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FINAL REPORT

- 1. ARO PROPOSAL NUMBER: P-12494-M
- 2. PERIOD COVERED BY REPORT: 8/15/75 thru 8/14/78
- 3. TITLE: DECISION MAKING, FUZZY SET THEORY AND COMPUTERS

4. CONTRACT OR GRANT NUMBER: DAA G29-76-G-0027

- 5. NAME OF INSTITUTION: University of Southern California
- 6. AUTHOR(S) OF REPORT: Richard Bellman and E. Stanley Lee
- 7. LIST OF MANUSCRIPTS SUBMITTED OR PUBLISHED UNDER ARO SPONSORSHIP DURING THIS PERIOD, INCLUDING JOURNAL REFERENCES:

"Systems Analysis, "R. Bellman, USCEE-TR75-9, 12/75, Encyclopedia of Science and Technology, 4th Ed., McGraw-Hill, New York (1976). "Mathematics and Systems," USCEE-TR76-1, Kybernetes, Blackburn, England, Vol. 7, No. 1 (1978), R. Bellman.

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8. SCIENTIFIC PERSONNEL SUPPORTED BY THIS PROJECT:

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Final Report

DECISION-MAKING, FUZZY SET THEORY AND COMPUTERS

INTRODUCTION

The advent of the computer age has stimulated a rapid expansion in the use of quantitative techniques for the analysis of military, economic, urban, social, engineering and other types of systems which in the past were too complex for analytical treatment without the aid of computers. However, the use of computers has proved to be highly effective in some areas and not in others. Generally, the areas in which computers have been less than effective are those which are concerned with or depend upon human perception, judgment and decision-making. This is particularly true of such problem areas as the summarization of information, machine translation of languages, pattern recognition, learning processes, and related fields. Progress in these problem areas has a direct bearing on our ability to understand the behavior of those large-scale systems, e.g., military strategy and operation systems, economic systems, urban systems, social systems, political systems, etc., in which human behavior plays an important role.

It appears that one of the basic reasons for the rather limited success of computers in dealing with the problems in question has to do with the fact that classical mathematics, which is used in computation, makes no provision for the phenomenon of fuzziness. By fuzziness, we mean a type of imprecision which relates not to randomness, but to a lack of sharp transition from membership to nonmembership in a class of objects. Thus, in classical mathematics an object either belongs to a specified set or does not. By contrast, most of the classes in the real world are fuzzy, in the sense that an object may belong to a class to an extent intermediate between full membership and nonmembership. For example, the class of qualified recruits and the class of intelligent men in a group are fuzzy sets, as are the classes of oval objects, black objects, etc.

The fuzzy set theory represents an attempt to construct a conceptual framework which can be more effective in dealing with real world problems than that of classical mathematics. Thus, by making it possible to treat problems which are too complex or too ill-defined to be susceptible for analysis by conventional means, the fuzzy set theory could provide a novel approach to important problem areas in fields such as military strategy, management science, economics, law psychology, linguistics, information retrieval, biology and medicine.

Fuzzy set theory has been applied to various fields. Some of these applications which are directly related to this project are the use of fuzzy set theory to automata theory, systems theory and decision-making, pattern recognition, linguistics, biological systems and large-scale system analysis.

However, only a start has been made in the development and application of fuzzy set theory. Much more needs to be done. In this work, the theory of dynamic programming was combined in various ways with the fuzzy set theory in a decision-making environment.

With simulation and fuzzy systems, we can now study many important problems in the social area.

In the following, a brief summary of the accomplishments of this project is given.

1. Artificial Intelligence

The book,

R. Bellman, An Introduction to Artificial Intelligence: Can Computers Think?, Boyd & Fraser Publishing Co., San Francisco, 1978.

appeared.

The table of contents is:

Chapter 1. Can Computers Think?

- 2. The Digital Computer
- 3. Decision Making
- 4. Puzzles
- 5. Uncertainty
- 6. Simulation
- 7. Learning
- 8. Consciousness
- 9. Humor

- 10. Local Logics
- 11. Mathematical Models of the Mind
- 12. Communication and Ambiguity
- 13. Can Computers Really Think?

The preface is:

Computers are here to stay. Like many other products of technology, fire, the wheel, the printing press, electricity, television, and nuclear energy, it has influenced everyone's life and the influence will increase.

Consequently, it is essential that everyone have some knowledge of what a computer can accomplish, what it cannot, and why. In the following, these questions are discussed.

In the first chapter we show how the verbal question. Can Computers Think?, gives rise to many mathematical questions. Furthermore, we show how the mathematician plays an important part in considering these questions. In Chapter 2, we consider the properties of the commercial digital computer which we will use. In Chapter 3, we turn to decision making. In Chapter 4. we show that this methodology can be used to treat many common puzzles. In Chapter 5, we consider decision making under uncertainty. In doing so, we discuss what is meant in many cases by uncertainty. In Chapter 6, we discuss simulation. In Chapter 7, we turn to learning. In Chapter 8, we consider consciousness. In Chapter 9, we consider humor as an aspect of consciousness. The consideration of humor gives us an opportunity to say some words about the paradoxes which occur in logic. These paradoxes have an important bearing on the possibilities that exist for the future use of digital computers. In the next chapter, Chapter 10, we consider local logics. In Chapter 11, we turn to a more mathematical discussion. Here, we are interested in mathematical models of the mind. In Chapter 12, we consider the important problem of communication. Finally, in the concluding chapter we ask the question, "Can Computers Think?".

Throughout, we have used one method. We have shown that many examples of thinking by computer can be regarded as tracing a path through a network. Obviously, other methods of human thinking exist. The method of tracing a path through a network has not worked well in pattern recognition, language translation, or theorem proving, to name a few. It is possible that a more adroit use of these techniques will be successful. It is more probable that new ideas are required, and it is even more probable that a digital computer may never be able to accomplish these particular tasks well.

Another way of putting it is that we have studied thinking as rational behavior. We have converted the question "Can computers think?" to "Can computers exhibit rational behavior?". What we have shown is that in many cases we can have a digital computer exhibit rational behavior. But, the point we stress is that the digital computer can only exhibit rational behavior if we understand the process.

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In many cases, the field of artificial intelligence, as it is often called is a very fascinating field for the young scientist. It contains many challenges and many interesting problems.

This book is intended for pure and applied mathematicians, physicists, engineers, computer scientists, system analysts, operations researchers, and all those interested in the use of a computer. The only prerequisite is a knowledge of function, a mathematical translation of "depends upon". Naturally, the more mathematics the reader has had the more comfortable he will be.

This manuscript was prepared under the sponsorship of the Army Research Office, "Decision Making, Fuzzy Set Theory and Computers," Grant No. DAHC-76-G-0027, headed by Dr. J. Chandra.

See also

R. Bellman, "Dynamic Programming and Artificial Intelligence," International Journal of Mathematics & Statistics, Brazil, Vol. 1, 1977.

2. Fuzzy Set Theory in Health Delivery Systems

With Austin Esogbue, we are preparing a book,

R. Bellman, A. Esogbue, and E. S. Lee, Dynamic Programming, Fuzzy Set Theory and Health Systems,

which summarizes our work on the application of fuzzy set theory to health delivery systems. This is essentially a multistage decision problem with fuzziness (not randomness).

Although the problem treated is health systems, the basic ideas can be applied equally well to other decision making processes such as military strategy, political systems, urban systems, and social systems.

The solution of multistage decision processes in a fuzzy environment is not simple. Although dynamic programming is naturally suited for obtaining their solutions, other techniques such as linear and nonlinear programming, functional gradient techniques, maximum principle and others are also used. The results appear to be very promising, however, many theoretical problems need to be studied. New and novel computational techniques which are especially suited for solving these problems are explored. Because of the special characteristics of fuzzy systems, the development of efficient iterative algorithms are one of the aims of this work. For example, an iterative technique was developed to solve the functional equation resulting from the problem with implicity defined termination time.

3. Fussy Set Theory in Computers and Linguistics

Translation, in the practice of human translators, is clearly a multistage decision process under the guidance of some principle of optimality or, at least, of feasibility, of the kind that has hardly been formulated at all. It exists in the "mind" of the translator, who, after many trials, recognizes by a convergent intuition that a word, or group of words, in one language, correspond suitably with a word, or group of words, in another, at least well enough for the purpose at hand.

Part of the research results is summarized in the following. Specific results will appear in a series of six papers.

Our purpose is to elaborate a program for the use of computers and mathematics in the translation of a domen or so ancient languages. First; the mathematical principles are applicable to all; secondly; there is a higher probability of succeeding in one of ten than in one of one; and thirdly, some of the items are quite likely interrlated in one way or another and a cascade effect may be found. For example, the key or better, the test of any key, to Cretan Linear-A is probably to be found in Cypro-Minoan.

In Sections b to f, a background in philology is given. There is also found a discussion of how the computer can be used.

In Section g, the uses of the computer are summarized. In Section h, some of the uses of mathematical techniques are given.

This work is highly inter-disciplinary. What is needed are philologists with a familiarity with computers and mathematics; and mathematicians versed in the use of computers and possessing a familiarity with philology. Philology in the broadest sense reconstructed societies.

We allude briefly to the problematic and to some of the history of this problem; and also to certain techniques of applied mathematics via computer that could be useful in its solution. The sort of work so far accomplished and proposed is indicated at the end. There is no intended connection of any of this material or problematic with so-called "machine translation." The probability, however cannot be excluded that a measure of success in what is here outlined could lead to useful results in MT an important field whose meager accomplishments up to now would seem to indicate that quite other theories of language and procedures could be advantageous to it.

Translation, in the practice of human translators, is clearly a <u>many-stage decision procedure</u> under the guidance of some princip'e of optimality or, at least, of feasibility, of the kind that has hardly been formulated at all. It exists in the "mind" of the translator, who, after many trials, recognizes by a convergent intuition that a word, or group of words, in one language, correspond suitably with a word, or group of words, in another, at least well enough for the purpose at hand. This principle, or principles, should be made explicit, to the degree possible. We have found the "literature" on this subject to be little concerned with logic, arbitrary, and discursive.

b. Problematic

A fundamental problem in the applied sciences and the humanities, say, to archaeology, cultural history, linguistics and philology is posed by the existence of a rather large number of completely undeciphered, or imperfectly deciphered, languages like Etruscan, Cretan Linear-A, Mayan, Meroitic, Indus Valley, and so on.^{*} Computers, and the techniques of applied mathematics that have been developed in connection with them, almost certainly will, in the future, be invaluable aids to decipherment. Among these techniques are the theories of "imbedding" (R. Bellman), "fuzzy sets" (Lotfi Zadeh), "kladistics" (R. Bellman and E. Marchi), and "dynamic simulation" (R. Bellman). All these techniques exist as extensions of the easier and more familiar use of the computer in information storage and retrieval, for example, for elaborating easily used corpora and concordances of inscriptional and

^{*}Some of these languages can be read but not translated. Some cannot be read or translated. There is no general term for translation and/or translation.

other available written material, as well as for statistical operations theron, e. g., finding morpheme boundaries, drawing up thesauroi with context, and allowing the largely statistical testing of <u>experimen-</u> tal values.

A language like Etruscan, to which we have devoted perhaps somewhat more attention than to any of the others mentioned below, seems ripe for these mathematical treatments. This important language is supposed popularly, and by very many scholars as well, to be impenetrably obscure. The fact is that there exists hardly any inscriptional material of this language of which the gist, at least, is not more or less clear. This sort of partial success has been almost completely due to painstaking "combinatorial", i.e., formal, analysis of morphemes, words, and sentences of the texts, rather than to the enthusiasts' "lost tribe of Israel" sort of speculation based upon one or two purely accidental, and mostly mistaken, coincidences between Etruscan lexemes and morphemes and those of some other (indeed very many other) well or better known languages.

It seems to us that Etruscan is a typically agglutinating language systematically resembling, to a marked degree, the kind of primitive Altaic that can be reconstructed from, say, the various Turkic-Chuvash, Tungua and Mongolian dialects, and modified greatly in vocabulary by the successive geographic stations of a set of nomads or mercenaries moving between the Altai and Tuscany. ^{*} And here it is not a question of one or two isoglosses but rather of dozens of words, morphemes and proper names. This "Altaic" thesis which could yield much of the hitherto lacking "semantic" context, in which Etruscan is embedded, must be tested by analysis of geographical and historical data yielding trajectories

^{*}The term "agglutinative" has long been out of favor with linguist's. But nothing better by way of terminology has come into use. We do not, however, confuse a number of different criteria of classification; morphophonemic complexity... and the elation of syntactic to morphologic phenomena" (R. J. Hall, 1964). By specifying "Altaic", our analysis categorizes words with little or no complex interior at the borders of inflectional elements; and produces a "stucture of strings of inflectionally bound forms of great simplicity.

of the route followed by the "Tyrrhenian" component of the culture which, at last intermingled with Villanovan and Italian indigenous, gave rise to the later Etruscan in situ. * This routing problem can be studied to great advantage with the aid of computer operations on well-known geographic maps. Up to now we have used plastic map-overlays; but the amount of information is rapidly growing too large not to force recourse to computers, even if the use of that device had not otherwise been thought of. The high possibility of this westward migration is clear. Almost as soon as we have written history at all, and with an increasing crescendo, we learn of the westward (and eastward) migrations of numerous bands of Altaic Tribes through Iranian linguistic regions into just those parts of Anatolia and points west from which most scholars still believe the Tyrrhenian element of the Etruscan culture came. The Austrian scholar, W. Brandenstein, suggested a generation ago that the Etruscan language, at least in its noun-declensions, much resembled Tatar-Turkish. But, what on the whole has been a meager followup of this hint has been carried out by very few persons, most notebly by the Austrian A. J. Pfiffig, who seems himself to display little firsthand knowledge of any of the Turkic languages, not to speak of a knowledge of other members of the Altaic family or their more primitive and reconstructable forebears. This nonsequel may be due in part to the fact that there are few scholars anywhere currently at work on Etruscan; and none in the United States. Most of the activity is in Italy among linguists who are more or less reluctant retainers of M. Pallottino of Rome, the imperious and gifted dean of

*"Trajectory" is a stronger version of "isogloss" than that commonly favored. At any point in the trajectory any given lexical item may be substituted by another that "fits" into this process of successive "pidgin"ization. Etruscology, who with the anti-diffusionist <u>companilismo</u> of most hypermodern archaeologists, has decreed an autochthonous Italian origin for the Etruscan language and culture.

(If we were to have any success along this line of investigation, an important bonus would be the aid given, in the opposite direction, to the reconstruction of the primitive Altaic referred to; by having clues to it from more than a millennium earlier than those we now possess from recorded specimens. Such aid might be small, however, since at least 50 percent of the lexical and grammatical forms of Turkic and Mongolian already prove, on analysis, to be held in common. One could easily reason in a circle in this; but that infirmity is generally true in science; and avoidance of it depends on the skill and the insight of the investigator.)

We might give an example here of the kind of aid one might well expect from the consideration of "linguistic trajectory." It is difficult on formal (so-called "combinatorial") grounds <u>alone</u> to distinguish the /l/ phoneme of the genitive from the /l/ plural particularly in view of the fact that some languages like Russian use "partitive" <u>singular</u> genitives with some numbers. But /l/ is well attested as a plural in Altaic; and the /l/ singular genitive is well known on our trajectory, for example in Anatolia, most importantly from Lydian. It was from Lydia that Herodotus tells us the Tyrrhenians migrated to Italy.

c. Historical

Historically, the great decipherments of the Nineteenth and Twentieth Centuries were based on (1) copious inscriptional material, and (2) the assumption, which often turned out to be correct, that the language(s) of the inscriptions were close to known languages. Thus;

(1) Egyptian Hieroglyphic, Babylonian Cuneiform and the Ugaritic alphabetic scripts were all amply attested. Also; (2) Egyptian was close enough to its descendant Coptic, and Akkadian (Babylonian), Ugaritic, and the recently discovered Eblaitic to related and known Semitic dialects to make rapid decipherment possible. Often bilinguals were available. The one decipherment apparently not fitting this condition was Linear-B. The material corpus was sparse and the language unknown. Michael Ventris made the brilliant supposition that the names of major Cretan cities, which were known to be non-Greek in origin, were the same in the Bronze Age as today. (He had spent ten years or more exploring a supposed affinity of Linear-A with Etruscan). This proved to be correct. The place-names, along with a vowel-consonant grid he had prepared, gave him the values of about thirty of sixty-five common signs, of which there were ninety in all. Applying these, he discovered the language to be an archaic Greek. From there on it was easy work. Ventris' work, it might be added here, could have been enormously expedited by a very simple use of a computer, e.g., in manipulating the grid.

Since Ventris' achievement (1952) there have been few important or conclusive decipherments of unknown scripts (except Eblaitic) though many have been claimed and some progress made on a few. Further, even more unknown scripts have come to light, mostly from Anatolia. The traditional methods, by means of their rapidly increasing cumbersomeness, will no longer serve in attacking these "mysterious" writing systems. All the still undeciphered scripts except Mayan, the Indus River script, and Etruscan (a special case, since it is written in an alphabet derived from an archaic Greek alphabet and which can be read with some but far from total certainty as to sound) are less sufficiently attested than Linear-B. Mayan and Indus Valley are mostly ideographic, and in the latter case the language's affinities are totally unknown. (Even if it were a Dravidian language pushed south by invading Indo-Aryan speakers, it would presumably be very far from modern Dravidian!). Etruscan is not clearly related to

any known language except perhaps as suggested above; and the combinatory method hithertofore used on it has suffered from the fact that this method is essentially mathematical; but few formally qualified scholars in this last respect exist. Even where a script is partially deciphered and the language tolerably well understood, as with Hieroglyphic Hittite, there are large areas of uncertainty, which, it may be noted have latterly seemed to grow, and not to diminish. (This is almost always the case!) Bilingual texts are often useful; but not so much as might be supposed. They are very seldom literal translations; and, if anything, most often raise more problems than they solve.

We, therefore, have proposed an attack on <u>all</u> these (and more) undeciphered scripts using the ability of computers to organize often too meager but, even so, staggering amounts of data, much of it nonlinguistic, say, archaeological.

Computers have been used on Indus Valley and Mayan; but this has occurred in Denmark and the U.S.S.R. where computer technology and its availability to linguistics is nowhere near as advanced as it is in the United States. In the United States concordances without meanings of Etruscan and Linear-A have been prepared with computer assistance and will probably prove to be valuable adjuncts to the imbedding of these languages in various linguistic and non-linguistic semantic contexts. * (The work of R. Bellman has made the concept of "imbedding" well understood and productive.)

*The two volume work: <u>Materials for the Study of the Etruscan Language</u>, M. Fowler and R. G. Wolfe, University of Wisconsin Press (1967), allows much more ready access to the <u>corpus</u> of Etruscan materials than is otherwise possible. This useful work is the kind of computer-based <u>propaedeutic that will prove in general necessary for translation proper</u>. It produces not only a <u>corpus</u> but also allows for vastly improved statistical manipulation of many kinds.

d. Simulation

Our purpose, then, is to analyze and apply simulation as well as other computer-based procedures to translations, e.g., of Etruscan, considering a translation as a <u>dialogue</u> between a human interrogator and a set of protocols than represent answers to wuestions that have already been explicitly put; to some questions that may be implicit and not at all a conscious part of the author's repertoire. (A detailed discussion of the methods to be used will be found in the book: R. Bellman and C.P. Smith, <u>Simulation in Human Systems: Decision-</u> <u>Making in Psychotherapy</u>, John Wiley & Sons, Inc., New York, 1973.)

We regard the "linguistic interview" as an interaction between two systems. The linguist wishes to elicit information which he does not yet possess. The relevant documents, in general, desire to make this information known. In this last respect there perhaps exists a fundamental difference from cryptanalysis where the document (or documents) are intended to keep information away from any arbitrarily given investigator, and to reserve it to the initiate. (Gome hierarchic languages, it must be remembered, <u>are of a partially and intentionally cryptic nature; and it may be that at least some Etruscan testimonia</u> may possess this property. It is well to bear in mind that some at least of the algorithms known to cryptanalysis may prove to be of important, and even of dramatic, assistance.)

In constructing a simulation, the following are required: description, sets of decisions, sets of equivalent "actions", (that is, meanings) cause and effect, and objectives. The method of introducing state variables ultimately depends on a theory (or theories) of linguistics. But, it is important to stress here that the methodology of simulation can be used even when the linguistic theories in question are widely divergent. It should be emphasized, too, that most of natural language's features are inherently "fuzzy".

Thus, L. Zadeh's theory of fuzzy systems should play an important role. * Therefore in making an Etruscan (computer-based) glossary, we classify possible and variant hypotheses about meaning under a number of headings, viz; "certain", "very likely", "likely", "little likely" and "unknown". These might correspond, for computer purposes, very roughly, to 100%, 75%, 50%, 25% and 0%, respectively.

There are difficulties with all the above. Two principal difficulties are "construction of responses"; and what might be called "branching".

The "construction of responses", using possible questions put by the linguist and answers by the documents, can be briefly alluded to in the book cited above.

Such construction, using questions put by the linguist and/or linguistics, requires as competent knowledge as possible of the system and sub-systems. This knowledge is obtained by intense interdisciplinary reading of texts; and by working with linguists, historians, archaeologists, and others interested in the problem-process involved.

Branching is a phenomenon as Bellman has recognized, encountered in <u>every</u> encounter with "artificial intelligence". It represents a principal difficulty not only in the use of the computer but perhaps even more so with unaided speculation. For example, if we permit only two possible questions and two possible answers, there are four possibilities at each stage. We easily see that if we allow but ten stages, there then exist more than a million possibilities. If we allow twenty

*L. Zadeh; A Linguistic (non-numerical) Approach to Decision Making and Systems Analysis (Berkeley, 1976).

stages, we see that we far exceed the capability of fast storage in a modern computer. This is so because of the rapid rise of factorial expressions. (21 -4; 101 =3, 628, 800; 20! is unbelievably large). How the human brain prevents branching in linguistics (or chess) is not at present clear. But its methods almost certainly are machine-like programs generating, in a complicated digital-analogue, computer environments of great complexity.

What is needed, therefore, is a method for keeping the process under control. The method will vary somewhat from system to system. It might be noted that in linguistics, time is an important consideration although it is seldom specifically referred to. ("Dynamic simulation" discusses at length some of the formal ways we could exploit to avoid excessive branching.) Linguistic, diachronic, theories of the origin of a language also serve to prevent branching and to assist us in maintaining control. If Etruscan is a set of repeatedly pidgins devenus creole languages, along the trajectory described, this theory would obviously narrow down the possible etymological and structural alternatives.* The procedures of A. Trombetti, who described, for example, in the Etruscan radical "men-" affinities with a dozen, or more, languages (e.g., Choctaw, Votyak, and even Gothic), drawn from nearly every part of the terrestrial globe, was ineffective precisely because it lost effective control of the translational procedure. For us, borrowings must be located along the trajectory, and follow what we know of the phonology at every point.

It is clear that even a false theory can also exert control by preventing branching; just as, trivially, the common conviction that Etruscan is and will remain "indecipherable" may in another way have the same effect.

³⁸Pidgin languages are constructed for simple purposes usually using the lowest common structural denominator of the vocabulary of one language and the grammar of another. When this pidgin becomes the only language of the speakers of it, it must grow to serve all the purposes of life. This subsequent process of complication is "creolization." Pidgins can be evolved very rapidly. The fact that the resulting lexical items represent lowest common linguistic denominators is not at all the same as thinking of pidgins as badly, or non-structured, gabble of civilized persons with primitives. It is obvious that neither a completely false theory nor the denial of the possibility of any theory would be especially useful: A linguistic theory of origin that was at least <u>partially</u> correct and, at the least, clear, would, however, contribute much to effective control and efficient translation. And this has been our first step. "Truth comes out of error much more quickly than from confusion."

A multiplicity of directions for further research exist. In the first place, we wish to consider different types of language systems beside Etruscan; for example, Meroitic, the Indus Valley script, Cretan Linear-A, etc. Firstly, we are in a position to construct computer "vignettes" in connect on with various known language systems. In the <u>second</u> place, there are ordinarily many different but guess-able types of communication, e.g., devotional, functory, etc. <u>Thirdly</u>, we wish to combine the scanning method utilized by K. Colby with the procedures described above.

The method can clearly be applied to other fields of "dialoguing", notably those that occur in medicine, law, and history. What we wish to emphasize, in addition to the attainment of satisfactory translations, is that we will create a powerful tool, not only for research, but also for education, say, in linguistics and/or philology.

e. Simulation Paradigms

The use by R. Bellman of so-called "vignettes" in connection with computer simulation of the psychiatric interview, suggested the potential usefulness of bilingual vignettes, or language paradigms for translation; provided that these could be composed, say, to represent translations between Etruscan and the oldest recorded Altaic (in the instance the Turkic inscriptions of the Yensisei and in the language of the old

Glossary and Commentary to Sentences 1 and 2 Above

mini: Etruscan and Turkic: accusative singular of the personal pronoun "I"; apa: Father (also exists in other languages probably onomatopoeically); tarchun-tarchon: Nominative sing., Tarchon was a principal deity of the Etruscans. Found in Anatolia Orchon Old Turkic inscriptions. Apa Tarchon is in Transbaikol the title of a high officer of the Royal entourage. Also, in consideration of the proposed geographic itinerary of the Tyrrhenians, notable as an important and widespread Anatolian deity of Hittite times and later. It is the base of Etruscan name Tarquinius; tinsi: Etruscan genitivus donandi of the kind regularly used in place of the Dative (which probably did not exist in Etruscan). Tinsi(y)e, Turk. Dative. Tins was the Zeus of the Etruscans, bu assimilation accommodation of two gods of Heaven. (Tins is also heaven in Etruscan.) The Altaic Tinsi seems to be derived from the same, or nearly the same, Chinese work T'ien-tse meaning son of heaven. (Note also the Greek word Tindareos, which almost certainly is a "Tyrrhenian" composition: i.e., Tinthur, i.e., the sons of Tin. Alpan: Etruscan and Turkic. Compound, durative, present-participle, from stem al- with the meaning "take", "buy", 'spenden" and the like. (cf., ali! = give in Mongolian). Turuke: Etruscan "gave". Tur- is the root, -u- is a participial particle, and -ke the ending of the preterite. Tirirke: is modern Turkish, cf., giderke(n) from verbal root gid-("go") for the same construction. It is true that tir- in Turkic means "collect" rather than "give"; but, recall, the collection in a church is the offering. The Etruscan -u- is the epsilon of Greek and so must probably be umlauted. Thus, tyr- or tur- is in all likelihood a better transcription. Sometimes "alpnu" with the S.

It seems remarkable indeed that two such sentences (representing, by appropriate substitutions, a class of identically-formed sentences) could be concocted in languages apparently as disconnected in time and space as Etruscan and primitive Altaic. It would be very difficult to do as much even in pairs of languages known to be closely related, as, for example, low German-English, or Spanish-Italian. (But see note below!)

Both Etruscan and Turkic are very conservative languages. The available Etruscan inscriptions display a language that scarcely varied over the 600 years of its inscriptional use; and the most ancient recorded Turkic is ver; like Turkish dialects of the present day. The eightcentury gap between the last Etruscan inscriptions and the first Turkic therefore takes on a diminished importance.

Note to the Above:

The above pair of paradigmatic sentences in Etruscan and Turkic, respectively, can be almost reproduced in Greek, with only a slightly different meaning, and suitable to an ex-voto offering, as follows:

κιν αππα ταρχων Δηνι αλφην εδωχε.

Transliterated into the standard modified-Latin alphabet already used, this would read: "Min apa Tarchon Teni alp(h) en etuke." (There are no double consonants or a vowel "o" in Etruscan.)

The meaning would be given as follows in English:

"The ancestor Tarchon gave it, a production, to Zeus." It cannot be pretended that this would be good Greek. And Etruscan only used the accusative morpheme |u| with pronouns.^{*} (But, it would be understandable enough.) For one reason: Greek has a definite article. Neither Etruscan nor Turkic possess one. Such a "telegra-

"However, Etruscan often took over the accusative of Greek words as a nominative. (The accusative is usually less eroded than the nominative.) phic" style would grate as such styles do.

Glossary to the Above Sentence

Min "He", "she", "it", accusative. Generally used enclitically, but it can be used non-enclitically if reflexivity is understood, i.e., "himself", "herself", "itself". <u>Appa</u>: Equals <u>Pappa</u>, "Grandfather" or "ancestor". <u>Tarchon</u>: A commonly known semi-divine or divine figure in the islands of the Southern Aegean, Crete, Cyprus, and the adjoining parts of Anatolia. <u>Teni</u>: A name of Zeus, particularly Cretan, equals th Dative case of Den-Etruscan Tin. (No voiced dental exists in Etruscan.) <u>Alphen</u>: Accusative of alphe - "something fetched" or "produced".^{**} <u>Edoke</u>: First aorist, indicative - "he gave" or "presented freely".

The parallelisms of most of these words and forms are striking. Others less so. For example, <u>apa</u> in all three languages is probably onomatopoeic and very widespread outside our East - West band. <u>Ten</u> is striking for Zeus (not perhaps the "T" from *TZ but certainly the final "n".) In the last word, the verb, it may be that -ke as a sign of the preterite in Etruscan was strengthened and therefore influenced by Greek; although it is almost certain that <u>originally</u> this morpheme arose elsewhere. Considering the geographically central position of Greek between Etruscan and Turkic on our proposed trajectory, the correspondences are more striking than the differences. Or is this Greek semi-equivalent merely a jeu d'esprit? One would rather think that in it way this possible since it showed a more ancient affinity between Indo-European and Altaic, a thesis that has attracted some scholars.

After I wrote the Etruscan vignette given above I recognized that alpan in Etruscan sometimes appears as alpnu with the same distribution. It is exciting to compare this with Turkic (e.g. Old Osman)

"But, if alp(h)en were taken from the Doric dialect of Greek it would appear as alp(h)an.

where a form <u>alpanu</u> (hitherto unanalyzed) substitutes for <u>alpan</u>. This lends credence to the supposition that /u/is in Etruscan a form meaning "his", "hers", "it", used, because of its heightened specification, in lieu of a definite article, as in Turkish. The Etruscan form <u>tularu</u> therefore means the boundaries. since <u>tular</u> is known to mean boundaries; and so forth.

f. Survey of Immediately Possible Applications

The following lists in a very summary fashion the tentative conclusions of a group of linguists concerning the possibility of using some or all of the techniques referred to above for the decipherment and/or translation of the language indicated. The group met, and will continue to meet, regularly in Santa Barbara and Santa Monica. Participants have included Professors H. Hetzron, J. Marvan, N. Shevoroshkin, J. Billigmeir, R. Bellman and J. Wilkinson.

ETRUSCAN: Can be read and partially understood. Prognosis: The use of computers by linguists and mathematicians is promising.

<u>CYPRO-MINOAN:</u> Very limited corpus, partly due to findings not yet having been published. Principal attack would be (1) to prepare concordance of all words accompanied by the words immediately before and after and (2) to do the same for signs. <u>Prognosis</u>: Computers at present would probably be of limited but important use.

<u>GUBLITIC (BYBLOG)</u>. Since there are no word dividers, it would first be necessary to use the computer to help determine morpheme boundaries by sorting out groups of 2, 3, 4, etc., signs that reoccur. These would be presumable morphemes. Since we know the writing-direction of inscriptions (mostly right to left), we could determine suffixes, and prefixes. If the language were Semitic, as seems likely, we could get

at the sounds, since the number of affixes is limited in Gemitic. In the second place the computer could tell us what syllables are CV and which VC. (VC could not be initial in Gemitic.) If we could tell what are CV and VC, a grid could be prepared. <u>Prognosis</u>: A <u>very</u> fruitful area in which to use computer technology.

HEROGLYPHIC HITTITTE: Though in large part deciphered, the values of some of the signs are hotly disputed. It would be a good idea to print out P. Meriggi's lexicon with different values according to different systems. <u>Prognosis</u>: Computer use could be very fruitful.

<u>CRETAN LINEAR-A</u>: Doubtfully a promising field for computers except for the kind of formal and combinatorial work already done by D. Packard. New texte must become available. Some already found not yet published. Publication of these last will probably increase our knowledge somewhat but not dramatically. <u>Leading problems</u>: Nearly all libation tables are severely damaged; so are many tablets. The <u>Hagia Triada</u> tablets contain mainly personal names. A close study using computers <u>might</u> help us to isolate place names, commodity names, etc. This last presupposes, however, that these rather sloppy-looking documents and archives of <u>Hagia Triada</u> possessed a greater degree of logical composition than they likely every did. <u>Prognosis</u>: Low priority for the moment.

LYDIAN AND LYCIAN: Mutatis mutandis the outlook is the same as for Etruscan. They, too, are written in a modified Greek alphabet; and some understanding of them has become available. Strong Indo-European influence. Its historical influence for a considerable time on Etruscan is undoubted.

INDUS VALLEY: The prognosis would seem favorable if and when a better corpus were available. (An Indian friend of R. Bellman, Vasudevan, is gathering this material.)

HITTITE: Itself contains many unsolved problems which the techniques of applied mathematics could presumably elucidate.

MEROITIC would seem at the moment nearly hopeless. However, practice (and some success) in using the techniques referred to above might dramatically alter what is at the moment an unfavorable prognosis.

URARTEAN and HURRIAN: Prospects excellent. Our supposition further is that success in one probably means a reverberation in some of the others. Old Armenian probably contains many words found in Etruscan.

g. Uses of the Computer

We hope to have made plausible that the computer will play a great role in philology.

It is an ideal device for the storage and retrieval of information. The researcher can save a great deal of time by storing a glossary in the computer. With a simple program and a CRT (cathode ray tube), he can very quickly see many desired words or combination of words.

But much more important is this is a quick way of <u>testing hypotheses</u>. With the aid of the stored glossary and the CRT, two languages can easily be compared. Many hypotheses can be tested.

Let us point out that with a CRT it is also possible to look at hieroglyphics. No transliteration will be required.

Communication with the computer at the present is by means of a typewriter which stores simple programs which are written in FORTRAN

and can easily be used all around the world. Also, we can communicate with the computer by means of a light pen. This technique which can be used to construct circuits is an ideal technique for constructing words or hieroglyphs.

In the future, more sophisticated techniques will be available. For example, at the present in various laboratories there are devices for using voice. Soon these will be commercially available.

The application of voice techniques to philology will be very powerful.

h. Uses of Mathematics

Mathematics can be used in many ways in philology and linguistics.

For example, we may want to see how a word changes from one language to another. A theory of branching processes will be valuable here. See

Harris, T. E., The Theory of Branching Processes, Springer-Verlag, New York, 1963.

The theory of invariant imbedding is a transliteration of the comparative method, so powerful in philology, to mathematics.

We often wish to trace the possible connections between one word in one language and another word in another language. Here, we can use the theory of <u>kladistics</u>, see:

Marchi, E., "Generalizations of the Parsimony Question in Evolution," Mathematical Biosciences, Vol. 17, No. 1/2, 1973, pp. 11-34.

(Many other references may be found there.)

The theory of "fuzzy" systems will play a large role in language translation. See:

Bellman, R. and L. Zadeh, "Decision-Making in a Fuzzy Environment," Management Science.Vol. 17, No. 4, pp. 141-164, 1970. D. Epstein, Ed., D. Reidel Publishing Co., Dordrecht, Germany.

We have discussed above the use of "simulation" techniques.

See also:

Wilkinson, J. and R. Bellman, <u>The Dynamic Simulation of Human Systems</u>, MSS Publications, New York, 1974 (ISBN 0-8422-8300-5). Contains a discussion of archetypes in language, dynamic programming and fuzzy sets.

In conclusion it ought to be pointed out that the decipherment and/or translation of the languages named above are a contribution to "philology" in the broad sense, which was, beyond its linguistic content proper, the task of reconstructing human societies.

4. Fuzzy Set Theory in Dynamic Programming and Large Systems

Dynamic programming and its application to multistage decision processes of stochastic and adaptive type is well-known. However, only a beginning has been made to apply dynamic programming to fuzzy systems. "Fuzziness" should not be confused with "randomness." For example, much of the decision-making in a real world takes place in a fuzzy-environment in which the goals, the constraints and the consequences of possible actions are not known precisely. To deal quantitatively with this imprecision, we usually employ the concepts and techniques of probability theory, and, more particularly, the tools provided by decision theory, control theory and information theory. In so doing, we are tacitly accepting the premise that imprecision can be equated with randomness. We are using the combined concepts of dynamic programming and fuzzy set theory to overcome this difficulty. For example, in the conventional approach to decision making, the principal ingredients of a decision process are goals, constraints and decisions. When we view a decision process from the broader perspective of decision making in a fuzzy environment, a different and perhaps a more natural conceptual framework suggests itself. The most important feature of this framework is its symmetry with respect to goals and constraints. For more detailed discussion, see

Bellman, R. E. and Zadeh, L. A., "Decision-Making in a Fuzzy Environment," <u>Management Science</u>, Vol. 17, No. 4, 1970, pp. 141-164.

In order to develop this concept further, actual physical systems are being used. The following is a brief summary of the various systems from the standpoint of applying fuzzy set theory.

(1) Energy Systems

Energy systems form typical examples of ill-defined systems and are well suited to illustrate the fuzzy concept.

The planning of energy resources is intimately connected with many social, legal, governmental, regional and environmental problems. In many of the planning aspects, therefore, the important factors to consider frequently are not engineering or economic factors. For example, we may cite the decision to use synthetic gas rather than coal in electric power generation due to severe air pollution conditions. Since these various social factors generally are not well defined and form fuzzy sets, they cannot be considered easily by classical mathematics. Furthermore, since these various factors generally have conflicting interests, multistage decision modeling in a fuzzy environment will be used. For example, certain decisions arrived at by particular groups of planners such as the energy agency, the public or the envir-

onmental group may have conflicting goals with each other. It is frequently desirable to know the consequences of these decisions. The multistage simulation model in a fuzzy environment would supply these consequences.

In using the fuzzy game simulation model there is an interaction between two major components, the decision makers who are responsible for the decisions in a particular region and the fuzzy mathematical model which represents this particular region. The decision makers represent different interests which include those of the politicians, the energy agency, the industrial or real estate developer and the environmental agency. Since the decision makers have different and even conflicting interests, a decision reached almost always constitutes a compromise. The optimization and simulation of this ill-defined qualitative system is not easy. Eynamic programming combined with the fuzzy concept is being used to treat this problem. The system can be represented by the following typical dynamic programming functional equation

$$\delta_{N}(\mathbf{x}) = \max_{\mathbf{u}} \left[\delta_{N-1}(\mathbf{x}) + \mathbf{f}(\mathbf{x}, \mathbf{u}) \right]$$

where x represents the state vector which is composed of both fuzzy and non-fuzzy states and u represents the alternatives. For the current work, u is assumed to be non-fuzzy.

(2) Water Resources Systems

Decause of the social, legal and environmental aspects of water usage, water resources systems forms another typical ill-defined large system. To illustrate and to further develop the fuzzy multistage decision processes, the optimization and simulation of water resources systems under fuzzy environments are being carried out.

(3) Fluidized Bed as a Fuzzy System

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Even a typical engineering system can be treated as a fuzzy system. This is because of our inability to describe the system precisely. Fluidized beds, which form the most important equipment in coal processing and in coal combustion, constitute one such system. Because of the complexity of the physical processes occurring inside, the fluidized bed is still an art and must be based on information obtained from building a sequence of prototype reactors. This procedure is not only costly and time consuming, but has also led to over-designing.

Fuzzy set theory combined with multistage concepts can be used to define this system more precisely. This approach allows isolated studies of the important aspects. Preliminary simulation results using fuzzy concepts are very encouraging.