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LOW PASS FILTERING OF DATA RECORDED ON MAGNETIC TAPE IN FREQUEN--ETC(U)

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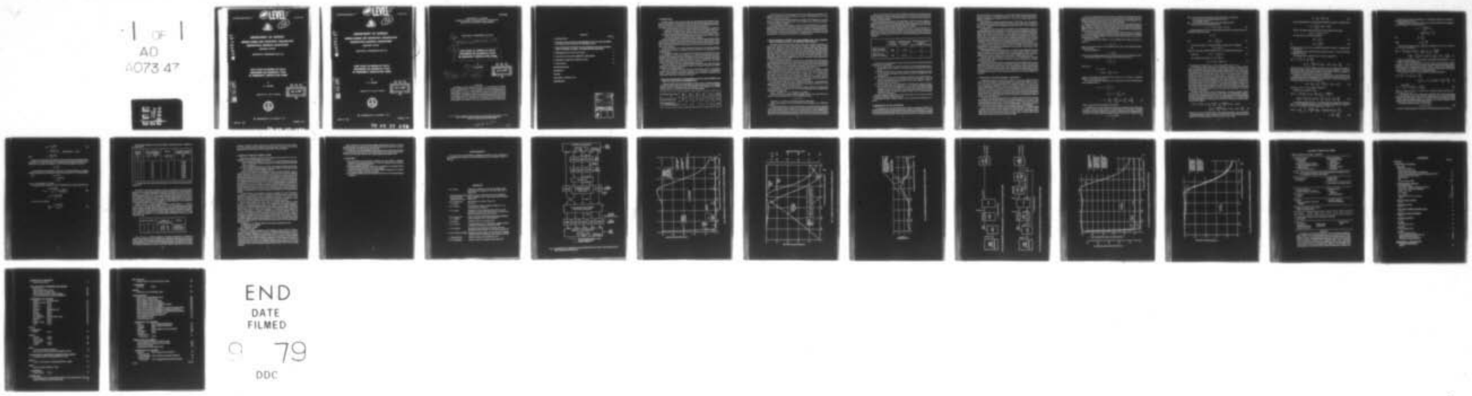

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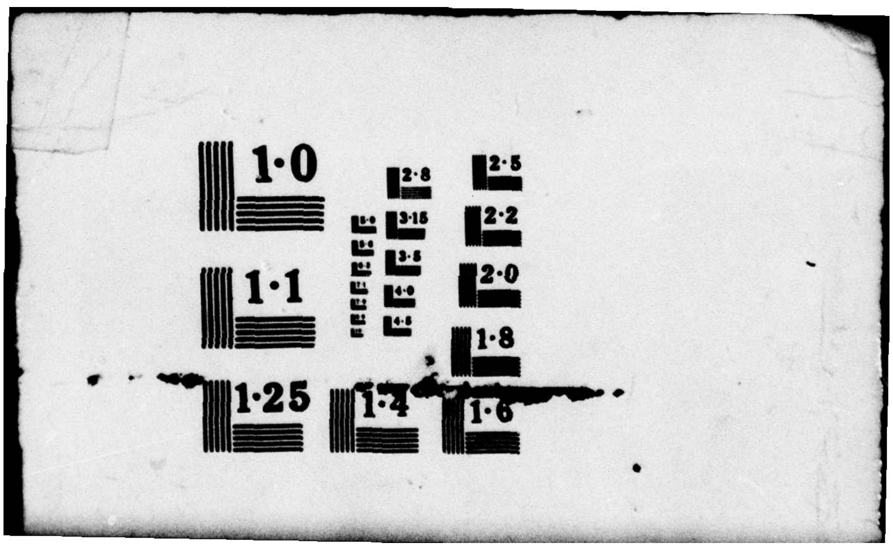
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MELBOURNE, VICTORIA

MECHANICAL ENGINEERING NOTE 373

LOW PASS FILTERING OF DATA RECORDED ON MAGNETIC TAPE IN FREQUENCY MODULATED FORM

by

K. F. FRASER

Approved for Public Release.

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MECHANICAL ENGINEERING NOTE 373

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LOW PASS FILTERING OF DATA RECORDED ON MAGNETIC TAPE IN FREQUENCY MODULATED FORM

by

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K. F. FRASER

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SUMMARY

Demodulators typically include low pass filters to recover signals from wideband frequency modulated carriers. Multispeed analogue tape machines normally require the carrier frequency to be set to a value proportional to tape speed. Reproducing hardware usually includes sets of filters, with bandwidth proportional to tape speed, for use with the demodulators. By dividing the frequency of the reproduced signal, a filter corresponding to a lower tape speed can be used and low pass filtering of recorded data signals can be readily achieved. However some unwanted noise components can be generated. These noise components are examined critically and a method of reducing them is demonstrated.

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CONTENTS

1. INTRODUCTION

The need sometimes arises to record signals over large bandwidth using frequency modulation (FM) techniques and subsequently low pass filter the reproduced signals. Such a requirement has recently arisen in the acquisition of vibration data from an aircraft fitted with acceleration sensors. There are two main areas of interest relating to the vibrations:

(i) To obtain information on the total vibration environment the amplitudes of acceleration components with frequencies up to about 2 kilohertz (kHz) are required. Time correlation between vibrations sensed at different points is not a requirement in this case.

(ii) Vibrations with frequency components up to about 30 Hz are of interest for flutter analysis. Amplitude and phase correlation between vibrations sensed at different points, or at the same point but in different directions, is necessary.

Normally the amplitude plots required for (i) would be obtained using a spectrum analyser and the analysis required in (ii) would be performed with the aid of a digital computer. In the latter instance the analogue vibration data must be regularly sampled and converted to digital form. To prevent the introduction of aliasing errors in this case low pass filtering of the reproduced data is necessary.

As tape recording tracks are usually in high demand in applications involving the acquisition of vibration data on aircraft, it is expedient to use the same sensor to provide signals to meet both requirements.

By dividing the frequency of the reproduced FM signal (before demodulation) the data can be transferred to a lower carrier frequency and a standard demodulator filter with correspondingly lower bandwidth can be used. Hardware[1] for such frequency division has been manufactured at these laboratories. Because the demodulator filters are usually fairly well matched in their amplitude and phase characteristics their use for data signal filtering, with requirements such as detailed in (ii) above, represents quite an advantage.

This report indicates how the standard demodulator filter can be used to meet the filtering requirements for the specific case cited above. However the techniques described can be readily extended to other filtering applications.

When the demodulator filters are used in this non-standard manner for data filtering purposes noise components of significant amplitude can be generated in the demodulator output. A detailed analysis of these noise components together with details on techniques which can be used to reduce them are included in this report.

2. DETAILED EXAMINATION OF REQUIREMENT FOR LOW PASS FILTERING AND TIME SCALE EXPANSION OF RECORDED DATA

Airborne magnetic tape recording equipment and ground station reproducing equipment, available at these laboratories for use in the application mentioned above (Sec. 1), are multispeed machines fitted with Intermediate Band[2] FM electronics. Carrier frequency and available bandwidth are given as a function of tape speed in the following table.

Tape Speed cm/s (Approx.) ips	152 (60)	76 (30)	38 (15)	19 (7·5)	9·5 (3·75)	4·75 (1·875)
Carrier Frequency kHz	108	54	27	13·5	6·75	3·375
Data Bandwidth kHz {±1 dB (decibel) passband}	20	10	5	2·5	1·25	0·625

To conserve tape, the lowest tape speed which will allow adequate data bandwidth to be obtained, is normally used. To accommodate the required 2 kHz bandwidth (Sec. 1) a tape speed of 19 cm/s ($7\frac{1}{2}$ ips) is selected.

For the flutter analysis under consideration, components with frequencies up to about 30 Hz are of interest. Low pass filtering with cut-off frequency above 30 Hz (say somewhere in the vicinity of 60 Hz) is required.

For the particular computer analysis to be used it may be assumed that the conversion rate per channel is limited to 50 samples per second. If six samples per cycle of the maximum frequency of interest (30 Hz at the time of recording) are assumed to be desirable then the maximum frequency of interest must not appear as more than about 8 Hz ($\approx$ 50/6 Hz) at the input to the converter. To satisfy this latter condition it is essential that the reproducing time scale be expanded by at least a factor of 4 relative to the recording time scale.

3. USE OF FREQUENCY DIVISION TO MAKE POSSIBLE LOW PASS FILTERING USING STANDARD MAGNETIC TAPE REPRODUCER HARDWARE

Intermediate bandwidth FM systems usually employ pulse averaging[3,4] demodulators as these are very suitable for handling wideband data. Normally a constant duration, constant height pulse is generated for every zero crossing of the reproduced FM signal. Because there are two zero crossings per cycle of the FM signal, the carrier frequency is effectively doubled. The train of pulses thus generated is taken to the input of the demodulator low pass filter which separates out a DC component (which must be backed off) and the modulating signal. Various other higher frequency sideband components are present in the input to the filter. In most applications separation of the modulating signal from the unwanted sideband components is the chief function of the low pass filter.

When the demodulator filters are considered for use in the filtering of recorded data their performance outside the passband takes on new significance. Because the frequency of modulating signal components may extend well beyond the carrier frequency which results after frequency division the generation of unwanted sideband components with frequencies within the filter passband can take place. These components will be analysed subsequently (Sec. 5).

The eight channel frequency divider[1], developed to facilitate low pass filtering of recorded data, provides division factors in binary steps from 1 to 32 (i.e. six factors). Division by a factor which is a power of 2 was adopted simply because the range of tape speeds available for the tape machine considered follows a binary sequence. In general division by factors which are not a power of 2 may be used but the carrier frequency, which results after division, would not then correpond to standard demodulator settings. In this latter case a DC shift with restrictions also on dynamic range would result when the data are demodulated.

It is convenient to define the "corner" frequency as the nominal ± 1 dB (decibel) filter passband limit which ranges from 20 kHz for 152 cm/s FM reproduction to 625 Hz for 4·75 cm/s FM reproduction as indicated in the table of Section 2.

To see whether a filter with corner frequency in the vicinity of 60 Hz can be obtained with the aid of the frequency divider, the ratio r_D of the recording band limit to the desired filter corner frequency is calculated.

Thus $r_D = 2500/60 = 41 \cdot 7$ for the application considered.

However the particular frequency divider available will allow only ratios which are a power of 2. The nearest actual ratio r_A which can therefore be used is

$$r_A = 32 = 2^5$$

In general $r_A = 2^N$ where N is the frequency division switch setting[1].

Using a division factor of 32 the low pass filter corner frequency is set to 2500/32 Hz (i.e. 78 Hz).

To perform the filtering in one pass of the tape without the introduction of an intermediate "copy" the recorded tape must be reproduced at 152 cm/s. Division of the FM signal frequency by 32 actually transfers the data to a lower carrier frequency (3375 Hz, equal to 108000/32 Hz). This transfer causes high frequency components to be irretrievably lost but the main frequency selection, with any associated amplitude and phase shifts, occurs when the signal is demodulated.

The act of reproducing the data at 152 cm/s (60 ips) rather than 19 cm/s ($7\frac{1}{2}$ ips) causes the data frequencies to appear 8 times higher than they were at the time of recording. Thus the 78 Hz cut-off frequency appears in real time as 625 Hz ($\approx 8 \times 78$ Hz) and the 30 Hz maximum frequency of interest as 240 Hz.

If the analysis system could handle a data bandwidth of 625 Hz it would be sufficient to reproduce the recorded data, perform the frequency division using a factor of 32, demodulate using a 4·75 cm/s ($1\frac{7}{8}$ ips) filter, and digitize the data without producing a filtered tape copy. In that case 8 seconds of recorded data would be digitized in 1 second.

For the flutter analysis the digital equipment will not allow a 625 Hz bandwidth to be achieved. As indicated in Section 2 the conversion rate is limited to 50 samples per second and the time scale at the time of the digital conversion must be expanded by at least a factor of 4 relative to the recording time scale. If a factor of 4 is adopted the figures given in the following table apply.

	Frequency at the time of recording	Frequency at the time of the digital conversion	Samples per cycle at 50 Hz sample rate (at time of digital conversion)
Signal of maximum frequency of interest	30 Hz	7·5 Hz	6·7
Signal at low pass filter corner frequency	78 Hz	19·5 Hz	2·5

To achieve a time scale expansion of 4 relative to that at the time of recording the following operations need to be performed:

(i) The FM signal which has its frequency divided by 32 for filtering purposes needs to be demodulated using a standard 4·75 cm/s ($1\frac{7}{8}$ ips) filter for intermediate band tape systems.

(ii) The demodulated output from (i) needs to be taken to an FM recording module aligned for 152 cm/s (60 ips) intermediate band (108 kHz) recording and plugged into a tape copying machine.

(iii) A "filtered" tape copy is to be produced by recording on the copying machine at 152 cm/s. This copying process sets the filtered data corner frequency to $\frac{1}{32}$ of the available bandwidth at this recording speed. The time scale "expansion" relative to that at the time of recording is $\frac{1}{8}$.

(iv) The filtered tape copy is to be reproduced at 4·75 cm/s and the output of the demodulator, aligned for that reproducing speed, is to be coupled to the analogue to digital converter. As the data are contained in the lowest $\frac{1}{32}$ of the available bandwidth negligible changes in the amplitude or phase characteristics should arise in this last demodulation stage. The time scale expansion relative to (iii) is 32 and that relative to the time of recording is 4.

With the arrangement detailed above it takes four seconds to convert every one second of recorded data (referred to original time scale).

Simultaneous handling of multiple channels of data is possible, thus allowing time correlation.

4. PERFORMANCE OF LOW PASS FILTER

The performance of a low pass filtering system meeting the requirements detailed in the previous section has been evaluated using hardware available at these laboratories. To do this a tape recording consisting of multiple tracks of simultaneously recorded swept sinewave signals, and a track on which was recorded a voltage proportional to frequency, was prepared. Recording using FM techniques was performed with a 7-track model FR1260 tape system manufactured by Ampex Corporation. Full amplitude (producing $\pm 40\%$ carrier deviation) swept sinewave signals were simultaneously recorded on tracks 1, 3, 5 and 6 and a voltage proportional to frequency on track 7.

Using the hardware arrangement indicated in Figure 1 a filtered tape "copy" was produced at 152 cm/s (60 ips) tape speed as outlined in the previous section. Information reproduced on tracks 3 and 7 was copied[1] without demodulation and that from tracks 1, 5 and 6 was filtered prior to being "copied".

Two frequency sweeps were recorded on the original tape (and subsequently copied), one extending well beyond the passband limits of the filter and one just encompassing the passband. When the filtered data were reproduced at 152 cm/s (using the wide frequency range sweep) together with a channel of data which had not been filtered, the response curves of Figure 2 resulted. Since, for the particular example under consideration, the passband of the filter had been located at approximately the lower $\frac{1}{32}$ of the available bandwidth of the "copy" channel the relative shape of the filter response curves (for both amplitude and phase) would be virtually independent of the speed (and hence the demodulator filter) used to reproduce the copy.

A series of double "humps" (Fig. 2) with maximum amplitudes near $2f_c$ (where f_c is the carrier frequency which results after frequency division and equals $3 \cdot 375$ kHz for 152 cm/s reproduction) is to be noted outside the passband of the low pass filter. These "humps" will be considered in more detail in Section 5.

Using the reduced frequency range sweep (up to $1 \cdot 75$ times the nominal bandwidth [response within ± 1 decibel] of the low pass filter) the low pass filter response of Figure 3 was automatically plotted. Because the characteristics of the various demodulator filters are not perfectly matched some variation in response curves would result if a different filter were used.

Using the signal copied (unfiltered) on track 3 (Fig. 1) as the phase reference, the phase of the filtered signal "copied" on track 5 was plotted as a function of frequency as indicated in the lower graph of Figure 3.

The extent of the phase mismatch between signals passed through different filters is of great significance especially where correlation of the signals is important. In Figure 4 the phase mismatch between signals recorded on tracks 1 and 5 (same head), and between signals recorded on tracks 5 and 6 (different head) is plotted. For the specific example considered here the filtered outputs from tracks 1, 5 and 6 match to within $2 \cdot 5$ degrees for 0 to 60 Hz bandwidth where the frequencies are referred to the recording time scale (19 cm/s for which the bandwidth is $2 \cdot 5$ kHz).

5. ANALYSIS OF UNWANTED SIDEBAND COMPONENTS

As indicated in Section 4 a series of double "humps" is observed in the amplitude response (filter output versus frequency of modulating signal) of the low pass filter. Nodes occur at $2f_c$, $4f_c$, $6f_c$ etc. where f_c is the carrier frequency which results after frequency division ($3 \cdot 375$ kHz for the example under consideration). In each case antinodes are observed at a separation f_x from the node frequencies. For each antinode f_x is approximately equal to $0 \cdot 2f_c$ (equal to about 670 Hz for $3 \cdot 375$ kHz carrier frequency). However the frequency of the components observed at the output of the low pass filter is f_x in each case. Since f_x is just beyond the corner frequency of the low pass filter, components at this frequency effectively fall within the filter passband and filtering of these unwanted components from desired components is impossible after frequency division as outlined in Section 4.

In the low pass filtering example of Section 4 a wideband signal was used to modulate a high frequency carrier and the frequency of the modulated signal was later divided to make the carrier frequency compatible with a filter having lower cut-off frequency. It has been shown experimentally that identical "humps" occur if the wideband signal is taken directly to a modulator set to the lower carrier frequency, and the modulated signal is later demodulated.

For the purpose of the analysis of unwanted sideband components it is therefore adequate to consider a low frequency carrier modulated with a wideband signal the frequency of which extends well above that of the carrier. When such a signal is taken to a pulse averaging demodulator (Sec. 3) the low pass filter will receive at its input a train of constant width, constant height pulses starting at each zero crossing of the FM signal.

In order to examine analytically the generation of sideband noise components a Fourier analysis may be performed on the train of pulses received at the low pass filter input. Any sideband components (excluding components at modulating frequency) with frequencies in the low pass filter passband will give rise to unwanted noise components.

Monson[5] has considered the frequency spectrum of the input to a low pass filter used in a pulse averaging demodulator. He analyses the output from an FM multivibrator system and shows that there is an equivalence between the multivibrator FM and pulse position modulation (PPM). It will now be shown that such equivalence applies equally well for pulses generated at the time of the zero crossings of a modulated sinewave.

To a first approximation the FM output from the tape reproducing head may be considered to be a constant amplitude frequency modulated sinewave signal. In practice the amplitude and phase will be frequency dependent but the net effect of such variations on the time of the zero crossings will be small.

The instantaneous radian frequency ω of the modulated carrier signal $e(t)$ may be expressed as

$$\omega = \omega_c + \omega_d$$

where ω_c is the radian frequency of the unmodulated carrier and ω_d is the instantaneous radian frequency deviation.

ω_d may be expressed as

$$\omega_d = A\, e_m(t)$$

where $e_m(t)$ is the modulating signal which is a function of time t and A is a modulation constant.

The instantaneous phase θ of the modulated carrier signal $e(t)$ is given by

$$\theta = \int_{-\infty}^{t} \omega\, dt$$

Thus we have

$$\begin{aligned} e(t) &= E \sin \theta \\ &= E \sin \left(\omega_c t + \int \omega_d\, dt + \phi_0\right) \qquad (1) \end{aligned}$$

where E is the amplitude of the modulated carrier signal and ϕ_0 is a constant of integration dependent on the initial value of $e(t)$.

If t_{r-1} and t_r are the times of consecutive zero crossings (two per cycle for the demodulator under consideration) then

$$\begin{aligned} \theta_r - \theta_{r-1} = \pi &= \int_{t_{r-1}}^{t_r} \omega\, dt \\ &= \omega_c (t_r - t_{r-1}) + \int_{t_{r-1}}^{t_r} \omega_d\, dt \end{aligned}$$

$$t_r - t_{r-1} = \frac{T_c}{2}\left[1 - \frac{1}{\pi}\left\{\int_0^{t_r} \omega_d\, dt - \int_0^{t_{r-1}} \omega_d\, dt\right\}\right] \qquad (2)$$

where T_c is the period of the unmodulated carrier signal.

For PPM the value of each instantaneous sample of the modulating wave is used to vary the position in time of a pulse relative to its unmodulated time of occurrence. If sampling coincides with the time of appearance of the position modulated pulse the sampling is said to be "natural". For PPM with natural sampling the change in position of the r^{th} pulse is given by

$$t_r - t_R = [\alpha\, S(t_r)]\, T_0 \qquad (3)$$

where t_R is the time of occurrence of the r^{th} pulse when there is no modulation,
$S(t_r)$ is the value of the modulating signal $S(t)$ at time t_r,
α is a modulation constant,
and T_0 is the repetition period of pulses in an unmodulated pulse train.

The time between successive pulses is given by

$$t_r - t_{r-1} = T_0 [1 + \alpha \{S(t_r) - S(t_{r-1})\}] \tag{4}$$

Comparing equation (4) with (2) the equivalence between the low pass filter input and a PPM signal is readily seen, giving

$$T_0 = \frac{T_c}{2}$$

and

$$\alpha S(t) = -\frac{1}{\pi} \int_0^t \omega_d \, dt \tag{5}$$

For the present analysis a cosinusoidal frequency deviation will be considered.

Let

$$\omega_d = \omega_D \cos \omega_m t \tag{6}$$

where ω_m is the radian modulation frequency and ω_D is the peak frequency deviation.

In this case

$$\alpha S(t) = -\frac{\omega_D}{\pi \omega_m} \sin \omega_m t \tag{7}$$

The modulated carrier signal $e(t)$ is now given by

$$e(t) = E \sin (\omega_c t + \frac{\omega_D}{\omega_m} \sin \omega_m t) \tag{8}$$

if θ is arbitrarily set equal to zero at $t = 0$ (i.e. a zero crossing occurs at $t = 0$).

The spectrum of a PPM signal has been considered by a number of authors including Krauss and Ordung,[6] Black[7] and Carlson[8]. As the sideband components of interest in the present investigation are generated when the modulating signal frequency is either in the vicinity of the repetition rate of the unmodulated pulse train or in excess of that rate, the simplifying assumptions which are commonly used are not valid here. The analysis by Black is suitable and will be considered here. However a detailed analysis of the PPM spectrum is not given by that author, but an expression (apparently derived from unpublished work by Edson) for computing the value of the Fourier coefficients is given. Black does provide a detailed analysis of the pulse duration modulation (PDM) spectrum for natural sampling using a double Fourier series approach based on work by Bennett[9] on the evaluation of modulation products which arise when two signals of different frequency are added together and applied to a rectifier. The close analogy between PPM and PDM is well known, and it will be shown that the spectrum of the former can be readily derived from that of the latter. Bell and Sergent[10] also consider the spectrum of a PDM signal.

Consider a PDM signal for which the leading edges of the pulses are spaced at regular intervals and for which the trailing edges are varied relative to the leading edges according to the value of the modulating signal at the time of the trailing edge transition (natural sampling). Thus the trailing edges follow exactly the same relationship (equation 3) as the PPM signal. The following expression for a PDM signal $F_1(t)$ with natural sampling and trailing edge modulation is given by Black.

$$F_1(t) = k + \frac{M}{2} \cos \omega_m t + \sum_{m=1}^{\infty} \frac{\sin m\omega_c t}{m\pi} - \sum_{m=1}^{\infty} \frac{J_0(m\pi M)}{m\pi} \sin \{m\omega_c t - 2m\pi k\}$$

$$- \sum_{m=1}^{\infty} \sum_{n=\pm 1}^{\pm\infty} \frac{J_n(m\pi M)}{m\pi} \sin\left(m\,\omega_c t + n\omega_m t - 2m\pi k - \frac{n\pi}{2}\right) \tag{9}$$

where $2\pi/\omega_c$ (or T_0) is the repetition period of the pulses (leading edges), $2\pi k/\omega_c$ is the duration of unmodulated pulses, m and n are integers, and $(M/2) \cos \omega_m t$ is the modulating signal as specified in the following expression for the duration T_r of the r^{th} pulse.

$$T_r = T_0\left(k + \frac{M}{2}\cos\omega_m t\right) \tag{10}$$

$J_n(z)$ is the Bessel function[11] of the first kind with argument z and order n. In integral[11] form

$$J_n(z) = \frac{1}{\pi}\int_0^\pi \cos(z\sin\phi - n\phi)\,d\phi$$

When n is an integer (which is the only case we need consider for this analysis)

$$J_{-n}(z) = (-1)^n J_n(z) = J_n(-z)$$

For small values of z as given by Hildebrand[12]

$$J_n(z) \sim \frac{1}{2^n\,n!}\,z^n \tag{11}$$

and

$$J_{-n}(z) \sim \frac{(-1)^n}{2^n\,n!}\,z^n \tag{12}$$

in the sense that the ratio of the two quantities connected by the symbol $\sim$ approaches unity as z approaches 0.

A leading edge of one of the PDM pulses is assumed to occur at $t = 0$ and the pulse height is considered to be unity.

The PDM spectrum of equation 9 may be written more generally as

$$F_1(t) = k + \frac{M}{2}\cos\omega_m t + \sum_{m=1}^{\infty}\frac{\sin\omega_c t}{m\pi}$$

$$-\sum_{m=1}^{\infty}\sum_{n=-\infty}^{\infty}\frac{J_n(m\pi M)}{m\pi}\sin\left\{m\left(\omega_c t - 2\pi k\right) + n\left(\omega_m t - \frac{\pi}{2}\right)\right\} \tag{13}$$

where the first term is the DC component, the second term is the modulation component, the third term represents a sawtooth wave of negative slope and abrupt transition at $t = 0$ (Krauss and Ordung consider pulse modulated signals of various forms as being the summation of sawtooth waves), and the fourth term varies with the modulation.

Consider now a PPM pulse train with pulses of duration $2\pi k/\omega_c$. Assume the leading edge is modulated according to natural sampling of the modulating signal (given by $(M/2)\cos\omega_m t$ as before), and further assume a leading edge transition occurs at $t = 0$. The time function $F_2(t)$ of the PPM signal can then be written simply as

$$F_2(t) = F_1\left(\omega_c t, \omega_m\left[t - \frac{2\pi k}{\omega_c}\right], k\right) - F_1\left(\omega_c t, \omega_m t, 0\right) \tag{14}$$

where $F_1(\omega_c t, \omega_m t, k) = F_1(t)$ as defined by equation 13.

Substitution into equation 13 yields

$$F_2(t) = k + M\sin\frac{\omega_m}{\omega_c}k\pi\sin\omega_m\left(t - \frac{k\pi}{\omega_c}\right) + \sum_{m=1}^{\infty}\sum_{n=-\infty}^{\infty}\frac{2J_n(m\pi M)}{m\pi}$$

$$\sin\left\{\left(m + n\frac{\omega_m}{\omega_c}\right)k\pi\right\}\cos\left\{m\omega_c\left(t - \frac{k\pi}{\omega_c}\right) + n\left[\omega_m\left(t - \frac{k\pi}{\omega_c}\right) - \frac{\pi}{2}\right]\right\} \tag{15}$$

If the modulating signal $(M/2)\cos\omega_m t$ is changed to $(M/2)\sin\omega_m t$ (equivalent to $(M/2)\cos(\omega_m t - \pi/2)$) the modified time function, $F_3(t)$ say, is obtained by replacing $\omega_m t$ in equation 15 with $\omega_m t - \pi/2$. The final term then becomes $n\{\omega_m(t - k\pi/\omega_c) - \pi\}$.

$$F_3(t) = (-1)^n\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\frac{J_n(m\pi M)}{m\pi}\sin\left\{\left(m + n\frac{\omega_m}{\omega_c}\right)k\pi\right\}\cos\left\{m\omega_c\left(t - \frac{k\pi}{\omega_c}\right)\right.$$

$$\left. + n\omega_m\left(t - \frac{k\pi}{\omega_c}\right)\right\} \tag{16}$$

The DC term is obtained by substituting $n = 0$ and letting m approach 0 in equation 16, and using relationships 11 and 12.

The spectrum of the low pass filter input can now be easily written down. From equations 3, 7 and 10 the shift of the r^{th} pulse is given by

$$t_r - t_R = -\frac{\omega_D T_0}{\pi\omega_m}\sin\omega_m t$$

$$= \frac{\omega_D T_0}{\pi\omega_m}\cos\left(\omega_m t + \frac{\pi}{2}\right)$$

and

$$\frac{M}{2} = \frac{\omega_D}{\pi\omega_m}$$

Define

$$\beta = \frac{\omega_D}{\omega_m} = \frac{\pi M}{2}$$

Thus the required spectrum $F_4(t)$ is obtained by substituting $2\omega_c$ for ω_c, $2\beta/\pi$ for M and $\omega_m t + \pi/2$ for $\omega_m t$ in equation 15.

$$F_4(t) = k + \frac{2\beta}{\pi}\sin\frac{\omega_m}{2\omega_c}k\pi\cos\left\{\omega_m\left(t - \frac{k\pi}{2\omega_c}\right)\right\} + \sum_{m=1}^{\infty}\sum_{n=-\infty}^{\infty}\frac{2J_n(2m\beta)}{m\pi}$$

$$\sin\left\{\left(m + n\frac{\omega_m}{2\omega_c}\right)k\pi\right\}\cos\left\{2m\omega_c\left(t - \frac{k\pi}{2\omega_c}\right) + n\omega_m\left(t - \frac{k\pi}{2\omega_c}\right)\right\} \quad (17)$$

If the zero time reference ($t = 0$) were shifted to the middle of the first pulse then $t - k\pi/\omega_c$ in equation 17 would be replaced with t.

The $(-1)^n$ factor in the PPM spectrum of equation 16 has not carried through to the FM demodulator low pass filter spectrum of equation 17 and indeed would not have been included in equation 16 if the modulating signal had been defined as $-(M/2)\sin\omega_m t$. Although it was of no consequence in his analysis, Monson seems to have overlooked this point as he indicated a negative sign for the component at the modulating frequency.

For convenience equation 17 may be replaced by the following equations

$$F_4(t) = \frac{A_{00}}{2} + A_{01}\cos\left\{2\pi f_m\left(t - \frac{t_0}{2}\right)\right\}$$

$$+ \sum_{m=1}^{\infty}\sum_{n=-\infty}^{\infty} A_{mn}\cos\left\{2\pi\left(2mf_c + nf_m\right)\left(t - \frac{t_0}{2}\right)\right\} \quad (18)$$

$$A_{mn} = \frac{2J_n\left(2m\frac{f_D}{f_m}\right)}{m\pi}\sin\left\{\left(2mf_c + nf_m\right)\frac{k\pi}{2f_c}\right\} \quad (19)$$

where $f_c = \omega_c/2\pi$, $f_m = \omega_m/2\pi$, $f_D = \omega_D/2\pi$ and t_0 is the pulse duration. Note that now $2\pi k = (2\omega_c)t_0$ or $k = 2f_c t_0$.

It was indicated, for the filter example under consideration, that for the major antinodes in the response (filter output versus modulating signal frequency) the observed filter output frequency was f_x (approximately equal to $0{\cdot}2 f_c$ or 670 Hz). The various values of f_m required to yield a significant sideband component at f_x frequency and the relative amplitude of such components are of interest and will be tabulated shortly.

The required sideband frequency can be expressed as

$$f_x = |2mf_c + nf_m| \quad (20)$$

and values of modulating frequency which produce that sideband frequency are given by

$$f_m = \frac{\gamma f_x - 2mf_c}{n} \tag{21}$$

$$= \frac{-2(m \mp 0\cdot1)}{n} f_c \qquad \text{(substituting } \gamma f_x = \pm 0\cdot2 f_c\text{)}$$

where $$\gamma = \frac{2mf_c + nf_m}{f_x}$$

Comparison of the amplitude of the sideband component with that of a modulating frequency component, for f_m small, will be made for a frequency deviation amplitude f_D equal to $0\cdot4 f_c$ in each case. For f_m small, the amplitude A_{01} of the component at frequency f_m is given by

$$A_{01} = \frac{f_D}{f_c} k$$

For the particular FM demodulator used the ratio k of the pulse duration t_0 to the pulse repetition period $1/(2f_c)$ for zero modulation was found by measurement to be $0\cdot45$ approximately.

Thus
$$\begin{aligned} A_{01} &= 0\cdot4\,k \\ &= (0\cdot4)\,(0\cdot45) \\ &= 0\cdot18 \end{aligned}$$

(i.e. 18% of the amplitude of the pulses).

From equation 19 the amplitude of the relevant sidebands may be written in general form as

$$A_{mn} = \frac{2\,J_n\left(0\cdot8m\dfrac{f_c}{f_m}\right)}{m\pi} \sin\left\{k\left(\frac{\gamma f_x}{2f_c}\right)\pi\right\} \tag{22}$$

$$\approx \gamma \frac{0\cdot2\,k\,J_n\left(0\cdot8m\dfrac{f_c}{f_m}\right)}{m}$$

The ratio A_{mn}/A_{01} becomes

$$\frac{A_{mn}}{A_{01}} = \gamma \frac{J_n\left(0\cdot8m\dfrac{f_c}{f_m}\right)}{2m} \tag{23}$$

The normalized amplitude of the relevant sideband components as given in equation 23 is tabulated below.

Sideband Component A_{mn} m	n	Ratio of Modulating to Carrier Frequency $\alpha = \frac{f_m}{f_c} = \frac{-2(m \mp 0 \cdot 1)}{n}$	Sign of γ $\gamma = \frac{2m + n\alpha}{\lvert 2m + n\alpha \rvert}$	Normalized Amplitude* of Sideband Component $\frac{A_{mn}}{A_{01}}$
1	−2	0·9	+1	+0·045
1	−2	1·1	−1	−0·031
1	−1	1·8	+1	−0·108
1	−1	2·2	−1	+0·088
2	−1	3·8	+1	−0·051
2	−1	4·2	−1	+0·046
3	−1	5·8	+1	−0·034
3	−1	6·2	−1	+0·031
4	−1	7·8	+1	−0·025
4	−1	8·2	−1	+0·024

* A negative sign simply indicates the initial phase of the cosinusoidal spectral component (equation 17).

The analysis of this section, the results of which are summarized in the preceding table, predicts that the most significant sideband component will be generated when the modulation frequency is $1 \cdot 8 f_c$ (equal to 6·08 kHz for 3·375 kHz carrier frequency) and that it will have a relative amplitude of about 10·8%. The calculated amplitude of the component which arises at this modulation frequency and of those at other antinodal frequencies agrees closely with the measured response as detailed previously in Figure 4. Some slight reduction in the amplitude of the f_x sideband component is to be expected as it passes through the low pass filter since the response has started to "roll off" at frequency f_x.

Consider now f_x as a variable frequency as defined by equation 20 for $n = -1$. When f_x is in the vicinity of the filter corner frequency (e.g. $f_x = 0 \cdot 2 f_c$ as considered above) the filter output will appear sinusoidal. When f_x is much lower than the filter corner frequency, the filter output will appear non-sinusoidal (noted experimentally) because of the presence of harmonics. As an example the summation of the particular spectral components listed in the following table results in the generation of a non-sinusoidal signal of fundamental frequency f_x at the output of the filter.

Value of m	Value of n	Frequency of Spectral Component	Comment
1	−1	f_x, $2f_c - f_m$	Fundamental
2	−2	$2f_x$, $4f_c - 2f_m$	Second harmonic
3	−3	$3f_x$, $6f_c - 3f_m$	Third harmonic

As $f_x \to 0$ (i.e. $f_m \to 2mf_c$) the amplitude of the spectral components (as predicted by equation 22) at frequency f_x, and of harmonics of f_x, approach zero thus giving rise to the nodes noted in Figure 2. The transmission through the filter will increase with separation of the modulating signal frequency f_m from the nodal modulating frequencies $2mf_c$ until limited by attenuation outside the passband of the filter. Thus the filter output corresponding to the antinodes of Figure 2 is expected to exhibit a frequency equal to that at which the $f\,A(f)$ function (where $A(f)$ is the amplitude response characteristic of the filter) peaks. The observed $0 \cdot 2 f_c$ (670 Hz approximately for the particular example considered) agrees with this prediction.

6. METHOD OF REDUCING SIDEBAND NOISE

Excessive data bandwidth at the time of the original recording may give rise to an unacceptable level of unwanted sideband components if:

(i) The modulating signal frequency at the time of the original recording is greater than about 1·6 times the carrier frequency (equivalent to the leading edge of the first sideband "hump" in Figure 2).

(ii) The modulating signal frequency at the time of the original recording is greater than about 1·6 times the carrier frequency arising after frequency division (which precedes low pass filtering of the type described in Sec. 4).

Consider, as an example of (i), an application in which a wideband transducer is used for vibration measurement, but only the lower frequency components are of interest for subsequent analysis. Assume that a tape speed and hence also carrier frequency are chosen such that the corresponding demodulator filter should restrict the data passband to the band of interest only. As a specific example assume that components with frequencies in excess of 10 kHz are present in the transducer output at the time of recording. Further assume that a 625 Hz cut-off frequency (as obtainable with 3·375 kHz carrier for 4·75 cm/s recording) adequately encompasses the frequency band of interest. A vibration component at 6·08 kHz frequency would give rise to an unwanted 670 Hz component at about 10% the amplitude of the 6·08 kHz component at the time of recording. In cases where the level of the unwanted sideband components which pass through the reproducing filter is excessive, frequency band limiting prior to recording would be essential.

A similar problem to that detailed in the previous paragraph may arise if recording is performed with an adequately high carrier frequency but frequency division and low pass filtering are performed when the data are reproduced (an example of (ii)). However in this latter case a modified frequency division and filtering procedure can be employed to reduce the level of the unwanted sideband components if that is considered necessary.

The first major sideband peak occurs when the modulating signal frequency is about 6·08 kHz, if a straight division by a factor of 32 is employed for the example considered in Section 4. For the modulating signal frequency below about 5·4 kHz the sideband contribution is virtually zero. It follows therefore that if the upper modulating signal frequency is kept below about 5·4 kHz (equal to 1·6 times carrier frequency or 8·6 times filter corner frequency for 4·75 cm/s reproduction) the sideband components should be largely eliminated. The latter condition can be satisfied if the data are pre-filtered prior to reaching the final filter. In Figure 5*a* the hardware arrangement for low pass filtering as discussed earlier is drawn, and in Figure 5*b* a modified arrangement which provides pre-filtering with 2·5 kHz corner frequency is drawn. An additional stage of modulation and demodulation (13·5 kHz carrier frequency) using standard hardware is required following a frequency divider with division factor of 8. The final frequency division factor is 4.

A comparison of the amplitude response curves obtained experimentally when:

(i) Pre-filtering is not included

(ii) Pre-filtering is included

is shown in Figures 6 and 7.

Reduction of sideband components with pre-filtering is readily observed in Figure 6. About 2% maximum sideband level is noted for a modulating signal frequency near 27 kHz. Significant sideband components will be generated at the 19 cm/s ($7\frac{1}{2}$ ips) pre-filter output in the vicinity of 27 kHz modulating frequency (twice carrier frequency for 19 cm/s recording). The amplitude of these sideband components, generated as the modulating signal frequency is varied, will increase from zero at zero sideband frequency to peak at about 2·7 kHz ($0 \cdot 2 f_c$) sideband frequency. Components with frequencies falling within the passband of the 4·75 cm/s ($1\frac{7}{8}$ ips) filter will give rise to the sideband components noted in Figure 6 near 27 kHz modulating frequency.

Some modification to the low pass filter amplitude response (Fig. 7) arises as a result of the additional stage of pre-filtering but for the particular filters used the difference is very small. Some modification to the phase response will also result.

To reduce the level of unwanted sideband components and also to avoid excessive departure in the low pass filter characteristic from the normal response a final frequency division factor of 4 (even if a division factor other than 32 is required) represents the optimum value.

7. CONCLUSION

a) Low pass filtering of data recorded on magnetic tape using frequency modulation techniques can be simply performed when the data are reproduced using a frequency divider and standard demodulator filter.

b) The filtering technique described can give rise to the generation of unwanted sideband components having frequencies within the filter passband.

c) Analytical expressions for the level of unwanted sideband components can be derived from a Fourier analysis of the filter input signal.

d) A pre-filtering technique can be used to greatly reduce the level of the unwanted sideband components.

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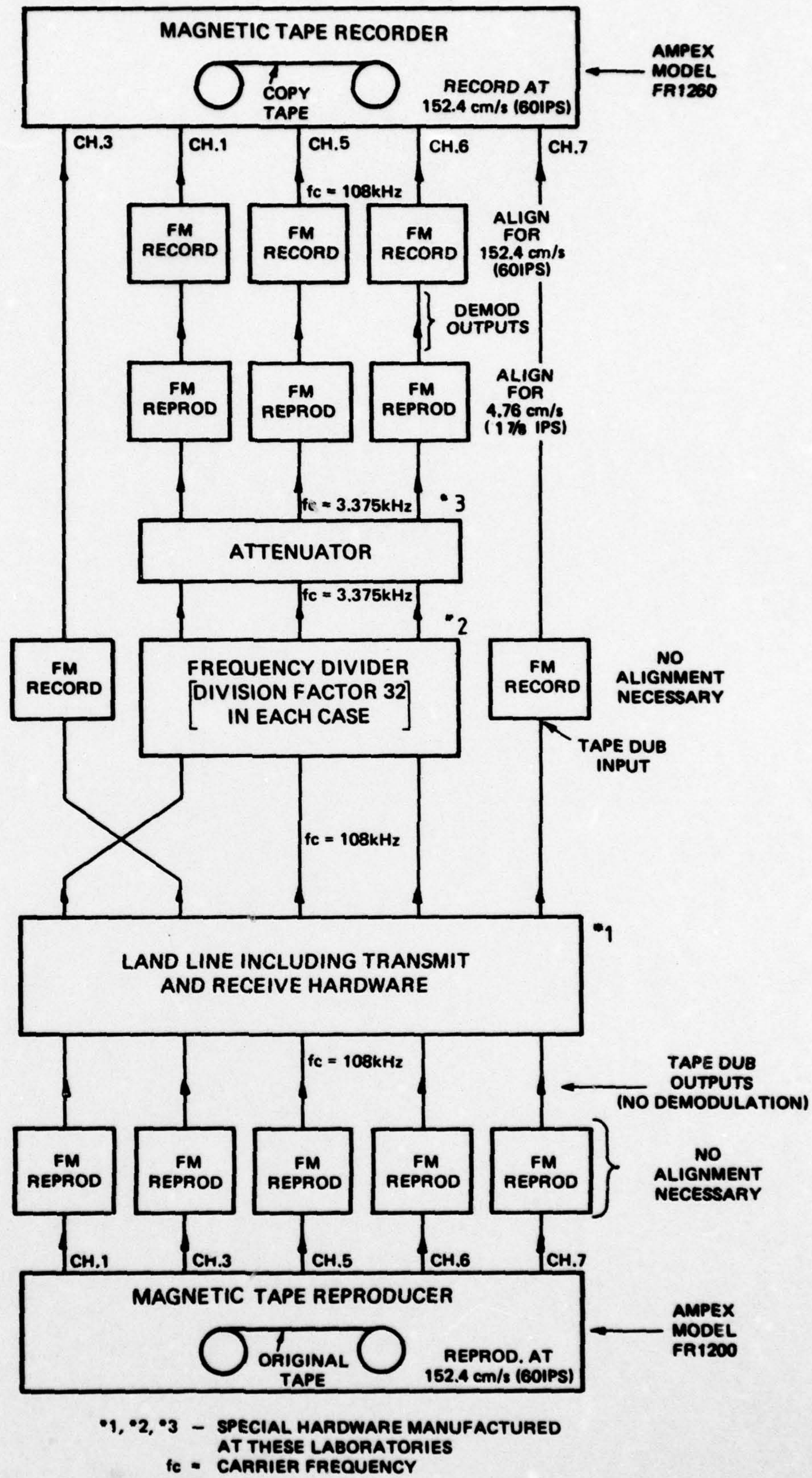


FIG. 1 HARDWARE TO LOW PASS FILTER RECORDED DATA AND TO RE-RECORD DATA WITH EXPANDED TIME SCALE

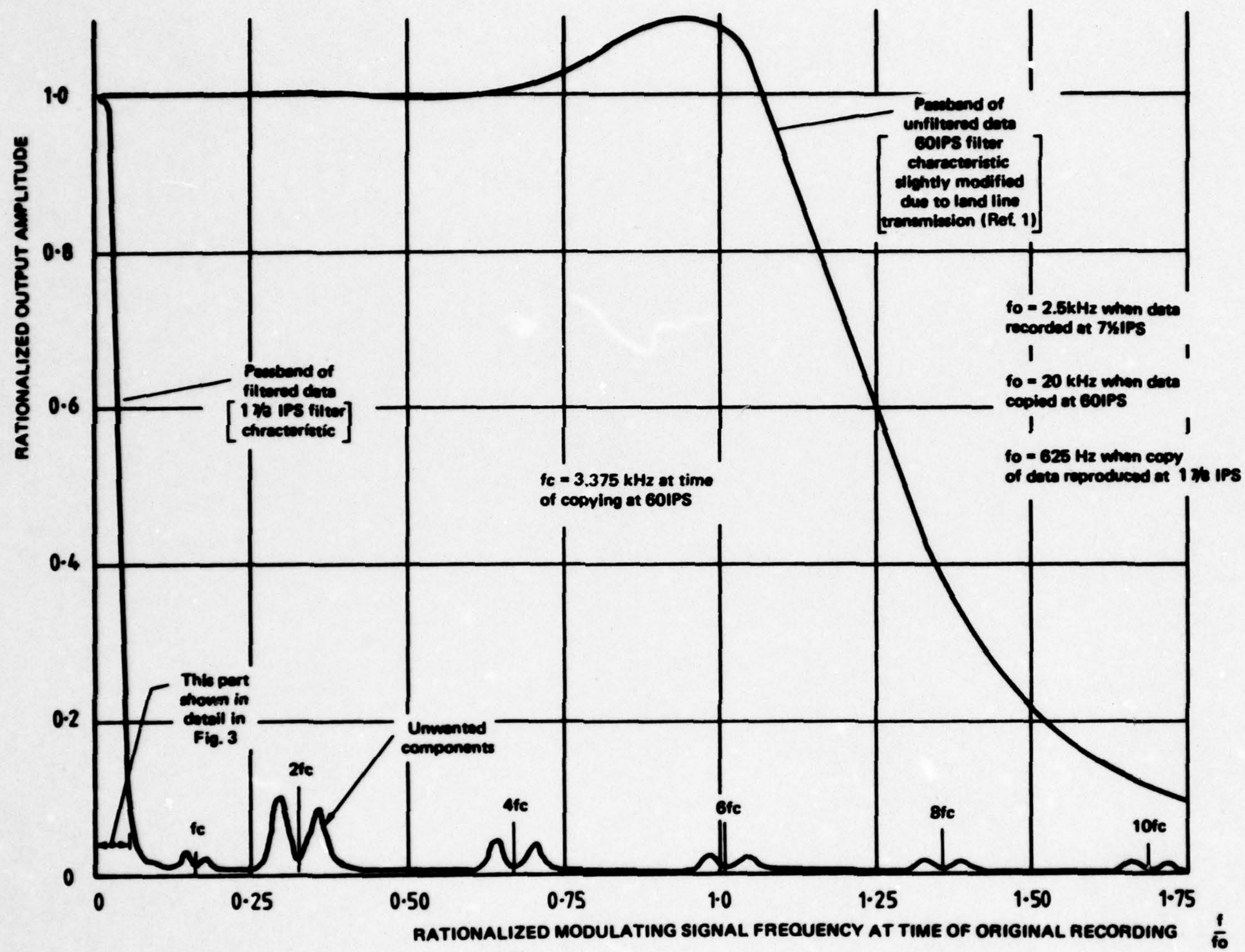
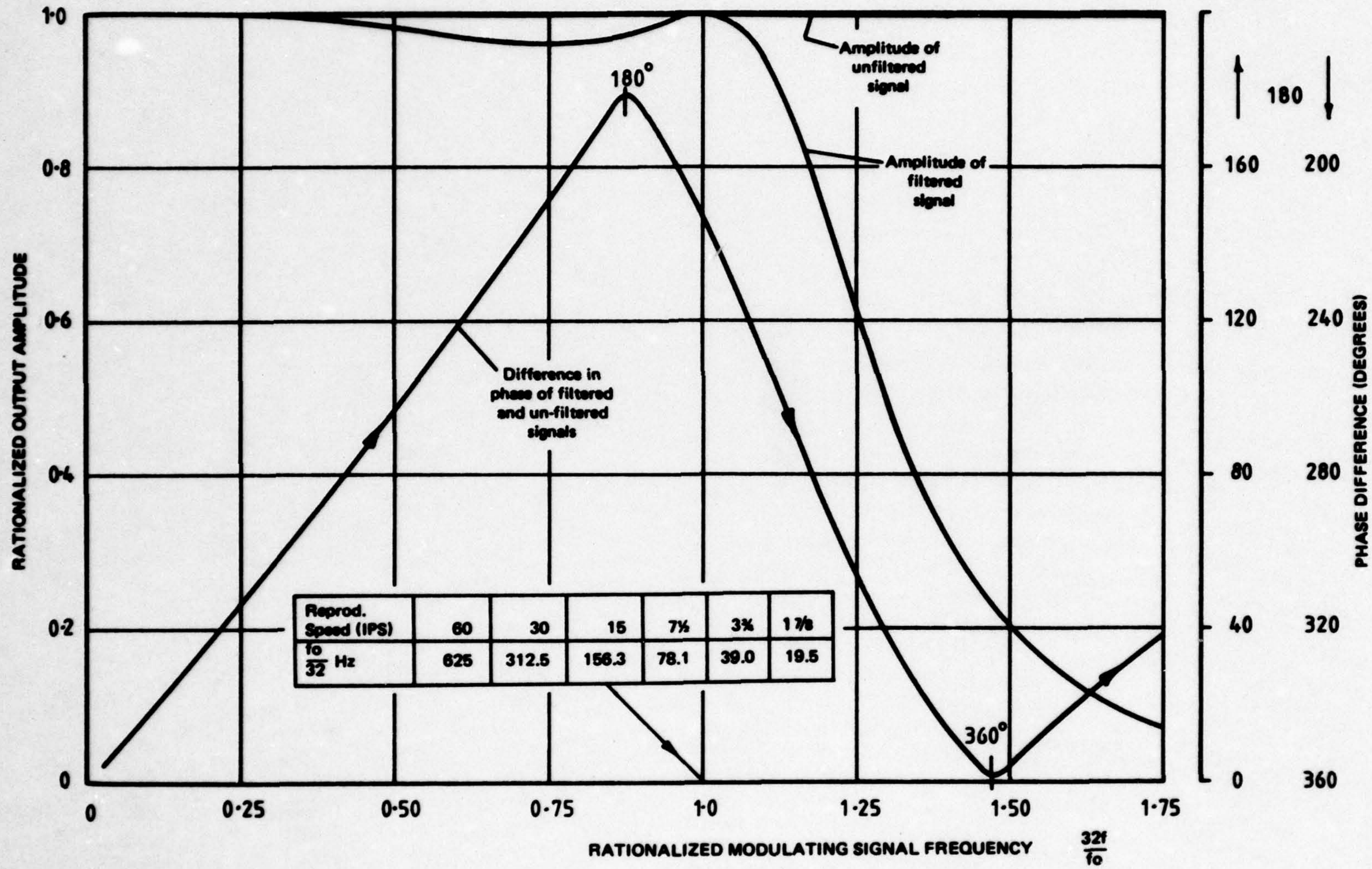


FIG. 2 COMPARISON OF FILTERED AND UN-FILTERED DATA PASSBANDS



Reprod. Speed (IPS)	60	30	15	7½	3¾	1 7/8
$\frac{fo}{32}$ Hz	625	312.5	156.3	78.1	39.0	19.5

FIG. 3 AMPLITUDE AND PHASE RESPONSES OF LOW PASS FILTER

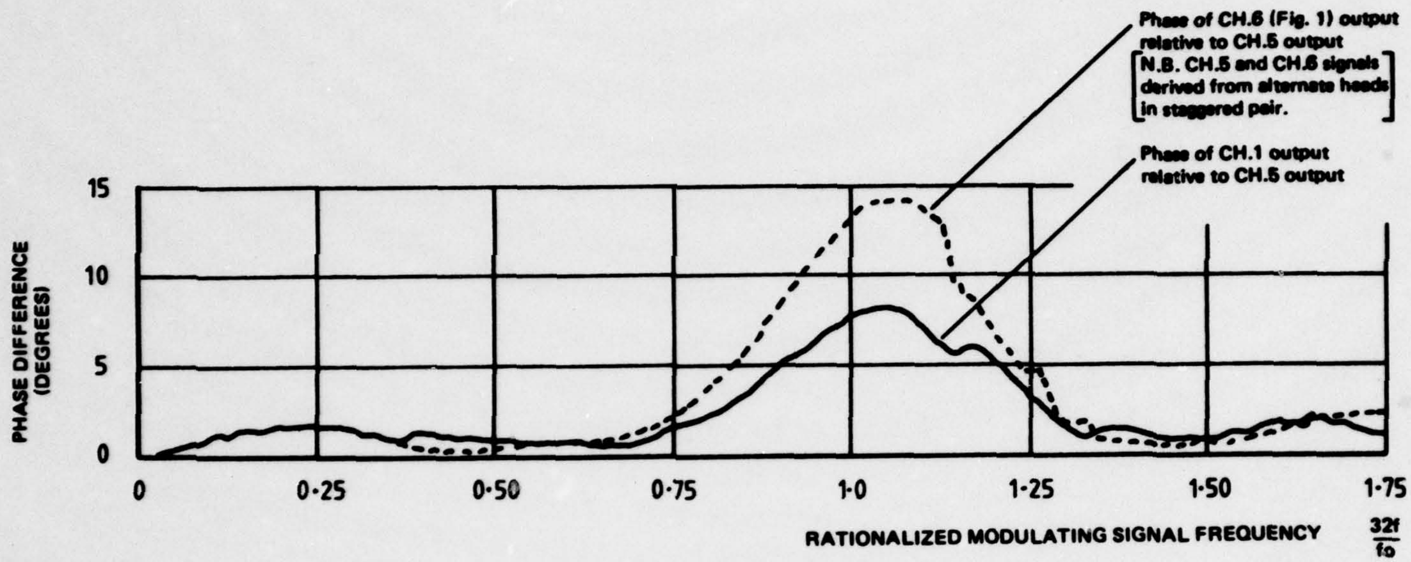


FIG. 4 EXTENT OF PHASE MISMATCH BETWEEN VARIOUS FILTERS

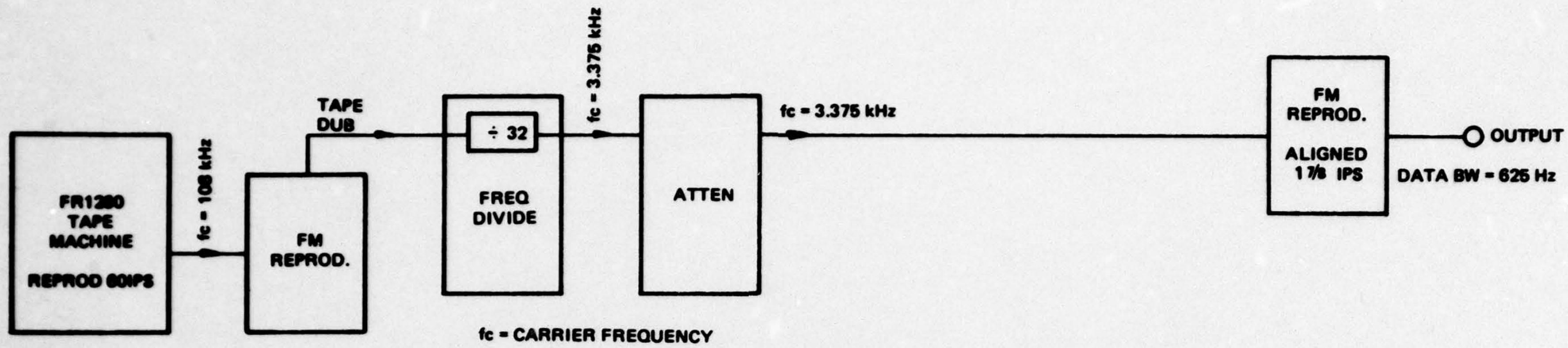


FIG. 5 (a) HARDWARE TO PRODUCE NORMAL FILTERING BUT WITH SIDEBAND NOISE

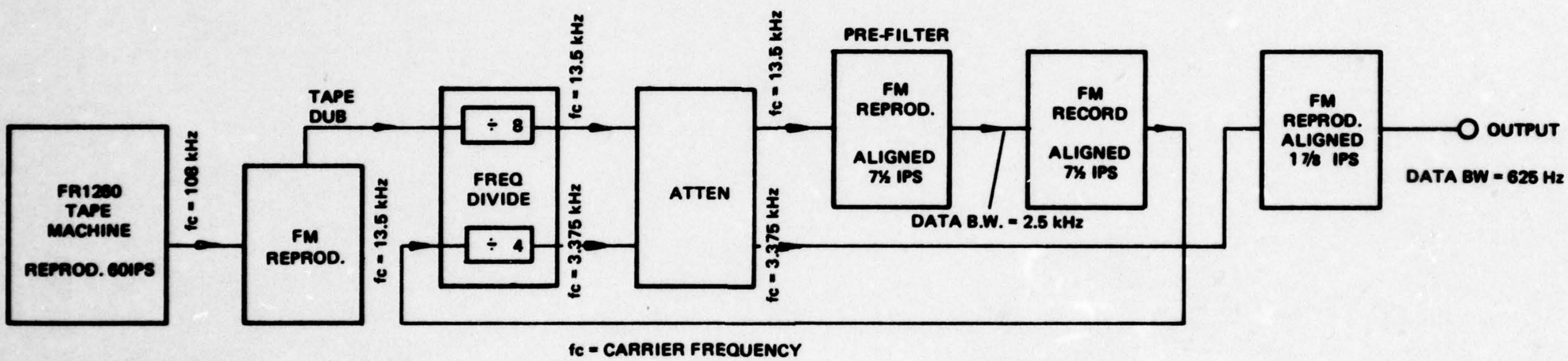


FIG. 5 (b) HARDWARE WHICH INCLUDES PRE-FILTER TO REDUCE SIDEBAND NOISE

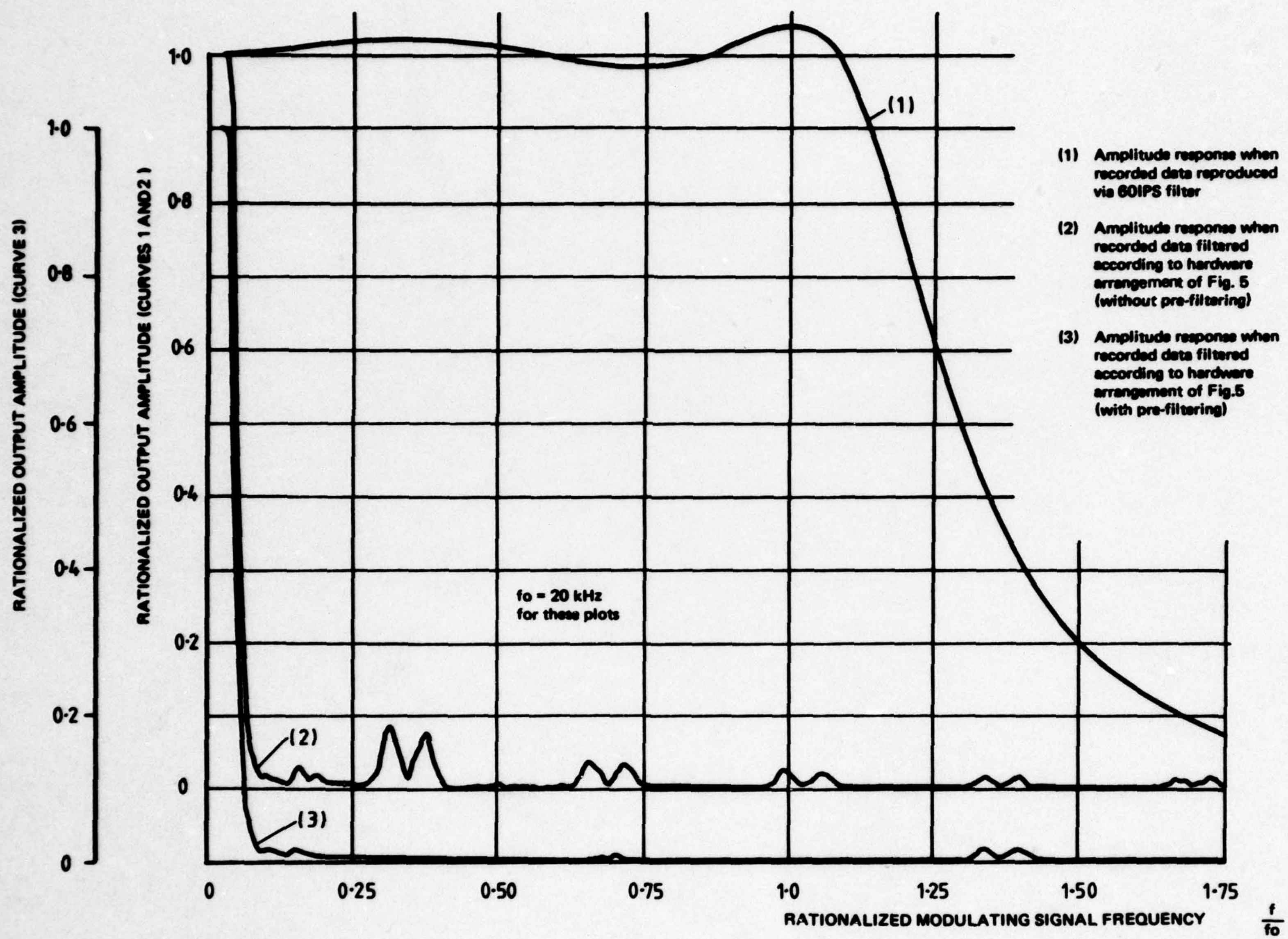


FIG. 6 REDUCTION OF SIDEBAND COMPONENTS WITH PRE-FILTERING

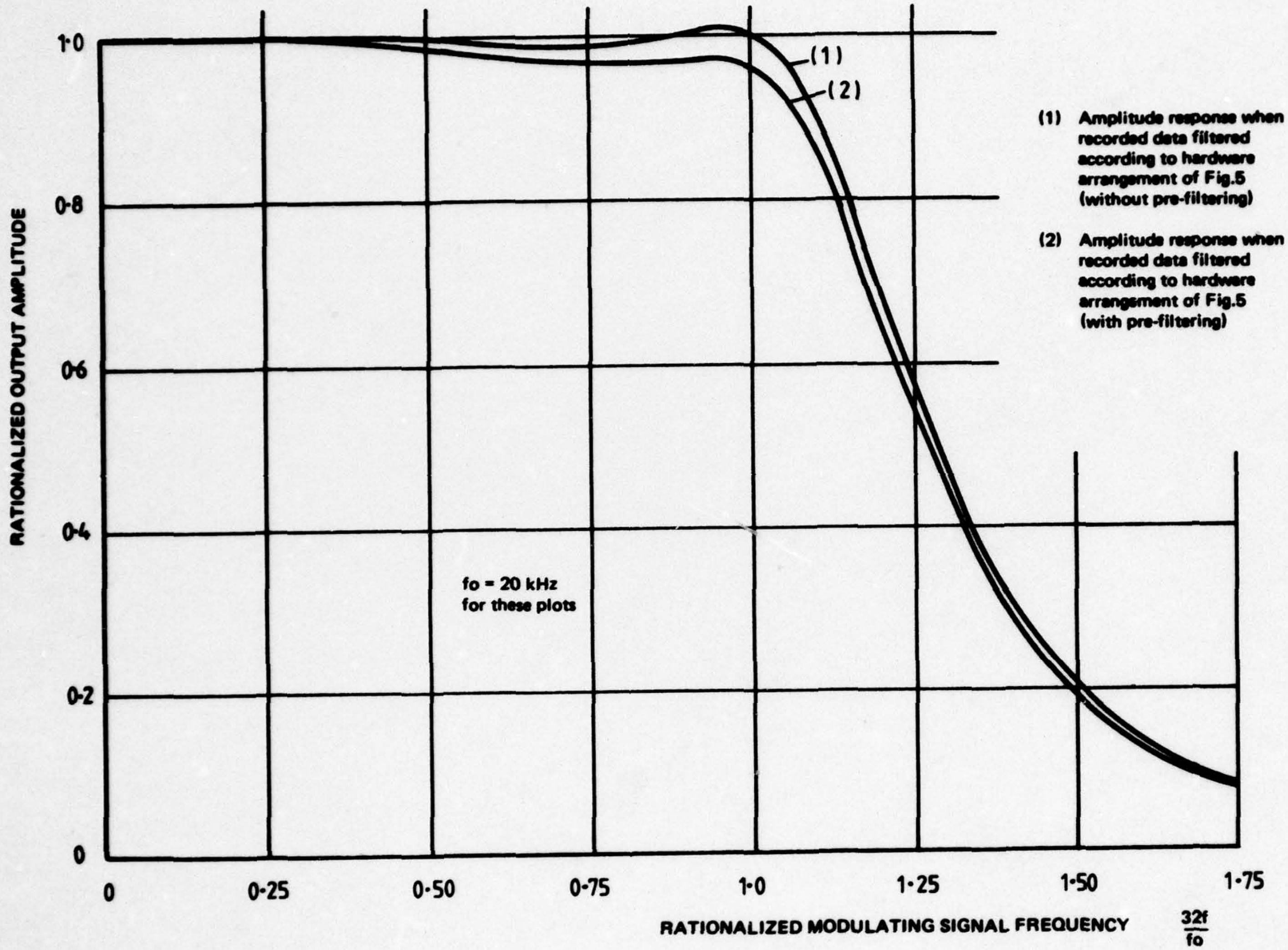


FIG. 7 EFFECT OF PRE-FILTERING ON LOW PASS FILTER AMPLITUDE RESPONSE

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ABSTRACT

Demodulators typically include low pass filters to recover signals from wideband frequency modulated carriers. Multispeed analogue tape machines normally require the carrier frequency to be set to a value proportional to tape speed. Reproducing hardware usually includes sets of filters, with bandwidth proportional to tape speed, for use with the demodulators. By dividing the frequency of the reproduced signal, a filter corresponding to a lower tape speed can be used and low pass filtering of recorded data signals can be readily achieved. However some unwanted noise components can be generated. These noise components are examined critically and a method of reducing them is demonstrated.

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