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Volume I

ANGULAR VIBRATION OF AIRCRAFT
Volume I — Executive Summary

ANAMET LABORATORIES, INC.
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AIR FORCE FLIGHT DYNAMICS LABORATORY
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This report describes development work in several distinct areas, all related to prediction of angular vibration of aircraft structures. Angular vibration in this context refers to dynamic rotations or changes in slope at specific points on a vibrating structure. It is of interest primarily in connection with high resolution optical and electro-optical systems. Efforts were directed at both low frequency vibration, where individual normal modes are known, as well as high frequency vibration where they are not. For low frequency

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predictions, improved accuracy per unit cost was sought by an evolutionary improvement to an existing finite element code. A state-of-the-art shell element, the Semi-Loof, was incorporated into NASTRAN by means of pre- and post-processors and DMAP instructions. Its accuracy is tested against both closed form solutions for simple cases and experiment for an actual fuselage. For high frequency predictions the method of Statistical Energy Analysis was pursued. A demonstration case involving two coupled plates is presented to show how SEA may be used to predict angular vibration in a situation where normal modes are too numerous to be predicted individually. Relationships between linear and angular vibration were developed for various structural forms. Theoretical error bounds are also derived for spectral measurements of angular vibration which are obtained by differencing of signals from translational sensors.

FOREWORD

This report describes an investigation into the prediction methods of angular vibration of aircraft, performed by Anamet Laboratories, Inc., San Carlos, California, for the Air Force Flight Dynamics Laboratory, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio, under Contract F33615-77-C-3050. This research was conducted under Project 2401, "Structural Mechanics," Task 240104, "Vibration Prediction and Control, Measurement and Analysis," Work Unit 24010408, "Angular Vibration of Aircraft." Lt. Michael W. Obal, (AFFDL/FBG) was the project engineer. This report is in two volumes: Volume I - Executive Summary; Volume II - Prediction Methods for Angular Vibration.

This program was conducted by the Applied Mechanics Division of Anamet Laboratories. Program Manager was Dr. Conor Johnson and Principal Investigators were Dr. Warren Gibson, Dr. David Kienholz, and Dr. Ernest Paxson. This research was performed between August 1977 and April 1979.

This report was submitted by the authors on 2 April 1979 for publication as an AFFDL Technical Report.

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INTRODUCTION

With the design and development of inertial sensing systems and laser beam control systems, angular vibration measurements and predictions have become as important, and in some cases more important, than translational vibration measurements and predictions. Also, very small amplitudes of angular vibration, on the order of a few microradians, have become important, especially in the design of airborne electro-optical systems. Since the electro-optical system is mounted to the aircraft, and the aircraft is subjected to many loads, such as gusts, turbulence, etc., the dynamics of the aircraft must be predicted accurately. The dynamic response characteristics of the aircraft that are used for design and analysis of optical systems involve both translational and angular vibrations. The elastic modes of an aircraft may be as low as 1 or 2 Hz. The lowest elastic modes of individual components of a laser system may be in the 150 to 300 Hz. range. Therefore, to perform complete frequency response analysis may require modal information from 1 Hz. to 1000 to 2000 Hz.

The objectives of this contract were (1) to develop techniques for predicting the low and high frequency angular response of aircraft; (2) to develop techniques for predicting the low and high frequency angular vibration of combined airframe and electro-optical systems; (3) to develop angular vibration measurement techniques adequate to support the analytical work; and (4) to demonstrate these techniques by applying them to an aircraft-like structure and comparing the results with measured data. For this report, low frequencies are defined as those frequencies for which individual normal mode shapes can be predicted accurately or determined by tests.

The work on this contract was subdivided into three phases. The objective and accomplishment of Phase I was to identify methods which could be used to predict angular vibration for low and high frequencies. Both manual and computer interactive literature searches were performed to obtain information on past experience in the area of angular vibration. Also searched were methods of analysis which may be useful in the prediction of angular vibration at low and high frequencies. For low frequencies, it was decided the main thrust should be directed toward obtaining more accuracy per degree of freedom in finite element analysis by bringing some advanced finite element technology from a "laboratory" status to a production environment. This approach satisfies all of the objectives for the low frequency method and also gives a method which is flexible and adaptable for application to complex structures. The Semi-Loof shell element and its companion beam element were selected as the most promising candidates to meet this objective. It was decided to join these elements with the finite element code, NASTRAN. The goal of NASTRAN compatibility was adopted as it became clear that any other approach to assembling finite element software would be prohibitive in terms of the effort required for coding, checking, training users, and gaining their acceptance.

Implicit in this approach is the proposition that angular problems are not fundamentally distinguished from those of translational vibrations. There is no way to divorce an angular deformation from the associated translational deformation. Mathematically, one is the derivative of the other. Hence, the pursuit of better angular vibration was embodied in a search for better finite element methods in general. In other words, an evolutionary approach was taken rather than revolutionary.

The method selected for combining components or structures, such as an aircraft and an electro-optical system, into a system analysis for low frequencies was component mode synthesis. The component mode synthesis technique treats each component or structure in terms of its modal description (obtained either from tests or analysis).

The selection of an approach for prediction of so-called high frequency angular vibration was conditioned by the definition of low vs. high frequency as previously stated. Modeling of a small stiff structure with a fundamental mode at 500 Hz. is not necessarily more difficult than modeling a larger structure with its first resonance at 5 Hz. The difficulty occurs when a model must be capable of predicting response to inputs over a frequency range which contains a very large number of modes. Higher order modes will be sensitive to structural details which are too small to model economically and may not even be identical for structures built from the same design. In effect, for high frequencies, one does not have a fixed description of the structure even in physical coordinates. The distinction between low and high frequencies for the purposes of this work is thus a functional one. Low frequency analysis implies that properties of individual vibration modes can be obtained, either by analysis or test. High frequency analysis presupposes that this level of detailed knowledge is unavailable.

Based on the above considerations, it was decided during Phase I that the method of Statistical Energy Analysis (SEA) was a promising candidate. While much of the specific SEA theory was unfamiliar to the investigators at this point, a number of attractive features were clear:

(1) The method does not necessarily require information about individual normal modes of a structure in order to make response predictions. The lack of such information inevitably introduces some uncertainty into the predictions but this may be acceptable for many cases. The point is that averaged descriptor quantities such as approximate modal density and total mass may be sufficient to make useful first estimates of response.

(2) High modal densities may actually be an advantage. Each mode contributing to response acts something like a statistical degree of freedom. As more modes contribute, their variability (i.e., the uncertainty as to the properties of any single mode) tends to become less important. For example, early SEA work

was often associated with room acoustics where mode counts in the audio band may be in the hundreds of thousands.

(3) It appeared that deterministic methods of analysis with which the investigators were intimately familiar could be used to good effect in SEA modeling. In particular, certain aspects of large scale finite element analysis and minicomputer-based experimental modal analysis appeared promising. It was suspected that a higher level of structural detail could be incorporated into an SEA model by utilizing these technologies which were not available when most of the basic SEA theory was worked out.

(4) SEA modeling can be attempted using structural descriptions of varying detail. This was considered essential if a method were to be usable during both preliminary and prototype stages of design.

The major emphasis of Phase II was the detailed development of the methods chosen during Phase I. For the low frequency method, the approach taken was to implement Semi-Loof into the finite element code, NASTRAN. As originally derived, the Semi-Loof element was for use on static problems only. Therefore, the mass matrices for the elements had to be incorporated. To be able to model complex structures such as airframes, other enhancements to the element had to be made. Among these were orthotropic material properties, offset beams, smeared stiffeners, variable thicknesses for the shell elements, non-prismatic beam geometry, and distributed loads. The input was developed using NASTRAN bulk data card formats. A stress recovery and a rotational recovery capability were added. The element is incorporated into NASTRAN by means of pre- and post-processors and NASTRAN DMAP instructions. The processors incorporate a matrix assembly routine, and extensive error checking capability. A User's Manual for Semi-Loof was written and is included as Appendix A of Volume II. During Phase II a number of sample problems were executed and the results were compared to NASTRAN solutions and tests. The results showed good improvement in accuracy per degree of freedom.

As stated earlier, statistical energy analysis (SEA) had been chosen to predict the angular vibration environment of aircraft when only an averaged or statistical description of the structures is known. SEA has been used as a tool for high frequency structural and acoustical analysis in the past but no work had been done toward employing this method to predict angular vibration. The theory of SEA was studied and the essential features of SEA which make it attractive for the present purpose were identified. A formula for estimating coupling loss factor by the wave transmission method was rederived without assuming response to be in the form of traveling waves. An experiment was conducted where the wave transmission method was used to predict transmitted power and the equilibrium energy ratio between two coupled plates. This experiment demonstrated that SEA could be used to predict the energy ratio and power transfer coefficient between coupled plates in a high frequency region where finite element modeling was impractical. A relation was derived between the r.m.s. angle at a typical point on a plate and the vibrational energy in frequency bands. An experiment was conducted to test this relation and to show that, at least for this simple case, r.m.s. angular displacement could be obtained from SEA results without detailed knowledge of individual mode shapes or natural frequencies. Software was developed for the minicomputer-based equipment used to perform these experiments.

A study was also conducted on relationships between linear and angular vibration for various simple structural forms. A detailed study of the simplest two degree-of-freedom structural system possessing both linear and angular degrees-of-freedom was conducted. The ratio of the mean square angular to mean square linear displacement was investigated for simply-supported beams, a free-free beam, a curved stiffened panel, a simply-supported flat plate, and an infinite beam. From this series of examples, it was concluded that the accuracy of angular-from-linear methods will decrease as one proceeds towards the low frequency end of the spectrum, i.e. where modes are few and well spaced.

In order to verify the prediction methods developed, reliable experimental data had to be obtained, especially dynamic rotations at specific points. The measurement method used to obtain these rotations was differencing of translational acceleration signals. A better quantitative understanding of limitations and error sources was desirable. Therefore, theoretical derivations were developed for estimating errors introduced by noise in individual channels, frequency-dependent gain and phase mismatching between channels, and flexure of the mounting surface. Effects of the first two error sources were demonstrated by experiment and it was shown that mismatch error can be reduced by appropriate data processing.

The major objective of Phase III was to apply the methods developed to a complex structure. A fuselage section of a fighter aircraft was chosen as the test structure. This structure was chosen because it required a fairly complex analytical model. A finite element model of the fuselage was developed using the Semi-Loof elements. All of the features that were developed for this element were employed in the modeling. An eigenvalue analysis of the structure was performed to determine the normal modes. A frequency response analysis was performed with random noise input at several different points. The responses at a number of translational and rotational degrees of freedom were output. A test of the fuselage was also performed. The fuselage was supported on a low stiffness mounting system and driven by a small shaker at the same points. Both force to linear acceleration and force to angular acceleration transfer functions were measured for drive and response points corresponding to points in the Semi-Loof model. After processing the data on the minicomputer-based modal analysis equipment, the experimental data was compared with results predicted by Semi-Loof.

The sections that follow in this report give a brief summary of the work performed on this contract. Volume II of this report gives the details.

SECTION II
LOW FREQUENCY METHODS

The finite element method is the dominant analytical tool for low frequency structural vibration analysis. The wide versatility of the method and its natural adaptability to computer coding primarily account for this popularity. There have been two major thrusts in finite element research and development in recent years. One is a pursuit of more accurate and versatile elements, particularly shell elements. Second is the development of very large software packages, such as NASTRAN, that make finite element technology accessible to engineers in a useful and economical form. This chapter summarizes an effort to adapt two advanced curved shell elements for use with NASTRAN. The specific purpose of this effort was to make these elements available for aircraft angular vibration studies. However, Semi-Loof elements have proved valuable in many other applications, as well.

The common measure of finite element cost effectiveness is accuracy per degree of freedom. The number of degrees of freedom governs the size of the matrices for the problem which, in turn, controls the major cost of the computer run. This does not account for factors which are less quantifiable, such as convenience of use, adaptability to other elements, and accessibility of software. Full attention was paid to these factors during development, but the accuracy per degree of freedom criterion was used in quantitative assessments of Semi-Loof. These considerations are, of course, equally applicable to angular vibration problems or to any other structural analysis problems.

The technical details concerning the Semi-Loof elements can be found in Volume II. Briefly, this element is a curved shell, so that curvature effects are directly incorporated in the element. By contrast, the conventional NASTRAN flat plate elements can simulate shell curvature effects only through interelement connections. Besides being a cruder representation than a true curved shell, a model made up of faceted flat elements approaches a stiffness singularity in the limit as the grid is refined.

The Semi-Loof elements achieve a high degree of accuracy by means of an unusual nodal configuration and a combination of constraints concerning transverse shear strains. These constraints make it possible to reduce out unwanted degrees of freedom, leaving a high-performance element with a rather modest degree-of-freedom count for a curved element (32 for the quadrilateral, 24 for the triangle). It is a rather unorthodox element in that it violates the rule of interelement compatibility that was once considered an essential requirement for any finite element. Semi-Loof passes the "patch test" which has gained acceptance among many researchers as a more practical form of convergence test. Typical quadrilateral, triangular, and beam elements are shown in Figures 1 through 3.

Volume II reports a number of assessments of Semi-Loof elements compared to other elements. When compared to standard NASTRAN QUAD4 and QUAD2 elements, small test cases suggest that comparable accuracy can be achieved with roughly half the number of degrees-of-freedom when using Semi-Loof. These encouraging results are not representative enough to warrant a general conclusion for a wide class of applications, both large and small. Nevertheless, they are sufficiently encouraging, along with the theoretical advantages of the Semi-Loof elements, to indicate that these elements constitute a significant addition to the methods available for analysis of aircraft angular vibration and other shell-type analysis problems.

Large finite element software systems are multi-million dollar undertakings, and most of this investment is devoted to matters other than pure finite element mathematics. Solution

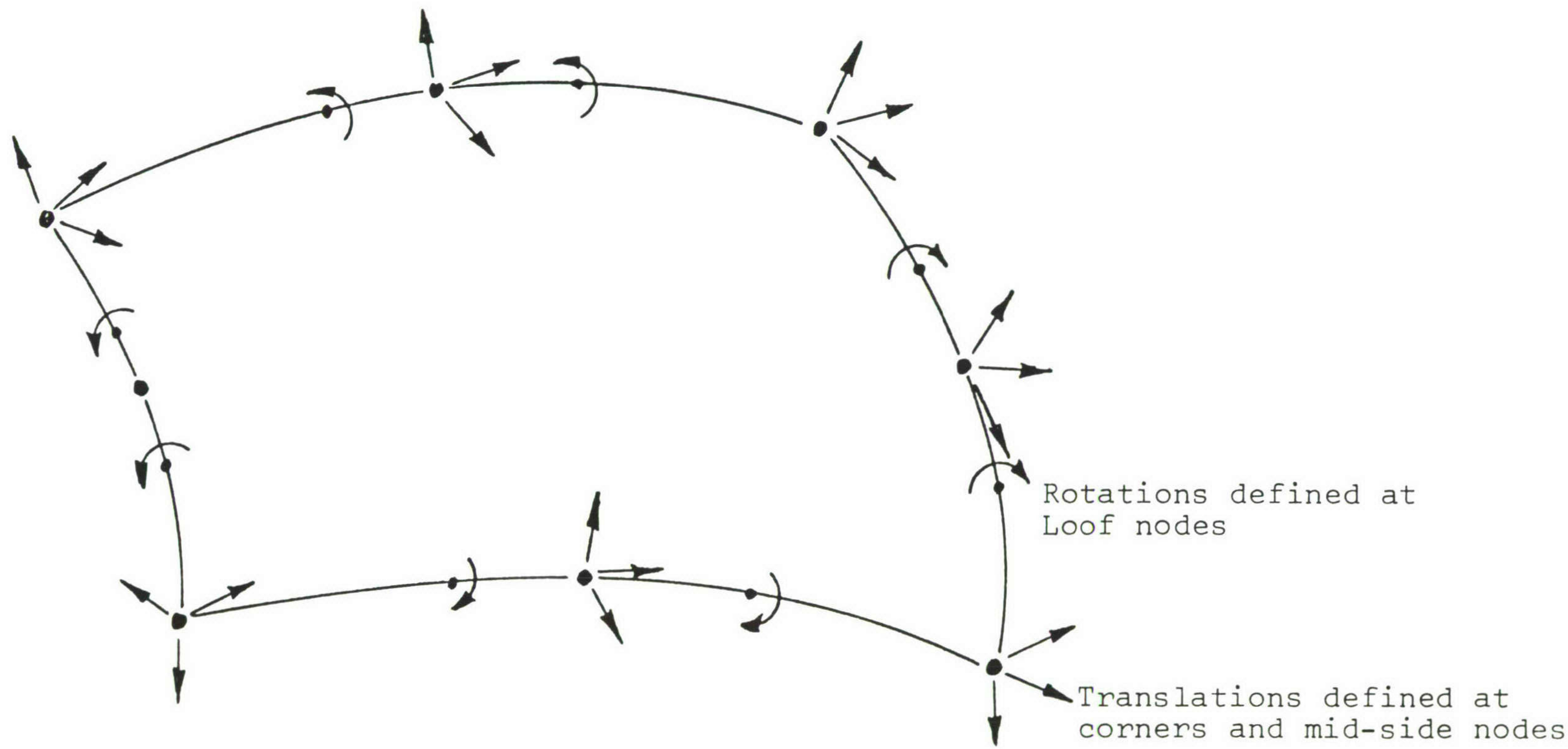


Figure 1   Semi-Loof Quadrilateral Element Degrees of Freedom

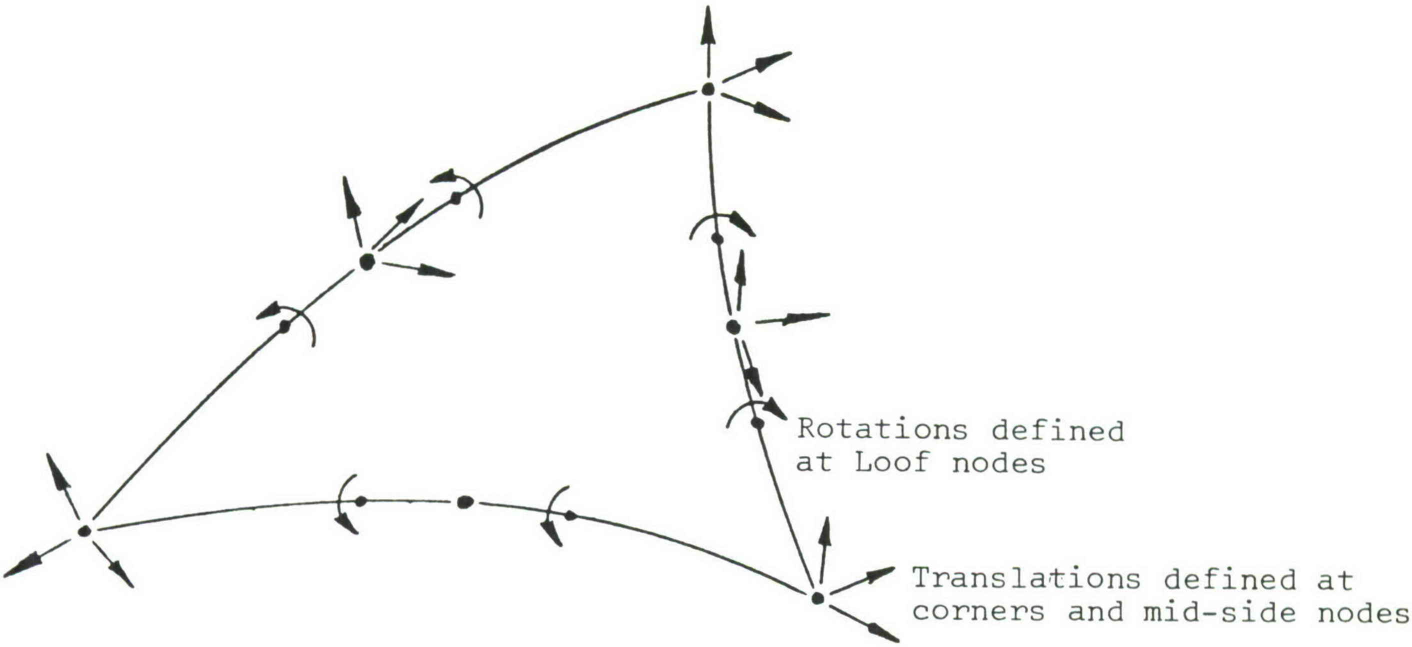


Figure 2   Semi-Loof Triangular Element Degrees of Freedom

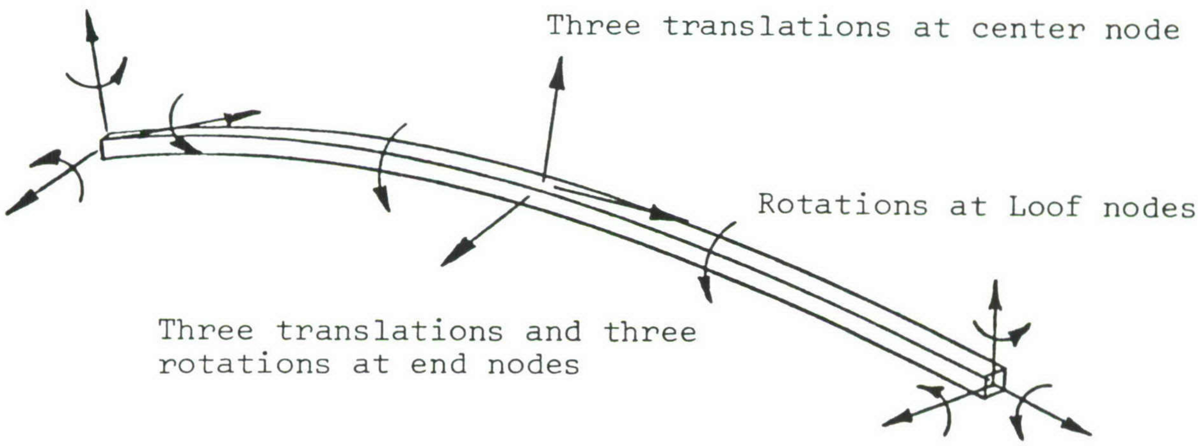


Figure 3   Semi-Loof Beam Element Degrees of Freedom

strategies, input sorting and verification, storage allocation and matrix partitioning, matrix algebra subroutines, and output formatting are examples of some of these concerns. Thus, it is mandatory that new elements be implemented within an established framework so as to take advantage of the investment in all of these capabilities. The programming effort documented in Volume II was devoted to implementation of Semi-Loof element in a form usable in NASTRAN. The NASTRAN code, itself, was not modified. Instead, preprocessor and post-processor routines were written and made to interface with NASTRAN via NASTRAN's "DMAP" matrix abstraction language. These routines are compatible with either the COSMIC or MSC versions of NASTRAN.

A computer program was obtained from Professor Bruce Irons, the principal developer of Semi-Loof. This program was primarily intended for student use and was not suited for production work. The heart of the code (shape function generators) was stripped out, and was used as a nucleus for a new code, "PRELOOF". The following are some of the features added to the original capability:

(1) Mass matrix generation.

(2) Distributed load generation.

(3) User-defined coordinate systems.

(4) Orthotropic materials.

(5) Effects of smeared, unsymmetric stiffeners.

(6) Arbitrary grid point numbers and element numbers.

(7) Offsets and arbitrary cross-section orientation added to curved beam elements.

(8) Error checking.

(9) Input formats compatible with NASTRAN bulk data conventions.

(10) Assembly of global stiffness matrix, mass matrix and load vectors, written to a binary file in formats suitable either for MSC/NASTRAN or COSMIC NASTRAN.

Two sets of DMAP Alters were coded for each of the commonly used rigid formats in both MSC/NASTRAN Level 44 and COSMIC NASTRAN Level 17. These enable the Semi-Loof user to take advantage of the full range of NASTRAN capabilities.

Finally, POSTLOOF was written as a postprocessor to recover angular deformations and/or stresses.

The following items are not incorporated in the Semi-Loof elements at this time:

(1) Geometric stiffness.

(2) Thermal loads.

(3) Heat transfer matrices.

(4) Line loads.

It is believed that the present effort has made available to knowledgeable structural analysts an improved tool for assessing angular vibration of aircraft and other shell-type structural analysis problems. The Semi-Loof element has been implemented in a convenient manner, allowing the user full access to NASTRAN capabilities.

The following drawbacks should be noted:

(1) The degrees of freedom are defined in an unorthodox manner, making it difficult to attach Semi-Loof elements to other NASTRAN bending elements.

(2) There is an anomaly, discussed in Volume II, concerning certain spurious mechanisms which sometimes arise. This problem can be circumvented at the cost of some additional user awareness and some additional computing time.

SECTION III
HIGH FREQUENCY METHODS

A basic assumption from the beginning of Anamet's work on angular vibration has been that deterministic methods of structural modeling would not, by themselves, be sufficient. Experience with airborne optical systems has shown that high frequency, low amplitude motion of optical components can seriously degrade system performance. This high frequency motion represents the aggregate contribution of numerous high order modes of vibration which are too sensitive to small details of construction to be reliably modeled by deterministic methods. While motions induced by high frequency disturbances may be small compared to low frequency contributions, the active servomechanisms used to control the optical beam have their own frequency limitations. They cannot effectively suppress and may actually amplify the effect of disturbances at frequencies beyond a few hundred Hertz.

It may be shown for various simple structural forms that a modal series solution for forced response will converge more slowly for a rotational degree of freedom than for a translational degree of freedom. This implies that deterministic prediction of angular response will be more difficult than linear response in that the properties of a larger number of modes must be calculated. The situation is similar to prediction of strains as opposed to displacements in finite element analysis by the direct stiffness method. For a given element formulation and mesh size, displacements will invariably be predicted more accurately than strains which, like rotations, are inherently differential quantities.

As the number of modes contributing to response becomes intractably large, one is led inevitably to methods which attempt to predict the aggregate response of groups of modes without necessarily finding individual contributions. In doing so, one is essentially discarding (or never acquiring) some information about the detailed construction of the structure. This is done in appreciation of the fact that the detail being ignored may be beyond the practical ability of deterministic methods to use it and may not even be identical for structures built from the same nominal design. The structural description may thus be thought of as correct only in some averaged or statistical way.

The distinction between so-called high frequency and low frequency prediction methods which has been adopted in the current studies is thus a functional one and does not necessarily involve specific frequency ranges. High frequency prediction methods are defined simply as those which do not necessarily involve knowledge of individual normal modes of a structure. Low frequency methods are those which are based upon such knowledge whether it be obtained through analysis or test.

The difficulties of high frequency vibration analysis are due to the difficulties of predicting large numbers of normal modes, many of which are sensitive to small physical details. The problem is by no means confined to angular vibration. During the 1960's, a body of theory was developed for structural-acoustic problems which specifically addresses these difficulties. It is called Statistical Energy Analysis (SEA).

Early in this study it was decided to pursue the method of SEA for predicting high frequency angular vibration. A significant portion of the total effort in Phase II, as well as Phase I, was expended in gaining an understanding of the theoretical and practical basis of the method. This, in itself, proved to be a formidable task, mainly because the theory is made up of contributions from several related but diverse fields: classical vibrations, random processes, wave mechanics, mechanical impedance methods, and acoustics. Reference [1], a technical report prepared originally for Air Force Flight Dynamics Laboratory (AFFDL), was found to be a most useful document in this regard due to its thoroughness and extensive bibliography.

Because SEA draws heavily on more familiar areas of deterministic analysis, it is useful to recognize just what is basic to the method. The following are the principal assumptions and characteristics of the method:

(1) It is based on a linear, small displacement theory.

(2) Component energies and interelement power flows are treated as primary variables rather than forces and displacements.

(3) All modes of a given component are assumed to be equally energetic. Only their energy sum is explicitly sought.

(4) It is based on steady-state, stationary excitation. Periodic forcing is not excluded, although the underlying assumptions are better suited to broadband random input.

(5) Internal energy dissipation rate for a component is assumed to be proportional to resident energy.

(6) Power flow between components is taken to be proportional to a weighted difference of component energies. This relation can be derived from Newton's laws for special cases, but is itself taken as a basic law in SEA.

(7) Temporal mean energies and power flows are divided into contributions by frequency. Energy equilibrium equations are written for finite-width frequency bands with coefficients assumed to be constant for each band.

The practical difficulties of SEA revolve mainly around estimating power flow coefficients in the energy equilibrium equations and in recovering displacement response estimates once these equations have been solved for component energies.

Efforts under this contract were relatively basic and exploratory. They focused on three main activities:

(1) Acquiring a basic working knowledge of the existing theory and determining how this theory may be applied to predicting angular vibration of aircraft.

(2) Recasting the wave transmission method for computing coupling loss factors into a format that is understandable in terms of finite coupled structures of large but finite modal density.

(3) Testing the theory experimentally for the case of two structures coupled at a single degree of freedom. The structures were chosen to be physically simple (uniform plates) but still exhibit mode counts too great to allow analysis by conventional deterministic means such as finite elements.

In the process of carrying out these three main activities, several peripheral areas were also addressed.

(1) It was demonstrated that for a uniform structure with a high modal density, recovery of r.m.s. angular response from equilibrium energy is not difficult.

(2) It was demonstrated that global component internal loss factors could be conveniently and accurately obtained from coupling point admittance measurements.

(3) Various software routines were developed for data reduction tasks which have a high probability of recurring in the development of more general SEA applications to actual airborne optical system. The tools of digital signal processing were used extensively in the experiment.

Principal results from the wave transmission experiment are shown in Figures 4 to 6. Referring to Figure 5, it was found that the energy of an indirectly excited plate (normalized on the energy of the directly excited plate) could be predicted within about 3 dB (factor of 2) over quite a wide frequency range using either an approximate analytical or an experimental description of the plates. This implies prediction of spatially averaged r.m.s. velocity within 41%. It is

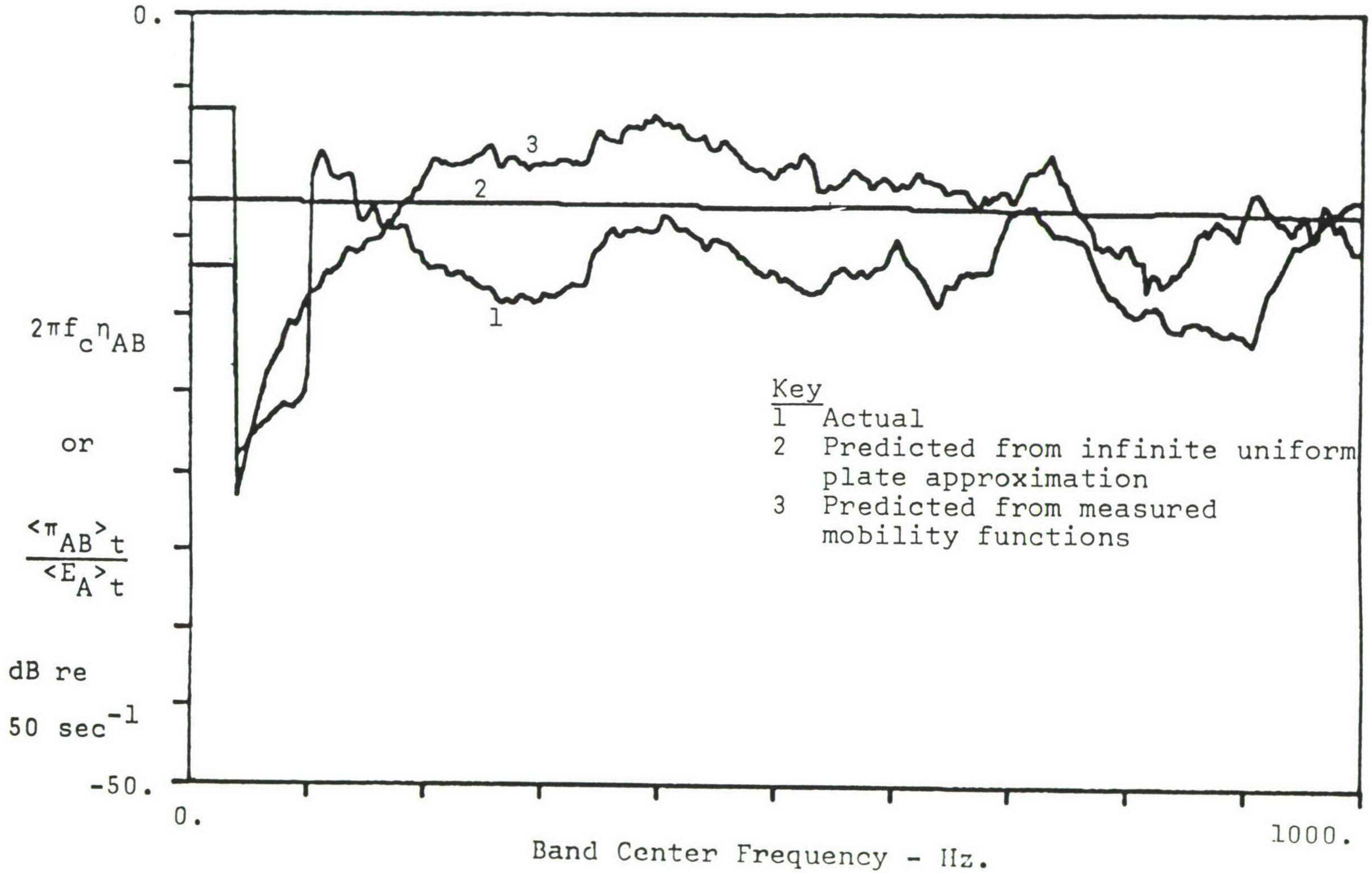


Figure 4 Normalized Transmitted Power $\langle \pi_{AB} \rangle_t / \langle E_A \rangle_t$

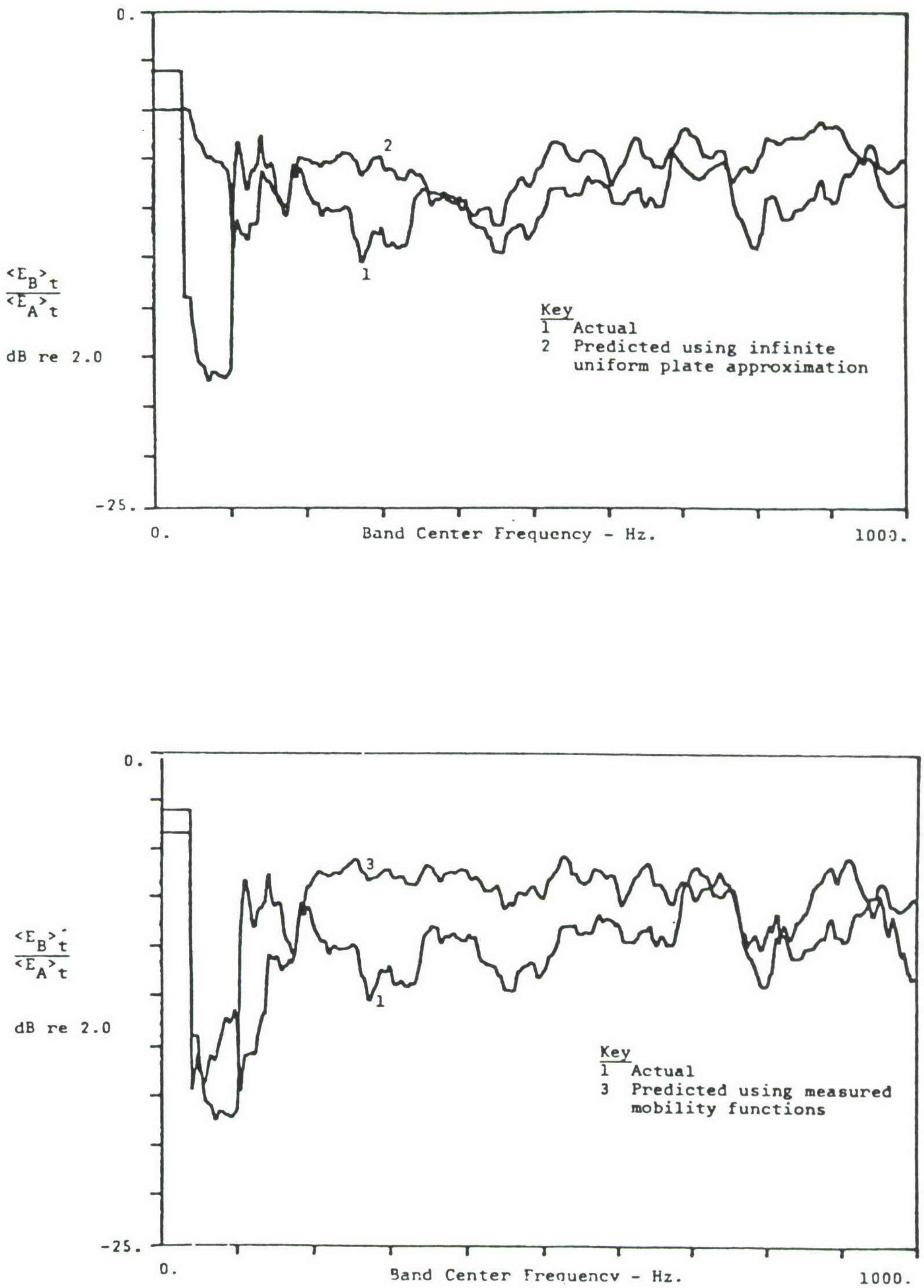


Figure 5 Normalized Energy of Indirectly Excited Body. Actual Damping Used.

suspected that some of this discrepancy is due to the test procedure itself. In Figure 6, angular response at a randomly chosen point on a single plate is estimated from known energy and compared with a direct measurement. Agreement between r.m.s. values is within 8%.

Several inherent limitations of the Statistical Energy Analysis method have already been mentioned indirectly. The most important drawbacks all stem from the statistical nature of the approach. Since response predictions are made in terms of spatial and temporal averages, the possibility exists that response at a certain critical point may deviate unacceptably from the mean. This situation is of particular concern if one of the components exhibits a low mode count in a particular modeling frequency band. With only a few modes contributing, the equipartition assumption may be violated or modal responses may add in-phase rather than randomly (the so-called modal coherence effect). A similar situation may exist for periodic excitation since the discrete frequency input will excite primarily only the modes with natural frequencies within one modal bandwidth of the driving frequencies. The practical result of these limitations is that the analyst should be aware of what component mode counts he is dealing with in SEA modeling of any real structure. It is also advisable to calculate an estimate of the spatial variance of response about the SEA-predicted mean. This calculation is based on the anticipated mode count and assumed typical mode shapes (See Chapter 4 of Reference [1]) and may be used to estimate confidence bands around the SEA predictions.

The work described here on high frequency angular vibration has not produced a general purpose tool with anything like the flexibility and accuracy which large finite element codes possess for low frequency analysis. Given the definition of the problem, that goal may never be reached. Nonetheless, what has been done constitutes a necessary step towards a method of

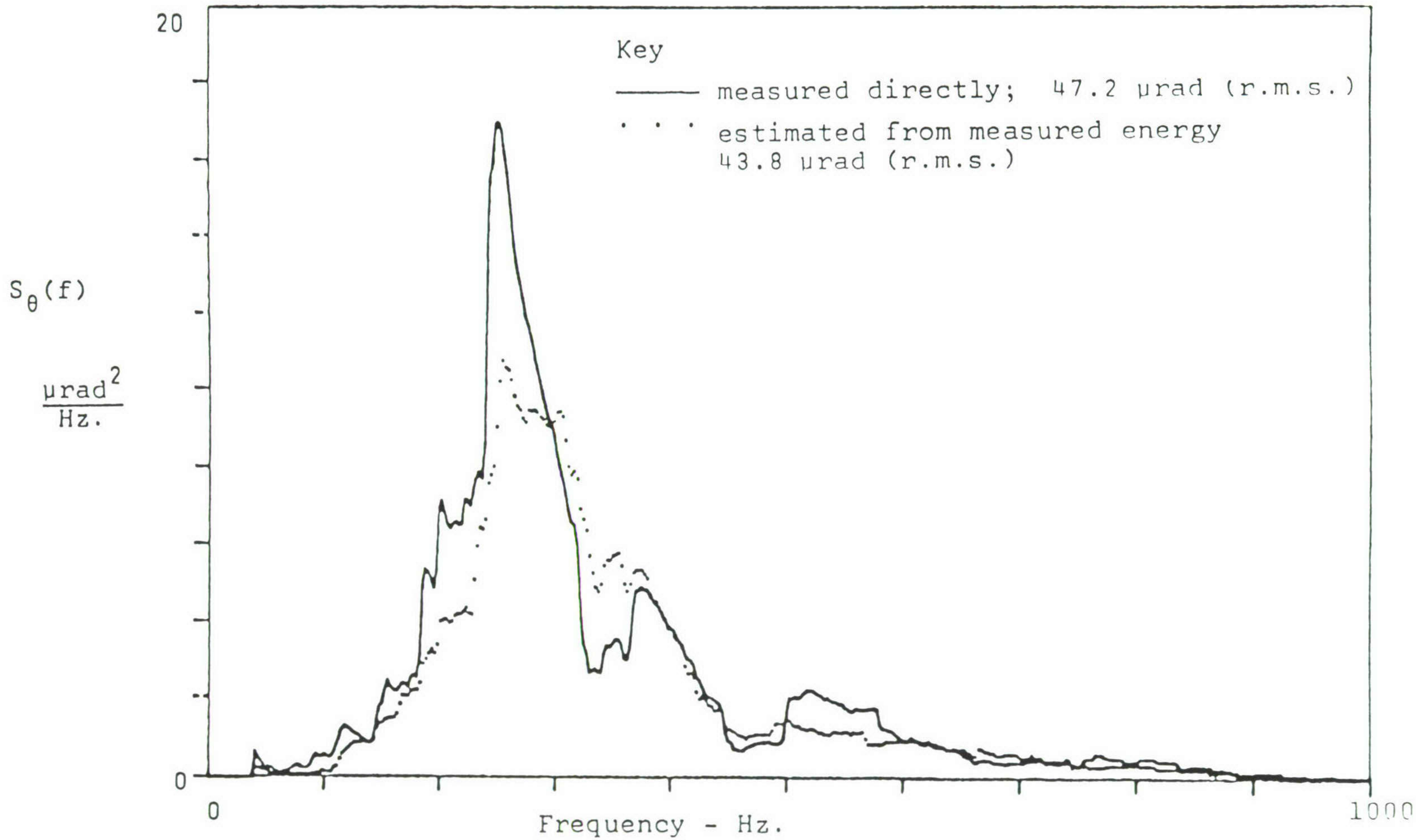


Figure 6 Smoothed Power Spectral Density and R.M.S. Value of High-Frequency Angular Vibration at a Typical Point of a Uniform Rectangular Plate

sufficient accuracy and efficiency of effort to justify its development cost. A demonstration case has been presented to show the essential elements of using SEA to predict angular vibration in a high frequency situation; i.e., one where individual normal modes are not known with any precision. In the course of this exercise, several specific areas have been identified where additional effort could be worthwhile.

(1) It may be possible to model individual normal modes as SEA components. This seems natural when an optical system with known normal modes is connected to a complex airframe with modes which are numerous and not individually known.

(2) The basic idea of the wave transmission method might be extended to cover the case where components are connected at more than one degree of freedom. An "equivalent source" model assembled from measured free mount point motions and the measured mount point admittance matrix might be used to describe the directly excited body (the aircraft). The use of frequency-averaged admittances, as demonstrated in the SEA experiment, could ease computation requirements for high modal density structures under stationary input.

(3) It might be possible to incorporate finite element results into an SEA model. An interface format suggested by work to date is the mechanical admittance function averaged with respect to frequency. It is suspected on theoretical grounds that this quantity can be predicted by a finite element model which is too coarse to accurately predict individual normal modes.

(4) The coupled plate experiment suggested numerous improvements in the acquisition and processing of experimental data to obtain coupling loss factors. Several avenues were identified but not pursued due to time restrictions.

SECTION IV
RELATIONSHIPS BETWEEN LINEAR AND ANGULAR VIBRATION IN AIRCRAFT STRUCTURAL COMPONENTS

Because of the predominant availability of aircraft translational (or "linear") vibration data as compared to that for angular vibration, it has been suggested that some general relationship between the two might exist. If angular vibration at specific locations and flight conditions could be estimated from available linear data, the available data base of angular vibration would be effectively enlarged.

It seemed like an appropriate way to attack the problem would be to investigate first the behavior of the simplest two degree-of-freedom (DOF) model of a structural component, which would be a spring-supported rigid bar, subjected to a temporally random concentrated load. In this way one could begin to obtain a feel for how a linear-to-angular relationship would depend upon the load and structural parameters (i.e. mass, stiffness, geometry) for a simple system. The assumptions made in the analysis of this structure are (1) no gravity forces are present, (2) angular displacements are small such that $\sin\theta \simeq \theta$, and (3) the concentrated force has a constant spectral density, i.e. white noise forcing. The detailed results are contained in Reference [2]. For the special case where the spring constants of the supporting springs are inversely proportional to their respective distance from the CG of the rigid bar, one would obtain the ratio of r.m.s. angular-to-r.m.s. linear displacement to be

$$\frac{\theta_{r.m.s.}}{x_{r.m.s.}} = \frac{\eta}{\alpha_1 \alpha_2 L} \tag{4.1}$$

where θ - angular displacement of bar

x - vertical displacement of center of gravity (CG) of bar

L - length of bar

α_1, α_2 - ratios of length L between ends and CG of bar such that $\alpha_1 + \alpha_2 = 1$

η - ratio of length L between P(t) and CG of bar

which points out how the linear-to-angular relationship may vary with geometry and position of load.

The next step was to proceed to more complex structural components. Linear-to-angular relations were derived for a beam with simple support boundary conditions, a plate with simple-support (SS) boundaries, and a beam with free-free (FF) boundary conditions all subjected to temporally random, spatially deterministic loading conditions. In each case linear, small-deflection theory neglecting shear deformations and rotatory inertia was used to model the behavior of the beam or plate. The detailed analyses are contained in Reference [2], but results from the beam studies are summarized in Figure 7 in which the ratio of r.m.s. angular to r.m.s. linear displacements along the length of the beam have been plotted for a beam of specific dimensions and material parameters with various loading and boundary conditions.

As one studies the results of loading conditions on a beam with simple support, one may conclude that any load which is symmetrically distributed with respect to the center of the beam will have a variation of θ_{rms}/y_{rms} between, at least, 0 and $+\infty$. Here, the use of a spatially averaged θ_{rms}/y_{rms} value could be unreasonable; but, in this case the support conditions of $y(o) = y(L) = o$ and the spatial symmetry of loading are the main source of the extreme variance. For this reason it was thought that a beam in the free-free vibration condition would furnish a more representative variation of θ_{rms}/y_{rms} relative to beam station in comparison to aircraft structural components, which do not normally have simple-support conditions. Figure 7 appears to bear this out. The spatially averaged value is that proposed by Lee and Whaley [3].

In order to examine a structural form somewhat more representative of an aircraft structural component, a NASTRAN

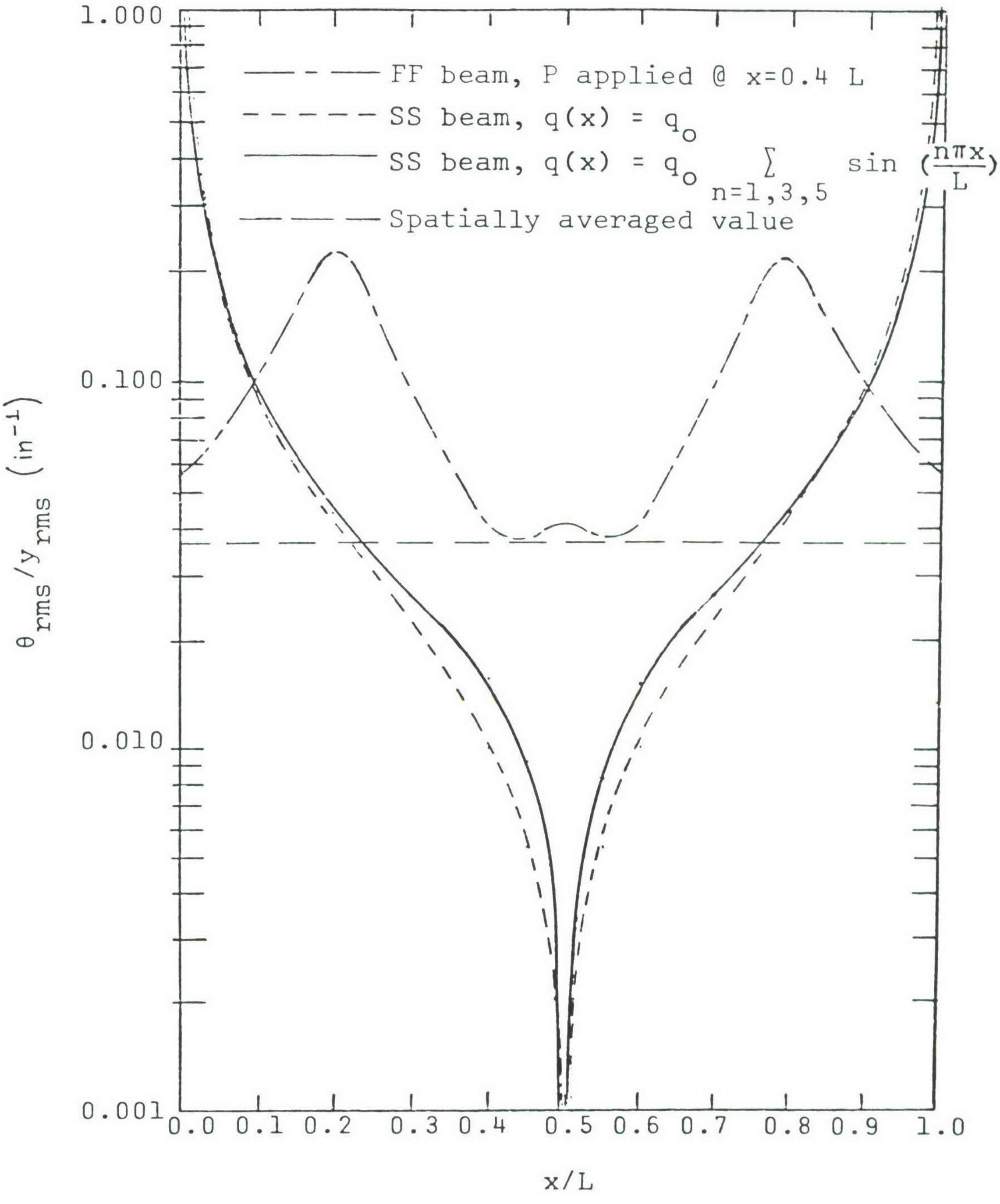


Figure 7 Plot of θ_{rms}/y_{rms} for Simply-Supported and Free-Free Beams Subjected to Temporally Random Loads

analysis of a stiffened curved panel subjected to a concentrated temporally random load was performed. The model consists of 726 degrees-of-freedom, 100 QUAD4 membrane-bending elements, and 30 beam elements. This panel was assumed to be in the free-free condition with a random concentrated load applied in the radial direction on one of the stiffeners. The response of the curved panel was evaluated for a 20-2560 Hz. band width and for the seven octaves in between. The output of the NASTRAN analysis included the following quantities:

(a) w_{rrms} - temporal average of displacement normal to plate

(b) $\theta_{\theta rms}$ - temporal average of angular displacement about θ-axis

(c) θ_{zrms} - temporal average of angular displacement about z-axis

at each grid point along three lines (one axial circumferential, one along the shell, and one circumferential along a stiffener).

The subsequent plots (Figure 8) present the angular-displacement-component-to-linear-displacement ratios of $\theta_{\theta rms}/w_{rrms}$ and θ_{zrms}/w_{rrms} as a function of grid point stations along a stiffener for the frequency band width of 20-2560 Hz. and selected octaves in between. In each of these plots, the straight lines represent the spatial average of the RMS angular component divided by the spatial average of the RMS linear displacement. For the 20-2560 Hz. case, there are two additional lines plotted which represent the spatial average of <u>all</u> grid points considered in this study on the curved panel.

Additional plots for other grid point stations may be found in Reference [4]. Also, a study where mass was added to the panel was reported in this reference.

A simple limiting case is available where, at least for single point excitation, a relationship exists between linear and angular response which is independent of space. For an infinitely long beam with sinusoidal excitation at one point, a simple traveling wave solution will have the form

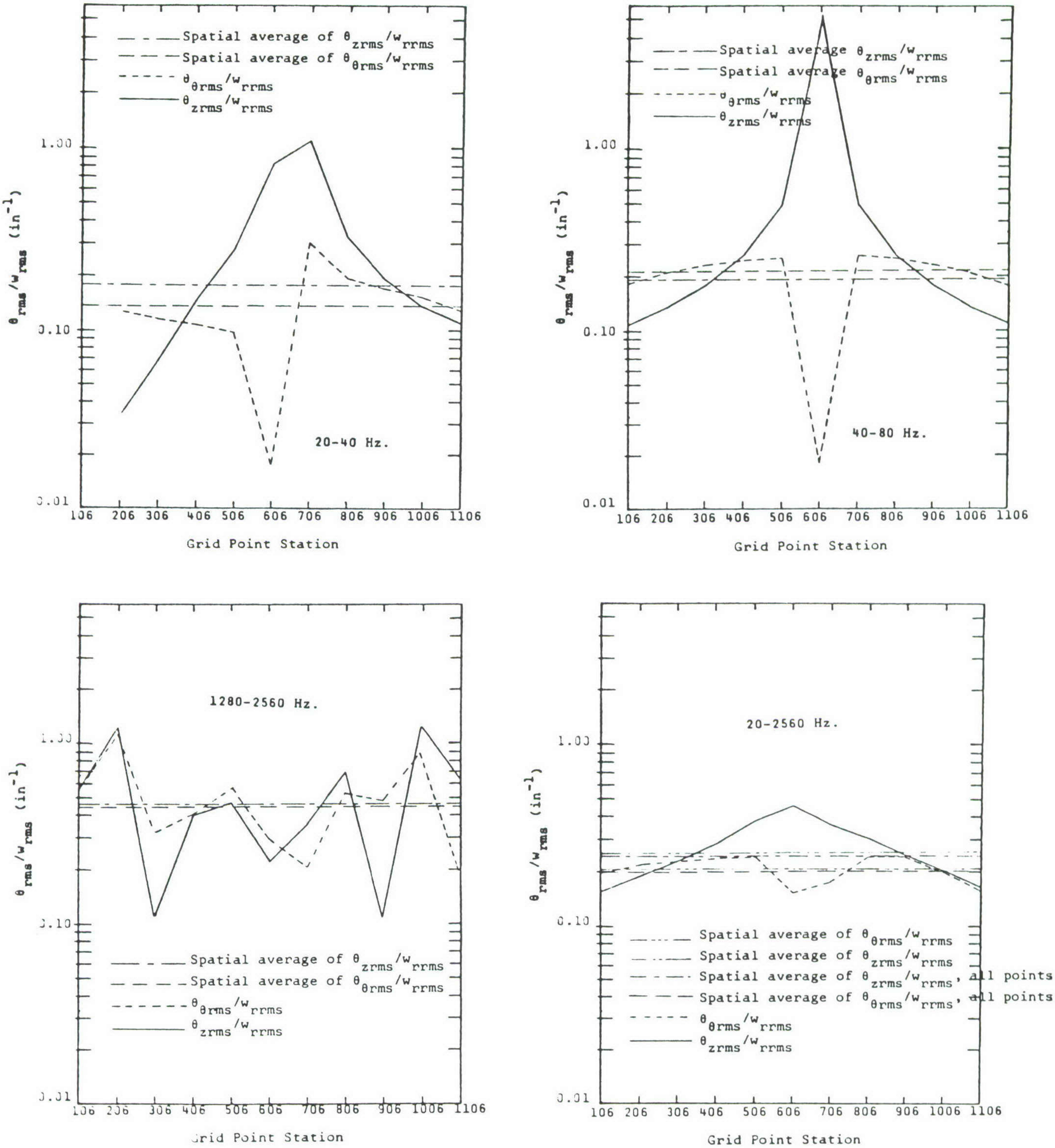


Figure 8 Ratio of RMS Angular Displacement to RMS Linear Displacement vs. Grid Point Station for Curved Plate with Stiffeners for Various Frequency Band Widths

$$y = Ae^{i(kx-\omega t)} \tag{4.2}$$

The wavenumber k is related to frequency f by the usual dispersion relation for flexural waves in a beam.

$$(2\pi f)^2 = k^4 \kappa^2 c_\ell^2 \tag{4.3}$$

In this expression κ is the radius of gyration of the beam cross-section and c_ℓ is the extensional wave speed. By differentiating Eq. (4.2) and using $\theta = \frac{dy}{dx}$

$$\frac{\theta}{y} = ik$$

The factor $i = \sqrt{-1}$ simply comes from using complex arithmetic to keep track of phase in time and space. It will drop out when ratios of r.m.s. values or power spectra are calculated. Introducing the definition of wavenumber $k = 2\pi/\lambda$ where λ is wavelength leads to

$$\frac{\theta_{rms}}{y_{rms}} = \frac{2\pi}{\lambda} = \sqrt{\frac{2\pi f}{\kappa c_\ell}}$$

For random steady state excitation at a point we note that, by linearity, the portion of response in frequency band Δf is due only to the excitation in that band so we may write

$$\frac{S_\theta(f)}{S_y(f)} = \frac{2\pi f}{\kappa c_\ell}$$

This ratio does not vary in space and requires only intensive beam parameters κ and c_ℓ rather than a global length L. It can be expected to hold for beams of finite length at frequencies sufficiently high that $L >> \lambda$.

Finally, an example of estimating angular response from measured translational response is described in Section 4.4.1 of Volume II of this report. The structure under consideration was a thin uniform plate and the frequency range was such that a very large number of modes (≈300) contributed to angular response. While the estimation formula is derived in terms of discrete normal modes, the large modal density and frequency range are exactly the conditions which would admit an estimate based on a traveling wave solution. This example may thus be considered to fall between the stiffened, curved panel and the infinite beam but is mathematically closer to the infinite beam. Not surprisingly, the agreement with experiment was excellent.

The usefulness of estimating angular response from known (usually measured) translational response has been examined in terms of a series of fairly simple examples. The examples have ranged from the simplest two degree-of-freedom system through continuous systems with discrete, known normal modes to a system with discrete but very dense modes and finally to an infinite system where natural frequencies form a continum and traveling wave methods are applicable. In addition, the use of ratios of spatially averaged angular-to-linear response for estimating local angular response has been evaluated. The following conclusions may be drawn.

(1) Variation of θ_{rms}/y_{rms} about a spatial average will be quite severe if response is dominated by a few modes. Dependence on loading will also be greater for structures with discrete, well spaced modes. As more modes contribute, spatial variance will decrease.

(2) Within the limits of Bernonlli-Euler beam theory, S_θ/S_y can be predicted exactly for single point excitation of an infinite uniform beam. The ratio depends on frequency and local beam parameters. This may be considered a limiting case with respect to modal density.

(3) Any estimate of the θ/y ratio must be based on some approximation as to structural form. Such an estimate may be useful if only a rough estimate of θ_{rms} is needed but no precision better than order-of-magnitude can be guaranteed for aircraft structures at low frequency.

In attempting to predict angular vibration by the angular-from-linear empirical approach just described, it is important to bear in mind the motivating physical problem. If one is concerned with the angular vibration environment of a small airborne electronic or sensor package, then extrapolation from measured linear flight data taken at the mount points of an unloaded aircraft may be directly useful. However, if the system to be mounted is physically large, the vibration at the mount points will, in general, be altered by its presence. Significant changes can be expected, particularly in the mid to high frequency range, where the unloaded vibration comes primarily from fairly local modes. It has been the author's experience that the latter case is typical of the current generation of airborne high-power electro-optical systems.

If one suspects that significant alteration of mount point vibration will occur, then some accounting must be made for the dynamic properties of both the airframe and the added system. Possible methods for accomplishing this include component mode synthesis for low frequency and SEA for high frequency. A third possibility exists which was not investigated for this report, but which is not inherently limited to either low or high frequencies. Measured or extrapolated linear and angular vibration at unloaded mount points might be used, along with measured or calculated mount point mechanical impedances of the airframe and added systems to predict response of the added system.

SECTION V
ANGULAR VIBRATION MEASUREMENT

The major objective of the effort described here and in Volume II of this report was the development of methods for prediction of angular vibration. However, it was known from the beginning that some attention would have to be given to purely measurement problems.

The measurement of dynamic rotations at specific points on an elastic structure is not new although concern about angular displacements at the microradian level seems to be confined to optical system applications. Historically, three basic methods of measurement have been used.

(1) Outputs of translational motion sensors mounted a known distance apart may be differenced.

(2) A light beam may be reflected off the point in question and its lateral displacement measured.

(3) Inertia torque on a suspended seismic mass may be sensed.

The most critical angular measurements on airborne optical or laser systems are generally those where motion of a reflecting surface is transduced. Typical requirements in this case may include:

(1) Bandwidth. As usual in structural dynamics, the frequency composition of unwanted motion often contains information which is highly useful to the designer attempting to reduce such motion. Frequency components in the 0.1 to 1.0 kHz range may be particularly important since, in general, they cannot be effectively removed by active servo systems.

(2) Size and Weight. Mirror assemblies as small as 0.1 m (4 in.) and weighing less than 1 kg (2.2 lb.) must be instrumented. Transducers must neither change the dynamic properties of the assembly nor interfere with the intended optical function.

(3) Flexibility of use. During development of component assemblies, sensors must be installed and removed quickly and easily without the requirement of elaborate fixturing. This tends to favor inertially-

referenced methods over optical sensing. Furthermore, the sensing of displacement by optical systems rather than velocity or acceleration may make high frequency measurements difficult.

(4) Cost. As usual, the use of standard devices made in production quantities and usable for other purposes is desirable.

It quickly became clear that differencing of translational acceleration signals was the most appropriate method for the present effort. Other investigators (References [5] and [6]) have used the method and have concluded that it is quite practical. However, it appeared that a better quantitative understanding of limitations and error sources was desirable. To this end, a modest amount of theoretical and experimental work was performed.

A formula was derived which expresses the combined effects of intra-channel random noise and inter-channel gain and phase mismatch. A typical result is shown in Figure 9. $S_{NR}(f)$ represents the narrow-band (i.e., frequency dependent) signal-to-noise ratio for a channel obtained by differencing of two transduced signals. δ is a complex function of frequency which quantifies the inter-channel mismatch. $\delta=0$ implies perfect matching. $1-\alpha$ is a real function of frequency which represents a non-dimensional magnitude of a difference signal. It varies from $1-\alpha=0$ for signals with no difference to $1-\alpha=2$ for signals having equal magnitude and opposite phase (at a given frequency). It was shown by example that either effect can seriously degrade the accuracy of angular acceleration measurements.

A simple digital processing scheme is demonstrated for reducing errors due to inter-channel mismatch. Results are shown in Figure 10 for a case where the PSD of angular acceleration could be obtained accurately by non-differential sensing and used to check measurements made by differencing. Figure 10 represents a difficult measurement case ($1-\alpha=0.0008$) which, it was concluded, is the only time that the added complexity of digital correction is justified. Such cases may be

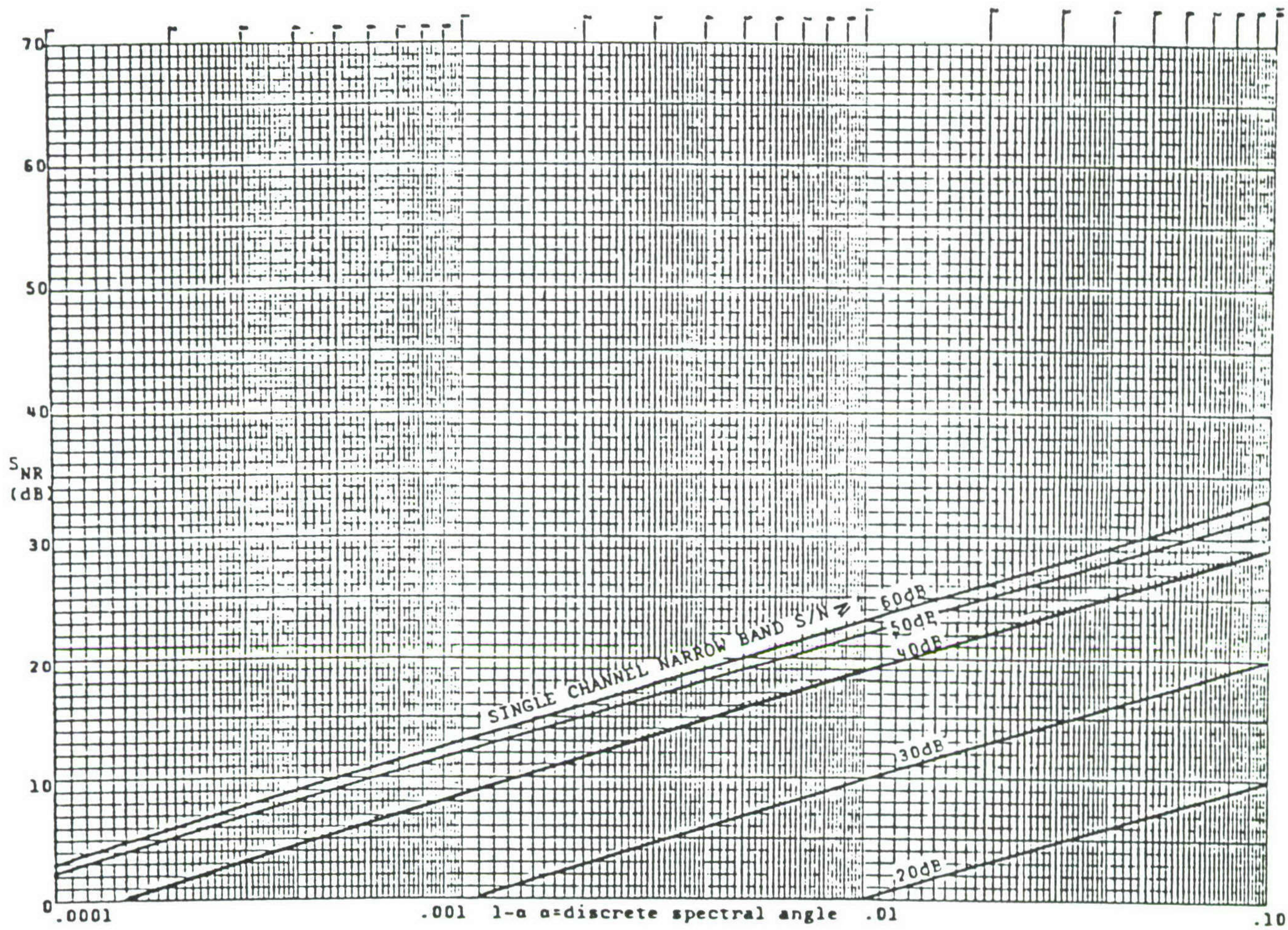
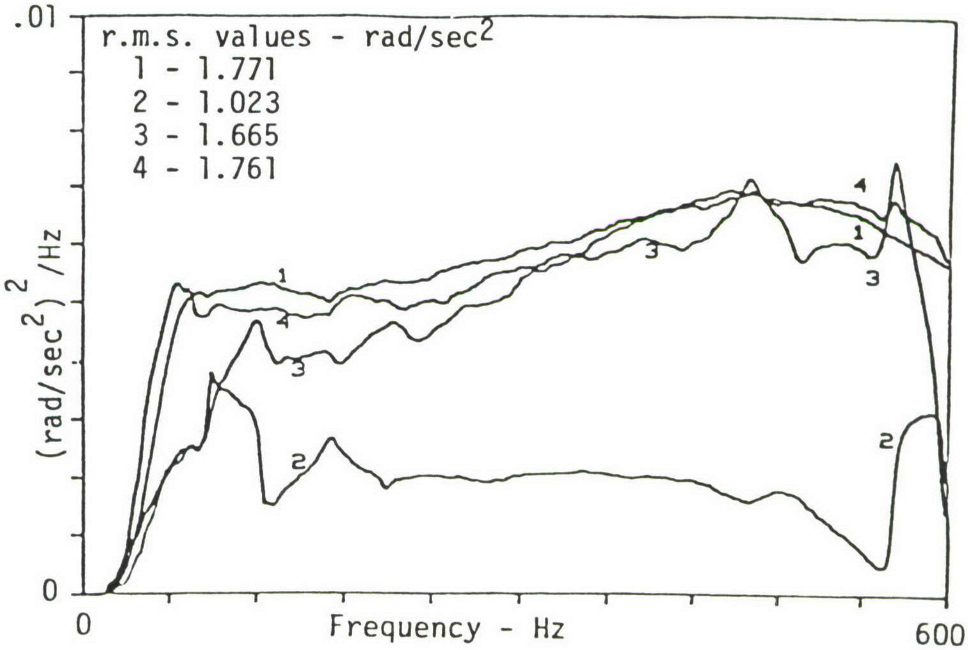


Figure 9 Narrow-Band Signal-to-Noise Ratio of Angular Acceleration Estimate Obtained by Differencing of Linear Accelerations
$|\delta| = 0.01$



<u>Key</u>
1 = scaled from single point acceleration
2 = analog differenced with nominal channel gains
3 = analog differenced with gains matched at 300 Hz.
4 = digital differenced in frequency domain after correction

Figure 10 Measured PSD of Angular Acceleration of a Rigid Body with One Axis Fixed identified as those involving high frequencies (beyond a few hundred Hz), small transducer separations (a few cm. or less), or high common mode signals.

A formula is derived for estimating frequency dependent error due to bending of the surface whose angular acceleration is being measured. A non-dimensional chart, given here as Figure 11, presented to facilitate its use. In Figure 11, κ represents the radius of gyration for the cross section of a beam or plate on which transducers are mounted. Δx is transducer separation, f is frequency, and C_ℓ is extensional wave speed $\sqrt{E/\rho}$ for the material.

The motivation for undertaking the measurement work was quite specific and has already been stated. It is felt that the objectives have been met and that the angular vibration measurements needed in the course of developing prediction methods can be made with confidence.

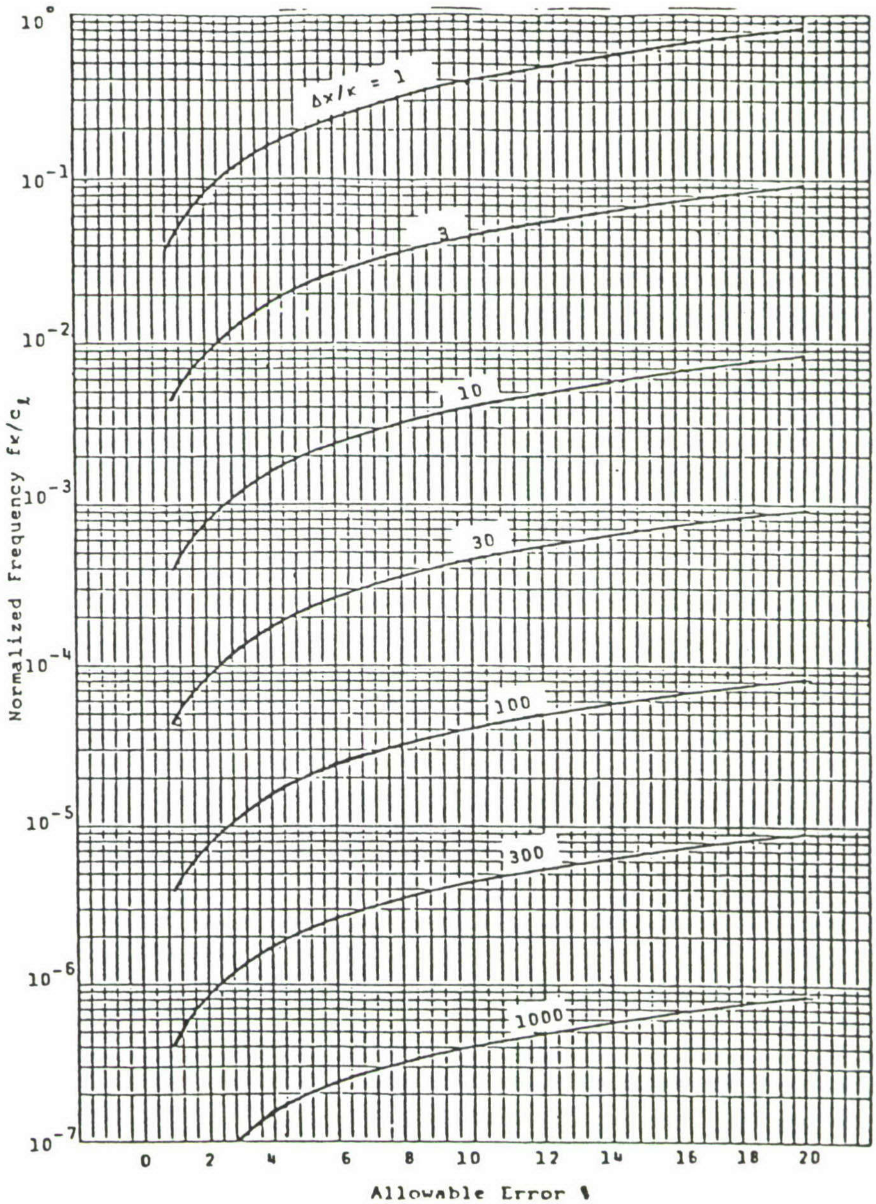


Figure 11 Upper Frequency Limit for Differential Sensing of Angular Vibration

SECTION VI
TEST AND EVALUATION OF PREDICTION METHOD

During Phase III of this contract, a test was performed to assess the accuracy and usability of prediction techniques for angular vibration which were developed during Phases I and II. The structure chosen as a test case for the Semi-Loof element was a rear fuselage section of a 1950 vintage Marine fighter aircraft. The fuselage is shown in Figure 12, after it had been prepared for testing. The preparation consisted of removing all wiring and hydraulics, general cleaning, and removal of several feet from the front and rear, as well as the vertical stabilizer. The structure is made up primarily of flat and singly curved panels with extensive stiffeners. The prepared structure weighed 334 pounds.

A test was carried out with forced vibration input at three selected points. Angular response PSD's were computed at a number of points, in addition to translational PSD's. It was felt that PSD's would provide a more representative summary of the dynamic characteristics of the structure than would lists of natural frequencies and mode shapes. The PSD's were computed in the range of 40 to 160 Hz so as to eliminate the finite stiffness of the supports, which only approximated a free-free condition.

A Semi-Loof element model of the fuselage was prepared. Figure 13 shows the shell elements of the model for half of the symmetric structure. Natural frequencies and mode shapes were calculated. The model was then driven with uniform "white noise" input at selected points, and PSD plots were obtained. All of these results were then compared with experimental results. Figures 14 and 15 are typical of the plots that were produced.

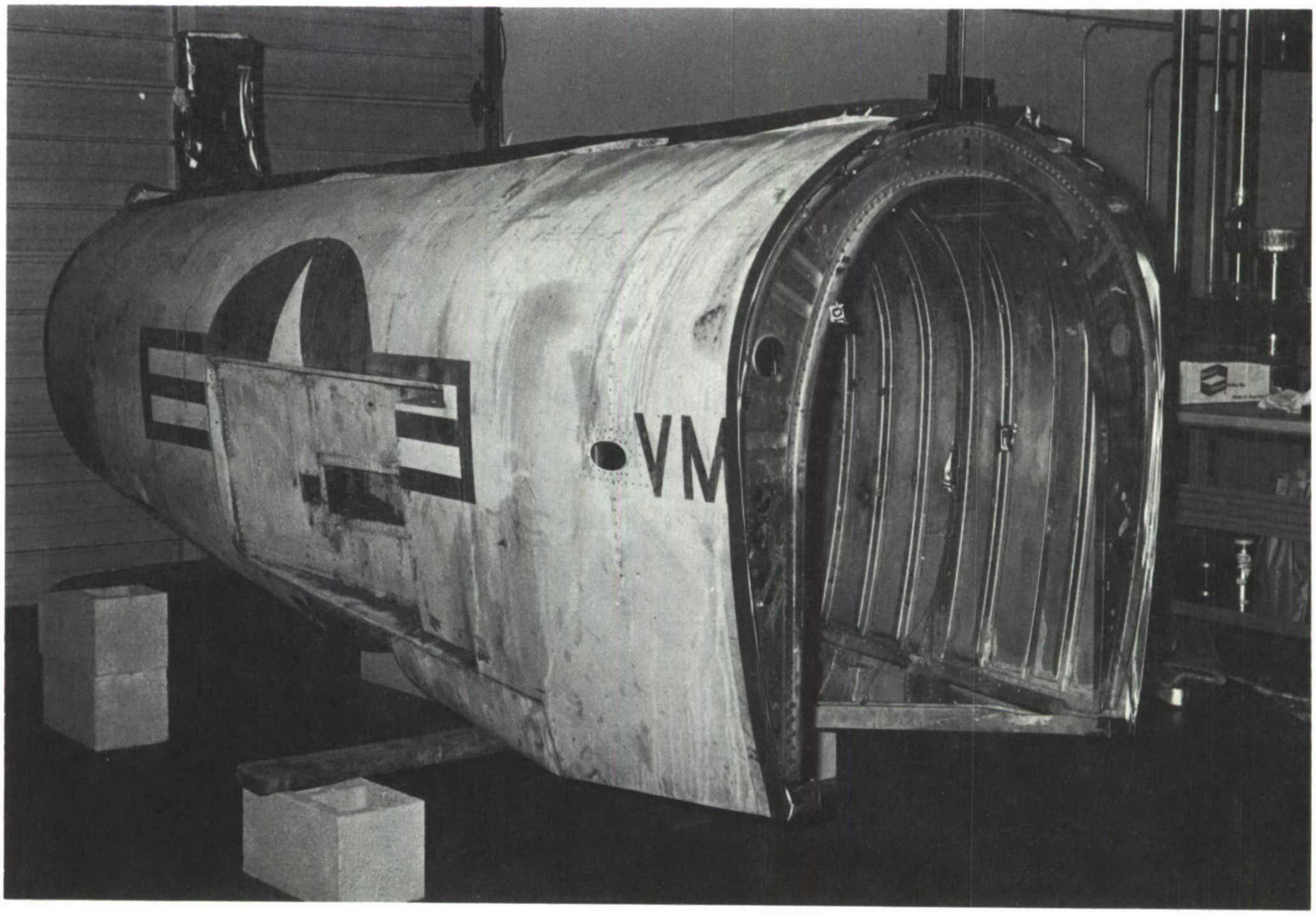


Figure 12 External View of Prepared Test Fuselage

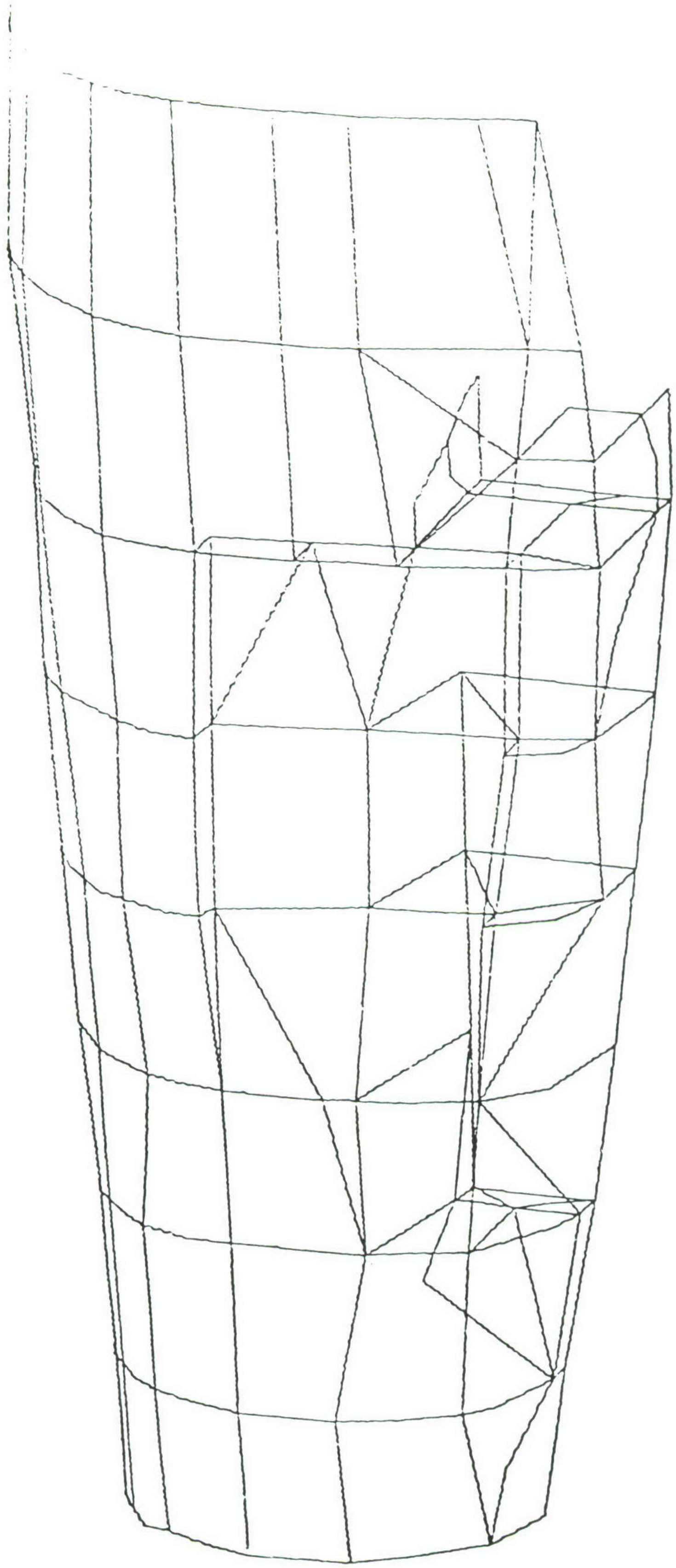


Figure 13 Semi-Loof Fuselage Model

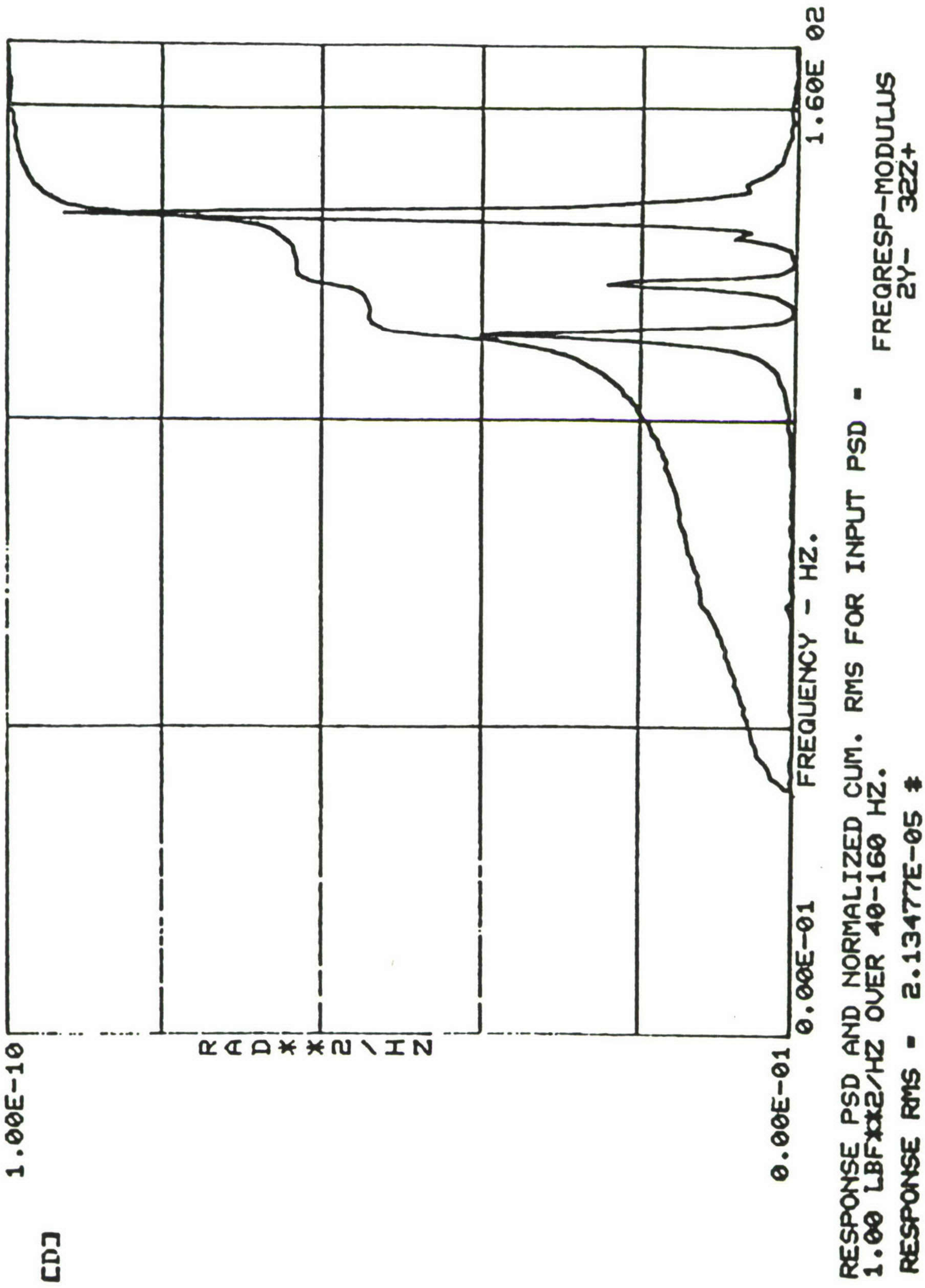


Figure 14 Experimental Results
Response Point 32, Rotation about Z
Driving Point 2, Y Translation

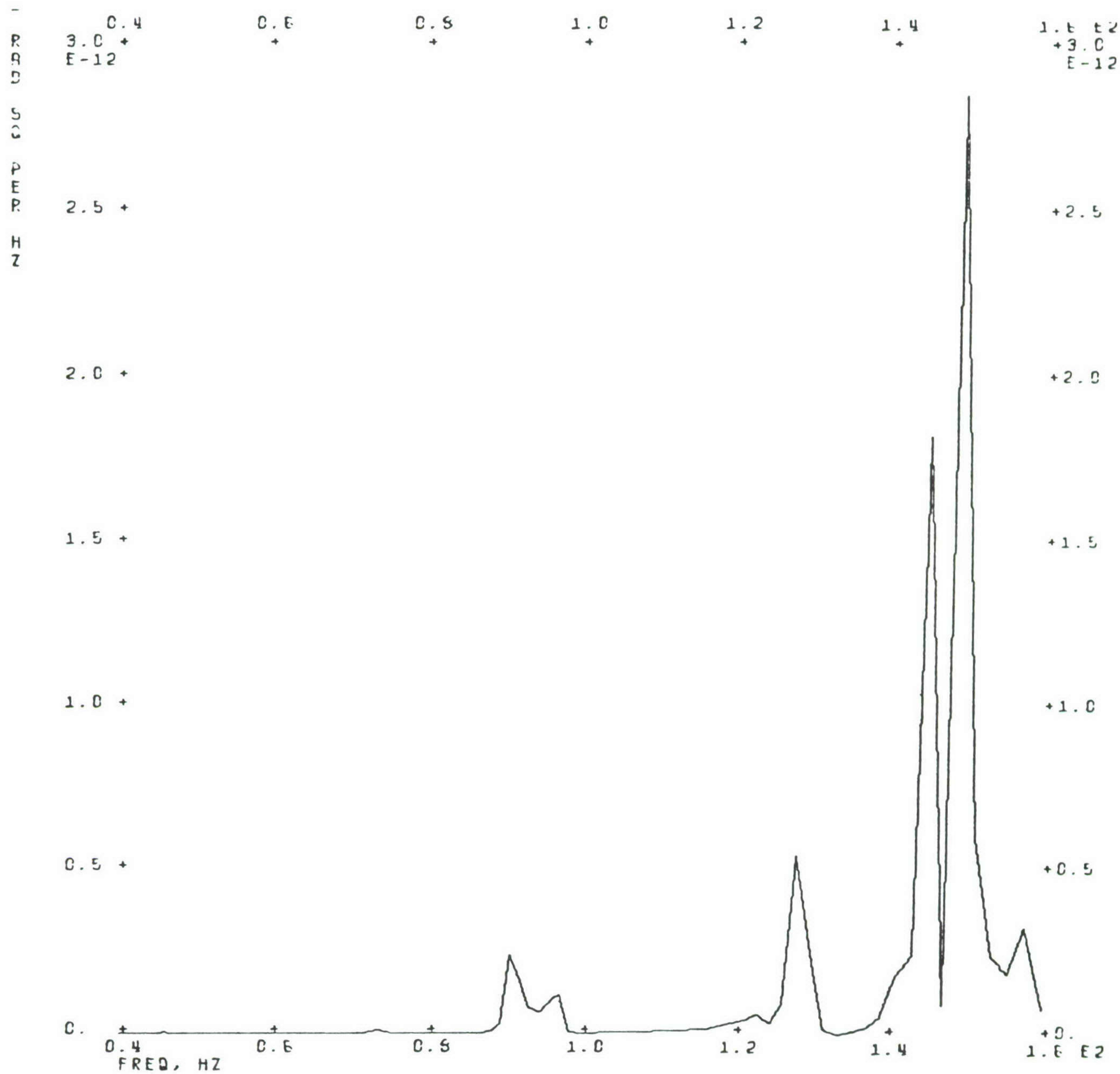


Figure 15 Analytical Results
Response Point 32, Rotation about Z
Driving Point 2, Y Translation

Although correlation was fairly good for the natural frequencies, PSD plots showed only qualitative correlation. One would have to call these results inconclusive with respect to evaluation of the Semi-Loof elements. The performance of the elements was masked by a number of other considerations which dominated this exercise.

SECTION VII
SUMMARY AND CONCLUSIONS

Angular vibration problems in aircraft have been studied in two distinct realms of vibration frequency. Low frequency problems have been studied from a deterministic viewpoint; that is, under the assumption that individual natural frequencies and mode shapes can be determined in some detail by analytical methods. Higher frequency problems have been approached in a statistical manner. The assumption here is that properties of individual modes cannot, and should not, be examined, but that aggregate response of mode groups can be estimated given only a limited structural description.

Considerable research and literature searching was carried out, but there was little to be found in the way of methods specifically addressed to angular vibration. Upon further reflection, it was decided that development work would have to be more general in nature, meaning that any new or improved methods would, for the most part, be relevant to vibration problems in general, and not just angular vibration.

The finite element method is the principal analytical tool currently in use for structural analysis. A number of advanced shell elements have been reported in the literature, but little of this progress has reached production users who mostly employ large, widely distributed codes such as NASTRAN. One set of curved shell elements known as "Semi-Loof" elements seemed especially promising. A skeleton computer program was obtained from its developer, Professor Bruce Irons. A preprocessing code was built around Irons' code to make it possible to use these elements with NASTRAN. This enables users to take full advantage of NASTRAN's capabilities. Small test problems showed encouraging results. A small stiffened panel was tested with results showing good agreement with Semi-Loof predictions. A large fuselage model was analyzed and tested, but with inconclusive results. A mathematical irregularity was encountered in the quadrilateral element involving spurious mechanism modes. The problem was found to be avoidable, but still annoying.

The Semi-Loof pre- and post-processing programs are considered to be in production status, with careful attention having been given to input formats and error checking. Still, much more user experience will be needed before conclusive evaluations of these elements can be made. It is hoped that a real contribution has been made, not only to angular vibration methods, but to other shell analysis problems, as well.

For high frequency analysis, there is no widely accepted method with the flexibility and accuracy which finite elements allow in low frequency work. This is not surprising, considering the previously stated definition of the high frequency problem. During early literature searching, it was found that a rather diverse body of theory under the general name of Statistical Energy Analysis (SEA) had been developed during the 1960's in response to the difficulties of high frequency analysis. The method draws heavily on classical deterministic theories of acoustics, wave mechanics, and normal mode methods, and yet takes quite a unique view of vibration problems generally in that it uses component energies and intercomponent power flows as primary variables. SEA appeared to be well suited to the problem of predicting indirectly excited high frequency vibration of a piece of equipment mounted in an aircraft. The method is quite general and is not restricted to angular vibration.

It was decided that, due to the inexperience of the investigators with SEA, a small demonstration problem should be undertaken. The wave transmission method of SEA was used to predict equilibrium energy ratio and power transmission coefficient between two uniform plates coupled at a single degree of freedom. Comparison with experiment showed agreement which was quite encouraging in the light of the extensive assumptions involved. While physically simple, the experiment embodied the typical factors which make high frequency predictions difficult. Mode counts were very high (over 600 total for the two plates) and individual mode shapes could not be determined. It was demonstrated how interactive digital FFT processing of measured data could be used to obtain the so-called coupling loss factors and internal loss factors of SEA. It was also demonstrated, and verified by experiment, that the basic assumptions of the wave transmission method can be utilized to estimate r.m.s. angular response given the component total energy for a uniform plate. While some software was developed in the course of the SEA experiment, it cannot yet be considered a production tool for the prediction of high frequency angular vibration.

Because of the extensive use of experimental methods for prediction of angular vibration, some effort was devoted to purely measurement problems. Differencing of output signals from translational accelerometers was identified as the most generally useful method of angular vibration measurement. Theoretical error analyses were performed to quantify the effects of intrachannel noise, interchannel gain and phase mismatch, and flexure of the mounting surface. It was concluded the measurement requirements involved in development of prediction methods could be satisfied by the differencing technique.

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