A MATHEMATICAL MODEL TO CALCULATE VOLUMES OF LUMBER AND RESIDUE PRODUCED IN SAWMILLING

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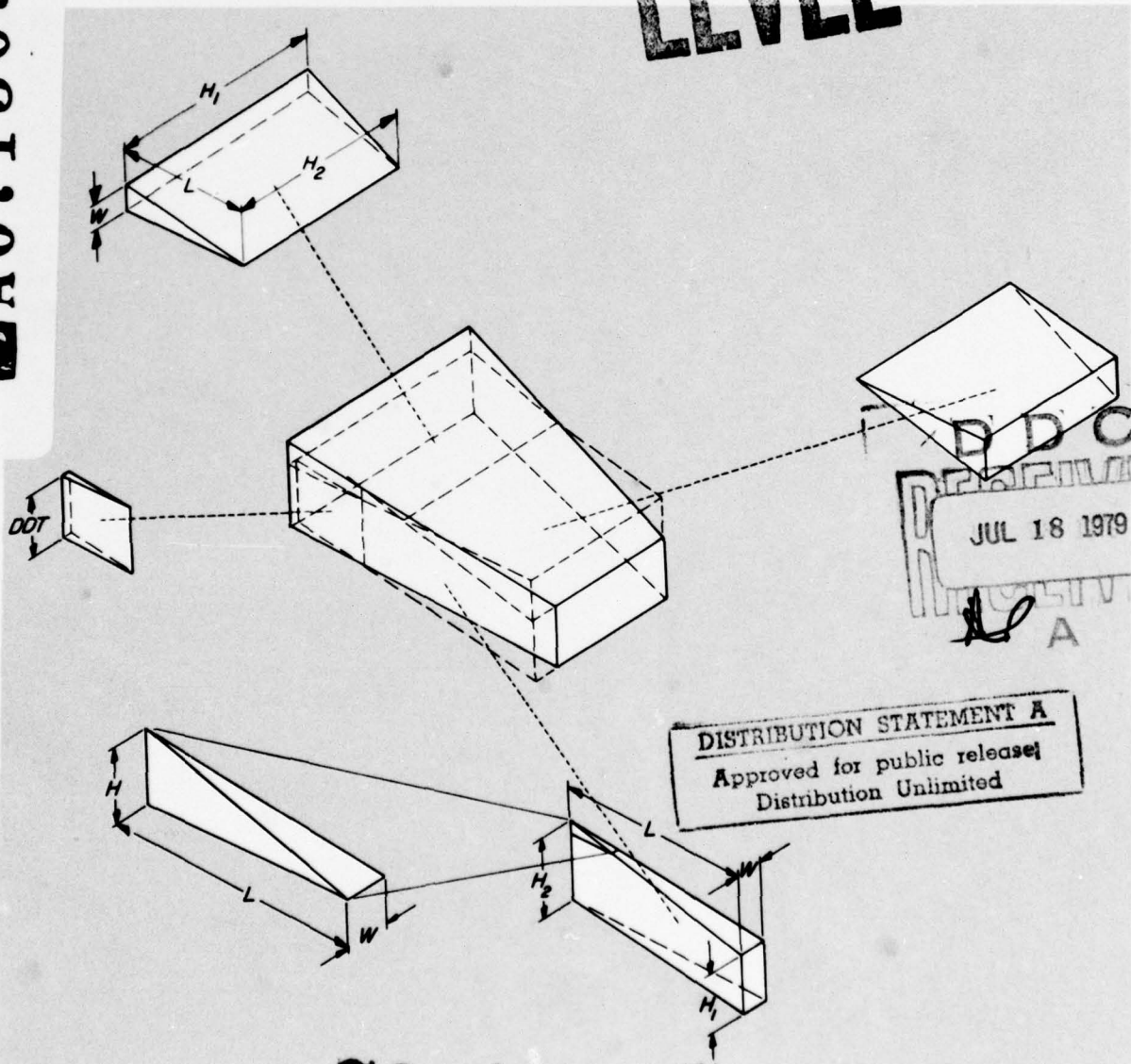
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Abstract

The need to measure the volumes of all materials produced in the saw-milling process is becoming more important as the value of these materials increases.

This paper introduces a geometric model with which to calculate the volumes of these materials with a minimum of data gathering. Methods to calculate the volumes of green lumber, dry lumber, green chips, green sawdust, and dry planer shavings are given.

The mathematical and geometric theory making up the model is illustrated by equations and drawings.

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A MATHEMATICAL MODEL TO CALCULATE VOLUMES OF LUMBER
AND RESIDUES PRODUCED IN SAWMILLING

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Introduction

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The importance of residues as a byproduct of sawmilling has been growing yearly as their utilization increases. Utilization and the resulting importance should continue to increase. The supply of commercially desirable sawtimber species is not keeping pace with demand and smaller diameter trees that at one time were being pulped are now being sawn for lumber. Some products now made with lumber will in the future be made with lower cost byproducts of the milling process--chips, planer shavings, sawdust, and bark. As the prices of fossil fuels climb, mills are increasing their use of residues as a source of energy.

Historically, sawmill residues had little or perhaps even a negative value, because their disposal often incurred a cost. As with all such valueless or "free" resources, the need to measure them did not arise. With an increase in their value, however, accurate estimates of residues becomes important.

There are many ways that an accurate method for calculating residue volumes can be used. For example: (1) mills with computerized lumber tallying equipment can automate the inventory of the residues they produce, as well as the lumber; (2) the economic feasibility of a sawmill in the planning stage can be determined more accurately if a good estimate of residue volumes is available; (3) the engineering design of

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a sawmill in the planning stage can take into account the volume of residue production and better estimate the requirements for conveyor capacity and volume; and (4) energy production from residues for sawmills currently operating and those in the planning stage can be accurately estimated so expectations of energy output are met and equipment of the proper capacity is built into the mill design.

This paper presents the geometric and mathematical theory making up a model to calculate residue volumes. The model being presented allows accurate calculation of the volumes of lumber, chips, planer shavings, and sawdust. It is meant to be applied to a specific sawmilling situation--to any mill with any type of operating characteristics sawing any species. It can be used to analyze a currently operating mill or a proposed mill.[3/]

Planning for the Model

Review of Literature

Residue studies made in the past entailed a considerable amount of labor involving measuring and then weighing the log and all components produced in the sawing process that are of interest. These studies have attempted to bypass the labor involved in future studies by using regression equations ([4],[6],[7],[8],[14],[15],[21],[22],[23],[25],[26],[27],[28]), conversion or percentage

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[3/] When this model is used to predict residues generated by a proposed mill, the necessary data have to be generated. This requires assumptions about the operating characteristics of the mill (kerf, variation, and fixed head planing cut) and the nature of the logs to be sawn (diameter distribution, length, and percent of defect). A less obvious requirement is the need to derive a lumber tally that fits the log distribution sawn. Knowing the expected Lumber Recovery Factor (LRF) of the mill, a reasonable lumber tally can be estimated. A more exacting approach would be to use a computer program that calculates the maximum yield from each log, such as the Best Opening Face (BOF) ([12]) program. Again, these maximum yields would have to be adjusted according to the expected LRF of the proposed mill.

factors ($\underline{1}$,$\underline{2}$,$\underline{5}$,$\underline{9}$,$\underline{10}$,$\underline{17}$,$\underline{19}$,$\underline{20}$,$\underline{24}$,$\underline{30}$), mathematical models ($\underline{3}$,$\underline{13}$,$\underline{29}$) or
reports of absolute residue volumes ($\underline{16}$,$\underline{18}$) as predictors. These studies
based their predicting models on one or more mill studies.

The various methods used by all of these studies have one weakness in
common; relatively few of the many variables that can affect residue
volumes are considered. Most use only diameter or diameter and length
of the tree or log as their independent variables. Three studies ($\underline{4}$,$\underline{17}$,
$\underline{24}$) include log grade and one study ($\underline{28}$) includes log grade and taper as
additional variables.

An accurate prediction of a sawmill's residue production requires
including in the analysis all important variables[4] that can affect this
production. Some of the more important variables that should be
accounted for, besides log or tree diameter, length, grade, and taper,
are:

> Product mix
> Kerf width
> Sawing variation
> Rough green lumber size
> Size of planed lumber
> Condition and maintenance of mill equipment
> Ability, conscientiousness, and fatigue level of the sawyer and
>   other mill personnel.

An actual mill study takes all of these sources of variation into account.
The problem has been that such a mill study has required a large amount
of time and labor. The model presented here allows a mill study to be
conducted with a relatively small amount of labor and data collection.[5]
No weighing of logs and residues is necessary. An accurate residue
analysis, considering all sources of variation, can therefore be run
with a minimum of effort.

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[4] The model presented here takes certain variables into account
implicitly (log grade, condition of mill equipment, personnel factors).
This is because a mill study is required. Even though these variables
are not quantified and used as data in the calculations, their effects
show up in the lumber and residue recovery figures.

[5] The Henley and Hoopes publication ($\underline{13}$) provides a computer
program to calculate residue volumes. It appears to use a model and
requires data similar in some respects to the one described in this
paper. Their model differs in that planer shavings volume is not
calculated and the methods used to calculate lumber volumes and residues
appear to be less exacting.

With these data, the calculations for an accurate estimate of residues can be made. As can be seen, the data are relatively easy to gather with no special equipment necessary.

Data Needed for Model

Length of each sample log and maximum and minimum diameters of both large and small ends are measured. Lumber resulting from the sawing of sample logs is tallied. Sample lumber is measured for size and variation. The thickest and thinnest measurements for each of 100 boards of each thickness cut, and the widest and narrowest measurements for each of 20 boards of each width cut, are considered to be an adequate sample. In mills planing their lumber, the depth of cut made by the fixed heads in the planer is needed.

For extreme accuracy in mills planing dry lumber, the moisture content of dry lumber can be measured, but an estimate is usually adequate. The tangential shrinkage value for each species studied is necessary and can be found in the Wood Handbook (11) published by the U.S. Forest Products Laboratory.

Abbreviations of Variables
Used in Model

Abbreviations will be used for simplicity in the equations that describe the model. The following is a list of these abbreviations and their meanings as used in the equations.

ACTVOL	=	Actual Dry Dressed Volume of Lumber
DDT	=	Dry Dressed Thickness
DDW	=	Dry Dressed Width
DDVOL	=	Dry Dressed Volume of Lumber
F	=	Conversion Factor = 1/144 = 0.0069444
FHC	=	Fixed Planer Head Cut
L_M	=	Nominal Lumber Length + MLTA
L_O	=	Nominal Lumber Length + MLTA + Average Log Overlength
MLTA	=	Minimum Lumber Trim Allowance
NOMVOL	=	Nominal Volume of Lumber
NSVTHK	=	Negative Sawing Variation--Thickness
NSVWTH	=	Negative Sawing Variation--Width
PLNSHV	=	Dry Planer Shavings Volume
PSVTHK	=	Positive Sawing Variation--Thickness
PSVWTH	=	Positive Sawing Variation--Width
RDT	=	Rough Dry Thickness
RDW	=	Rough Dry Width

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RDVOL   = Rough Dry Volume of Lumber
RGT   = Rough Green Thickness
RGW   = Rough Green Width
RGVOL   = Rough Green Volume of Lumber

SHRINK   = Shrinkage Factor = $\dfrac{(100 - \text{Tangential Shrinkage for Species Analyzed})}{100}$

SKPVOL   = Volume of Skips in Planing
TSVTHK   = Total Sawing Variation--Thickness
TSVWTH   = Total Sawing Variation--Width
WTDKRF   = Weighted Average Kerf

Specifics of the Model

Introduction of Model

All of the calculations shown are based on the assumption that the mill
being studied is of the typical softwood type that planes its lumber dry
and produces dry dressed lumber.

There are three other possible conditions in which a mill can sell its
lumber: Dry rough, green dressed, or green rough. The residue calcula-
tions for these three cases can be made with minor modifications to the
calculations given here.

A geometric model was created to make practical the application of
mathematical calculations to obtain the volume of each product produced
in milling a quantity of lumber. This model of the lumber produced is
based on the following assumptions:

(1) Each thickness and width class of lumber can be represented by a
theoretical piece of lumber made up of the total lengths of lumber in
that class. For example, if the nominal 2 by 4 thickness and width
class contains 10 pieces of lumber each 10 feet long, then the entire
class is treated as one 2 by 4, 100 feet long.

(2) This theoretical 2 by 4 has its smallest dimensions on one end and its largest on the other. In gathering data each piece of lumber sampled is measured for maximum and minimum thickness in both dimensions. From these measurements, a total sawing variation, positive sawing variation, negative sawing variation, and average width or thickness are calculated.[6] Knowing these values, the small end dimensions are equal to the average dimension minus negative sawing variation; the large end dimensions are equal to the average dimension plus positive sawing variation. Figure 1 illustrates this, showing some hypothetical dimensions and variations applied in the 2 by 4 example given above.

Lumber Length (L_M and L_O)

In many of the equations to follow, the total length of lumber for each thickness and width class must be known. Two lengths will be used in these solutions. L_M is the total nominal lumber length plus minimum lumber trim allowance (MLTA) per thickness and width class. L_O is L_M plus average overlength per log.

L_M reflects the true length of trimmed lumber, since each piece of lumber is at least the nominal length plus the MLTA.

$$L_M = \text{(Nominal Lumber Length + MLTA)} \times \text{(Number of Boards per Nominal Lumber Width and Thickness Class)} \quad (1)$$

L_O reflects the length of each piece of lumber prior to end trimming. This length is used when the distance of the travel of the saw through a log is necessary in an equation.

$$L_O = \text{(Nominal Lumber Length + MLTA + Average Log Overlength)} \times \text{(Number of Boards per Nominal Lumber Width and Thickness Class)} \quad (2)$$

Rough Green Lumber Volume (RGVOL)

Before calculating green lumber volume, it is necessary to break down the geometric model of lumber into two separate geometric solids. The volumes of these solids can then be mathematically determined. Figure 2

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[6] An explanation of the meaning of negative, positive, and total sawing variation is contained in Appendix I.
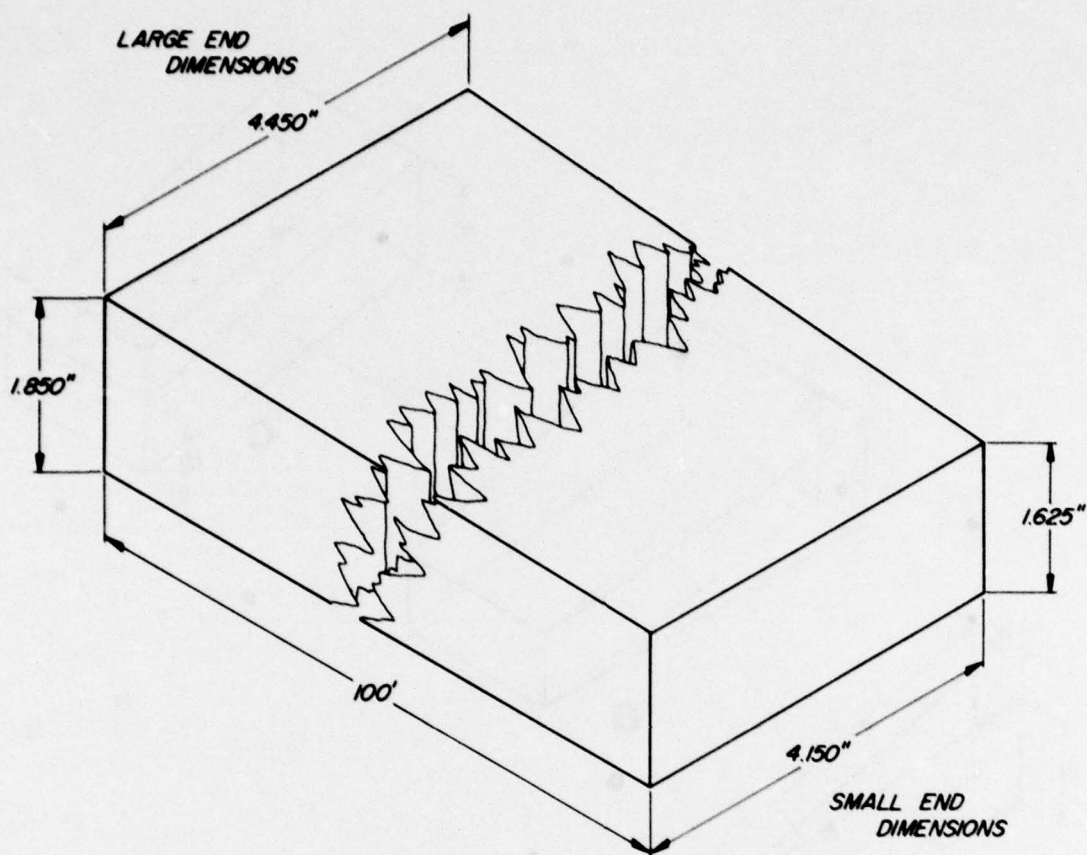
Figure 1.--Geometric model of a nominal 2- by 4-inch width and thickness
class of lumber with hypothetical variations used to calculate large
and small end dimensions.

	Width (In.)	Thickness (In.)
Total Sawing Variation	≈ 0.300	0.225
Positive Sawing Variation	= .200	.100
Negative Sawing Variation	= .100	.125
Average Dimension	= 4.250	1.750

Small End Dimension = Average Dimension - Negative Variation
       Width = 4.250 - .100   Thickness = 1.750 - .125
             = 4.150   = 1.625

Large End Dimension = Average Dimension + Positive Variation
       Width = 4.250 + .200   Thickness = 1.750 + .100
             = 4.450   = 1.850

(M 146 633)

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H_1

L_M

W_1

c

H_2
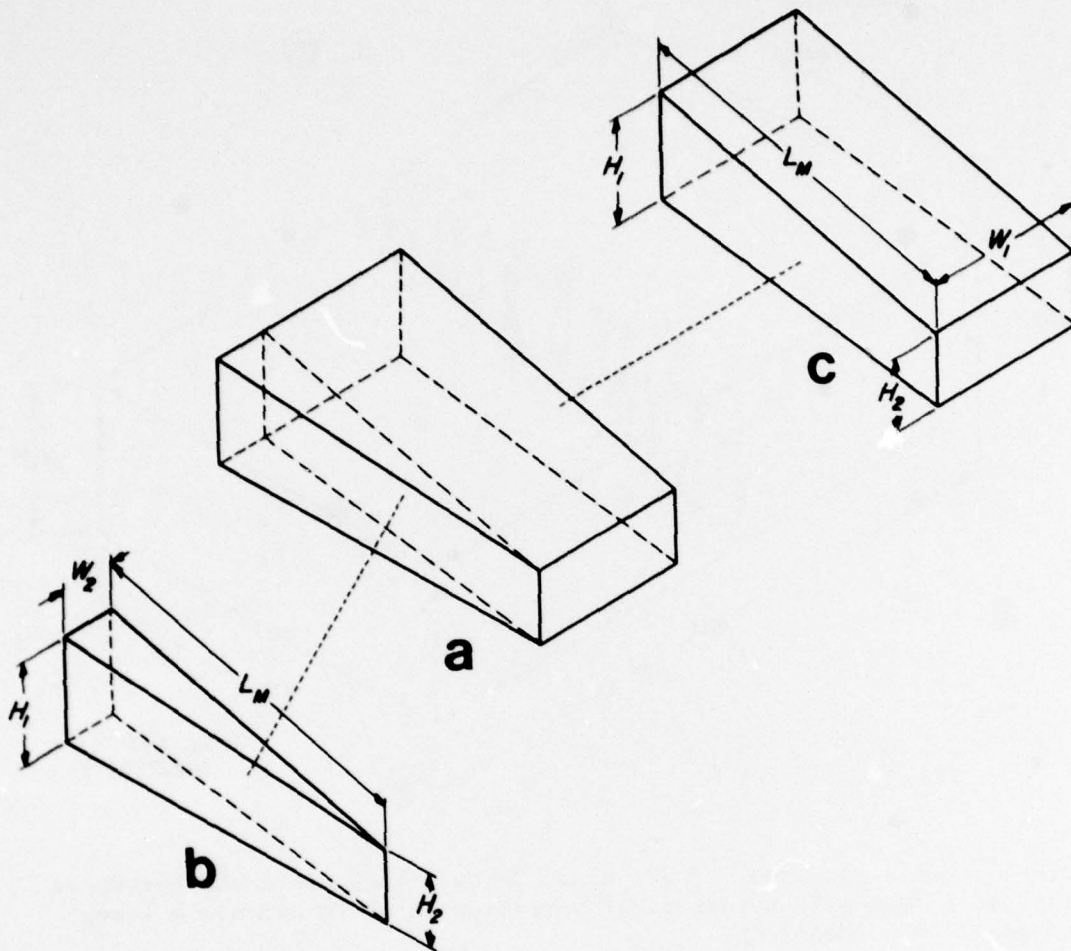
a

W_2

L_M

H_1

b

H_2

Figure 2.--Breakdown of lumber model (a) into a wedge (b) and a
  trapezoidal solid (c).

(M 146 634)

illustrates this breakdown, and equations (3), (4), and (5) are derived from the variables defined in these figures.

The volume of the trapezoidal solid (fig. 2c) is given by the general equation:

$$W_1 L_M (\frac{H_1 + H_2}{2}) \tag{3}$$

The volume of the wedge (fig. 2b) is given by the general equation:

$$\frac{W_2 L_M}{2} (H_1 + \frac{H_2 - H_1}{3}) \tag{4}$$

The equation to calculate rough green lumber then becomes:

$$RGVOL = \left[W_1 L_M (\frac{H_1 + H_2}{2}) + \frac{W_2 L_M}{2} (H_1 + \frac{H_2 - H_1}{3}) \right] \times F \tag{5}$$

where:

$$H_1 = RGT + PSVTHK$$
$$H_2 = RGT - NSVTHK$$
$$W_1 = RGW - NSVWTH$$
$$W_2 = (RGW + PSVWTH) - (RGW - NSVWTH)$$

Rough Dry Lumber Volume (RDVOL)

Rough dry lumber volume is calculated in the same manner and using the same equations as rough green lumber volume but with dry dimensions replacing those for green. Dry dimensions are obtained by multiplying the shrinkage factor (SHRINK) times the rough green dimensions as in equation (6).

$$RDW = SHRINK \times RGW$$
$$RDT = SHRINK \times RGT \tag{6}$$

The equation for rough dry lumber volume becomes:

$$RDVOL = \left[W_1 L_M \left(\frac{H_1 + H_2}{2} \right) + \frac{W_2 L_M}{2} \left(H_1 + \frac{H_2 - H_1}{3} \right) \right] \times F \qquad (7)$$

where:

$H_1 = RDT + PSVTHK$

$H_2 = RDT - NSVTHK$

$W_1 = RDW - NSVWTH$

$W_2 = (RDW + PSVWTH) - (RDW - NSVWTH)$

Dry Dressed Lumber Volume (DDVOL)

Dry dressed lumber volume is calculated using the target widths and thicknesses the planer is producing. For example, the dry dressed volume of a class of lumber that is 1.50 inches by 3.50 inches and 100 feet long would be 1.50 inches x 3.50 inches x 100 feet x 1/144 = 3.65 cubic feet.

The general equation to calculate dry dressed lumber volume is:

$$DDVOL = DDW \times DDT \times L_M \times F \qquad (8)$$

Actual Dry Dressed Lumber Volume (ACTVOL)

Actual dry dressed volume is equal to the total amount of wood present in the particular piece of lumber being discussed.

It is important to distinguish between this value and that of dry dressed volume. The dry dressed volume of 3.65 cubic feet of the lumber class given above (1.50 x 3.50 in.) would in most cases not be the actual dry dressed lumber volume. The two would be equal only if there were no planing skips on the lumber. This equality is unlikely because the lumber from most mills will have some surface skips. Thus, the actual dry dressed volume in the example above will probably be something less than 3.65 feet.

Actual dry dressed volume is a useful concept and serves to remove scantness (planing skips) as a variable when comparing the conversion efficiencies of mills. It provides a precise basis on which a comparison

can be made because actual dry dressed volume gives the true volume of
lumber produced. Neither lumber recovery factor (board feet of lumber
as a percent of total log volume) nor dry dressed volume as a percent of
total log volume are as precise.

No equation has been developed to calculate actual dry dressed lumber
volume directly; the volume depends on the skips present on the lumber
after planing. One method is given in the next section, along with the
equations to calculate planer shavings.

Planer Shavings Volume (PLNSHV)

The possibility of a mill undersizing its lumber introduces some diffi-
culties into calculating planer shavings volume. To simplify this
initially, the discussion will be limited to situations that could occur
on one face of a piece of lumber.

To illustrate what is occurring on a single face with reference to planing
lines intersecting that face, it is useful to show the adjacent face.
The dimension of the adjacent face determines how the planer head inter-
acts with a given face. That is, the variability and dimensions of the
narrow face of a piece determine the amount of planer shavings taken
from the wide face and vice-versa. Figure 3 shows the face adjacent to
the single face dealt with when illustrating how the variability and
dimension of the piece interact with the lines of planing.

Three possible situations can occur on a single face with respect to the
intersection of planing lines with that face:

> (1) The variable plane can be in wood the entire length of
> the face. When there is enough oversizing to compensate for any
> variability, no planing skips are produced. Figure 3a illustrates
> the path of a plane intersecting the face of a piece of lumber under
> this condition. The side view shows that the plane remains in the
> piece on its entire pass. This situation will be referred to here-
> after as OVERPLANE.

> (2) The variable plane can entirely miss hitting wood when the
> piece passes through. This situation (fig. 3b) rarely occurs for
> it would indicate that the rough dry lumber size is less than the
> dry dressed size plus fixed head cut. Such lumber would be planed
> on two sides only by the planing heads making the fixed cut.

> This situation does indicate a lower limit to the volume of planer
> shavings that will be produced when lumber is planed. This lower
> limit exists because, as lumber passes through the planer, the
> fixed heads will take their cut from one wide face and one narrow

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PATH OF VARIABLE PLANE

a

PATH OF FIXED PLANE

PATH OF VARIABLE PLANE

b

PATH OF FIXED PLANE

PATH OF VARIABLE PLANE
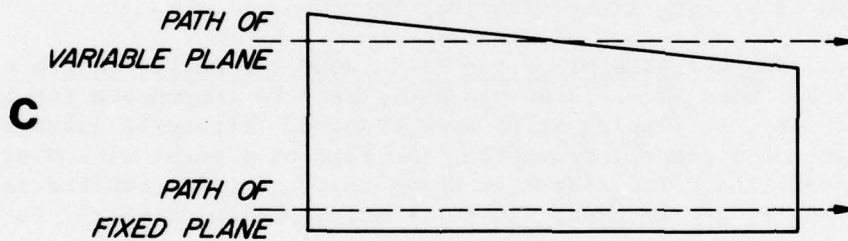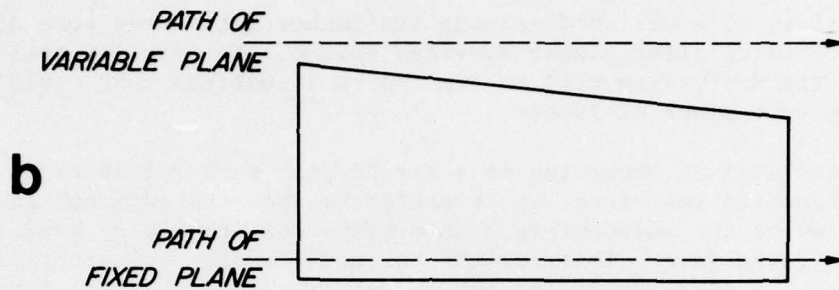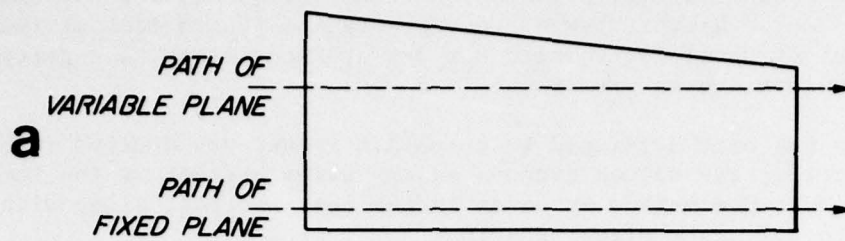
c

PATH OF FIXED PLANE

Figure 3.--Path of variable plane: (a) in wood the entire length of face (OVERPLANE); (b) entirely missing wood (UNDERPLANE); (c) passing through wood and air alternately (PARTIAL PLANE).

(M 146 635)

face. Therefore, the minimum amount of shavings produced is equal
to that taken off by the fixed head cut on one thickness and one
width. This situation will be referred to hereafter as <u>UNDERPLANE</u>.

(3) <u>The variable plane can pass through wood and air alternately,</u>
<u>producing skips on the lumber.</u> Probably the most common and
usually desirable situation encountered in a mill is shown in
figure 3c. Because some skips are allowed under the grading rules,
maximum LRF will be produced when the skips are produced up to the
limits of these rules. This situation will be referred to hereafter
as <u>PARTIAL PLANE</u>.

Each of the three cases can occur on each of two faces on the geometric
model of lumber because all variation in each dimension is represented
as occurring on a single face. The result is that each piece of lumber
can be described by one of nine possible situations when it is planed:

Case	Thickness	Width
(1)	Overplane	Overplane
(2)	Underplane	Underplane
(3)	Partial plane	Partial plane
(4)	Underplane	Overplane
(5)	Overplane	Underplane
(6)	Partial plane	Overplane
(7)	Overplane	Partial plane
(8)	Partial plane	Underplane
(9)	Underplane	Partial plane

Each of these nine cases requires a specific equation to calculate the
volume of planer shavings produced. Before the appropriate equation can
be applied, a test must be made to determine which case has occurred.
The tests for each case are given below followed by the appropriate
equations to be used if a lumber thickness and width class fits a case.
Figures illustrating the geometric relations involved in each case are
included as an aid to understanding the derivation of the equations.

Two types of planing paths are referred to--variable and fixed. The
variable head removes a variable thickness of wood from the lumber
surface being planed. The fixed head removes a constant amount from
each piece--usually just enough to remove saw marks, or 1/32 to 2/32 of
an inch. The fixed heads always remove shavings from two faces of each
four-sided piece. There are always one fixed and one variable head for
thickness and another fixed and another variable head for width.

Because the path of the fixed head cut (FHC) is a constant, it will be illustrated in the figures to follow only when it is necessary in calculating planer shaving volume. Usually the volume of planer shavings can be calculated without it. The path of the variable head cut, in relation to the two faces of the model within which all of the variation has been represented, determines which of the nine cases is involved and is the one that it is important to illustrate.

Case 1.--Thickness overplane; width overplane (fig. 4).

Test for case--

IF: (RDT - NSVTHK) $\geq$ (DDT + FHC)
AND: (RDW - NSVWTH) $\geq$ (DDW + FHC)
THEN: Use this equation to calculate volume of planer shavings:

ACTVOL = DDVOL
PLNSHV = RDVOL - DDVOL

Case 2.--Thickness underplane; width underplane (fig. 5).

Test for case--

IF: (RDT + PSVTHK) $\leq$ (DDT + FHC)
AND: (RDW + PSVWTH) $\leq$ (DDW + FHC)
THEN: Use these equations to calculate volume of planer shavings:

$$PLNSHV = \left\{ \left[\left(\frac{2RDT + PSVTHK - NSVTHK}{2} \right) \times FHC \right] + \left[\left(\frac{2RDW + PSVWTH - NSVWTH}{2} - FHC \right) \times FHC \right] \right\} \times L_M \times F$$

ACTVOL = RDVOL - PLNSHV

Case 3.--Thickness partial plane; width partial plane (figs. 6-10).

In the five cases where partial planing is involved, the calculation of planer shavings volume becomes more complex. Figure 6 illustrates why.
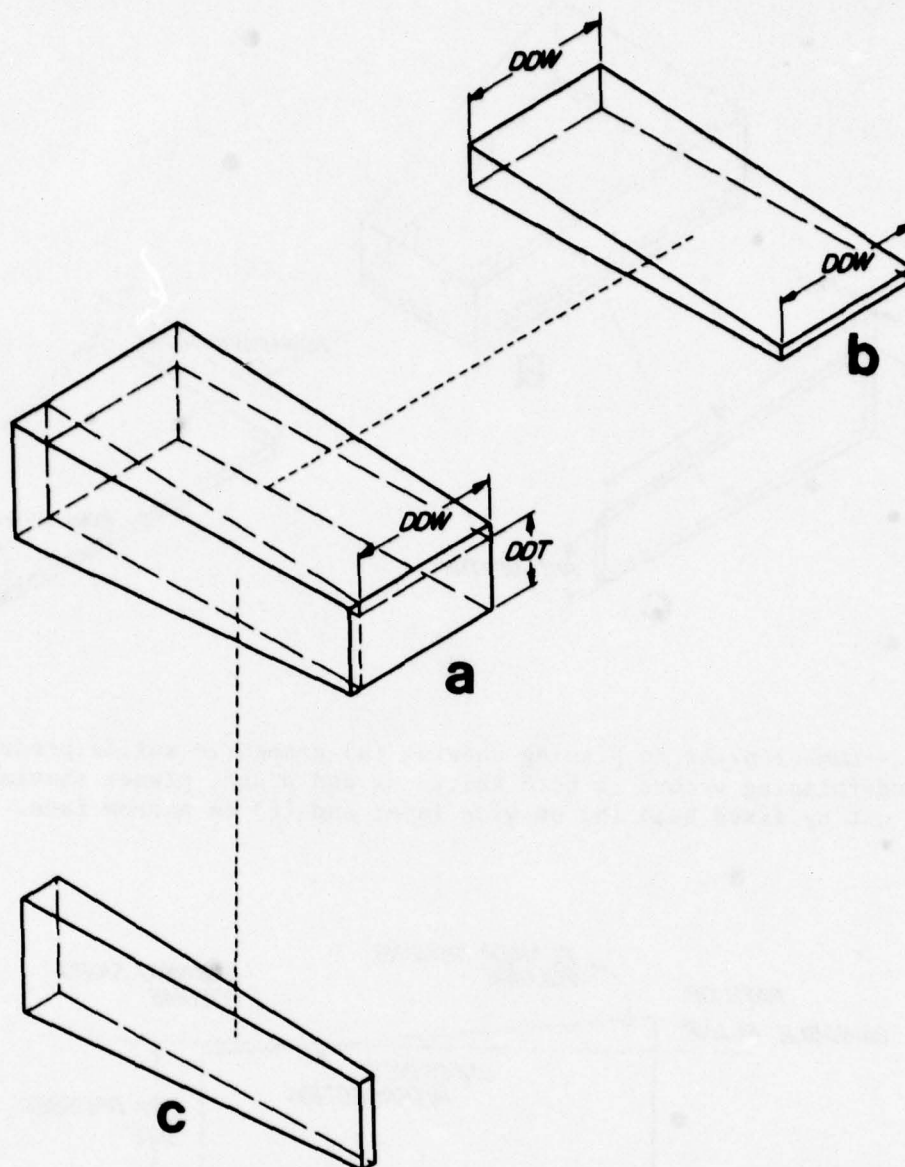
Figure 4.--Lumber prior to planing showing (a) geometric solids produced when overplaning occurs in both thickness and width; planer shavings volume by variable plane on (b) wide face; and (c) narrow face.
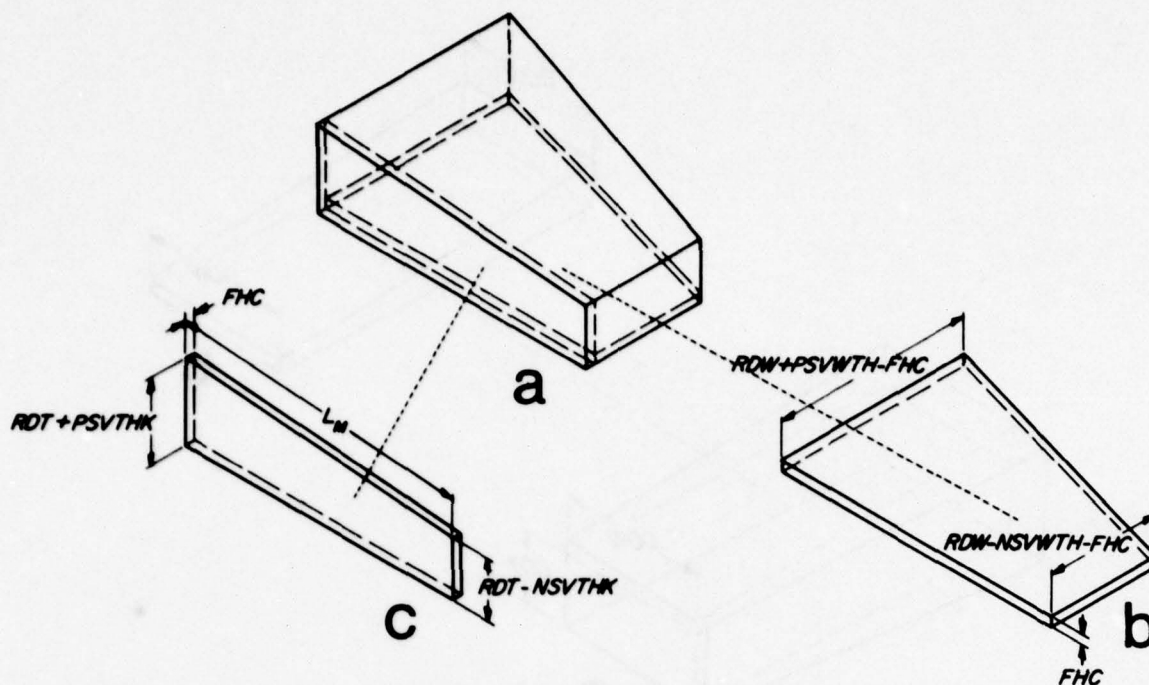
(M 146 636)

Figure 5.--Lumber prior to planing showing (a) geometric solids produced when underplaning occurs in both thickness and width; planer shavings volume cut by fixed head (b) on wide face; and (c) on narrow face.
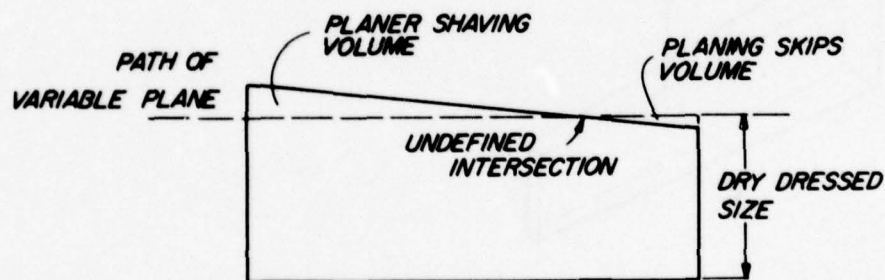

Figure 6.--Path of plane intersecting model, illustrating partial plane situation.
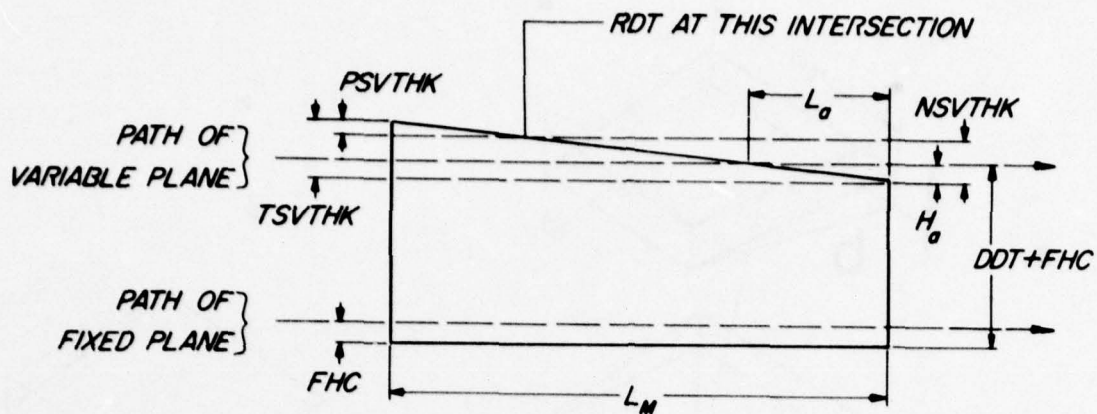
(M 146 638)

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Figure 7.--Path of plane intersecting wide face and definition of unknown dimensions necessary to solve for volume of skips. (Shown from perspective of narrow face.)
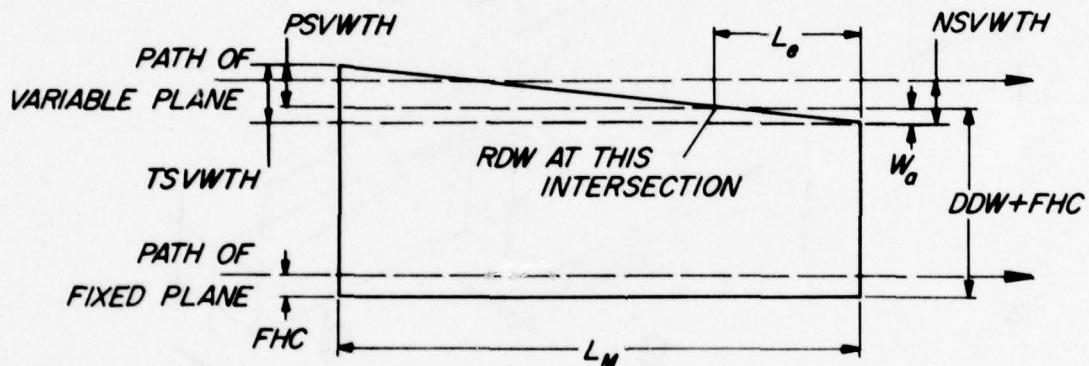
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Figure 8.--Path of plane intersecting narrow face and definition of unknown dimensions necessary to solve for volume of skips. (Shown from perspective of wide face.)
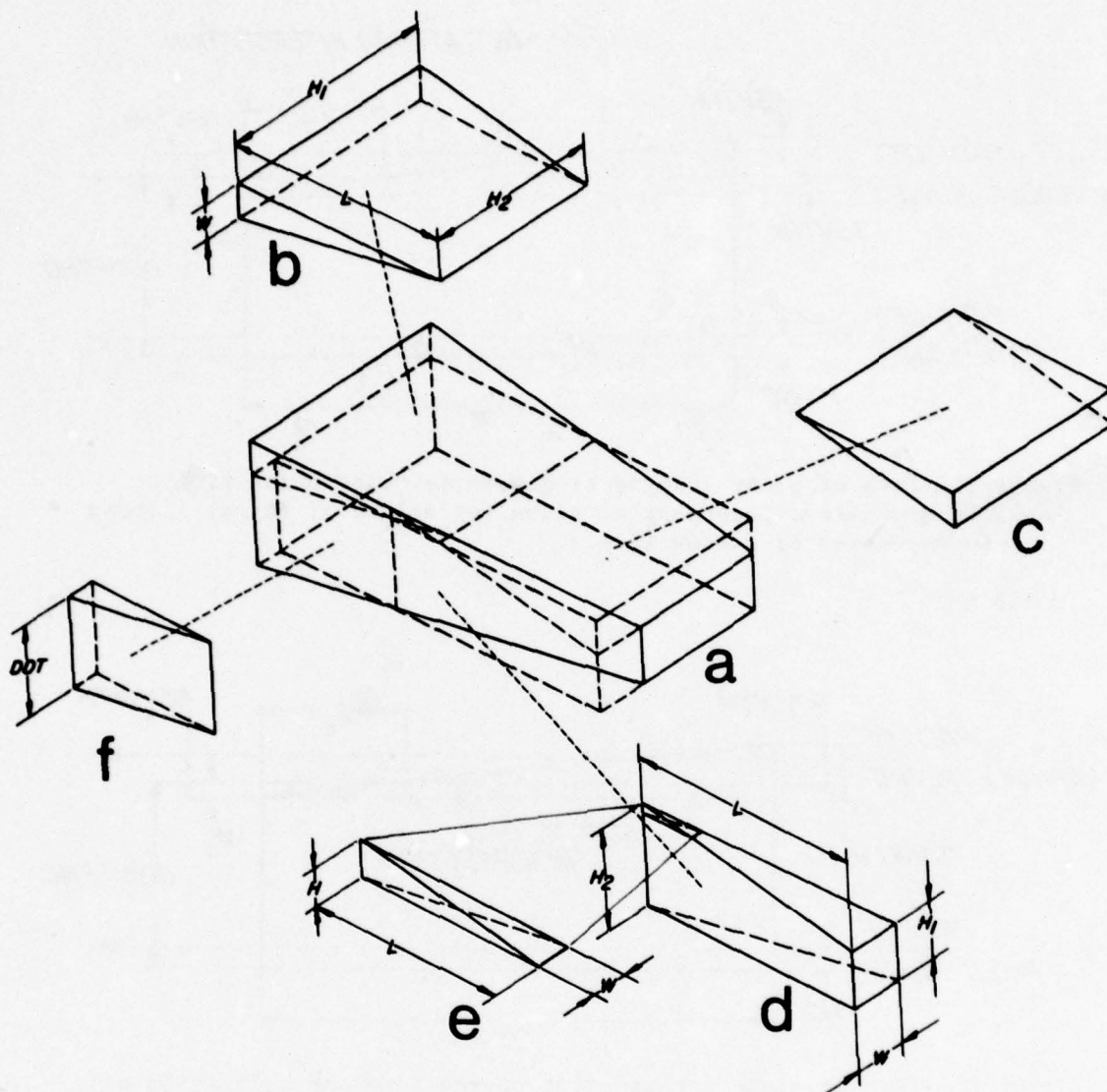
(M 146 639)

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Figure 9.--Lumber prior to planing showing (a) geometric solids produced
when partial planing occurs in both thickness and width; (b) planer
shavings volume by variable plane on wide face; planer skips volume
defined by (c) wide face and path of plane and (d) narrow face and path
of plane; (e) shavings volume shown as planing skips; and (f) planer
shavings volume by variable plane on narrow face.
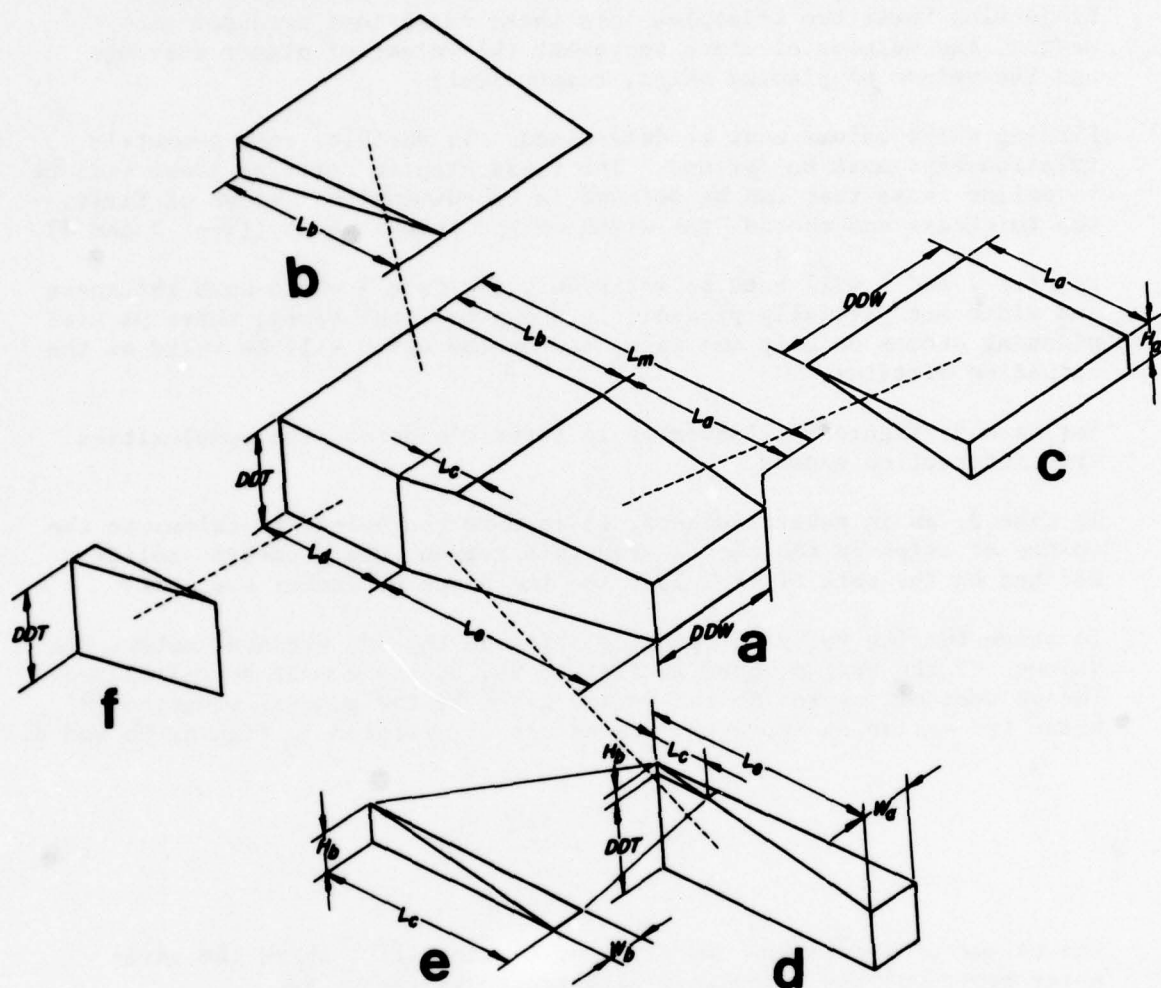
(M 146 629)

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Figure 10.--Lumber showing (a) geometric shape of wood after solids
removed in planing are broken away; (b) planer shavings volume by
variable plane on wide face; planing skips volume defined by (c) wide
face and path of plane, and (d) narrow face and path of plane;
(e) shavings volume shown as planing skips; and (f) planer shavings
volume by variable plane on narrow face.

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Note that the path of the plane in a partial planing situation (fig. 6) cuts the sloping line connecting the largest dimension of the trapezoid and its smallest dimension at an undefined point. This leaves the two triangles labled planing skips volume and planer shavings volume. Projecting these two triangles into three dimensions produces two wedges, the volumes of which represent the volume of planer shavings and the volume of planing skips, respectively.

Planing skips volume must be determined. To do this, some geometric relationships must be defined. The first step in defining these will be to define those that can be defined in two-dimensional views of first, the thickness and second, the width of the lumber model (figs. 7 and 8).

Figures 7 and 8 will both be valid only for Case 3 where both thickness and width are partially planed. In the other four cases, where partial planning occurs on only one face, one or the other will be valid as the situation dictates.

For Case 3, figure 9 illustrates in three dimensions the complexities this interaction causes.

In Case 3, as in several others, it is more convenient to calculate the volume of skips on the piece. For this reason, the geometric solids defined by the path of the plane and the piece of lumber are shown.

To solve for the volume of planing skips on the dry dressed lumber, the volumes of the wedges shown in figures 9b, d, and e must be calculated. The volumes of figures 9b and 9d are given by the general equation (9) where the variables represent dimensions illustrated in figures 9b and d.

$$\frac{WL}{2} (H_1 + \frac{H_2 - H_1}{3}) \qquad (9)$$

The volume of figure 9e[7/] is given by equation (10) where the variables represent the dimensions illustrated in figure 9e.

$$\frac{WLH}{6} \qquad (10)$$

7/ Note that the geometric solid depicted in figure 9e and 10e is planed off as the plane passes through the wide face and is a volume of planer shavings, not skips. Its volume must be calculated and subtracted from the total volume of skips.

To derive the values of the variables to use in general equations (9) and (10), figure 9 was redrawn with variables included in figure 10.

Where partial planing occurs in the cases to follow, certain variables defined in this section will again be referenced. The reader should refer to this section, and especially figure 10, for an understanding of their meaning.

The calculation of planing skips (SKPVOL) for Case 3 requires the definition of variables not previously defined. These variables are shown in figure 10.[7/]

The test and equations to solve for Case 3 can now be given.

Test for case --

IF: (RDT - NSVTHK) < (DDT + FHC)
AND: (RDT + PSVTHK) > (DDT + FHC)
AND: (RDW - NSVWTH) < (DDW + FHC)
AND: (RDW + PSVWTH) > (DDW + FHC)

THEN: Use the following equations to calculate volume of planer shavings:

$$SKPVOL = \left\{ \left[\overbrace{\left(\frac{H_a \times L_a \times DDW}{2} \right)}^{(Vol.\ of\ fig.\ 10c)} \right] \right.$$

$$+ \left[\left(\frac{W_a \times L_e}{2} \right) \left\{ \overbrace{(DDT - H_a) + \frac{(DDT + H_b) - (DDT - H_a)}{3}}^{(Vol.\ of\ fig.\ 10d)} \right\} \right]$$

$$\left. - \left[\overbrace{\left(\frac{W_b \times L_c \times H_b}{6} \right)}^{(Vol.\ of\ fig.\ 10e)} \right] \right\} \times F$$

ACTVOL = DDVOL - SKPVOL
PLNSHV = RDVOL - ACTVOL

-21-

where:

$$H_a = (DDT + FHC) - (RDT - NSVTHK)$$

$$H_b = \left\{ \left[(RDT + PSVTHK) - (DDT + FHC) \right] \quad L_c \right\} / L_b$$

$$L_a = \frac{H_a \times L_M}{TSVTHK}$$

$$L_b = L_M - L_a$$

$$L_c = L_M - (L_d + L_a)$$

$$L_d = L_M - L_e$$

$$L_e = \frac{W_a \times L_M}{TSVWTH}$$

$$W_a = (DDW + FHC) - (RDW - NSVWTH)$$

$$W_b = (L_c \times W_a) / L_e$$

Case 4.--Thickness underplane; width overplane (fig. 11).

    Test for case --

IF: $(RDT + PSVTHK) \leq (DDT + FHC)$
AND: $(RDW - NSVWTH) \geq (DDW + FHC)$
THEN: Use the following equations to calculate volume of
         planer shavings:

$$ACTVOL = WL_M \left(\frac{H_1 + H_2}{2} \right) \times F$$

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U.S. Forest Products Laboratory.

A mathematical model to calculate volumes of lumber and residues produced in sawmilling, by Philip H. Steele and Hiram Hallock. Res. Pap. FPL 336, For. Prod. Lab., For. Serv., U.S. Dept. Agric., Madison, Wis. 45 p.

Introduces a geometric model with which to calculate the volumes of materials produced in sawmilling, but with a minimum of data gathering. Methods are given to calculate volumes of green or dry lumber, green chips, green sawdust, and dry planer shavings.

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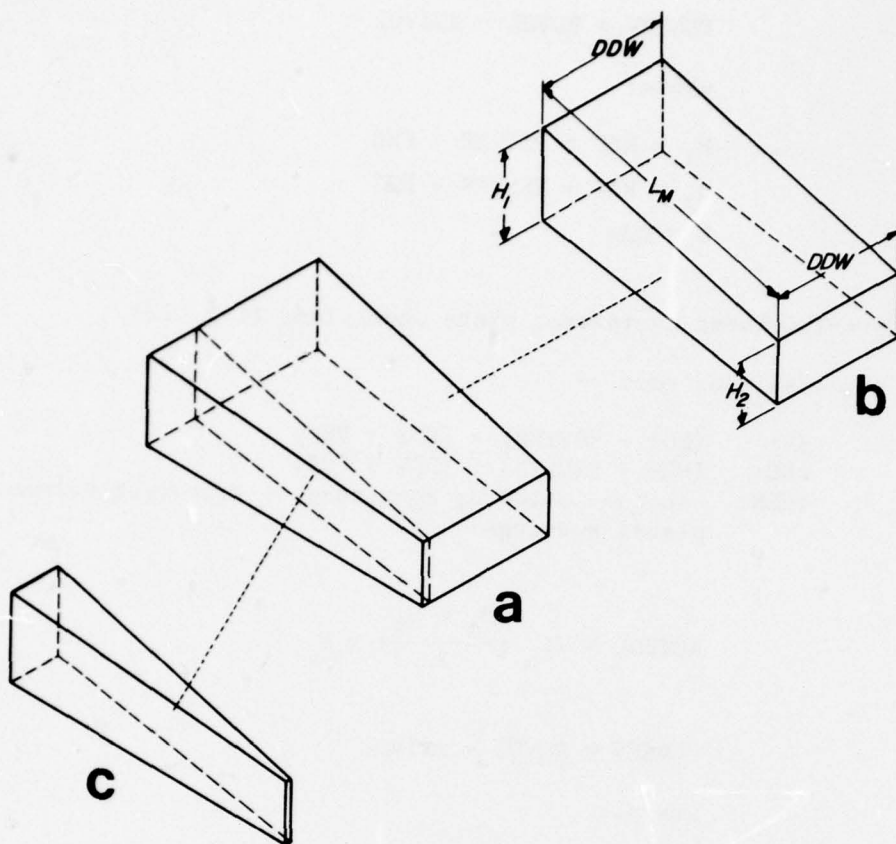
Figure 11.--Lumber prior to planing showing (a) geometric solids produced when underplaning occurs in thickness and overplaning in width; (b) planed lumber; and (c) planer shavings volume by variable plane on narrow face.

(M 146 641)

$$PLNSHV = RDVOL - ACTVOL$$

where:

$$H_1 = RDT + PSVTHK - FHC$$
$$H_2 = RDT - NSVTHK - FHC$$
$$W = DDW$$

Case 5.--Thickness overplane; width underplane (fig. 12).

Test for case --

IF: $(RDT - NSVTHK) \geq (DDT + FHC)$
AND: $(RDW + PSVWTH) \leq (DDW + FHC)$
THEN: Use the following equations to calculate volume of planer shavings:

$$ACTVOL = WL_M \left(\frac{H_1 + H_2}{2}\right) \times F$$

$$PLNSHV = RDVOL - ACTVOL$$

where:

$$H_1 = RDW + PSVWTH - FHC$$
$$H_2 = RDW - NSVWTH - FHC$$
$$W = DDT$$

Case 6.--Thickness partial plane; width overplane (fig. 13).

Test for case --

IF: $(RDT - NSVTHK) < (DDT + FHC)$
AND: $(RDT + PSVTHK) > (DDT + FHC)$
AND: $(RDW - NSVWTH) \geq (DDW + FHC)$
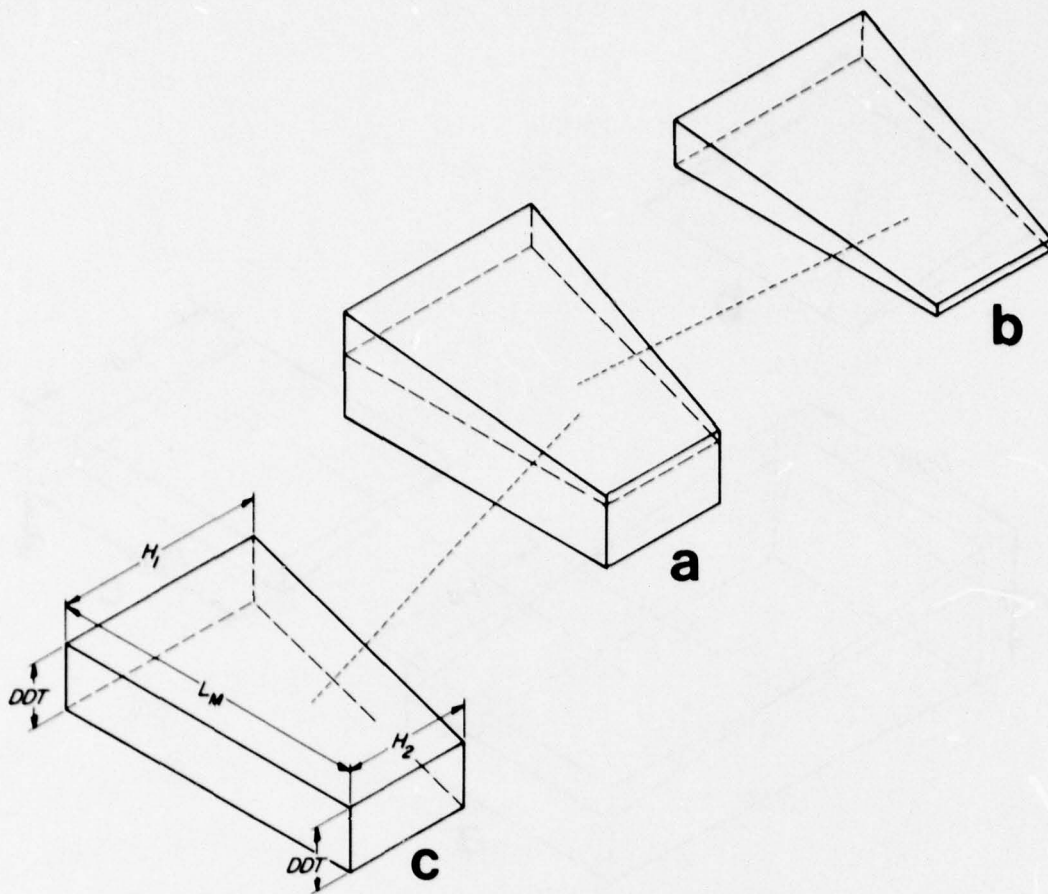THEN: Use the following equations to calculate volume of planer shavings:

Figure 12--Lumber prior to planing showing (a) geometric solids produced when overplaning occurs in thickness and underplaning occurs in width; (b) planer shavings volume by variable plane on wide face; and (c) planed lumber.
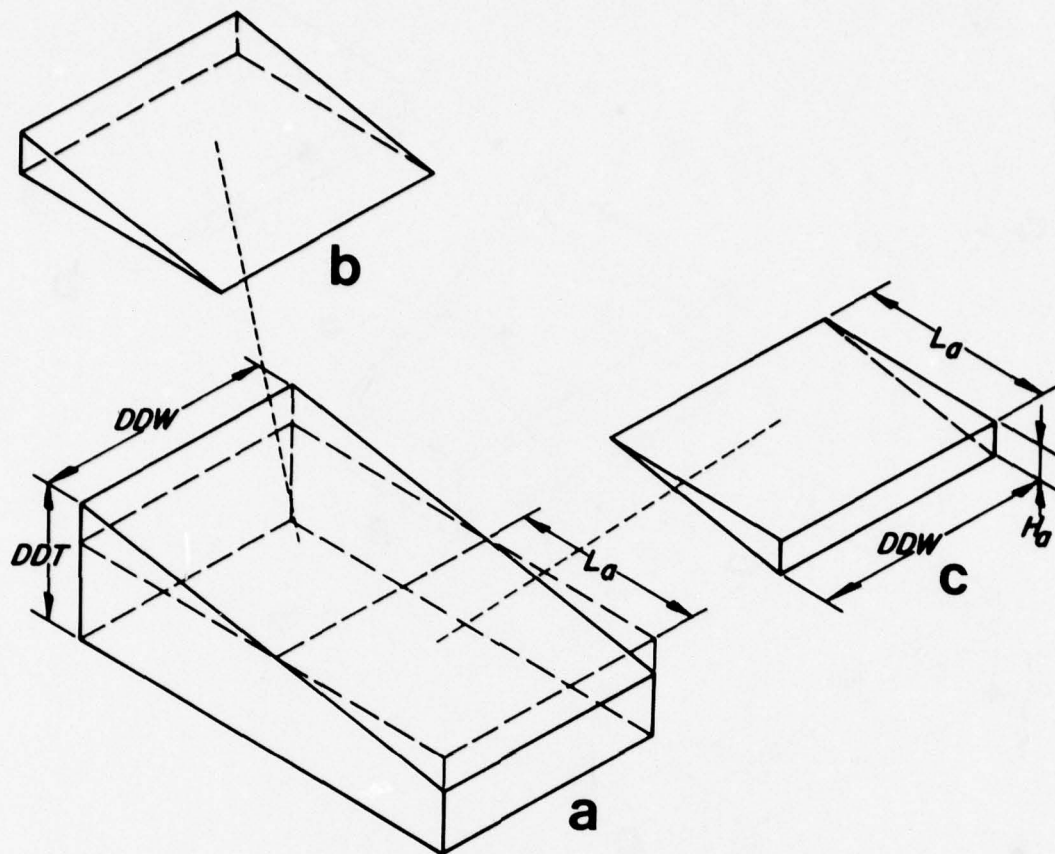
(M 146 643)

Figure 13.--Lumber prior to planing showing (a) geometric solids necessary to calculate planer shavings volume when partial planing occurs in thickness and overplaning occurs in width; (b) planer shavings volume by variable plane on wide face; and (c) planing skips volume defined by wide face and path of plane.

(M 146 642)

$$SKPVOL = \frac{H_a \times L_a \times DDW}{2} \times F$$

ACTVOL = DDVOL - SKPVOL
PLNSHV = RDVOL - ACTVOL

Case 7.--Thickness overplane; width partial plane (fig. 14).

    Test for case --

    IF: (RDW - NSVWTH) < (DDW + FHC)
    AND: (RDW + PSVWTH) > (DDW + FHC)
    AND: (RDT - NSVTHK) $\geq$ (DDT + FHC)
    THEN: Use the following equations to calculate volume of
           planer shavings:

$$SKPVOL = \frac{W_a \times L_e \times DDT}{2} \times F$$

ACTVOL = DDVOL - SKPVOL
PLNSHV = RDVOL - ACTVOL

Case 8.--Thickness partial plane; width underplane (fig. 15) shows solids before breakaway; figure 16, shows solids after breakaway.

Planer shavings volume is obtained in this case by direct calculation of the volume of the wedge of shavings removed on the wide face where partial planing occurs plus the amount of shavings removed by the fixed head. Calculating the volume of shavings in the wedge of shavings requires definition of variables not yet defined. These variables are shown in figure 16.

    Test for case --

    IF: (RDW + PSVWTH) $\leq$ (DDW + FHC)
    AND: (RDT - NSVTHK) $\bar{<}$ (DDT + FHC)
    AND: (RDT + PSVTHK) > (DDT + FHC)
    THEN: Use the following equations to calculate volume of
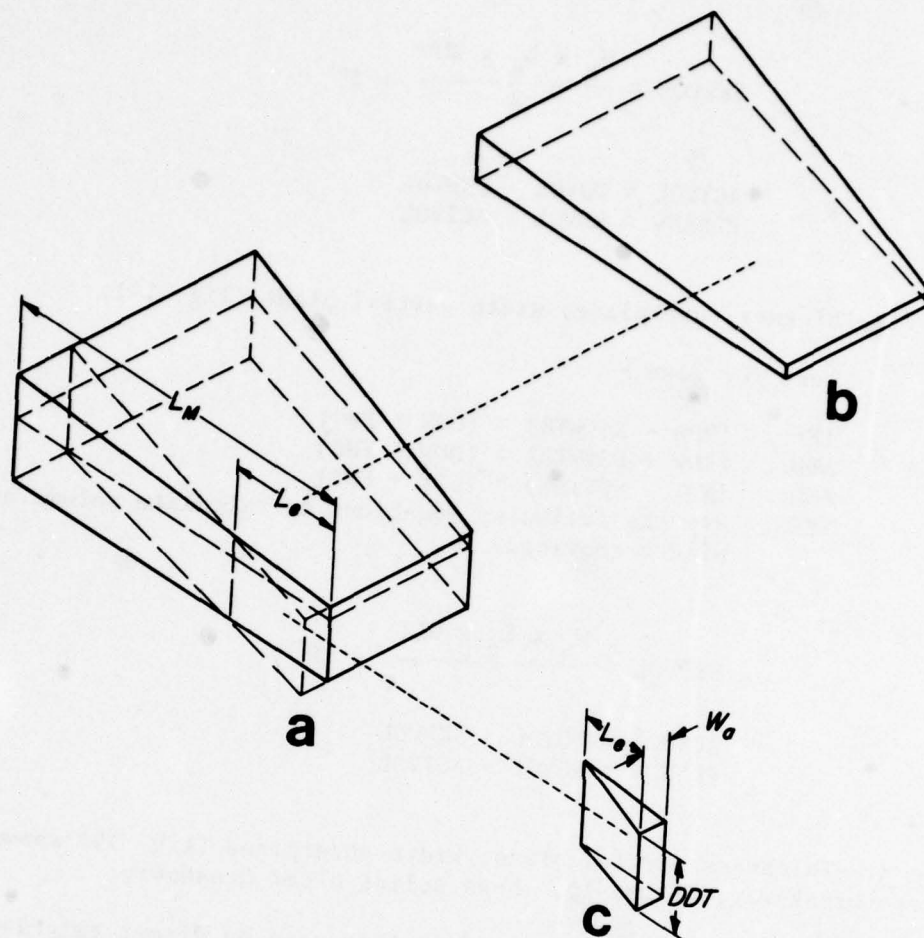   planer shavings:

Figure 14.--Lumber prior to planing showing (a) geometric solids produced when overplaning occurs in thickness and partial planing occurs in width; (b) planer shavings volume by variable plane on wide face; and (c) planing skips volume defined by narrow face and path of plane.
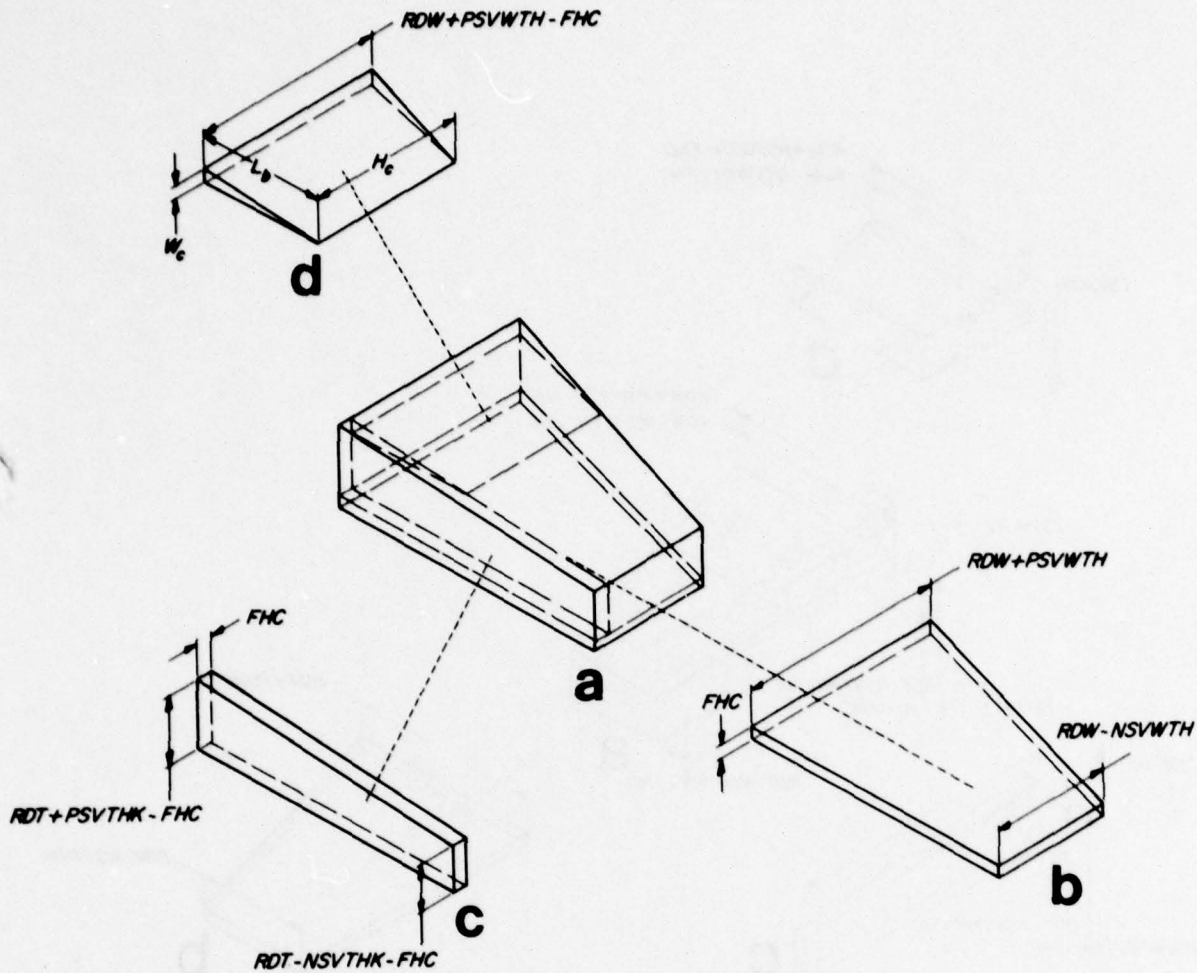
(M 146 644)

Figure 15.--Lumber prior to planing showing (a) geometric solids produced when partial planing occurs in thickness and underplaning occurs in width. Planer shavings volume from fixed head (b) cut on wide face; (c) cut on narrow face; and (d) partial planing by variable plane on wide face. (The geometric solid produced by underplaning in the width is not shown since its volume does not need to be calculated for this case.)
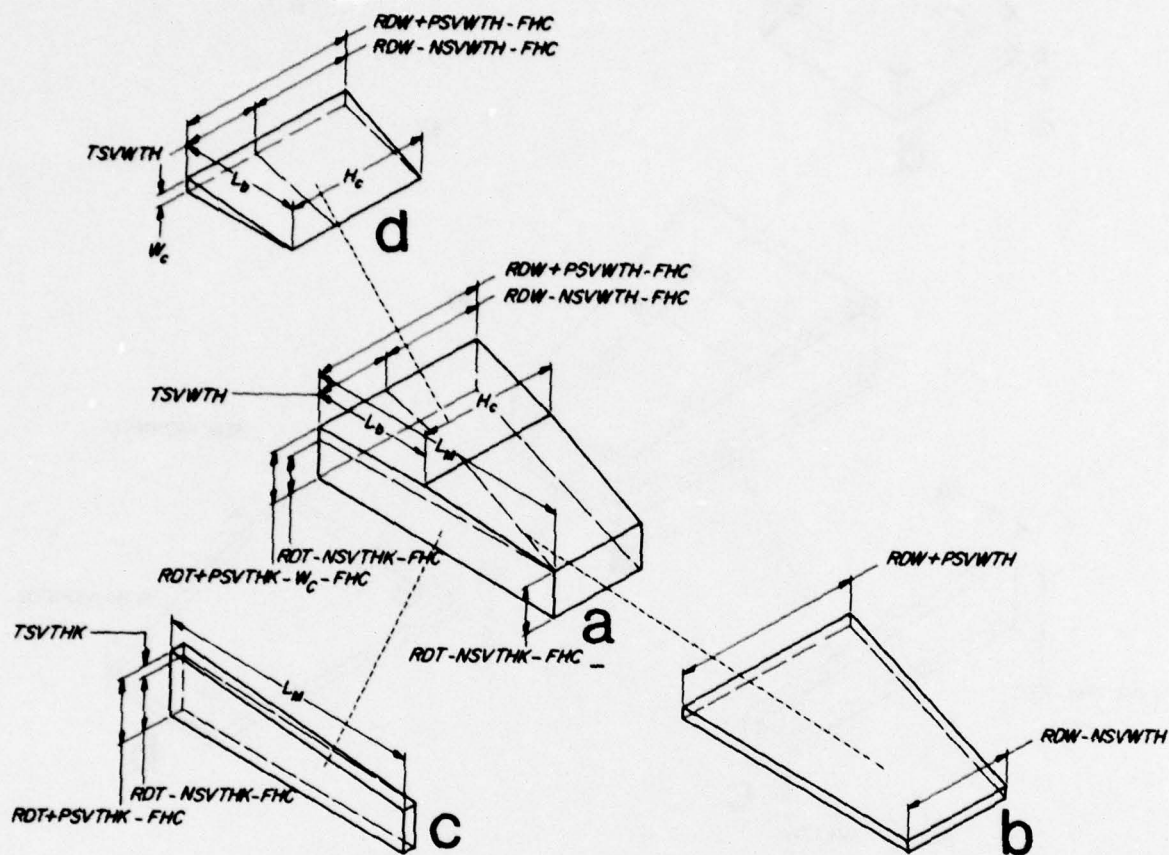
(M 146 645)

Figure 16.--Lumber showing (a) geometric shape of wood after solids removed in planing are broken away; planer shavings volume by (b) fixed head cut on wide face; (c) fixed head cut on narrow face; and (d) variable plane on wide face. (The geometric solid produced by underplaning in the width is not shown since its volume does not need to be calculated for this case.)

(M 146 630)

$$\text{PLNSHV} = \left\{ \left[\overbrace{\frac{W_c L_b}{2} \left(H_d + \frac{H_c - H_d}{3} \right)}^{\text{Vol. of fig. 16d}} \right] \right.$$

$$+ \left[\overbrace{\text{FHC} \times L_M \left(\frac{K_1 + K_2}{2} \right)}^{\text{Vol. of fig. 16c}} \right]$$

$$\left. + \left[\overbrace{\text{FHC} \times L_M \left(\frac{K_3 + K_4}{2} \right)}^{\text{Vol. of fig. 16b}} \right] \right\} \times F$$

$$\text{ACTVOL} = \text{RDVOL} - \text{PLNSHV}$$

where:

$$H_c = \text{RDW} - \text{NSVWTH} - \text{FHC} + \frac{\text{TSVWTH} (L_M - L_b)}{L_M}$$

$$H_d = \text{RDW} + \text{PSVWTH} - \text{FHC}$$
$$K_1 = \text{RDT} + \text{PSVTHK} - \text{FHC}$$
$$K_2 = \text{RDT} - \text{NSVTHK} - \text{FHC}$$
$$K_3 = \text{RDW} + \text{PSVWTH}$$
$$K_4 = \text{RDW} - \text{NSVWTH}$$

$$W_c = \frac{\text{TSVWTH} \times L_b}{L_M}$$

Case 9.--Thickness underplane; width partial plane (fig. 17).

This case is identical to Case 8 except that the partial planing is on the narrow face rather than the wide face. Figure 17a and d show the additional variables necessary to be defined to calculate shavings for this case.
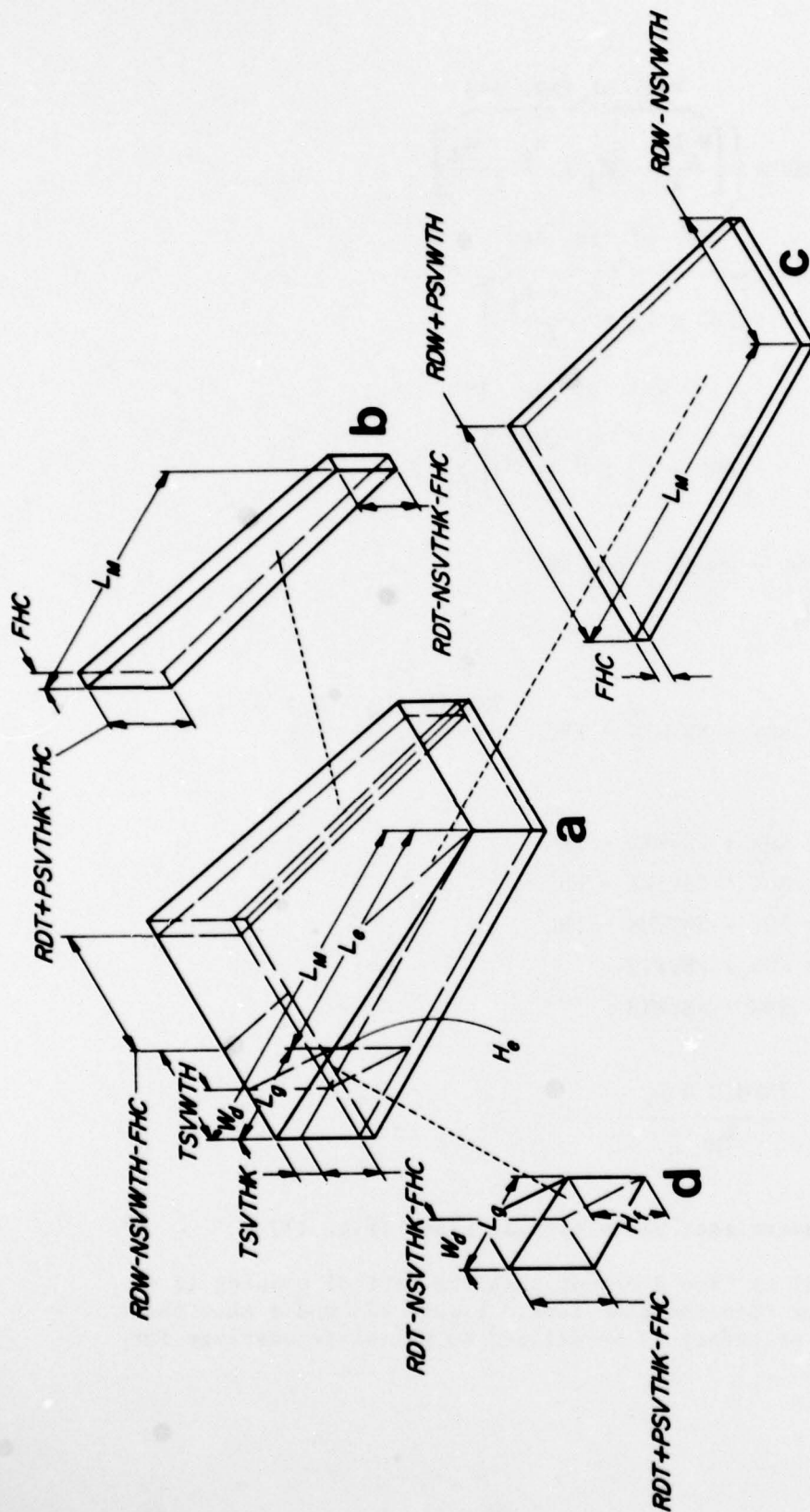
Figure 17.--Lumber prior to planing showing (a) geometric solids produced when underplaning occurs in thickness and partial planing occurs in width; planer shavings volume by (b) fixed head cut on narrow face; (c) fixed head cut on wide face; and (d) variable plane on narrow face. (The geometric solid produced by underplaning in the thickness is not shown since its volume does not need to be calculated for this case. Also note that the planing line that defines (d) should in fact be parallel to the planing line defining (b). This deliberate distortion of the sketch allows the determination of the relationships to be more apparent.)

(M 146 632)

-32-

Test for case --

IF: (RDW - NSVWTH) < (DDW + FHC)
AND: (RDW + PSVWTH) > (DDW + FHC)
AND: (RDT + PSVTHK) $\leq$ (DDT + FHC)
THEN: Use the following equations to calculate volume of
       planer shavings:

$$PLNSHV = \left\{ \left[\overbrace{\frac{W_d L_g}{2} \left(K_1 + \frac{H_f - K_1}{3} \right)}^{\text{Vol. of fig. 17d}} \right] \right.$$

$$+ \left[\overbrace{FHC \times L_M \left(\frac{K_1 + K_2}{2} \right)}^{\text{Vol. of fig. 17b}} \right]$$

$$\left. + \left[\overbrace{FHC \times L_M \left(\frac{K_3 + K_4}{2} \right)}^{\text{Vol. of fig. 17c}} \right] \right\} \times F$$

ACTVOL = RDVOL - PLNSHV

where:

$$H_e = \frac{TSVTHK \times L_e}{L_M}$$

$H_f = H_e + RDT - NSVTHK - FHC$

$K_1 = RDT + PSVTHK - FHC$

$K_2 = RDT - NSVTHK - FHC$

$K_3 = RDW + PSVWTH$

$K_4 = RDW - NSVWTH$

$L_g = L_M - L_e$

$$W_d = \frac{TSVWTH \times L_g}{L_M}$$

It is necessary to carry out the calculations set forth here for each thickness and width class according to which of the nine cases each situation falls in. The sum of the volumes calculated for each class gives total planer shavings volume for the run of logs and lumber studied.

Actual dry dressed volume is summed in the same manner for each thickness and width class to obtain total volume.

Weighted Sawkerf (WTDKRF)

In a mill that uses more than one thickness of saw to break down its logs, a weighted average sawkerf must be determined. The calculation of this weighted kerf depends on an estimation of how much of the sawing is done on each machine.

The estimations as to percentage of sawing each machine is responsible for in breaking down a mill's logs can be made by any convenient method. The method used depends entirely on the accuracy that is desired in the estimate of sawdust volume. It must be stressed that the accuracy of this volume calculation is a function of the accuracy with which the sawkerf is weighted and the relative difference in the kerf widths. Once the estimate is made, equation (11) gives the weighted sawkerf.

WTDKRF = (Estimate of Percentage Sawn on
Machine 1 x Kerf of Machine 1) +
(Estimate of Percentage Sawn on   (11)
Machine 2 x Kerf of Machine 2) + ...
+ (Estimate of Percentage Sawn
on Machine n x Kerf of Machine n).

Adjacent Board Kerf

To determine the volume of sawdust removed when the saw makes a cut adjacent to each piece of lumber, L_0 must be used in the equations.

This is because the saw cuts the adjacent kerf from the log and the log length or L_0 determines the length of the cut.

Equation 12 solves for the volume of kerf adjacent to pieces of lumber by width and thickness class.

$$
\begin{aligned}
\text{(Adjacent Board} \\
\text{Kerf Volume)} \quad = \quad & [RGW + (2 \times WTDKRF) \\
& + (2 \times RGT)] \quad \times WTDKRF \qquad (12) \\
& \times L_0 \times F
\end{aligned}
$$

The amount of kerf that is accounted for in the log by the above equation can be seen in figure 18 for live sawing and for cant sawing. The total adjacent board kerf is the sum of the solutions for each width and thickness class.

Edgings and Adjacent Kerfs Volume

As seen in figure 18, a significant amount of kerf shown as dotted is contained in the edgings of the logs as diagramed that is not accounted for by equation (12). The actual amount of sawdust contained in the kerfs shown there is approximated by equation (13):

$$
\begin{aligned}
\text{(Total Kerfs Adjacent} \\
\text{to Edgings Volume)} \quad = \quad & [WTDKRF/(RGT + WTDKRF)] \\
& \times \text{(Total Edgings and} \qquad (13) \\
& \text{Adjacent Kerfs Volume)}
\end{aligned}
$$

To solve equation (13), it is first necessary to know the volume of edgings and adjacent kerf.

$$
\begin{aligned}
\text{(Total Lumber and Adjacent} \\
\text{Board Kerf Volume)} \quad = \quad & \text{(Total Green Lumber Volume)} \\
& + \text{(Total Adjacent Board} \qquad (14) \\
& \text{Kerf Volume)}
\end{aligned}
$$

$$
\begin{aligned}
\text{(Total Edgings and Adjacent} \\
\text{Kerfs Volume)} \quad = \quad & \text{(Total Log Volume)} \\
& - \text{(Total Lumber and} \qquad (15) \\
& \text{Adjacent Board Kerf} \\
& \text{Volume)}
\end{aligned}
$$

The solution to equation (15) is then placed into equation (13).
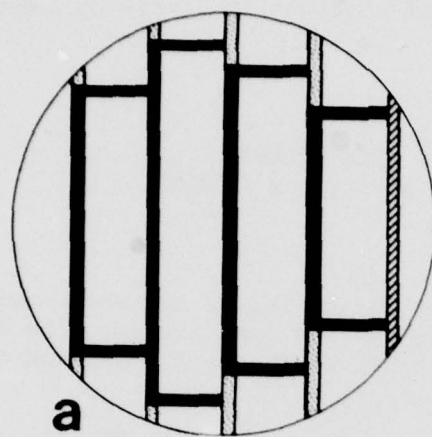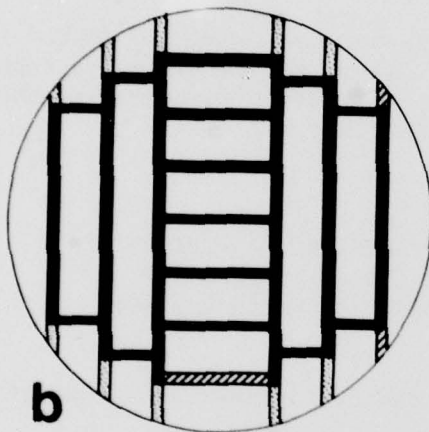
Figure 18.--Kerfs created in sawing log by: (a) live sawing method; (b) cant sawing method.

(M 146 647)

An additional problem will be encountered if more than one thickness of lumber is produced by a mill. When a mill does produce more than one thickness, calculate an average lumber thickness by the following equation.

$$
\begin{aligned}
\text{(Average Thickness} \\
\text{of Lumber)} \quad = \quad &[\text{(Number of Pieces of Lumber of} \\
&\text{Thickness 1)} \times \text{(Thickness 1)}] + \\
&[\text{(Number of Pieces of Lumber of} \\
&\text{Thickness 2)} \times \text{(Thickness 2)}] \ldots \\
&+ [\text{(Number of Pieces of Lumber of} \\
&\text{Thickness n)} \times \text{(Thickness n)}] \div \\
&\text{(Total Number of Boards of Thick-} \\
&\text{ness 1 + 2 \ldots + n)}
\end{aligned} \quad (16)
$$

This average thickness should then be placed in equation (13) where rough green thickness appears.

Trimmer Kerf Volume

The amount of kerf removed by the trim saw can be accurately calculated when the kerf is known. An estimate of this kerf will be acceptable if the kerf width is not known since the amount of trim sawdust is a relatively small part of the total.

$$
\begin{aligned}
\text{(Trimmer Kerf} \\
\text{Volume)} \quad = \quad 2 \; &[\text{RGW} \times \text{RGT} \times \text{(Trimmer Kerf)} \\
&\times \text{(Number of Boards per Width and} \\
&\text{Thickness Class)}] \; \times \text{F}
\end{aligned} \quad (17)
$$

The sum of the calculations for each width and thickness class will equal the total volume of sawdust produced from the trimmer.

Chips Volume from a Chipping Headrig

When a chipping headrig is used in a mill, it is necessary to take this factor into account in the algorithms to prevent an overestimation of sawdust volume and a corresponding underestimation of chip volume. An equation to calculate the volume of chips can be written if two variables in addition to those already known can be determined. These variables are the average number of log faces chipped and the average width of the faces.

-37-

As with percent of sawing done at each breakdown machine in order to weight the kerf, the number of log faces chipped and the average width of each face chipped is an estimate. The accuracy of the calculations of chips depends on the accuracy of this estimation, so relative care should be exercised in obtaining it depending on the accuracy desired in the study.

Equation (18) calculates the amount of kerf produced in chips by a chipping headrig.

$$(\text{Kerf Volume Produced in Chips}) = \Big[(\text{Average Width of Faces Chipped}) \times (\text{Number of Faces Chipped}) \times \text{WTDKRF} \times (\text{Total Length of Logs Milled})\Big] \times F \quad (18)$$

Total Sawdust and Chip Volumes

Given the equations above and total rough green lumber volume calculated previously, it is now possible to solve for total sawdust and chip volumes.

$$
\begin{aligned}
&\text{(From equation 12)}\\
(\text{Total Sawdust Volume}) = &(\text{Total Adjacent Board Kerf Volume})\\[1em]
&\text{(From equation 13)}\\
+ &(\text{Total Kerfs Adjacent to Edgings Volume})\\[1em]
&\text{(From equation 17)} \qquad\qquad (19)\\
+ &(\text{Total Trimmer Kerf Volume})\\[1em]
&\text{(If applicable, value from equation 18)}\\
- &(\text{Kerf Volume Produced in Chips})
\end{aligned}
$$

$$(\text{Total Chip Volume}) = (\text{Total Log Volume}) - \Big[(\text{Total Sawdust Volume}) + (\text{Total Green Lumber Volume})\Big] \quad (20)$$

Kerf Exclusion and Overestimation of Kerf

In figure 18, a segment of kerf is shown in hatch marks that is not calculated by any previous equations. This results in a small underestimation of total adjacent board sawdust volume. Figure 19 illustrates that equation (13) overestimated the volume of kerfs adjacent to edgings.

The dark portions of the figures represent slabs that were included in the volume of adjacent edgings. Because this extra slab volume was added to adjacent edgings volume, the overestimation of kerfs adjacent to edgings occurred. This overestimation can be calculated and is equal to the amount arrived at by equation (21).

$$\left[WTDKRF/(RGT + WTDKRF)\right] \times (\text{Total Volume of Slabs}/2) \qquad (21)$$

From the data available in a mill study, the width of the cut that produces the slabs and last uncalculated kerf are not known. This makes impossible an accurate mathematical calculation of the underestimation and overestimation involved. The solution followed here was to consider these compensating errors. For the limits of accuracy of this study, this is a reasonable assumption.

An examination of figure 19 shows why. When a thick slab is produced, the vertical length of the kerf that produced the slab increases as well as the volume of the slab. Similarly, when a thin slab is cut, the vertical kerf length is reduced. When slab volumes are high, as in figure 19a, an overestimation of kerf results from including slab volume with adjacent edgings volume but an offsetting underestimation of adjacent board kerf also occurs, since the uncalculated kerf next to the slab is longer and therefore has a larger volume. The same reasoning applies when slab volumes are low (fig. 19b). A low slab volume causes a smaller overestimation of kerf but the underestimation of the uncalculated adjacent board kerf volume is smaller as well. (The volume of slab overestimated is not the total darkened area, but a small fraction of this volume--the fraction being WTDKRF/(RGT + WTDKRF), as shown in equation (21). If kerf equals 0.25 and thickness equals 2.0, this fraction would equal 0.111.)

The relationship between slab thickness and kerf length is not a direct one. The volume of sawdust involved, however, is very small and the errors largely compensating. An error of no more than 1 percent in the total volume of sawdust produced would be expected.
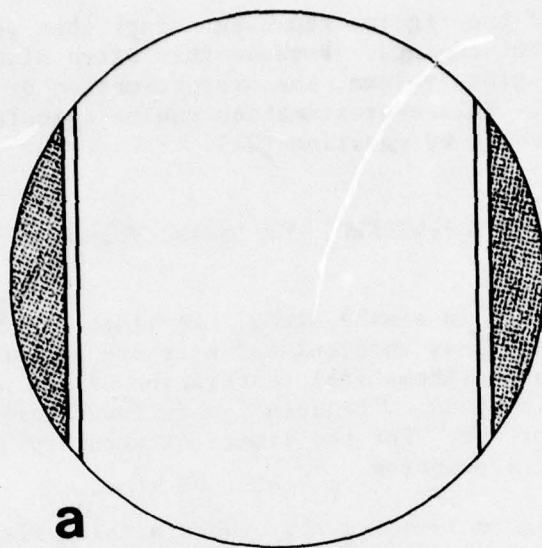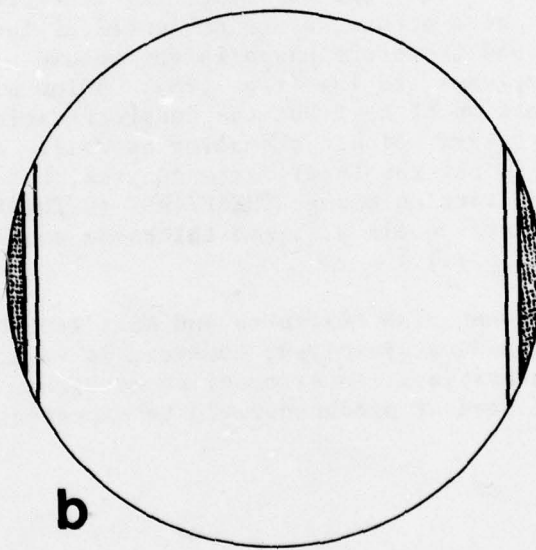
Figure 19.--Kerfs produced from: (a) thick slab; (b) thin slab.

(M 146 646)

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Appendix I

The calculation of negative, total, and positive sawing variations assume
a normal distribution of maximum and minimum measurements. A statistical
test is then made to an appropriate confidence limit.

The model has been used successfully with a 95% confidence interval.
When extreme variation occurs in the tails of the distribution, however,
this limit should be reduced to avoid distortion of the model. Tests
are now being conducted to determine the level this confidence limit
should be for a given amount of variation found in the distribution.

Given the proper choice of confidence interval, total sawing variation
is the distance between the lower and upper confidence limits of the
distribution. Negative sawing variation is the distance from the mean
size of all measurements to the lower confidence limit. Positive sawing
variation is the distance from the mean size to the upper confidence
limit. If a perfectly bell-shaped normal distribution were being dealt
with, negative and positive sawing variations would be equal. This is
rarely the case, however, for the distributions are most often somewhat
skewed.