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AN ABBREVIATED INSTRUCTIVE DESCRIPTION OF THE RESISTIVE HOSE IN--ETC(U)
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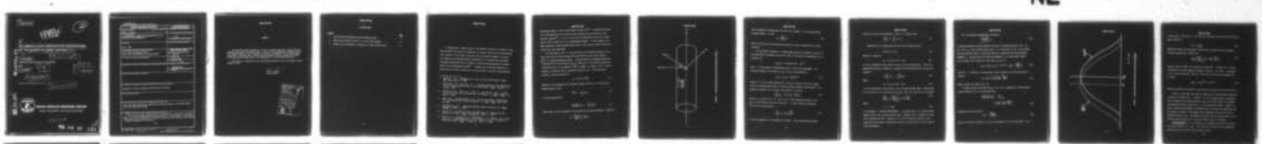

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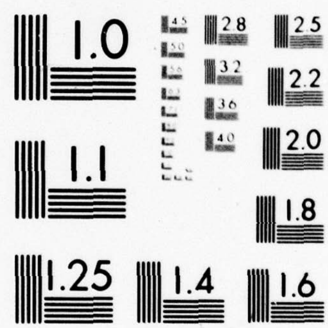

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(14) NSWC-TR-79-255

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AN ABBREVIATED INSTRUCTIVE DESCRIPTION OF THE RESISTIVE HOSE INSTABILITY.

(9) Final rept.

BY (10) HAN S. UHM

RESEARCH AND TECHNOLOGY DEPARTMENT

(11) 11 JUNE 1979

(12) 19p.

(16) ZR00001

(17) ZR0000101

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NSWC

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391 596

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER NSWC TR 79-255	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) AN ABBREVIATED INSTRUCTIVE DESCRIPTION OF THE RESISTIVE HOSE INSTABILITY		5. TYPE OF REPORT & PERIOD COVERED Final
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Han S. Uhm		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Surface Weapons Center White Oak, Silver Spring, Maryland 20910		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 61152N; ZR000-01-01; 0; 9R01AA
11. CONTROLLING OFFICE NAME AND ADDRESS		12. REPORT DATE 11 June 1979
		13. NUMBER OF PAGES 16
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

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18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Resistive Hose Instability, Relativistic Electron Beam, Self-pinched Beam, Collision-dominated Plasma.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The resistive hose instability of a self-pinched, relativistic electron beam in a collision-dominated background plasma is investigated. Previously known results are recovered in a simple instructive way, without going through a lengthy mathematical derivation.

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SUMMARY

The resistive-hose instability is one of the most important instabilities of intense relativistic electron beams in the collision-dominated background plasma. This report examines the resistive-hose stability properties for a self-pinched relativistic electron beam. Without going through a lengthy mathematical derivation, previously known results are recovered in a simple instructive way.

This work was supported by the Independent Research Fund at the Naval Surface Weapons Center.

PAUL R. WESSEL
By direction

ILLUSTRATIONS

In recent years, there has been a considerable increase in interest in the equilibrium[1,2] and stability properties of intense relativistic electron beams. This interest is the result of several diverse research programs, including (a) research on collective-effect accelerators[3], (b) research on high-power microwave generation[4], and (c) studies of electron beam propagation through a neutral gas or background plasma[5-7]. Perhaps one of the most important instabilities of intense relativistic electron beams in the collision-dominated

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1. Bennett, W.H., "Magnetically Self-Focusing Streams," Phys. Rev. 45, 1934, p. 890.

2. Davidson, R.C. and Uhm, H.S., "Thermal Equilibrium Properties of an Intense Relativistic Electron Beam," Phys. Fluids, in press, 1979.

3. Destler, W.W., Uhm, H.S., Kim, H., and Reiser, M.P., "Study of Collective Ion Acceleration in Vacuum," J. Appl. Phys., in press, 1979.

4. Uhm. H.S., and Davidson, R.C., "Cyclotron Maser Instability for Intense Solid Electron Beams," J. Appl. Phys. 50. 1979. p. 696.

5 Rosenbluth, M.N., "Long-Wavelength Beam Instability," Phys. Fluids 3 1960, p. 932.

6. Lee, E.P., "Resistive Hose Instability of a Beam with the Bennett Profile," Phys. Fluids 21, 1978, p. 1327.

7. Lauer, E.J., Briggs, R.J., Fessenden, T.J., Hester, R.E., and Lee, E.P., "Measurements of Hose Instability of a Relative Electron Beam," Phys. Fluids 21, 1978, p. 1344.

background plasma is the resistive hose instability[5,6]. Although the theoretical investigation of the resistive hose instability can be found in the previous studies[5,6], it is still difficult to understand the physical mechanism of this instability in a straightforward manner. In this regard, this paper develops a simple sketchy description of the resistive hose instability within the rigid beam model.

As illustrated in Fig. 1, the equilibrium configuration consists of intense relativistic electron beam with equilibrium axial current density $J_b^o(r)$ in a dense background plasma. The beam electrons propagate along the z-direction with axial velocity $\beta_b c \hat{e}_z$, where $\hat{e}_z$ is a unit vector along the z-direction and c is the speed of light in vacuo. As shown in Fig. 1, we introduce a cylindrical polar coordinate system (r,θ,z). The total beam current I_b can be expressed as

$$I_b = 2\pi\int_o^\infty drr\, J_b^o(r). \tag{1}$$

Moreover, the θ-component of the Maxwell equation for the equilibrium azimuthal magnetic field

$$B_\theta^o(r) = -\frac{d}{dr} A_o(r) \tag{2}$$

is also obtained from

$$\frac{1}{r}\frac{d}{dr}r\frac{d}{dr} A_o(r) = -\frac{4\pi}{c} J_b^o(r), \tag{3}$$

where $A_o(r)$ is the z-component of the equilibrium vector potential. Defining

$$\lambda_b = \frac{2\pi\gamma_b m}{\beta_b cq}\int_o^\infty drrJ_b^o(r),$$

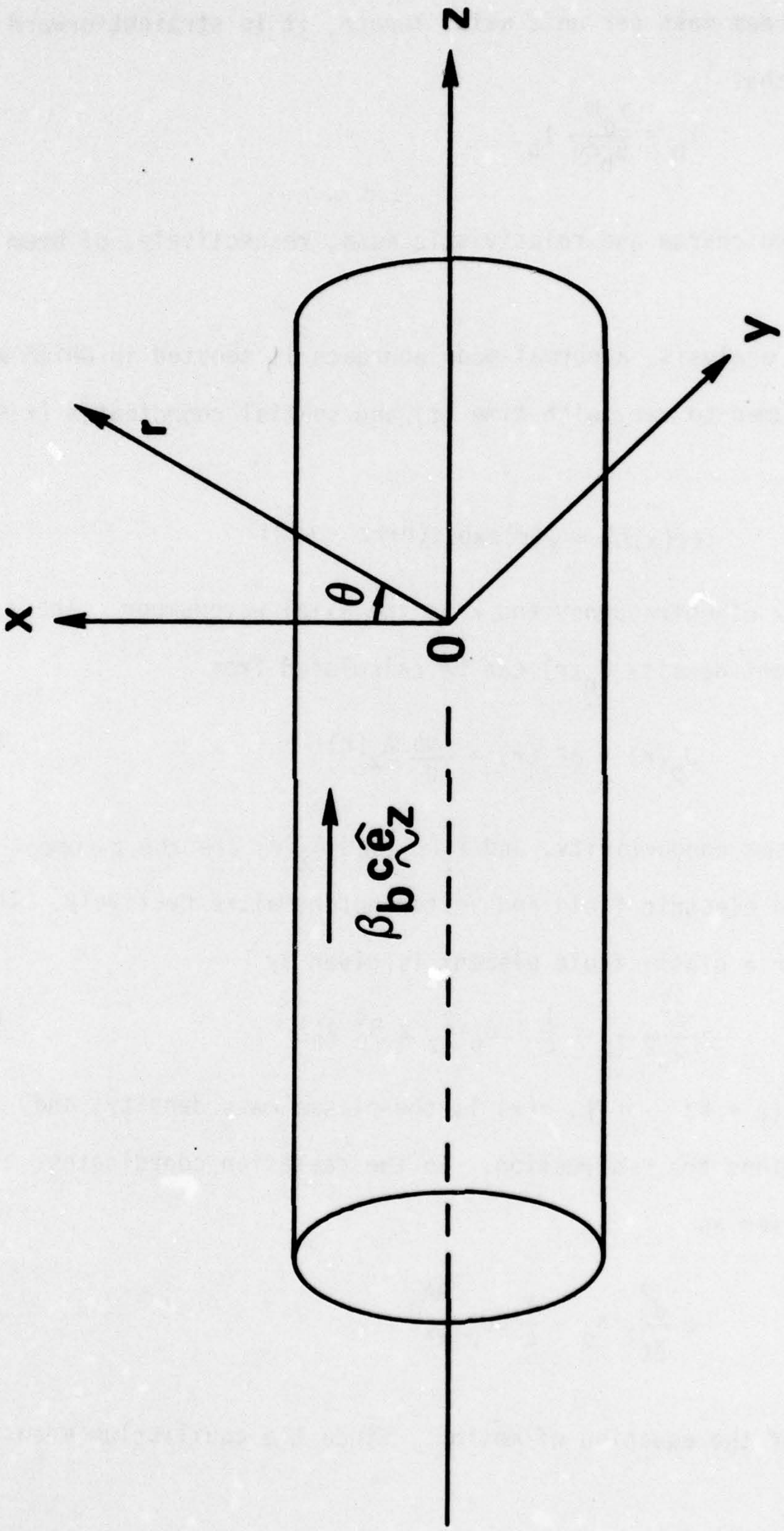

FIGURE 1 EQUILIBRIUM CONFIGURATION AND COORDINATE SYSTEM

which represents the beam mass per unit axial length, it is straightforward to show from Eq. (1) that

$$\lambda_b = \frac{\gamma_b m}{\beta_b c q} I_b, \tag{4}$$

where q and $\gamma_b m$ are the charge and relativistic mass, respectively, of beam electrons.

In the stability analysis, a normal-mode approach is adopted in which all perturbations are assumed to vary with time (t) and spatial coordinates (r,θ,z) according to

$$\delta\psi(\underset{\sim}{x},t) = \psi(r)\exp[i(\theta+kz - \omega t)].$$

Here, ω is the complex eigenfrequency and k is the axial wavenumber. The perturbed plasma current density $J_p(r)$ can be calculated from

$$J_p(r) = \sigma E_z(r) = \frac{i\omega\sigma}{c} A_z(r) \tag{5}$$

where $\sigma(r)$ is the plasma conductivity, and $E_z(r)$ and $A_z(r)$ are the z-components of the perturbed electric field and vector potential, respectively. The equation of motion for a plasma fluid element is given by

$$\rho \frac{d^2}{dt^2} \underset{\sim}{x}_p = \frac{1}{c} (\delta J_p \, \hat{\underset{\sim}{e}}_z \times B_\theta^o \, \hat{\underset{\sim}{e}}_\theta) \tag{6}$$

where $\delta J_p = J_p \exp[i(\theta + kz - \omega t)]$, $\rho(r)$ is the plasma mass density, and $\hat{\underset{\sim}{e}}_\theta$ is a unit vector along the θ-direction. In the Cartesian coordinates, Eq. (6) can be expressed as

$$\rho \frac{d^2}{dt^2} x_p = \frac{1}{c} \delta J_p \frac{\partial A_o}{\partial x} \tag{7}$$

for the x-component of the equation of motion. Since the equilibrium mean velocity of plasma fluid element is zero, it is obvious that

$$\rho \frac{d^2}{dt^2} x_p = \frac{d^2}{dt^2} (\rho x_p). \tag{8}$$

Defining λ_p as a plasma mass per unit axial length, we have

$$\int ds \rho x_p = \lambda_p \bar{x}_p, \tag{9}$$

where ds = drrdθ and

$$\bar{x}_p = \hat{x}_p \exp [i (kz - \omega t)] \tag{10}$$

is the x coordinate of the center of mass for the plasma column. Since the beam-plasma system is completely isolated from the outside world, we have the relation

$$\lambda_p \frac{d^2}{dt^2} \bar{x}_p = - \lambda_b \frac{d^2}{dt^2} \bar{x}_b, \tag{11}$$

where

$$\bar{x}_b = \hat{x}_b \exp [i(kz - \omega t)] \tag{12}$$

is the x coordinate of the center of mass for the electron beam. Substituting Eq. (5) into Eq. (7), and making use of Eqs. (4), (8), (9) and (11), we obtain

$$\left(\frac{\Omega^2}{i\omega}\right) \hat{x}_b = \frac{q\beta_b \pi}{\gamma_b m I_b} \int_0^\infty drr \frac{\sigma}{c} A_z \frac{dA_o}{dr}, \tag{13}$$

where

$$\Omega = \omega - k\beta_b c \tag{14}$$

is the Doppler - shifted eigenfrequency. In obtaining Eq. (14), use has been made of $dz/dt = \beta_b c$ for the electron beam. Equation (13) is identical to the result obtained by Lee[6]. Equation (13) is one of the main results of this paper and can be used to investigate stability properties for a broad range of system parameters.

For low frequency perturbation satisfying

$$\left|\frac{4\pi\sigma\omega R_b^2}{c^2}\right| \ll 1, \tag{15}$$

a closed expression of the dispersion relation is obtained from Eq. (13). In Eq. (15), R_b is the effective beam radius. As illustrated in Fig. 2, we assume that the electron beam is displaced along the x-direction with the displaced segment $\hat{x}_b$. Assuming that the displaced segment $\hat{x}_b$ is infinitismally small, we can express the displaced current density $J_b(r)$ as

$$J_b(r) = J_b^o\{[(x-\hat{x}_b)^2 + y^2]^{1/2}\} = J_b^o(r) - \left(\frac{dJ_b^o}{dr}\right)\hat{x}_b \tag{16}$$

from Fig. 2. Therefore, the perturbed current density of the electron beam is given by

$$J_{bz} = \frac{c}{4\pi}\hat{x}_b \frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dA_o}{dr}\right)\right], \tag{17}$$

where use has been made of Eq. (3).

In lowest order consistent with Eq. (15), the z-component of the perturbed vector potential $A_z(r)$ is calculated from

$$\begin{aligned}\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(rA_z\right)\right] &= -\frac{4\pi}{c}J_{bz}\\ &= -\hat{x}_b\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dA_o}{dr}\right)\right]\end{aligned} \tag{18}$$

thereby giving the solution

$$A_z(r) = -\left(\frac{dA_o}{dr}\right)\hat{x}_b, \tag{19}$$

which is a typical example of a rigid displacement of an electron beam[5]. Sub-

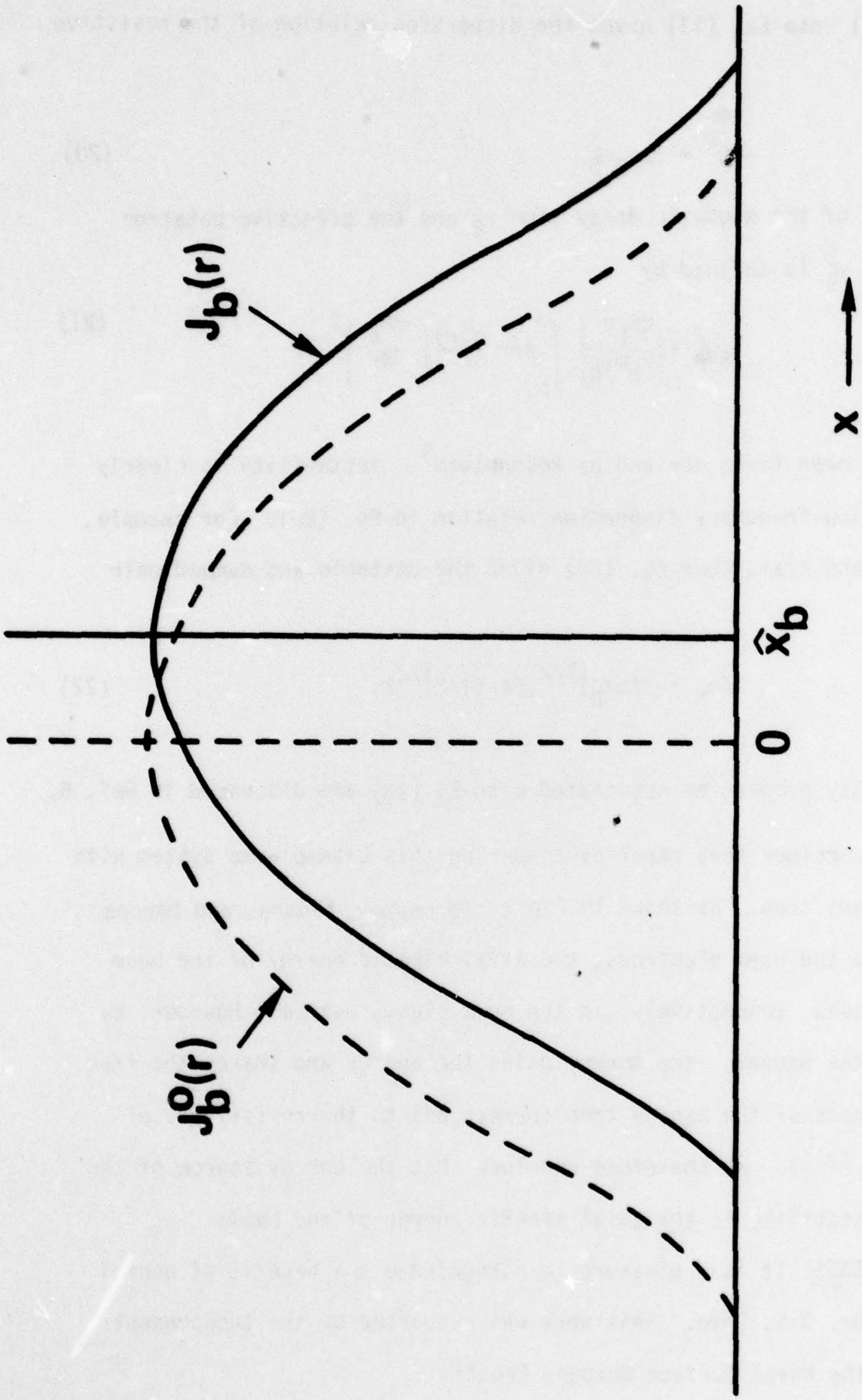

FIGURE 2 BEAM CURRENT DENSITY PROFILE IN A RIGID DISPLACEMENT

stituting Eq. (19) into Eq. (13) gives the dispersion relation of the resistive hose instability

$$-\Omega^2 = i\omega\tau_B\omega_B^2 \tag{20}$$

where the product of the magnetic decay time τ_B and the effective betatron frequency-squared ω_B^2 is defined by

$$\tau_B\omega_B^2 = \left(\frac{q\beta_b\pi}{\gamma_b m I_b}\right)\int_0^\infty drr\,\frac{\sigma(r)}{c}\left(\frac{dA_o}{dr}\right)^2. \tag{21}$$

Equation (20) has been first derived by Rosenbluth[5]. Instability is clearly indicated by the low-frequency dispersion relation in Eq. (20). For example, if ω is positive and real, then Eq. (20) gives the unstable and damped pair of roots,

$$\Omega/\omega_B = \pm(\omega\tau_B)^{1/2}[(i-1)/2^{1/2}]. \tag{22}$$

Additional stability properties associated with Eq.(20) are discussed in Ref. 6.

Finally, we conclude this paper by comparing this beam-plasma system with a monkey on a banana tree. As shown in Fig.3,the monkey, banana, and banana tree correspond to the beam electrons, the axial kinetic energy of the beam and background plasma, respectively, in the beam-plasma system. However, by slowly consuming the bananas, the monkey gains the energy and shakes the tree. The bending stiffness of the banana tree corresponds to the resistivity of background plasma ($1/\sigma$). We therefore conclude that the energy source of the resistive hose instability is the axial kinetic energy of the beam.

ACKNOWLEDGEMENTS: It is a pleasure to acknowledge the benefit of useful discussions with Dr. D.L. Love. This work was supported by the Independent Research Fund at the Naval Surface Weapons Center.

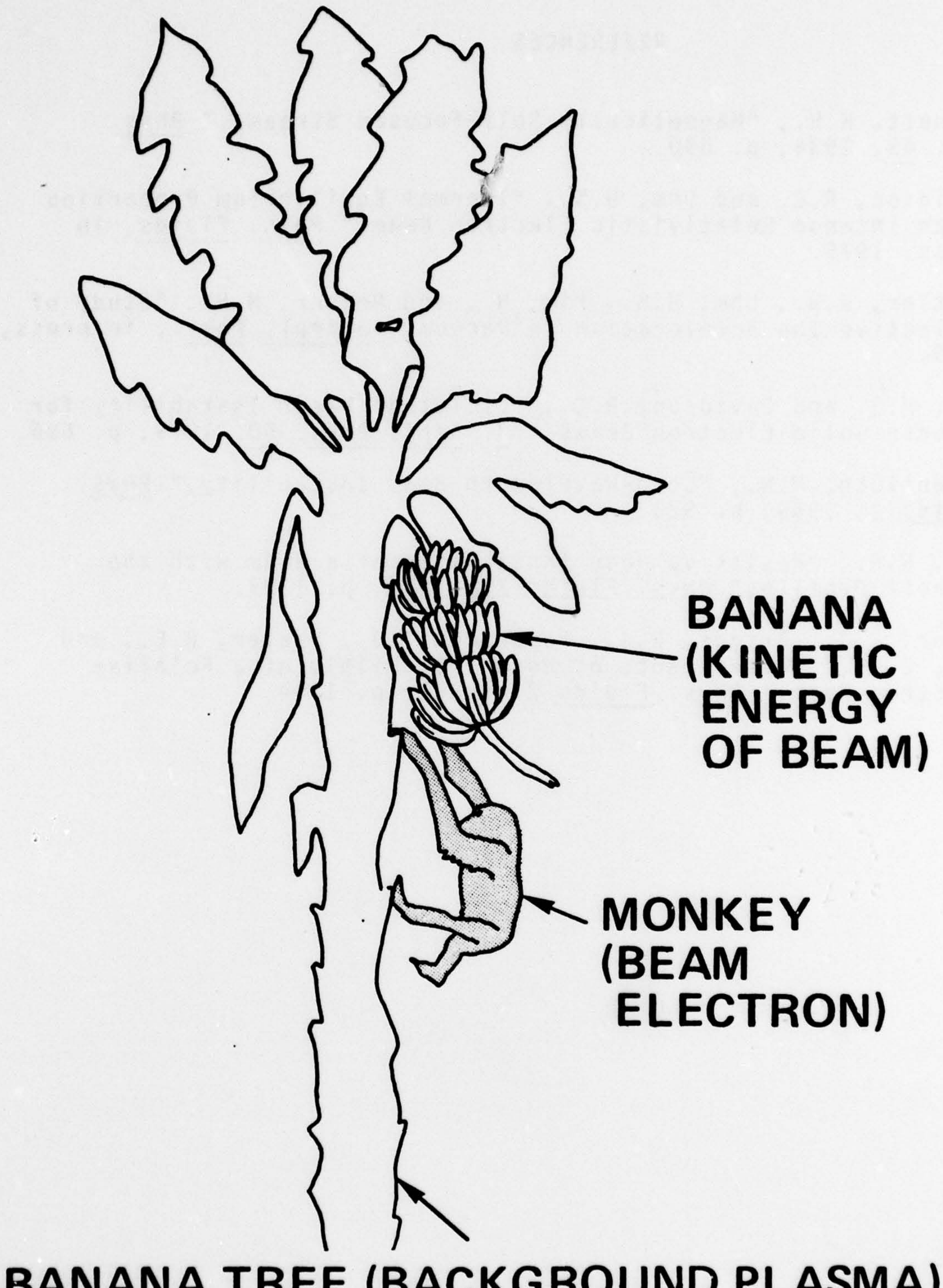

FIGURE 3 BANANA TREE WITH MONKEY IN ANALOGY OF A BEAM-PLASMA SYSTEM

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