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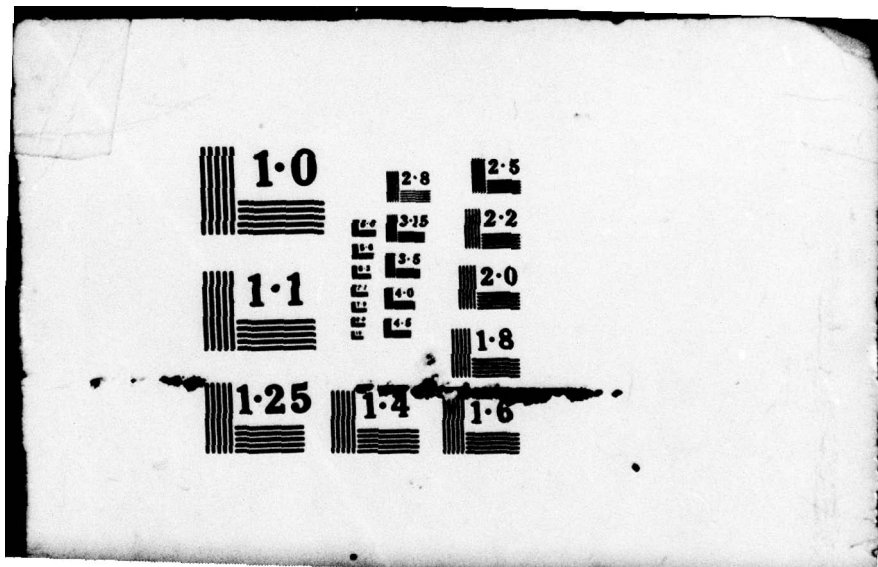
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TWO-DIMENSIONAL MESH ADJUSTMENT ALGORITHM

Prepared for:

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| 20. ABSTRACT (Continue on reverse side if necessary and identify by block number) This second year of research effort has produced a mesh adjustment algorithm for the solution of one-dimensional boundary-value problems based on an arbitrarily given monitor function. A second, more complex algorithm simultaneously finds the solution and the optimal monitor to minimize the truncation error. The results provide an immediately applicable technique for the solution of stiff differential systems such as arise in reactive duct flow problems as well as a firm basis for the extension to the higher dimensional problems of viscous duct flow or external lifting flows. |                                   |   |

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## I RESEARCH OBJECTIVES

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The general objective of this research is to establish a transformation of coordinates that facilitates the finite difference solution of partial differential equations in two dimensions. The objective can be attained by completion of the following tasks.

### A. Campylotropic Coordinate Definition

A best coordinate net would be composed of orthogonal families of curves on the surface, with the curves of a family coming closer together where the curvature of the curves of the other family increases. The families of lines of curvature have the orthogonality property but may not be the most campylotropic. Analytical effort is warranted to define the best coordinate net.

### B. System Organization

The differential equations form a system that can be presented in various ways. In the one-dimensional case, the given differential equation and the equation for the campylotropic coordinate were combined into a quasilinear system with coefficient determinant properly bounded and bounded away from zero. A rearrangement of the two-dimensional equations into uniquely solvable form needs to be derived.

### C. Numerical Algorithm

Possible methods of approximation of the derivatives by finite differences and solving the resulting algebraic system need to be explored.

### D. Applicability

By analogy with the one-dimensional case, this formidable numerical method is needed only if the solution has regions of sharp variation. The method may be inapplicable to solution surfaces with shock or derivative discontinuities and may also be inferior to a characteristics method for hyperbolic problems. The range of applicability needs investigation.

## II STATUS OF THE RESEARCH EFFORT

During this second year the campylotropic mesh adjustment algorithm for the solution of one-dimensional boundary-value problems has been generalized and simplified, a mesh adjustment scheme that optimally satisfies a minimum truncation error criterion has been defined, and quite direct ways of applying the one-dimensional method to higher dimensional problems have been developed.

### A. Campylotropic Coordinate Definition

The transformation of independent variable to a campylotropic coordinate, defined by arc length and curvature of the solution curve, has been shown<sup>1\*</sup> to be effective in the finite difference solution of differential equations of boundary-layer type. The campylotropic coordinate takes on equally spaced values at the adjusted nodal points. It is therefore a particular case of the monitor functions defined by White,<sup>2</sup> who showed that use of properly defined monitor functions provided a convergent solution method where the Keller box scheme<sup>3</sup> converges.

The conditions to be satisfied by a monitor function so that it minimizes the sum of the squares of the truncation error at each node have been reduced to a differential equation and boundary conditions. Where heuristically suggested monitor functions prove ineffective, it may be advantageous to add this equation to the system being solved.

### B. System Organization

A general way of organizing the given differential equation and boundary conditions as a first-order system in block tridiagonal form has been presented by Keller.<sup>3</sup> The transformation to a monitor coordinate complicates the system by the addition of one differential equation and by the multiplication of each component of the derivative by a factor. This added complexity is minor for the high order nonlinear systems, where the method properly applies.

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\*References are listed on page 4.

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### C. Numerical Algorithm

The method is being developed by using finite difference approximations of low order, to minimize special handling of mesh intervals near the boundaries. This minimization is increasingly important as the dimension is raised.

The difference scheme provides a system of nonlinear algebraic equations. Solution by Newton's iterative method is being implemented, with full advantage taken of the block tridiagonal form of the Jacobian matrix.<sup>4</sup>

For problems in two dimensions, the solution region is mapped by the coordinate change into a region paved with rectangular mesh tiles and right triangle half tiles. This is the form of domain handled by subroutine REGION of the ITPACK scheme.<sup>5</sup> Use of this routine or its general framework will be considered for the geometric part of the two-dimensional algorithm.

### D. Applicability

The mesh adjustment scheme in one dimension is most suitable for stiff differential systems with solutions having sharp variations in layers at a boundary or in the interior. The rate of energy production in steady flow through a chemical reactor is modeled by such a boundary-value problem. Sharp variations in reaction rate occur, for example, when short-lived intermediate species are produced, or where a cold boundary quenches the reaction.

In higher dimensions, it is expected that the freedom in the coordinate transformation to expand a singular point into a line segment or to smooth out a boundary layer will be a great aid to computation.

## III PUBLICATION LIST

1. C. M. Ablow and S. Schechter, "Campylotropic Coordinates," Journal of Computational Physics (June 1978).
2. C. M. Ablow, S. Schechter and W. E. Zwisler, "Node Selection for Boundary Value Problems," in preparation.



#### IV PERSONNEL

Mr. W. E. Zwisler, an exceptionally competent mathematician-programmer, has joined the coinvestigators, Drs. C. M. Ablow and S. Schechter, in the research effort. His biography is attached.

#### V INTERACTIONS

The contents of the second paper in the publications list will be presented at the fall meeting of S.I.A.M. in Denver, November 12-14, 1979. The abstract is attached.

#### VI SPECIFIC APPLICATIONS

The one-dimensional algorithm is currently applicable to steady reactive flow calculations used in the analysis of jet energy thrusters, gas generators, or interior ballistics. The two-dimensional algorithm, when completed, should facilitate calculation of steady conical flows for supersonic airframe design, reactive flow fields in shaped axisymmetric ducts, or flows influenced by skin friction and erosion.

#### VII REFERENCES

1. C. M. Ablow and S. Schechter, "Campylotropic Coordinates," Journal of Computational Physics, Vol 27, No. 3 pp. 351-362 (June 1978).
2. A. B. White, Jr., "On the Selection of Equidistributing Meshes for Two-Point Boundary-Value Problems," S.I.A.M. Journal of Numerical Analysis, Vol. 16, No. 3, pp. 472-502 (January 1979).
3. H. B. Keller, Numerical Solution of Two-Point Boundary Value Problems (S.I.A.M., Philadelphia, 1976).
4. S. Schechter, "Quasi-Tridiagonal Matrices and Type-Insensitive Difference Equations," Quarterly of Applied Mathematics, Vol. 18, No. 3, pp. 285-295 (October 1960).
5. D. R. Kincaid and R. G. Grimes, "Numerical Studies of Several Adoptive Interactive Algorithms," Report CNA126, University of Texas, Austin (August 1977).



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Manager, Applications Programming group  
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System data analysis and computer programs for Experimental Orbiting Geophysical Observatory Satellite; established system to analyze large files of information and order real-time data, stored time data, and orbital information  
Establishment of system to project employment and population of U.S. cities  
Development of TIGER code, a chemical equilibrium and hydrodynamics code  
Design of command and control system for U.S. European Command Headquarters  
Liaison between Mathematical Sciences Department and other SRI departments to promote efficient utilization of Institute computing facilities  
Implementation of a generalized circuit analysis program  
Design and implementation of control optimization program using parallel tangent gradient method  
Testing of virial equation of state for gaseous mixtures  
Development of an assembler for the DDP124  
Use of data-base management systems  
Design and implementation of a large scale logistics information system, MAGTF

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**ACADEMIC BACKGROUND**

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## NODE SELECTION FOR BOUNDARY VALUE PROBLEMS\*

by

C. M. Ablow, S. Schechter, and W. H. Zwisler  
SRI International, Menlo Park, California 94025

### ABSTRACT

The solutions of two-point boundary-value problems often have narrow regions of sharp variation, or boundary layers, that can occur in any part of the interval. A finite difference method of numerical solution will require more closely spaced nodes in the boundary layers than elsewhere. An automatic method is needed for achieving the irregular spacing when the location of the boundary layer is not known in advance. Several automatic node-insertion or node-movement methods have been proposed. A new node-movement method is presented that is optimal under the criterion of producing the least sum of squares of the truncation errors at the nodes. For the box scheme applied to a system of  $N$  coupled first-order differential equations, this truncation error minimizing (TEM) method increases the system size to  $(N + 6)$  equations. The campylotropic coordinate transformation method and other published methods based on heuristically derived monitor functions are node-movement methods that involve systems of only  $(N + 1)$  or  $(N + 2)$  first-order equations. A comparison is made of the accuracies of several such methods and the TEM method in the solution of a standard problem.

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\*To be presented at S.I.A.M. fall meeting, Denver, November 12-14, 1979.