1.0
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DEPARTMENT OF PSYCHOLOGY

The University of Michigan, Ann Arbor

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Intersession Variability and Stress: A Monte Carlo Study.

ELLIOT NOMA

N00014-76-C-0648

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Technical Report. No. 63

June 1979

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INTERSESSION VARIABILITY AND STRESS:

A MONTE CARLO STUDY

Elliot Noma

HUMAN PERFORMANCE CENTER TECHNICAL REPORT NO. 63

June, 1979

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Two Monte Carlo studies explore the relation of the tau measure of inter-session response variability and the stress of the corresponding multi-dimensional scaling solution, thereby providing a statistical basis for evaluating the goodness-of-fit of a spatial configuration. In the first Monte Carlo study, the stress and tau of 10-, 16-, and 30-point configurations in 1, 2, 3, and 4 dimensions are shown to be linear functions of the internal error level. In the second study, these relations are shown to be relatively invariant with respect to the particular configurations. Three methods are

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proposed for establishing acceptable levels of stress for heuristic and for constrained multidimensional scaling.

Intersession variability and stress:
A Monte Carlo study [1]

Elliot Noma

The University of Michigan

April, 1979

Abstract

Two Monte Carlo studies explore the relation of the tau measure of inter-session response variability and the stress of the corresponding multidimensional scaling solution, thereby providing a statistical basis for evaluating the goodness-of-fit of a spatial configuration. In the first Monte Carlo study, the stress and tau of 10-, 16-, and 30-point configurations in 1, 2, 3, and 4 dimensions are shown to be linear functions of the internal error level. In the second study, these relations are shown to be relatively invariant with respect to the particular configurations. Three methods are proposed for establishing acceptable levels of stress for heuristic and for constrained multidimensional scaling.

Statistical guidelines for evaluating stress are essential for the successful application of multidimensional scaling. For this reason, Monte Carlo studies have been published establishing correspondences between internal error levels and stress values (Young, 1970; Sherman, 1972; Spence & Graef, 1974; Cohen & Jones, 1974). Unfortunately, all previous investigations have short-comings (see Arabie, 1973): 1) inflated stress values due to local-minimum prob-lems, 2) scaling of interpoint distances plus noise or random rank order dissimi-larities does not guarantee recovery of the interpretation of the original configura-tion, 3) currently there is no way to independently estimate the error level to the distribution of stresses, and 4) all previous Monte Carlo studies concentrate only on local-minimum solutions, but scaling with constraints (Noma & Johnson, 1977) often produces suboptimal-stress solutions. In this paper a method around these complications is proposed.

In section 2, it is argued that the latent configuration is best assumed equivalent to the scaled configuration. This assumption avoids both inflated stresses due to recovery of suboptimal solutions and the recovery of non-representative solutions. The Monte Carlo methodology relating error to stress and error to intersession variability is introduced in section 3. In section 4, the results of two Monte Carlo studies are presented. Ways of applying intersession variability to evaluate stress appear in section 5.

2. The Latent Configuration

The multidimensional scaling methodology has been applied in two ways: 1) heuristic standard multidimensional scaling searches for structures in the data; 2) constrained multidimensional scaling with constraints emphasizes hypothesis testing. When multidimensional scaling is used as a heuristic tool, it is customarily assumed that the algorithm constructs a configuration that approximates a latent or "true" configuration. Also, only one scaled configuration is of

interest: the local-minimum solution. Scaling with constraints, however, produces configurations that are often suboptimal in terms of stress level. In addition, from a single dissimilarity set, many different configurations are produced by varying the constraints placed on interpoint distances (Borg & Lingoes, 1978), point coordinates (Bentler & Weeks, 1978; Bloxom, 1978), or order of point coordinates (Noma & Johnson, 1977). Each configuration may also have a stress comparable to that of the local-minimum solution yet illuminate a different structure in the data. This means that potentially many configurations could be representative of structure in the data. Since any one of these configurations, or none of them, may be the latent configuration, the latent configuration is best defined as the configuration produced by the scaling algorithm. This simplifying assumption also allows the separation of the recovery of the original structure and the production of the lowest attainable stress level. That is, by dictating that the structure is perfectly recovered, the stress may be examined alone. Also there is no possibility of suboptimal stress for a given dissimilarity set.

3. Methodology

By equating the latent and scaled configurations, the question is, given a configuration (C), how much noise must be added to the interpoint distances (D) to produce a given stress (S_1). That is, a matrix of interpoint distances plus noise (denoted by D_{Σ}) is computed for a given configuration. The matrix and the given configuration are input to a scaling program which computes a stesss value after zero iterations.

To generate the distance plus noise matrix the procedure described by Sherman (1972; Hefner, 1958; Ramsay, 1969) is used. Briefly, the procedure may be summarized as follows: 1) After specifying the number ofpoints (N) and dimensionality (d), a configuration is randomly generated in a d-dimensional

unit hypercube. 2) A number called the level of noise is computed by multiplying a specified error level (E), times the variance of the N*d coordinates (σ_c^2). 3) The elements of the dissimilarity matrix are generated by adding noise to the Euclidean distance between all $N(N-1)/2$ pairs of points:

$$d_{ij} = \sum_{k=1}^{k} (x_{ik} - x_{jk} + \varepsilon_{ijk})^2$$

where ε_{ijk} is a random variable distributed as $N(0, 2\sigma_c^2 E^2)$. 4) From these dissimilarities, the stress of the latent configuration is computed:

$$S_1 = f(C, D_\Sigma)$$

For a given number of points, dimensionality, and configuration, many simulated dissimilarity sets at a given error level will map out a distribution of stress values.

One measure of noise in the data to be scaled is the intersession variability. Due to the assumed ordinal nature of the input to the multidimensional scaling algorithm, the tau statistic (Kendall, 1962) is used as the measure of the correlation between dissimilarity sets from one session to another. By averaging taus from all pairs of dissimilarity sets one can derive the expected error level. For instance, intersession taus near unity imply that the error level is low so only configurations with near-zero stresses are acceptable. Configurations with stresses outside acceptable error bounds are considered inadequate representations of the data.

4. Results

Two Monte Carlo studies were done. The first characterizes the relationship of error level to mean stress and mean tau for specific configurations. The second determines the sensitivity of the error-stress and error-tau relationships

to different configurations.

In the first, arbitrary configurations were chosen with 10, 16, and 30 points in 1, 2, 3, and 4 dimensions. For each of the 12 configurations, five dissimilarity sets were generated at error levels increasing from E = .025 by steps of .025. Figures 1 and 2 show typical error-tau and error-stress relationships from E = .025 to E = 1.5. Note that the functions are nearly linear up to about E = .750 before reaching asymptotes at stress = 45% and tau = 0. These relationships seem to typify all curves produced since all regressions using E values in the range .025 to .500 had correlations in excess of .94. Since scaling solutions would be excluded from further analysis with stress over 45% or intersession tau near 0, all further analysis was done for error levels from .025 to .5.

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Figures 1 and 2 about here
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The second Monte Carlo study explores the relation of the slopes of the error-stress and error-tau functions to the number of points, dimensionality, and specific configurations. Fifty dissimilarity sets were generated at each combination of 5 different random configurations at N=10, 16, 30, d=1, 2, 3, and 3 error levels (the error levels were picked to produce mean taus of approximately .5, .75, and .9 as predicted by the regression coefficients obtained in the first Monte Carlo study). For each of the 135 configurations, (5 x 3 x 3 x 3), means and variances of the stress distributions were computed for the set of 50 dissimilarities. To save computation, taus were computed only between the first 15 of the 50 dissimilarity sets. Means and variances of these 105 values, (15(15 - 1)/2), were computed for each configuration.

In accordance with the results of the first Monte Carlo study, for each

ERROR

TAU
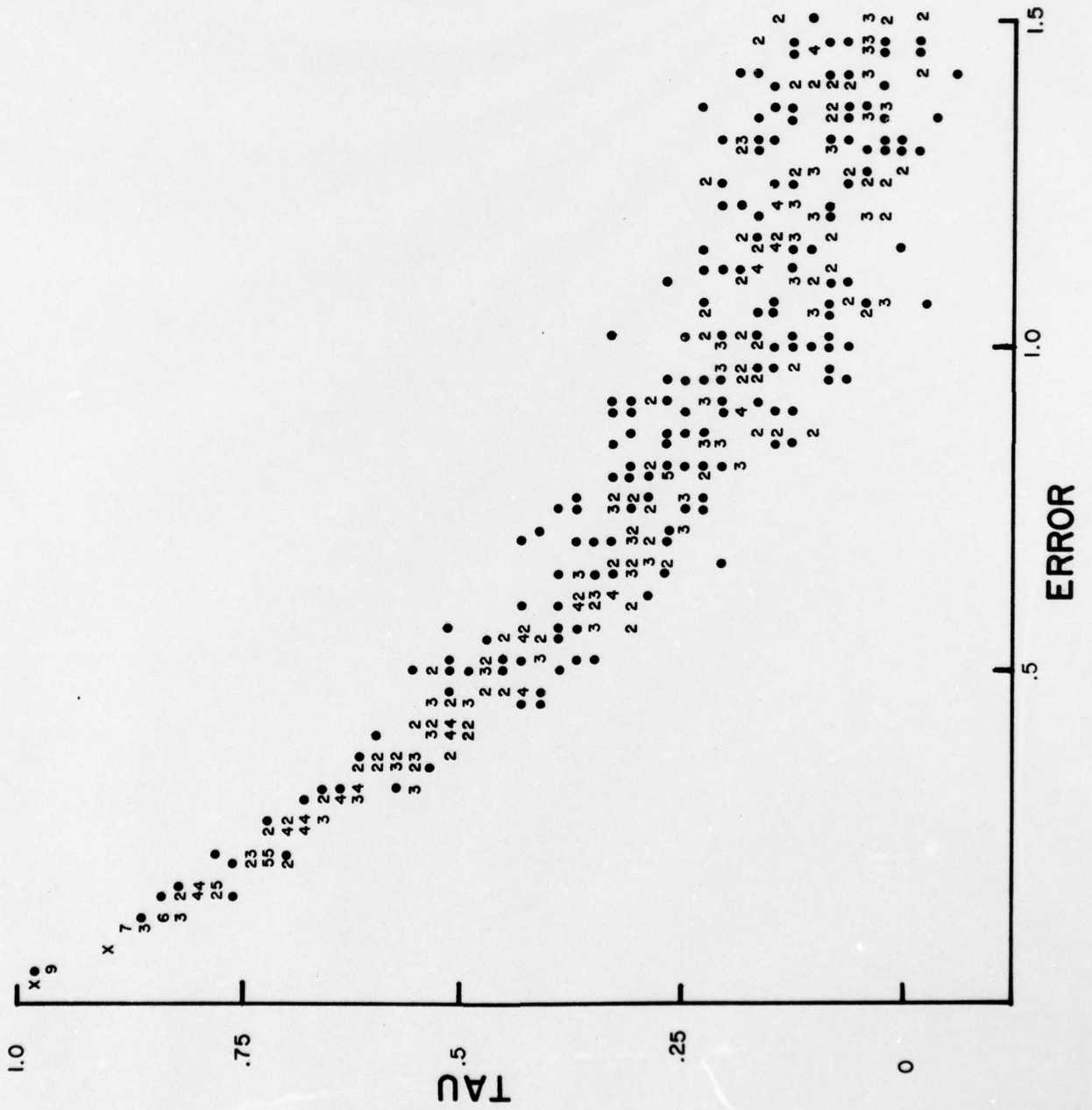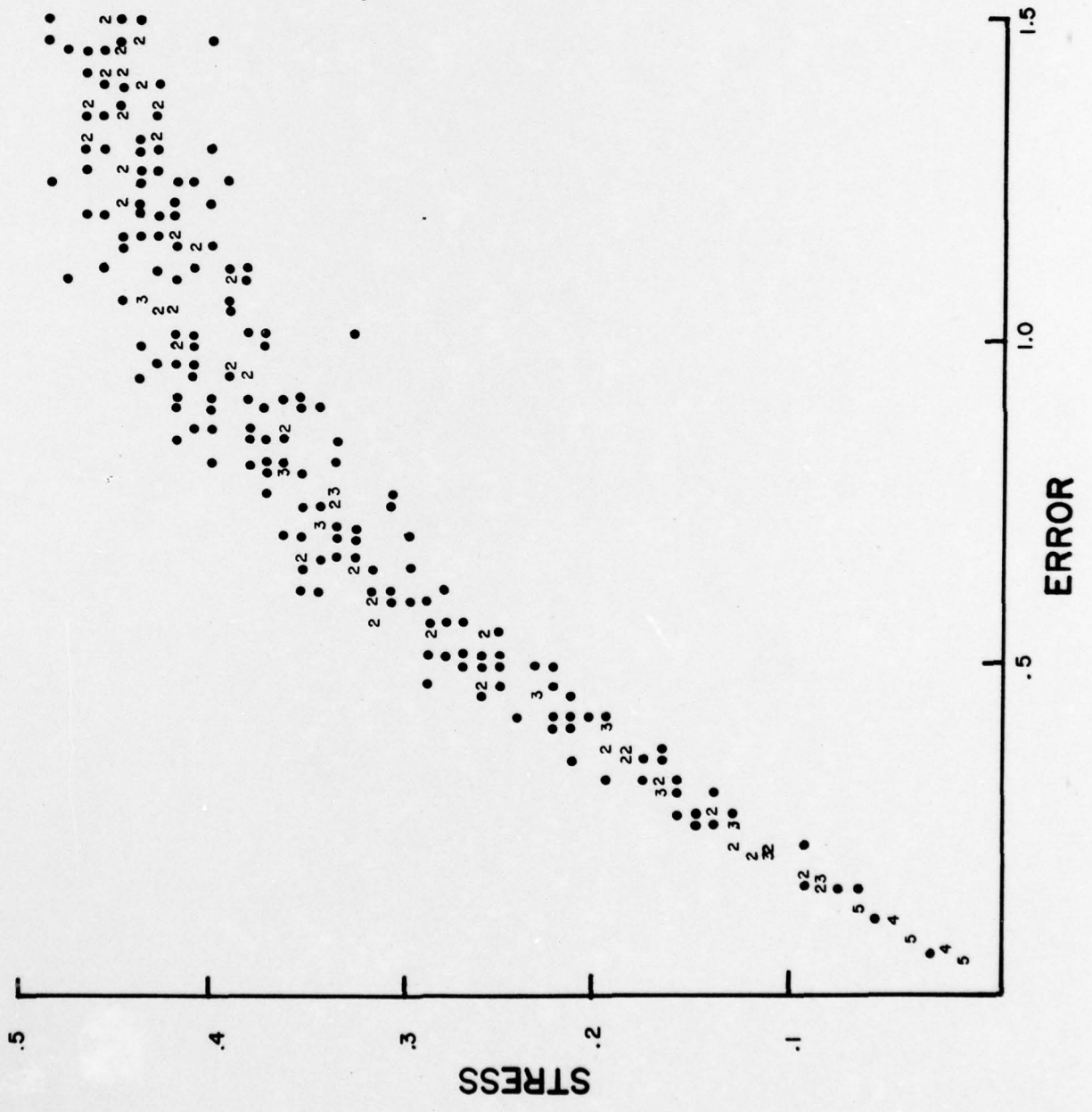
fixed level of N and d, the error-mean stress and error-mean tau correlations were all in excess of .98. However, the 27 analyses of variance of the stress distributions across the five configurations for a given N, d, and error level were all significant at $p < 05$. Therefore it must be concluded that the two functions, the one relating error and mean stress and the one relating error and mean tau, are only relatively invariant with respect to specific configurations. Two other issues are of interest: the mean tau-to-mean stress relation and the variance of the taus and stresses for a given level of tau and stress. To estimate the slope of the tau-stress function, $\frac{\mu \text{Stress}}{1-\mu \text{tau}}$ was computed for each of the 135 configurations. Figure 3 shows the geometric means of these values for all combinations of N and d. The log ($\frac{\mu \text{Stress}}{1-\mu \text{tau}}$) values are then regressed on log (N) and log (d) yielding the following equation:
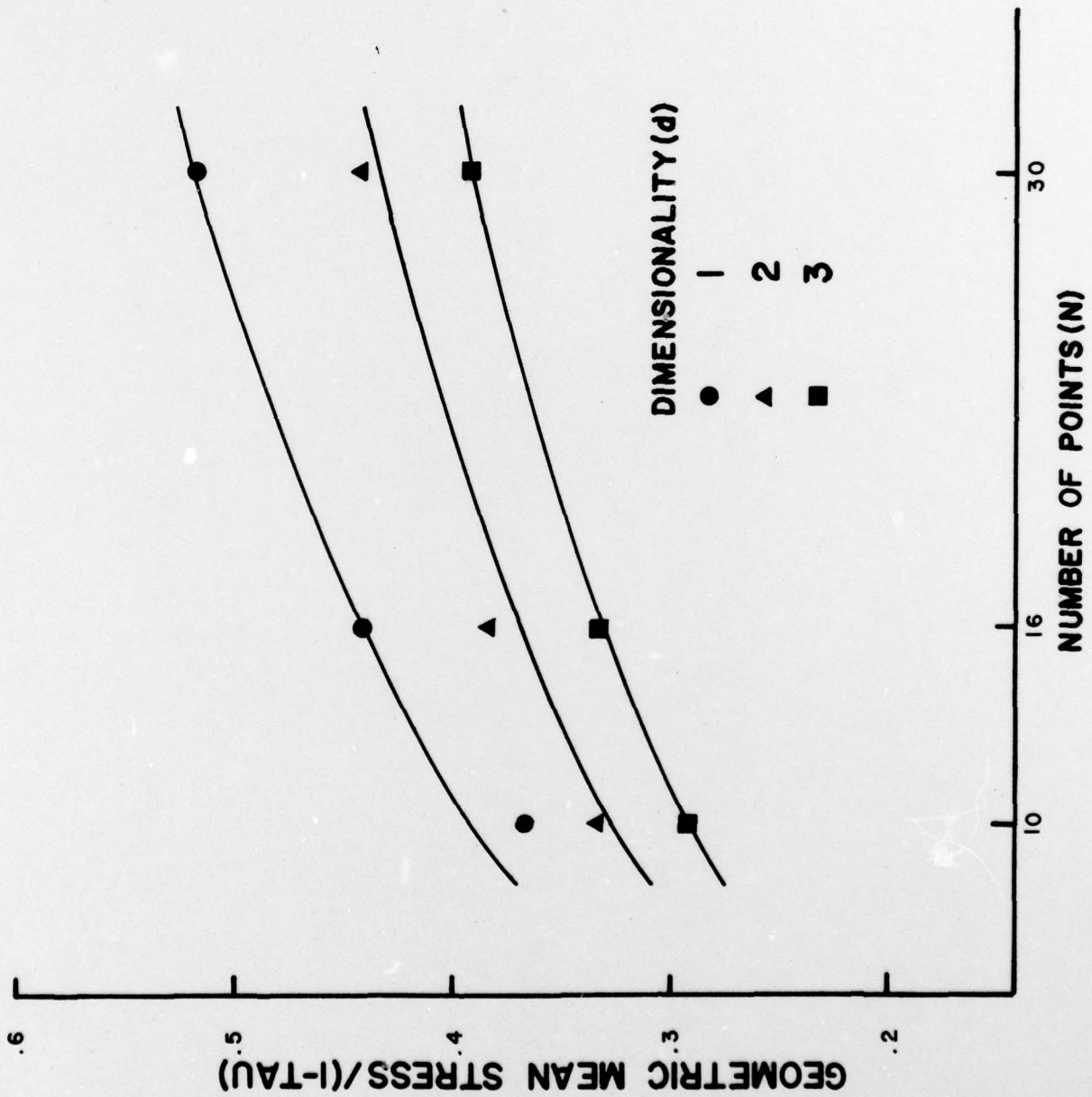
$$(1) \qquad \frac{\mu \text{Stress}}{1-\mu \text{tau}} = e^{-1.5301} N^{.25377} d^{-.25498} \quad r = .729$$

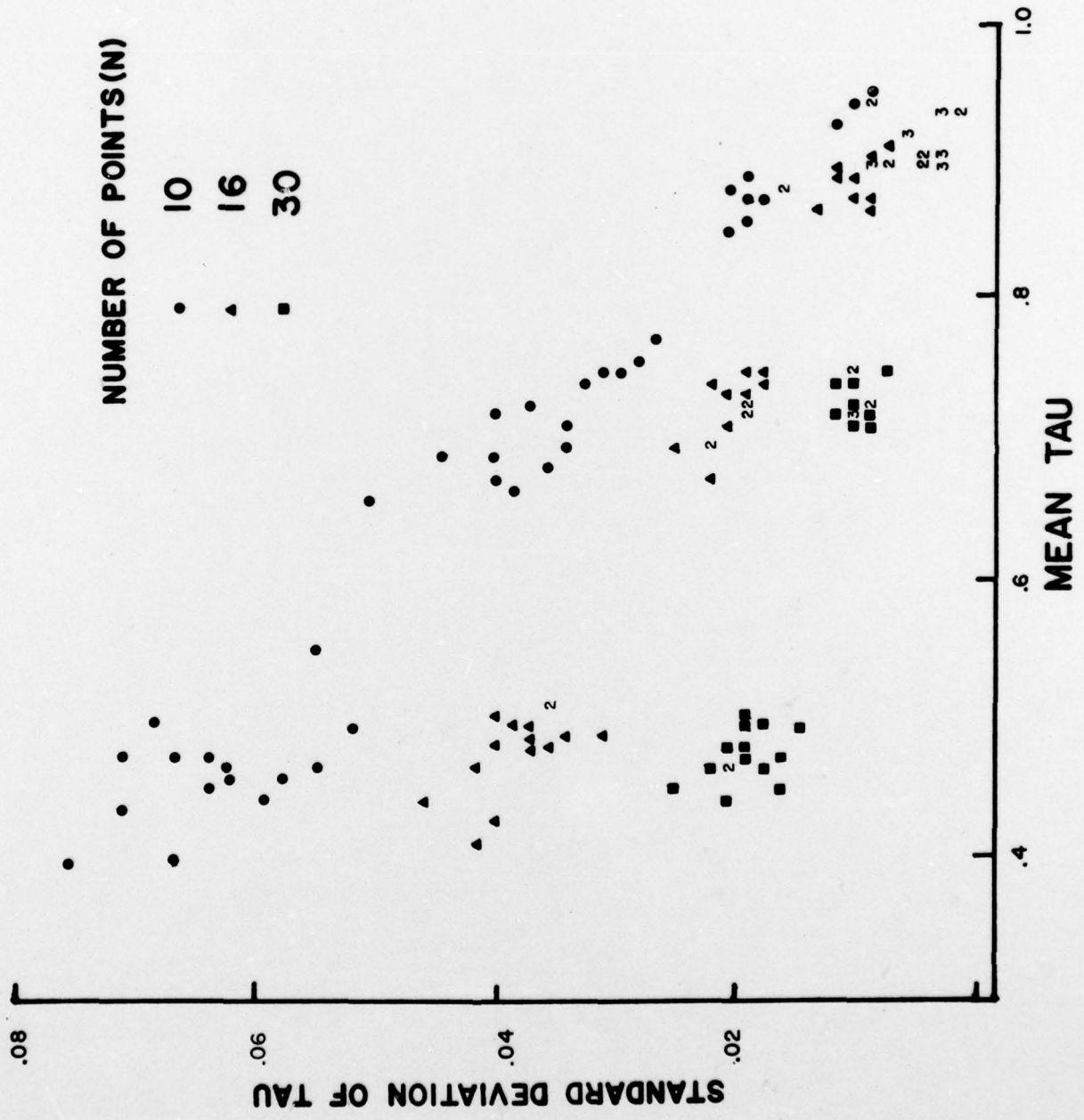Similarly, the standard deviation-to-mean ratios of tau and stress (see Figures 4 and 5) were computed as these equations:
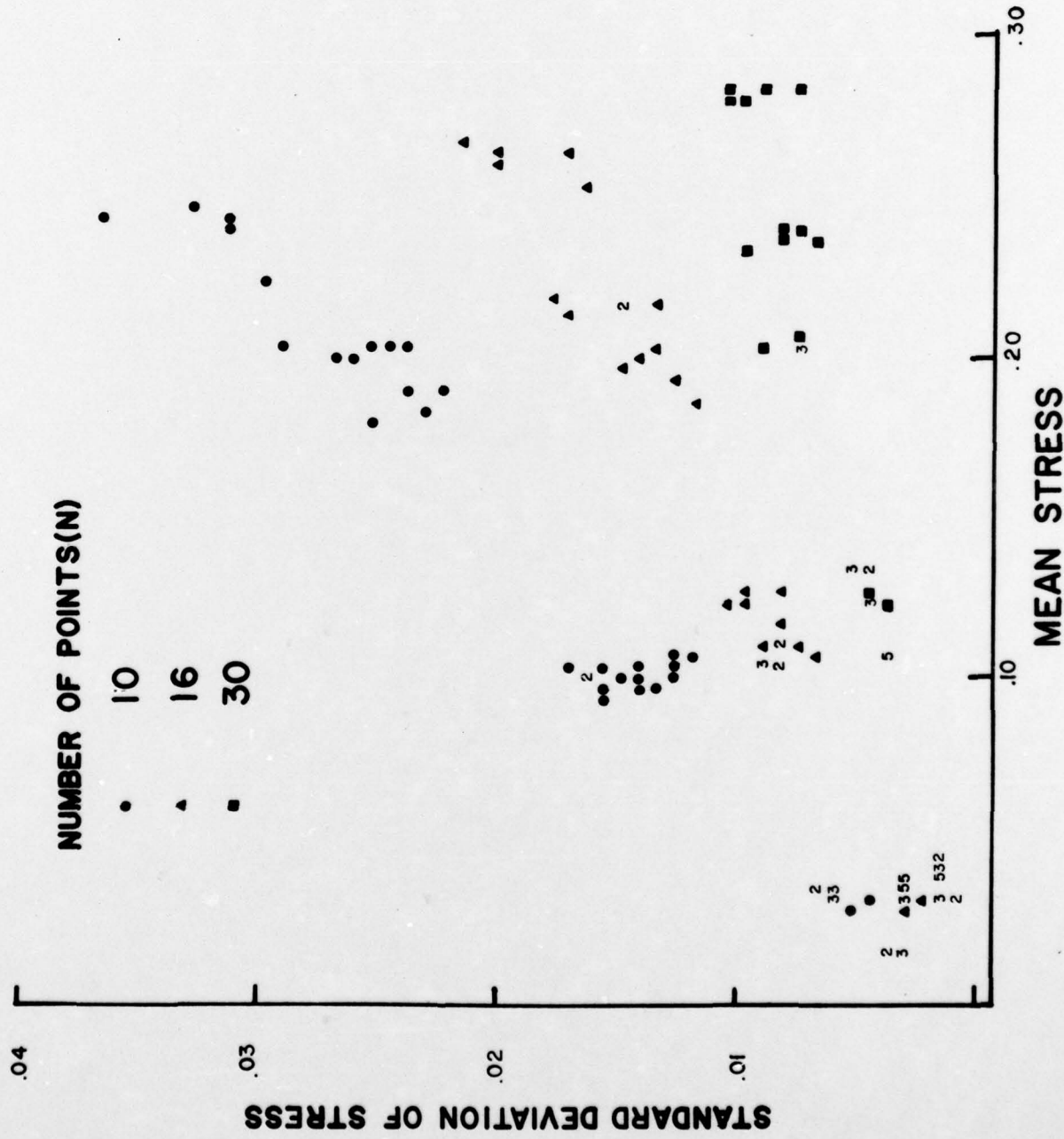
$$(2) \qquad \frac{\sigma \text{tau}}{1-\mu \text{tau}} = e^{.66718} N^{-1.1713} \qquad r = .970$$

$$(3) \qquad \frac{\sigma \text{Stress}}{\mu \text{Stress}} = e^{1.1278} N^{-1.3123} \qquad r = .964$$

Figures 3, 4, and 5 about here

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5. Discussion

Previous techniques for statistically evaluating stress are inadequate for a variety of reasons. Fixed criteria (Kruskal, 1964) are affected by the number of points and the dimensionality. "Looking for the elbow" requires the existence of such an elbow. Other Monte Carlo studies have the shortcomings of inflated stresses due to local-minimum problems (Arabie, 1973; Spence, 1974). Evaluating the output of constrained multidimensional scaling programs is even more difficult since the scaled configurations are usually not a local-minimum solution. Therefore all previous Monte Carlo studies are inappropriate since they deal only with local-minimum solutions. Attempts to extend the Monte Carlo results by counting the number and type of constraints also appear inadequate (see Noma & Johnson, 1977). In this section, three different methods are proposed for establishing acceptable bounds on stress in heuristic multidimensional scaling. The first two are also applicable to constrained multidimensional scaling.

All three methods are based on comparisons of mean stress and mean tau. To compute mean stress $(\overline{S}_1)$, a single configuration (C') is produced using some average of responses over replications in a two-way analysis or a group space from a three-way analysis (e.g. INDSCAL - Carroll & Wish, 1974). The stresses are then computed for each of the r replications with the same configuration:

$$S_{1i} = f(C', D_i) \qquad i = 1,\ldots,r$$

and the mean stress is computed. Mean tau $(\overline{\tau})$ is computed by averaging the taus for all pairs of replications:

$$\tau_{ij} = (D_i, D_j) \qquad i = 1, \ldots, r \qquad j = 1,\ldots,r \qquad i \neq j$$

In method one, the tau predicts the mean stress $(\overline{S}_1)$ using either equation 1 or the appropriate ratio of $\dfrac{\mu_{Stress}}{1-\mu_{tau}}$ in Figure 3. The empirical mean stress

($\overline{S}_1$) must fall within specified confidence bounds of $\hat{S}$ for the configuration to be acceptable.

In method two, varying amounts of error are added to the interpoint distances (D_Σ) of the scaled configuration (C') to determine an error stress curve. Assuming this curve is linear within a reasonable range of error values, the error value ($\hat{E}$) for the empirical stress ($\overline{S}_1$) is derived from the regression equation. The range of compatible taus is then easily computed using the formula (see Figure 6):

$$\hat{\tau} = -1.2725 \ \hat{E} + 1 \qquad\qquad r = .987$$

and the variance of τ is found by using equation (2).

Figure 6 about here

The third method can be applied only to scaled local-minimum solutions. In contrast to the first two methods, no assumptions are made as to the relationship between the latent and the scaled configurations. One only assumes that a latent configuration exists. Previous Monte Carlo studies (e.g. Sherman, 1972) are first used to estimate the error level ($\hat{E}$) given the mean stress ($\overline{S}_1$). This error level is then used, as in method two, to determine a range of acceptable taus.

In all three methods, stresses that are too high indicate an inadequate configuration. In this case, an attempt should be made to scale the configuration in a higher dimensional space. Stresses that are too low indicate a fit that is too good and the scaling should be done in a lower dimensional space or with constraints.

r=.987

TAU = -1.2725·ERROR + 1

MEAN TAU

ERROR

1.0   .8   .6   .4

.200   .400   .600
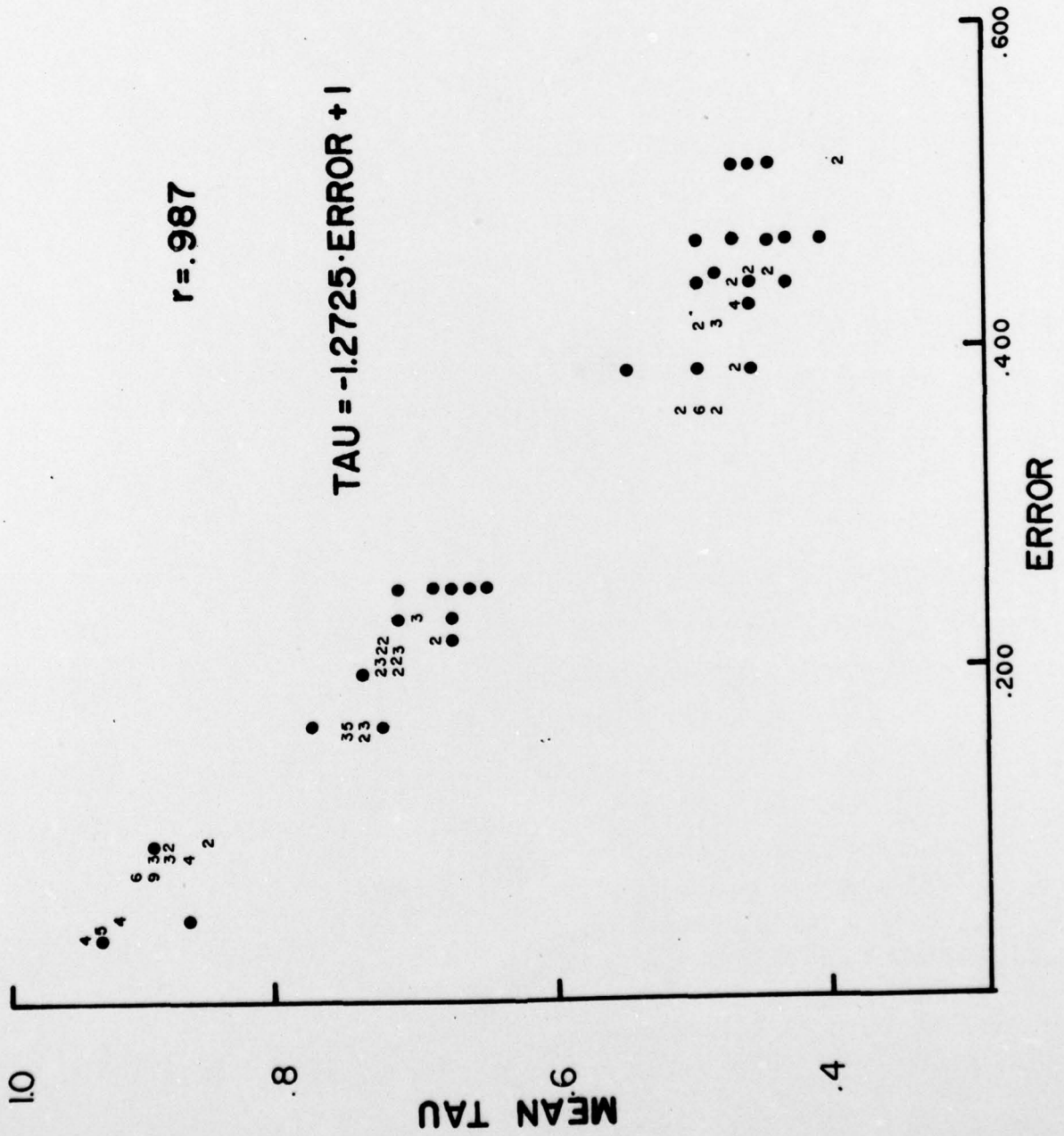
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Footnote

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Figure Captions

1. Simulated taus for an arbitrary 16 point configuration in two dimensions.

2. Simulated stresses for an arbitrary 16 point configuration in two dimensions.

3. Mean slope of the stress-tau relationship for the mean stress and tau values of the 135 random configurations. Lines describe the best fitting log-linear function of N and d (see text).

4. The standard deviation of the tau distribution as a function of the mean tau of the 135 random configurations.

5. The standard deviation of the stress distribution as a function of the mean stress of the 135 random configurations.

6. Mean tau as a function of the error level for the 135 random configurations.

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