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REDUCTION OF QUANTIZATION NOISE IN PCM IMAGE CODING

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ABSTRACT

A new technique to reduce the effect of quantization in PCM image coding is presented in this report. The new technique consists of Roberts' pseudonoise technique followed by a noise reduction system. The technique by Roberts effectively transforms the signal dependent quantization noise to a signal independent additive random noise. The noise reduction system that follows reduces the additive random noise. Some examples are given to illustrate the performance of the new quantization noise reduction system.

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I. INTRODUCTION

In coding an image by a Pulse Code Modulation (PCM) technique, an image is sampled and a bit rate reduction is achieved by reducing the number of quantization levels assigned to each sample. If the number of quantization levels is not sufficiently large, noticeable gray scale contouring results due to a luminance jump in the reconstructed image in a region where the original image luminance varies slowly. For typical monochrome images, experience (1,2) has shown that at least five bits per sample are generally necessary to avoid gray scale contouring. If the degradation due to quantization can be reduced, then images of equivalent quality may be generated with a smaller number of quantization levels. Thus, it is of interest to develop techniques to reduce the degradation due to quantization.

Various techniques have been proposed (1,3,4,5) in the literature to reduce the visual effect of gray scale contouring. In most cases, the general approach has been to transform gray scaling contouring to a different type of degradation which is more easily tolerated by human viewers. One such example is Roberts' pseudonoise technique (3). Even though such an approach has been successful in eliminating gray scale contouring, the quantization noise in the mean square sense generally increases and the degradation due to quantization may be less annoying but nevertheless is quite visible. In this paper, we develop a new quantization noise reduction technique by attempting to eliminate the quantization noise.

Gray scale contouring due to quantization is a signal dependent degradation and therefore it is difficult to apply signal processing techniques for its elimination. Roberts (3), however, has shown that gray scale contouring due to quantization can be transformed to a signal independent additive random noise with a slightly larger mean square error by adding a known random noise to the sampled image before quantization and subtracting the same random noise after quantization. Once the quantization effect is converted to a signal independent additive random noise, then the quantization noise may be more easily dealt with by currently available signal

processing techniques. This is the basis for the new quantization noise reduction technique presented in this paper. As will be seen later, such an approach to reducing the quantization noise has the potential to eliminate gray scale contouring, reduce the mean square error, and generate higher quality images than existing techniques.

The overall objectives of this paper are to present a new technique to reduce the quantization noise in PCM image coding and illustrate its performance. In Section II, we describe a basic PCM image coding technique, Roberts' pseudo-noise technique and a new quantization noise reduction technique that have been implemented for their performance comparison. In Section III, we compare and discuss the performance of the techniques described in Section II.

II. A NEW TECHNIQUE FOR QUANTIZATION NOISE REDUCTION

In this section, we develop a new technique to reduce the quantization noise in PCM image coding. We also describe a basic PCM coding technique and Roberts' pseudonoise technique which will be compared in performance with the technique developed in this section.

The basic PCM image coding technique that has been implemented is shown in Figure 1. In the Figure, $f(n_1,n_2)$ represents a noise-free digital image and $q(n_1,n_2)$ represents an image quantized by a uniform quantizer.



Fig. 1. A Basic PCM Coding Technique

In the pseudonoise technique by Roberts, a known random noise is added to the noise-free image $f(n_1, n_2)$ before quantization and the same random noise is subtracted after quantization, as is shown in Figure 2. In the



Fig. 2. Roberts' Pseudonoise Technique

figure, $w(n_1, n_2)$ represents white noise generated by a uniform probability density;

$$P_{w(n_1,n_2)}(w) = \frac{1}{\Delta} \text{ for } |w| \leq \frac{\Delta}{2}$$

$$0 \text{ otherwise}$$
(1)

where Δ represents the spacing between reconstruction levels. The sequence $r(n_1, n_2)$ represents the reconstructed image.

The new technique for quantization noise reduction consists of Roberts' pseudonoise technique followed by a noise reduction system, as is shown in Figure 3. The sequence $s(n_1,n_2)$ represents the reconstructed image. The technique by Roberts effectively transforms the signal dependent quantization noise to a signal independent additive random noise (3). The noise reduction system that follows reduces the additive random noise. For



Fig. 3. A New Technique for Quantization Noise Reduction

the noise reduction system, we have implemented on a short space basis the "Spectral Subtraction Image Restoration" (SSIR) technique recently proposed by Lim (6). The SSIR technique was chosen due to its effectiveness in reducing additive random noise without significant signal distortion.

In the short space SSIR technique, the image $r(n_1,n_2)$ in Figure 3 is divided into many subimages each of which is restored separately and then the restored images are combined to form $s(n_1,n_2)$. More specifically, let $r(n_1,n_2)$ be represented by

$$r(n_1, n_2) = f(n_1, n_2) + d(n_1, n_2)$$
(2)

where $d(n_1, n_2)$ denotes the additive random noise that results from Roberts' pseudonoise technique. By applying a 2-D window function $w_{ij}(n_1, n_2)$ to equation (2),

$$r(n_1, n_2) \cdot w_{ij}(n_1, n_2) = f(n_1, n_2) \cdot w_{ij}(n_1, n_2) + d(n_1, n_2) \cdot w_{ij}(n_1, n_2)$$
. (3)

Rewriting equation (3),

 $r_{ij}(n_1, n_2) = f_{ij}(n_1, n_2) + d_{ij}(n_1, n_2)$ (4)

where $r_{ij}(n_1,n_2)$ represents $r(n_1,n_2) \cdot w_{ij}(n_1,n_2)$, and $f_{ij}(n_1,n_2)$ and $d_{ij}(n_1,n_2)$ are similarly defined. To estimate the noise-free subimage $f_{ij}(n_1,n_2)$ from $r_{ij}(n_1,n_2)$ in equation (4), $F_{ij}(\omega_1,\omega_2)$, the discrete space Fourier transform* of $f_{ij}(n_1,n_2)$, is first estimated and then inverse Fourier

* The definition of discrete space Fourier transform, power spectrum and energy spectrum, and the determination of the normalization constant "k" can be found in references (6) and (7). transformed. The discrete space Fourier transform $F_{ij}(\omega_1, \omega_2)$ is estimated by a particular form of spectral subtraction (6);

$$\hat{\mathbf{F}}_{ij}(\omega_{1},\omega_{2}) = \left(\left|\mathbf{R}_{ij}(\omega_{1},\omega_{2})\right|^{2} - \alpha \cdot \mathbf{k} \cdot \mathbf{P}_{d}(\omega_{1},\omega_{2})\right)^{1/2} \cdot \mathbf{e}^{j \mathbf{K}} \mathbf{R}_{ij}(\omega_{1},\omega_{2})$$
for $\left|\mathbf{R}_{ij}(\omega_{1},\omega_{2})\right|^{2} \ge \alpha \cdot \mathbf{k} \cdot \mathbf{P}_{d}(\omega_{1},\omega_{2})$
(5)

and 0 otherwise

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where $F_{ij}(\omega_1, \omega_2)$ represents an estimate of $F_{ij}(\omega_1, \omega_2)$, $R_{ij}(\omega_1, \omega_2)$ represents the discrete space Fourier Transform of $r_{ij}(n_1, n_2)$, $\bigstar R_{ij}(\omega_1, \omega_2)$ represents the phase of $R_{ij}(\omega_1, \omega_2)$, " α " is a constant, "k" is a scaling factor that normalizes the power and energy spectral densities, and $P_d(\omega_1, \omega_2)$ represents the power spectrum of the additive random noise. From the estimated $f_{ij}(n_1, n_2)$, $s(n_1, n_2)$ in Figure 3 is obtained by combining the restored subimages;

$$(n_1, n_2) = \sum_{i j} \hat{f}_{ij}(n_1, n_2)$$
 (6)

where $\hat{f}_{ij}(n_1, n_2)$ represents the estimated $f_{ij}(n_1, n_2)$.

In implementing the short space SSIR technique discussed above, a separable 2-D triangular window of size 16 x 16 pixels overlapped with its neighboring window by half the window duration in each dimension was used for $w_{ij}(n_1,n_2)$ and the value of " α " was assumed to be 2. The additive random noise $d(n_1,n_2)$ was approximated as white noise with the same

statistics as $w(n_1, n_2)$ in equation (1), and thus the power spectrum $P_d(\omega_1, \omega_2)$ was approximated to be flat with the spectral amplitude of $\Delta^2/12$ where Δ represents the spacing between reconstruction levels. Further details on the theoretical development, implementation and performance of the SSIR technique can be found in (6).

III. EXAMPLES AND DISCUSSIONS

In this section, we present a few examples to illustrate the performance of the quantization noise reduction technique developed in Section II. In Figure 4 are shown three original images of 256x256 pixels with each pixel represented by 8 bits. The three images will be referred to as Image 1,2 and 3. In Figure 5 are shown the reconstructed images from Image 1. Specifically, in Figures 5(a), (b) and (c) are shown the reconstructed images $q(n_1,n_2)$ by the basic PCM coding technique in Figure 1, $r(n_1,n_2)$ by Roberts' pseudonoise technique in Figure 2 and $s(n_1, n_2)$ by the quantization noise reduction technique in Figure 3, with three bits per sample assigned to the uniform quantizer. Figures 5(d), (e) and (f) are equivalent to Figures 5(a), (b) and (c) except that two bits per sample were assigned to the uniform quantizer. Figures 6 and 7 are equivalent to Figure 5 except that the images were reconstructed from Images 2 and 3, respectively. From the above examples, it is clear that Roberts' pseudonoise technique eliminates gray scale contouring but the reconstructed images appear to be noisy. By cascading an effective noise reduction system, however, noticeable noise reduction can be achieved without significant signal distortion.

Even though a lower mean square error does not necessarily imply higher image quality, the mean square error is a useful measure in that it is easy to compute and has at least some correlation with image quality. For each of the examples illustrated in this section, a normalized mean square error (NMSE) was computed and is tabulated in Table 1. The NMSE in % between an original image $f(n_1, n_2)$ and a reconstructed image $\hat{f}(n_1, n_2)$ is defined by

$$\text{NMSE}(f(n_1, n_2); \hat{f}(n_1, n_2)) = 100 \cdot \frac{\sum_{\substack{n_1 \ n_2}} \sum_{\substack{n_1 \ n_2}} (f(n_1, n_2) - \hat{f}'(n_1, n_2))^2}{\sum_{\substack{n_1 \ n_2}} \sum_{\substack{n_1 \ n_2}} (f(n_1, n_2) - E[f(n_1, n_2)])^2}$$
(7)

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Fig. 4. Original Images

(a) Image 1





(b) Image 2

(c) Image 3





Fig. 5. (a) 3 bit PCM System



(d) 2 bit PCM System





(b) 3 bit Roberts' Pseudonoise System (e) 2 bit Roberts' Pseudonoise System



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(c) ³ bit New Quantization Noise Reduction System



(f) 2 bit New Quantization Noise Reduction System

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Fig. 6. (a) 3 bit PCM System



(b) 3 bit Roberts' Pseudonoise System



(c) ³ bit New Quantization Noise Reduction System



(d) 2 bit PCM System



(e) ² bit Roberts' Pseudonoise System



(f) 2 bit New Quantization Noise Reduction System

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Fig. 7. (a) 3 bit PCM System



(b) 3 bit Roberts' Pseudonoise System



(c) 3 bit New Quantization Noise Reduction System



(d) 2 bit PCM System



(e) 2 bit Roberts' Pseudonoise System



(f) 2 bit New Quantization Noise
 Reduction System

	Bits per			
Images	Sample	$(f(n_1, n_2); q(n_1, n_2))$	$(f(n_1, n_2); r(n_1, n_2))$	(f(n ₁ ,n ₂);s(n ₁ ,n ₂)
Image 1	3	2.39	2.6	0.78
	2	5.59	10.61	2.57
Image 2	3	4.83	4.87	2.40
	2	16.53	18.29	6.66
Image 3	3	2.34	3.02	0.63
	2	11.33	13.70	2.96
Average	3	3.19	3.50	1.27
	2	11.15	14.20	4.06

TABLE 1 NORMALIZED MEAN SQUARE ERROR

where $f'(n_1, n_2) = a.f(n_1, n_2)+b$ with "a" and "b" chosen such that $E[f(n_1, n_2)] = E[f'(n_1, n_2)]$ and Var $[f(n_1, n_2)] = Var [f'(n_1, n_2)]$. $E[f(n_1, n_2)]$ and Var $[f(n_1, n_2)]$ represent mean and variance of $f(n_1, n_2)$, and $E[f'(n_1, n_2)]$ and Var $[f'(n_1, n_2)]$ are similarly defined. The definition of NMSE given by equation (3) has the property that linearly scaling and adding a bias to $f(n_1, n_2)$ or $f(n_1, n_2)$ do not affect NMSE $(f(n_1, n_2); f(n_1, n_2))$, and NMSE $(f(n_1, n_2); f(n_1, n_2))$ equals NMSE $(f(n_1, n_2); f(n_1, n_2))$. From Table 1, it is clear that images reconstructed by the quantization noise reduction technique developed in this paper have significantly lower NMSE than images reconstructed by a basic PCM coding technique or Roberts' pseudonoise technique.

In this paper, we have considered reducing the quantization noise by first transforming gray scale contouring to an additive random noise and then applying a noise reduction system to reduce the random noise. Even though a detailed subjective evaluation has not been performed to measure the image quality improvement, the results in this paper are quite encouraging and indicate that such an approach to reduce the quantization noise in PCM image coding can potentially aid in developing new image coding techniques or improving existing coding techniques (8,9) which incorporate PCM coding as an integral part.

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