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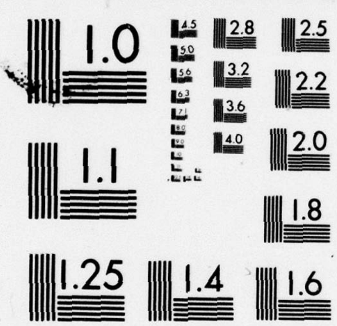
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AERONAUTICAL RESEARCH LABORATORIES

MELBOURNE, VICTORIA

AERODYNAMICS NOTE 377

AN INTRODUCTION TO DYNAMIC DERIVATIVES

(2) THE EQUATIONS OF MOTION FOR WIND TUNNEL PITCH-YAW OSCILLATION RIGS

by

G. F. FORSYTH

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AN INTRODUCTION TO DYNAMIC DERIVATIVES
(2) THE EQUATIONS OF MOTION FOR WIND TUNNEL PITCH-YAW OSCILLATION RIGS

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SUMMARY

The equations of motion are developed for a simplified free flight pitch-yaw system and for spring-mounted and rigidly-driven wind tunnel systems. For the spring-mounted system both initial displacement and forced-oscillation conditions are examined. A simplified cable towed system is also derived. A bibliography is included.

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CONTENTS

NOTATION

C_x, C_y, C_z	aero-normal coefficient of the force X, Y or Z given by
C_x	$= X/(\frac{1}{2}\rho_e V_e^2 S)$, for example
C_m	aero-normal coefficient of the moment m $= m/(\frac{1}{2}\rho_e V_e^2 S l_0)$
$C_{x_{\dot{\alpha}}}$, etc.	derivatives based on aero-normal force coefficients and aero-normal variables $= \frac{\partial C_x}{\partial \dot{\alpha}} = \frac{1}{\frac{1}{2}\rho_e V_e^2 S}\frac{\partial X}{\partial \dot{\alpha}} = \frac{1}{\frac{1}{2}\rho_e V_e l_0 S}\frac{\partial X}{\partial \dot{\alpha}}$
$C_{m_{\dot{\alpha}}}$, etc.	derivatives based on aero-normal moment coefficients and aero-normal variables $= \frac{\partial C_m}{\partial \dot{\alpha}} = \frac{1}{\frac{1}{2}\rho_e V_e^2 S l_0}\frac{\partial m}{\partial \dot{\alpha}} = \frac{1}{\frac{1}{2}\rho_e V_e S l_0^2}\frac{\partial m}{\partial \dot{\alpha}}$
I_y	moment of inertia about a y-axis
j	$\sqrt{-1}$
K	a constant, derivatives used as spring rates, K_θ & $K_{\dot{\theta}}$
l_0	representative length
m	pitching moment about a y-axis
m_e	mass in stability axes
$\bar{m}_e = \mu$	aero-normal mass
m_α	pitching moment incidence derivative
q	pitch rate; equal to $\dot{\theta}$
S	representative area
T	Tension or Torque; amplitude is T_0
V_e	datum velocity
α	incidence angle, usually sine definition as per Figure 1
Δ	change
η	pitch motivator deflection
θ	pitch angle
θ_c	cable pitch angle
μ	density parameter, aero-normal mass
λ	real part of an exponential solution
ρ_e	datum air density
ω	frequency in radians/second
ω_0	natural frequency, ω for undamped system

Superscript bar means aero-normal terms

1. INTRODUCTION

To measure the forces and moments on a manoeuvring aircraft by taking scale model measurements in a wind-tunnel, it is necessary to know the relationship between the equations of motion for the free flight aircraft and the wind-tunnel model. The reasons for such measurements were discussed[86] in the first Note of this Series.

The wind-tunnel model system normally will have additional restraints and constraints on its flight. This note will investigate the equations of motion and resulting forces for several such restraints starting with a simplified version of the free-flight arrangement. It is hoped that this will shed light on the advantages and disadvantages of the various methods of oscillating test models which will be studied in the next Note of this Series.

This Note concludes with a bibliography of introductory texts. By no means an exhaustive list, this covers publications before 1970 which are considered complementary to this series of Notes. Lists of later publications can be easily obtained from any of the computer abstracting services.

A single degree of freedom system will be considered throughout and for convenience it will always be a pitch system. A yaw system would be identical, with the appropriate different moment, force and inertia terms, apart from the lack of weight. In fact, any system which measures pitch derivatives can measure yaw derivatives (by, for example, rolling the model ninety degrees). There may be difficulties with freely suspended and cable towed models if all possible modes are not stable, but in general a sensible choice of pivot location will solve these problems.

1.1 The Aero-Normal Notation

For convenience and simplicity of equations and relationships between quantities, most aerodynamic results are expressed in non-dimensional terms. Several schemes for making quantities "normal" exist and the ones of major interest were considered in detail in an earlier publication in this series.[86] The system used throughout this publication is called the aero-normal notation and is identical to the usual "C" notation for a static case. Coefficients are changed only in that ρ and V become ρ_e and V_e which are the values at some initial condition where the stability axes are fixed. Lengths are divided by l_0, velocities by V_e, time derivatives by V_e/l_0 (for each differentiation) and mass by $\frac{1}{2}\rho_e S l_0$ which is the mass of a reference volume. Aero-Normal mass is called μ but other quantities are denoted by a superscript bar.

2. CONSTANT VELOCITY UNDAMPED SYSTEM

The equations of motion of a single degree of freedom system may be expressed in terms of Newton's equation for a force and a moment:

force = mass × acceleration

moment = moment of inertia × angular acceleration.

Ignoring velocity terms, the Z force may be expressed

$$Z = Z_\alpha \alpha + Z_0 \qquad (2.1)$$

and the moment as

$$m = m_\alpha \alpha + m_0. \qquad (2.2)$$

For a body of mass m_e and moment of inertia I_y, Newton's equations then become:

$$Z_\alpha \alpha + Z_0 = m_e \ddot{z} \qquad (2.3)$$

and

$$m_\alpha \alpha + m_0 = I_y \ddot{\theta}. \qquad (2.4)$$

If the initial condition is stable and primed variables are changes from this initial condition, these equations may be written

$$Z_\alpha \, \alpha' = m_e \, \ddot{\bar{z}} \tag{2.5}$$

and

$$m_\alpha \, \alpha' = I_y \, \ddot{\theta}. \tag{2.6}$$

Each term in equations 2.5 and 2.6 can be made non-dimensional by multiplying and/or dividing by appropriate constants, as follows:

$$\frac{Z_\alpha}{\frac{1}{2}\rho_e V_e^2 S} \alpha' = \frac{m_e}{\frac{1}{2}\rho_e S l_0} \frac{l_0}{V_e^2} \ddot{z} \tag{2.7}$$

which is written[86]

$$C_{z_\alpha} \, \alpha' = \mu \, \ddot{\hat{z}}. \tag{2.8}$$

Similarly for the moment expression

$$\frac{m_\alpha}{\frac{1}{2}\rho_e V_e^2 S l_0} \alpha' = \frac{I_y}{\frac{1}{2}\rho_e S l_0^3} \frac{l_0^2 \ddot{\theta}}{V_e^2} \tag{2.9}$$

which becomes

$$C_{m_\alpha} \, \alpha' = \bar{I}_y \, \ddot{\hat{\theta}}. \tag{2.10}$$

This non-dimensional or aero-normal notation is used throughout this work except where particular cases note otherwise.

These equations of motion (2.8 and 2.10) are connected by the geometric relation

$$\theta = \alpha + \dot{z}/V_e = \alpha + \dot{\hat{z}}. \tag{2.11}$$

This leaves two independent variables with two separate additive solutions. One way to arrive at each solution is to choose appropriate initial conditions.

2.1 Heave Mode

If initial conditions

$$\theta = 0 \tag{2.12}$$

$$q = \dot{\theta} = 0 \tag{2.13}$$

and

$$\dot{z} = K \tag{2.14}$$

are imposed on the system of equations 2.8, 2.10 and 2.11, a trial solution can be tried:

$$z = z_0 + z_1 \, e^{j\omega t} \tag{2.15}$$

$$\alpha = \alpha_0 + \alpha_1 \, e^{j(\omega t + \zeta)} \tag{2.16}$$

$$\theta = \theta_0 + \theta_1 \, e^{j(\omega t + \xi)}. \tag{2.17}$$

Equation 2.14 means, in conjunction with 2.15, that

$$K = j\omega z_1 \, e^{j\omega(0)} = j\omega z_1. \tag{2.18}$$

Similarly 2.17 and 2.12 yield

$$0 = \theta_0 + \theta_1 \, e^{j\xi} \tag{2.19}$$

and 2.17 and 2.13 give

$$0 = j\omega\theta_1 \, e^{j\xi} \tag{2.20}$$

2.19 and 2.20 further imply

$$\theta_0 = \theta_1 = 0 \tag{2.21}$$

so θ remains at its initial zero value. Then equation 2.11 becomes

$$\alpha = -\dot{\hat{z}} \tag{2.22}$$

and hence

$$\alpha_0 + \alpha_1 \, e^{j(\omega t + \zeta)} = -(K/V_e) e^{j\omega t} \tag{2.23}$$

which means, after evaluating this expression for t not zero and $t = 0$, that

$$\alpha_1 = -K/V_e = -\bar{K}$$

and

$$\alpha_0 = -\alpha_1 \, e^{j\zeta}. \tag{2.24}$$

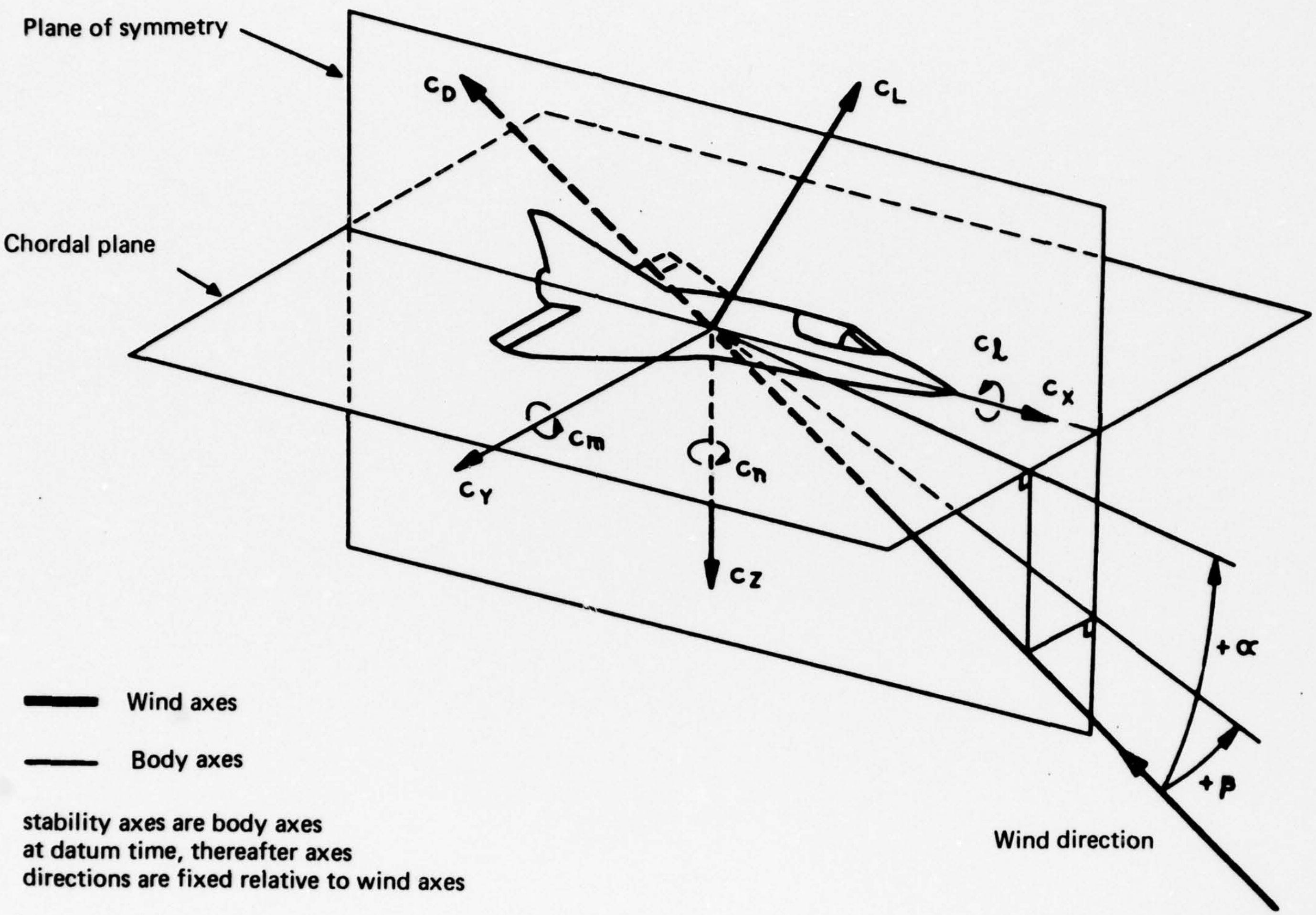

FIG. 1. FORCE AND MOMENT AXES SYSTEM
(Sine definitions of angles)

In equation 2.10, the right hand side term $\ddot{\theta}$ is zero but the left hand side contains no zero terms. The "heave" mode must then assume infinite moment of inertia I_y, and for a constant velocity system equation 2.8 becomes:

$$-C_{z_\alpha} \frac{K}{V_e} e^{j\omega t} = \frac{\mu \, l_0}{V_e^2} j\omega K \, e^{j\omega t}. \tag{2.25}$$

Cancelling terms,

$$j\omega = -(V_e \, C_{z_\alpha})/(\mu \, l_0), \text{ which is real, } \quad = \lambda, \text{ say.} \tag{2.26}$$

The heave mode is now seen to be an exponential response with

$$z = z_0 + (K/\lambda) \, e^{\lambda t} \tag{2.27}$$

$$\alpha = -\bar{K} \, e^{\lambda t} = -(K/V_e) \, e^{\lambda t} \tag{2.28}$$

where

$$\lambda = -V_e/(\mu \, l_0) \, C_{z_\alpha}.$$

2.2 Pitch Mode

Taking new initial conditions

$$\left.\begin{aligned} z(0) &= 0 \\ q(0) &= K \\ \text{and} \quad \theta(0) &= \alpha(0) = 0 \end{aligned}\right\} \tag{2.29}$$

then equations 2.15 to 2.17, the same trial solution as before, give

(*a*)

$$\dot{z}(0) = 0 = j\omega z_1 \, e^{j\omega 0} \tag{2.30}$$

$$\therefore \; z = z_0 + z_1 \cos \omega t \tag{2.31}$$

and

$$\alpha = \alpha_1 \sin \omega t \tag{2.32}$$

and

$$\theta = \theta_1 \sin \omega t. \tag{2.33}$$

(*b*)

$$\dot{\theta} = \omega \theta_1 \cos \omega t$$

$$\theta(0) = \omega \theta_1 = K \tag{2.34}$$

$$\theta_1 = K/\omega$$

$$\therefore \; \theta = (K/\omega) \sin \omega t. \tag{2.35}$$

Substituting back in our equations of motion,

$$2.10 \rightarrow C_{m_\alpha} \alpha_1 \sin \omega t = -\bar{I}_y \, (K/\omega) \, (l_0^2/V_e^2) \, \omega^2 \sin \omega t$$

$$\therefore \; C_{m_\alpha} \alpha_1 = -\bar{I}_y \, (K/\omega) \, \bar{\omega}^2. \tag{2.36}$$

In equation 2.8, the right hand side term $\ddot{z}$ is zero but the left hand side contains no zero terms. The "pitch" mode must then assume infinite aero-normal mass μ. This further implies that

$$\alpha_1 = \theta_1 = K/\omega. \tag{2.37}$$

Substituting for α_1 in 2.36 and clearing,

$$\bar{\omega}^2 = -C_{m_\alpha}/I_y. \tag{2.38}$$

The pitch mode is thus a steady sinuosoidal oscillation in both pitch and incidence.

The system of this section is not physically realizable. Real systems have losses (damping) and in the next section damping terms will be considered.

3. THE DAMPED CONSTANT VELOCITY SYSTEM

The equations of motion considering velocity terms in the aerodynamic forces become

$$C_{z_\alpha} \alpha' + C_{z_{\dot{\alpha}}} \dot{\alpha} + C_{z_q} \bar{q} = \mu \, \ddot{z} \tag{3.1}$$

and

$$C_{m_\alpha} \alpha' + C_{m_{\dot{\alpha}}} \dot{\alpha} + C_{m_q} \bar{q} = I_y \, \ddot{\theta} \tag{3.2}$$

with $$\theta = \alpha' + \dot{\bar{z}} \tag{3.3}$$

where primed variables are changes from initial conditions and all terms are aero-normal. The same two sets of initial conditions are considered for this system as for the undamped system in the previous section.

3.1 Heave Mode

Assume initial conditions

$$\left.\begin{array}{l}\theta = 0\\ \dot{\theta} = 0\\ \dot{z} = K.\end{array}\right\} \tag{3.4}$$

As in section 2.1, the first two conditions are met by

$$\theta(t) = 0 \tag{3.5}$$

and a solution of the form

$$z = z_0 + z_1\, e^{\lambda t} \tag{3.6}$$

$$\alpha = \alpha_0 + \alpha_1\, e^{\lambda t} \tag{3.7}$$

is tried.

Equation 3.3 gives $$0 = \alpha_1\, e^{\lambda t} + (z_1\, \lambda\, e^{\lambda t})/V_e \tag{3.8}$$

$$\therefore \quad \alpha_1 = -\bar{z}_1\, \bar{\lambda}. \tag{3.9}$$

From 3.4, $$\dot{z}(0) = z_1\, \lambda\, e^{\lambda 0} = K$$

$$= z_1\, \lambda$$

$$\therefore \quad z_1 = K/\lambda \quad \text{or} \quad \bar{z}_1 = \bar{K}/\bar{\lambda}. \tag{3.10}$$

Hence from 3.9 and 3.10,

$$\alpha_1 = -\bar{K}. \tag{3.11}$$

Substituting back in equation 3.1,

$$-\bar{K}\, C_{z_\alpha}\, e^{\lambda t} - \bar{K}\, \bar{\lambda}\, C_{z_{\dot{\alpha}}}\, e^{\lambda t} = \mu\, (\bar{K}\, \bar{\lambda})\, e^{\lambda t} \tag{3.12}$$

$$\text{so} \quad \bar{\lambda} = -\, C_{z_\alpha}/(\mu + C_{z_{\dot{\alpha}}}). \tag{3.13}$$

3.2 Pitch Mode

Assuming new initial conditions

$$\left.\begin{array}{l}\dot{z}(0) = 0\\ q(0) = K\\ \theta(0) = \alpha'(0) = 0\end{array}\right\} \tag{3.14}$$

and trial solutions

$$\left.\begin{array}{l}z = z_e\\ \alpha' = \theta = \alpha_0\, e^{\lambda t}\, e^{j\omega t}\end{array}\right\} \tag{3.15}$$

then $$q(0) = K = \alpha_0\, \lambda_e{}^{\lambda 0}\, e^{j\omega 0} + j\omega\, \alpha_0\, e^{\lambda 0}\, e^{j\omega 0} \tag{3.16}$$

and $$\theta(0) = 0 = \alpha_0\, e^{\lambda 0}\, e^{j\omega t}$$

$$= \alpha_0 \sin(\omega 0) \tag{3.17}$$

as the cosine term must be zero also.

From 3.17

$$\alpha' = \theta = \alpha_0\, e^{\lambda t} \sin(\omega t) \tag{3.18}$$

and 3.16 becomes $$K = \alpha_0\, \lambda\, 0 + \alpha_0\, e^0\, \omega \cos(\omega 0)$$

$$= \omega\, \alpha_0$$

$$\therefore \quad \alpha_0 = K/\omega \tag{3.19}$$

and the equation of motion becomes 3.2

$$C_{m_\alpha}(\bar{K}/\omega)\,e^{\lambda t}\sin(\omega t) + (C_{m_{\dot\alpha}} + C_{m_q})\,\{[(\bar{K}\bar{\lambda})/\bar{\omega}]\,e^{\lambda t}\sin(\omega t) + \bar{K}\,e^{\lambda t}\cos(\omega t)\} =$$

$$= I_y\,\bar{K}\,\{2\bar{\lambda}\cos(\omega t) + [(\bar{\lambda}^2 - \bar{\omega}^2)/\bar{\omega}]\sin\omega t\}\,e^{\lambda t}. \qquad (3.20)$$

By equating the sine and cosine terms, and clearing,

$$C_{m_{\dot\alpha}} + C_{m_q} = 2\bar{\lambda}\,I_y \qquad (3.21)$$

and

$$C_{m_\alpha} + (C_{m_{\dot\alpha}} + C_{m_q})\,\lambda = I_y\,(\bar{\lambda}^2 - \bar{\omega}^2). \qquad (3.22)$$

Substituting 3.21 into 3.22 then gives

$$C_{m_\alpha} = -(\bar{\lambda}^2 + \bar{\omega}^2)\,I_y \qquad (3.23)$$

$$= -\bar{\omega}_0{}^2\,I_y. \qquad (3.24)$$

This is of the same form as in the previous section (Eqn 2.38) but ω is now the complex quantity ω_0. For a dynamically (as well as statically) stable system, the real part of ω_0 must be negative. If positive, i.e. $\lambda > 0$, any disturbance in pitch will grow.

The two modes, heave and pitch, of 3.1 and 3.2, exist concurrently, and a suitable choice of initial conditions shows both. However, as long as the time scales of the two disturbances are sufficiently different, say one more than forty percent longer than the other, the resultant remains simply the sum of the two disturbances.

The damped pitch disturbance occurs in aircraft (fixed controls) and can be simulated in a wind-tunnel with low friction pivots. The heave mode does not however occur in this constant velocity form. In flight, speed will change as the aircraft climbs or falls modifying the result. In a wind tunnel it is difficult to simulate translations; a rotation about a point not at the reference centre will be considered instead.

4. SPRING RESTRAINED PITCH SYSTEM

Consider a system where, as well as the aerodynamic terms, there are structural stiffness (K_θ) and damping ($K_{\dot\theta}$) terms but heaving motions are constrained. For the wind-off or tare case, but non-dimensionalizing as for the wind-on case to be considered next,

$$I_y\,\ddot\theta = \bar{K}_\theta\,\theta + \bar{K}_{\dot\theta}\,\dot\theta \qquad (4.1)$$

which has the solution

$$\theta = \theta_0\,e^{(\lambda_t + j\omega_t)t} \qquad (4.2)$$

with

$$\bar{K}_{\dot\theta} = 2\bar{\lambda}_t\,I_y \qquad (4.3)$$

and

$$\bar{K}_\theta = -(\bar{\omega}_t{}^2 + \bar{\lambda}_t{}^2)\,I_y \qquad (4.4)$$

where

$$\bar{\lambda}_t = \lambda_t\,l_0/V_e \quad \text{and} \quad \bar{\omega}_t = \omega_t\,l_0/V_e$$

with V_e referring to the wind-on case.

Since a pitch system with $\alpha = \theta$ is being considered, the wind-on case becomes:

$$I_y\,\ddot\theta = (\bar{K}_\theta + C_{m_\alpha})\,\theta + (\bar{K}_{\dot\theta} + C_{m_{\dot\alpha}} + C_{m_q})\,\dot\theta \qquad (4.5)$$

which has the solution

$$\theta = \theta_0\,e^{(\lambda + j\omega)t} \qquad (4.6)$$

with

$$\bar{K}_{\dot\theta} + C_{m_{\dot\alpha}} + C_{m_q} = 2\bar{\lambda}\,I_y \qquad (4.7)$$

and

$$\bar{K}_\theta + C_{m_\alpha} = -(\bar{\omega}^2 + \bar{\lambda}^2)\,I_y. \qquad (4.8)$$

From equations 4.3 and 4.7

$$C_{m_{\dot\alpha}} + C_{m_q} = 2(\bar{\lambda} - \bar{\lambda}_t)\,I_y \qquad (4.9)$$

and from equations 4.4 and 4.8

$$C_{m_\alpha} = -(\bar{\lambda}^2 - \bar{\lambda}_t{}^2 + \bar{\omega}^2 - \bar{\omega}_t{}^2)\,I_y \qquad (4.10)$$

$$= (\bar{\omega}_{0_t}{}^2 - \bar{\omega}_0{}^2)\,I_y \qquad (4.11)$$

using the results of Section 3.

The terms on the left in 4.9 and 4.10/11 are the same as those responsible for the behaviour of the pitch system in Section 3 and the equations are the difference between two sets of results of the form of that Section. This convenient relationship means that this system is often used in wind-tunnel testing.

5. SPRING RESTRAINED PITCHING AND PLUNGING SYSTEM

If the centre of rotation is moved from the origin as in Section 4 to $(-x_0, 0, 0)$ in the body co-ordinates, then

$$\theta = \alpha + \dot{\bar{z}} \tag{5.1}$$

$$= \alpha + \dot{z}/V_e$$

and

$$\theta = z/x_0. \tag{5.2}$$

The equation of motion is far more complicated

$$\mu \bar{x}_0^2 \ddot{\theta} + I_y \ddot{\theta} = \bar{K}_\theta \theta + \bar{K}_{\dot{\theta}} \dot{\theta} + C_{m_\alpha} \alpha + C_{m_{\dot{\alpha}}} \dot{\alpha} + C_{m_q} \dot{\theta} + C_{z_\alpha} \bar{x}_0 \alpha + C_{z_{\dot{\alpha}}} \bar{x}_0 \dot{\alpha} + C_{z_q} \bar{x}_0 \dot{\theta}. \tag{5.3}$$

Equations 5.1 and 5.2 give

$$\theta = \alpha + \bar{x}_0 \dot{\theta}. \tag{5.4}$$

The moment of inertia about the new y_1 axis

$$I_{y_1} = I_y + \mu \bar{x}_0^2. \tag{5.5}$$

Substituting 5.4 and 5.5 in equation 5.3 gives

$$I_{y_1} \ddot{\theta} = (\bar{K}_\theta + C_{m_\alpha} + \bar{x}_0 C_{z_\alpha}) \theta + (\bar{K}_{\dot{\theta}} - \bar{x}_0 C_{m_\alpha} + C_{m_{\dot{\alpha}}} + C_{m_q} - \bar{x}_0^2 C_{z_\alpha} + \bar{x}_0 C_{z_{\dot{\alpha}}} + \bar{x}_0 C_{z_q}) \dot{\theta} - $$

$$- \bar{x}_0 (C_{m_{\dot{\alpha}}} + \bar{x}_0 C_{z_{\dot{\alpha}}}) \ddot{\theta}. \tag{5.6}$$

Tare values give

$$\bar{K}_{\dot{\theta}} = 2\bar{\lambda}_t I_{y_1} \tag{5.7}$$

and

$$\bar{K}_\theta = -(\bar{\omega}_t^2 + \bar{\lambda}_t^2) I_{y_1} = -\bar{\omega}_{0_t}^2 I_{y_1}. \tag{5.8}$$

For the wind-on case, try a solution

$$\theta = \theta_0 e^{(\lambda + j\omega)t}. \tag{5.9}$$

Then $\bar{K}_{\dot{\theta}} + C_{m_{\dot{\alpha}}} + C_{m_q} + \bar{x}_0 (C_{z_{\dot{\alpha}}} + C_{z_q} - C_{m_\alpha}) - \bar{x}_0^2 C_{z_\alpha} = 2\lambda (I_{y_1} + \bar{x}_0 C_{m_{\dot{\alpha}}} + \bar{x}_0^2 C_{z_{\dot{\alpha}}})$ (5.10)

and

$$\bar{K}_\theta + C_{m_\alpha} + \bar{x}_0 C_{z_\alpha} = -(\bar{\lambda}^2 + \bar{\omega}^2)(I_{y_1} + \bar{x}_0 C_{m_{\dot{\alpha}}} + \bar{x}_0^2 C_{z_{\dot{\alpha}}}) =$$

$$= -\omega_0^2 (I_{y_1} + \bar{x}_0 C_{m_{\dot{\alpha}}} + \bar{x}_0^2 C_{z_{\dot{\alpha}}}). \tag{5.11}$$

Substituting 5.8 into 5.11,

$$C_{m_\alpha} + \bar{x}_0 C_{z_\alpha} = -(\bar{\omega}_0^2 - \bar{\omega}_{0_t}^2) I_{y_1} - \bar{\omega}_0^2 \bar{x}_0 (C_{m_{\dot{\alpha}}} + \bar{x}_0 C_{z_{\dot{\alpha}}}). \tag{5.12}$$

Similarly 5.7 into 5.10 gives

$$(C_{m_{\dot{\alpha}}} + \bar{x}_0 C_{z_{\dot{\alpha}}}) + (C_{m_q} + \bar{x}_0 C_{z_q}) - \bar{x}_0 (C_{m_\alpha} + \bar{x}_0 C_{z_\alpha}) = 2(\bar{\lambda} - \bar{\lambda}_t) I_{y_1} + 2\bar{\lambda} \bar{x}_0 (C_{m_{\dot{\alpha}}} + \bar{x}_0 C_{z_{\dot{\alpha}}}). \tag{5.13}$$

Using 5.12, equation 5.13 becomes:

$$(C_{m_{\dot{\alpha}}} + \bar{x}_0 C_{z_{\dot{\alpha}}})(1 + \bar{x}_0^2 \bar{\omega}_0^2 - 2\bar{\lambda} \bar{x}_0) + (C_{m_q} + \bar{x}_0 C_{z_q}) = [2(\bar{\lambda} - \bar{\lambda}_t) - \bar{x}_0 (\bar{\omega}_0^2 - \bar{\omega}_{0_t}^2)] I_{y_1}. \tag{5.14}$$

This is complicated. However if values for $x_0 = 0$ are known it allows measurement of $C_{z_{\dot{\alpha}}}$ and estimation of the split between $C_{m_{\dot{\alpha}}}$ and C_{m_q} which have appeared together up to now.

6. FORCED SPRING-RESTRAINED PITCH SYSTEM

To the system of Section 4 a torque $\bar{T}_0 e^{j\Omega t}$ is added. The tare case becomes:

$$I_y \ddot{\theta} = \bar{K}_\theta \theta + \bar{K}_{\dot{\theta}} \dot{\theta} + \bar{T}_0 e^{j\Omega t}. \tag{6.1}$$

The homogeneous solution ($T_0 = 0$) was given in Section 4

$$\theta = \theta_0 e^{(\lambda_t + j\omega_t)t} \tag{4.2}$$

where $$\bar{K}_{\dot{\theta}} = 2\bar{\lambda}_t \, \bar{I}_y \tag{4.3}$$

and $$\bar{K}_\theta = -(\varpi_t^2 + \bar{\lambda}_t^2) \, \bar{I}_y \tag{4.4}$$

$$= -\varpi_{0_t}^2 \, \bar{I}_y. \tag{6.2}$$

The complete solution in the tare case is thus

$$\theta = \theta_0 \, e^{(\lambda_t + j\omega_t)t} + \theta_1 \, e^{j(\Omega t + \xi)} \tag{6.3}$$

where it may be shown that

$$\xi = \tan^{-1} [(-\bar{K}_{\dot{\theta}} \, \bar{\Omega})/(\bar{I}_y \, \bar{\Omega}^2 + \bar{K}_\theta)], \text{ which using 4.3 and 4.4,}$$

$$= \tan^{-1} [(-2\bar{\lambda}_t \, \bar{\Omega})/(\bar{\Omega}^2 - \varpi_{0_t}^2)] \tag{6.4}$$

and also $$\theta_1 = -\frac{\bar{T}_0}{\bar{I}_y \, [4\bar{\lambda}_t^2 \, \bar{\Omega}^2 + (\bar{\Omega}^2 - \varpi_{0_t}^2)^2]^{\frac{1}{2}}}. \tag{6.5}$$

Note that for zero tare damping, i.e. $\lambda_t = 0$, then $\xi = 0$ so that the oscillation is in phase with the applied torque, and

$$\theta_1 = \bar{T}_0/[\bar{I}_y \, (\bar{\varpi}_{0_t}^2 - \bar{\Omega}^2)] \tag{6.7}$$

which is infinite at resonance.

The wind-on equation of motion is

$$\bar{I}_y \, \ddot{\theta} = \bar{K}_\theta \, \theta + \bar{K}_{\dot{\theta}} \, \dot{\theta} + C_{m_\alpha} \, \theta + C_{m_{\dot{\alpha}}} \, \dot{\theta} + C_{m_q} \, \dot{\theta} + \bar{T}_0 \, e^{j\Omega t}. \tag{6.8}$$

The solution† is of the form

$$\theta = \theta_0 \, e^{(\lambda + j\omega)t} + \theta_1 \, e^{j(\Omega t + \xi)} \tag{6.9}$$

where $$C_{m_\alpha} = -(\varpi_0^2 - \varpi_{0_t}^2) \, \bar{I}_y \tag{6.10}$$

$$C_{m_{\dot{\alpha}}} + C_{m_q} = 2(\bar{\lambda} - \bar{\lambda}_t) \, \bar{I}_y \tag{6.11}$$

and $$\xi = \tan^{-1} [(2\bar{\lambda} \, \bar{\Omega})/(\varpi_0^2 - \bar{\Omega}^2)] \tag{6.12}$$

remembering $$\varpi_0^2 = \varpi^2 + \bar{\lambda}^2 \tag{6.13}$$

and where $$\theta_1 = \bar{T}_0/\{\bar{I}_y \, [4\bar{\lambda}^2 \, \bar{\Omega}^2 + (\bar{\Omega}^2 - \varpi_0^2)^2]^{\frac{1}{2}}\}. \tag{6.14}$$

For zero damping ($\lambda = 0$) equation 6.14 reduces to

$$\theta_1 = \bar{T}_0/[\bar{I}_y \, (\varpi_0^2 - \bar{\Omega}^2)] \tag{6.15}$$

which again gives infinite amplitude for the resonance condition $\omega_0 = \Omega$.

For non-zero damping, the resonance condition gives $\xi = \tan^{-1} \infty = 90$ degrees for both tare and wind-on cases. In the tare case,

$$\bar{\Omega}_t = \varpi_{0_t} \tag{6.16}$$

then $$\theta_1 = \bar{T}_0/(2\bar{I}_y \, \bar{\lambda}_t \, \bar{\Omega}). \tag{6.17}$$

For wind-on

$$\bar{\Omega} = \varpi_0 \tag{6.18}$$

then $$\theta_1 = \bar{T}_0/(2\bar{I}_y \, \bar{\lambda} \, \bar{\Omega}). \tag{6.19}$$

Combining these equations

$$C_{m_{\dot{\alpha}}} + C_{m_q} = \bar{T}_0/(\bar{\Omega} \, \theta_1) - \bar{T}_{0_t}/(\bar{\Omega}_t \, \theta_{1_t}). \tag{6.20}$$

If tare and wind-on tests have the same amplitude, and stiffness can be varied to obtain the same frequency, then, providing $K_{\dot{\theta}}$ is the same,

$$C_{m_{\dot{\alpha}}} + C_{m_q} = (\bar{T}_0 - \bar{T}_{0_t})/(\bar{\Omega} \, \theta_1). \tag{6.21}$$

This is similar to the results obtained for free oscillation with the decay rate replaced by a torque input. It has an advantage in that the output is steady and measurements can be averaged over long time periods for high accuracy. As with the free system an estimation of the split of $C_{m_{\dot{\alpha}}}$ and C_{m_q} requires multiple axis locations.

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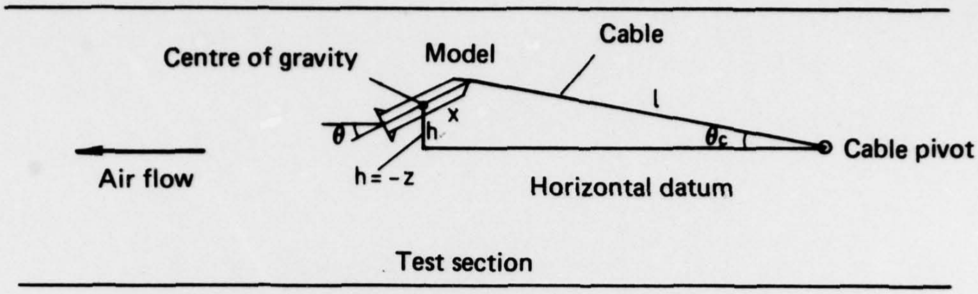
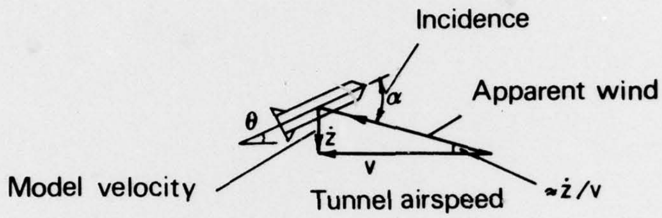
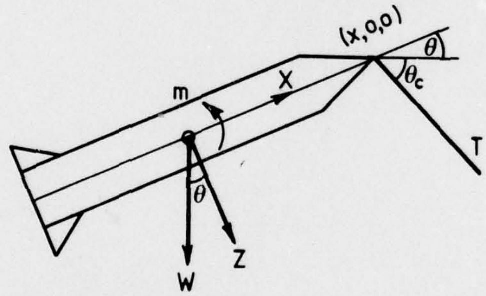
† S. Timoshenko, Vibration Problems in Engineering. Van Nostrand 1928.



(a) Relations, angles and lengths for towed models ($h \triangleq -x\,\theta + l\,\theta_c$)



(b) Relations, angles and velocities for towed models ($\alpha = \theta - \dot{z}/v$)



(c) Forces and moment Towed model ($T = -x\cos(\theta + \theta_c) - Z\sin(\theta + \theta_c)$
$+ mx\sin(\theta + \theta_c) - W\sin\theta_c$

FIG. 2. RELATIONSHIPS REQUIRED FOR SECTION 7

7. CABLE TOWED MODELS

7.1 Pitch/Incidence System

Consider a wind tunnel model on a long, flexible cable. The constraint conditions are taken to be:

(i) The cable is inextensible (length l), massless and dragless;

(ii) No moment about the point of attachment is provided by the cable which is attached distance x ahead of the model centre of gravity;

(iii) No disturbance in roll or yaw exists implying that the roll-yaw system must be statically and dynamically stable and have natural frequencies sufficiently different from those of the pitch/incidence system to minimize cross-excitation of one system on the other.

Condition (i) means that the force on the cable must equal the tension T in the cable. Resolution of all the forces acting leads (Fig. 2*a*) to the expression

$$T = -X \cos(\theta + \theta_c) - Z \sin(\theta + \theta_c) + mx \sin(\theta + \theta_c) - W \sin \theta_c \tag{7.1}$$

for the tension in the cable, and

$$-\Delta L = Z \cos \theta + W - X \sin \theta + T \sin \theta_c \tag{7.2}$$

for the force unbalance in the vertical direction. This will equal zero for a static equilibrium.

Similar resolution yields

$$\Delta m = m + Tx \sin(\theta + \theta_c) \tag{7.3}$$

for the resulting pitching moment. In each of these three equations, the terms have not been made non-dimensional as yet, in order to show the precise nature of some approximations made.

For θ and θ_c both small, from Fig. 2,

$$T \sin \theta_c \approx [-X - Z(\theta + \theta_c) + mx(\theta + \theta_c) - W\theta_c]\,\theta_c$$

$$\approx -X\theta_c \tag{7.4}$$

being first order in θ_c, and similarly

$$Tx \sin(\theta + \theta_c) \approx -Xx(\theta + \theta_c). \tag{7.5}$$

The nett vertical force is then

$$-\Delta L \approx Z + W - X(\theta + \theta_c) \tag{7.6}$$

and the nett moment about the y-axis is

$$\Delta m \approx m - Xx(\theta + \theta_c). \tag{7.7}$$

Now θ, θ_c, z and α are related (Fig. 2*b*) by

$$z = x\theta - l\,\theta_c \tag{7.8}$$

and

$$\alpha = \theta - \dot{z}/V. \tag{7.9}$$

From equations 7.8 and 7.9 it is clear that there are still only two independent variables as in Sections 2 and 3. So comparisons can be made with the equations derived in those sections, the equations of motion are considered in non-dimensional form.

If the initial conditions represent equilibrium, then

$$C_{z_0} + \bar{W} = 0 \quad (\text{"lift"} = \text{weight}) \tag{7.10}$$

and

$$C_{m_0} + C_{m_\eta} \cdot \eta = 0 \quad (\text{zero moments}) \tag{7.11}$$

and the equations of motion become

$$C_{z_\alpha}\alpha + C_{z_{\dot\alpha}}\dot\alpha + C_{z_q}\bar{q} - (\theta + \theta_c)(C_{x_0} + C_{x_\alpha}\alpha + C_{x_{\dot\alpha}}\dot\alpha + C_{x_q}\bar{q}) = \mu\ddot{z} \tag{7.12}$$

and

$$C_{m_\alpha}\alpha + C_{m_{\dot\alpha}}\dot\alpha + C_{m_q}\bar{q} - x(\theta + \theta_c)(C_{x_0} + C_{x_\alpha}\alpha + C_{x_{\dot\alpha}}\dot\alpha + C_{x_q}\bar{q}) = I_y\ddot{\theta}. \tag{7.13}$$

It is assumed that the solution of these equations can be divided into two additive modes as in Sections 2 and 3. It is further assumed that one way of representing these modes is by separating Equation 7.9 by zeroing one, then the other, of the right hand terms. These modes are then the "pitch" and "heave" modes of Sections 2 and 3, and the equations of motion become:

$$- C_{z_\alpha} \dot{\bar{z}} - C_{z_{\dot{\alpha}}} \ddot{\bar{z}} - (\bar{z}/\bar{l})(-C_{x_0} + C_{x_\alpha} \dot{\bar{z}} + C_{x_{\dot{\alpha}}} \ddot{\bar{z}}) = \mu \ddot{\bar{z}} \tag{7.14}$$

for the mode with

$$\dot{\bar{z}} = -\alpha \tag{7.15}$$

and

$$C_{m_\alpha} \theta + (C_{m_{\dot{\alpha}}} + C_{m_q}) \dot{\theta} - x\theta [C_{x_0} + C_{x_\alpha} \theta + (C_{x_{\dot{\alpha}}} + C_{x_q}) \dot{\theta}] = I_y \ddot{\theta} \tag{7.16}$$

for the mode with

$$\theta = \alpha. \tag{7.17}$$

Both 7.14 and 7.16 are non-linear differential equations. The effect of the tow cable is to introduce a non-linear additional stiffness to each mode. If terms of more than first order can be ignored, these equations become:

$$- C_{z_\alpha} \dot{\bar{z}} - C_{z_{\dot{\alpha}}} \ddot{\bar{z}} + C_{x_0} (\bar{z}/\bar{l}) = \mu \ddot{\bar{z}} \tag{7.18}$$

and

$$(C_{m_\alpha} - x\, C_{x_0})\, \theta + (C_{m_{\dot{\alpha}}} + C_{m_q})\, \dot{\theta} = I_y \ddot{\theta}. \tag{7.19}$$

Equation 7.19 is identical in form to the spring restrained pitch system of Section 4 with spring constant

$$K_\theta = -x\, C_{x_0} \tag{7.20}$$

and spring damping

$$K_{\dot{\theta}} = 0. \tag{7.21}$$

8. FORCES AND MOMENTS ON A RIGIDLY-DRIVEN MODEL

If a model of mass m_e and of moment of inertia about its centre of mass I_y is driven rigidly, then a force and moment balance between the model and the driving mechanism measures inertial as well as aerodynamic terms. The balance axis system is a body-axis set, and not the fixed "stability axes" used for other methods of driving oscillatory models. Two positions of the axis of rotation are considered as well as a simple translation. The balance is considered to be rigid in comparison with the movement applied.

8.1 Centre of Rotation at Reference Centre (0, 0, 0)

For a rigid sinusoidal pitch oscillation about the y-axis,

$$\alpha = \alpha_0 + \theta_1 \sin \omega t \tag{8.1}$$

$$\theta = \theta_0 + \theta_1 \sin \omega t \tag{8.2}$$

$$\dot{\alpha} = \dot{\theta} = \varpi\, \theta_1 \cos \omega t \tag{8.3}$$

$$\ddot{\alpha} = \ddot{\theta} = -\, \varpi^2\, \theta_1 \sin \omega t. \tag{8.4}$$

The z force coefficient has the form, where g is the acceleration due to gravity and μ is the non-dimensional model mass ($\bar{m}_e$),

$$C_z = \mu\, \bar{g} \cos(\alpha_0 + \theta_1 \sin \omega t) + C_{z_0} + C_{z_\alpha} \theta_1 \sin \omega t + (C_{z_{\dot{\alpha}}} + C_{z_q})\, \varpi\, \theta_1 \cos \omega t. \tag{8.5}$$

For a small amplitude θ_1 this may be approximated by

$$C_z = (\mu\, \bar{g} \cos \alpha_0 + C_{z_0}) + (C_{z_\alpha} - \mu\, \bar{g} \sin \alpha_0)\, \theta_1 \sin \omega t + (C_{z_{\dot{\alpha}}} + C_{z_q})\, \varpi\, \theta_1 \cos \omega t \tag{8.6}$$

where the three terms are the static, in-phase and quadrature components of the force with respect to the motion, respectively.

The pitching moment coefficient is given by

$$C_m = (C_{m_0} + C_{m_\eta} \eta') + (C_{m_\alpha} - \varpi^2 I_y)\, \theta_1 \sin \omega t + (C_{m_{\dot{\alpha}}} + C_{m_q})\, \varpi\, \theta_1 \cos \omega t \tag{8.7}$$

where the same comment about the three terms applies and where η' is a fixed control deflection.

8.2 Centre of Rotation Shifted Along the x-axis (x_0, 0, 0)

If the model is rotated about an axis x_0 ahead of the y-axis, α and θ are no longer co-incident quantities. As always, from the definitions of the angles of pitch and incidence,

$$\theta = \alpha + \dot{\bar{z}}, \quad \text{i.e.} = \alpha + \dot{z}/V_e. \tag{8.8}$$

In this case

$$\theta = z/x_0, \text{ also.} \tag{8.9}$$

Thus $$\theta = \alpha + \bar{x}_0\,\bar{q} \tag{8.10}$$

which is the same as equation 5.4 for the spring-restrained system.

The model now has moment of inertia

$$I_{y_1} = I_y + x_0^2\, m_e$$

i.e. $$\bar{I}_{y_1} = \bar{I}_y + \bar{x}_0^2\, \mu. \tag{8.11}$$

The oscillating model now has a linear acceleration term in its force equation. The equations of motion are now

$$\theta = \theta_0 + \theta_1 \sin \omega t \tag{8.12}$$

and $$\alpha = \theta_0 + \theta_1 \sin \omega t - \bar{x}_0\,\varpi\,\theta_1 \cos \omega t \tag{8.13}$$

with derivatives

$$\dot{\theta} = \bar{q} = \theta_1\,\varpi \cos \omega t \tag{8.14}$$

$$\dot{\alpha} = \theta_1\,\varpi \cos \omega t + \bar{x}_0\,\theta_1\,\varpi^2 \sin \omega t \tag{8.15}$$

$$\ddot{\bar{z}} = -\,\bar{x}_0\,\varpi^2\,\theta_1 \sin \omega t \tag{8.16}$$

and $$\ddot{\theta} = \dot{\bar{q}} = -\,\theta_1\,\varpi^2 \sin \omega t. \tag{8.17}$$

Then the z force coefficient has the form

$$C_z = C_{z_0} + \mu\,\bar{g} \cos \theta + \mu\,\ddot{\bar{z}} + C_{z_\alpha}\,\alpha + C_{z_{\dot{\alpha}}}\,\dot{\alpha} + C_{z_q}\,q. \tag{8.18}$$

If the oscillation amplitude θ_1 is small

$$\cos \theta \approx \cos \theta_0 - \sin(\theta_0)\,\theta_1 \sin \omega t. \tag{8.19}$$

Including all the constant terms in C_{z_0} to make C_{z_0}' and substituting for the angles in 8.18,

$$C_z = C_{z_0}' + [C_{z_\alpha} + \mu\,(\bar{x}_0\,\varpi^2 - \bar{g} \sin \theta_0) + C_{z_{\dot{\alpha}}}\,\bar{x}_0\,\varpi^2]\,\theta_1 \sin \omega t + $$
$$+ (C_{z_{\dot{\alpha}}} + C_{z_q} - \bar{x}_0\,C_{z_\alpha})\,\varpi\,\theta_1 \cos \omega t. \tag{8.20}$$

The pitching moment is even more complicated. The moment about the new axis is given by

$$m' = m + x_0\,Z \tag{8.21}$$

i.e. $$C_{m'} = C_m + \bar{x}_0\,C_z. \tag{8.22}$$

The balance however measures moments about the reference axis, i.e. C_m. The expression for this is exactly as before:

$$C_m = (C_{m_0} + C_{m_\eta}\,\eta') + (C_{m_\alpha} - \varpi^2\,\bar{I}_y)\,\theta_1 \sin \omega t + (C_{m_{\dot{\alpha}}} + C_{m_q})\,\varpi\,\theta_1 \cos \omega t. \tag{8.7}$$

Moments about the new axis can be obtained using Equations 8.7, 8.20 and 8.22.

8.3 Rigid Translation Along the z-axis

If the motion inexorably impressed on the model is a z translation and not a rotation, pitch angle θ remains a constant, say zero. Then from equation 8.8

$$\alpha = -\dot{\bar{z}} \tag{8.23}$$

and for a motion $$z = z_0 + z_1 \sin \omega t \tag{8.24}$$

$$\alpha = -\bar{z}_1\,\varpi \cos \omega t \tag{8.25}$$

and $$\dot{\alpha} = \bar{z}_1\,\varpi^2 \sin \omega t. \tag{8.26}$$

The z-force and pitching moment coefficients are given by, ignoring static valves,

$$C_z = C_{z_\alpha}\,\alpha + C_{z_{\dot{\alpha}}}\,\dot{\alpha} + \mu\,\ddot{\bar{z}} \tag{8.27}$$

and $$C_m = C_{m_\alpha}\,\alpha + C_{m_{\dot{\alpha}}}\,\dot{\alpha} \tag{8.28}$$

and using 8.25, 8.26 these become

$$C_z = -\,C_{z_\alpha}\,\bar{z}_1\,\varpi \cos \omega t + (C_{z_{\dot{\alpha}}} - \mu)\,\bar{z}_1\,\varpi^2 \sin \omega t \tag{8.29}$$

and $$C_m = -\,C_{m_\alpha}\,\bar{z}_1\,\varpi \cos \omega t + C_{m_{\dot{\alpha}}}\,\bar{z}_1\,\varpi^2 \sin \omega t. \tag{8.30}$$

9. CONCLUSION

The equations of motion have been developed for a simplified free flight pitch-yaw system and for spring-mounted and rigidly-driven wind tunnel systems. For the spring-mounted system both initial displacement and forced-oscillation conditions have been examined. A simplified cable towed system has also been derived. These equations will be used in later Notes of this Series.

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16. *ABSTRACT*

The equations of motion are developed for a simplified free flight pitch-yaw system and for spring-mounted and rigidly-driven wind tunnel systems. For the spring-mounted system both initial displacement and forced-oscillation conditions are examined. A simplified cable towed system is also derived. A bibliography is included.

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