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NONLINEAR WAVES, SOLITONS, AND INVERSE SPECTRAL THEORY.(U)

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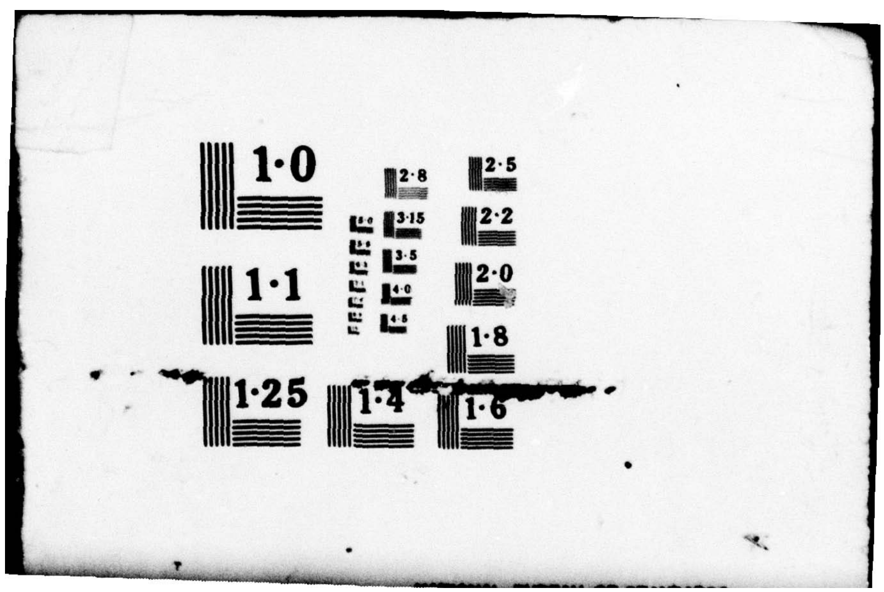
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REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER (19) 15288.4-M	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) (6) Nonlinear Waves, Solitons, and Inverse Spectral Theory		5. TYPE OF REPORT & PERIOD COVERED (9) Final Report. 1 Feb 78 – 31 Jan 79
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) (10) H. Flaschka, D. W. McLaughlin (12) 12p.		8. CONTRACT OR GRANT NUMBER(s) (15) DAAG29-78-G-0059
9. PERFORMING ORGANIZATION NAME AND ADDRESS University of Arizona, Tucson, Arizona 85721		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS U. S. Army Research Office, P. O. Box 12211, Research Triangle Park, NC 27709 (11)		12. REPORT DATE June 1979
		13. NUMBER OF PAGES 11
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

LEVEL

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

The view, opinions, and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

- nonlinear pulses
- lasers
- periodic sine-Gordon equation
- soliton annihilation
- soliton creation
- nonlinear wave trains
- deformation theory

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report summarizes the results of investigations in the following general areas: Nonlinear Pulses, Nonlinear Wave Trains, Dispersive Smoothing of Shocks, Connections between Spectral Preserving Deformations and Monodromy Preserving Deformations.

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Nonlinear Waves, Solitons, and Inverse Spectral Theory

Final Report

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D. W. McLaughlin

June 7, 1979

U. S. Army Research Office

Grant Number - DAAG29-78-G-0059

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LIST OF MANUSCRIPTS SUBMITTED OR PUBLISHED UNDER ARO SPONSORSHIP DURING THIS PERIOD, INCLUDING JOURNAL REFERENCES:

1. "Fluctuations in Nonlinear Swept Gain Amplifiers" F. A. Hopf and E. A. Overman , Phys. Rev. A. 19, 1180-1191 (1979).

2. "Concrete Periodic Inverse Spectral Transform," W. E. Ferguson, Jr., H. Flaschka, and D. W. McLaughlin, Proc. Kyoto Conference on Solitons, ed. M. Toda, Pg. 171-230 (1978).

3. "Multiphase Averaging and the Inverse Spectral Solution of K.dV." H. Flaschka, G. Forest, and D. W. McLaughlin, submitted to Comm. Pure. Appl. Math.

4. "Canonical Variables for Periodic sine-Gordon Equation and a Method of Averaging," G. Forest & D. W. McLaughlin, about to be submitted to J. Math. Phy.

5. "Monodromy and Spectral Preserving Deformations, I", H. Flaschka and A. C. Newell, about to be submitted.

6. "A Commutator Representation of Painleve Equations", H. Flaschka, submitted.

SCIENTIFIC PERSONNEL SUPPORTED BY THIS PROJECT AND DEGREES AWARDED DURING THIS REPORTING PERIOD:

H. Flaschka (5-28-78/6-20-78)
D. W. McLaughlin (5-22-78/6-20-78)
E. A. Overman (02-01-78/09-30-78)
G. Forest (09-01-78/01-31-79)

E. A. Overman - All requirements for Ph.D. completed - August 1978.
G. Forest - Requirements for Ph.D. expected to be completed at thesis defense - August 1979.

FLUCTUATIONS IN NONLINEAR SWEPT-GAIN AMPLIFIERS

by

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Abstract

We investigate, theoretically, fluctuations in coherent steady-state pulses in a nonlinear swept-gain amplifier. In the super-radiant steady-state, we predict large energy and power fluctuations which are found to be largely independent of the magnitude of the fluctuating force.

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†Work supported by NSF Contract Number GP 40370

*Supported in part by Army Research Office Grant DAAG29-78-G-0059. This paper is based on a thesis submitted by E. A. Overman, II, to The University of Arizona in partial fulfillment of the requirements for the Ph.D. degree.

Thesis on:

MACROSCOPIC FLUCTUATIONS IN SWEPT GAIN LASER PULSES*

by

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ABSTRACT. Fluctuations in coherent steady-state pulses are investigated, theoretically, in the superradiant regime of a swept-gain laser amplifier. Spontaneous emission is modelled by letting the initial polarization be a complex stochastic variable. Energy fluctuations are found in the steady-state region. These fluctuations differ from peak power fluctuations in a laser oscillator in four ways:

(1) these energy fluctuations are macroscopic ($\sim$10% of the classical value);

(2) they are unidirectional in that energies lower than the semi-classical value are always involved;

(3) they are episodic in character and clearly different from "random walk" processes about a classical mean;

(4) they are largely independent of the magnitude of the initial polarization.

It is shown, analytically as well as numerically, that these energy fluctuations arise from fluctuations in the phase of the initial polarization. An energy distribution is also derived, which is predicted to be observed in the output of this laser. It is also shown that these fluctuations are fundamentally different from the macroscopic fluctuations in Dicke superradiance, which have been reported previously by Bonifacio, Schwendimann, and Haake and also by Degiorgio.

*Supported in part by A.R.O. Grant DAAG29-78-G-0059.

CONCRETE PERIODIC INVERSE SPECTRAL TRANSFORM

by

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ABSTRACT. The periodic inverse spectral transform (IST) is described in terms of the inverse scattering transform. This description is used to predict the qualitative behavior of the Toda lattice under periodic boundary conditions. Soliton and radiation components are identified. A computer experiment of Zabusky is described analytically using IST to identify soliton and group velocities. The behavior of the lattice with one light mass imperfection is described with IST. Finally, the theory is extended to a discrete nonlinear Schrödinger equation.

*Supported in part by N.S.F. Grant MPS75-07530, A.R.O. Grant DAAG29-78-G-0059, and N.S.F. Exchange Travel Grant INT76-21516. This paper is an extended version of two lectures, one by H. Flaschka and one by D. W. McLaughlin, which were presented at a Conference on Solitons held at Kyoto University in January, 1978. Support by the N.S.F. of the United States and by the government of Japan is gratefully acknowledged.

MULTIPHASE AVERAGING AND THE INVERSE SPECTRAL SOLUTION OF K.dV.

by

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ABSTRACT

Inverse spectral theory is used to prescribe and study equations for the slow modulations of N-phase wave trains for the Korteweg de Vries (K.dV.) equation . An invariant representation of the modulational equations is deduced. This representation depends upon certain differentials on a Riemann surface. When evaluated near ∞ on the surface, the invariant representation reduces to averaged conservation laws; when evaluated near the branch points, the representation shows that the simple eigenvalues provide Riemann invariants for the modulational equations; integrals of the invariant representation over certain cycles on the Riemann surface yield "conservation of wave action." Explicit formulas for the characteristic speeds of the modulational equations are derived. These results generalize known results for a single phase traveling wave, and indicate that complete integrability can induce enough structure into the modulational equations to diagonalize (in the sense of Riemann invariants) their first-order terms.

*Supported in part by N.S.F. under contract No. MPS75-07530 and in part by U. S. Army under contract No. DAAG29-78-G-0059.

*†Supported in part by U. S. Department of Energy during academic year 1978-79 as a consultant to T-Division, Los Alamos Scientific Laboratory.

CANONICAL VARIABLES FOR PERIODIC SINE-GORDON EQUATION
AND A METHOD OF AVERAGING

by

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ABSTRACT

Canonical variables and action-angle variables are computed for the periodic sine-Gordon equation. These variables are used to prescribe a method of averaging for N-phase wave trains which is compared with a method of Whitham. In particular the simple periodic and anti-periodic spectra of the Takhatajian-Faddeev eigenvalue problem are shown to provide Riemann invariants for the single-phase modulational equations.

*Supported in part by N.S.F. under contract No. MPS75-07530 and in part by U. S. Army under contract No. DAAG29-78-G-0059.

*†Supported in part by U. S. Department of Energy during academic year 1978-79 as a consultant to T-Division, Los Alamos Scientific Laboratory.

Monodromy and Spectrum Preserving Deformations, I.

by

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Abstract

A method for solving certain nonlinear ordinary and partial differential equations is developed. The central idea is to study monodromy preserving deformations of linear ordinary differential equations with regular and irregular singular points. The connections with isospectral deformations and with classical and recent work on monodromy preserving deformations are discussed. Specific new results include the reduction of the general initial value problem for the second Painleve equation and a special case of the third type Painlevé equation to a system of linear singular integral equations. Several classes of solutions are discussed, and in particular the general expression for rational solutions for the second Painlevé equation family is shown to be $-d/dx \ln(\Delta+/\Delta-)$, where $\Delta+$ and $\Delta-$ are determinants. The basic ideas presented here are applicable to a broad class of ordinary and partial differential equations; additional results will be presented in a sequence of future papers.

Body of the Report

During the time period of this grant (02-01-78/01-31-79) we have investigated topics in the following general areas: Nonlinear Pulses, Nonlinear Wave Trains, Dispersive Smoothing of Shocks, Connections between Spectral Preserving Deformations and Monodromy Preserving Deformations. Our results are described below and in the enclosed abstracts. It should be remarked that we believe the most significant work is described in II.A and III.

I. Nonlinear Pulses.

I.A. Steady State Pulses of a Swept Gain Laser (Overman, McLaughlin,(Hopf)). Hopf, in his numerical simulations of a swept gain amplifier, observed rather large fluctuations in the steady state pulses. Both the origin and unusual properties of these fluctuations were unclear from his simulations. Overman, in his thesis, showed that these fluctuations originate from the phase of the stochastic initial polarization. Their unusual properties are now understood, and can be traced to the power balance (nonlinear diffusion) nature of the pulses. (See paper and thesis mailed earlier.)

I.B. Soliton Annihilation and Creation (McLaughlin and Overman). When a structural perturbation destroys or creates solitons, it does so by coupling the solitons to radiation modes of the system. In order to describe this coupling analytically, yet simply, one needs a concise representation of those radiation modes which participate in the interaction. We had conjectured that "resonances," that is poles of the spectral transform of the wave in the lower half spectral plane, will provide such a representation. (Poles in the upper half plane represent solitons.) To verify this conjecture, we have used the integral equations of IST to construct from pure resonances the waves which they represent. These wave forms were then compared with the numerical solution of the structurally perturbed nonlinear evolution equation. In particular, we investigated numerically the annihilation of a sine-Gordon breather by dissipation. By combining numerical evaluations of the space-time wave form with numerical evaluations of its scattering transform, we find that breather annihilation can be accurately and concisely described as the transferal of a pole from the upper half complex plane into the lower half plane (where it describes radiation).

II. Nonlinear Wave Trains

II.A. Modulations of N-phase Wave Trains for Korteweg-de Vries Equation. (Flaschka, McLaughlin, Forest) Inverse spectral theory is used to develop an invariant representation of modulational equations. This representation depends upon certain differentials on a Riemann surface. When evaluated near ∞ on the surface, the representation reduces to averaged conservation laws; when evaluated near the branch points, it shows that the branch points provide Riemann invariants; integrals of the representation yield "conservation of waves". Explicit formulas for the characteristic speeds of the modulational equations are derived. These results generalize results of G. B. Whitham for a single phase traveling wave of K.dV. They indicate that complete integrability can induce enough structure into the modulational equations to diagonalize (in the sense of Riemann invariants) their first-order terms. In addition to their theoretical importance, the formulas are so explicit that they will be of use in practical applications.

II.B. Periodic Boundary Conditions. (Flaschka, McLaughlin, Ferguson) Although IST applies under periodic boundary conditions, its current mathematical description is too complicated for applications. For the Toda lattice under periodic boundary conditions, we have learned to simplify the description of the transform and, thus, to extract physical information. (See manuscript mailed earlier).

II.C. Periodic Sine-Gordon Equation (McLaughlin and Forest). We are adapting the methods of II.A and II.B to the sine-Gordon equation, which, under periodic boundary conditions, is important in modeling nonlinear oscillations for applications in low temperature memory devices and in condensed matter physics. Of particular importance in these applications is the identification of decay processes for kink-antikink wave trains and the processes of coupling to exterior circuits. For example, the current-voltage characteristics of a Josephson oscillator can be computed using a special class of periodic solutions called "separable solutions". Because of its importance in basic and applied physics, we are extending the results of II.A,B to the sine-Gordon case. First, we worked out concrete information about IST for sine-Gordon, none of which was available in the literature. As two examples, we found the frequencies and wave numbers in terms of spectral data, and we described the spectral representation of "separable solutions". We developed an action-angle description. With this necessary background material complete, we are beginning to adapt (II.A) to study modulations of N-phase sine-Gordon wave trains.

III. Deformation Theory (Flaschka and (Newell)). Two groups of authors (Barouch, McCoy, Tracy, Wu and Sato, Miwa, Jimbo) have uncovered deep connections between Painleve equations, the Ising model, and Fermi quantum fields. We have found a relation between the deformation theory of Sato et al. and IST. This new approach leads to a "scattering problem" in which the scattering matrix is replaced by Stokes multipliers. (See enclosed abstracts).

IV. Dispersive Smoothing of Shocks in Toda Lattice (Flaschka, McLaughlin, (Holian)). The Toda lattice is a chain of unit masses connected by nonlinear springs. We study one semi-infinite Toda lattice moving rigidly at constant velocity v, impinging upon a second semi-infinite Toda lattice at rest. The impact creates two symmetrical shock waves. The problem is to understand the behavior behind the shock. Holian and Straub (Los Alamos) have made computer studies; they find either decaying oscillations or a high-frequency steady-state, depending on the size of v. Using the inverse scattering transform, we identify the transition analytically and quantitatively. In addition, we fit the "nonlinear normal modes" of the Toda lattice to their data; thus, we interpret their observations and identify parameter dependence. Our results agree with the numerical experiments.