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USL Problem
No. 1-650-01-00
(S-F001 03 098056)

U. S. NAVY UNDERWATER SOUND LABORATORY FT. TRUMBULL, NEW LONDON, CONNECTICUT

TOWED BODY SOLUTIONS USING CRAPHS OF NON-DIMENSIONAL CABLE FUNCTIONS. 195Fap14349

9 Matthew F./Borg USL-T/
USL/Technical Memorandum No. 933-27-63

JUL 16 1979

INTRODUCTION

(2) 18p.

1 April 1963

Values of the non-dimensional cable functions used in computations to obtain towcable tensions, towed body depth and towed body trail are listed in Table I of DTMB Report No. 687, (reference (a)). This memorandum contains a compendium of the non-dimensional cable functions in the form of a single graph and examples of typical solutions in towed body applications. The graphical plot and typical solutions are published so as to amplify and simplify the utility of reference (a): The cable functions when used in the abridged form shown will enable engineers to obtain approximate solutions to towed body problems. This memorandum is not intended to supersede reference (a).

#### BACKGROUND

The cable functions shown in reference (a) resulted from analyses performed at the David Taylor Model Basin by L. Pode. These functions are exact when used for a non-streamlined circular towline (i.e., a bare towcable). For a streamlined or faired towcable, the tables in reference (a) have been supplanted by the more exact analyses made by Whicker in reference (b). Whicker's solution in the limit for a circular cross-section, becomes identical with Pode's solution.

USL is preparing an IBM-704 computer program in which the theoretical analyses of reference (b) are used. Therefore, at USL, towed body situations involving either a bare or faired towcable should be solved accurately and expeditiously by the computer program.

In the process of designing a towline system, or of checking the response of an existing towed system, the engineer may not desire a rigorous computer solution. The graphical plot will fill the need for an approximate solution. If a more accurate solution is needed, use should be made of either the tables of reference (a) or of the computer program.

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Figure 1 is a plot of the non-dimensional cable functions,  $\tau$ ,  $\sigma$ ,  $\xi$ , . (See "Glossary of Terms" for definitions of symbols). The functions are plotted on the ordinate scale. The abscissa scale is the cable angle  $\phi$  . The functions are plotted through the range of critical angles,  $\phi_c$  , from 10 to 85 degrees, at 5-degree intervals. The plotted functions cover the range of f (ratio of the tangential to the normal cable drag coefficients) from 0.01 through 0.03, with negligible differences. The range of f from 0.01 through 0.03 covers problems concerned with bare circular cables. (Pode, in a later report (reference (c)), modified the hydrodynamic loading functions in order to approximate the forces on a faired cable. However, reference (b), by being more exact, still remains the acceptable solution for faired cables.) For faired cables, Pode recommended (see reference (b)), values of f as high as 0.5. Notwithstanding the variance in f values (the low values for bare circular cables and the higher values for faired cables), the graphical plot in Figure 1 can be used for approximate solutions to both bare and faired cables. An advantage of using this graphical plot is in being able to interpolate between the critical angle parameter.

The non-dimensional cable functions, with respect to the towstaff point of the coordinate system, are by definitions (references (a) and (b)):

$$\tau_{\circ} = T/T_{\circ} \tag{1}$$

$$\sigma_{o} = Rs/T_{o} \tag{2}$$

$$\xi_{o} = Rx/T_{o} \tag{3}$$

$$\eta_{o} = Ry/T_{o} \tag{4}$$

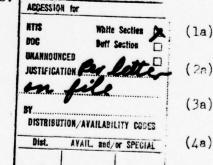
Transferring the reference point from the towstaff to any other point on the towcable leads to the following equations:

$$\tau_{o}/T_{o} = \tau/T$$

$$Rs/T_{o} = \frac{(\sigma - \sigma_{o})}{\tau_{o}}$$

$$Rx/T = (\xi - \xi_0)/\tau_0$$

$$Ry/T_o = (\eta - \eta_o)/\tau_o$$



where:

T = the tension at any point on the cable;

- $\phi$  = the angle between a tangent to the cable and the direction of motion at any point on the cable;
- $\phi_c$  = constant value of angle  $\phi$  when cable by itself is towed freely;
- T = tension at towpoint (cable attachment point on towed body);
- a = amount of cable between any point on the cable and towed body attachment point;

- x = trail distance (horizontal projection of cable length s);
- y = nominal depth of towed body (vertical projection of cable lengths);
- R = cable drag per unit length when cable is normal to direction of motion;
- $R = C_D \rho/2 d V^2;$
- CD = coefficient of normal drag, see Figure 2;
- d = diameter of cable;
- $\rho$  = mass density of sea water (  $\approx$  2 in the English system), and
- V = speed of tow.

The subscript zero refers to the towstaff location.

Figure 2 is a plot of circular cable drag coefficients versus Reynolds Number ( $N_R$ ) = V d/ $\nu$ : V is the velocity of tow (ft./sec), d is the largest cross-sectional diameter of the bare or faired tow-cable (ft.) and  $\nu$  is the kinematic viscosity of sea water which, at atmospheric pressure and a temperature of 60 degrees Fahrenheit, is approximately equal to 1.2 x  $10^{-5}$  ft<sup>2</sup>/sec.

Figure 3 is a plot of drag coefficient as a function of Reynolds Number for several bodies of revolution. This graph is useful for determining the drag coefficient for both a streamlined or faired cable and a towed object. Figure 3 is useful for illustrating the range of drag coefficients for various bodies of revolution. If possible, the actual drag coefficient of the faired cable or towed body (if different from Figure 3) should be used.

Typical examples using the graphs of Figures 1, 2, and 3 are shown illustrated directly on Figure 1. The procedures for solving the parameters of an existing or new design type towline are delineated below. Representative solutions are illustrated on Figure 1.

### COMPUTATIONS FOR AN EXISTING TOWLINE

#### A. Basic Information Needed

- (a) Diameter of the cable or largest cross-sectional thickness of fairing;
- (b) Weight per linear foot of cable in the water;
- (c) Water weight of towed object;

- (d) Largest diameter (D) and largest cross-sectional area (A) of towed object; and
- (e) Range of tow speeds.

The following can be obtained:

- (1) Reynolds number at each tow speed;
- (2) Approximate coefficient of form drag of the towed object at each tow speed (use Fig. 3 if necessary); and
- (3) Approximate coefficient of normal drag of the towline cable at each tow speed (use Fig. 2 if necessary).

# B. Preliminary Computations

With the basic information above, the following is next computed:

- (a) The normal drag force per unit length on the towline;
- (b) The critical angle  $\phi_e$  , obtained from the table on Fig. 1;
- (c) The normal drag force on the towed object;
- (d) The towed body towstaff angle, and
- (e) The total cable tension at the towstaff.

#### C. Solutions Available Using the Graphs

With the above preliminary computations and by means of Fig. 1, the following may be obtained:

- (a) For constant amount of cable payed-out and different speeds of tow:
  - (1) Depth of towed object;
  - (2) Trail of towed object;
  - (3) Towcable tension at water surface:
  - (4) Scope of the towline (the shape of the towline curve), and
  - (5) The horizontal and vertical projection of the towline.
- (b) For constant speed of tow and different amounts of cable payed out:
  - (1) Same as items (a)-(1) through (a)-(5) above.

- (b) For constant depth of the towed object and different speeds of tow, to obtain:
  - (1) Amount of cable payed-out;
  - (2) Trail of towed object;
  - (3) Towcable tensions; and
  - (4) Scope of the towline.

# - COMPUTATIONS FOR A NEW DESIGN OF A TOWLINE

## A. Basic Information Needed

- (a) Range of diameter of cable or largest thickness of fairing;
  - (b) Range of weight per linear foot of cable in water;
  - (c) Water weight of towed object;
- (d) Largest diameter (D) and cross-sectional area (A) of towed object; and
  - (e) Range of tow speeds.

The following can be obtained:

- (1) Reynolds number at each tow speed;
- (2) Approximate coefficient of form drag of the towed object at each tow speed (use Fig. 3 if necessary).
- (3) Approximate coefficient of normal drag of the towline cable at each tow speed. (Use Figure 2 if necessary)

### B. Preliminary Computations

For each proposed towline, the following is next computed:

- (a) The normal drag force per unit length of towline;
- (b) The critical angle  $\phi_c$ , obtained from the table on Fig. 1;
- (c) The normal drag force on the towed object;
- (d) The towed body towstaff angle, and
- (e) The total cable tension at the towstaff.

# C. Solutions Available Using the Graphs

The same solutions outlined previously can be obtained. The solutions obtained are for a <u>preliminary</u> design; more rigorous solutions should be performed for the particular towline selected.

## TYPICAL PROBLEM SOLUTION

For an existing towline, the following is known:

- (a) Largest cross-section diameter of fairing = 1.75 inches;
- (b) Weight per linear foot of cable in water = 7 lbs./foot;
- (c) Water weight of towed object = 25,000 lbs.;
- (d) Largest diameter of towed object = 12 feet;
- (e) Largest cross-sectional area of towed object = 113 ft.2;
- (f) Towing speed = 19.5 knots = 33 fps.

#### Revnolds Number

The Reynolds numbers for the towline and towed object are:

For the towline: 
$$N_R = Vd/\nu = \frac{33 (1.75/12)}{1.2 \times 10^{-5}} = 4 \times 10^5$$
 (5)

For the towed body: 
$$N_R = VD/\nu = \frac{33(12)}{1.2 \times 10^{-5}} = 3.3 \times 10^7$$
 (6)

#### Coefficients of Drag:

For the towline: Assuming the cable is bare, from Figure 2 at

$$N_R = 4 \times 10^5$$
;  $C_D \cong 1$ .

Assuming the cable is faired and in the shape of an airship hull, from Figure 3 at

$$N_R = 4 \times 10^5$$
;  $C_D = 0.08$ .

For the towed body: Assuming an airship hull shape, from Figure 3 at  $N_R = 3.3 \times 107$ ;  $N_R$  falls outside the limits of Figure 3.

The value of  $C_D = 0.08$  is extrapolated from an extended curve.

(7)

# Drag Forces

For the cable: 
$$R = C_D \rho/2 dV^2$$

For the towed body:  $F_p = C_p \rho/2 \text{ AV}^2$ 

Normal Drag on the bare cable towline = 
$$R = 1.0 (\rho/2) \frac{1.75}{12} (33)^2 = 159 \text{ lbs./ft.}$$
(8)

Normal Drag on faired towline = R = 0.08 
$$(\rho/2) \frac{1.75}{12} (33)^2 = 12.6 \text{ lbs./ft.}$$
  
where  $\rho/2 \approx 1$ 

These calculations of cable drag show that it is important to streamline the cable for high speed tows. For the remaining calculations, the towline will be assumed faired.

Towed Body Drag Force = 0.08 (
$$\rho$$
/2) (113) (33)<sup>2</sup>  $\stackrel{\sim}{=}$  10,000 lbs. (9)

Towstaff Tension = T = [(Towed Body Weight)<sup>2</sup> + (Drag Force)<sup>2</sup>]<sup>5</sup>
= 27,000 lbs.

The Towstaff Angle is: 
$$\phi_o = \cot^{-1} F_D/Body$$
 Weight  $\simeq 68^{\circ}$ 

Critical Angle of Cable: W/R = 7/12.6 = 0.56; from Table in Fig. 1,  

$$\phi_{\circ} \approx 40^{\circ}$$
 (10)

# Non-Dimensional Cable Functions at Towed Body:

For towstaff angle of  $68^{\circ}$  and  $\phi_{e}$  equal to  $40^{\circ}$ , the following is obtained

from Figure 1: 
$$\tau_o = 1.28$$
 (1.2836)°  $\sigma_o = 0.525$  (0.5214) (11)  $\xi_o = 0.41$  (0.1098)  $\eta_o = 0.50$  (0.5065)

\*Values in parenthesis are taken from the tables of Reference (a) for f = 0.02. They are shown for comparison purposes. The largest discrepancy should be on the order of 4 to 5%.

# Towing Conditions

Solutions will be obtained for the following conditions:

# A. Cable Payed Out of 600 Feet

(a) At the water surface (from equation 
$$2a$$
);  

$$\sigma_1 = \frac{Rs\tau_0}{T_0} + \sigma_0 = \frac{(12.6)(600)(1.28)}{27000} + 0.525 = 0.879$$
(12)

where subscript (1) refers to the location at the water surface.

From Figure (1) with  $\phi_{\rm e}$  equal to  $40^{\circ}$  and  $\sigma_{\rm i}$  equal to 0.879, the angle at the water surface is found to be  $\phi_{\rm i}$ , equal to 60 degrees. The remaining cable functions at the surface are therefore:

$$\tau_1 = 1.45$$
 (1.4534)  
 $\xi_1 = 0.25$  (0.2591)  
 $\eta_1 = 0.80$  (0.8088)

(b) Depth of towed body (from equation (4a)):

Depth = y = 
$$\frac{T_o(\eta_1 - \eta_o)}{R\tau_o} = \frac{27000(0.80 - 0.50)}{12.6(1.28)} = 502 \text{ feet}$$
 (14)

(c) Trail of towed body, (from equation (3a)):

Trail = 
$$x = \frac{T_o (\xi_1 - \xi_o)}{R \tau_o} = \frac{27000 (0.25 - 0.11)}{12.6 (1.28)} = 251 \text{ feet}$$
 (15)

(d) Towline tension at surface. (from equation (1a)):

Tension = 
$$T_1 = \tau_1 T_o / \tau_0 = \frac{1.45}{1.28} (27000) = 31000 lbs$$
 (16)

## B. Towing Depth of 350 feet

Since the cable function constants at the towstaff are unchanged:

(a) At the water surface, from equation (4a):

$$\eta_1 = \frac{\text{Ry}\,\tau_o}{\text{T}} + \eta_o = \frac{(12.6)\,(350)\,(1.28)}{27000} + 0.50 = 0.71$$
(17)

\*\*By varying the length s, the parameters along the towcable length can be obtained. In all the illustrations, only the parameters at top and bottom of the towline cable are computed.

From Figure (1), with  $\phi_e$  equal to 40 degrees and  $\eta_e$  equal to 0.71, the angle at the surface is found to be 62 degrees.

The remaining cable functions are:

$$\tau_1 = 1.40$$
 (1.4047)  
 $\sigma_1 = 0.75$  (0.7597)  
 $\xi_1 = 0.20$  (0.1901) (18)

(b) Length of cable necessary to maintain depth of 350 feet, from equation (2a):

$$\sigma_{1} = \frac{Rs \tau_{o}}{T_{o}} + \sigma_{o}$$
Cable Length =  $s = \frac{T_{o}}{R \tau_{o}} (\sigma_{1} - \sigma_{o}) = \frac{27000 (0.75 - 0.525)}{12.6 (1.28)}$ 

$$= 377 \text{ feet}$$
(19)

(c) Trail of towed object from equation (3a):

Trail = 
$$x = \frac{T_o}{R \tau_o} (\xi_1 - \xi_o) = \frac{27000 (0.2 - 0.11)}{12.6 (1.28)}$$
= 151 feet

(d) Towline tension at water surface from equation (1a)):

$$T_1 = \tau_1 T_0 / \tau_0$$
  
=  $\frac{1.40}{1.28}$  (27000)  
= 29500 lbs.

#### CONCLUSIONS

The non-dimensional cable functions tabulated originally by L. Pode have been graphed for the range of f (ratio of tangential to normal drag coefficients) of 0.01 to 0.03, and cable angle  $\phi$  from 10 through 90 degrees. The critical angle parameter,  $\phi_c$ , is represented in 5-degree intervals through 10 to 85 degrees. Typical calculations are shown on the graphed cable functions. Plots of coefficients of drag for bare cables and some bodies of revolution are also included.

MATTHEW F. BONG Mechanical Engineer

# GLOSSARY OF TERMS

| <u>Symbol</u>  | Definition  | Units               |
|----------------|---|---------------------|
| A              | Largest cross-sectional area of towed body                                      | sq.ft.              |
| CD             | Coefficient of Normal Drag  | -                   |
| D              | Largest diameter of towed body  | ft.                 |
| Do or FD       | Horizontal drag force against towed body  | lbs.                |
| d              | Diameter of cable   | ft.                 |
| L <sub>o</sub> | Water weight of towed body  | lbs.                |
| NR             | Reynolds number   | -                   |
| R              | Cable drag when cable is normal to direction of motion                          | lbs./ft.            |
| s              | Length of cable between any point on the cable and towed body attach-ment point | ft.                 |
| T              | Tension at any point on the cable   | lbs.                |
| To             | Tension at towpoint (at towstaff) (Also T reference)                            | lbs.                |
| V              | Velocity of tow   | ft./sec.            |
| W              | Unit weight of towline in water Non-dimensional ratio: W/R                      | lbs./ft.            |
| w<br>*         | Horizontal Projection of cable length (Trail distance)                          | ft,                 |
| у              | Vertical projection of cable length (Depth distance)                            | ft.                 |
| η              | Cable function equal to Ry/T reference  | non-<br>dimensional |
| ν              | Kinematic Viscosity of sea water  | ft.2/sec.           |

### LIST OF REFERENCES

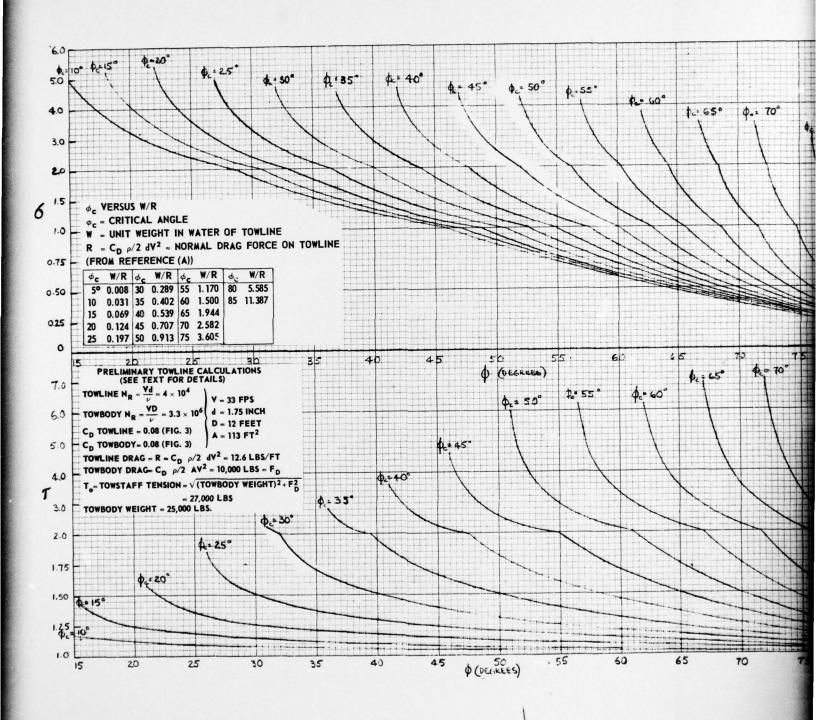
- (a) L. Pode, "Tables for Computing the Equilibrium Configuration of a Flexible Cable in a Uniform Stream," DTMB Report 687, March 1951.
- (b) L. F. Whicker, "Oscillatory Motion of Cable-Towed Systems," Ph.D. Dissertation, University of California, 16 July 1957.
- (c) L. Pode, "The Configuration and Tension of a Light Flexible Cable in a Uniform Stream," DTMB Report 717, and Appendix 1, March 1956. (USL Accession No. 10312).

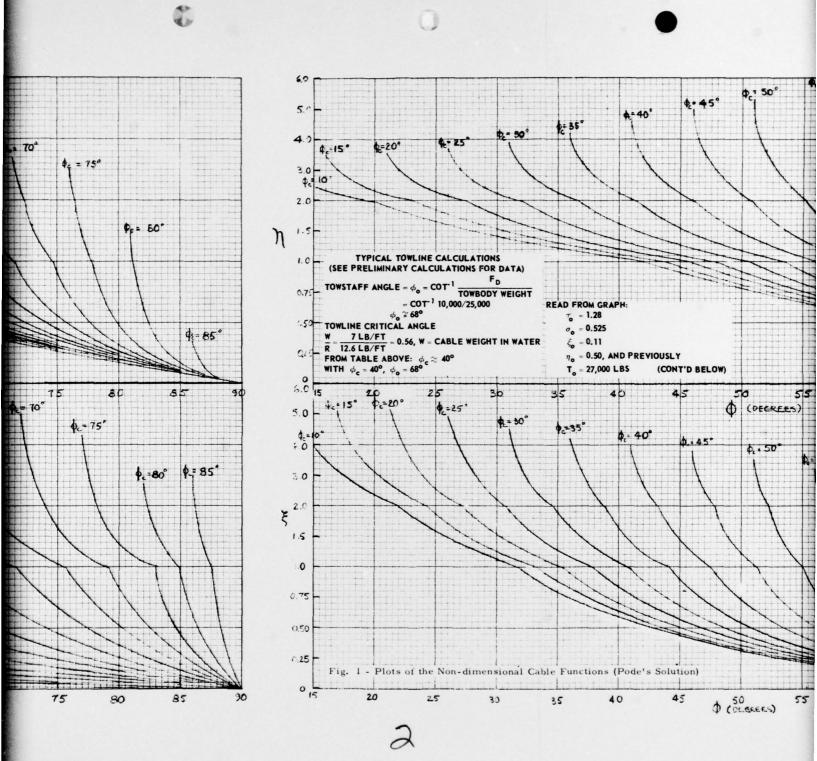
| Symbol   | <u>Definition</u>   | Units             |
|----------|---|-------------------|
| ξ        | Cable Function equal to Rx/T reference  | non-dimensional   |
| ρ        | Mass density of sea water   | $lbssec^2/ft^4$   |
| σ        | Cable function equal to Rs/T reference  | non-dimensional   |
| φ        | Angle between tangent to the cable and the direction of motion, at any point on the cable | degrees           |
| $\phi_c$ | Value of Angle $\phi$ when cable (by is towed freely $(\phi_c = F(W/R))$                  | tself)<br>degrees |

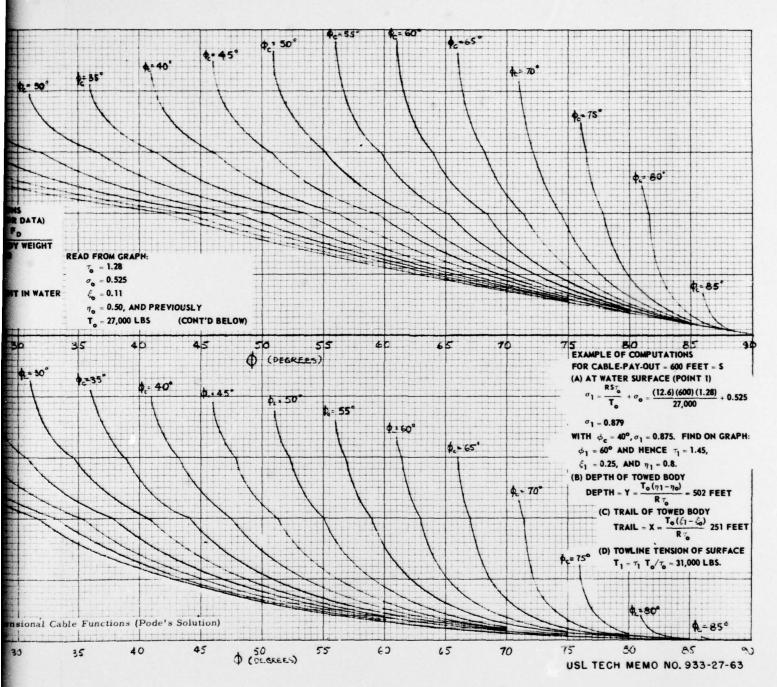
Subscript zero refers to towstaff point.

Subscript one refers to point at water surface.









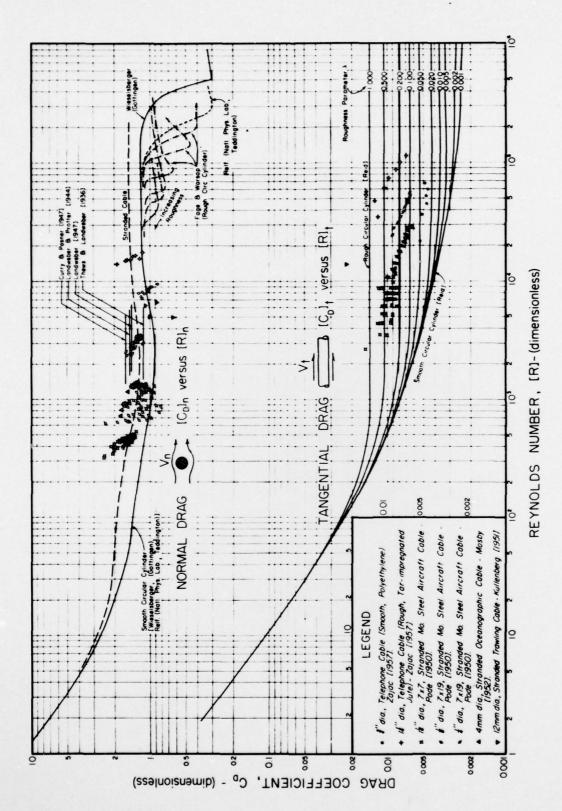


Fig. 2 - Dependence of Drag Coefficient on Reynolds Number for Flows Normal and Tangential to Smooth and Rough Circular Cylinders

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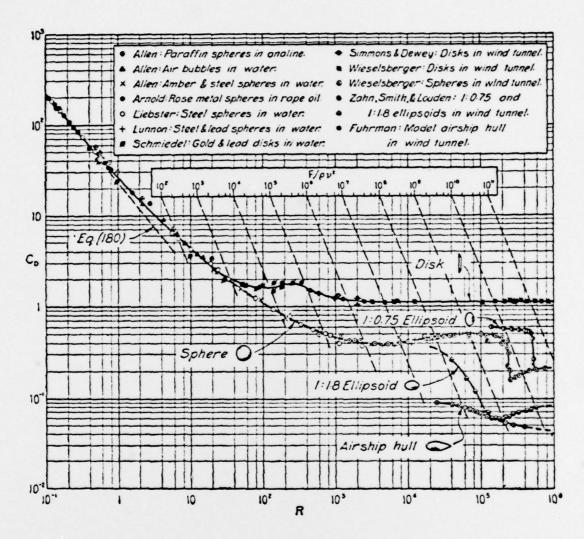


Fig. 3 - Drag Coefficient as a Function of Reynolds Number for Several Bodies of Revolution.

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