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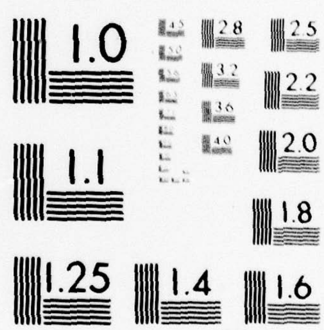

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SIDELOBE LEVELS FOR ARRAYS WHICH HAVE TAPERED EXCITATIONS WITH RANDOM ERRORS

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Submitted to:
Navy Electronics Laboratory
San Diego, Calif.

Approved:	Approved:
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ABSTRACT

The probabilities that the sidelobes will exceed some specified level are compared for arrays with uniform and tapered excitations when random errors are present in the amplitudes and phases of the array element excitations. While in the absence of errors the tapered excitation will usually yield lower sidelobes than the uniform excitation, with random errors present the situation can be reversed, i.e., due to the errors it may be more probable that the tapered array sidelobes will exceed some level than that the uniform array sidelobes will do so. Numerical results are presented for a "triangular" excitation distribution. A new expression for the probability that a sidelobe will exceed a specified level is also obtained.

ACKNOWLEDGMENT

The study reported in this paper has been performed under Contract NObsr-93023 with the U.S. Navy, Bureau of Ships. The opinions expressed are those of the author, and not necessarily those of the sponsoring agency.

The author is indebted to Walton Graham for the suggestion of the problem, and to John Gobins for programming the numerical calculations on a computer.

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A. INTRODUCTION

The effects which random errors in the array element excitations have on the far field patterns of the arrays are fairly well known.[1-4] In general, one can expect that the presence of random errors will increase the far field sidelobe levels. One might seek to reduce the sidelobe levels by choosing an element excitation distribution which without the presence of errors will have a low sidelobe level to begin with, hoping that the increase in sidelobe levels due to the errors will be tolerable. Theoretically almost any sidelobe level can be achieved if errors are absent; however, to achieve this, complicated nonuniform (or "shaded" or "tapered") distributions are necessary, and for a given array element rms error excitation σ the sidelobe levels of the nonuniform distribution arrays may be increased more than the levels of a uniform distribution array.[5] Thus it is not at all obvious that the sidelobe levels can be improved by tapering the excitation distribution, if random errors are present; i.e., while the no-error levels are lower for tapered arrays, the increase in levels due to errors is larger than for uniform excitation arrays, and therefore, it is not clear whether the final level with errors present is lower or higher for tapered arrays. The effects of tapering will be discussed in this study, and some numerical results will be presented. Although the expression "tapered excitation" is usually used to describe an excitation which has a maximum amplitude at the center of the array, and which decreases uniformly as one approaches the edges of the array, for simplicity we will use the word "tapered" to describe any nonuniform excitation. Moreover, most of the time we will be concerned with arrays in which the excitation phase distributions are the same as in a uniform excitation array, and only the excitation magnitudes are nonuniform, although some of the results will apply also to arrays in which the phases are varied. To be able to make comparisons, the uniform excitation array will be chosen as the standard against which the tapered arrays will be tested.

If the sidelobe levels in a specified direction in the far field with errors present are F_u and F_t for the uniform and tapered distributions respectively, and if $P(F > Y_o)$ is the probability with which a level F may exceed some specified value Y_o, we may define an improvement by tapering I_T as

$$I_T = P(F_u > Y_o) - P(F_t > Y_o) \tag{1}$$

which will be positive if tapering has decreased the probability that the far field pattern in the given direction will exceed the level Y_o for a specified rms error excitation σ.

Let us consider what we know about I_T without performing any involved analyses. When the array element rms error excitation σ approaches zero, we have no errors, and the sidelobe level is lower for the tapered array than for the uniform array, therefore

I_T should be positive (this is mathematically obvious if $F_{ou} > Y_o > F_{ot}$, where F_{ou} and F_{ot} are the sidelobe levels with errors absent, because then $P(F_u > Y_o) \rightarrow 100\%$ and $P(F_t > Y_o) \rightarrow 0\%$ as $\sigma \rightarrow 0$). When $\sigma \rightarrow \infty$, the element excitation is completely incoherent, it does not matter what the no-error distribution is, tapered or uniform, because the no-error excitation is insignificant compared to the error excitation, therefore

$$P(F_u > Y_o) \rightarrow P(F_t > Y_o) \text{ or } I_T \rightarrow 0 \text{ as } \sigma \rightarrow \infty$$

Thus we might expect the graph of I_T vs. σ to have one of the shapes shown in Figure 1, where we have also indicated the points at $\sigma = 0$ and $\sigma \rightarrow \infty$. Of course, more complicated curves are also possible. We intend to show that at least in some cases I_T approaches zero for $\sigma \rightarrow \infty$ from the negative side, therefore in those cases the curve must cross the $I_T = 0$ axis, and there have to be values of σ for which I_T is positive, and there are also values of σ for which it is negative, i.e., the curve might be similar to the dashed curve in Figure 1.

We will first examine the general equations, and then we will calculate I_T for some special cases.

B. THE GENERAL EQUATIONS

Let us now consider an array of arbitrary configuration. In the absence of errors the far field pattern can be written as

$$F_o(\Omega_o,\Omega) = \frac{1}{N|E_o|} \sum_{n=1}^{N} E_{no}(\Omega_o) e^{i\gamma_n(\Omega)} \qquad (2)$$

where N is the number of elements, $E_{no}(\Omega_o)$ is the complex excitation of the nth element designed so as to point the array main beam in the direction Ω_o, $\gamma_n(\Omega)$ is the phase of the signal from the nth element arriving at a far field point in the direction Ω, and E_o is some reference excitation used to normalize the pattern so that $F_o(\Omega_o, \Omega_o) = 1$; i.e., if we consider only amplitude tapering, so that the phase of $E_{no}(\Omega_o)$ is $-\gamma_n(\Omega_o)$ then

$$N|E_o| = \sum_{n=1}^{N} \left| E_{no}(\Omega_o) \right| \quad .$$

In the presence of random errors Δ_n, δ_n or r_n, α_n, see Figure 2, the far field pattern is

$$F(\Omega_o,\Omega) = \frac{1}{N|E_o|} \sum_{n=1}^{N} E_{no}(\Omega_o)(1 + \Delta_n) e^{i\gamma_n(\Omega) + i\delta_n} \qquad (3)$$

The phase error α_n is assumed to be uniformly distributed from $-\pi$ to π, and the amplitude error r_n is assumed to have a Rayleigh distribution

$$q\ (r_n) = (2r_n/\sigma^2)\ e^{-r_n^2/\sigma^2} \quad , \qquad (4)$$

where σ^2 is the variance, and σ the root-mean-square error.

The probability that a sidelobe with errors present will exceed a level Y_o is given by [1-4]

$$P(F>Y_o) = \frac{2}{\nu^2} \int_{Y_o}^{\infty} df\ f\ e^{-(F_o^2 + f^2)/\nu^2}\ I_o\left(\frac{2F_o f}{\nu^2}\right) \tag{5}$$

where for simplicity we have let $F = \left|F(\Omega_o,\Omega)\right|$ and $F_o = \left|F_o(\Omega_o,\Omega)\right|$, I_o is the modified Bessel function of the first kind, order zero, and

$$\nu^2 = \frac{\sigma^2}{\left(N\left|E_o\right|\right)^2} \sum_{n=1}^{N} \left|E_{no}\right|^2 = \sigma^2 \frac{\sum_{n=1}^{N} \left|E_{no}\right|^2}{\left(\sum_{n=1}^{N} \left|E_{no}\right|\right)^2} \tag{6}$$

$P(F>Y_o)$ is usually[1-4] expressed as an infinite integral as shown in Eq. (5). However, for numerical calculations it is sometimes more convenient to have an integral over a finite interval, therefore let us transform Eq. (5) into another form. Since

$$I_o(x) = (1/\pi) \int_{o}^{\pi} d\beta\, e^{x\cos\beta}, \tag{7}$$

Eq. (5) can be written as

$$P(F>Y_o) = \frac{2}{x\pi\nu^2} \int_{Y_o}^{\infty} df \int_{o}^{\pi} d\beta\ f\ e^{-(F_o^2 + f^2 - 2F_o f\cos\beta)/\nu^2} \tag{8}$$

The integral in Eq. (8) is over an infinite plane outside a circular area of radius Y_o, see Figure 3; f is the radial coordinate, and β the angular coordinate. Since

$$V^2 = F_o^2 + f^2 - 2F_o f \cos\beta \tag{9}$$

it is advantageous to shift from the f, β coordinate system to the V, γ coordinate system in Figure 3 in which case Equation (8) becomes

$$P(F>Y_o) = \frac{2}{\pi\nu^2} \int_0^{\pi} d\gamma \int_{V_o}^{\infty} dV\, V e^{-V^2/\nu^2} \tag{10}$$

where (for $Y_o \geq F_o$)

$$V_o = -F_o \cos\gamma + (Y_o^2 - F_o^2 \sin^2\gamma)^{1/2} \tag{11}$$

is the value of V at $f = Y_o$.

The integration over V is elementary, thus

$$P(F>Y_o) = (1/\pi) \int_0^{\pi} d\gamma \exp\left\{-\left[\left(Y_o^2 - F_o^2 \sin^2\gamma\right)^{1/2} - F_o \cos\gamma\right]^2 / \nu^2\right\} \tag{12}$$

which is a new and sometimes more suitable expression for the probability that a sidelobe level F will exceed a specified level Y_o in the presence of element excitation errors with the rms value σ.

The integration can be performed explicitly if $Y_o = F_o$. We then have

$$P(F>F_o) = (1/\pi) \int_0^{\pi} d\gamma \exp\left\{-(F_o/\nu)^2 \left(\left|\cos\gamma\right| - \cos\gamma\right)^2\right\}$$

$$= \frac{1}{2} + (1/\pi) \int_{\frac{1}{2}\pi}^{\pi} d\gamma\, e^{-(2F_o/\nu)^2 \cos^2\gamma}$$

$$= \frac{1}{2} + (1/2\pi)\, e^{-2(F_o/\nu)^2} \int_0^{\pi} d\beta\, e^{2(F_o/\nu)^2 \cos\beta} \tag{13}$$

The integral in Eq. (13) is of the same form as in Eq. (7); therefore finally

$$P(F>F_o) = \frac{1}{2} + \frac{1}{2}\, e^{-2(F_o/\nu)^2}\, I_o\left[2(F_o/\nu)^2\right] \tag{14}$$

The probability with which the sidelobe level F will exceed the designed level F_o is shown in Figure 4 vs. 2 $(F_o/\gamma)^2$. As $\sigma \to o$, also $\nu \to o$, and $P(F>F_o) \to 1/2$, while for $\sigma \to \infty$ $P(F>F_o) \to 1$.

One can also derive an approximation for $P(F>Y_o)$ from Eq. (12) when $F_o \ll Y_o$ and $(F_o/\nu)^2 \ll 1$. We replace V_o^2 in the argument of the exponential in Eq. (12) by

$$V_o^2 \approx Y_o^2 - 2Y_oF_o \cos\gamma \tag{15}$$

The integral in Eq. (12) then yields

$$P(F>Y_o) \approx P_\infty\ (F>Y_o)$$

$$= e^{-(Y_o/\nu)^2}\, I_o(2Y_oF_o/\nu^2),\ F_o \ll Y_o, (F_o/\nu)^2 \ll 1. \tag{16}$$

In Figure 5 the exact and approximate probabilities are compared for $\mu = \sqrt{2}F_o/\nu = 0.5$, 1, and 2. The agreement is excellent for $\mu = 0.5$, and rather poor for $\mu = 2$, but for the latter we are violating our original condition: $(F_o/\nu)^2 \ll 1$, and therefore good agreement cannot be expected. The condition $(F_o/\nu)^2 \ll 1$ is necessary because in the integrand of Eq. (12) we are replacing terms of the form exp $\left[\pm(F_o/\nu)^2\right]$ by 1.

C. TAPERED VS. UNIFORM EXCITATION

Let us use the subscripts u and t on the quantities F, F_o, and ν to distinguish the uniform excitation and the tapered excitation quantities.

For the uniform excitation $|E_{no}| = |E_o|$ and Eq. (6) becomes

$$\nu^2{}_u = \sigma^2 /N \tag{17}$$

From Schwarz's inequality[6]

$$\left(\sum_{n=1}^{N} |E_{no}|\right)^2 \leq N \sum_{n=1}^{N} |E_{no}|^2 , \tag{18}$$

where the equality sign holds only for the uniform excitation; therefore from Eq. (6)

$$\nu_t^2 > \nu_u^2 \tag{19}$$

Consequently,

$$Y_o/\nu_t < Y_o/\nu_u \tag{20}$$

$$F_{ot}/\nu_t < F_{ou}/\nu_u \tag{21}$$

As σ increases, ν increases and F_o/ν decreases; therefore as $\sigma \rightarrow \infty$ we can use Eq. (16) to determine L_T if $F_o \ll Y_o$. Eq. (1) becomes

$$L_T \approx e^{-(Y_o/\nu_u)^2} I_o(2Y_oF_{ou}/\nu_u^2)$$

$$-e^{-(Y_o/\nu_t)^2} I_o(2Y_oF_{ot}/\nu_t^2) \tag{22}$$

Because of Eq. (20) $\quad e^{-(Y_o/\nu_u)^2} < e^{-(Y_o/\nu_t)^2}$ (23)

while as $\sigma \to \infty$, $2Y_oF_o/\nu^2 \to 0$, and

$I_o(2Y_oF_o/\nu^2) \to 1$; consequently for large σ

$$I_T < 0,\ F_o \ll Y_o\ . \tag{24}$$

Of course, since eventually $e^{-(Y_o/\nu)^2} \to 0$ as $\sigma \to \infty$; $I_T \to 0$ also, but I_T will approach zero from the negative side, thus there is at least some interval of σ for which it is not possible to improve upon the sidelobe level by tapering.

D. A SPECIAL CASE

Let us consider a line array of M + 1 = N omnidirectional elements (M even) with the "triangular" excitation distribution shown in Figure 6,

$$E_{no} \Big/ \left|E_o\right| = \left[2\ (M+1) \Big/ (M+2)\right] \left[1 - |n| \Big/ (\tfrac{1}{2} M + 1)\right] \tag{25}$$

for $n = o, \pm 1, \ldots, \pm \frac{1}{2} M$. The array is steered to broadside, and $\gamma_n\ (\Omega) = -\ nkd \sin \theta$, where θ is the angle between a normal to the array and the observation direction in the far field, k is the wave number, and d is the distance between elements. The far field pattern can be evaluated to give [7,8]

$$F_{ot} = \frac{\sin^2 \left[\frac{1}{2}\ (\frac{1}{2} M + 1)\ kd \sin \theta\right]}{(\frac{1}{2} M + 1)^2 \sin^2 (\frac{1}{2} kd \sin\theta)} \tag{26}$$

while for a uniform excitation array of M + 1 elements

$$F_{ou} = \frac{\sin \left[\frac{1}{2}\ (M + 1)\ kd \sin \theta\right]}{(M + 1) \sin\ (\frac{1}{2} kd \sin \theta)} \tag{27}$$

From Eq. (6) we obtain [9]

$$\nu_t^2 = \frac{4\sigma^2}{3} \bullet \frac{M^2 + 4M + 6}{(M + 2)^3} \tag{28}$$

while

$$\nu_u^2 = \sigma^2/(M + 1) \tag{29}$$

One can now obtain L_T for specified M, kd, etc. However, at some angle θ one might not get a meaningful comparison of the uniform and tapered distributions because, for example, at that particular θ F_{ou} might have a null while F_{ot} might have a maximum. Thus instead of the actual F_{ou} and F_{ot} one should use the envelopes of the far field patterns $\hat{F}_{ou}$ and $\hat{F}_{ot}$ in Eqs. (5), (12), and (1),

$$\hat{F}_{ou} = \frac{1}{(M+1)\ \sin(\frac{1}{2}\ kd\ \sin\theta)} \tag{30}$$

$$\hat{F}_{ot} = \frac{1}{(\frac{1}{2}M+1)^2\ \sin^2(\frac{1}{2}\ kd\ \sin\theta)} \tag{31}$$

L_T and $L_T/P(F_u > Y_o)$ vs. the rms element excitation error σ are shown in Figs. 7-10 for M = 200; kd = 3; θ = 1°, 6°, and 45°; and $Y_o = Q\ \hat{F}_{ou}$, Q = 0.8, 1.0, 1.3, and 2.0. The values of $\hat{F}_{ou}$ and $\hat{F}_{ot}$ are given in Table I. In all cases there is a region in which L_T is positive, and for large σ L_T is negative. For a fixed θ the value of σ at which L_T becomes negative seems to be almost independent of the value of Y_o, see Figs. 8-10.

Note that the choice of Q = 2, for example, means that we are examining the probabilities of exceeding the designed sidelobe level for the uniform excitation array $\hat{F}_{ou}$ by 6 db or more, regardless of the value of θ or the value of $\hat{F}_{ou}$. For a given rms error σ the probability of exceeding a sidelobe level originally designed to be -15 db by 6 db or more will be very small, while the probability of exceeding a designed level of -50 db by 6 db or more will be relatively large; therefore the same value of Q for different values of θ does not imply similar situations. Moreover, one frequently does not care whether the -50 db sidelobe is increased to -44 db, but one does care whether the -15 db sidelobe is increased to -9 db. However, there are only a few sidelobes of a relatively high level (say -15 db), while there are many low level (say -50 db or lower) sidelobes. Thus to evaluate the improvement or impairment offered by a certain element excitation distribution at all angles in the far field one would first have to assign relative importance and tolerable sidelobe levels to different directions in the far field. Such an evaluation is beyond the scope of this study.

E. CONCLUSIONS

Obviously it is not always possible to improve upon the sidelobe level by tapering if random errors in the excitation distribution are present. The details of the improvement by tapering will depend on the array configuration, the type of tapering, the rms error in the excitation, etc. For the particular triangular tapering which we have considered in the previous section, an improvement is obtained for rms errors from zero to some value σ_o. The value of this σ_o decreases as the no-error sidelobe level decreases (i.e., as θ increases). For rms errors greater than this σ_o there is no improvement.

Thus the decision whether to taper or not will depend on the relative importance of the sidelobe levels in different regions in space. For example, assume that the rms error $\sigma = 0.3$, and we are considering the triangular tapering discussed in the previous section. Then from Fig. 7 we have a definite improvement by tapering at $\theta = 6°$, a negligible improvement at $\theta = 1°$, and an impairment at $\theta = 45°$. If the improvement at 6° and angles nearby outweighs the impairment at 45° and other similar angles, then the tapering should be chosen. However, such an investigation of the relative importance and the tolerable sidelobe levels in different regions in space is beyond the scope of this study. We merely wished to show that it is not always possible to reduce the sidelobe levels by tapering if random errors are present, and the results of the present study give ample support to that contention.

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TABLE I

Envelope Values

θ	$\hat{F}_{ou}$, db	$\hat{F}_{ot}$, db
1°	-14.4	-16.9
6°	-29.9	-47.9
45°	-44.9	-77.8

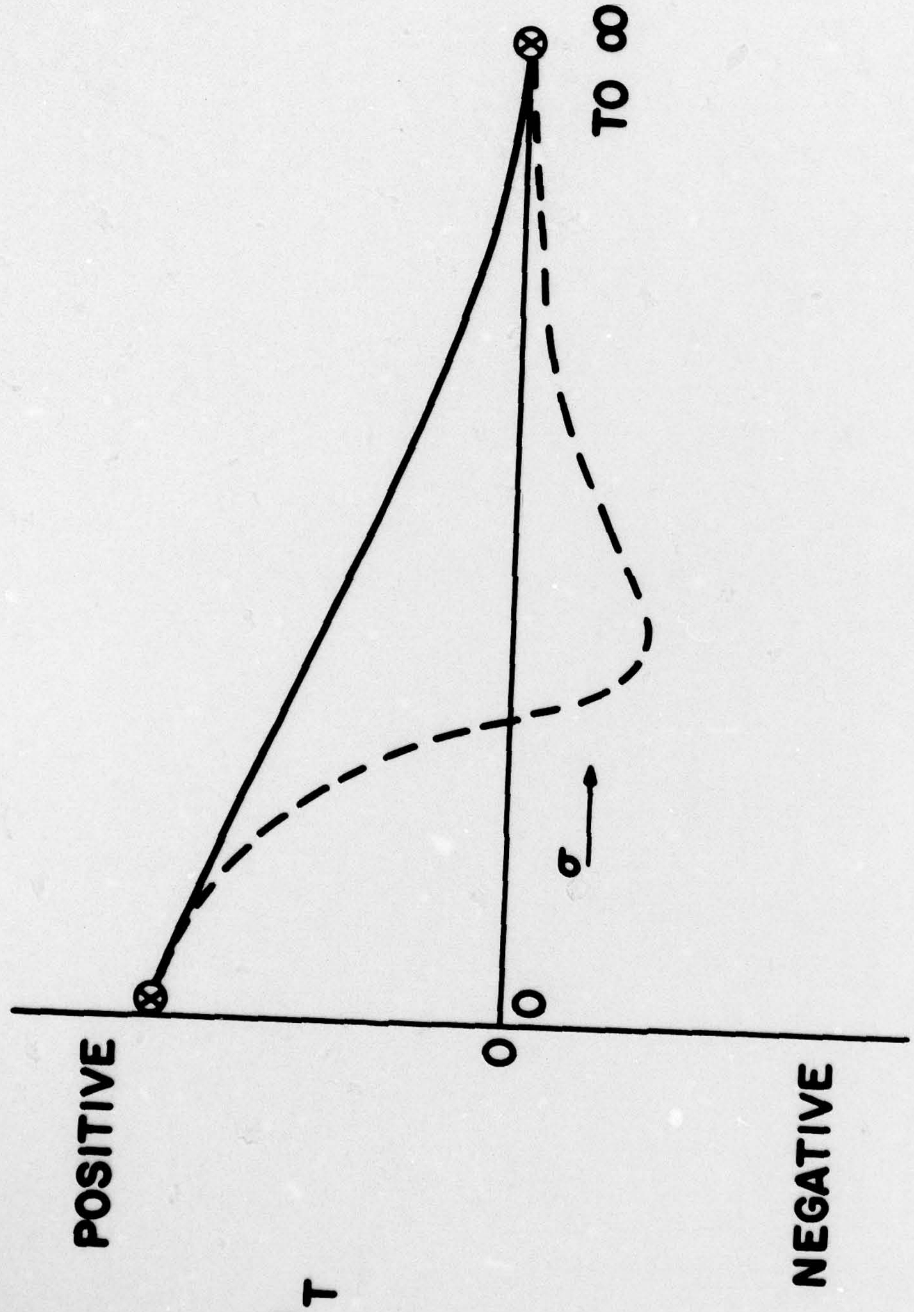

FIGURE 1. SOME POSSIBLE GRAPHS OF L_T VS. σ

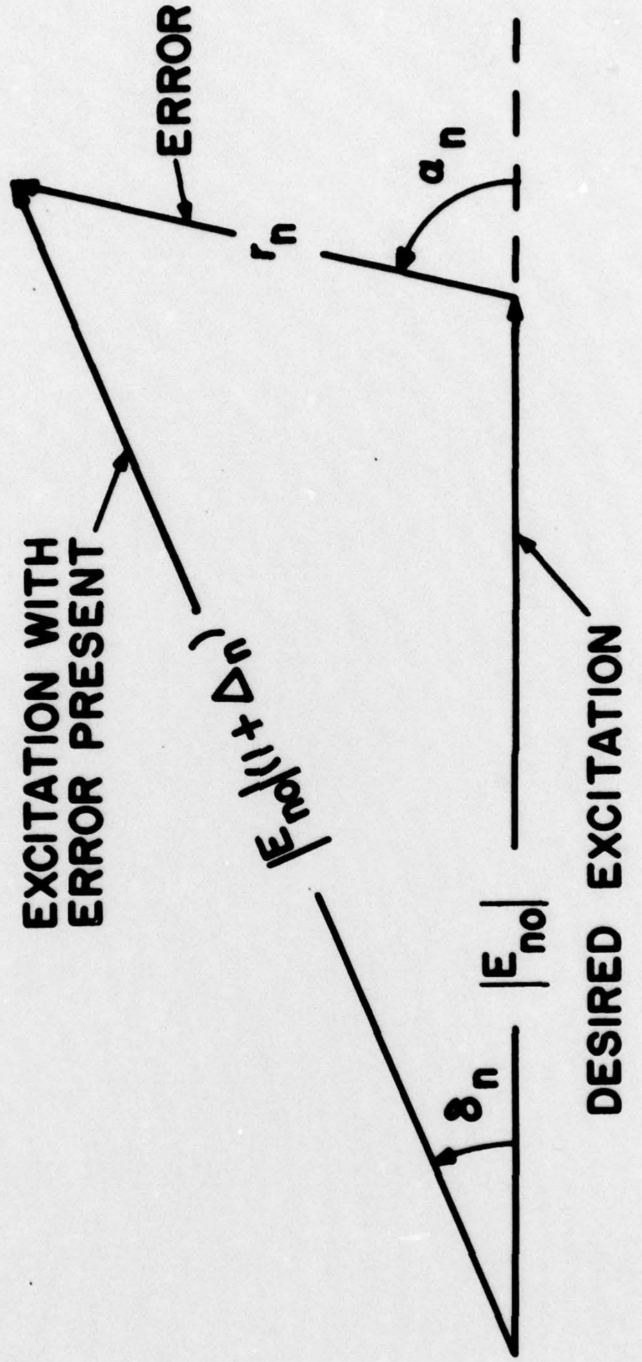

FIGURE 2. EXCITATION OF THE n^{th} ELEMENT

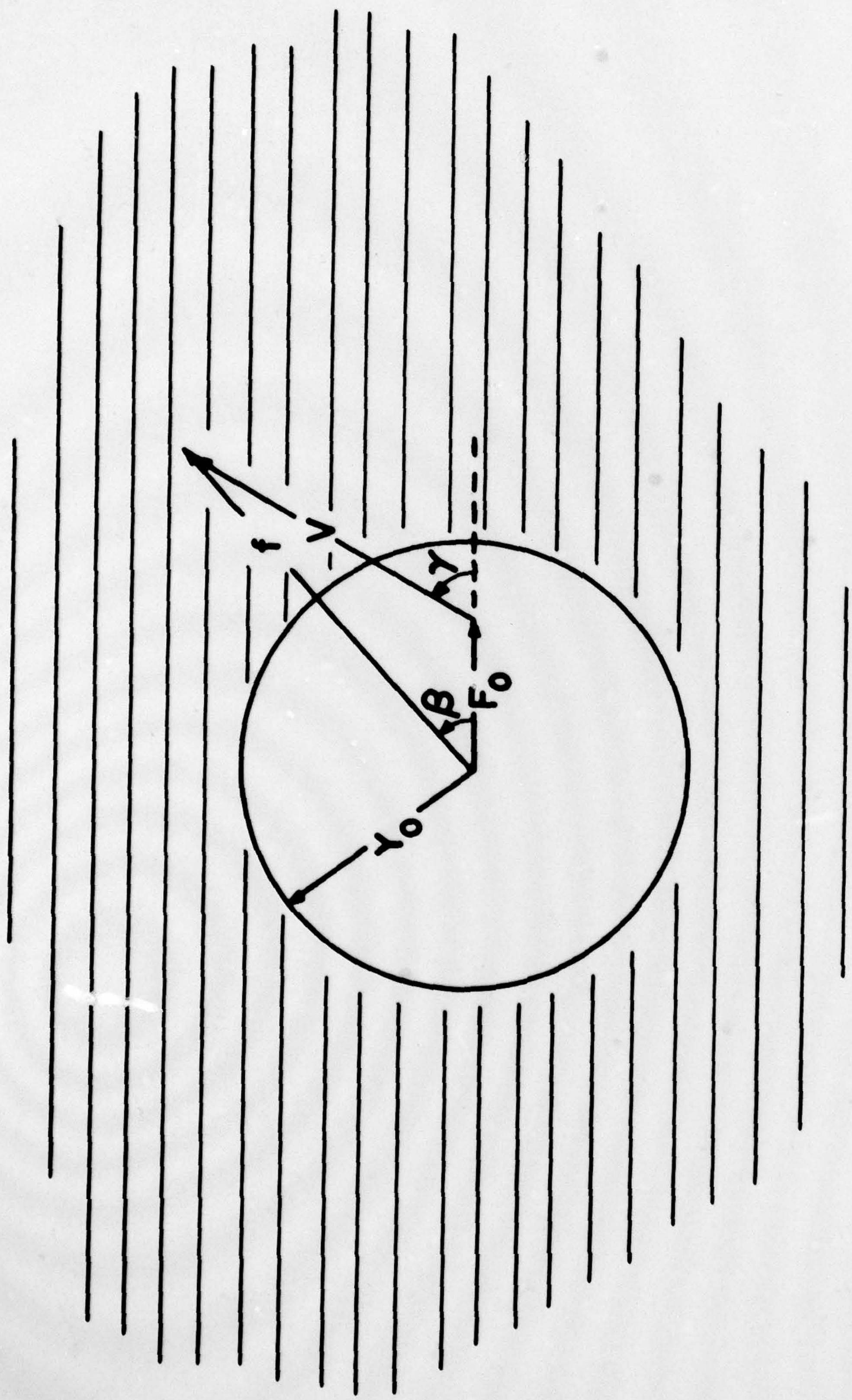

FIGURE 3. THE COORDINATE SYSTEMS IN THE INTEGRATION PLANE

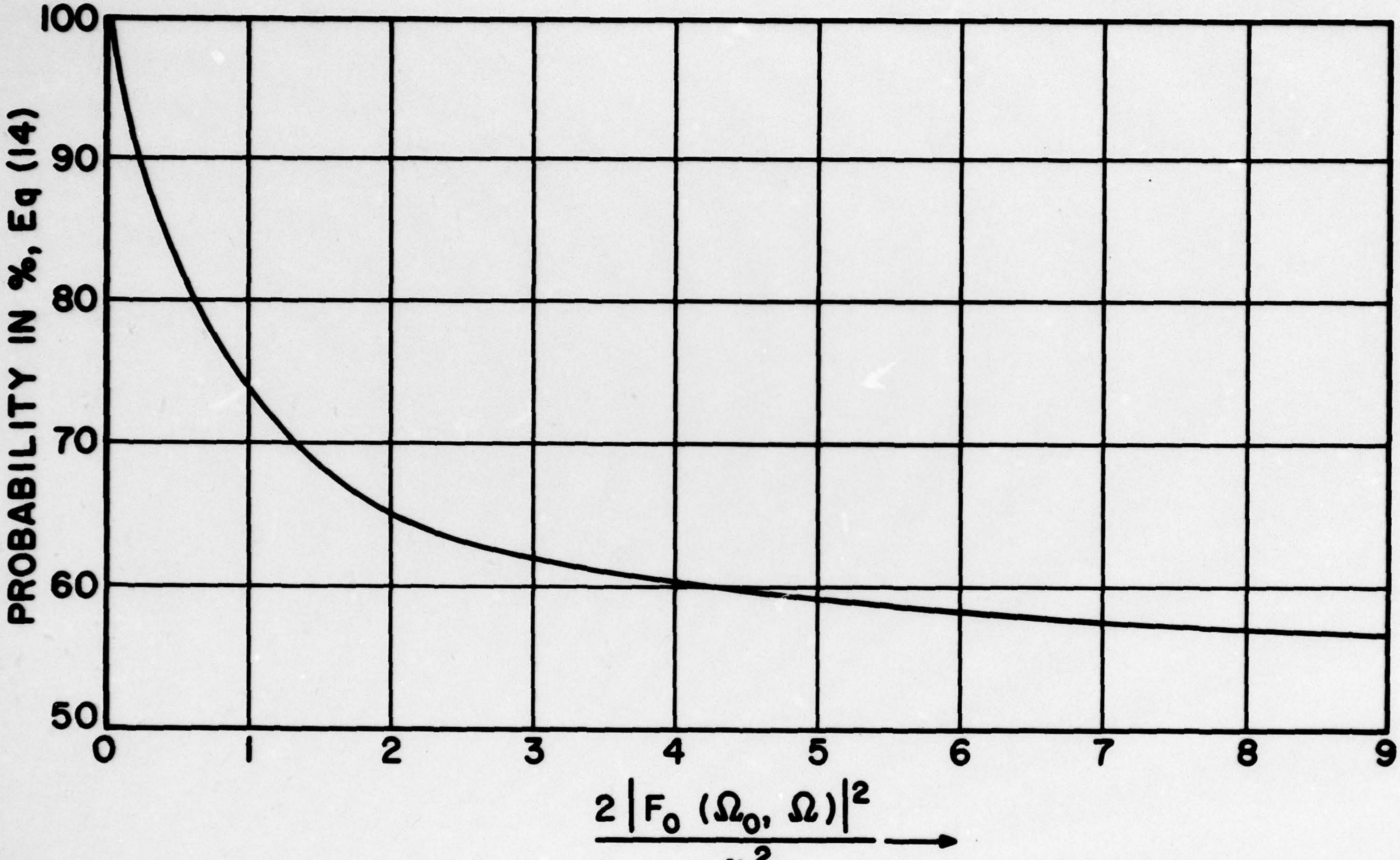

FIGURE 4. THE PROBABILITY THAT $|F(\Omega_0,\Omega)| > |F_0(\Omega_0,\Omega)|$ vs. $2|F_0(\Omega_0,\Omega)|^2/\nu^2$

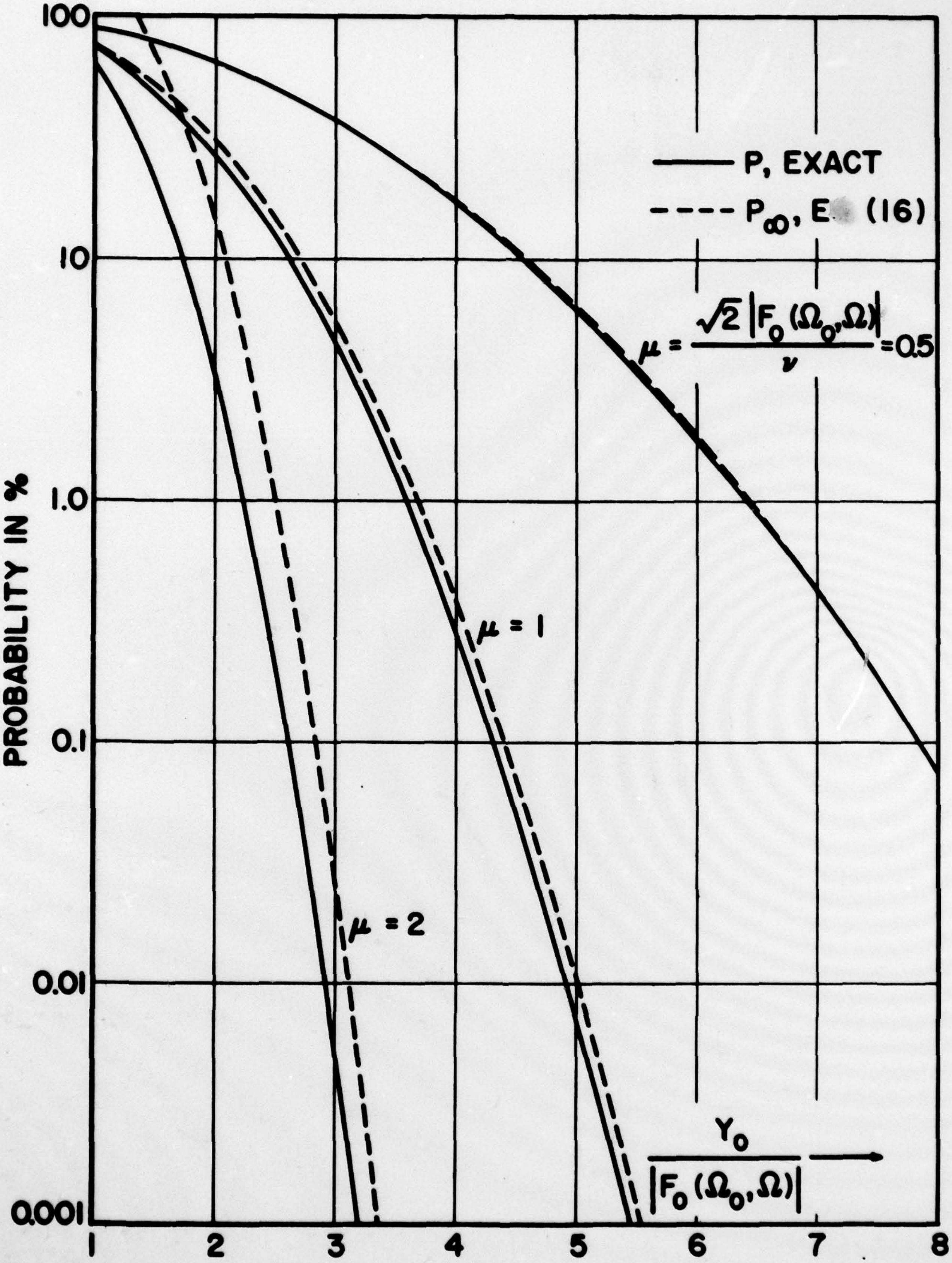

FIGURE 5. COMPARISON OF THE EXACT (P) AND APPROXIMATE (P_∞) PROBABILITIES THAT $|F(\Omega_0,\Omega)| > Y_0$ VS. $Y_0/|F_0(\Omega_0,\Omega)|$

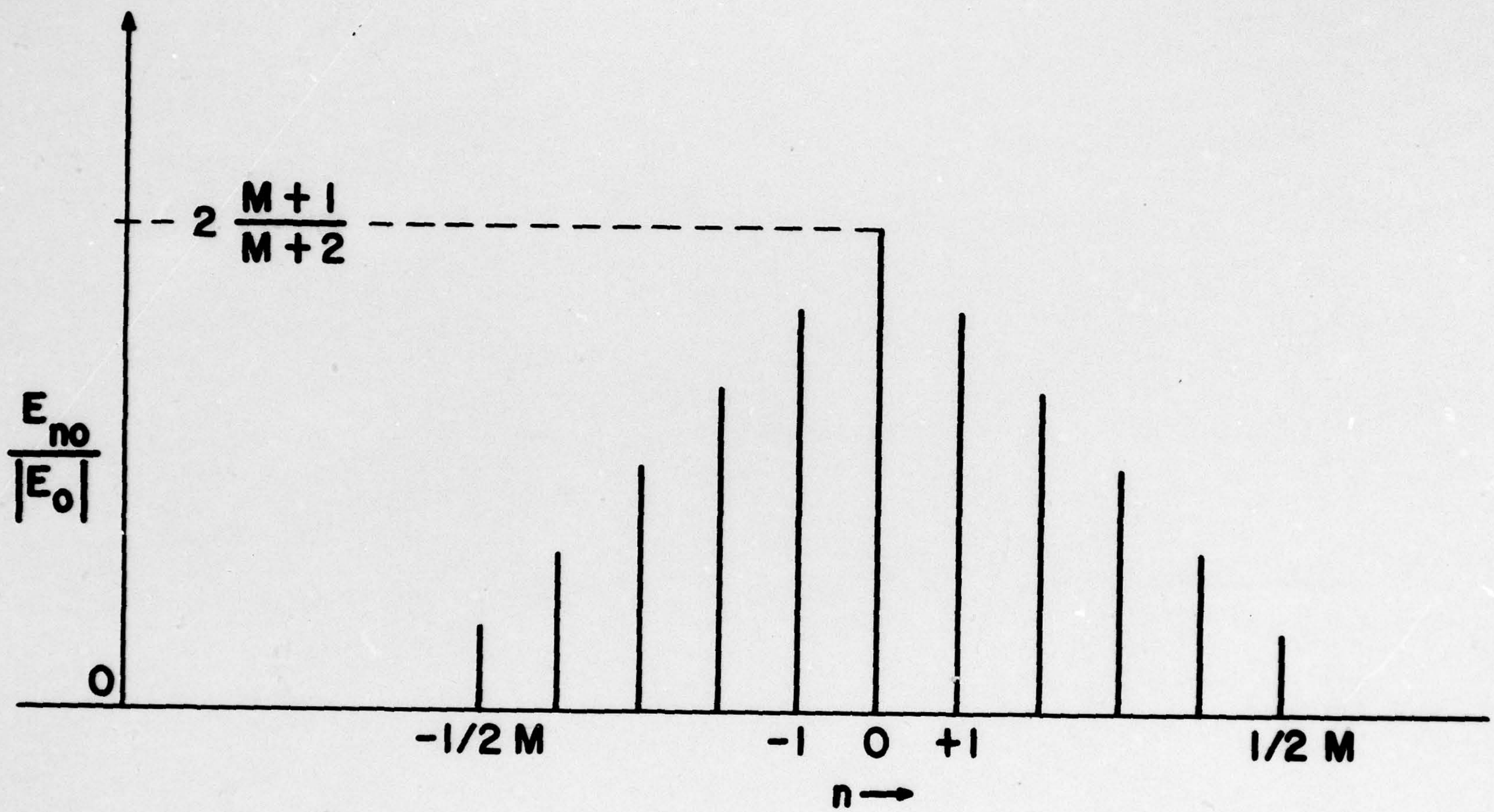

FIGURE 6. TRIANGULAR EXCITATION DISTRIBUTION

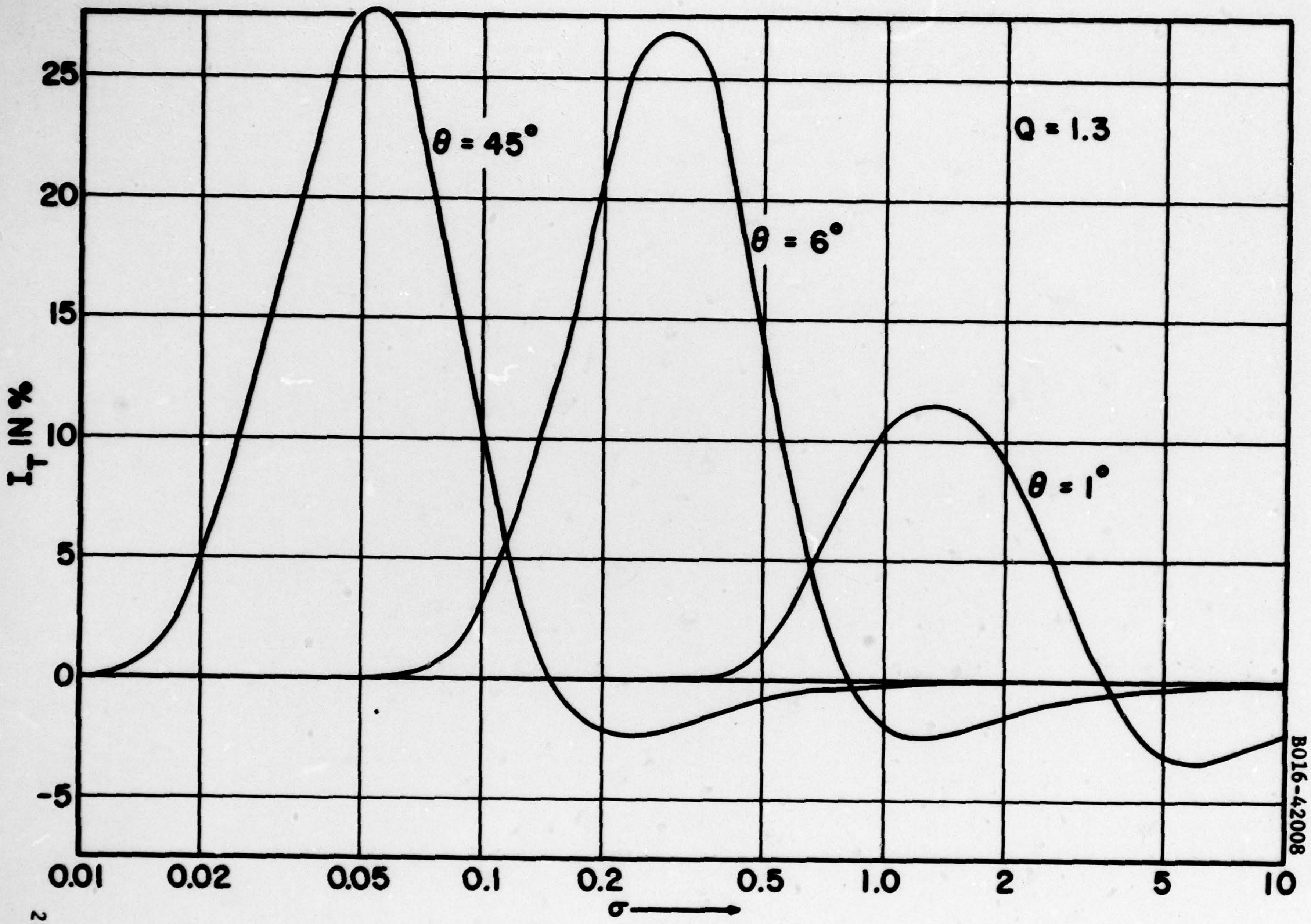

FIGURE 7. L_T VS. σ FOR Q = 1.3

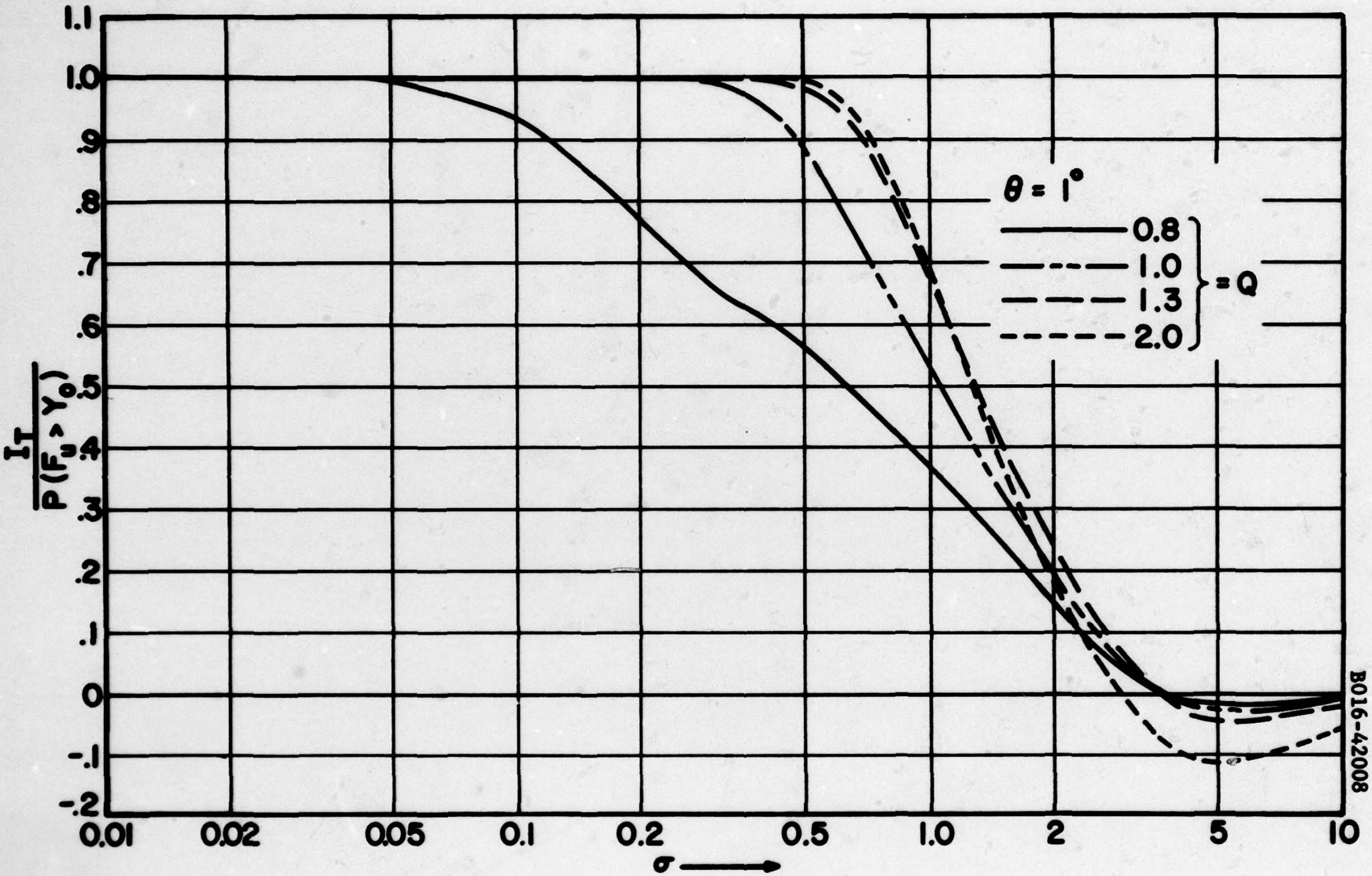

FIGURE 8. $L_T/P(F_u > Y_o)$ VS. σ FOR $\theta = 1°$

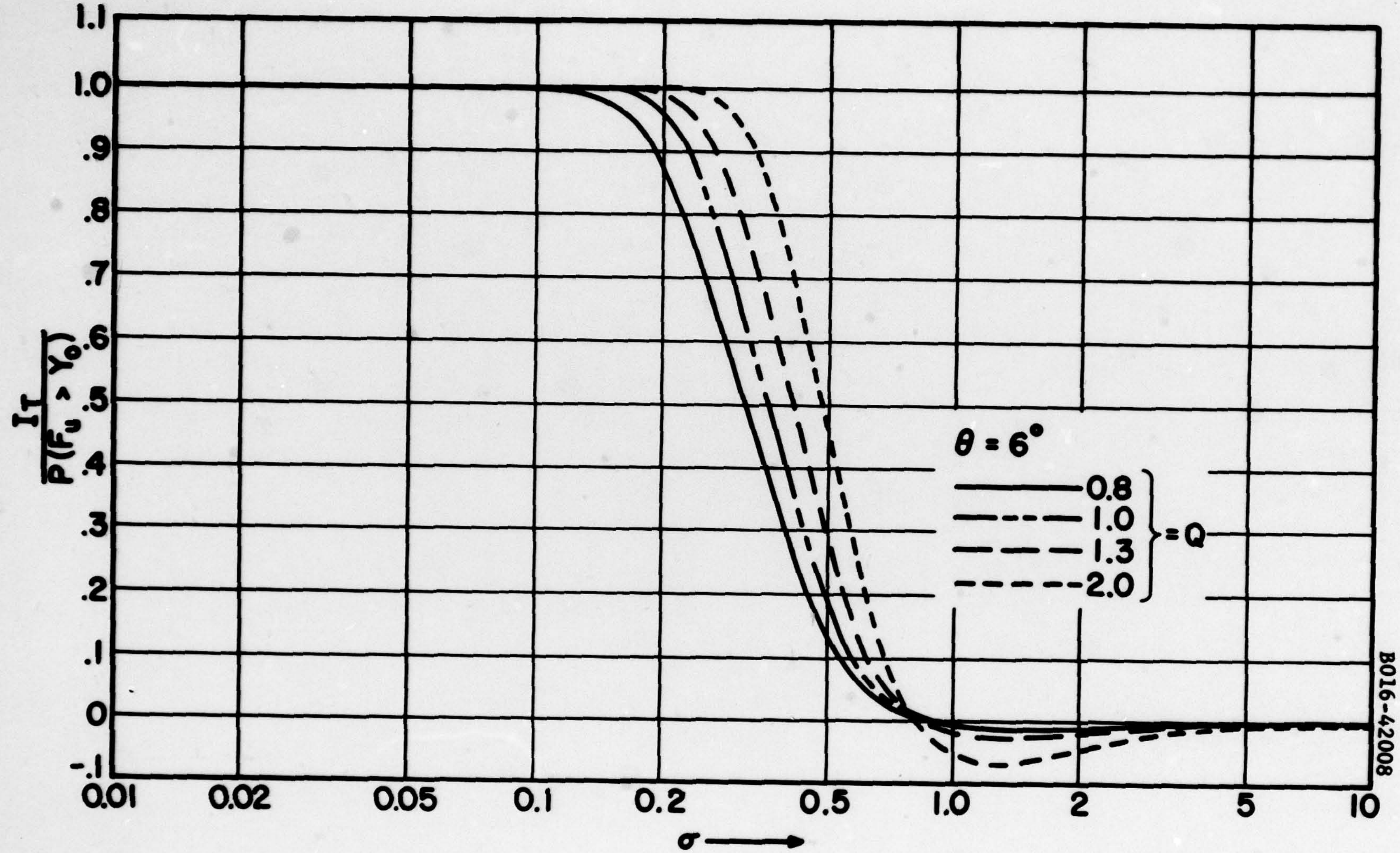

FIGURE 9. $L_T/P(F_u > Y_o)$ VS. σ FOR $\theta = 6°$

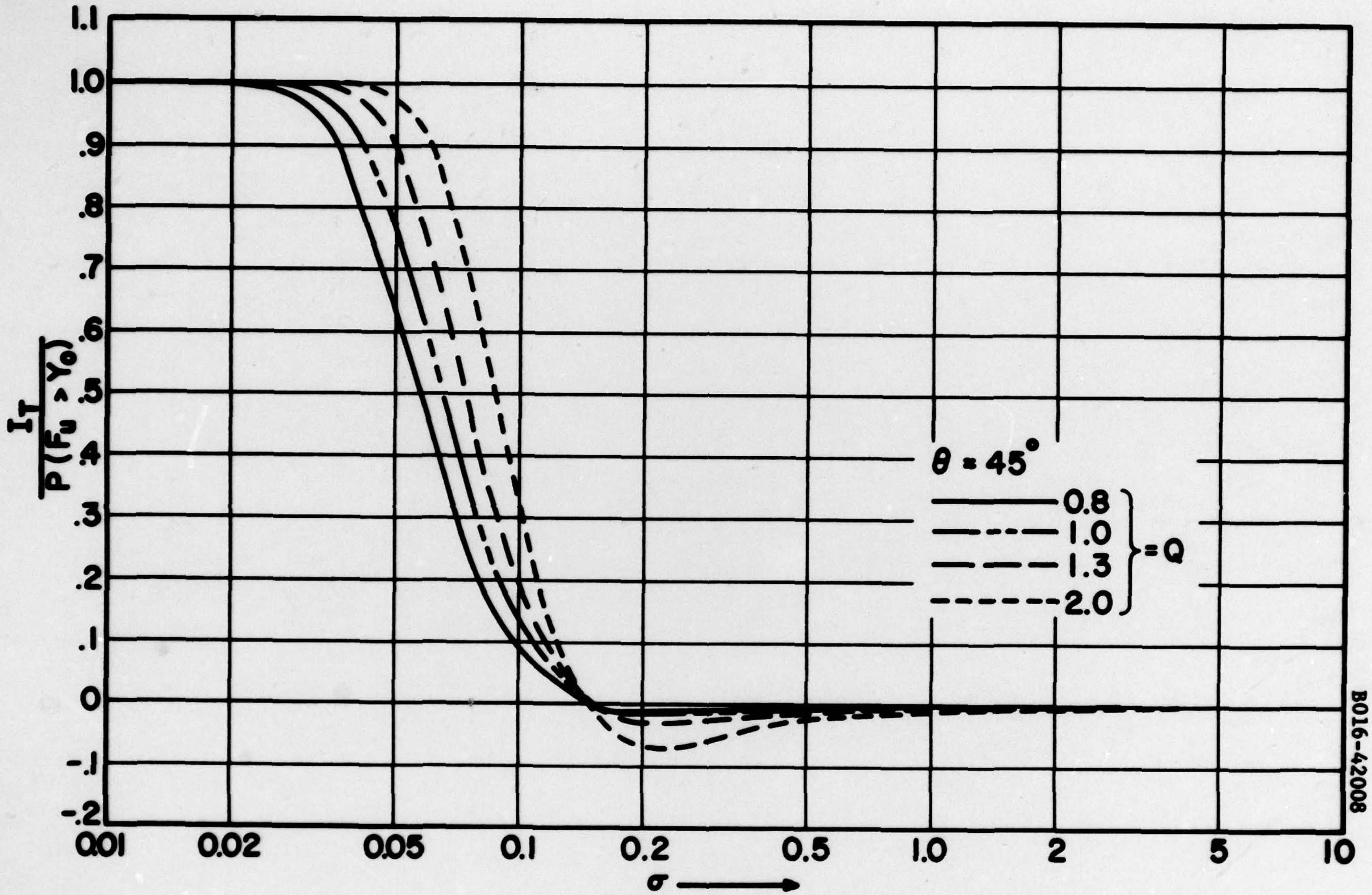

FIGURE 10. $I_T/P(F_u > Y_o)$ VS. σ FOR θ = 45°