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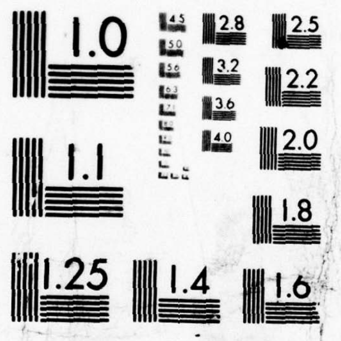

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FLUCTUATIONS IN SYSTEMS WITH MULTIPLE STEADY STATES. APPLICATION TO LANCHESTER EQUATIONS

Marc Mangel

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The ideas expressed in this paper are those of the author. The paper does not necessarily represent the views of the Center for Naval Analyses.

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ABSTRACT

We consider the effects of small random perturbations on deterministic systems of differential equations. The systems of interest have a steady state that is a saddle point. A first exit problem is formulated. The quantity of basic interest is the probability of exit from a band around the deterministic separatrix through a specified boundary, conditioned on initial position. A technique for the approximate calculation of this probability is given. As an example, we show how the theory applies to the calculation of the probability of victory in a combat that has a stochastic component.

SECTION I

FORMULATION OF THE EXIT PROBLEM

Let $X(t,\varepsilon)$ be a random variable that represents the state of a system at time t. We assume that, given $X(t,\varepsilon) = x$, the increment dX is normally distributed with mean $b(x)dt + o(dt)$ and covariance $\varepsilon a(x)dt + o(dt)$. The process $X(t,\varepsilon)$ will satisfy the Ito equation:

$$dX = b(X)dt + \sqrt{\varepsilon a(X)}\, dW \qquad X = (X^1, X^2) \tag{1}$$

We consider the analysis, for small ε, of the above stochastic equation when the reduced deterministic system:

$$dx^i = b^i(x)dt \qquad i = 1, 2 \tag{2}$$

has a steady state, X_s, that is a saddle point. Such equations arise, for example, in chemical reaction dynamics (Chang and Schmitz, 1972; Mangel, 1977), population dynamics (Ludwig, 1975), and in Lanchester's equations of combat. We will discuss the last case in section III. The saddle point gives rise to a deterministic separatrix, S (see figure 1). We surround the separatrix by a band with boundaries I and II, as shown in figure 1. Let $T(x) = \min\,(t : X(t) \in I$ or $X(t) \in II$, conditioned on $X(0) = x)$. We have suppressed the ε in the above definition

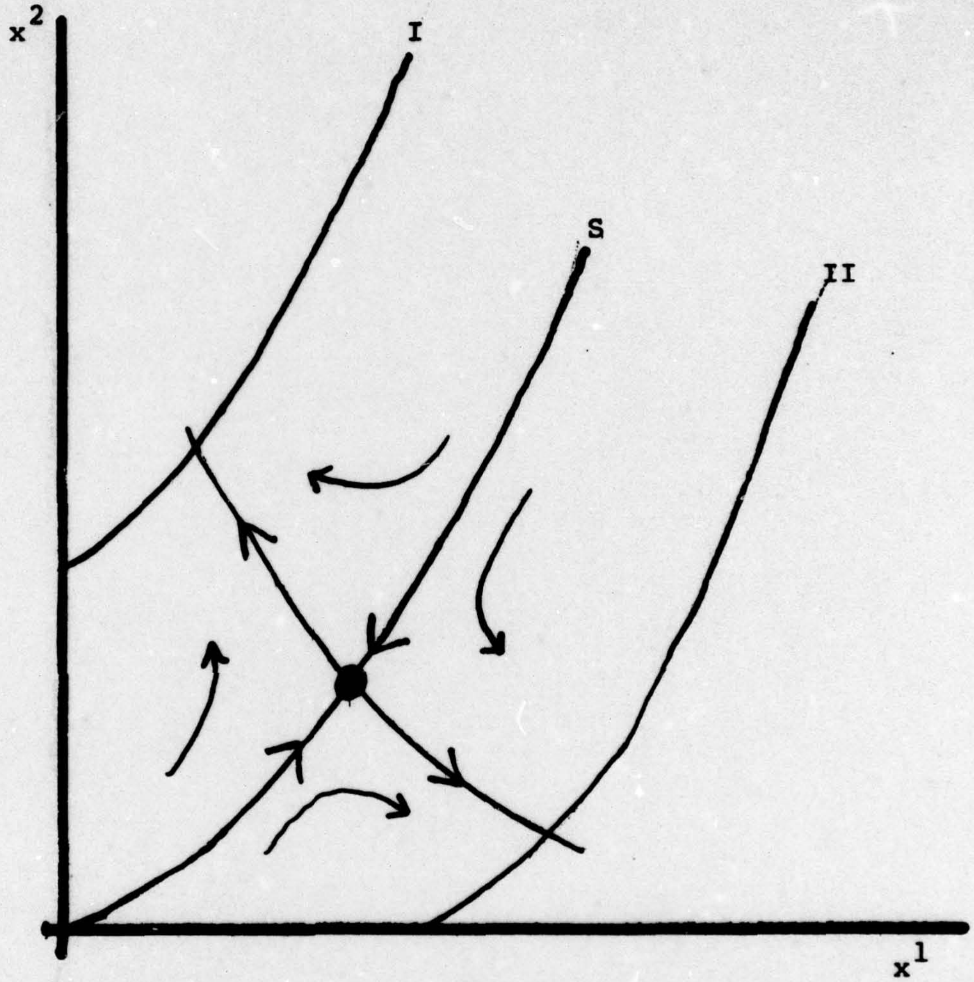

FIGURE 1: THE FIRST EXIT PROBLEM FOR A PLANAR DYNAMICAL SYSTEM

of $T(x)$. Let $u(x)$ be the probability that $X(T(x)) \in II$. Namely, $u(x)$ is the probability that the process $X(t)$ first exits from the band around the separatrix through boundary II, given that $X(0) = x$.

Then $u(x)$ satisfies the backward equation (Feller, 1971)

$$0 = \frac{\varepsilon a^{ij} u_{ij}}{2} + b^i u_i, \; u(x) = 0, \; x \in I; \; u(x) = 1, \; x \in II. \tag{3}$$

In (3), subscripts indicate differentiation and repeated indices are summed form 1 to n (n=2 in this case). Feller (1971) gives interpretations for b^i and εa^{ij} in terms of the averages of functions of dX. Equation (3) is a singularly perturbed elliptic equation. If $\varepsilon=0$, equation (3) becomes

$$0 = b^i u_i \; , \; u(x) = 0, \; x \in I; \; u(x) = 1, \; x \in II \tag{4}$$

Let A denote the region between I and S, not including S; let B denote the region between II and S, not including S. The solution of equation (4) is then:

$$u(x) = 0 \text{ for } x \in A; \quad u(x) = 1 \text{ for } x \in B$$
$$u(x) \text{ indeterminate for } x \in S.$$

Thus, for $\varepsilon=0$ (the deterministic case), $u(x)$ is a discontinuous function. For small ε, we expect that the discontinuous solution will be "smeared out" into a 'boundary' layer around the deterministic separatrix. This is indeed the case.

SECTION II

AN APPROXIMATE SOLUTION

In this section, we will construct an approximate solution of equation (3) by a generalized "ray method". We consider the one-dimensional canonical problem:

$$0 = \frac{\varepsilon a u}{2}{}_{xx} + b u_x \; , \qquad u(r_0) = 0 \; ; \quad u(r_1) = 1 \; . \tag{5}$$

Assume that $b(x) = 0$ has a solution, X_s, and the $b'(X_s) > 0$. Then X_s is a "one-dimensional" saddle point. The solution of equation (5) is

$$u(x) = \frac{\displaystyle\int_{r_0}^{x} \exp\left\{ -\int^{s} \frac{2b(y)}{\varepsilon a(y)}\, dy \right\} ds}{\displaystyle\int_{r_0}^{r_1} \exp\left\{ -\int^{s} \frac{2b(y)}{\varepsilon a(y)}\, dy \right\} ds} \; . \tag{6}$$

For small values of the parameter ε, the above solution can be analyzed by Laplace's method (Olver, 1974). We find that

$$\begin{aligned} u(x) = {} & k(x,\varepsilon) E\left((b'(X_s))^{\frac{1}{2}} (x - X_s) / (\varepsilon a(X_s))^{\frac{1}{2}} \right) \\ & + \varepsilon^{\frac{1}{2}} h(x,\varepsilon) E'\left((b'(X_s))^{\frac{1}{2}} (x - X_s) / (\varepsilon a(X_s))^{\frac{1}{2}} \right) . \end{aligned} \tag{7}$$

In equation (7), $k(x,\varepsilon)$ and $h(x,\varepsilon)$ are power series in ε, and $E(z)$ is the error function. It satisfies the differential equations

$$E'(z) = \exp(-z^2/2) \quad \text{and} \quad E''(z) = -z\,E'(z) \quad . \tag{8}$$

The value of z in equation (8) varies over a (possibly infinite) range $z_0 < z < z_1$. This result suggests that a possible solution of equation (3) is

$$u(x) = g(x,\varepsilon)E(\psi(x)/\varepsilon^{\frac{1}{2}}) + \varepsilon^{\frac{1}{2}}h(x,\varepsilon)E'(\psi(x)/\varepsilon^{\frac{1}{2}}) \quad , \tag{9}$$

where g, h, and ψ are to be determined as functions of x. We assume that g and h will have power series of the form $g(x,\varepsilon) = \sum g^n(x)\,\varepsilon^n$; $h(x,\varepsilon) = \sum h^n(x)\,\varepsilon^n$. When derivatives are evaluated, equation (8) is used to replace $E''(\psi/\varepsilon^{\frac{1}{2}})$ by the product $-E'(\psi/\varepsilon^{\frac{1}{2}})\,\psi/\varepsilon^{\frac{1}{2}}$. After substitution into equation (3), terms are collected according to powers of ε. The leading terms will vanish if the following equations are satisfied (also see Mangel and Ludwig, 1977; Mangel, 1977):

$$0 = b^i\psi_i - \frac{a^{ij}}{2}\psi_i\psi_j\psi \quad . \tag{10}$$

$$0 = b^i g_i^0 \quad . \tag{11}$$

Equation (10) is a first order partial differential equation. It can be solved by the method of characteristics (Courant, 1962). The characteristics are called rays (see Ludwig, 1975; Mangel, 1977; for a fuller interpretation of the rays). Hence, we call this technique a generalized ray method. Initial data for the function ψ can be determined by Hamilton-Jacobi theory; we find that $\psi = 0$ on the deterministic separatrix. Equation (11) indicates that g^0 is constant on deterministic trajectories. The values of g^0 and the endpoints z_0, z_1 are chosen so that the leading order term in the asymptotic solution (i.e., $u(x) \sim g^0 E(\psi(x)/\varepsilon^{\frac{1}{2}}) + O(\varepsilon^{\frac{1}{2}})$) satisfies the boundary conditions $u(x) = 0$ on I and $u(x) = 1$ on II. Once g^0 and $\psi(x)$ are known, contours of equal first exit probability can be constructed by solving the equation $\psi(x) =$ constant (see Mangel and Ludwig, 1977; Mangel, 1977).

SECTION III

STOCHASTIC LANCHESTER EQUATIONS: PROBABILITY OF VICTORY

As an example of the above theory, we shall consider a model of the combat between two forces (e.g., Morse and Kimball, 1946). The deterministic equations used to model the combat are:

$$dx^1 = (ax^1 - bx^1x^2)dt + o(dt); \quad x^1(0) = x^1_0 \tag{12}$$

$$dx^2 = (cx^2 - dx^1x^2)dt + o(dt); \quad x^2(0) = x^2_0. \tag{13}$$

The constants a, b, c, d are all assumed to be greater than zero. Equations similar to (12) and (13) arise in theoretical population dynamics (e.g., the completion between two species for one resource). The first terms on the right hand side of each equation represent the rate at which forces are added to the combat; the latter terms represent the losses due to combat. The point $(X^1_s, X^2_s) = (c/d, a/b)$ is a steady state of the system (12,13). It is a saddle point.

In order to account for stochastic effects during the combat, we might replace (12, 13) by the following stochastic equations:

$$dX^1 = (aX^1 - bX^1X^2)dt + \sqrt{\varepsilon a(X)}dW_1 \tag{14}$$

$$dX^2 = (cX^2 - dX^1X^2)dt + \sqrt{\varepsilon a(X)}dW_2 \tag{15}$$

In the above equations, $W_1(t)$ and $W_2(t)$ are two independent Wiener processes (Brownian motion). If boundary I is the x^2 axis and boundary II is the x^1 axis, the $u(x_0)$, as defined in previous sections, is the probability that if a combat starts with $X^1(0) = x_0^1$ and $X^2(0) = x_0^2$, then force 1 will be victorious. The probability of victory can be calculated by using the technique described in section II. The results given here generalize the work of Brown (1963).

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