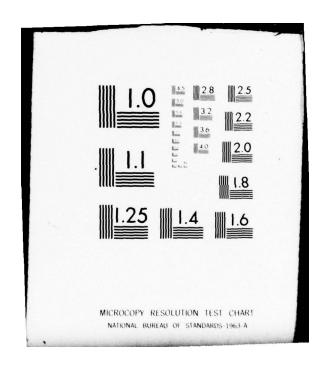
AERONAUTICAL RESEARCH LABS MELBOURNE (AUSTRALIA) F/G 20/11 SIMPLE FORMULAE FOR CALCULATING THE VIBRATIONAL FREQUENCIES OF --ETC(U) AD-A071 639 JUN 78 R JONES ARL/STRUC-371 UNCLASSIFIED NL | OF | AD A07/639 DATE FILMED 8-79



AD A 071639

GUE FILE COPY



# DEPARTMENT OF DEFENCE DEFENCE SCIENCE AND TECHNOLOGY ORGANISATION AERONAUTICAL RESEARCH LABORATORIES

MELBOURNE, VICTORIA

STRUCTURES REPORT 371

## SIMPLE FORMULAE FOR CALCULATING THE VIBRATIONAL FREQUENCIES OF PLATES, SHELLS AND MEMBRANES

by

R. JONES

Approved for Public Release.





C COMMONWEALTH OF AUSTRALIA 1978

COPY No 11

JUNE 1978

79 07 24 017

### APPROVED

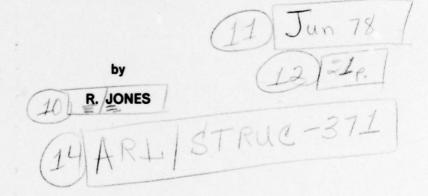
### FOR PUBLIC RELEASE

THE UNITED STATES NATIONAL
TECHNICAL INFORMATION SERVICE
IS AUTHORISED TO
REPRODUCE AND SELL THIS REPORT

## DEPARTMENT OF DEFENCE DEFENCE SCIENCE AND TECHNOLOGY ORGANISATION AERONAUTICAL RESEARCH LABORATORIES



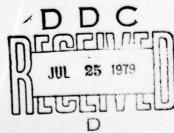
## SIMPLE FORMULAE FOR CALCULATING THE VIBRATIONAL FREQUENCIES OF PLATES, SHELLS AND MEMBRANES



#### SUMMARY

Simple approximate formulae for estimating the vibrational frequencies of plates, shells and membranes are presented. These formulae take the general form  $F = A/W_{\rm max}$  where F is a frequency parameter, A is a numerical parameter having a constant value for many different problems, and  $W_{\rm max}$  is the maximum deflection of an associated static deflection problem.

The approach is particularly simple and the agreement between the frequencies calculated using the approximate formulae and those calculated using standard vibration analyses is excellent.



POSTAL ADDRESS: Chief Superintendent, Aeronautical Research Laboratories, Box 4331, P.O., Melbourne, Victoria, 3001, Australia.

008 650

500

#### CONTENTS

	Page No.
NOTATION	
1. INTRODUCTION	1
2. FORMULA FOR PLATES	1
3. FORMULA FOR MEMBRANES AND SHEELS	4
4. FORMULA FOR ANISOTROPIC PLATES	9
5. CONCLUSIONS	10
REFERENCES	
DOCUMENT CONTROL DATA	
DISTRIBUTION	

Acces	sion For	,
DDC 1	GRA&I TAB counced fication	督
Ву		
Distr	ibution/	
Avei	lability	Codes
lat	Avall and specia	
n		

#### NOTATION

P	Mass per unit area
h	Thickness of the plate, membrane or shell
ν	Poisson's ratio
ω	Frequency
W'max	Maximum deflection due to a uniformly distributed load
Ŵ <sub>max</sub>	Maximum deflection due to a load varying linearly with the distance from the centroid
k	Curvature of a shallow cylindrical or spherical shell
ωο	Frequency parameter for a flat plate with Poisson's ratio equal to zero
E	Young's Modulus
$D_x$ , $D_y$ , $H$	Flexural rigidities for an orthotropic plate
8	Aspect ratio

#### 1. INTRODUCTION

Static deflection modes, due to particular pressure distributions, have long been used for vibration and flutter analysis, and are commonly used in conjunction with the Ritz method to obtain estimates of the vibrational frequencies. However, in a recent paper, Jones¹ proposed a new procedure which related the fundamental frequency of a vibrating plate to the maximum deflection of the corresponding, uniformly loaded plate. Unlike the Ritz method, this technique has the advantages that it is not necessary to perform any integration, and that the approximation is explicitly independent of the support conditions.

The aim of the present report is to discuss the developments of this technique, which have enabled the method to be extended so as to predict the frequencies of several higher modes as well as the frequencies of membrane and shell structures.

#### 2. FORMULA FOR PLATES

In a recent article, Jones<sup>1</sup> presented an approximate expression for the fundamental frequency of a vibrating plate: viz.

$$\rho h \omega^2 = (10 \cdot 2151)^2 / 64 W'_{\text{max}} \tag{1}$$

where  $\rho$  is the mass per unit area, h is the plate thickness,  $\omega$  the frequency of vibration and  $W'_{\text{max}}$  is the maximum deflection under unit loading of the corresponding uniformly loaded plate. Although the accuracy of this expression was clearly shown in Reference 1, this paper will include a more detailed investigation into its accuracy. In Table 1, the predicted values of the frequency, along with corresponding values obtained from the literature are shown for a large range of plates of various geometrical shapes and subject to a combination of boundary conditions.

TABLE 1

Frequency Parameter  $\omega a^2 \sqrt{\rho h/D}$  for Fundamental Mode for Various Plate Problems,  $\nu = 0.3$  (unless otherwise stated)

Problem	Equation (1)	Literature, e.g. Ref. 3
Simply supported equilateral triangle, sides of length $2a/\sqrt{3}$	39.80	39.48
Clamped rectangular plate, sides of		
length a, b		
b/a = 1.0	35.97	35.99
= 1.5	27.22	26.59
= 2.0	25.33	24.58
Simply supported rectangular plate		
b/a = 1.0	20.04	19.74
= 1.5	14.53	14.26
$=2\cdot0$	12.67	12.34
Clamped semicircular plate radius a	28.40	27.?
•		(too inaccurate)

TABLE 1—continued

Problem	Equation (1)	Literature, e.g. Ref. 3
Clamped elliptic plate, semi major axis		
of length a, semi minor axis of length b		
a/b = 1.0	10.22	10.22
= 1.5	17.20	17.20
= 2·0 = 3·0	27·74 58·68	27·74 58·68
= 3.0	36.09	36.06
Clamped rhombic plate, sides of length $a$ , an included angle of $\theta$		
$\theta = 75^{\circ}$	38 · 10	38 · 19
= 60°	46.03	46.17
= 45°	65.77	65 · 59
= 30°	122 · 79	121 · 29
Simply supported rhombic plate		
$\theta = 60^{\circ}$	24.23	23.70
= 45°	33-31	31.90
Rectangular plate, two opposite edges simply supported, two clamped. Length of clamped sides a, simply supported sides of length b		
b/a = 1.0	29 · 14	28.95
= 1.5	17.52	17.37
= 2.0	13.90	13 · 69
= 3.0	11.82	11.36
Circular plate of radius $a$ clamped on the boundary over angle $\theta$ , remaining		( 0.25)
boundary simply supported	4.00	$(\nu = 0.25)$ $4.87$
$\theta = 0^{\circ}$	4.98	
= 45°	6.01	5.87
= 90°	6.49	6.35
= 135° = 180°	7·00 7·59	6·88 7·51
= 180° = 225°	8.31	8.23
= 225° = 270°	9.13	9.12
= 2/0 = 315°	9.13	9.12
= 315 = 360°	10.22	10.22
_ 500	10.22	10 22

TABLE 1-continued

Problem	Equation (1)	Literature, e.g. Ref. 3
Annular plate free on the inner edge $r = b$ and either		
(a) clamped on outer edge $r = a$ ,		
a/b = 1.25	93.49	85.32
= 2.00	17.39	17.51
= 4.00	10.36	10.86
= 5.00	9.97	10.34
or (b) simply supported on outer edge $r = a$		
a/b = 1.25	9.72	9.43
= 2.00	5.117	5.04
= 4.00	4.58	4.625
= 5.00	4.625	4.726
Square plate, sides simply supported, free, simply supported and clamped (SS-F-SS-C)	12.01	12.69
Square plate, three sides simply supported, the other free (SS-SS-SS-F)	11.26	11-68

It is interesting to note that Equation (1) does not depend explicitly upon the geometry or the support conditions at the edges of the plate. Furthermore it should also be noted that, in order to obtain an estimate of the frequency, it is not necessary to perform any integration. For example, let us consider a square plate clamped on all of its sides. In this case the value of  $W'_{\rm max}$  is given by Timoshenko<sup>2</sup> as

$$W'_{\text{max}} = 0.00126 \frac{a^4}{D} \tag{2}$$

where a is the total length of a side of the plate. Substituting for  $W'_{\text{max}}$ , as given in Equation (2), into Equation (1), yields

$$\rho h \omega^2 = \frac{D(10 \cdot 2151)^2}{(64a^4) \cdot 00126} \tag{3}$$

and this subsequently gives

$$\omega a^2 \sqrt{\frac{\rho h}{D}} = 35.97 \tag{4}$$

which differs from the value of 35.99 given by Leissa,3 by less than 0.1%.

As a second illustration, let us consider the same rectangular plate but with two opposite sides clamped and the remaining sides simply supported. In this case the value of  $W'_{\max}$  given in Reference 2 is

$$W'_{\max} = \frac{0.00192a^4}{D} \,. \tag{5}$$

Substituting for  $W'_{max}$  in Equation (1) now gives

$$\omega^2 a \sqrt{\frac{\rho h}{D}} = 29 \cdot 14 \tag{6}$$

which differs by less than 1% from the value of 28.95 as given by Leissa.3

The two illustrative examples considered above clearly indicate the ease of aplpying the method, while Table 1 shows its remarkable accuracy.

It should be mentioned that the formula—Equation (1)—was discovered when the author was investigating the vibration of elliptical plates, and the constant was chosen so as to give the exact value of the fundamental frequency for the case of a clamped circular plate.

The ease and accuracy of this technique prompted Johns<sup>4</sup> to extend the method to the calculation of the frequency parameter for the first anti-symmetric mode. In this case the approximate expression

$$\rho h \omega^2 = \frac{40}{25\sqrt{5}\,\hat{W}_{\text{max}}}\tag{7}$$

was found to hold where  $\hat{W}_{max}$  is the maximum deflection of the plate due to a load distribution which is proportional to the distance from the centroid. The accuracy of this approximation can also be judged by examining Table 2.

TABLE 2
Frequency Parameter  $\omega a^2 \sqrt{\rho h/D}$  for the First Anti-symmetric Mode for Various Plate Problems

	Equation (7)	Literature, e.g. Ref. 3
Circular plate of radius a		
(1) clamped	21.90	21 · 20
(2) simply supported	14.00	13.90
Square plate sides of length a		
(1) clamped	75.30	73 · 40
(2) simply supported	47.60	49.30

#### 3. FORMULA FOR MEMBRANES AND SHELLS

Following the success of Johns'<sup>4</sup> extension of Jones'<sup>1</sup> original work, Jones<sup>5,6</sup> subsequently extended the approximation so as to predict the fundamental frequencies of membranes and shells of arbitrary shape. The governing equation was found to take the following generalized form

$$\rho h \omega^2 = A/W_{\text{max}} \tag{8}$$

where  $W_{\text{max}}$  is the maximum value of the corresponding static deflection mode, and A is a constant dependent upon the particular problem and the mode under consideration.

For plates and shallow shells,  $A = (10 \cdot 2151)^2/64$ , for the fundamental mode, and  $A = 40/25\sqrt{5}$  for the first antisymmetric mode, while when predicting the fundamental frequency of a membrane of arbitrary shape,

$$A = (2 \cdot 4048)^2/4. \tag{9}$$

When predicting the fundamental frequencies of membranes, plates and shells,  $W_{\text{max}}$  is the maximum deflection which the structure would experience when subjected to a uniform loading of unity. When predicting the frequency of the first antisymmetric mode, the load distribution is to be taken as proportional to the distance from the centroid.

The accuracy of Equation (7) in predicting the frequencies of plates has already been established in Tables 1 and 2. Its accuracy in predicting the fundamental frequency of membranes can be seen in Table 3 while Table 4 shows the accuracy of the method in predicting the fundamental frequency of a simply supported cylindrical shell of curvature k. Here the fundamental

frequency  $\omega$  has been compared to the exact frequency in the special case for which k=0 and  $\nu=0$ . The accuracy of the method can also be judged from Figures 1, 2 and 3 which show plots of the fundamental frequency of a shallow spherical shell versus the non-dimensional parameter  $ka^2/2h$ , where h is the shell thickness, a is the radius of the base of the shell and k is the curvature

TABLE 3
Fundamental Frequency  $\omega a^2 \sqrt{\rho h/E}$ \ of Various Shaped Membranes

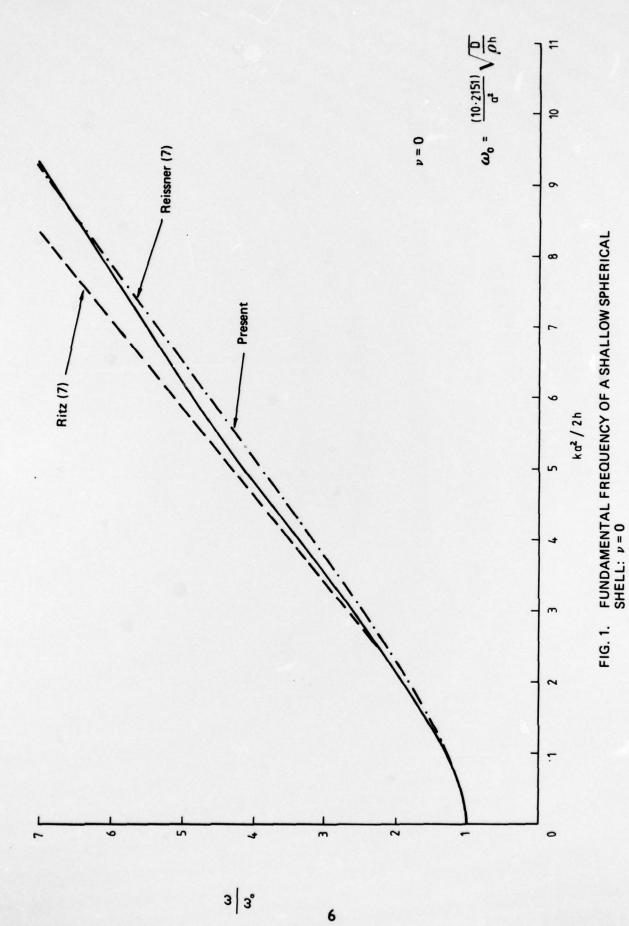
Problem	Equation (9)	% difference from the literature
Elliptical membrane major axis length a minor axis length b		
b/a = 1.0	2.405	0.0
= 1.2	2.134	0.0
= 1.5	2.044	0.0
= 2.0	1.792	0.0
= 5.0	1.734	0.0
= ∞	1.701	0.0
Rectangular; sides of length 2a, 2b		
b/a = 1.0	2.223	0.0
= 1·1	2.117	0.2
= 1 · 2	2.041	0.2
= 1.5	1.894	0.3
= 2.0	1 · 787	1.8
= 3.0	1.716	3.7
= 4.0	1 · 707	5.4
Semicircular radius a	3.848	0.3
Equilateral triangle	6.248	0.6

TABLE 4 Fundamental Frequency Parameter  $\omega a^2 \sqrt{\frac{12\rho}{E\pi^4h^2(1+a^2/b^2)^2}}$  for a Cylindrical Shallow Shell

ka²/h		1	2			3		4	5		
a/b	Pred.	Exact	Pred.	Exact	Pred.	Exact	Pred.	Exact	Pred.	Exact	
1	1.519	1.520	1.062	1.063	1.076	1.081	1.095	1.105	1.108	1.136	
1	1.072	1.072	1.136	1.140	10 1-221 1	1 · 246	1·246 1·308 1 1·351 1·513 1		1.389	1.536	
1	1.089	1.086	1.198	1.193	1.341	1.351			1.674	1.766	
ł	1.098	1.093	1.234	1.219	1.387	1 · 403	1.647	1.627	1.880	1.875	

of the shell. Exact values of the frequency are shown as given by Reissner<sup>7</sup> as well as the corresponding Ritz approximation (also given by Reissner<sup>7</sup>). In calculating the fundamental frequency of the spherical shell, exact values of the maximum deflection were not available and so use was made of the approximate expression, given in Reference 4, of

$$W'_{\max} = \frac{a^4}{128D} \left\{ \frac{1}{[1+k^2a^4(1+\nu)/8h^2]} + \frac{1}{(1+72k^2/6h^2)a^4(1+\nu)^2[0\cdot 1+(1+\nu)/18(1-\nu)]} \right\}. \tag{10}$$



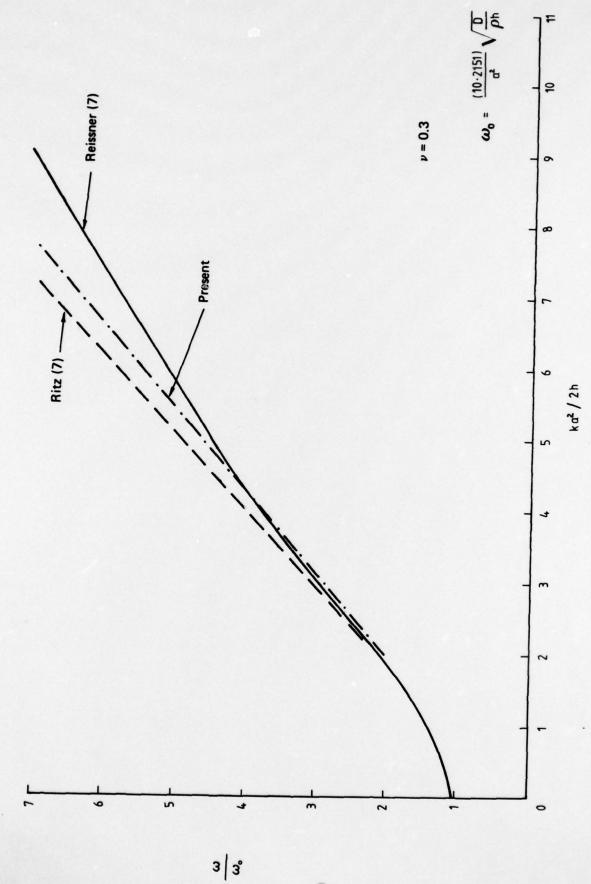
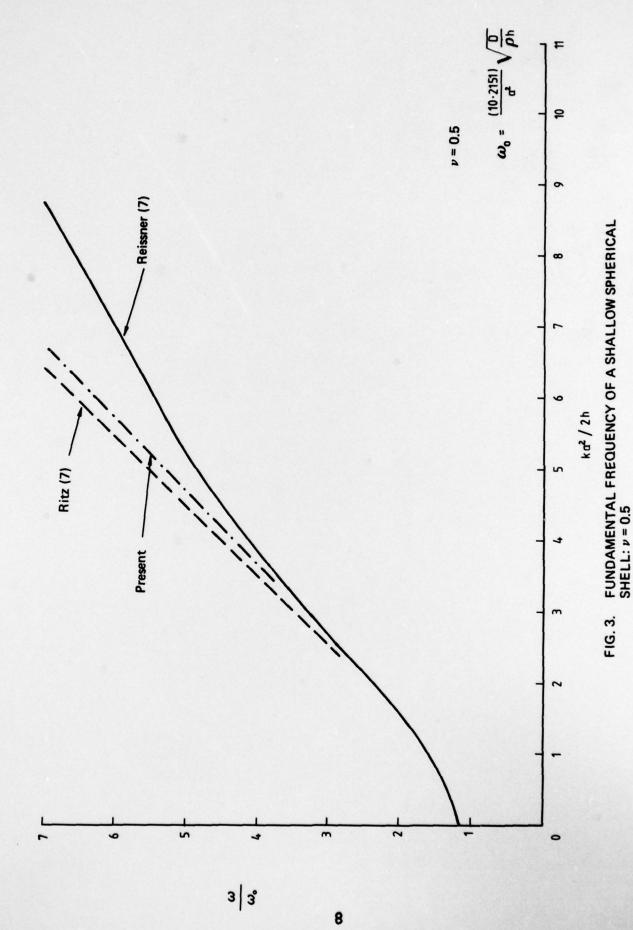


FIG. 2. FUNDAMENTAL FREQUENCY OF A SHALLOW SPHERICAL SHELL:  $\nu = 0.3$ 



Tables 3 and 4 and Figures 1, 2 and 3 can be found in either Reference 5 or in Reference 6. We thus see that Equation (8) represents an exceptionally simple means of estimating the fundamental frequency of plates, shells and membranes.

#### 4. FORMULAE FOR ANISOTROPIC PLATES

One very important structural form which has not yet been discussed is that of a plate consisting of orthotropic material. Let us now apply the approximate Equation (8) to the problem of a clamped elliptical plate made of composite material. In this case

$$W_{\text{max}} = \frac{a^4}{8(3D_x + 3\delta^4 D_y + 2\delta^2 H)}$$
 (11)

where  $\delta = a/b$  is the aspect ratio of the ellipse, a, b are the lengths of the major and minor axes of the ellipse and  $D_x$ ,  $D_y$ , H are the rigidities of the plate. This yields

$$\omega a^2 \sqrt{\rho h} = (10.2151) \left( \frac{3D_x}{8} + \frac{3\delta^4 D_y}{8} + \frac{2\delta^2 H}{8} \right)$$
 (12)

which is in exact agreement with the recent analytical results of Dharmarajan and Fang Hui Chou.<sup>8</sup> As a second illustration, let us consider the case of a simply supported rectangular composite plate with sides of length a and b respectively. In this case the fundamental frequency is known to be given by the relationship

$$\rho h \omega^2 = D_y \pi^4 (D_x / D_y a^4 + 2H / a^2 b^2 D_y + 1) \tag{13}$$

while the values of  $W'_{\text{max}}$  are found in Reference 3 for the particular case  $H = \sqrt{D_x D_y}$ . The values of the fundamental frequency  $\omega b^2 \sqrt{\rho h}/D_y$  thus predicted are shown in Table 5 along with exact values given by Equation (13) above. As in all previous cases, the agreement between these two sets of values is excellent.

TABLE 5
Fundamental Frequency.
Parameter  $\omega b^2 \sqrt{\rho h/D_y}$  for an Orthotropic, Simply Supported Rectangular Plate;  $\epsilon = a/b \sqrt[4]{D_y/D_x}$ 

$\stackrel{\epsilon}{\omega b^2}\sqrt{ ho h/D_y}$	1.0	1.5	2.0	3.0
Predicted	20.01	14.53	12.68	11.54
Exact	19.74	14.22	12.33	10.96

We thus see that the initial work of Jones<sup>1</sup> has led to a new technique for estimating the fundamental frequency of common structural elements. As can be seen in the recent review article of Leissa,<sup>9</sup> this procedure is one of the few new techniques which may be applied to plates of arbitrary plan form with mixed support conditions.

Although we have concentrated on the positive aspects of this technique, there are two areas in which the technique is lacking.

The most important lack is the development of a rigorous mathematical basis for the approximation. So far, the only work in which this has been attempted is that of Sundarajan. <sup>10</sup> Unfortunately it is the opinion of the author that Reference 10 fails to provide this mathematical basis.

Again, the approximation fails in the field of large amplitude vibrations. The author has attempted to apply the approximation to the large amplitude vibration of plates but found that the predicted frequencies were in considerable disagreement with the values given in Reference 3. The method at present can only be applied when vibration amplitudes are small.

#### 5. CONCLUSIONS

We have seen that the initial approximation developed by Jones<sup>1</sup> can be significantly extended so that it now provides a very simple, yet accurate, means for calculating the fundamental frequencies of membranes, plates and shallow shells vibrating at small amplitudes.

#### REFERENCES

- Jones, R. An approximate expression for the fundamental frequency of vibration of elastic plates. J. Sound and Vibration 38, 503-504, 1975.
- Timoshenko, S. P., and Wornowsky Krieger, S. Theory of plates and shells. McGraw Hill, 2nd edition, 1959.
- 3. Leissa, A. W. Vibration of plates. NASA SP-160, 1969.
- Johns, D. J. Comments on "An approximate expression for the fundamental frequency of vibration of elastic plates". J. Sound and Vibration 41, 385, 1975.
- Jones, R. Approximate expressions for the fundamental frequency of vibration of several dynamic systems. J. Sound and Vibration 44, 475-78, 1976.
- Jones, R. Approximate methods for the linear and non-linear analysis of plates and shallow shells. J. Structural Mechanics 3, 5, 1977.
- Reissner, E. On axisymmetrical vibrations of shallow spherical shells. Q. Appl. Maths. 13, 279-90, 1955.
- Dharmarajan, S. and Fang Hui Chou. Vibration of composite plates of arbitrary shape by the method of constant deflection lines. Shock and Vibration Bulletin, Part 2, 99-106 September 1977.
- Leissa, A. W. Recent research in plate vibrations: classical theory. The Shock and Vibration Digest 9, 13-24, October 1977.
- 10. Sundarajan, C. Relationship between the fundamental frequency and the static response of elastic systems. J. Sound and Vibration 51, 4, 493-499, 1977.

#### DOCUMENT CONTROL DATA SHEET

													_		_
1	Document Num	bers					2. 5	ecur	ity C	lassif	icatio	n			
	(a) AR Number	r:													
	AR-001-290	)					(a) Complete document: Unclassified								
	(b) Document S	eries an	d Nu	ımber	:		(b) Title in isolation:								
	Structures R	eport 37	71					U	nclas	sified	1				
	(c) Report Num				_		(c) Summary in isolation:								
	ARL-Struc.	-Repor	t-37	1 -				U	nclas	sified					
3.	Title: SIMPLE FREQU	FORM	IUL.	AE F	OR	CAL S, SH	CUL	ATIN	NG T	HE MEM	VIBI BRA	RATI	ONA	L	
4.	Personal Author	(s):				_	5. I	Oocu	ment	Date	:				_
R. Jones							J	une,	1976		/				
							6. 7	ype	of Re	port	and	Perio	d Co	vered	:
7.	Corporate Author	or(s):					8. I	Refer	ence	Num	bers				_
	Aeronautical Research Laboratories						8. Reference Numbers (a) Task:								
_							(b) Sponsoring Agency:								
9	Cost Code: 21 6901														
10.	Imprint						11. C								
	Aeronautical Res Melbourne	search L	abor	atorie	es,		(	i itie(	s) and	d lan	guage	(s)):			
12.	Release Limitation Approved for pu			ocum	ent)										
12-	0. Overseas:	No.		P.R.	1	A		В		С		D		Е	
13.	Announcement L	imitatio	ns (c	of the	info	mati	on on	this	page	): No	limi	tatio	n		
14.	Descriptors:									15	. Co	sati C	Codes	: 201	1
	Vibration			Mer	nbra	nes								120	1
	Frequencies						athem	atics	)					131	3
	Shells (Structural				nputa										
	Plates (Structural	membe	rs)	Vib	ration	ana	lysis								
16.					A	BSTR	ACT								
	Simple appro	ximate	form	ulae j	for es	stima	ting t	he vi	bratio	nal f	reque	ncies	of p	lates,	
	shells and membro	anes are	prese	ented.	Thes	e fore	nulae	take	the g	enera	l for	n F =	= A/S	Vmax	
	where F is a freque														
	for many different	problen	is, ar	nd Wn	nax is	the n	naxim	um d	efleci	on o	f an a	ssoci	ated s	tatic	
	deflection problem	1.													
	The approach	ı. h is par	ticul	arly s	imple	e and	the	agree	ment	betv	veen	the f	reque	ncies	

#### DISTRIBUTION

AUSTRALIA	Copy No.
DEPARTMENT OF DEFENCE	
Central Office Chief Defence Scientist Executive Controller, ADSS Superintendent, Defence Science Administration Australian Defence Scientific and Technical Representative (UK) Counsellor Defence Science (USA) Defence Library Assistant Secretary, DISB JIO	1 2-3 4 5 6 7 8-23 24
Aeronautical Research Laboratories	
Chief Superintendent	25 26
Library Superintendent, Structures Division	27
Divisional File, Structures Division	28
Author: R. Jones	29
Materials Research Laboratories Library	30
Defence Research Centre Library	31
Central Studies Establishment Library	32
Engineering Development Establishment Library	33
RAN Research Laboratory Library	34
Defence Regional Office Library	35
Air Office	
Air Force Scientific Adviser	36
Engineering (CAFTS) Library	37 38
D. Air Eng.	39
RAAF Point Cook Aircraft Research and Development Unit	40
Army Office Army Scientific Adviser	41
	42
US Army Standardisation Group	43
Navy Office Naval Scientific Adviser	44
Royal Military College US Army Standardisation Group Navy Office	4

DEPARTMENT OF PR	ODUCTIVITY		
Government Aircraft		45	
DEPARTMENT OF TR	ANSPORT	46	
	Secretary/Library		
Airworthiness Group	(Mr. R. Ferrari)	47	
STATUTORY, STATE	AUTHORITIES AND INDUSTRY		
Australian Atomic Energy Commission, Director		48	
CSIRO Central Library		49	
CSIRO Division of Mechanical Engineering, Chief		50	
CSIRO Materials Science Division, Director		51	
Applied Engineering Pty. Ltd. Commonwealth Aircraft Corporation, Manager		52	
	53		
Commonwealth Aircraft Corporation, Manager of Engineering Conzinc Riotinto of Australia (Dr. Worner)		54 55	
Hawker de Havilland Pty. Ltd., Librarian, Bankstown		56	
	Hawker de Havilland Pty. Ltd., Manager, Lidcombe		
ICI Australia Ltd., L		57 58	
	alia Pty. Ltd. (Mr. O. Mosley)	59	
UNIVERSITIES AND C	OLLEGES		
Adelaide	Barr Smith Library	60	
	Prof. J. Mazundar, Applied Mathematics	61	
Australian National	Library	62	
Flinders	Library	63	
James Cook	Library	64	
La Trobe	Library	65	
Melbourne	Engineering Library	66	
Monash Newcastle	Library Library	67 68	
New England	Library	69	
New South Wales	Library	70	
New South Wates	Assoc. Prof. R. W. Traill-Nash	71	
Sydney	Engineering Library	72	
by anot	Dr. W. B. Fraser, Applied Mathematics	73	
Queensland	Library	74	
	Dr. V. G. Hart	75	
Tasmania	Engineering Library	76	
Western Australia	Library	77	
	Dr. P. B. Chapman	78	
RMIT	Library	79	
CANADA			
CANADA  Physics and Matallus	gy Research Labs. (Dr. A. Williams)	80	
CAARC Co-ordinate		81	
	National Aeronautical Establishment NRC, Library		
	al Engineering NRC, Director	82 83	
Universities and College			
McGill	Library	84	
Toronto	Institute of Aerophysics	85	
FRANCE			
ONERA, Library		86 87	
AGARD, Library			
Service de Documentation, Technique de l'Aeronautique			

GERMANY		
ZLDI	89	
INDIA		
CAARC Co-ordinator, Structures		
Civil Aviation Department, Director		
Defence Ministry, Aero Development Establishment, Library		
Hindustan Aeronautics Ltd., Library		
Indian Institute of Science, Library Indian Institute of Technology, Library		
National Aeronautical Laboratory, Director	95 96	
ISRAEL		
Technion—Israel Institute of Technology, Prof. J. Singer	97	
ITALY		
Fiat Co., Dr. G. Gabrielli	98	
Associazione Italiana di Aeronautica e Astronautica (Prof. A. Evla)	99	
JAPAN		
National Aerospace Laboratory, Library	100	
Universities		
Tohoku (Sendai) Library	101	
Tokyo Institute of Space and Aerospace	102	
NETHERLANDS		
Central Org. for Applied Science Research TNO, Library	103	
National Aerospace Laboratory (NLR) Library	104	
NEW ZEALAND		
Air Department, RNZAF Aero Documents Section	105	
Universities		
Canterbury Library	106	
SWEDEN		
Aeronautical Research Institute	107 108	
Chalmers Institute of Technology, Library		
Kungl. Tekniska Hogscholens		
SAAB, Library Research Institute of the Swedish National Defence	110 111	
Research institute of the Swedish National Defence	***	
SWITZERLAND		
Brown Boverie, Management Chairman	112 113	
Escher-Wyss Ltd., Manager Institute of Aerodynamics ETH		
Histitute of Actodynamics 2111	114	
UNITED KINGDOM	116	
CAARC NPL, Secretary Royal Aircraft Establishment, Library, Farnborough	115 116	
Royal Aircraft Establishment, Library, Parnoorough Royal Aircraft Establishment, Library, Bedford		
Military Vehicles and Experimental Establishment		
National Engineering Laboratories, Superintendent		
British Library, Science Reference Library		
British Library, Lending Division		
Naval Construction Research Establishment, Superintendent		
CAARC Co-ordinator, Structures	123	

Eulmas Dasaarch Instit	tuta I td. Dassarch Director	124
Fulmer Research Institute Ltd., Research Director Ricardo ii. Co., Manager		
Rolls Royce (1971) Ltd., Aeronautics Div. (Chief Librarian)		
Science Museum (Library)		126 127
	ration (Holdings) Ltd., Military Aircraft Div.	128
Universities and Colleges		
Bristol	Library, Engineering Dept.	129
	Dr. W. Chester, Mathematics Dept.	130
Cambridge	Library, Engineering Dept.	131
	Sir William Hawthorne, Engineering Dept.	132
Manchester	Professor of Applied Mathematics	133
Nottingham	Library	134
Southampton	Library	135
Strathclyde	Library	136
Cranfield Institute of	Library	137
Technology	The Mand	120
Imperial College	The Head	138
Y amakhasamah Ilai	Prof. B. G. Neal, Struct. Engineering Prof. D. J. Johns, Dept. of Transport Engineering	139 140
Loughborough Uni. of Technology	Prof. D. J. Johns, Dept. of Transport Engineering	140
UNITED STATES OF A		
	Technical Information Facility	141
Sandia Group Research Organisation		142
	Aeronautics and Astronautics	143
Applied Mechanics Review		144
The John Crerar Library		145
Allis Chalmers Inc. (Director)		146
Boeing Co., Industrial Production Division		147
Esso Research Laboratories (Director)		148
Westinghouse Laboratories (Director)		149
United Technologies Corp. Pratt and Whitney Aircraft Group Battelle Memorial Institute (Library)		150 151
	nute (Library)	131
Universities and Colleges Brown	Prof. R. E. Meyer	152
Cornell (New York)	Library, Aeronautical Laboratories	153
Florida	Aero. Engineering Department	154
Harvard	Prof. G. F. Carrier, Applied Mathematics	155
	Dr. S. Goldstein	156
Johns Hopkins	Prof. S. Corrsin	157
Iowa State	Dr. G. K. Serory, Mech. Eng. Dept.	158
Princeton	Prof. G. L. Mellor, Mechanics Department	159
George Washington	Prof. Freudenthal	160
Brooklyn	Library, Polytech. Aero. Lab.	161
California	Library, Guggenheim Aero, Lab.	162
	Dr. M. Holt, Dept. of Aerosciences	163
Ohio State	Prof. A. W. Leissa, Dept. of Mech. Eng.	164
Illinois	Prof. N. M. Newmark, Talbot Laboratories	165
Massachusetts	Prof. W. A. Nash, Dept. of Civil Engineering	166
Stanford	Dept. of Aeronautics, Library	167
Wisconsin	Memorial Library, Serials Dept.	168

Spares 169-178