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DTNSRDC/SPD-0393-09

DAVID W. TAYLOR NAVAL SHIP RESEARCH AND DEVELOPMENT CENTER

Bethesda, Md. 20084

DTNSRDC REVISED STANDARD SUBMARINE EQUATIONS OF MOTION

by

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SHIP PERFORMANCE DEPARTMENT

June 1979

DTNSRDC/SPD-0393-09

DTNSRDC REVISED STANDARD SUBMARINE
EQUATIONS OF MOTION

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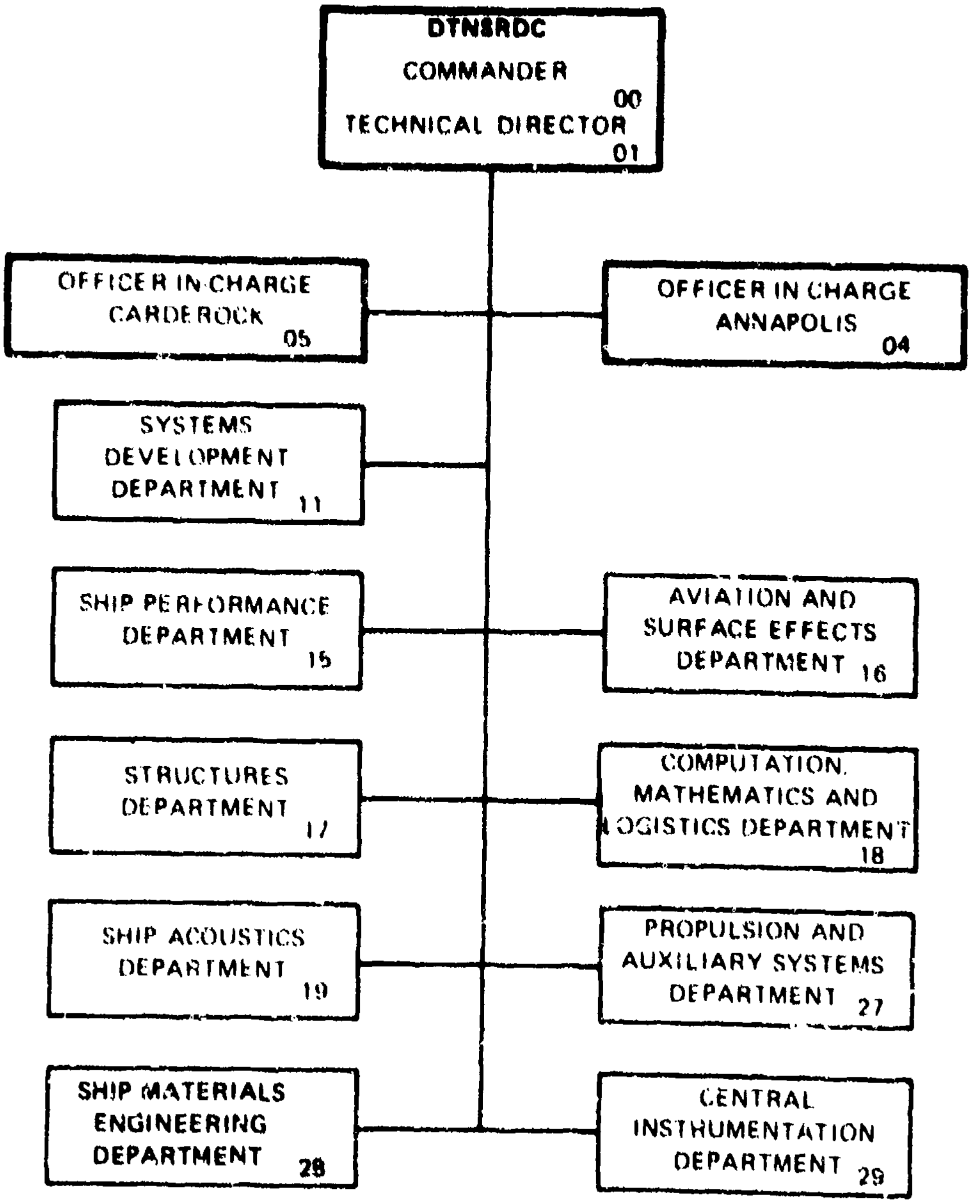

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: 14 DTNSRDC/SPD-0393-09
2. GOVT ACCESSION NO.
3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle): 6 DTNSRDC Revised Standard Submarine Equations of Motion.
5. TYPE OF REPORT & PERIOD COVERED: 9 Final rept.
6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s): 10 J. Feldman
8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS:
David W. Taylor Naval Ship R&D Center
Bethesda, Maryland 20084
10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: 17
PE 63561N
TA S0207001
Work Unit 1564-096
11. CONTROLLING OFFICE NAME AND ADDRESS:
Naval Sea Systems Command (Code 32R)
Washington, DC 20362
12. REPORT DATE: 11 Jun[redacted]79
13. NUMBER OF PAGES: 30
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office): 12 34p
15. SECURITY CLASS. (of this report): UNCLASSIFIED
15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report):

APPROVED FOR PUBLIC RELEASE: DISTRIBUTION UNLIMITED

16 S0207

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)
18. SUPPLEMENTARY NOTES
19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

(U) Submarine Motions
(U) Mathematical Models
(U) Computer Simulation

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Revised standard equations of motion are presented for use in computer simulations of submarine motions in six degrees of freedom conducted for the US Navy.

389 694

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE
S/N 0102-LF-014-6601

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INTRODUCTION

It is highly desirable that the motions of the Navy's modern submarines be predicted in advance of full-scale trials and operations to establish their safe operating envelope and their ability to perform specific maneuvers effectively. To predict these motions and to establish valid control strategies it is necessary both to develop an accurate mathematical model of the submarine and to determine accurate values of the hydrodynamic forces and moments acting on the submarine hull and appendages which are required in the mathematical model.

The David W. Taylor Naval Ship R&D Center (DTNSRDC) provided a standard set of equations of motion for use in submarine computer simulation in Reference 1. These equations have been used to simulate the trajectories and responses of submarines in six degrees of freedom resulting from various types of normal maneuvers as well as for extreme maneuvers such as those associated with emergency recoveries from a sternplane jam.

Reference 2 gives a general description of the effort at the Center to predict, evaluate, and improve the stability, control, and maneuvering characteristics of the Navy's submarines, including modifications and improvements made to the equations of motions given in Reference 1. The improvements to the equations of motion outlined in Reference 2 have resulted in better correlation with full-scale trial data.

This report has been prepared to provide those working in the field of submarine stability, control, and maneuvering with documentation of the current interim mathematical model. This report defines the notation, axes systems, and sign conventions used for the equations and presents the DTNSRDC Revised Standard Submarine Equations of Motion for performing computer simulation.

NOTATION

NOTATION

a_i, b_i, c_i		Sets of constants used in the representation of combined thrust and drag in the axial equation
A_y	$A_y' = \frac{A_y}{\ell^2}$	Projected area of hull plus deck in xz-plane
A_z	$A_z' = \frac{A_z}{\ell^2}$	Projected area of hull in xy-plane
b(x)	$b(x)' = \frac{b(x)}{\ell}$	Local beam of hull in xy-plane $\int_\ell b(x)dx = A_z$
B	$B' = \frac{B}{\frac{1}{2}\rho\ell^2U^2}$	Buoyancy force of envelope displacement, positive upward
C		Variable coefficient used in scaling model thrust and drag data to full-scale. Function of ΔX
C_6, C_7, C_8		Constants used in computing C
CB		Center of buoyancy of submarine
C_d	$C_d = \frac{CFD}{\frac{1}{2}\rho A_z U^2}$	Coefficient used in integrating forces and moments along hull due to local cross-flow
CFD		Cross-flow drag
CG		Center of mass of submarine
$\bar{C}_L$		Modified nondimensional sectional lift-curve slope used in computing the effects of the hull-bound vortex due to lift on the bridge fairwater
h(x)	$h(x)' = \frac{h(x)}{\ell}$	Local height of hull plus deck in xz-plane $\int_\ell h(x)dx = A_y$

I_x	$I_x' = \frac{I_x}{\frac{1}{2}\rho\ell^5}$	Moment of inertia of submarine about x axis
I_y	$I_y' = \frac{I_y}{\frac{1}{2}\rho\ell^5}$	Moment of inertia of submarine about y axis
I_z	$I_z' = \frac{I_z}{\frac{1}{2}\rho\ell^5}$	Moment of inertia of submarine about z axis
I_{xy}	$I_{xy}' = \frac{I_{xy}}{\frac{1}{2}\rho\ell^5}$	Product of inertia with respect to the x and y axes
I_{yz}	$I_{yz}' = \frac{I_{yz}}{\frac{1}{2}\rho\ell^5}$	Product of inertia with respect to the y and z axes
I_{zx}	$I_{zx}' = \frac{I_{zx}}{\frac{1}{2}\rho\ell^5}$	Product of inertia with respect to the z and x axes
K	$K' = \frac{K}{\frac{1}{2}\rho\ell^3 U^2}$	Hydrodynamic moment component about x axis (rolling moment)
K_*	$K_*' = \frac{K_*}{\frac{1}{2}\rho\ell^3}$	Coefficient used in representing K as a function of u^2
K_i	$K_i' = \frac{K_i}{\frac{1}{2}\rho\ell^3}$	Coefficient used in representing K due to interference effects of vortices from the bridge fairwater on the stern control surfaces
K_p	$K_p' = \frac{K_p}{\frac{1}{2}\rho\ell^4}$	Coefficient used in representing K as a function of up. Does not include effects of vortices from the bridge fairwater on the stern control surfaces
$K_{\dot{p}}$	$K_{\dot{p}}' = \frac{K_{\dot{p}}}{\frac{1}{2}\rho\ell^5}$	Coefficient used in representing K as a function of $\dot{p}$
$K_{p\|p\|}$	$K_{p\|p\|}' = \frac{K_{p\|p\|}}{\frac{1}{2}\rho\ell^5}$	Coefficient used in representing K as a function of $p\|p\|$

K_{qr}	$K_{qr}' = \frac{K_{qr}}{\frac{1}{2}\rho\ell^5}$	Coefficient used in representing K as a function of qr
K_r	$K_r' = \frac{K_r}{\frac{1}{2}\rho\ell^4}$	Coefficient used in representing K as a function of ur. Does not include effects of vortices from bridge fairwater on stern control surfaces
$K_{\dot{r}}$	$K_{\dot{r}}' = \frac{K_{\dot{r}}}{\frac{1}{2}\rho\ell^5}$	Coefficient used in representing K as a function of $\dot{r}$
K_{vR}	$K_{vR}' = \frac{K_{vR}}{\frac{1}{2}\rho\ell^3}$	Coefficient used in representing K as a function of uv. Does not include effects of vortices from bridge fairwater on stern control surfaces
$K_{\dot{v}}$	$K_{\dot{v}}' = \frac{K_{\dot{v}}}{\frac{1}{2}\rho\ell^4}$	Coefficient used in representing K as a function of $\dot{v}$
K_{wp}	$K_{wp}' = \frac{K_{wp}}{\frac{1}{2}\rho\ell^4}$	Coefficient used in representing K as a function of wp
$K_{\delta r}$	$K_{\delta r}' = \frac{K_{\delta r}}{\frac{1}{2}\rho\ell^3}$	Coefficient used in representing K as a function of $u^2\delta_r$
$K_{\delta r\eta}$	$K_{\delta r\eta}' = \frac{K_{\delta r\eta}}{\frac{1}{2}\rho\ell^3}$	Coefficient used in representing K as a function of $u^2\delta_r\left(\eta - \frac{1}{C}\right)C$
K_{4S}	$K_{4S}' = \frac{K_{4S}}{\frac{1}{2}\rho\ell^3 U_S{}^2}$	Coefficient used in representing K due to ϕ_S at the stern control surfaces
K_{8S}	$K_{8S}' = \frac{K_{8S}}{\frac{1}{2}\rho\ell^3 U_S{}^2}$	Coefficient used in representing K due to ϕ_S at the stern control surfaces
ℓ	$\ell' = 1$	Overall length of submarine

m	$m' = \frac{m}{\frac{1}{2}\rho\ell^3}$	Mass of submarine, including water in free-flooding spaces
M	$M' = \frac{M}{\frac{1}{2}\rho\ell^3U^2}$	Hydrodynamic moment component about y axis (pitching moment)
M_*	$M_*' = \frac{M_*}{\frac{1}{2}\rho\ell^3}$	Coefficient used in representing M as a function of u^2
M_q	$M_q' = \frac{M_q}{\frac{1}{2}\rho\ell^4}$	Coefficient used in representing M as a function of uq
$M_{\dot{q}}$	$M_{\dot{q}}' = \frac{M_{\dot{q}}}{\frac{1}{2}\rho\ell^5}$	Coefficient used in representing M as a function of $\dot{q}$
M_{rp}	$M_{rp}' = \frac{M_{rp}}{\frac{1}{2}\rho\ell^5}$	Coefficient used in representing M as a function of rp
M_w	$M_w' = \frac{M_w}{\frac{1}{2}\rho\ell^3}$	Coefficient used in representing M as a function of uw
$M_{\dot{w}}$	$M_{\dot{w}}' = \frac{M_{\dot{w}}}{\frac{1}{2}\rho\ell^4}$	Coefficient used in representing M as a function of $\dot{w}$
$M_{\lvert w\rvert}$	$M_{\lvert w\rvert}' = \frac{M_{\lvert w\rvert}}{\frac{1}{2}\rho\ell^3}$	Coefficient used in representing M as a function of $u\lvert w\rvert$
$M_{w\lvert w\rvert R}$	$M_{w\lvert w\rvert R}' = \frac{M_{w\lvert w\rvert R}}{\frac{1}{2}\rho\ell^3}$	Coefficient used in representing M as a function of $w\lvert(v^2 + w^2)^{1/2}\rvert$
M_{ww}	$M_{ww}' = \frac{M_{ww}}{\frac{1}{2}\rho\ell^3}$	Coefficient used in representing M as a function of $\lvert w(v^2 + w^2)^{1/2}\rvert$
$M_{\delta b}$	$M_{\delta b}' = \frac{M_{\delta b}}{\frac{1}{2}\rho\ell^3}$	Coefficient used in representing M as a function of $u^2\delta_b$
$M_{\delta s}$	$M_{\delta s}' = \frac{M_{\delta s}}{\frac{1}{2}\rho\ell^3}$	Coefficient used in representing M as a function of $u^2\delta_s$

N	$N' = \frac{N}{\frac{1}{2}\rho\ell^3U^2}$	Hydrodynamic moment component about z axis (yawing moment)
N_*	$N_*' = \frac{N_*}{\frac{1}{2}\rho\ell^3}$	Coefficient used in representing N as a function of u^2
N_p	$N_p' = \frac{N_p}{\frac{1}{2}\rho\ell^4}$	Coefficient used in representing N as a function of up
$N_{\dot{p}}$	$N_{\dot{p}}' = \frac{N_{\dot{p}}}{\frac{1}{2}\rho\ell^5}$	Coefficient used in representing N as a function of $\dot{p}$
N_{pq}	$N_{pq}' = \frac{N_{pq}}{\frac{1}{2}\rho\ell^5}$	Coefficient used in representing N as a function of pq
N_r	$N_r' = \frac{N_r}{\frac{1}{2}\rho\ell^4}$	Coefficient used in representing N as a function of ur
$N_{\dot{r}}$	$N_{\dot{r}}' = \frac{N_{\dot{r}}}{\frac{1}{2}\rho\ell^5}$	Coefficient used in representing N as a function of $\dot{r}$
N_v	$N_v' = \frac{N_v}{\frac{1}{2}\rho\ell^3}$	Coefficient used in representing N as a function of uv
$N_{\dot{v}}$	$N_{\dot{v}}' = \frac{N_{\dot{v}}}{\frac{1}{2}\rho\ell^4}$	Coefficient used in representing N as a function of $\dot{v}$
$N_{v\|v\|R}$	$N_{v\|v\|R}' = \frac{N_{v\|v\|R}}{\frac{1}{2}\rho\ell^3}$	Coefficient used in representing N as a function of $v\|(v^2+w^2)^{1/2}\|$
$N_{\delta r}$	$N_{\delta r}' = \frac{N_{\delta r}}{\frac{1}{2}\rho\ell^3}$	Coefficient used in representing N as a function of $u^2\delta_r$
$N_{\delta r\eta}$	$N_{\delta r\eta}' = \frac{N_{\delta r\eta}}{\frac{1}{2}\rho\ell^3}$	Coefficient used in representing N as a function of $u^2\delta_r\left(\eta - \frac{1}{C}\right)C$
p	$p' = \frac{p\ell}{U}$	Angular velocity component about x-axis relative to fluid (roll)

$\dot{p}$	$\dot{p}' = \frac{\dot{p}\ell^2}{U^2}$	Angular acceleration component about x-axis relative to fluid
q	$q' = \frac{q\ell}{U}$	Angular velocity component about y-axis relative to fluid (pitch)
$\dot{q}$	$\dot{q}' = \frac{\dot{q}\ell^2}{U^2}$	Angular acceleration component about y-axis relative to fluid
Q_p		Contribution of propeller torque to K and machinery equation
r	$r' = \frac{r\ell}{U}$	Angular velocity component about z-axis relative to fluid (yaw)
$\dot{r}$	$\dot{r}' = \frac{\dot{r}\ell^2}{U^2}$	Angular acceleration component about z-axis relative to fluid
S_1, S_2		Constants used in computing x_2
t	$t' = \frac{tU}{\ell}$	Time
U	$U' = \frac{U}{U}$	Velocity of origin of body axes relative to fluid
U_s	$U_s' = \frac{(u^2+v_s^2+w_s^2)^{1/2}}{U}$	Velocity of sternplane x-coordinate relative to the fluid
u	$u' = \frac{u}{U}$	Component of U in direction of the x-axis
$\dot{u}$	$\dot{u}' = \frac{\dot{u}\ell}{U^2}$	Time rate of change of u in direction of the x-axis
u_c	$u_c' = \frac{u_c}{U}$	Command speed: steady value of ahead speed component u for a given propeller rpm when body angles (α,β) and control surface angles are zero. Sign changes with propeller reversal

v	$v' = \frac{v}{U}$	Component of U in direction of the y-axis
$\dot{v}$	$\dot{v}' = \frac{\dot{v}\ell}{U^2}$	Time rate of change of v in direction of the y-axis
v_s	$v_s' = \frac{v_s}{U}$	Velocity component in the y-axis direction at the quarter chord of the sternplanes. $v_s = v + x_s r$
$\bar{v}_{FW}$	$\bar{v}_{FW}' = \frac{\bar{v}_{FW}}{U}$	Velocity component in the y-axis direction at the starting position of the hull-bound vortex due to lift on the bridge fairwater. $\bar{v}_{FW} = v + x_1 r - z_{FW} p$
v_{FW}	$v_{FW}' = \frac{v_{FW}}{U}$	Velocity component in the y-axis direction at the bridge fairwater. $v_{FW} = v + x_{FW} r - z_{FW} p$
$\bar{v}_{FW}(t - \tau[x])$		Value of $\bar{v}_{FW}$ at time $= t - \tau(x)$
$v_{FW}(t - \tau_T)$		Value of v_{FW} at time $= t - \tau_T$
$v(x)$	$v(x)' = \frac{v(x)}{U}$	Velocity component in the y-axis direction of any x coordinate $v(x) = v + xr$
w	$w' = \frac{w}{U}$	Component of U in the direction of the z-axis
$\dot{w}$	$\dot{w}' = \frac{\dot{w}\ell}{U^2}$	Time rate of change of w in the direction of the z-axis
w_s	$w_s' = \frac{w_s}{U}$	Velocity component in the z-axis direction at the quarter chord of the sternplanes $w_s = w - x_s q$
$w(x)$	$w(x)' = \frac{w(x)}{U}$	Velocity component in the z-axis direction of any x coordinate $w(x) = w - xq$

W	$W' = \frac{W}{\frac{1}{2}\rho\ell^2U^2}$	Weight of submarine, including water in free flooding spaces
x	$x' = \frac{x}{\ell}$	Longitudinal body axis; also the coordinate of a point relative to the origin of the body axis
x_1	$x_1' = \frac{x_1}{\ell}$	The x coordinate of the starting position of the hull-bound vortex due to lift on bridge fairwater
x_2	$x_2' = \frac{x_2}{\ell}$	The x coordinate of the aft-most position of the hull-bound vortex due to lift on bridge fairwater
x_B	$x_B' = \frac{x_B}{\ell}$	The x coordinate of the CB
x_G	$x_G' = \frac{x_G}{\ell}$	The x coordinate of the CG
x_o	$x_o' = \frac{x_o}{\ell}$	A coordinate of the displacement of the origin of the body axis relative to the origin of a set of fixed axes
x_R	$x_R' = \frac{x_R}{\ell}$	The x coordinate of the quarter chord of the rudders
x_S	$x_S' = \frac{x_S}{\ell}$	The x coordinate of the quarter chord of the sternplanes
x_T	$x_T' = \frac{x_T}{\ell}$	The x coordinate of the average location of the sternplanes and rudders. $x_T = \frac{1}{2}(x_S + x_R)$
x_{AP}	$x_{AP}' = \frac{x_{AP}}{\ell}$	The x coordinate of the after perpendicular
x_{FW}	$x_{FW}' = \frac{x_{FW}}{\ell}$	The x coordinate of the quarter chord of the bridge fairwater

X	$X' = \frac{X}{\frac{1}{2}\rho\ell^2U^2}$	Hydrodynamic force component along x-axis (longitudinal, or axial, force)
ΔX		Variable coefficient used in scaling model thrust and drag data to full scale
ΔX_1, ΔX_2, ΔX_3		Constants used in computing ΔX
X_{qq}	$X_{qq}' = \frac{X_{qq}}{\frac{1}{2}\rho\ell^4}$	Coefficient used in representing X as a function of q^2.
X_{rp}	$X_{rp}' = \frac{X_{rp}}{\frac{1}{2}\rho\ell^4}$	Coefficient used in representing X as a function of rp
X_{rr}	$X_{rr}' = \frac{X_{rr}}{\frac{1}{2}\rho\ell^4}$	Coefficient used in representing X as a function of r^2
$X_{\dot{u}}$	$X_{\dot{u}}' = \frac{X_{\dot{u}}}{\frac{1}{2}\rho\ell^3}$	Coefficient used in representing X as a function of $\dot{u}$
X_{vr}	$X_{vr}' = \frac{X_{vr}}{\frac{1}{2}\rho\ell^3}$	Coefficient used in representing X as a function of vr
X_{vv}	$X_{vv}' = \frac{X_{vv}}{\frac{1}{2}\rho\ell^2}$	Coefficient used in representing X as a function of v^2
X_{wq}	$X_{wq}' = \frac{X_{wq}}{\frac{1}{2}\rho\ell^3}$	Coefficient used in representing X as a function of wq
X_{ww}	$X_{ww}' = \frac{X_{ww}}{\frac{1}{2}\rho\ell^2}$	Coefficient used in representing X as a function of w^2
$X_{\delta b\delta b}$	$X_{\delta b\delta b}' = \frac{X_{\delta b\delta b}}{\frac{1}{2}\rho\ell^2}$	Coefficient used in representing X as a function of $u^2\delta_b^2$

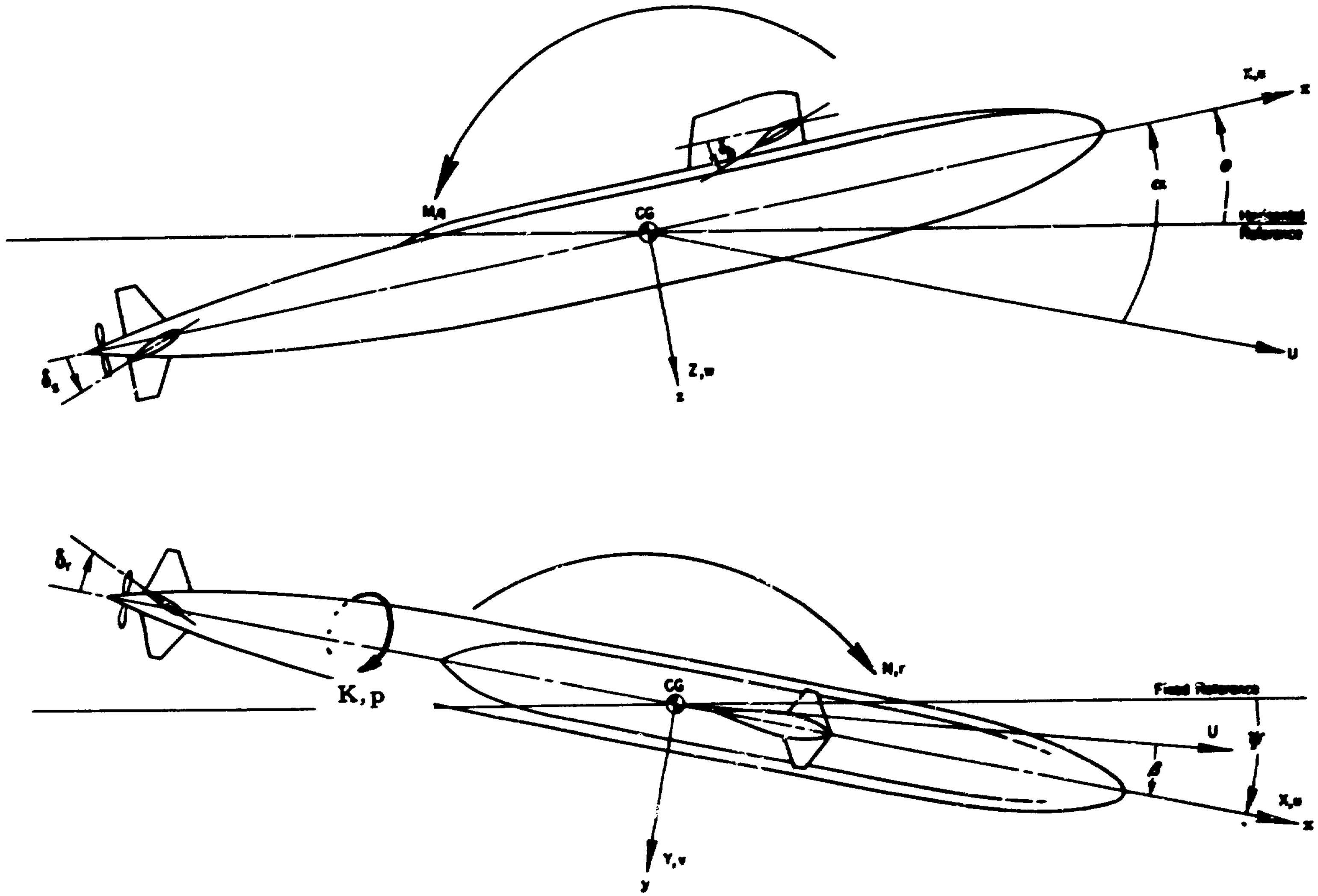
$X_{\delta r\delta r}$	$X_{\delta r\delta r}' = \frac{X_{\delta r\delta r}}{\frac{1}{2}\rho\ell^2}$	Coefficient used in representing X as a function of $u^2\delta_r^{\ 2}$
$X_{\delta s\delta s}$	$X_{\delta s\delta s}' = \frac{X_{\delta s\delta s}}{\frac{1}{2}\rho\ell^2}$	Coefficient used in representing X as a function of $u^2\delta_s^{\ 2}$
y'	$y' = \frac{y}{\ell}$	Lateral body axis; also the coordinate of a point relative to the origin of body axes
y_B	$y_B' = \frac{y_B}{\ell}$	The y coordinate of CB
y_G	$y_G' = \frac{y_G}{\ell}$	The y coordinate of CG
y_o	$y_o' = \frac{y_o}{\ell}$	A coordinate of the displacement of the origin of the body axis relative to the origin of a set of fixed axes
Y	$Y' = \frac{Y}{\frac{1}{2}\rho\ell^2U^2}$	Hydrodynamic force component along y axis (lateral force)
Y_*	$Y_*' = \frac{Y}{\frac{1}{2}\rho\ell^2}$	Coefficient used in representing Y as a function of u^2
Y_p	$Y_p' = \frac{Y_p}{\frac{1}{2}\rho\ell^3}$	Coefficient used in representing Y as a function of up
$Y_{\dot{p}}$	$Y_{\dot{p}}' = \frac{Y_{\dot{p}}}{\frac{1}{2}\rho\ell^4}$	Coefficient used in representing Y as a function of $\dot{p}$
$Y_{p\|p\|}$	$Y_{p\|p\|}' = \frac{Y_{p\|p\|}}{\frac{1}{2}\rho\ell^4}$	Coefficient used in representing Y as a function of $p\|p\|$
Y_{pq}	$Y_{pq}' = \frac{Y_{pq}}{\frac{1}{2}\rho\ell^4}$	Coefficient used in representing Y as a function of pq

Y_r	$Y_r' = \frac{Y_r}{\frac{1}{2}\rho\ell^3}$	Coefficient used in representing Y as a function of ur
$Y_{\dot{r}}$	$Y_{\dot{r}}' = \frac{Y_{\dot{r}}}{\frac{1}{2}\rho\ell^4}$	Coefficient used in representing Y as a function of $\dot{r}$
Y_v	$Y_v' = \frac{Y_v}{\frac{1}{2}\rho\ell^2}$	Coefficient used in representing Y as a function of uv
$Y_{\dot{v}}$	$Y_{\dot{v}}' = \frac{Y_{\dot{v}}}{\frac{1}{2}\rho\ell^3}$	Coefficient used in representing Y as a function of $\dot{v}$
$Y_{v\|v\|R}$	$Y_{v\|v\|R}' = \frac{Y_{v\|v\|R}}{\frac{1}{2}\rho\ell^2}$	Coefficient used in representing Y as a function of $v\|(v^2 + w^2)^{1/2}\|$
Y_{wp}	$Y_{wp}' = \frac{Y_{wp}}{\frac{1}{2}\rho\ell^3}$	Coefficient used in representing Y as a function of wp
$Y_{\delta r}$	$Y_{\delta r}' = \frac{Y_{\delta r}}{\frac{1}{2}\rho\ell^2}$	Coefficient used in representing Y as a function of $u^2\delta_r$
$Y_{\delta r\eta}$	$Y_{\delta r\eta}' = \frac{Y_{\delta r\eta}}{\frac{1}{2}\rho\ell^2}$	Coefficient used in representing Y as a function of $u^2\delta_r\left(\eta - \frac{1}{C}\right)C$
z	$z' = \frac{z}{\ell}$	Normal body axis; also the coordinate of a point relative to the origin of the body axis
z_1	$z_1' = \frac{z_1}{\ell}$	The z coordinate of the hull centerline
z_B	$z_B' = \frac{z_B}{\ell}$	The z coordinate of the CB
z_G	$z_G' = \frac{z_G}{\ell}$	The z coordinate of the CG

z_o	$z_o' = \frac{z_o}{\ell}$	A coordinate of the displacement of the origin of the body axis relative to the origin of a set of fixed axes
z_{FW}	$z_{FW}' = \frac{z_{FW}}{\ell}$	The z coordinate of the 42-percent span of the bridge fairwater
Z	$Z' = \frac{Z}{\frac{1}{2}\rho\ell^2U^2}$	Hydrodynamic force component along z-axis (normal force)
Z_*	$Z_*' = \frac{Z_*}{\frac{1}{2}\rho\ell^2}$	Coefficient used in representing Z as a function of u^2
Z_q	$Z_q' = \frac{Z_q}{\frac{1}{2}\rho\ell^3}$	Coefficient used in representing Z as a function of uq
$Z_{\dot{q}}$	$Z_{\dot{q}}' = \frac{Z_{\dot{q}}}{\frac{1}{2}\rho\ell^4}$	Coefficient used in representing Z as a function of $\dot{q}$
Z_{vp}	$Z_{vp}' = \frac{Z_{vp}}{\frac{1}{2}\rho\ell^3}$	Coefficient used in representing Z as a function of vp
Z_w	$Z_w' = \frac{Z_w}{\frac{1}{2}\rho\ell^2}$	Coefficient used in representing Z as a function of uw
$Z_{\dot{w}}$	$Z_{\dot{w}}' = \frac{Z_{\dot{w}}}{\frac{1}{2}\rho\ell^3}$	Coefficient used in representing Z as a function of $\dot{w}$
$Z_{\lvert w\rvert}$	$Z_{\lvert w\rvert}' = \frac{Z_{\lvert w\rvert}}{\frac{1}{2}\rho\ell^2}$	Coefficient used in representing Z as a function of $u\lvert w\rvert$
Z_{ww}	$Z_{ww}' = \frac{Z_{ww}}{\frac{1}{2}\rho\ell^2}$	Coefficient used in representing Z as a function of $\lvert w(v^2 + w^2)^{1/2}\rvert$
$Z_{\delta b}$	$Z_{\delta b}' = \frac{Z_{\delta b}}{\frac{1}{2}\rho\ell^2}$	Coefficient used in representing Z as a function of $u^2\delta_b$

$Z_{\delta s}$	$Z_{\delta s}' = \frac{Z_{\delta s}}{\frac{1}{2}\rho\ell^2}$	Coefficient used in representing Z as a function of $u^2\delta_s$
$Z_{\delta s\eta}$	$Z_{\delta s\eta}' = \frac{Z_{\delta s\eta}}{\frac{1}{2}\rho\ell^2}$	Coefficient used in representing Z as a function of $u^2\delta_s\left(\eta - \frac{1}{C}\right)C$
α		Angle of attack
β		Angle of drift
β_s		Geometric in-flow angle at the sternplanes $\beta_s = \tan^{-1}\frac{(v_s^2 + w_s^2)^{1/2}}{u}$
β_{ST}		Value of β at which hull-bound vortex separates from the hull given in radians
δ_b		Deflection of bowplane or sailplane
δ_r		Deflection of rudder
δ_s		Deflection of sternplane
η		The ratio $\frac{u_c}{u}$
θ		Angle of pitch
ψ		Angle of yaw
ϕ		Angle of roll
ϕ_s		Hydrodynamic roll angle at the sternplanes $\phi_s = -\tan^{-1}\frac{w_s}{v_s}$
τ_T		Time interval required for vortex to travel from x_{FW} to x_T. It is implicitly defined as: $\int_{t-\tau_T}^{t} u(t)\,dt = x_{FW} - x_T$

$\tau(x)$	Time interval required for vortex to travel from x_1 to any x coordinate aft of x_1. It is implicitly defined as: $\int_{t-\tau(x)}^{t} u(t)dt = x_1 - x$
ρ	Mass density of water



Sketch Showing Positive Directions of Axes, Angles, Velocities, Forces, and Moments

DTNSRDC REVISED STANDARD SUBMARINE EQUATIONS OF MOTION

AXIAL FORCE EQUATION

$$m\left[\dot{u} - vr + wq - x_G(q^2 + r^2) + y_G(pq - \dot{r}) + z_G(pr + \dot{q})\right] =$$

$$+ \frac{\rho}{2} \ell^4 \left[X_{qq}{}' \, q^2 + X_{rr}{}' \, r^2 + X_{rp}{}' \, rp\right]$$

$$+ \frac{\rho}{2} \ell^3 \left[X_{\dot{u}}{}' \, \dot{u} + X_{vr}{}' \, vr + X_{wq}{}' \, wq\right]$$

$$+ \frac{\rho}{2} \ell^2 \left[X_{vv}{}' \, v^2 + X_{ww}{}' \, w^2\right]$$

$$+ \frac{\rho}{2} \ell^2 \left[X_{\delta r\delta r}{}' \, u^2\delta_r{}^2 + X_{\delta s\delta s}{}' \, u^2\delta_s{}^2 + X_{\delta b\delta b}{}' \, u^2\delta_b{}^2\right]$$

$$-(W-B)\sin\theta + F_{xp}$$

$$F_{xp} = \begin{cases} T_p - \text{DRAG} \\ \frac{\rho}{2}\ell^2 \quad \left[(a_i + \Delta X)\, u^2 + b_i \, Cuu_c + c_i \, C^2 u_c{}^2\right] \end{cases}$$

$$\text{where} \quad \Delta X = \Delta X_1 + \frac{\Delta X_2}{(\Delta X_3 + \log_{10} u)^2}$$

$$C = C_6 + (C_7 + C_8 \, \Delta X)^{1/2}$$

Note: F_{xp} is represented by T_p - DRAG when propulsion characteristics are available. Otherwise the second expression for F_{xp} is used.

LATERAL FORCE EQUATION

$$m\left[\dot{v} - wp + ur - y_G(r^2 + p^2) + z_G(qr - \dot{p}) + x_G(qp + \dot{r})\right] =$$

$$+ \frac{\rho}{2}\ell^4\left[Y_{\dot{r}}'\,\dot{r} + Y_{\dot{p}}'\,\dot{p} + Y_{p|p|}'\,p|p| + Y_{pq}'\,pq\right]$$

$$+ \frac{\rho}{2}\ell^3\left[Y_r'\,ur + Y_p'\,up + Y_{\dot{v}}'\,\dot{v} + Y_{wp}'\,wp\right]$$

$$+ \frac{\rho}{2}\ell^2\left[Y_*'\,u^2 + Y_v'\,uv + Y_{v|v|R}'\,v|(v^2 + w^2)^{1/2}|\right]$$

$$+ \frac{\rho}{2}\ell^2\left[Y_{\delta r}'\,u^2\delta_r + Y_{\delta r\eta}'\,u^2\delta_r\left(\eta - \frac{1}{c}\right)c\right]$$

$$- \frac{\rho}{2}C_d\int_\ell h(x)\;v(x)\;\left\{[w(x)]^2 + [v(x)]^2\right\}^{1/2}dx$$

$$- \frac{\rho}{2}\ell\bar{C}_L\int_{x_2}^{x_1} w(x)\;\bar{v}_{FW}(t - \tau[x])\;dx$$

$$+ (W - B)\cos\theta\,\sin\phi$$

NORMAL FORCE EQUATION

$$m\left[\dot{w} - uq + vp - z_G(p^2 + q^2) + x_G(rp - \dot{q}) + y_G(rq + \dot{p})\right] =$$

$$+ \frac{\rho}{2} \ell^4 Z_{\dot{q}}' \, \dot{q}$$

$$+ \frac{\rho}{2} \ell^3 \left[Z_{\dot{w}}' \, \dot{w} + Z_q' \, uq + Z_{vp}' \, vp\right]$$

$$+ \frac{\rho}{2} \ell^2 \left[Z_*' \, u^2 + Z_w' \, uw\right]$$

$$+ \frac{\rho}{2} \ell^2 \left[Z_{|w|}' \, u|v| + Z_{ww}' \, \left|w(v^2 + w^2)^{1/2}\right|\right]$$

$$+ \frac{\rho}{2} \ell^2 \left[Z_{\delta s}' \, u^2\delta_s + Z_{\delta b}' \, u^2\delta_b + Z_{\delta s\eta}' \, u^2\delta_s \left(\eta - \frac{1}{C}\right) C\right]$$

$$- \frac{\rho}{2} C_d \int_\ell b(x) \, w(x) \left\{[w(x)]^2 + [v(x)]^2\right\}^{1/2} dx$$

$$+ \frac{\rho}{2} \ell \bar{C}_L \int_{x_2}^{x_1} v(x) \, \bar{v}_{FW}(t - \tau[x]) \, dx$$

$$+ (W - B) \cos\Theta \cos\phi$$

ROLLING MOMENT EQUATION

$$
\begin{aligned}
& I_x \dot{p} + (I_z - I_y)qr - (\dot{r} + pq)I_{zx} + (r^2 - q^2)I_{yz} + (pr - \dot{q})I_{xy} \\
& + m\left[y_G(\dot{w} - uq + vp) - z_G(\dot{v} - wp + ur)\right] = \\
& + \frac{\rho}{2}\ell^5\left[K_{\dot{p}}'\,\dot{p} + K_{\dot{r}}'\,\dot{r} + K_{qr}'\,qr + K_{p|p|}'\,p|p|\right] \\
& + \frac{\rho}{2}\ell^4\left[K_p'\,up + K_r'\,ur + K_{\dot{v}}'\,\dot{v} + K_{wp}'\,wp\right] \\
& + \frac{\rho}{2}\ell^3\left[K_*'\,u^2 + K_{vR}'\,uv + K_i'\,uv_{FW}(t - \tau_T)\right] \\
& + \frac{\rho}{2}\ell^3\left[K_{\delta r}'\,u^2\delta_r + K_{\delta r\eta}'\,u^2\delta_r\left(\eta - \frac{1}{C}\right)C\right] \\
& + \frac{\rho}{2}\ell^3(u^2 + v_S{}^2 + w_S{}^2)\,\beta_S{}^2\left[K_{4S}'\sin 4\phi_S + K_{8S}'\sin 8\phi_S\right] \\
& + \frac{\rho}{2}\ell^2 z_1'\,\overline{C}_L\int_{x_2}^{x_1} w(x)\,\overline{v}_{FW}(t - \tau[x])\,dx \\
& + (y_G W - y_B B)\cos\theta\cos\phi - (z_G W - z_B B)\cos\theta\sin\phi \\
& - Q_p
\end{aligned}
$$

PITCHING MOMENT EQUATION

$$I_y \, \dot{q} + (I_x - I_z)rp - (\dot{p} + qr)I_{xy} + (p^2 - r^2)I_{zx} + (qp - \dot{r})I_{yz}$$

$$+ m \left[z_G(\dot{u} - vr + wq) - x_G(\dot{w} - uq + vp) \right] =$$

$$+ \frac{\rho}{2} \ell^5 \left[M_{\dot{q}}' \, \dot{q} + M_{rp}' \, rp \right]$$

$$+ \frac{\rho}{2} \ell^4 \left[M_{\dot{w}}' \, \dot{w} + M_q' \, uq \right]$$

$$+ \frac{\rho}{2} \ell^3 \left[M_*' \, u^2 + M_w' \, uw + M_{w|w|R}' \, w \left| (v^2 + w^2)^{1/2} \right| \right]$$

$$+ \frac{\rho}{2} \ell^3 \left[M_{|w|}' \, u|w| + M_{ww}' \, \left| w \, (v^2 + w^2)^{1/2} \right| \right]$$

$$+ \frac{\rho}{2} \ell^3 \left[M_{\delta s}' \, u^2\delta_s + M_{\delta b}' \, u^2\delta_b + M_{\delta s\eta}' \, u^2\delta_s\left(\eta - \frac{1}{C}\right) C \right]$$

$$+ \frac{\rho}{2} C_d \int_{\ell} x \, b(x) \, w(x) \left\{ [w(x)]^2 + [v(x)]^2 \right\}^{1/2} dx$$

$$- \frac{\rho}{2} \ell \bar{C}_L \int_{x_2}^{x_1} x \, v(x) \, \bar{v}_{FW}(t - \tau[x]) \, dx$$

$$- (x_G \, W - x_B \, B) \cos\Theta \cos\phi - (z_G \, W - z_B \, B) \sin\Theta$$

YAWING MOMENT EQUATION

$$I_z \dot{r} + (I_y - I_x)pq - (\dot{q} + rp)I_{yz} + (q^2 - p^2)I_{xy} + (rq - \dot{p})I_{zx}$$

$$+ m\left[x_G(\dot{v} - wp + ur) - y_G(\dot{u} - vr + wq)\right] =$$

$$+ \frac{\rho}{2} \ell^5 \left[N_{\dot{r}}' \dot{r} + N_{\dot{p}}' \dot{p} + N_{pq}' pq\right]$$

$$+ \frac{\rho}{2} \ell^4 \left[N_p' up + N_r' ur + N_{\dot{v}}' \dot{v}\right]$$

$$+ \frac{\rho}{2} \ell^3 \left[N_*' u^2 + N_v' uv + N_{v|v|R}' v\left|(v^2 + w^2)^{1/2}\right|\right]$$

$$+ \frac{\rho}{2} \ell^3 \left[N_{\delta r}' u^2\delta_r + N_{\delta r\eta}' u^2\delta_r\left(\eta - \frac{1}{C}\right) C\right]$$

$$- \frac{\rho}{2} C_d \int_{\ell} x\, h(x)\, v(x) \left\{[w(x)]^2 + [v(x)]^2\right\}^{1/2} dx$$

$$- \frac{\rho}{2} \ell \bar{C}_L \int_{x_2}^{x_1} x\, w(x)\, \bar{v}_{FW}(t - \tau[x])\, dx$$

$$+ (x_G W - x_B B) \cos\Theta \sin\phi + (y_G W - y_B B) \sin\Theta$$

AUXILIARY EQUATIONS

$$\dot{\phi} = p + \dot{\psi} \sin\theta$$

$$\dot{\theta} = q \cos\phi - r \sin\phi$$

$$\dot{\psi} = (r \cos\phi + q \sin\phi)/\cos\theta$$

$$\dot{x}_o = u \cos\theta \cos\psi + v (\sin\phi \sin\theta \cos\psi - \cos\phi \sin\psi) + w (\sin\phi \sin\psi + \cos\phi \sin\theta \cos\psi)$$

$$\dot{y}_o = u \cos\theta \sin\psi + v (\cos\phi \cos\psi + \sin\phi \sin\theta \sin\psi) + w (\cos\phi \sin\theta \sin\psi - \sin\phi \cos\psi)$$

$$\dot{z}_o = - u \sin\theta + v \cos\theta \sin\phi + w \cos\theta \cos\phi$$

$$U = (u^2 + v^2 + w^2)^{1/2}$$

$$x_2 = \begin{cases} x_{AP} \text{ for } |\beta| \leq \beta_{ST} \\ x_1 - (x_1 - x_{AP}) (S_1 + S_2|\beta|) \text{ for } |\beta| > \beta_{ST} \end{cases}$$

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