_a)$ - forced convective heat transfer at both surfaces

iii. $T = T_a$ at $t \leq 0$ - the rubber pads start the process at ambient temperature.

III. Mathematical description of the simulation model

The starting point for the solution of any heat transfer problem is the equation of energy, which in terms of the temperature field is

$$\rho C_p \frac{\partial T}{\partial t} = -\rho C_p \vec{v}\cdot\vec{\nabla T} + k\nabla^2 T - T\left(\frac{\partial P}{\partial T}\right)_V(\vec{\nabla}\cdot\vec{v}) - \underset{\sim}{\tau}:\nabla\underset{\sim}{v}$$

The term on the left is the rate of accumulation of thermal energy and the terms on the right-hand side are, respectively, the rate of heat convected by flow, rate of heat conducted, rate of heat involved in P-V (expansion) work, and rate of heat dissipated.

Keeping the accumulation, conduction and viscous dissipation terms, the only non-zero quantities is the simulation problem, we obtain

$$\rho C_p \frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial z^2} - \tau_{xx}\frac{\partial v_x}{\partial x}$$

The above, as well as the boundary and initial conditions can be put in dimensionless form

$$\frac{\partial \Theta}{\partial \tau} = \frac{\partial^2 \Theta}{\partial \xi^2} - \frac{\ell^2}{kT_a}\tau_{xx}\frac{\partial v_x}{\partial x}$$

$$\frac{\partial \Theta}{\partial \xi} = 0 \quad \text{at} \quad \xi = 0 \qquad \text{B.C \#1}$$

$$\frac{\partial \Theta}{\partial \xi} + \frac{h\ell}{k}\Theta = 0 \quad \text{at} \quad \xi = 1 \qquad \text{B.C \#2}$$

$$\Theta = 0 \quad \text{at} \quad \tau = 0 \qquad \text{I.C}$$

Θ, τ and ξ are defined in the appendix.

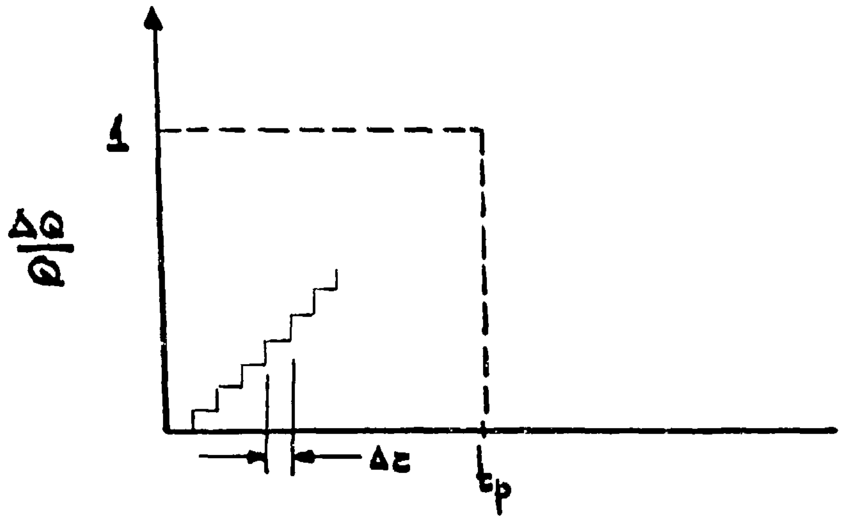
The term $-\tau_{xx}\frac{\partial v_x}{\partial x} = -\tau_{xx}\dot{\varepsilon}$, responsible for the heat up, can be calculated as follows: $\dot{\varepsilon} = \varepsilon_0\omega \cos \omega t$ and $\tau_{xx} = -\mu\dot{\varepsilon}$. Only the viscous contribution of the Voigt-Kelvin element is taken into account because the elastic part does not contribute to the dissipated energy. The viscosity is a function of temperature $\mu = a\ e^{-bT}$. Letting μ_0 be the viscosity at T_a, we obtain

$$\mu = \mu_0\ e^{-b(T - T_a)} = \mu_0\ e^{-b\theta T_a}$$

If the viscosity is taken to be constant during each cycle, but different from cycle to cycle, then the heat dissipated per cycle period t_p is

$$\Delta Q = \pi\omega\ \varepsilon_0^2\ \mu_0\ e^{-b\theta T_a}$$

Choosing the dimensionless time interval $\Delta\tau = \frac{\alpha t_p}{\ell^2}$, where $\Delta\tau \ll t_p$, assuming that the heat generated in each period is equally divided in each dimensionless time interval $\Delta\tau$, as shown



we obtain for each dimensionless time interval Δτ , a quantity of heat generated (in dimensionless form)

$$\frac{\ell^2}{K_o T_a} \pi\omega\varepsilon_o^2 \mu_o e^{-b\theta T_a} \cdot \frac{\Delta\tau}{t_p}$$

This problem can be formulated in terms of finite differences (Taylor series expansion) as follows:

$$\frac{\partial^2\theta}{\partial\xi^2} = \frac{\theta_{i-1} - 2\theta_i + \theta_{i+1}}{(\Delta\xi)^2}$$

$$\frac{\partial\theta_i}{\partial\tau} = \frac{\theta_i^* - \theta_i}{\Delta\tau}$$

$$\theta_i^* = \theta_i + \frac{\Delta\tau}{(\Delta\xi)^2}\left[\theta_{i-1} + \theta_{i+1} - 2\theta_i\right] + \frac{\ell^2}{k_o T_a} \pi\omega\varepsilon_o^2\mu_o e^{-b\theta T_a} \cdot \frac{\Delta\tau}{t_p}$$

Example : $\Delta\xi = 0.05$ [divide half thickness into 20 parts]

$$\theta_1^* = \theta_3^* \quad [\text{B.C } \#1]$$

$$\theta_2^* = \theta_2 + \frac{\Delta\tau}{(\Delta\xi)^2}(\theta_1 - 2\theta_2 + \theta_3) + \frac{\ell^2}{K_o T_a} \pi\omega\varepsilon_o^2\mu_o e^{-b\theta T_a} \cdot \frac{\Delta\tau}{t_p}$$

$\vdots$

$$\theta_{22} = \frac{\theta_{21}^*}{1 + h L \Delta\xi / k} \quad [\text{B.C } \#2]$$

IV. Results and discussion

The "design expression" for heat generation is

$$\Delta Q = \pi \varepsilon_o^2 \omega \mu$$

and one can see the effects of the material variable μ and the operation variables ε_o , and ω . This heat generation term is "in competition" with the conductive heat transfer rate, which, inside the pad is controlled by the thermal conductivity k and at the pad surface by the heat transfer coefficient h .

The results below indicate the effects of the variables ε_o , ω , μ , k and h on the temperature distribution after 3,000 cycles. Specifically:

a. (Fig. 1) - Keep μ_o constant, choose a small h_o value: Vary k_o to $10k_o$ and $100k_o$, the result shows very small effects. That is, the heat generated can not be dissipated to the outside if h_o is small.

b. (Fig. 2) - Keep μ_o constant, choose h_o to be nearly the same magnitude as $100k_o$: Vary k_o to $10k_o$ and $100k_o$. The temperature rise for large k_o may decrease by about 25% than for small k_o .

c. (Fig. 3) - Keep μ_o constant, choose a small k_o value: Vary h_o to $10h_o$, and $100h_o$. Because of k_o is small, the heat still can not be transferred outside. Large h_o only lowers the temperature near the rubber pad surface.

d. (Fig. 4) - Keep μ_o constant, let k_o be large: Vary h_o to $10h_o$ and $100h_o$. The large h_o can reduce the temperature rise in the rubber by about 25% than small h_o .

e. (Fig. 5, Fig. 6) - Keep k_o and h_o value constant: Vary viscosity μ_o to $0.1\mu_o$; the result shows a significiant effect on reducing the temperature rise. The temperature rise is only about 20% of the original value. Of course, larger k_0 and h_0 will still help in transferring heat and lowering the temperature rise.

f. Vary the frequency and the amplitude give the same result as varying viscosity. It is important to note that ΔQ depends on the <u>square</u> of the deformation amplitude ε_0 .

From the above we can conclude that the following are desirable design decisions on the elastomeric track pads and their overall support system design:

a. Since increasing the thermal conductivity reduces the temperature rise inside the pads, we should investigate the effects of incorporating conductive additives to the mechanical properties of elastomers and decide whether it is desirable to use them or not.

b. Consider track fender (cover) designs which would increase the convective heat transfer during the rest period; forced air systems under the fender may be good design conditions

c. The role of viscosity is predominant. We should consider the effect of material parameters, such as chain flexibility and crosslink density on the value of both the viscosity and the flow activation energy b .

d. The effect of the frequency should be related to the field effect of "rough" roads. The presence of obstacles increase the frequency of deformation.

e. The large effect of the amplitude of deformation on the rate of heat generation may be related to the presence of sharp (small contact area) obstacles. The smaller the contact area, the larger the pressure (τ_{xx}) and thus the larger the deformation. The exact relationship between the obstacle sharpness and ε_0 is not clear because of the viscoelastic nature of the pad.

The above are simple, approximate design guides useful as such, and in providing us with directions for extending the work into the non-linear response regime and is devising the appropriate tests to compare with field test results.

B. MOLECULAR CONSIDERATION OF THE HEAT GENERATION PROBLEM

I. Dependence of Elastomer Properties on Molecular Parameters

In addition to its influence on chemical degradation there is an additional effect of chemical structure on the basic strength properties of a vulcanized elastomer, since fracture involves some mechanical rupture of molecular chains by rupture of chemical bonds along the chain, as has been verified by electron spin resonance (ESR) measurements, and chemical bond strength varies with the type of chemical bond. One of the most important parameters of molecular structure is the degree of cross-linking. The type of cross-linking is also a variable which may be significant in certain cases (e.g., where chemical degradation is involved).

In representing viscoelastic properties in our matrix, no distinction is made between linear and non-linear viscoelasticity. This is not a critical point where trends are being discussed, but it immediately becomes critical when quantitative calculations are made. Non-linear viscoelasticity is one of the difficult areas in the theory of mechanical properties of polymers. There is no unified theory to treat this type of behavior (as there is for linear viscoelasticity). There is therefore a great need for research in this area, and for the working out of even empirical methods to handle this type of behavior, since it is the type of behavior encountered in many use applications (including the tank tread application under consideration here) and in many of the areas of polymer processing, such as injection molding and other fabrication methods.

Another point of importance is that the matrix presented here refers to the molecular parameters of the polymer itself. Other components are usually also involved in a rubber compound, such as carbon black, antioxidants, curing agents, etc. However, since only trends are specified, the relationships shown here should remain valid even for commercial compounds containing other ingredients besides the elastomer itself.

In developing an algorithm to predict service life from molecular parameters (an ambitious, but very attractive goal, related to the concept of "tailor-making" molecules), the molecular parameters that will be most useful are again those referring to the general characteristics of the molecules (the "physical" characteristics, one might say) rather than their exact chemical structure. Put another way, polymer chemistry usually must be translated into polymer physics before physical and mechanical properties can be predicted successfully.

In relating the mechanical behavior of elastomers to molecular parameters, six different areas of description can be distinguished:

1) Chemical structure (atomic composition and chemical bonds)
2) Molecular characteristics (chain stiffness, symmetry, freedom of rotation of side groups, stereospecificity, molecular weight)
3) Intermolecular forces (secondary bonding, hydrogen bonding, dipole-dipole interactions)
4) Supermolecular structure (cross-linking, aggregation and crystallization)
5) Bulk properties (glass transition temperature, thermal conductivity, density, etc.)
6) Mechanical behavior (heat generation, viscoelastic properties, fracture strength)

These categories are sometimes closely related, and may even overlap in some ways, but they clearly represent a hierarchy connecting the basic chemical constitution of the molecule and the mechanical properties observed under use conditions. The possibility of making this connection is the basis of the concept of "tailor-making" polymer molecules to produce desired final properties. This can, in fact, be carried out in practice in some cases. An example is the successful development by DuPont recently of Qiana fiber, which was done by a highly logical approach in which the proper molecular parameters were brought together in order to produce a type of nylon fiber which was more like silk. The resulting fiber obtained was exactly what was predicted and desired.

In an approximate way, the above list of six categories could be reduced to two, in which the first four categories could all be lumped as "molecular parameters" and the last two as "properties." Our interest here is basically in viscoelastic (V-E) properties and strength properties, since these are most closely related to the problem of the service life of tank track shoes. However, heat-up during use depends not only on the viscoelastic properties (the effective "internal viscosity" used in the heat generation calculations in the other section of this report) but also on such properties as thermal conductivity (also used in these heat generation calculations); these other properties of secondary importance must therefore also be taken into account. However, as noted there, the internal viscosity (a viscoelastic property) is the property of major and overriding importance in the heat generation.

Present theory does not allow a quantitative calculation of properties from molecular parameters (this would be an ultimate goal for polymer scientists). However, solid and quantitative theoretical connections can be made between the viscoelastic properties of a material (as expressed by the experimentally measured relaxation time spectrum) and the heat generated in a cyclic deformation experiment. The relaxation time spectrum itself, however, cannot at the present time be predicted from a knowledge of the chemical structure of the polymer molecule.

Qualitative connections can, however, be made between molecular parameters (preferably the general characteristics of the molecule rather than the exact chemical structure) and observed bulk properties. As a concise way of doing this, we have set up a matrix relating significant general molecular parameters to viscoelastic and strength properties, and also to the other properties involved in the heat generation calculations given in the other section of this report. To indicate trends, we have used a + sign to indicate that the property increases or improves as the molecular parameter increases, and a - sign to indicate a decrease. Parentheses are used when the effect is small, and a 0 is given where there is no appreciable effect. This is shown on Table I.

One important molecular parameter which cannot be included in this table is the chemical composition of the molecule, since this is something that cannot be described in terms of plus or minus trends. However, it is a very important parameter in terms of thermal and oxidative (both oxygen and ozone) degradation, since these are chemical processes specifically related to the chemical composition of the molecule. This degradation has a very serious effect on the fracture strength properties of elastomers; even a relatively small amount of degradation will produce an appreciable decrease of tensile strength. Here is a case also where the properties themselves interact, since heat generation from viscoelastic deformation will raise the temperature of the elastomer, and this in turn will speed up the rate of chemical degradation. Another case of properties which are related are the glass transition temperature (T_g) and he V-E activation energy; a lower value of T_g for the polymer will produce a lower value of E_{act} (defined by the Arrhenius rate equation) for any given experimental temperature.

TABLE I

Bulk Props.	MOLECULAR PROPERTIES Chain Stiffness	Flexibility of Side Groups	Degree of Cross-linking	Molecular Weight	Intermolecular Forces	Molecular Symmetry
Dynamic Viscosity	+	+	(+)	(-)	+	0
Density	0	(+)	(+)	(+)	+	+
Heat Capacity	-	+	(-)	0	(+)	0
Heat Conductivity	-	+	(-)	0	(+)	0
Heat Transfer Coeff.	-	+	(-)	0	(+)	0
V-E Activation Energy	+	-	(+)	0	+	(-)
Glass Transition Temp. (Brittle Temp.)	+	-	+	+	+	-
Mech. Fracture Strength (Abrasion Resistance)	(+)	(-)	+	+	+	0
Crystallization or Aggregation	-	+	-	(-)	+	+

II. Heat Generation - Molecular Phenomena

In relating heat generation to molecular structure (which would be one of our first goals in making a connection with molecular parameters), a number of factors would have to be analyzed and taken into account. The nature of the chemical structure itself may not allow a direct prediction of the internal viscosity (or heat generation) of the rubber. As an illustration of this point, it can be pointed out that two rubbers which are known to show opposite extremes in heat generation -- natural rubber, which has very low heat generation, and butyl rubber, which shows unusually high heat generation -- are both hydrocarbon rubbers. Our explanation which has been offered for the high internal friction of butyl rubber is the "bumpiness" of the molecules, which contain very many methyl side groups, in contrast to the natural rubber molecule, which contains relatively few methyl side groups. This suggests attempting a correlation with the number and size of the side groups on the polymer chain. Some further suggestive evidence in this direction is the fact that SBR rubber (styrene/butadine copolymer) is also known to have a relatively high heat generation, and this polymer has large pherryl rings as side groups. Following out this reasoning, the polymer with the lowest heat dissipation would be predicted to be linear (non-branched) polyethylene. Unfortunately this polymer (usually known as high density polyethylene, or HDPE) is crystalline, and is therefore a plastic rather than a rubber. Even branched polyethylene is crystalline and not an elastomer. A certain nuumber of small side group seems to be necessary on a polyethylene chain to make it a rubber; copolymerization of ethylene with propylene (in approximately 50/50 ratio) gives a polymer which is an elastomer (called EPR rubber). A full analysis of heat generation from this point of view should be carried out, and we would plan to do this in the next phase of this project.

The "bumpiness" of the polymer molecules produces some "interlocking" of adjacent molecules, which would be an analog at the molecular level "surface roughness" which is known to be an important factor in the sliding friction of solid surfaces. This type of interlocking might be reflected in an identifiable way in the bulk density of the rubber. A possible correlation of heat generation with bulk density should therefore also be checked.

III. HEAT GENERATION MEASUREMENTS

(Linear Sinusoidal Deformation)

Equipment has been designed and constructed to measure heat generation in the linear range during sinusoidal deformation of a rubber sample. The instrument is being operated in shear, but can easily be adapted to operate in tension, flexure or compression. A temperature box has also been built around the sample so that experiments can be carried out at different ambient temperatures. A sketch of the equipment is shown in an attached figure.

The sample is deformed by a 1/8 HP motor fitted with an eccentric of adjustable radius to allow the amplitude of deformation to be varied. The frequency can be varied by a variable speed drive attached to the motor which changes the RPM of the motor by a simple manual dial. The frequency is the same as the motor RPM, which is measured with ± 1% error by an accurate stroboscope.

The rubber sample used for shear measurements is in the form of a sandwich with two thick rubber blocks on opposite sides of a central driving plate (attached by a connecting rod to the motor eccentric), with two metal clamping plates on the outside of the sandwich. The rubber blocks are one-inch thick, made by cementing two pieces of rubber cut from a cured sheet 1/2 inch thick with epoxy glue. The cross-sectional area is a little over one spuare inch. The two rubber blocks are cemented to the center plate and the outside plates by epoxy, and a copper-constantan thermocouple is inserted in the interior of one of the blocks. Interval sample temperature is read using a leeds and Northrup potentiometer.

Results of a heat generation measurement are shown in an attached graph. The exact quantitative nature of this graph should not be overemphasized because this experiment was carried out using a friction grip between the rubber blocks and the central plate (with emery cloth glued to the surfaces of the central plate), and it is believed that some slippage took place at this point which would add a frictional contribution to the heat generation, in addition to the internal viscoelastic lossed (sometimes called "internal friction" since it arises from the polymer molecules rubbing past each other). However, the general shape of such a heat generation curve is evident. There is an initial rise (almost linear) as heat starts to be developed, and heat loss from the sample is negligible. There is then a curving off as heat loss (due to thermal conductivity) starts to become significant; and then a final leveling off of the curve as a steady state is reached in which heat generation is just balanced by heat loss from the sample. Under the experimental conditions used here, the temperature rise observed is only a few degrees centigrade. The amount of heat generated could of course be increased by using larger amplitudes of deformation. This would require the use of a heavier-duty motor (greater HP), since the work of deformation would be greater. This would also take us into the non-linear region, as we move into the region of greater power and correspondingly greater lossed (of work or energy converted into heat). And this is something that we will, in fact, want to do as our studies extend into the non-linear region. But these linear experiments, at relatively small deformations, will allow comparisons to be made with a linear viscoelastic model, and related heat capacity and thermal conductivety data.

The linear viscoelastic model discussed in a previous section (Voigt Model) involved a single elastic spring and viscous dashpot and can be expressed in terms of an effective viscosity coeffecient, ν, as a representation of the "internal viscosity" (molecular friction effects) of the system. In the linear region, this value is a constant. Actually, it is known that this internal friction arises from a whole spectrum of molecular responses having different relaxation times, λ. The energy losses can be expressed in a more detailed way by a "loss modulus", E'', which can in turn be expressed as an integral:

$$E'' = \int_0^\infty H(\lambda)\,\frac{w\,\lambda}{1+w^2\ {}^2}\,d\lambda$$

where the function $H(\lambda)$ is the linear viscoelastic "distribution of relaxation times" function, w is circular frequency and λ is the relaxation time ($\lambda = \nu/E$ in a simple model).

This is the theoretically complete and rigorous formulation for the linear case, but in order to use it, it is obviously necessary to know the distribution function $H(\lambda)$. This function cannot be calculated at the present time from a prior theory, but can only be determined by experimental measurement. A necessary part of our experimental program will therefore be the measurement of this relaxation time spectrum for the rubber samples used in our heat generation measurements. This can be done to some extent using the Rheovibron dynamic mechanical instrument which we already have (in which different fixed frequencies can be used, but where a continuous temperature scan is the primary variable). But this should be supplemented by measurements in which frequency can be varied as a primary variable, at a fixed temperature. Although the $H(\lambda)$ function should in principle be known from $\lambda = 0$ to $\lambda = \infty$ for substitution in the above integral , the multiplying function in the integral is such that a knowledge of the $H(\lambda)$ function for values of ($\lambda = 1/\omega$) one or two orders of magnitude above and below the experimental frequency, , is all that is necessary in actual practice.

We will eventually want to measure <u>non-linear</u> viscoelastic spectra as a function of frequency and temperature. Commercial instruments are now available which are able to do such measurements.

C. Mathematical Fitting of Non-Linear Stress-Strain Curves

One approach which is being developed to treat the non-linear case is an empirical mathematical fitting of the stress-strain curve using a power series. By fitting both the extension and retraction parts of the stress-strain cycle by such a series, the energy loss (or heat generation) per cycle can be calculated as the difference in area under the two curves, and this can also be expressed as a power series, where the coefficients are simply obtained by subtraction of the corresponding coefficients for the extension and retraction parts of the cycle.

This is a very simple and convenient type of mathematical representation, and the question is how adequately actual experimental stress-strain cycles can be fitted in this way. We plan to measure stress-strain cycles ourselves on the Instron machine, as soon as rubber samples become available. However, we have made some preliminary tests of the method using stress-strain cycles from the published literature. It is known that the stress-strain relation for an elastomer is very non-linear, with a strongly pronounced S-shape when the stress or strain is taken to high values. The non-linear shape at lower stresses and strains can be interpreted by the Theory of Ideal Rubber Elasticity (developed by Ruhn in the 1930's). This predicts a stress-strain function of the form:

$$\sigma = N\,k\,T\,[\alpha - \frac{1}{\alpha^2}]$$

where σ is stress, N is the number of "network chains" per unit volume (these are the sections of molecules between cross-links), k is Boltzmann's constant, T is absolute temperature and α is the extension ratio, or ratio between stretched and unstretched length of the sample ($\alpha = \ell/\ell_0$). This theory applies to a vulcanized rubber, and describes the equilibrium elastic behavior only, not the viscoelastic behavior. This therefore gives no description of the heat generation phenomenon, since the extension and

and retraction parts of a stress-strain cycle would be identical, according to this theory, so that there would be no energy loss or heat generation.

A stress-strain function of slightly different shape is obtained by modifying the above equation into the form:

$$\frac{\alpha}{[\alpha - \frac{1}{\alpha^2}]} = C_1 + \frac{C_2}{\alpha}$$

This is known as the Mooney-Rivlin Equation. However this still represents only the first curvature of the overall S-shape. The large upswing of the curve at high strains is usually attributed to a combination of two effects: crystallization of the rubber, and the fact that the network chains are reaching their limit of extensibility. The relative importance of these two factors will be different for different rubbers, since the tendency to crystallize varies considerably for different rubbers. No quantitative theory has been proposed to explain this final up-moving region of the stress-strain curve.

It is clear, in addition, that no elastic theory (even if one were available which could represent the entire S-shaped stress-strain curve) would be able to predict the heat generation effect. For this, a difference in extension and retraction curves is necessary, and this type of behavior requires a viscoelastic theory. A viscoelastic theory for this high-strain behavior is similarly not available at the present time. In view of these various difficulties in applying currently available rubber elasticity theory to the problem of heat generation during cycling to large strains, the idea of developing a purely empirical approach which provides the needed mathematical representation and which can then be used in practical calculations seems to be a very reasonable and useful approach.

In checking the effectiveness of the power series representation of actual high-strain experimental curves (where the S-shape is very pronounced), a power series representation up to the sixth power:

$$\alpha = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5 + a_6 x^6$$

was first tried, and the accuracy of representation was found to be surprisingly good. The term a_0 is always equal to zero and can be dropped, since the curve starts at the origin. The coefficients a_i are obtained by choosing six points on the experimental curve to be fitted exactly, and then solving the corresponding matrix with a computer program. This calculation can be carried out so easily that the polynomial can be readily extended to additional terms. We have also checked the fit obtained using a polynomial up to the eighth power ($a_8 x^8$) and up to the tenth power ($a_{10} x^{10}$). The fit is a little improved by using the eighth-power rather than the sixth-power polynomial, but the use of a tenth-power polynomial seems to give very little additional improvement. The eighth power polynomial gives a very satisfactory fit.

The sort of fit obtained is illustrated in the attached figure showing the first stress-strain cycle of a natural rubber sample (taken from Harwood, Mullins, and Payne, Rubber Chem. Tech., 39, 814 (1966)), using an eighth order polynomial. The strain parameter used was the fraction of the maximum strain ($x = \varepsilon/\varepsilon_{max}$) in this particular case. The points used for curve fitting are shown as circles, and the solid curves are both experimental and calculated curves, since there is essentially a negligible difference between them (experimental and calculated curves superimpose). The coefficients obtained for the extension and retraction parts of the cycle are listed below the graph.

A similar fit for the second cycle of extension (not shown here) was equally satisfactory. There is a gradual change of shape of the cycle as cycling continues, but this finally settles down to a steady-state shape, corresponding to the experimental conditions (here, a constant rate of length change, as in most stress-strain tests today). This steady state corresponds to a constant loss per cycle.

Further investigation of this method of curve-fitting is being carried out. The shape of the stress-strain curve is affected by strain rate, temperature, degree of maximum strain, elastomer type, and degree of cross-linking. The usefulness of this power series method can be extended by learning how to take these factors into account. The results obtained thus far verify that this approach can be a very useful one, particularly in handling realistic, high-strain situations such as are encountered with tank treads and with other heavy-duty service conditions.

D. EFFECTS OF MANUFACTURING ON PAD STRUCTURE

During the injection molding of track pads, a prepolymer, or linear polymer, together with additives and a crosslinking agent are loaded into hot molds where "curing" occurs. The reacting pad is under non-uniform temperature during molding because of the heat transfer and the exothermic heat of reaction. This fact leads to the real possibility of structural (crosslink density) non-uniformities of the pads. These non-uniformities, in turn, may contribute to the pad performance and failure.

The problem may be simulated by considering a mold very wide in the x and y directions and of thickness 2H in the z direction. Assuming:

a. A well mixed heating fluid

b. No flow

c. No diffusion

d. Constant conductivity,

we arrive at the following equations governing the reactive species and energy balances.

$$\frac{dc_A}{dt} = R_A$$

$$\rho C_p \frac{\partial T}{\partial t} = K \frac{\partial^2 T}{\partial z^2} + \Delta H_R R_A$$

where $R_A = K_r c_A^n$, the reaction rate

H_R = is the heat of raction

$K_r = K_r^o e^{E_r/RT}$ is the reaction rate constant.

The initial conditions are

$$T(x_i, 0) = T_0$$

$$C_A(x_i, 0) = C_{A_0}$$

The thermal boundary conditions are:

$$\frac{\partial T}{\partial z}(x, y, z = 0, t) = 0 \quad \text{(symmetry)}$$

and

$$T(x, y, t, z = \pm H) = Tw \quad \text{(isothermal)}$$

or

$$\frac{\partial T}{\partial z}(x, y, t, z = \pm H) = 0 \text{ (adiabatic)}$$

or

$$K \frac{\partial T}{\partial z}(x, y, t, z = \pm H) = h(T_{z=\pm H} - T_{mold})$$

where h = is the heat transfer coefficient.

Experimentally, we need to determine K, C_p and H and R_A. The last two quantities we plan to determine using the Differential Scanning Calorimeter and/or the Mooney Viscosimeter.

The goal of this simultaneous work is to yield a relation such as:

$$C_A(x_i, t) = \nu (x_i, t) = f (K, C_p, K_r, n, H_R, T_W, h)$$

where: ν ----crosslink density.

which would be the "molding design equation." Work is in progress in this area.

APPENDIX: Values of material and operation variables used in the simulation; dimensionless variables

VIS: viscosity $\mu_0 = 10^5$ poise $= 10^5$ g/sec cm
CP: heat capacity $C_v = 0.5$ cal/kg
DEN: density $\rho = 0.94$ g/cm^3
CON: thermal conductivity $K_0 = 5 \times 10^{-4}$ cal/cm sec °K
CVE: convective heat transfer coefficient $h_0 = 5.6 \times 10^{-4}$ cal/cm^2 sec °K
G: modulus $G = 10^6$ g/cm sec^2
AE: activation energy $b = 1/50$ °K^{-1}

AMP: amplitude $\varepsilon = 0.05$
FRE: frequency $\omega = 100$ cycle/sec
TA: temp. of surrounding $T_a = 25$ °C
TINC: increment time 0.1 sec $= t_{in}$
TFIN: final time $t_f = 30$ sec
DIS: length of the sample/2 dis $= 2$ cm
PER: period of oscillation $t_p = 0.01$ sec.

Dimensionless groups:

Viscosity: $\mu' = \frac{\mu}{\mu_0}$

Thermal conductivity: $k' = \frac{k}{k_0}$

Convective coefficient: $h' = \frac{h}{h_0}$

Temp.: $\theta = \frac{T - T_a}{T_a}$

Time: $\tau = \frac{\alpha t}{l^2}$ $\left(\alpha = \frac{k}{\rho C}\right)$

Length: $\xi = \frac{z}{l}$

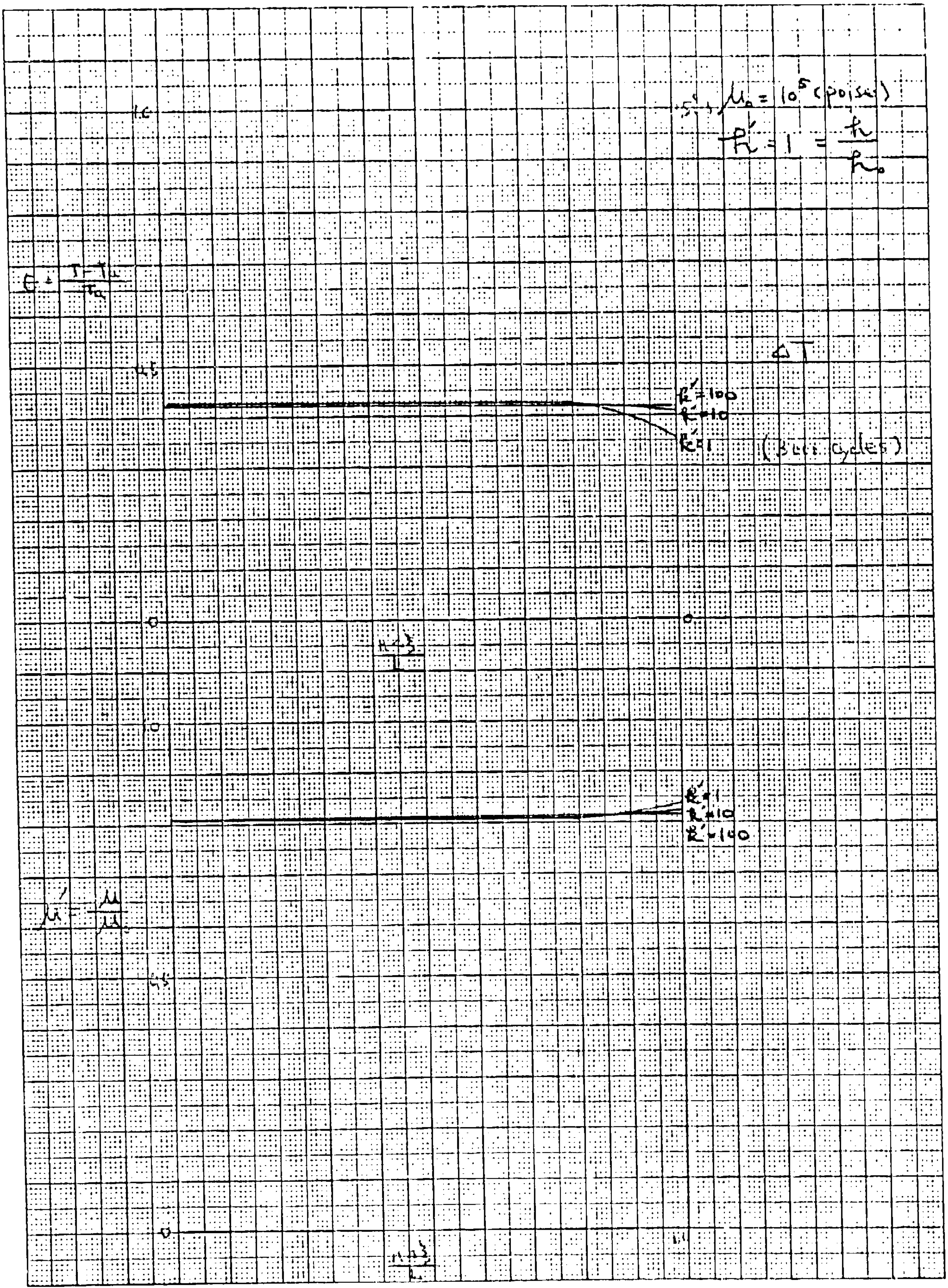
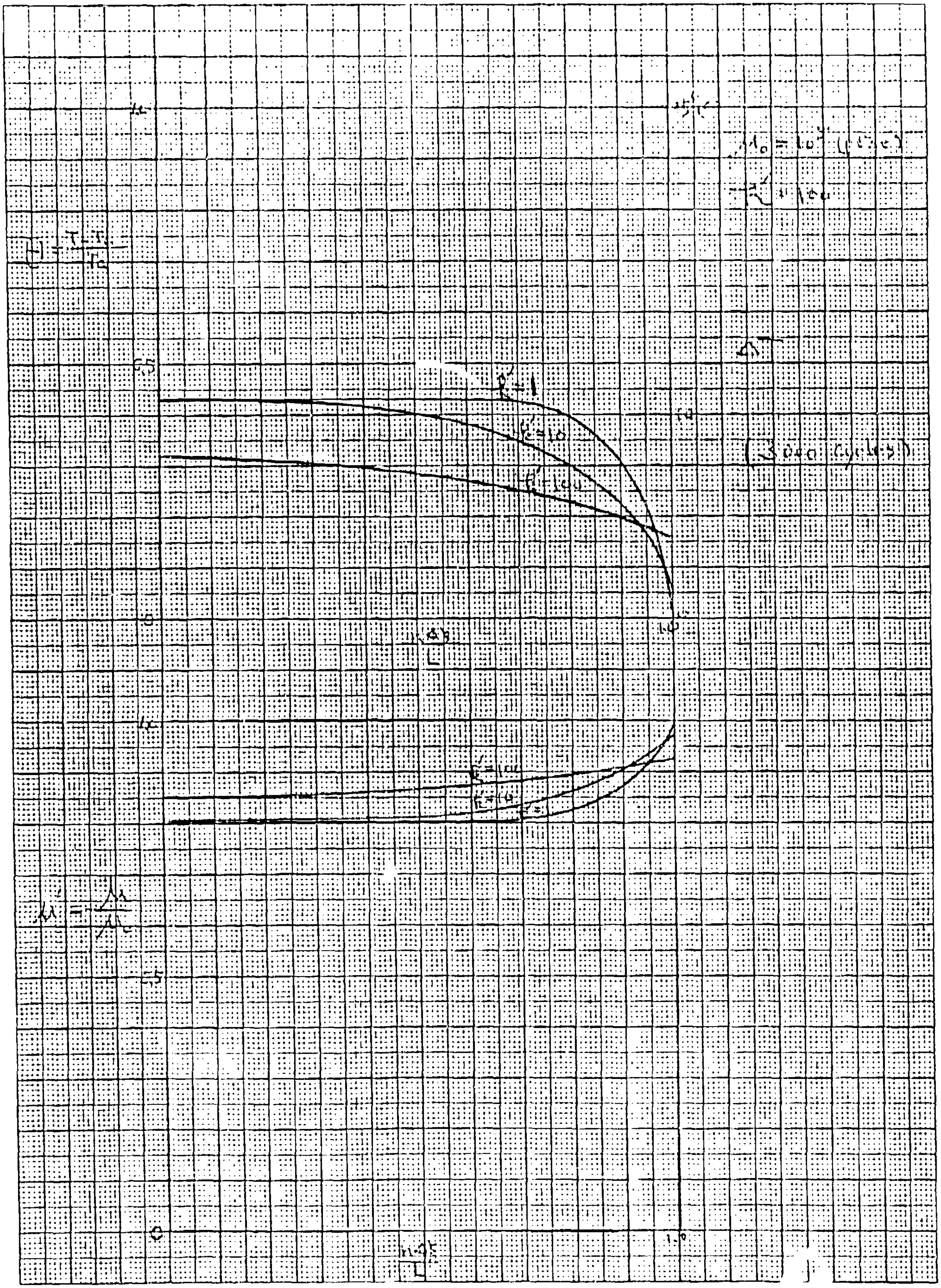

$\mu_0 = 10^5$ (poise)
$k' = 1 = \frac{k}{k_0}$
1.0
$\theta = \frac{T - T_a}{T_a}$
0.5
ΔT
$k' = 100$
$k' = 10$
$k' = 1$
(3000 cycles)
0
$k' = 1$
$k' = 10$
$k' = 100$
$\mu' = \frac{\mu}{\mu_0}$
0.5
0

Fig. 1


1.0
Θ = (T - T_i)/T_c
.5
ρ'=1
ρ'=10
ρ'=100
μ₀ = 10³
ρ' = 100
(3040 cycles)
0
1.0
-1.0
ρ'=100
ρ'=10
ρ'=1
μ' = μ/μ_c
-.5
0
1.0
n-.25/L

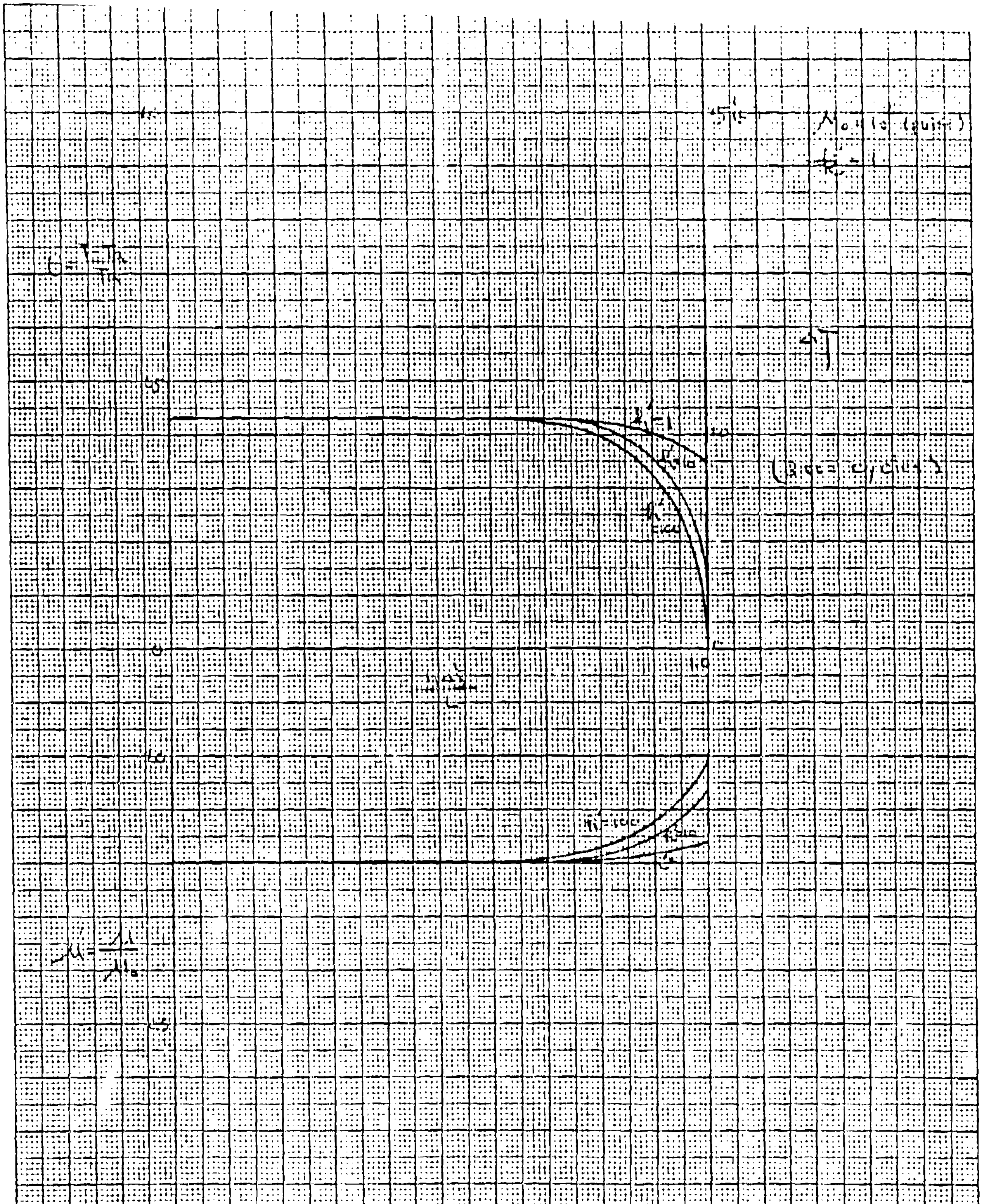


Fig. 3

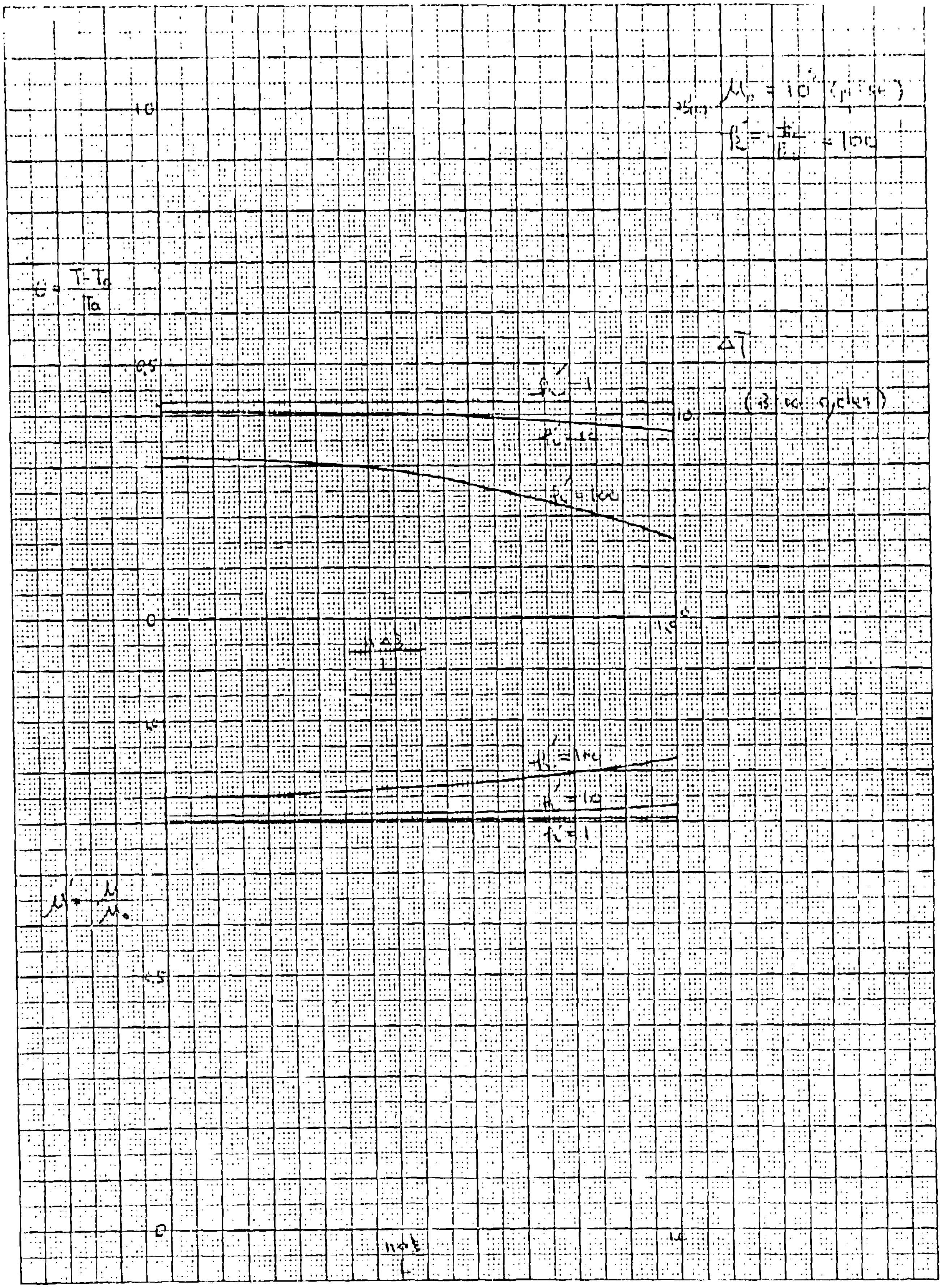

μ₀ = 10⁻² (poise)
β' = 100
1.0
θ = (T−T₀)/T₀
0.5
ΔT
(10 °C)
β' = 1
β' = 10
β' = 100
1.0
0
1.0
μ' = μ/μ₀
.5
0
1.0

Fig. 4

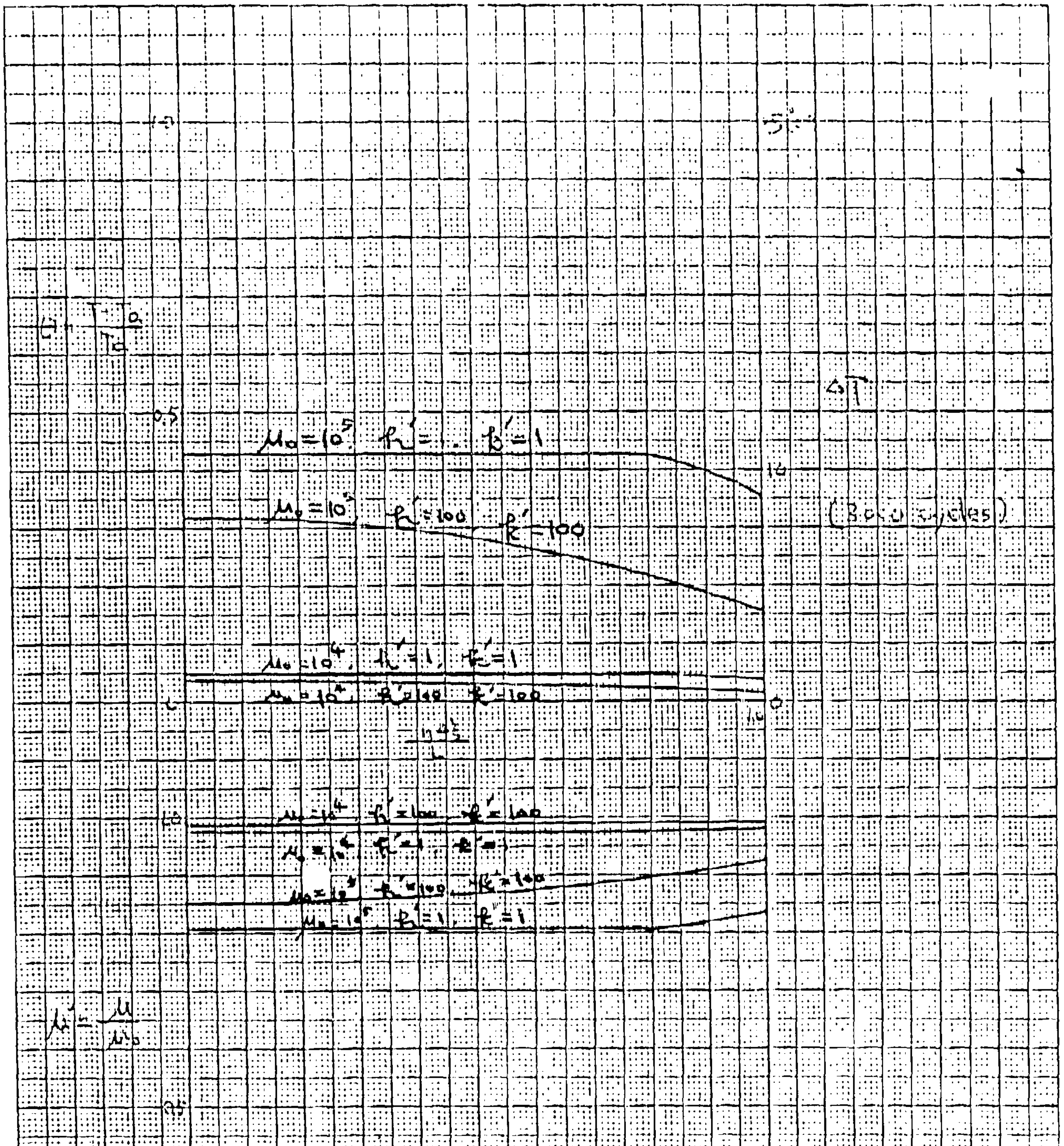

θ = (T − T0)/T0
1.5
5
ΔT
0.5
μ0 = 10^5, k' = 1, k' = 1
10
μ0 = 10^5, k' = 100, k' = 100
(Bo... cycles)
μ0 = 10^4, k' = 1, k' = 1
μ0 = 10^4, k' = 100, k' = 100
1.0
μ0 = 10^4, k' = 100, k' = 100
μ0 = 10^4, k' = 1, k' = 1
μ0 = 10^5, k' = 100, k' = 100
μ0 = 10^5, k' = 1, k' = 1
μ' = μ/μ0
0
10

Fig. 5

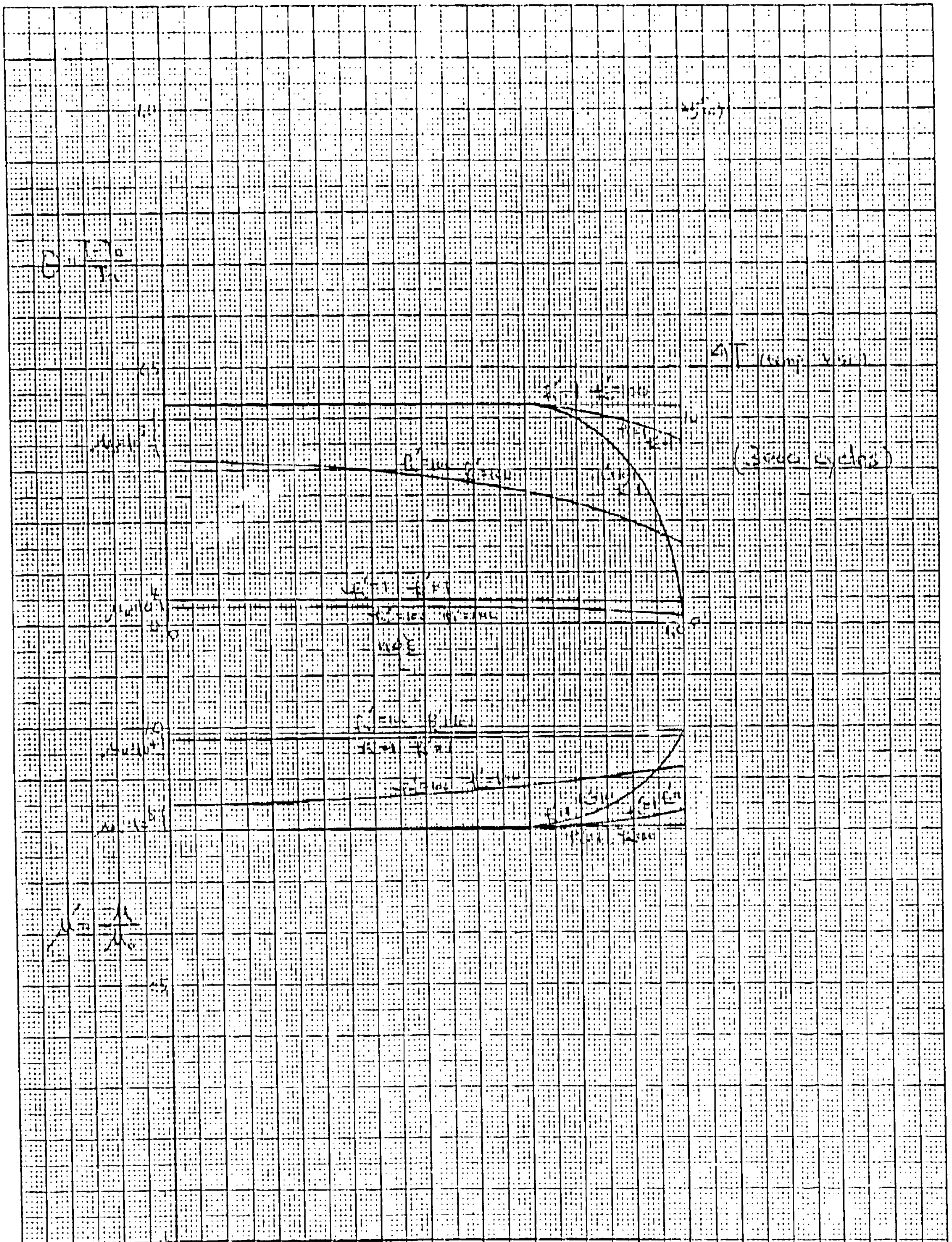


Fig. 1

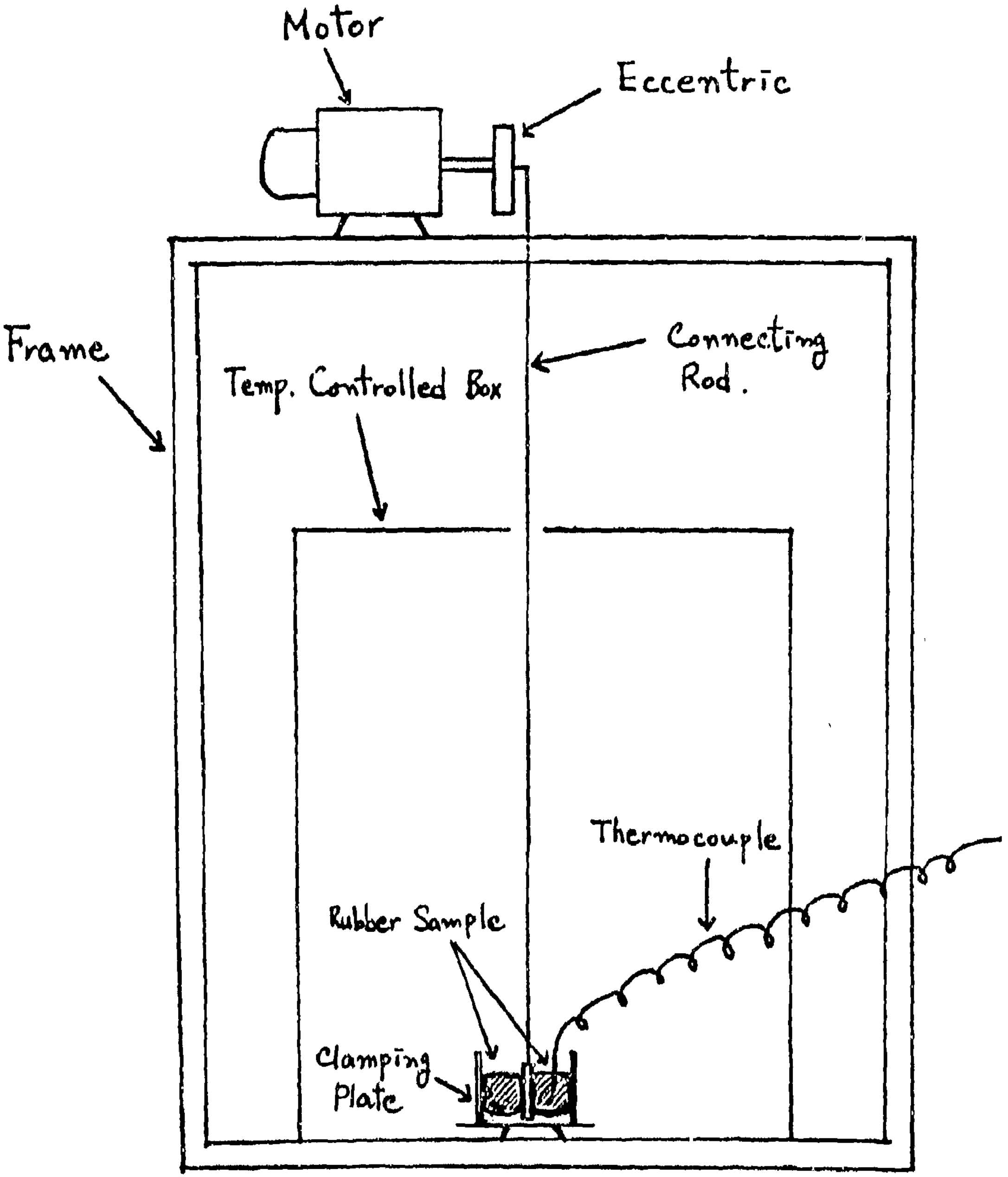


Fig – 7 : Instrument for Heat Generation Measurement.

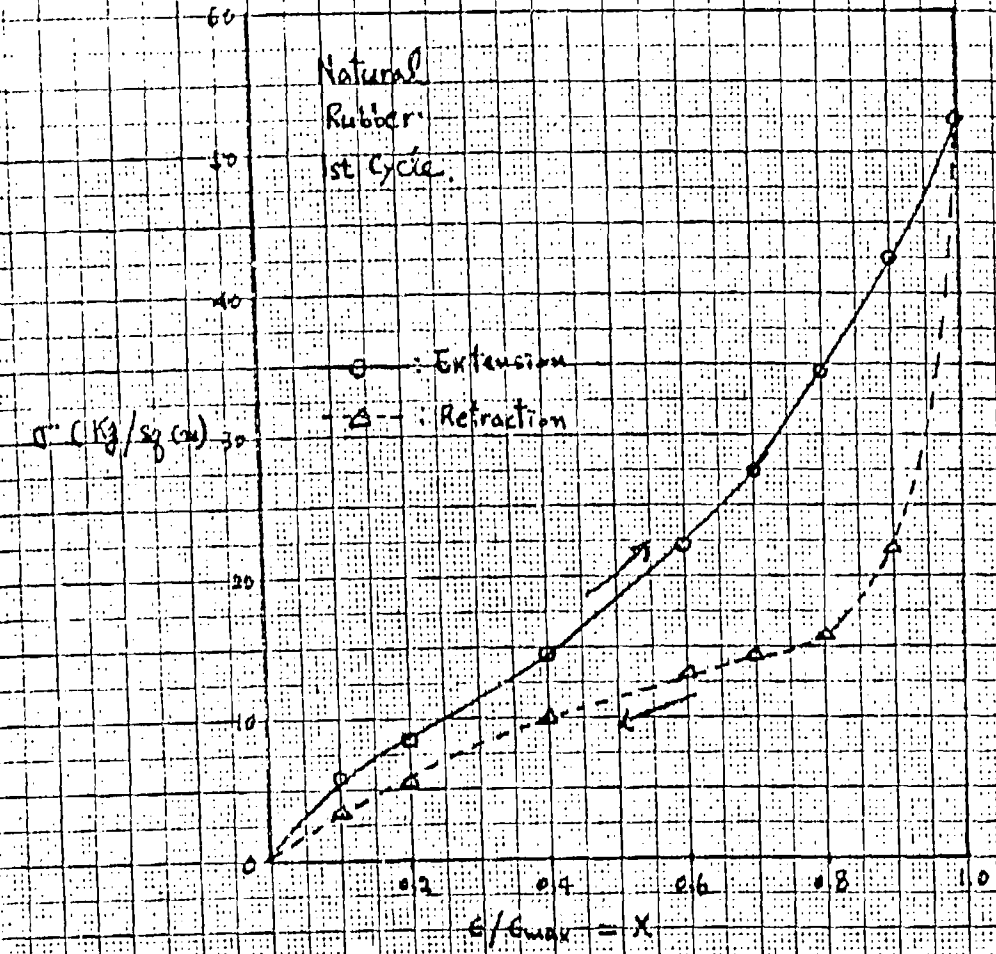


Fig-8: Stress-Strain Loop of Natural Rubber
(1st Cycle)

Empirical Eq. of S-S curve

$$\sigma(x) = \sum_{n=1}^{8} a_n x^n$$

Coefficient	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
Extension	108.4	-764.2	3244	-7255	9169	-6476	2394	-367.6
Retraction	45.47	-85.93	107.5	157.1	-1406	3580	-7038	1692