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SCINTILLATION STRUCTURE IN A SEVERELY DISTURBED SCATTERING ENVIRONMENT

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October 1978

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This report presents a complete treatment of the second-order statistics of intensity under conditions of strong scattering. A three-dimensional powerlaw striation model is used in the analysis. For completeness, the special forms for a strictly two-dimensional medium are also derived. Asymptotic formulas for the fade coherence time are derived. Simple asymptotic results can be derived only for three-dimensional spectral indices less than 4. However, naturally occurring striations seem to fall in this range, and the simple formulas are generally applicable.

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CONTENTS

ILLUSTRATIONS

TABLE

I EXECUTIVE SUMMARY

In severely disturbed propagation environments, the scintillation structure asymptotically approaches a form that can be characterized by simple formulas. For example, the intensity statistics are accurately approximated by a Rayleigh distribution. Since the performance of both coherent and non-coherent systems in a Rayleigh fading environment is critically dependent on the fade coherence time, the second-order statistics must also be specified to complete the phenomenology-systems analysis link.

This report presents a complete treatment of the second-order statistics of intensity in a power-law scattering medium. A three-dimensional striation model is used, so that the propagation angle dependence in a three-dimensional anisotropic medium can be properly evaluated. For completeness, however, the special forms that apply to a strictly two-dimensional medium are also included. Thus, the results of the study are applicable to the Wideband satellite data base as well as the various numerical simulations that have been performed.

An important finding from this study is that the asymptotic behavior of the second-order statistics of intensity under strong scatter conditions is critically dependent on the power-law index. The theory admits three-dimensional striation spectral densities of the form $q^{-\alpha}$ where $2 < \alpha < 6$. When $\alpha < 4$, the contribution of large-scale structures is strongly suppressed by Fresnel filtering. When $\alpha \geq 4$, the large-scale structures are only weakly suppressed, and that the strong-scatter behavior begins to show some characteristics of scattering in a medium that contains a single dominant scale size.

When $\alpha < 4$, the second-order statistics asymptotically approach a form that is independent of the Fresnel radius, and a simple expression for the fade coherence time can be obtained. In light of data from the

Wideband satellite, it appears that the effective three-dimensional spectral index is indeed somewhat less than 4. Thus, the simpler asymptotic results are applicable. When $\alpha \geq 4$, the intensity statistics retain a dependence on the Fresnel radius, and simple limiting forms cannot be obtained.

A detailed summary of the results is given in Section VI as is a comparison with earlier results based on a gaussian spectral density function. Under conditions of weak scatter, the results converge to the weak-scatter forms that were independently derived in Rino and Matthews (1978). The intermediate range between weak and strong scattering must be treated by numerical computations or more elaborate asymptotic approximations.

II INTRODUCTION

A full treatment of multiple scattering in an extended three-dimensional medium is now well formulated [see, for example, Rino (1978) and the references cited therein]. However, it is impractical to solve the necessary vector differential equations numerically except in certain special cases. Thus, simplified models and/or asymptotic results are invariably used to interpret data. Fante (1975) has reviewed the various asymptotic results that have been applied to laser beam propagation where isotropic scattering is an appropriate idealization.

Ionospheric scattering, by comparison, is complicated by the fact that the irregularities are highly anisotropic. Moreover, the ionospheric outer-scale cutoff, q_o, is sufficiently small that the condition $q_o \sqrt{Z} << 1$, where $Z = \lambda\tilde{z} \sec \theta/(4\pi)$ is the Fresnel area, holds for all observing geometries at the lowest frequencies of interest ($\lambda \leq 3m$). This fact can be used to greatly simplify the theory.

In Rino and Matthews (1978) (hereafter Paper I), the weak-scatter theory was reformulated to show explicitly the ramifications of the very large outer scale $\ell_o = 2\pi/q_o$. In particular, a closed form analytic expression for the intensity scintillation index S_4 was derived by taking the limit of the integral expression as q_o approaches zero. The resulting expression is valid when $q_o\sqrt{Z} << 1$, which is a very good approximation as noted above.

In this paper, the results for the second-order statistics of intensity are extended to accommodate strong scattering. The analysis is based on the formulation of the gaussian phase-screen model developed by Gochelashvily and Shishov (1971). The general results are reviewed in Section III, where it is shown that, to calculate the second-order statistics of intensity, only the phase-structure function need be specified.

In Section IV of this paper, the limiting forms of the general results are computed as q_o approaches zero, as was done in Paper I for weak scatter. The analysis is identical to that presented by Rumsey (1975). For completeness, however, the special forms for a one-dimensional phase screen are also included. The scattering from a power-law phase screen is governed by a single "universal" strength parameter, U, which combines the effects of perturbation strength and distance from the scattering medium. The relationship between U and the intensity scintillation index is discussed.

In Section V, asymptotic results are derived for weak and strong scattering. The weak-scatter results were, of course, already deduced in Paper I. Under conditions of strong scattering, the asymptotic forms are critically dependent on the power-law spectral index, ν. [The three-dimensional SDF has the form $C_s q^{-(2\nu+1)}$ for $q \gg q_o$].*

When $\nu < 1.5$, simple asymptotic formulas can be derived for the autocorrelation function of intensity. Indeed, the S_4 scintillation index approaches unity from below and $\langle II' \rangle - 1 = \exp\{-2D(y)\}$, where D(y) is the phase structure function. Thie behavior is demonstrated for a special case of one-dimensional scattering in which an exact result has been obtained.

When $\nu \geq 1.5$, however, the intensity statistics retain an explicit dependence on the Fresnel parameter Z, and S_4 can exceed unity. This particular finding is evidently new. In effect, the behavior of the scattering for the more steeply sloped spectra ($1.5 \leq \nu < 2.5$) is transitional between that for a power-law environment in which large-scale structures are strongly suppressed by Fresnel filtering ($0.5 < \nu < 1.5$) and a medium dominated by a single scale size where strong focusing can occur.

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*The form of the three-dimensional index $q^{-\alpha}$ where $\alpha = 2\nu + 1$ is largely historical. The corresponding one-dimensional spectral index is $\alpha - 2 = 2\nu - 1$. For scintillation studies, it is the one-dimensional <u>phase</u> spectral index, p, that can be measured. In terms of ν, $p = 2\nu$. Thus, if $\nu = 1.5$, $\alpha = 4$ and $p = 3$.

In Section VI the results are summarized and compared to the earlier results (e.g., Salpeter, 1967) that apply strictly to scattering in a medium dominated by a single scale size, as opposed to a power-law continuum of scale sizes. A simple formula for the intensity coherence time is derived that fully accounts for angle effects in a highly anisotropic medium.

III GENERAL RESULTS FOR A GAUSSIAN PHASE SCREEN

Mercier (1962) developed a mathematically cumbrous but general formula for the second-order statistics of the intensity fluctuations of a wavefield as it propagates away from a phase-changing screen. A more compact formulation was developed by Gochelashvily and Shishov (1971). Alternative derivations of the Gochelashvily and Shishov result have been presented by Taylor (1972) and Rumsey (1975). Here we review only the principal results.

In the phase-screen model, a phase perturbation, $\delta\phi(\vec{\rho})$, is imparted to a wavefield at some plane, say $z = z_o$. Diffraction effects cause intensity fluctuations (scintillation) to develop as the wavefield propagates beyone $z = z_o$. The structure of intensity scintillation is characterized by the correlation function

$$R_I(\vec{\Delta\rho};z) = \langle I(\vec{\rho},z)\ I(\vec{\rho}',z)\rangle \quad , \tag{1}$$

where $I(\vec{\rho},z) = |u(\vec{\rho},z)|^2$, $u(\vec{\rho},z)$ is the complex wavefield at z, and the angle brackets denote mathematical expectation.

The free-space propagation of $R_I(\vec{\Delta\rho};z)$ from z_o to z can be calculated from the integral expression [the Gochelashvily and Shishov (1971) result]:

$$R_I(\vec{\Delta\rho},z) = \frac{k}{\tilde{z}}^2 \iiiint M_4(\vec{\alpha},\vec{\Delta\rho}-\vec{\beta},0;z_o)\ \exp\{-i\vec{\alpha}\cdot\vec{\beta}\ k/\tilde{z}\}d\vec{\alpha}\ d\vec{\beta} \tag{2}$$

where $k = 2\pi/\lambda$ and $\tilde{z} = z-z_o$. The quantity $M_4(\vec{\alpha}^{(2)},\vec{\alpha}^{(3)},\vec{\alpha}^{(4)};z_o)$ is the fourth-order coherence function of the wavefield in the plane $z = z_o$, evaluated in terms of the variables

$$\vec{\alpha}^{(2)} = \frac{1}{2}[\vec{\rho}^{(1)} - \vec{\rho}^{(2)} - \vec{\rho}^{(3)} + \vec{\rho}^{(4)}] \quad (3a)$$

$$\vec{\alpha}^{(3)} = \frac{1}{2}[\vec{\rho}^{(1)} + \vec{\rho}^{(2)} - \vec{\rho}^{(3)} - \vec{\rho}^{(4)}] \quad (3b)$$

$$\vec{\alpha}^{(4)} = \frac{1}{2}[\vec{\rho}^{(1)} - \vec{\rho}^{(2)} + \vec{\rho}^{(3)} - \vec{\rho}^{(4)}] \quad . \quad (3c)$$

Two assumptions are used in deriving Eq. (2). First, the scattering of the principal components of the angular spectrum must be confined to a narrow cone about the direction of the reference wave. Under this assumption, the propagation effects are governed by the free-space form of the parabolic wave equation

$$\frac{\partial u(\vec{\rho},z)}{\partial \ell} = -i \frac{1}{2k} \nabla_T^2 u(\vec{\rho},z) \quad , \quad (4)$$

where $\partial u/\partial \ell$ is the directional derivative along the propagation path (Rino, 1978). Second, M_4 cannot depend on the "centroid" variable $\vec{\alpha}^{(1)} = \frac{1}{2}[\vec{\rho}^{(1)} + \vec{\rho}^{(2)} + \vec{\rho}^{(3)} + \vec{\rho}^{(4)}]$. That is, M_4 is statistically homogeneous, or independent of where it is measured. Note that this neither implies nor requires that lower order moments be statistically homogeneous.

To evaluate M_4, the phase-screen model is used to obtain

$$M_4(\vec{\rho}^{(1)},\vec{\rho}^{(2)},\vec{\rho}^{(3)},\vec{\rho}^{(4)};z_o) = \langle \exp\{i[\delta\phi(\vec{\rho}_1) - \delta\phi(\vec{\rho}_2) + \delta\phi(\vec{\rho}_3) - \delta\phi(\vec{\rho}_4)]\}\rangle . \quad (5)$$

If $\delta\phi(\vec{\rho})$ is a zero-mean gaussian field, then Eq. (5) can be evaluated by using the well-known result

$$\langle \exp\{x\}\rangle = \exp\{-\frac{\langle x^2\rangle}{2}\} \quad . \quad (6)$$

The final form of Eq. (5) can then be conveniently written in terms of the phase structure function,

$$D(\vec{\Delta}\rho_{ij}) = \langle[\delta\phi(\rho^{(i)}) - \delta\phi(\rho^{(j)})]^2\rangle \quad , \quad (7)$$

as

$$M_4(\vec{\rho}^{(1)},\vec{\rho}^{(2)},\vec{\rho}^{(3)},\vec{\rho}^{(4)};z_o) = \exp\{\frac{1}{2}[D(\Delta\vec{\rho}_{12}) - D(\Delta\vec{\rho}_{13}) + D(\Delta\vec{\rho}_{14})$$

$$- D(\Delta\vec{\rho}_{24}) + D(\Delta\vec{\rho}_{34}) + D(\Delta\vec{\rho}_{23})]\} \quad . \tag{8}$$

By making the appropriate substitutions from Eq. (3), it follows immediately that M_4 does not depend on $\vec{\alpha}^{(1)}$. Moreover,

$$M_4(\vec{\alpha}^{(2)},\vec{\alpha}^{(3)},0;z_o) = \exp\{D(\vec{\alpha}^{(2)}) + D(\vec{\alpha}^{(3)}) - \frac{1}{2}D(\vec{\alpha}^{(2)} + \vec{\alpha}^{(3)})$$

$$- \frac{1}{2}D(\vec{\alpha}^{(2)} - \vec{\alpha}^{(3)})\} \quad . \tag{9}$$

Thus, the homogeneity of M_4 follows from the assumption that the structure function depends only on the difference variable $\vec{\Delta\rho}_{ij}$. It will be shown, however, that because of the symmetry of the variables in Eq. (9), Eq. (2) is well defined even if $D(\vec{\Delta\rho})$ does not exist.

The spectral domain version of Eq. (2) takes a particularly simple form. By direct computation

$$\Phi_I(\vec{\kappa}) = \iint R_I(\vec{\Delta\rho}) \exp\{i\vec{\kappa}\cdot\vec{\Delta\rho}\}\, d\vec{\Delta\rho}$$

$$= \iint M_4(-\vec{\kappa}\frac{\tilde{z}}{k},\vec{\xi};0,z_o) \exp\{-i\vec{\kappa}\cdot\vec{\xi}\}\, d\vec{\xi} \quad . \tag{10}$$

By substituting Eq. (9) into Eq. (10), one obtains

$$\Phi_I(\vec{\kappa}) = \iint \exp\{-g(\vec{\xi},\vec{\kappa}\frac{\tilde{z}}{k})\} \exp\{-i\vec{\kappa}\cdot\vec{\xi}\}\, d\vec{\xi} \quad , \tag{11}$$

where

$$g(\vec{\xi},\vec{\eta}) = D(\vec{\xi}) + D(\vec{\eta}) - \frac{1}{2}D(\vec{\xi}+\vec{\eta}) - \frac{1}{2}D(\vec{\xi}-\vec{\eta}) \quad , \tag{12}$$

which is the principal result that will be used in later sections.

To investigate the general behavior of $g(\vec{\xi},\vec{\eta})$, the spectral representation,

$$D(\vec{\xi}) = \iint [1 - \cos(\vec{q} \cdot \vec{\xi})] \, \Phi_{\delta\phi}(\vec{q}) \, d\vec{q}/(2\pi)^2 \quad , \tag{13}$$

can be used where $\Phi_{\delta\phi}(\vec{q})$ is the two-dimensional phase SDF. If this form is substituted into Eq. (12), after some straightforward manipulations, one obtains the equivalent representation

$$g(\vec{\xi},\vec{\eta}) = 8 \iint \Phi_{\delta\phi}(\vec{q}) \sin^2 (\vec{\eta} \cdot \vec{q}/2) \sin^2 (\vec{\xi} \cdot \vec{q}/2) \frac{d\vec{q}}{(2\pi)^2} \quad . \tag{14}$$

From the form of Eq. (14), it follows that if $\Phi_{\delta\phi}(\vec{q}) \propto q^{-\alpha}$, then $g(\vec{\xi};\vec{\eta})$ is well defined as long as $2 < \alpha < 6$. From Eq. (13), however, it is clear that the structure function itself is only defined for $2 < \alpha < 4$.

To carry the development one step further, consider the special case in which $|g(\vec{\xi},\vec{\eta})| \ll 1$ for all significant $\vec{\eta}$ values. The necessary conditions are stated in Section V. By using the approximation $\exp\{-g\} \simeq 1 - g$, substituting from Eq. (14), and finally using the identity $\sin^2 x = (1 - \cos x)/2$, it follows from Eq. (11) that

$$\Phi_I(\vec{\kappa}) \simeq \delta(\vec{\kappa}) + 4\Phi_{\delta\phi}(\vec{\kappa}) \sin^2 [\kappa^2 \tilde{z}/(2k)] \quad , \tag{15}$$

which is the well known weak-scatter result.

To summarize, in the gaussian phase-screen model, the intensity SDF can be computed by evaluating a single integral, Eq. (11). Only the phase structure function need be specified for the computation. However, the diffraction effects, in conjunction with the size distribution of the irregularities, are completely characterized by the function $g(\vec{\xi},\vec{\kappa}\,\tilde{z}/k)$ [Eq. (12)], which is well defined independently of an outer-scale cutoff for phase SDFs of the form $q^{-\alpha}$ with $2 < \alpha < 6$.

IV SCATTER IN STRICT POWER-LAW ENVIRONMENTS

In Section III, we showed that the second-order statistics of intensity are completely characterized by the function $g(\vec{\xi},\vec{\eta})$, as defined by Eq. (12) in terms of the phase structure function. If the power-law model in Paper I is used,

$$\Phi_{\delta\phi}(\vec{q}) = \frac{abC_p}{\left[q_o^2 + q^2\right]^{\nu+1/2}} \quad , \tag{16}$$

where

$$C_p = r_e^2\lambda^2 \, L \sec\theta \, C_s \tag{17}$$

and

$$q^2 = A'\kappa_x^2 + B'\kappa_x\kappa_y + C'\kappa_y^2 \quad . \tag{18}$$

The coefficients A′, B′, and C′ are defined in Paper I (Eqs. 26a, 26b, and 26c). The three-dimensional irregularity SDF has the same form as Eq. (16), except that abC_p is replaced by C_s. The parameters a and b are axial ratios along and transverse to the principal irregularity axis. A complete discussion of this model is given in Rino and Fremouw (1977).

The general form of the two-dimensional phase SDF corresponding to Eq. (16) is

$$R_{\delta\phi}(y) = \frac{GC_p}{2\pi\Gamma(\nu + 1/2)} \left|\frac{q_o y}{2}\right|^{\nu-1/2} K_{\nu-1/2}(q_o y)/q_o^{2\nu-1} \quad , \tag{19}$$

where the geometric factor G is defined by Eq. (10) in Paper I. Hereafter, we shall consider only isotropic irregularities (a = b = 1). The general case is easily retrieved by reintroducing G and interpreting y appropriately. By using the small-argument formula,

$$K_{\nu-1/2}(x) \sim 1/2\, \Gamma(\nu - 1/2)\left|x/2\right|^{-(\nu-1/2)} \quad ,$$

it is readily shown that

$$R_{\delta\phi}(0) = \frac{C_p \Gamma(\nu - 0.5)}{4\pi\Gamma(\nu + 0.5)\, q_o^{2\nu-1}} \quad . \tag{20}$$

For completeness, a one-dimensional phase screen for which

$$\Phi_{\delta\phi}(q) = \frac{C_p'}{\left[q_o^2 + q^2\right]^{p/2}} \quad , \tag{21}$$

$$R'_{\delta\phi}(y) = \frac{C_p'}{\sqrt{\pi}\,\Gamma(p/2)} \left|\frac{q_o y}{2}\right|^{\frac{p-1}{2}} K_{\frac{p-1}{2}}(q_o y)/q_o^{p-1} \quad , \tag{22}$$

and

$$R'_{\delta\phi}(0) = \frac{C_p' \quad \Gamma\left(\frac{p-1}{2}\right)}{2\sqrt{\pi}\Gamma(p/2)\, q_o^{p-1}} \tag{23}$$

will also be considered. The one-dimensional model is appropriate for propagation across highly elongated irregularities. The relationship between p and ν is $p = 2\nu$.

Whenever the phase autocorrelation function (ACF) exists, the phase structure function can be written as

$$D(\vec{\Delta}\rho) = 2[R_{\delta\phi}(0) - R_{\delta\phi}(\vec{\Delta}\rho)] \quad . \tag{24}$$

Thus, from Eqs. (19) and (20),

$$D(y) = \frac{C_p\ \Gamma(\nu - 0.5)}{2\pi\ \Gamma(\nu + 0.5)} \left[\frac{1 - 2\left|\frac{q_o y}{2}\right|^{\nu-1/2} K_{\nu-1/2}(q_o y)/\Gamma(\nu - 0.5)}{q_o^{2\nu-1}}\right] \tag{25}$$

and

$$D'(y) = \frac{C'_p \, \Gamma\left(\frac{p-1}{2}\right)}{\sqrt{\pi}\,\Gamma(p/2)} \left[\frac{1 - 2\left|\frac{q_o y}{2}\right|^{(p-1)/2} K_{(p-1)/2}(q_o y)/\Gamma\left(\frac{p-1}{2}\right)}{q_o^{p-1}} \right] . \quad (26)$$

As q_o approaches zero, the correlation functions $R_{\delta\phi}(y)$ and $R'_{\delta\phi}(y)$ become singular. However, for a limited range of ν and p values, D(y) and D'(y) remain well defined.

The limiting forms of D(y) and D'(y) as q_o approaches zero are not immediate, since $\lim_{q_o \to 0} D(y) = 0/0$. Thus, one applies L'Hospital's rule, whereby the limit is obtained as the limit of the ratio of the separate derivatives of the numerator and the denominator. The results are

$$\lim_{q_o \to 0} D(y) = \lim_{q_o \to 0} \frac{C_p}{2\pi\Gamma(\nu + 0.5)} \left[\frac{2\left|\frac{q_o y}{2}\right|^{\nu-1/2} K_{\nu-3/2}(q_o y)\, y}{(2\nu - 1) q_o^{2\nu-2}} \right]$$

$$= \frac{C_p}{2\pi} \frac{2\,\Gamma(1.5 - \nu)}{\Gamma(\nu + 0.5)(2\nu - 1)\, 2^{2\nu-1}} |y|^{2\nu-1} \quad (27)$$

and

$$\lim_{q_o \to o} D'(y) = \lim_{q_o \to 0} \frac{C'_p}{\sqrt{\pi}\Gamma(p/2)} \left[\frac{2\left|\frac{q_o y}{2}\right|^{(p-1)/2} K_{(p-3)/2}(q_o y)\, y}{(p - 1) q_o^{p-2}} \right]$$

$$= \frac{C'_p}{\sqrt{\pi}} \frac{2\Gamma\left(\frac{3-p}{2}\right)}{\Gamma(p/2)(p - 1)\, 2^{p-1}} |y|^{p-1} \quad . \quad (28)$$

It follows from Eqs. (27) and (28) that the respective structure functions are well defined as q_o approaches zero, as long as $0.5 < \nu < 1.5$ and $1 < p < 3$.

However, it was shown in Section II that $g(\vec{\xi},\vec{\eta})$ is well defined beyond $\nu = 1.5$. Similarly, $g'(\vec{\xi},\vec{\eta})$ is well defined beyond $p = 3$. Hence, the most general limiting form cannot be deduced by substituting

Eq. (27) or (28) into Eq. (12). Two methods of avoiding this dilemma have been discussed by Rumsey (1975). The most straightforward procedure is to apply the finite q_o form of D(y) or [D′(y)] and use L'Hospital's rule to evaluate the limiting form of $g(\vec{\xi},\vec{\eta})$. Thus,

$$g(\vec{\xi},\vec{\eta}) = \frac{C_p}{2\pi}\frac{\Gamma(\nu - 0.5)}{\Gamma(\nu + 0.5)}\left|1 - \frac{1}{\Gamma(\nu - 0.5)}\left[2|q_o y1/2|^{\nu-1/2} K_{\nu-1/2}(q_o y_1)\right.\right.$$

$$+ 2|q_o y2/2|^{\nu-1/2} K_{\nu-1/2}(q_o y_2)$$

$$- |q_o y3/2|^{\nu-1/2} K_{\nu-1/2}(q_o y_3)$$

$$\left.\left. - |q_o y4/2|^{\nu-1/2} K_{\nu-1/2}(q_o y_4)\right]\right| /q_o^{2\nu-1} \quad , \qquad (29)$$

where $y_1 = |\vec{\xi}|$, $y_2 = |\vec{\eta}|$, $y_3 = |\vec{\xi} - \vec{\eta}|$, and $y_4 = |\vec{\xi} + \vec{\eta}|$. A similar expression can be generated for $g'(\vec{\eta},\vec{\xi})$. To evaluate Eq. (29) and the corresponding expression for $g'(\vec{\eta},\vec{\xi})$, L'Hospital's rule must be applied twice. The results of this computation are summarized in Table 1.

The results in Table 1 for $g(\vec{\xi},\vec{\eta})$ agree with Rumsey's (1975) Eqs. (18) and (19) for $0.5 < \nu \leq 1.5$, but not for $1.5 < \nu < 2.5$. The limiting forms of $g(\vec{\xi},\vec{\eta})$ for $\nu < 1.5$ and $\nu > 1.5$ cannot be simply combined. A term has evidently been omitted in Rumsey's computation for $\nu > 1.5$ ($\alpha > 4$). In any case, $\nu = 1.5$ marks an important transition in the scattering behavior, which will be shown in Section IV.

To illustrate the significance of these results, let $\vec{\xi} = C(\nu)^{-1/(2\nu-1)}\vec{\xi}$ in Eq. (11) where

Table 1

LIMITING FORMS OF $g(\vec{\xi},\vec{\eta})$ AND $g'(\xi,\eta)$

Two-Dimensional $g(\vec{\xi},\vec{\eta})$	
$\frac{C_p}{2\pi} \frac{\Gamma(1.5 - \nu)}{\Gamma(0.5 + \nu)(2\nu - 1)2^{2\nu-1}} \left[2\lvert\vec{\xi}\rvert^{2\nu-1} + 2\lvert\vec{\eta}\rvert^{2\nu-1} - \lvert\vec{\xi} - \vec{\eta}\rvert^{2\nu-1} - \lvert\vec{\xi} + \vec{\eta}\rvert^{2\nu-1}\right]$	$0.5 < \nu < 1.5$
$\frac{C_p}{2\pi} \left[- \lvert\vec{\xi}\rvert^2 \log \lvert\vec{\xi}\rvert - \lvert\vec{\eta}\rvert^2 \log \lvert\vec{\eta}\rvert + \frac{1}{2}\lvert\vec{\xi} - \vec{\eta}\rvert^2 \log \lvert\vec{\xi} - \vec{\eta}\rvert + \frac{1}{2}\lvert\vec{\xi} + \vec{\eta}\rvert^2 \log \lvert\vec{\xi} + \vec{\eta}\rvert\right]$	$\nu = 1.5$
$\frac{C_p}{2\pi} \frac{\Gamma(2.5 - \nu)}{\Gamma(0.5 + \nu)(2\nu - 1)(2\nu - 2)2^{2\nu-2}} \left[- 2\lvert\vec{\xi}\rvert^{2\nu-1} - 2\lvert\vec{\eta}\rvert^{2\nu-1} + \lvert\vec{\xi} - \vec{\eta}\rvert^{2\nu-1} + \lvert\vec{\xi} + \vec{\eta}\rvert^{2\nu-1}\right]$	$1.5 < \nu < 2.5$
One-Dimensional $g^{(1)}(\xi,\eta)$	
$\frac{C'_p}{\sqrt{\pi}} \frac{\Gamma\left(\frac{3 - p}{2}\right)}{\Gamma(p/2)(p - 1)2^{p-1}} \left[2\lvert\xi\rvert^{p-1} + 2\lvert\eta\rvert^{p-1} - \lvert\xi - \eta\rvert^{p-1} - \lvert\xi + \eta\rvert^{p-1}\right]$	$1 < p < 3$
$\frac{2C'_p}{\sqrt{\pi}} \left[- \lvert\xi\rvert^2 \log \lvert\xi\rvert - \lvert\eta\rvert^2 \log \lvert\eta\rvert + \frac{1}{2}\lvert\xi - \eta\rvert^2 \log \lvert\xi - \eta\rvert + \frac{1}{2}\lvert\xi + \eta\rvert^2 \log \lvert\xi + \eta\rvert\right]$	$p = 3$
$\frac{C'_p}{\sqrt{\pi}} \frac{\Gamma(5/2 - p/2)}{\Gamma(p/2)(p - 1)(p - 2)2^{p-2}} \left[- 2\lvert\xi\rvert^{p-1} - 2\lvert\eta\rvert^{p-1} + \lvert\xi - \eta\rvert^{p-1} + \lvert\xi + \eta\rvert^{p-1}\right]$	$3 < p < 5$

$$C(\nu) = \begin{cases} \frac{C_p}{2\pi} \frac{\Gamma(1.5 - \nu)}{\Gamma(0.5 + \nu)(2\nu - 1)2^{2\nu-1}} & 0.5 < \nu < 1.5 \\ \frac{C_p}{2\pi} & \nu = 1.5 \\ \frac{C_p}{2\pi} \frac{\Gamma(2.5 - \nu)}{\Gamma(0.5 + \nu)(2\nu - 1)(2\nu - 2)2^{2\nu-2}} & 1.5 < \nu < 2.5 \end{cases} \qquad (30)$$

Then, with an appropriate substitution for $\vec{q}$,

$$C(\nu)^{1/(\nu-0.5)} \Phi_I\left(\vec{q}' C(\nu)^{1/(2\nu-1)}\right) =$$

$$\iint \exp\left\{-h\left(\vec{\xi}', \tilde{z}/k\ C(\nu)^{1/(\nu-0.5)}\ \vec{q}'\right)\right\} \cos(\vec{\xi}' \cdot \vec{q}')\, d\vec{\xi}' \qquad (31)$$

where

$$h(\vec{\xi},\vec{\eta}) = \begin{cases} \pm 2|\vec{\xi}|^{2\nu-1} \pm 2|\vec{\eta}|^{2\nu-1} \mp |\vec{\xi} - \vec{\eta}|^{2\nu-1} \mp |\vec{\xi} + \vec{\eta}|^{2\nu-1} & \nu \neq 1.5 \\ -|\vec{\xi}|^2 \log|\vec{\xi}| - |\vec{\eta}|^2 \log|\vec{\eta}| + 1/2|\vec{\xi} - \vec{\eta}|^2 \log|\vec{\xi} - \vec{\eta}| \\ \quad + 1/2|\vec{\xi} + \vec{\eta}|^2 \log|\vec{\xi} + \vec{\eta}| & \nu = 1.5 \end{cases} \qquad (32)$$

The form of Eq. (31) suggests the definition

$$U = (\tilde{z}/k)^{\nu-0.5}\ C(\nu) = C_p Z^{\nu-0.5}\ F(\nu) \qquad (33)$$

where

$$Z = \lambda\tilde{z}/4\pi \qquad (34)$$

and

$$F(\nu) = 2^{\nu-0.5}\ C(\nu)/C_p \quad . \qquad (35)$$

The reason for introducing the new definitions, Z and $F(\nu)$, will become clear shortly.

It now follows that the second-order statistics of intensity are completely characterized by the integral

$$I(\vec{q};U) = \iint \exp\left\{-h\left(\vec{\zeta},U^{\left(\frac{1}{\nu-0.5}\right)}\vec{q}'\right)\right\} \cos(\vec{\zeta}\cdot\vec{q})\, d\vec{\zeta} \quad . \tag{36}$$

Similarly, for the one-dimensional phase screen

$$I'(q';U') = \int \exp\left\{-h'\left(\zeta,U'^{\left(\frac{2}{p-1}\right)}q\right)\right\} \cos(\zeta q)\, d\zeta \tag{37}$$

completely specifies the second-order statistics of intensity, where

$$\begin{aligned} U' &= (\tilde{z}/k)^{(p-1)/2}\, C'(p) \\ &= C'\, Z^{(p-1)/2}\, F'(p) \quad , \end{aligned} \tag{38}$$

$$C'(p) = \begin{cases} \dfrac{C'_p}{\sqrt{\pi}} \dfrac{\Gamma\left(\frac{3-p}{2}\right)}{\Gamma(p/2)(p-1)2^{p-1}} & 1 < p < 3 \\[2ex] \dfrac{2C'_p}{\sqrt{\pi}} & p = 3 \\[2ex] \dfrac{C'_p}{\sqrt{\pi}} \dfrac{\Gamma(5/2 - p/2)}{\Gamma(p/2)(p-1)(p-2)2^{p-2}} & 3 < p < 5 \quad , \end{cases} \tag{39}$$

and

$$F'(p) = 2^{(p-1)/2}\, C'(p)/C'_p \quad . \tag{40}$$

The definition of $h'(\zeta,\eta)$ is readily deduced from Table 1.

The single parameter U determines the general behavior of the second-order intensity statistics. In Section V asymptotic results will be derived for weak-scatter conditions ($U \ll 1$) and strong scatter conditions ($U \gg 1$). Before doing so, however, it is instructive to compare the U parameter to the intensity scintillation index S_4

$$[S_4^2 = R_I(0;z) - 1] \quad .$$

Paper I showed that for isotropic irregularities

$$S_4^2 = C_p Z^{\nu-0.5} \left[\frac{\Gamma\left(\frac{2.5-\nu}{2}\right)}{2\sqrt{\pi}\,(\nu-0.5)\,\Gamma\left(\frac{\nu+0.5}{2}\right)} \right] \quad . \tag{41}$$

Thus, from Eq. (33),

$$S_4^2 = U \left[\frac{\Gamma\left(\frac{2.5-\nu}{2}\right)}{2\sqrt{\pi}\,\Gamma\left(\frac{\nu+0.5}{2}\right) F(\nu)} \right] \quad . \tag{42}$$

Similarly, for the one-dimensional phase-screen,

$$S_4^2 = U' \left[\frac{2\Gamma\left(\frac{5-p}{4}\right)}{\sqrt{\pi}\,(p-1)\,\Gamma\left(\frac{p+1}{4}\right) F'(p)} \right] \quad . \tag{43}$$

For a fixed value of ν (or p), S_4^2 and U admit the same functional dependence on C_p and Z. However, the functional dependences of S_4^2 and U on ν are radically different.

From Eq. (41) it follows that S_4^2 is a continuous monotonically increasing function of ν. This happens because as the slope of the phase spectrum steepens with C_p fixed, the spectral intensity of wavenumbers corresponding to the Fresnel radius rapidly increases relative to the smaller-scale structures. The parameter U, by comparison, is sharply discontinuous at $\nu = 1.5$ [U' is discontinuous at $p = 3$]. This is illustrated in Figure 1, where S_4^2/U is plotted as a function of ν.

In Section III it was shown that Fresnel filtering suppresses the influence of large-scale structures in intensity scintillation as long as $\nu < 2.5$. The behavioral transition at $\nu = 1.5$ can be interpreted as a boundary between regimes in which large-scale structures are strongly suppressed ($\nu < 1.5$) and those in which they are weakly suppressed ($\nu > 1.5$). The asymptotic behavior of $R_I(\vec{\Delta\rho};z)$ for large U is very different in these two regimes, as we show in Section V.

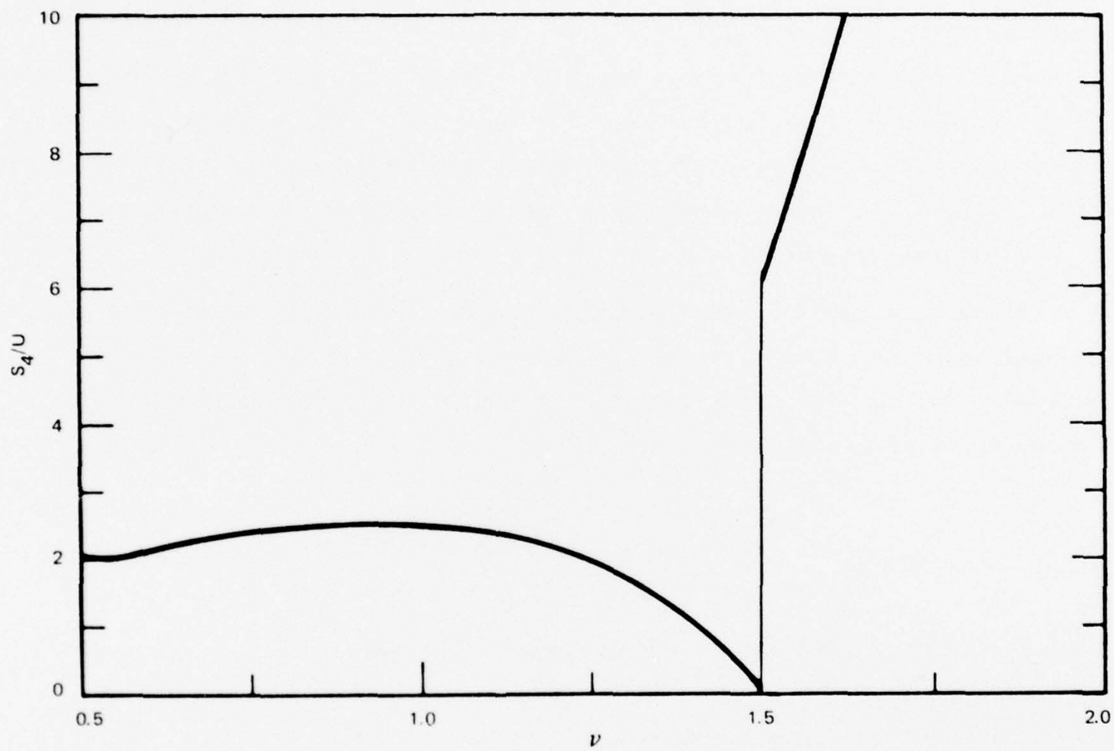

FIGURE 1 RATIO OF S_4 TO U vs. ν SHOWING DISCONTINUITY AT ν = 1.5

V ASYMPTOTIC RESULTS FOR LARGE AND SMALL VALUES OF U

Following the work of Gochelashvily and Shishov (1971), a number of authors have derived asymptotic formulas for $\Phi_I(\vec{\kappa};z)$ and/or $R_I(\vec{\Delta\rho};z)$ by using series expansions of $q(\vec{\xi};\vec{\eta})$ (e.g., Taylor and Infosino, 1976; Buckley, 1971a, 1971b). Difficulties can arise, however, if $g(\vec{\xi},\vec{\eta})$ is defined in terms of an implicit structure function. For example, moments do not exist unless an inner-scale cutoff is introduced. To do so, however, does not give physically meaningful results, since the basic integral [Eq. (11)] is well defined independent of either the inner-scale or the outer-scale cutoff.

Proper treatment of this problem demands that series approximations be applied to Eq. (32) directly, as Rumsey (1975) did for the special case of $\nu = 1$ ($\alpha = 3$). First note that $h(\vec{\xi},0) = 0$ irrespective of ν. Hence, for sufficiently small $\vec{q}$,

$$\exp\left\{-h\left(\vec{\xi},\ U^{\frac{1}{\nu-0.5}}\vec{q}\right)\right\} \simeq 1 - h\left(\vec{\xi},\ U^{\frac{1}{\nu-0.5}}\vec{q}\right) \quad . \tag{44}$$

Substituting Eq. (44) into Eq. (36) gives the so-called low-frequency approximation

$$\begin{aligned} I_{LF}(\vec{q};U) &= \delta(\vec{q}) - \iint h\left(\vec{\xi};\ U^{\frac{1}{\nu-0.5}}\vec{q}\right)\cos(\vec{q}\cdot\vec{\xi})d\xi \\ &= \delta(\vec{q}) + 4\left(C_p\ C(\nu)^{-1}\right) q^{-(2\nu+1)} \sin^2\left(U^{\frac{1}{\nu-0.5}} q^2/2\right) \quad . \end{aligned} \tag{45}$$

Note that the product $C_p\ C(\nu)^{-1}$ is independent of C_p.

The low-frequency approximation is valid for $q \ll \tilde{q} = U^{-\frac{1}{\nu-0.5}}$. This is essentially equivalent to Gochelashvily and Shishov's result when $\nu = 4/3$ ($\alpha = 11/3$). The situation is more complicated when $\vec{q} >> \tilde{q}$. For $\nu \neq 1.5$ and $\vec{\eta} >> \vec{\xi}$ the starting point is

$$h(\vec{\xi},\vec{\eta}) \simeq \pm 2|\xi|^{2\nu-1} \mp 2 \frac{(\nu-0.5)\xi^2 + 2(\nu-0.5)(\nu-1.5)(\vec{\xi} \cdot \hat{a}_\eta)^2}{\eta^{3-2\nu}} \tag{46}$$

where $\hat{a}_\eta = \vec{\eta}/|\vec{\eta}|$. Ordering the terms in Eq. (46) and retaining only the lowest order terms in ξ/η gives the result

$$h(\vec{\xi},\vec{\eta}) \simeq \begin{cases} 2|\xi|^{2\nu-1} & 0.5 < \nu < 1.5 \\ (2\nu-1)[\xi^2 + (2\nu-3)(\vec{\xi} \cdot \hat{a}_\eta)^2]\, \eta^{2\nu-3} & 1.5 < \nu < 2.5 \quad . \end{cases} \tag{47}$$

For $\nu < 1.5$, the high-frequency approximation takes the particularly simple form

$$I_{HF}(\vec{q};U) = \iint \exp\left|-2|\xi|^{2\nu-1}\right| \cos(\vec{q} \cdot \vec{\xi})\, d\vec{\xi} \quad . \tag{48}$$

It follows from Eq. (48) *that* $I_{HF}(\vec{q};U)$ converges rapidly to zero for $\vec{q} >> 1$. Thus, if $U << 1$, the bulk of the intensity SDF is contained in the low-frequency approximation. For $U >> 1$, $\tilde{q}$ is very small, and the bulk of the intensity SDF (except the singularity at $\vec{q} = 0$) is contained in the high-frequency approximation. Thus,

$$I(\vec{q};U) \simeq \begin{cases} I_{LF}(\vec{q};U) & U << 1 \\ \delta(\vec{q}) + I_{HF}(\vec{q};U) & U >> 1 \quad . \end{cases} \tag{49}$$

This behavior is nicely illustrated by the one-dimensional model with $p = 2$, wherein $I(\vec{q};U)$ can be evaluated exactly. Indeed, for $p = 2$

$$h'(\xi,\eta) = \begin{cases} 2|\xi| & |\xi| \leq |\eta| \\ 2|\eta| & |\xi| > |\eta| \quad . \end{cases} \tag{50}$$

By substituting Eq. (50) into Eq. (34), it is readily shown that

$$I'(q;U') = \delta(q) + 2\int_{o}^{U'^2|q|} \exp\{-2|\xi|\}\cos(q\xi)\,d\xi$$

$$= \delta(q) + \left[\frac{8\sin^2(U'^2 q^2/2)}{4+q^2} + \frac{2q\sin(qU'^2|q|)}{4+q^2}\right]\exp\{-2U'^2|q|\}$$

$$+ \frac{4}{4+q^2}\left[1 - \exp\{-2U'^2|q|\}\right] . \qquad (51)$$

For $U' << 1$, and q small, the first $\sin^2$ term dominates Eq. (51) in agreement with the low-frequency approximation. For $U' >> 1$, the third term dominates and there is no further dependence on U'. The second term in the square brackets is significant only in the intermediate q range.

By using the high-frequency approximation it is easily shown that for large U

$$R_I(\xi) \cong \exp\{-2|\xi|^{2\nu-1}\} + 1 \quad . \qquad (52)$$

In terms of the original variables

$$R_I(y) - 1 \cong \exp\{-2C(\nu)y^{2\nu-1}\}$$

$$= \exp\{-D(y)\} \qquad (53)$$

[see Eq. (27)]. It follows that $S_4^2 = R_I(0) - 1$ approaches unity as U becomes arbitrarily large. Moreover, upon recalling Bramely's (1954) result for a gaussian phase screen,

$$\langle uu'^*\rangle = \exp\left|-\frac{1}{2}D(y)\right| \quad , \qquad (54)$$

it follows from Eq. (53) that $\langle II'\rangle - 1 = |\langle uu^*\rangle|^2$, again as U becomes arbitrarily large.

If $u = x + iy$, where x and y are uncorrelated, equal-variance gaussian processes, then it is well known that $I = |u|^2$ is Rayleigh distributed and $\langle II' \rangle - 1 = |\langle uu^* \rangle|^2$. Moreover, the phase of **U** is uniformly distributed. Now, while Eq. (53) alone cannot guarantee uniformly distributed phase (modulo 2π) and Rayleigh statistics for intensity, experimental results (Fremouw et al., 1978; Rino, 1978) suggest that the Rayleigh model is a good approximation.

For $\nu > 1.5$, the high-frequency approximation [see Eq. (47)] is considerably more complicated. In particular, $I_{HF}(\vec{q};U)$ retains an explicit dependence on U such that numeric computations must be used. Intuitively, however, it is clear that as ν increases from 1.5 to 2.5, the intensity statistics tend to behave more like the medium is being dominated by a single scale size, even though strictly speaking the outer scale is not important until ν exceeds 2.5.

When a single scale size is dominant, strong focusing can occur (Pisareva, 1958), which causes S_4 to exceed unity. The form of $I_{HF}(\vec{q};U)$ for $\nu > 1.5$ does not exclude this from happening in a power-law environment, although the conditions that lead to $S_4 > 1$ have generated some controversy (Rumsey, 1975; Taylor and Infosino, 1976).

VI SUMMARY AND DISCUSSION

In this paper the power-law phase-screen model has been generalized to accommodate strong scattering. The intensity SDF $\Phi_I(\vec{q})$ was derived in terms of a single integral expression $I(\vec{q};U)$:

$$\Phi_I(\vec{q}) = C(\nu)^{-1/(\nu-0.5)} \, I\left[\vec{q} \; C(\nu)^{-1/(2\nu-1)}, \; U\right] \quad , \tag{55}$$

where

$$U = C_p Z^{\nu-0.5} \; F(\nu) \tag{56}$$

has the same functional form as the weak-scatter formula for the scintillation index, S_4. [Recall that $C_p = r_e^2 \lambda^2$ (L sec θ) C_s; where C_s is the strength-of-turbulence as defined by Eq. (7) in Paper I.] In general

$$S_4^2 = \iint I(\vec{q}, U) \; d\vec{q}/(2\pi)^2 - 1 \quad , \tag{57}$$

whereas for $U \ll 1$,

$$S_4^2 \simeq C_p Z^{\nu-0.5} \left[\frac{\Gamma\left(\frac{2.5-\nu}{2}\right)}{2\sqrt{\pi}\;(\nu-0.5)\Gamma\left(\frac{\nu+0.5}{2}\right)} \right] \mathfrak{I} \quad , \tag{58}$$

where $\mathfrak{I}$ is defined by Eq. (34) in Paper I.

It was shown in Section IV that as long as ν does not lie in the range $1.4 \leq \nu < 1.5$, then $S_4^2 > U$, where S_4^2 is evaluated using Eq. (58). Thus, the weak-scatter condition $U \ll 1$ need not require that $S_4 \ll 1$. This is consistent with the experience of most experimentalists, namely that the weak-scatter S_4 formula accurately reproduces their data, even for moderately large scintillation levels.

As ν approaches 1.5, $F(\nu)$ approaches infinity. The $\nu = 1.5$ point marks an important transition from a scintillation behavior regime in which the contribution of structures larger than the Fresnel radius is strongly suppressed ($\nu < 1.5$) to one in which the contribution is only weakly suppressed ($\nu > 1.5$). When $\nu < 1.5$, the phase-structure function, for example, is well defined in the limit as the outer-scale wavenumber approaches zero.

In Section V an asymptotic analysis of $I(\vec{q},U)$ was performed for large and small q values. For U << 1, the low-frequency approximation

$$\Phi_I(\vec{q}) \simeq \delta(\vec{q}) + 4C_p q^{-(2\nu+1)} \sin^2 (Zq^2) \tag{59}$$

accounts for all significant Fourier components. As U increases, the high-frequency approximation accounts for all the significant Fourier components, except the delta function at $\vec{q} = 0$. The form of the high-frequency approximation, however, is very critically dependent on ν.

When $\nu < 1.5$, the high-frequency approximation is independent of U. Thus, under conditions of strong scattering, the intensity statistics converge; ultimately there is no further change with increasing Z. When this happens, a simple analytic form for $\Phi_I(\vec{q})$ cannot be obtained. Nonetheless, the intensity correlation function takes the particularly simple form

$$\langle II' \rangle = \exp\{-D(y)\} \tag{60}$$

where

$$D(y) = G\frac{C_p}{2\pi}\left[\frac{2\Gamma(1.5-\nu)}{\Gamma(\nu+0.5)(2\nu-1)2^{2\nu-1}}\right]|y|^{2\nu-1} \quad . \tag{61}$$

To evaluate the temporal autocorrelation function of intensity, y is replaced by $v_{eff}\delta t$, where v_{eff} is defined by Eq. (13) in Paper I. Thus, if τ_I is defined to be the time separation to achieve an intensity decorrelation of e^{-1}, then

$$\tau_I = v_{eff}\left[G\frac{C_p}{2\pi}\left(\frac{2\Gamma(1.5-\nu)}{\Gamma(\nu+0.5)(2\nu-1)2^{2\nu-1}}\right)\right]^{\frac{1}{2\nu-1}} \quad . \tag{62}$$

The formula becomes meaningless as ν approaches 1.5. For $\nu \geq 1.5$, no simple formula such as Eq. (60), exists and numeric computations must be employed. Indeed, in that case τ_I will depend on Z, as it does under conditions of weak scatter, in which there is also no simple formula.

It is instructive to compare these results to the earlier results for a gaussian phase screen with a gaussian ACF of the form $\exp\{-\Delta\rho^2/r_o^2\}$. For the gaussian ACF, the rms phase φ_o can be computed from the expression

$$\varphi_o^2 = r_e^2\lambda^2 \,(L \sec \theta)\, G \,\langle \Delta N_e^2 \rangle\, 2\pi^{-3/2}\, r_o \quad . \tag{63}$$

Under conditions of weak, isotropic scattering ($\varphi_o \ll 1$)

$$S_4^2 = 2\varphi_o^2 \left\{ 1 - \cos^2 \left(2\lambda\tilde{z}/(\pi r_o^2) \right) \right\} \quad . \tag{64}$$

If $\lambda z/r_o^2 \ll 1$,

$$S_4^2 \simeq 2\varphi_o^2 \left(\frac{2\lambda\tilde{z}}{\pi r_o^2} \right)^2 \quad , \tag{65}$$

which is similar to Eq. (58). If, on the other hand, $\lambda\tilde{z}/r_o^2 \gg 1$,

$$S_4^2 \simeq 2\varphi_o^2 \quad . \tag{66}$$

Moreover, Mercier (1962) has shown that the intensity statistics are Rician in this limit. By comparison, in a strict power-law environment, λz cannot be increased indefinitely without S_4 approaching unity. Put another way, in a strict power-law environment one always encounters strong scattering with increasing distance from the scattering region before S_4 saturates independent of $\tilde{z}$.

Under conditions of strong scattering, the relationship $\langle II' \rangle - 1 = \exp\{-2D(y)\}$ where

$$D(y) = \varphi_o^2 \left(1 - \exp\{-y^2/r_o^2\} \right) \tag{67}$$

will ultimately apply. However, it is known that when $\varphi_o > 1$ and

$$\varphi_o \frac{\tilde{z}}{k} Q_o^2 \sim 1 \quad , \tag{68}$$

where Q_o^4 is the fourth-order moment of the gaussian spectrum, the scintillation index S_4 exceeds unity (Salpeter, 1967). This is the strong focusing effect. This phenomenon has a counterpart in a power-law medium. However, since neither φ_o nor Q_o exist, the strong focusing condition cannot be simply characterized as with Eq. (68).

The scintillation behavior in a gaussian ACF environment is conveniently summarized in diagram form, as was done initially by Cohen et al. (1967). Figure 2, the usual representation, is reproduced from Singleton (1970). Regions I and II correspond to the near and far zones, respectively. Region III is the "fully modulated" or saturation region where S_4 is unity and the relationship $\langle II' \rangle - 1 = \exp\{-2D(y)\}$ holds. Region IV is the strong focusing region where $S_4 > 1$.

In a strict power-law environment, by comparison, there is no characteristic scale size akin to r_o, and the magnitude of the rms phase is irrelevant to the scattering behavior. Contours of constant S_4 have the form U = const., or, from Eq. (56),

$$C_p Z^{\nu-0.5} = \text{const.} \tag{69}$$

Thus, if one were to make a diagram similar to the Cohen-Singleton diagram for a power-law medium, the phase strength-of-turbulence, C_p, would replace φ_o and the Fresnel area $\lambda z/4\pi$ would replace $\lambda z/r_o^2$.

The degree to which Region II is present depends critically on the spectral index ν. If $\nu < 1.5$, S_4 does not significantly exceed unity. For $\nu \geq 1.5$, however, S_4 values substantially larger than 1 do occur. When this happens, the contours are still given by Eq. (69). Thus, the scattering behavior in a strict power-law environment can be similar to that predicted by the Cohen-Singleton diagram for $\lambda z/r_o^2 \ll 1$.

In fact, if a finite outer-scale is introduced, then the Cohen-Singleton diagram for a power-law medium is similar to that of a gaussian

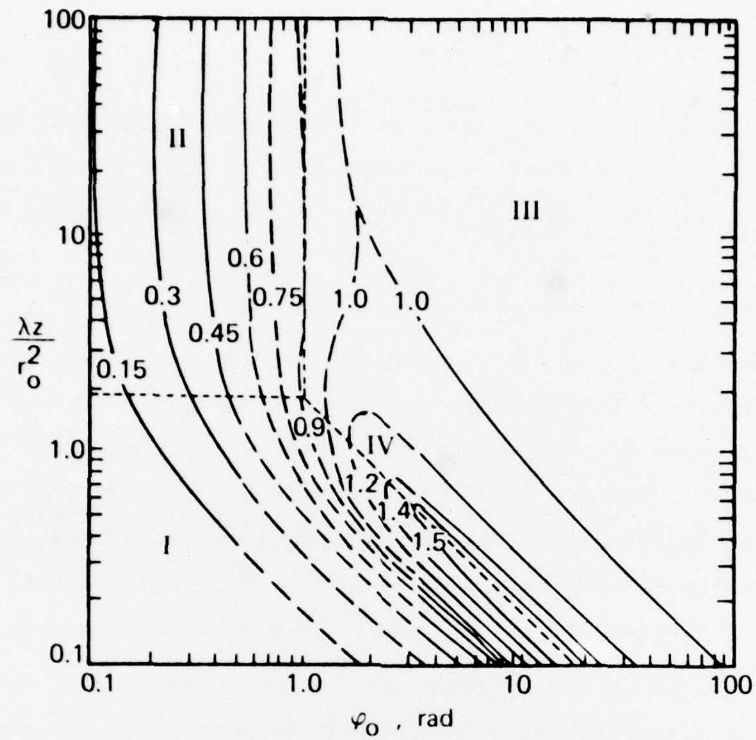

FIGURE 2 CURVES OF CONSTANT S_4 ON A PLOT OF $\lambda z/r_o^2$ vs. φ_o--TAKEN FROM SINGLETON (1970)

medium with the outer scale playing the role of r_o. Because the outer scale is at best indeterminant in the ionosphere, however, such an extrapolation is not useful for data interpretation.

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