





FOREWORD

This memorandum describes a balanced modulator with high linearity and high carrier rejection for the purpose of up or down shifting the spectrum of an audio signal. The memorandum has been prepared because it may be of interest to a number of people here at NUWC and NELC and possibly to a few people or activities outside of NUWC and NELC. It should not be construed as a formal report as its only function is to present, for the information of others, a small portion of the work being done in the area of signal conditioning for A to D conversion and signal analysis.





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THE PROBLEM

Develop a balanced modulator with high linearity and high carrier rejection for the purpose of up or down shifting the spectrum of an audio signal.

RESULTS

A circuit has been developed which will provide better than 30 db of carrier rejection while up or down shifting the spectrum to the desired values.

ADMINISTRATIVE INFORMATION

Work was performed by members of the Signal Recognition Division under SF-101-03-16, Task 8132 (NUWC E1-11).

The basic design of the modulator was developed by L. R. Weill. Circuit design and testing was performed by the author, working in conjunction with Mr. Weill.

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SYMBOLS

A	The sinusoidal input waveform.
Ā	The waveform which is the complement of A.
B	The square wave switching waveform.
vo	The output waveform.
ĸ	A constant scaling factor.
A(t)	
Ā(t) B(t)	The value of the indicated waveform at time t.
$V_0(t)$	
Ei	Input voltage of the operational amplifier.
Eo	Output voltage of the operational amplifier.
RL	Load resister.
ĸı	Maximum amplitude of B.
T	Period of B.
fB	Frequency of B.
an	The nth Fourier cosine coefficient of the Fourier series of B.
dn	The nth Fourier sine coefficient of the Fourier series of B.
ω _B	The frequency of B expressed in radians, $f_B = \frac{\omega_B}{2\pi^2}$
B _K (t)	The K th term of the Fourier series of B.
fA	Frequency of A.
K2	Maximum amplitude of A.
V _m (t)	The m th term of the Fourier series of Vo.
Nm	The m^{th} Fourier cosine coefficient of the Fourier series of V ₀ .
W _A	The frequency of A expressed in radians, $f_A = \frac{\omega_A}{2\pi}$

INTRODUCTION

In the conversion and analysis of active sonar data, it is often desirable to up or down shift the frequency spectrum of analog data to a more easily handled band. One method of doing this is to use an analog multiplier to heterodyne the signal. However, this method has the disadvantages of non-linearity, sensitivity to thermal changes and high cost. The above difficulties may be overcome by using a switching modulator. This device multiplies the input data signal by a square wave of any chosen frequency. The output of the modulator can then be passed through a bandpass filter to select the upper or lower sideband of the heterodyned signal. The filter output will be an up or down shifted version of the input signal.

DESCRIPTION OF THE MODULATOR

Basic Operation

Basically, the modulator consists of an inverting amplifier with a gain of -1, a non-inverting amplifier with a gain of 1, and a transistor switch as shown in Figure 1. The outputs A and \overline{A} of the amplifiers are selected by the switch at a rate determined by the square wave switching waveform B. If B is as shown in Figure 2 with period T, then from $t = \frac{-T}{4}$ to $t = \frac{T}{4}$, the switch selects the non-inverting amplifier and V_0 (t) = KA (t). From $t = \frac{T}{4}$ to $t = -\frac{3T}{4}$, the switch selects the inverting amplifier and V_0 (t) = KĀ (t). The resulting waveform, V_0 , can then be expressed as

$$V_{o}(t) = K(A(t) \times B(t))$$
 (1)

the time multiplication of A and B where K is a constant scaling factor.

Circuit Description

1. Inverting Amplifier

The inverting amplifier is a Philbrick P 65 A operational amplifier connected with a feedback network as shown in Figure 3. The feedback equation for an operational amplifier connected in this configuration is

$$\frac{E_0}{E_1} = \frac{-R_f}{R_1} \quad \text{or} \quad \frac{E_0}{R_1} = \frac{-R_f}{R_1} \quad \frac{E_1}{E_1}$$

When the potentiometer in the feedback branch is adjusted so that $R_f = R_i = 10K \Omega$ this reduces to $E_0 = -E_i$ or $\overline{A} = -A$. (2) The resistors shown are ±1%; and the offset of the operational amplifier is adjusted so that when A = 0, $\overline{A} = 0$.

2. Non-inverting Amplifier

The non-inverting amplifier is another P 65 A operational amplifier connected as a special case of the feedback network shown in Figure 4 a). The feedback equation for this network is $\frac{E_0}{E_1} = \frac{R_1 + R_f}{R_1}$ or $E_0 = \frac{R_1 + R_f}{R_1}$ E1. For the noninverting amplifier used in the modulator, Figure 4 b), R1 is an open circuit and Rf is a short circuit (i.e., $R_1 = \infty$, Rf = 0) so that the equation reducts to $E_0 = E_1$. The offset of this amplifier is also adjusted to zero.

3. Transistor Switch

To provide the switching function, the circuit shown in Figure 5 was developed, where Q1, Q2 and Q3 are pnp germanium transistors (2N1305's). Q3 operates as an inverter, supplying a switching waveform which is 180° out of phase with B to the base of Q2. Therefore, when B is negative, Q1 is on, and Q2 is off and the output can be determined by the simplified circuit of Figure 6. Therefore, the output can be stated as

$$V_{0}(t) \approx \frac{R_{L} || 10^{5}}{10^{5} + R_{L} || 10^{5}} \overline{A}(t) = \frac{10^{5} R_{L}}{10^{5} + R_{L}} \overline{A}(t)$$

$$= \frac{R_{L}}{10^{5} + 2R_{L}} \overline{A}(t)$$
(3)

When B is positive, Q1 and Q3 are off and Q2 is on and the output can be stated as

$$V_{0}(t) \simeq \frac{R_{L}}{10^{5} + 2R_{L}} A(t)$$

Combining equations 2, 3 and 4, it can be seen that with B positive

Vo (t) of A (t)

and with B negative

 $V_{o}(t) \propto \overline{A}(t) = -A(t)$

so that

 $Vo(t) \propto A(t) \times B(t)$

This can also be stated as

 $V_{0}(t) = K(A(t) \times B(t))$

(5)

(4)

where K is a constant scaling factor dependent on the circuit resistance values.

Theoretical Output Spectrum

Assume that B is a squarewave of the form

$$B(t) = \begin{cases} -K_{L}, & -T/2 \leq t \leq -T/4 \\ K_{L}, & -T/4 \leq t \leq T/4 \\ -K_{L}, & T/4 \leq t \leq T/2 \end{cases}$$

with period T and frequency $f_B = 1/T$. The Fourier series of B can be written as

 $B(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n a_n f_g t + b_n \sin n a_n f_g t)$ But, B(t) by definition is a zero mean even function. Thus, $a_0 = 0$ (zero mean) and $b_n = 0$ (odd functions not present). The expression for a_n is

$$\begin{split} a_n &= 2f_0 \int_{B(t)}^{\overline{z}F_0} B(t) \cos n z \pi f_0 t \, dt \\ &= \frac{i}{\overline{z}F_0} = dt, \text{ then} \\ a_n &= \frac{i}{n\pi} \int_{(-K_i)}^{-\frac{\pi}{2}} \cos y \, dy + \frac{i}{n\pi} \int_{K_i}^{\frac{\pi}{2}} \cos y \, dy \\ &+ \frac{i}{n\pi} \int_{(-K_i)}^{\pi} \cos y \, dy + \frac{i}{n\pi} \int_{\overline{z}}^{\frac{\pi}{2}} K_i \cos y \, dy \\ &+ \frac{i}{n\pi} \int_{\overline{z}}^{\pi} (-K_i) \cos y \, dy \end{split}$$

$$a_{n} = \frac{2K_{i}}{n\pi} \int_{c}^{\frac{\pi}{2}} \cos y \, dy - \frac{2K_{i}}{n\pi} \int_{c}^{\pi} \cos y \, dy$$

$$= \frac{2K_{i}}{n\pi} \left[s_{iny} \Big|_{0}^{\frac{\pi}{2}} - s_{iny} \Big|_{\frac{\pi}{2}}^{\frac{\pi}{2}} \right]$$

$$= \frac{RK_{i}}{n\pi} \left[s_{in\frac{\pi}{2}} - s_{in0} - s_{in\pi n + s_{in\frac{\pi}{2}}} \right]$$

$$= \frac{4K_{i}}{n\pi} \left[s_{in\frac{\pi}{2}} - s_{in0} - s_{in\pi n + s_{in\frac{\pi}{2}}} \right]$$

therefore,

$$\mathbf{a_n} = \begin{cases} 4 \ K_1/n \ \pi, \ n = 1, \ 5, \ 9, \dots, 4 \ K + 1, \dots \\ -4 \ K_1/n \ \pi, \ n = 3, \ 7, \ 11, \dots, 4 \ K + 3, \dots \\ 0 \qquad , \ n = \text{ even integers} \end{cases}$$

Thus, the Fourier series of B is

$$B(t) = \frac{4\pi}{\pi} \left[\cos(2\pi f_0 t) - \frac{1}{3} \cos[3(2\pi f_0 t)] + \frac{1}{5} \cos[5(2\pi f_0 t)] - \frac{1}{5} \cos[7(2\pi f_0 t)] + \frac{6}{3} \cos[7$$

which can be written in exponentials as

$$B(t) = \frac{4\kappa_1}{2\pi} \bigotimes \left[\left(e^{j\omega_0 t} + e^{-j\omega_0 t} \right) - \frac{1}{3} \left(e^{j3\omega_0 t} + e^{-j\omega_0 t} \right) + \frac{1}{5} \left(e^{j5\omega_0 t} + e^{j5\omega_0 t} \right) - \frac{1}{3} \left(e^{j3\omega_0 t} + e^{j\omega_0 t} \right) \right]$$

where $\omega_B = 2\pi f_B$ and the Kth term of B (t) is

$$B_{k}(t) = \frac{4\kappa_{1}(-1)^{k}}{2(2k-1)\pi} \left[e^{j(2k-1)\omega_{0}t} + e^{-j(2k-1)\omega_{0}t} \right](7)$$

Now assume A to be a pure cosinusoid of frequency f_A . A (t) can then be written as

$$A(t) = K_2 \cos 2\pi f_A t = \frac{K_2}{2} \left(e^{j\omega_A t} + e^{j\omega_A t} \right)$$
(8)

where $W_A : 2\pi f_A^2$. The mth term of $V_O(t)$, where $V_O(t) = K(A(t) \times B(t))$, is therefore:

$$V_{m}(t) = K (A(t) \times B_{m}(t))$$

$$= K \cdot \frac{K_{2}}{2} \cdot \frac{4 K_{1}(-1)^{m}}{2 \pi (2m-1)} \cdot (e^{j\omega_{A}t} + e^{-j\omega_{A}t})$$

$$\times (e^{j(2m-1)\omega_{B}t} + e^{-j(2m-1)\omega_{B}t})$$

$$= \frac{N_{m}}{2} (-1)^{m} \left\{ e^{j[(2m-1)\omega_{B} + \omega_{A}]t} + e^{-j[(2m-1)\omega_{B} + \omega_{A}]t} + e^{-j[(2m-1)\omega_{B} - \omega_{A}]t} \right\}$$

$$= N_{m}(-1)^{m} \left\{ \cos [(2m-1)\omega_{B} + \omega_{A}]t + e^{-j[(2m-1)\omega_{B} - \omega_{A}]t} \right\}$$

$$= N_{m}(-1)^{m} \left\{ \cos [(2m-1)\omega_{B} + \omega_{A}]t + \cos [(2m-1)\omega_{B} - \omega_{A}]t \right\}$$

$$= N_{m}(-1)^{m} \left\{ \cos 2\pi [(2m-1)f_{B} + f_{A}]t + \cos 2\pi [(2m-1)f_{B} - f_{A}]t \right\}$$

$$N_{m} = \frac{2 \cdot K \cdot K_{1} \cdot K_{2}}{N_{m}}$$

where $\pi(2m-1)$

Note (Figure 7) that the spectrum of V_0 (t) contains the sidebands which are $\pm f_A$ about the fundamental frequency and about the harmonics of B, but does not contain fB, its harmonic frequencies, or fA.

TEST RESULTS

The performance of the modulator was tested by choosing $f_A = 400$ Hz and $f_B = 1$ KHz and displaying the results with a Nelson-Ross model O21 spectrum analyzer plug-in unit in a Tektronix 564-A oscilloscope. The resulting waveforms and spectrum are

shown in the pictures at the end of this section. Figure 8 is a display of V_0 , the switched waveform that is the output of the modulator. In Figure 9, the spectrum of B, the lKHz squarewave switching waveform with the fundamental frequency and the odd harmonics, is displayed in a linear plot of signal amplitude. Figure 10 is a plot of the 400Hz sum and difference frequencies that make up the spectrum of V_0 . Figure 11 is an overlay of the 400Hz sum and difference frequencies on the spectrum of B. The amplitudes in Figures 10 and 11 are plotted logarithmically.

By displaying the spectrum of A x B with the spectrum analyzer and increasing its gain, it was found that the carrier rejection is approximately -30 db with respect to the amplitude of the sidebands about the lKHz fundamental frequency. This measurement is inexact due to the difficulty in obtaining sufficient sensitivity without instability in the trace. The signal to noise ratio is approximately 33 db. The voltage transfer ratio with A = 1v peak-to-peak and $B = \pm 3v$ is

$$VTR = \frac{|V_{0}|}{|A|} = \frac{50mv}{|v|} = .05$$
.

The modulator will operate for voltage ranges from 2 mv peak-to-peak to 10v peak-to-peak. At higher input levels, some clipping occurs due to the input characteristics of the operational amplifiers.

The use of an unsymmetrical switching waveform causes the appearance of even harmonics and a degradation of the carrier rejection. Other causes of carrier rejection degradation were found to be the presence of D.C. offset at the output of the operational amplifiers and slow switching speeds of the transistors used.

CONCLUSIONS

1. A comparison of Figure 7 with Figure 11 shows that the discribed modulator closely approximates the mathematical model described in the section on the theoretical output spectrum.

2. Since the transistor switches are being driven to cutoff and to saturation,

the non-linearities of the transistors have no effect on the operation of the modulator.

3. For the same reason, the sensitivity to thermal change is very slight.

4. Since standard components are used, the overall price of the modulator is small.
5. It can be concluded, therefore, that the use of a switching modulator is a practical method of up or down shifting the frequency spectrum of analog data which eliminates several of the drawbacks of multipliers.

RECOMMENDATIONS

1. This device should be useful in shifting the frequency of analog sonar data to a band more convenient for study.

2. Further study is indicated in the following directions:

- a. extending the useful range of the modulator by adding an additional stage to cancel the Third Harmonic side bands of the modulating signal (see Appendix),
- b. increasing the switching speed by using F. E. T.'s,
- c. increasing the symmetry of the switching waveform by the use of Flip-flops,
- d. investigating the requirements of the filters to be used to extract the desired band of frequencies.















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Figure 8: $V_0 = K (A \times B)$, the output waveform of the modulator. Vertical is .02 v/div. Horizontal is .5 msec/div. A is a 400Hz sine wave and the switching frequency, B is 1KHz.



Figure 9: The spectrum of a 1KHz square wave, the B input to the modulator. Amplitude is plotted linearly.



Figure 10: Spectrum of V_0 , the output of the modulator, showing the 400Hz sum and difference frequencies. Logrithmic plot, vertical is 2.5 db/div.



Figure 11: Overlay of the spectrums of B and $V_{\rm O}$ showing fundamental and odd harmonics of the 1KHz square wave and the 400Hz sum and difference frequencies of the modulator output.

APPENDIX:

Cancellation of Sidebands About the Third Harmonic

When f_A is approximately equal to f_B , a difficulty arises in distinguishing the upper sideband of the fundamental frequency from the lower sideband of the third harmonic. This section describes a possible method of eliminating this difficulty. The method calls for including an additional switching modulator operating at 3 times the switching frequency of the original circuit. Scaling the output of the additional modulator by 1/3 and summing with the output of the original modulator, the sidebands of the $3K^{th}$ (K = 1, 3, 5,...,) harmonics of V_0 should be eliminated (see Figure 12). The feasibility of this process is demonstrated by the following mathematical development. Let B (t) be the following:

$$B(t) = \begin{cases} -1, & -\frac{1}{2f_B} < t < -\frac{1}{4f_B} \\ -1, & -\frac{1}{4f_B} < t < -\frac{1}{4f_B} \\ -1, & -\frac{1}{4f_B} < t < -\frac{1}{4f_B} \\ -1, & -\frac{1}{4f_B} < t < -\frac{1}{2f_B} \end{cases}$$

The Fourier series of this is described in equation 6 as

$$f(t) = \frac{4}{3} \left\{ \cos 2\pi f_{B} t - \frac{1}{3} \cos 2\pi (3f_{B}) t + \frac{1}{5} \cos 2\pi (5f_{B}) t - \frac{1}{7} \cos 2\pi (7f_{B}) t + \dots \right\}$$

Now let C (t) be the same as B (t), but with a frequency of $3f_B$. Then

$$C(t) = \{\cos 2\pi (3f_{B})t - \frac{1}{3}\cos 2\pi (9f_{B})t + \frac{1}{3}\cos 2\pi (15f_{B})t - \dots \}$$

By summing B (t) with 1/3 C (t), the result is

$$B(t) + \frac{1}{5}C(t) = \frac{4}{5} \Big[\cos 2\pi f_B t + \frac{1}{5} \cos 2\pi (5f_B) t \\ -\frac{1}{5} \cos 2\pi (7f_B) t - \frac{1}{5} \cos 2\pi (11f_B) t + \dots \Big]$$

and the third harmonic has been eliminated.



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