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COORDINATION OF PRODUCTION SCHEDULES WITH SHIPPING SCHEDULES,

by

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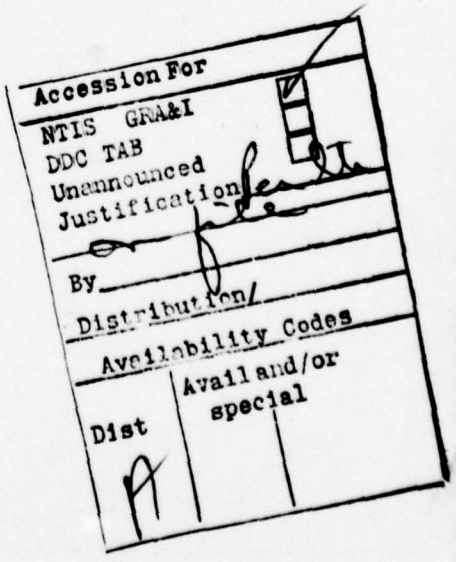
ABSTRACT

Scheduling logistics operations in a multi-echelon production system requires planning and coordinating production and transportation decisions for all facilities in a system. In this paper we show how these decisions can be made in an actual multi-facility system consisting of a component plant, at which products are produced, and a set of destinations, at which the end product, automobiles, are assembled. An assembly schedule for automobiles is specified in advance for each week in a planning horizon. The component plant is required to produce and ship the correct mix of products to each destination to meet this automobile assembly schedule on time. Our objective is to develop models and an algorithm that can be used to determine what products should be produced on the various production lines at the component plant during each period (either a shift or a week), what portion of a period's production and inventory on-hand at the beginning of a period at the component plant should be loaded into rail cars and shipped to the various destinations, and what portion of a period's production should be held in inventory at the component plant into the next period so as to minimize the appropriate costs subject to constraints on production and shipping. The structure of the problem is examined and is exploited in the proposed algorithm.



I. INTRODUCTION

Scheduling logistics operations in a multi-echelon production system requires planning and coordinating production and transportation decisions for all facilities in a system. Our goal in this paper is to show how these decisions can be made in an actual multi-facility system operated by a large automotive manufacturer. The system consists of a component plant, at which products are produced, and a set of destinations, at which automobiles are assembled. An assembly schedule for automobiles is specified in advance for each week in the planning horizon for each of the destinations. In the real environment, the planning horizon is normally 12 weeks, and the assembly schedule is not the same for each week of the horizon. The component plant is required to produce and ship the correct mix of products to each destination to meet the automobile assembly schedule on time.

The component plant produces products on independent production lines. Certain products can be produced on each line. Only one product is produced on a given line at a time; different or the same products can be produced on different lines at the same time. Changing production from one product to another on a given line is accomplished quickly and at virtually zero cost, and therefore can be ignored. Production lines are designed to produce products used in the assembly of a limited number of types of automobiles. These production lines can be divided into separate groups so that lines in one group are all capable of producing the same range of products; however, any product produced in the group cannot be produced on a production line in any other group. Only certain types of cars are assembled at each destination. Furthermore, the manner in which products have been assigned to production lines corresponds to the products used at the destinations. The destinations can be divided into groups such that a) each product produced in a group of production lines goes to only one group of destinations, and b) the products

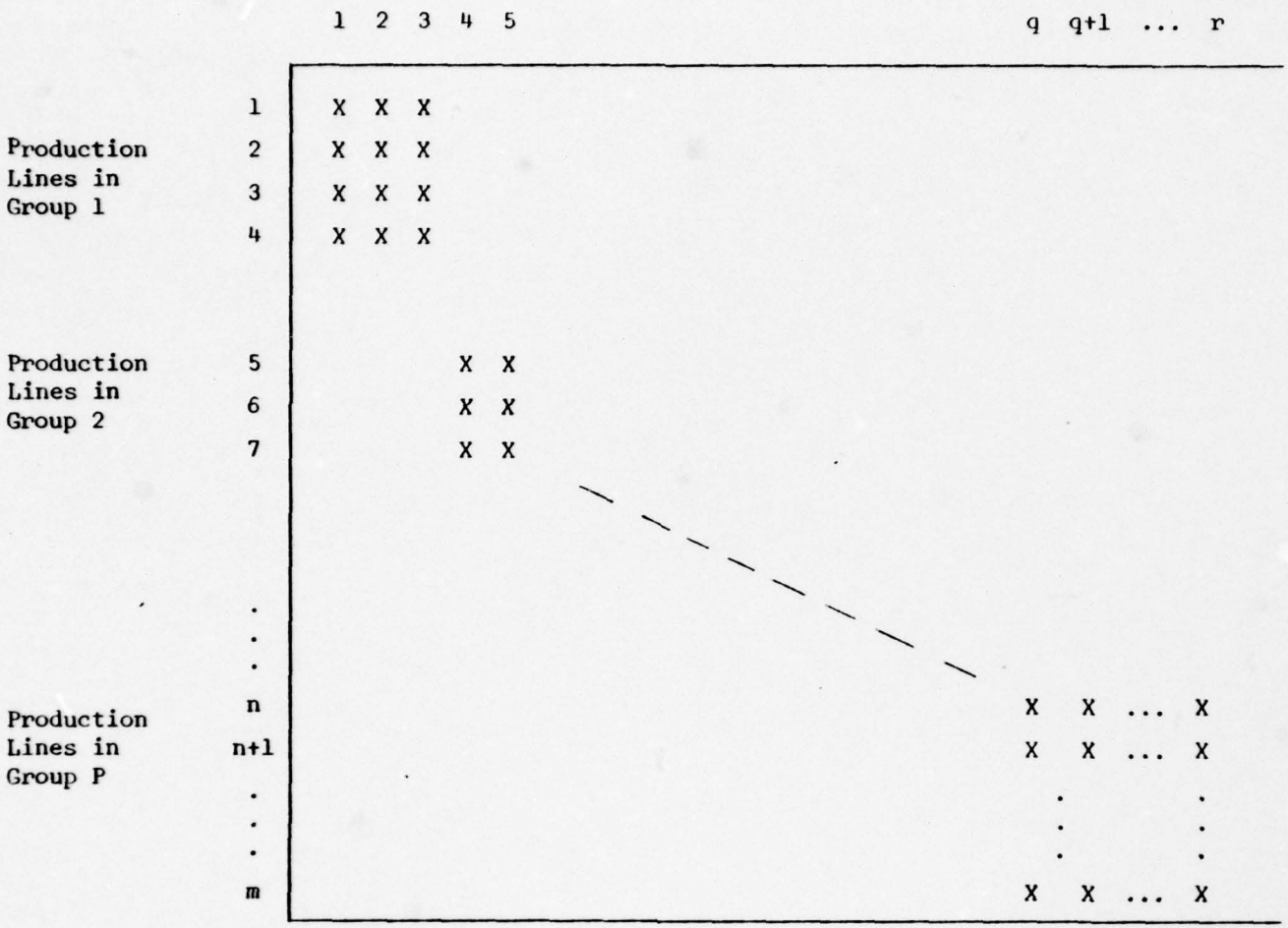
used at any destination are produced on only one group of production lines. This relationship is illustrated in Figure 1.

The products produced on all lines perform roughly the same function on each type of automobile. The production lines are designed to take advantage of the peculiarities of product manufacture that are dictated by the differences in the design of the automobiles. Due to the similarity in their basic design, the cost to manufacture each product on the same group of production lines is essentially the same.

The amount of a product produced on a particular production line is normally measured in container loads. The time required to produce a container load is essentially the same for all products. Once produced, the individual units are placed in containers which are transported to a warehouse for temporary storage prior to being shipped.

Each week in the planning horizon is subdivided into shifts. During each shift container loads of different products are loaded into rail cars, which are then sent to various destinations. For any given destination, one or more products can be loaded into a rail car; any integral number of rail cars can be loaded for a particular destination. However, one rail car goes to one and only one destination.

Assembly schedules can be expressed in terms of rail car equivalents. Thus a schedule can be stated in terms of the number of rail cars of various products that are needed at each destination by a specified time to carry out the planned assembly schedule. Due to the manner in which each product is produced on one of a group of production lines, which are, in turn, uniquely identified with a set of destinations, we can aggregate the requirements for each destination and express these requirements in terms of rail car equivalents. For example, we could state destination twenty's requirement as follows: by the end of week three we must have at least five rail cars shipped to destination



Relationship Among P Production Lines, M Products, and r Destinations

Figure 1

X indicates a particular combination is possible; a blank indicates the combination is not possible

twenty from the component plant. The component plant can disaggregate this rail car plan by recognizing the exact mix of automobiles that will be assembled at each destination. Thus the production goals for each type of product can be established for each week in the horizon given a weekly rail car shipping schedule and the assembly requirements at each destination. Furthermore, these weekly production goals can then be systematically assigned by product and ultimate destination to each shift throughout a given week.

There are a number of considerations in addition to the ones we have already mentioned that must be taken into account when preparing a production and rail car shipping schedule. First, no rail car will be shipped from the component plant unless it is full. Second, there are weekly and shift constraints on the number of rail cars that can be loaded and dispatched to destinations from the component plant. Third, a maximum number of container loads of products can be produced on a given line during a shift; however, this production capacity can be divided in any fashion among the products that can be produced on the line plus possibly slack time.

We have noted that production changeover costs are negligible at the component plant, and that the cost of producing a unit of any product is approximately the same. By union contract, employees are paid whether or not they work so that over the short run labor costs are independent of the production or shipping schedules. Also, the cost of production does not depend on the shipping schedule since there is adequate capacity on regular time to meet all demands at the component plant. (Overtime, in practice, would be used only when an unforeseen shortage occurs for parts used in making the products, or a quality problem occurs at the component plant.) However, the manner in which production and shipping activities occur at the component plant significantly affect inventory carrying costs at each location. If production takes place several

weeks prior to the time that units are needed, then carrying costs are incurred. These units are sometimes stored at the component plant, but may be shipped by rail car to assembly plants in advance of the time they are needed and stored there. Normally, if the units are stored at a destination, they are left in the rail cars. The cost of storing a unit, including the additional material handling cost at the component plant and the demurrage for a rail car used as a storage device at a destination, is assumed to be the same at either the component plant or a destination. We ignore the pipeline inventory of units in rail cars traversing from the component plant to a destination; this time has been taken into account in the assembly schedule at the component plant for a destination.

The production and shipping scheduling problem we have discussed can be modelled in several ways. One possible model would have as decision variables the amount of each product produced on each shift at the component plant and the number of rail cars shipped to each destination during each shift throughout the entire planning horizon. This type of model is easy to develop. However, the usefulness of this type of model is not certain. Predicting shift-by-shift production and shipping quantities at the component plant many weeks into the future is considered to be a meaningless exercise in the real environment due to the day-to-day dynamics. Consequently, this detailed model is not appropriate. Furthermore, computational requirements for generating an optimal solution to this type of problem are substantial.

Rather than tackling the detailed shift-by-shift problem for the entire horizon, we propose to separate the production and shipping scheduling problem into two parts. Since weekly assembly schedules are fixed at the destinations for many weeks in advance, we first propose to identify aggregate week-by-week production and shipping goals for the component plant. Aggregation is done over all products that are a) produced on a given group of lines, and b) shipped

to a particular set of destinations. Thus, rather than being concerned with particular products, production and shipping requirements will be expressed in terms of rail car equivalents for each group of products at each group of destinations over the planning horizon. This aggregation is possible since production capacity in each group of lines is interchangeable among the products produced on those lines, and each destination's requirements are produced in only one group of lines. The first model we develop will determine the number of rail car loads to produce at and ship from the component plant to the destinations each week so that the only cost under control, the carrying cost at the plant and the destinations, is minimized while satisfying constraints on a) meeting aggregate product demand at each destination, b) loading no more than a maximum number of rail cars each week at the component plant, and c) producing no more than capacity allows each week on a group of production lines at the component plant. The model must also not allow a rail car to be sent to more than one destination. Thus, the solution to this problem, which we will call the aggregate production and shipping scheduling problem, will indicate the number of rail cars to ship each week to each destination so that overall inventory carrying costs will be minimized.

Once this solution is available we can address the second part of the problem, namely, determining the detailed shift-by-shift production and shipping schedules. However, rather than determining these detailed schedules for the entire horizon, we will establish them for only the first week of the horizon. A detailed schedule can be developed for longer horizons if desired using the methods we will describe. Recall that the solution to the first problem establishes the weekly shipping plan. Given this plan, the second problem we propose to solve determines what products to produce and ship on each shift of the first week so that carrying costs are minimized over this period of time and the constraints on a) rail car loading capacity for each shift,

b) production capacity on each line in each shift, and c) the weekly shipping schedule as established in the solution to the aggregate production and shipping scheduling problem are all met. We also must ship only full rail cars, and individual rail cars can go to only one destination.

As we will see, each of these two problems has a special structure. In the next section we will state a mathematical model for the aggregate production and shipping scheduling problem, analyze the structure of this problem, develop an algorithm which exploits this structure, and present an example problem. In the third section, we will show how the solution to the aggregate production and shipping scheduling problem can be disaggregated. In the final section we summarize our results and discuss some possible extensions to the model.

II. AN ANALYSIS OF THE AGGREGATE PRODUCTION AND SHIPPING SCHEDULING PROBLEM

The aggregate production and shipping scheduling problem described in the last section can be formulated as a mathematical programming problem. The model we present takes special advantage of the relationship between a group of production lines and a set of destinations. Recall that a) all production lines within a group can produce the same products, b) products produced in one group of production lines cannot be produced in any other group of lines, c) each product produced in a group of lines is shipped to only one group of destinations, and d) the products used at a destination are produced in only one group of production lines. Hence there is a one-to-one correspondence between a group of production lines and a group of destinations. The weekly production capacity for each group of production lines can therefore be considered as the sum of the capacities of the lines within that group; also, the shipping requirements for all destinations within the same group can be aggregated since only full rail car shipments are made and the one-to-one correspondence exists between a group of destinations and a group of production lines.

We assume the planning horizon is W weeks in length; the number of production line groups, and therefore the number of destination groups, is G; the demand, measured in rail car loads, for all products used in destination group g in week w is D_{gw} (g=1, ..., G and w = 1, ..., W); and the maximum number of rail car loads that can be shipped from the component plant during week w is L_w.

The decision variables used in the model are

P_{gw} = the number of rail cars of products produced on production lines in group g during week w,

S_{gw} = the number of rail cars shipped from the component plant to destination group g in week w,

H_{gw} = the number of rail car loads of product produced on production lines in group g in inventory at the component plant at the end of week w,

E_{gw} = the number of rail car loads of product on hand at the end of week w at destinations in group g,

U_{gw} = the slack production capacity for production lines in group g during week w (measured in rail cars), and

V_w = the slack rail car loading capacity in week w.

Recall that the objective of the aggregate production and shipping scheduling problem is to determine a) the number of rail car loads to produce on each group of production lines during each week of the planning horizon, and b) the number of rail cars to send to each group of destinations each week of the horizon so as to minimize system carrying charges while satisfying production and shipping limitations at the component plant, and demand requirements for each group of destinations. As we have discussed, the carrying charges are proportional to the number of rail car equivalents worth of inventory carried in the system (excluding those in transit from the component plant to the destinations since there is no way to reduce this quantity). The model can be stated as:

(1) $\min \quad Z = \sum_g \sum_w H_{gw} + \sum_g \sum_w E_{gw}$

(minimize the total rail car loads of inventory carried at the component plant and the destinations)

subject to

(2) (Inventory balance constraints at the component plant)

$$H_{g,w-1} + P_{gw} = H_{gw} + S_{gw}$$

(3) (Inventory balance constraints at destination group g)

$$E_{g,w-1} + S_{gw} = E_{gw} + D_{gw}$$

(4) (Production capacity constraints at the component plant)

$$P_{gw} + U_{gw} = C_{gw}$$

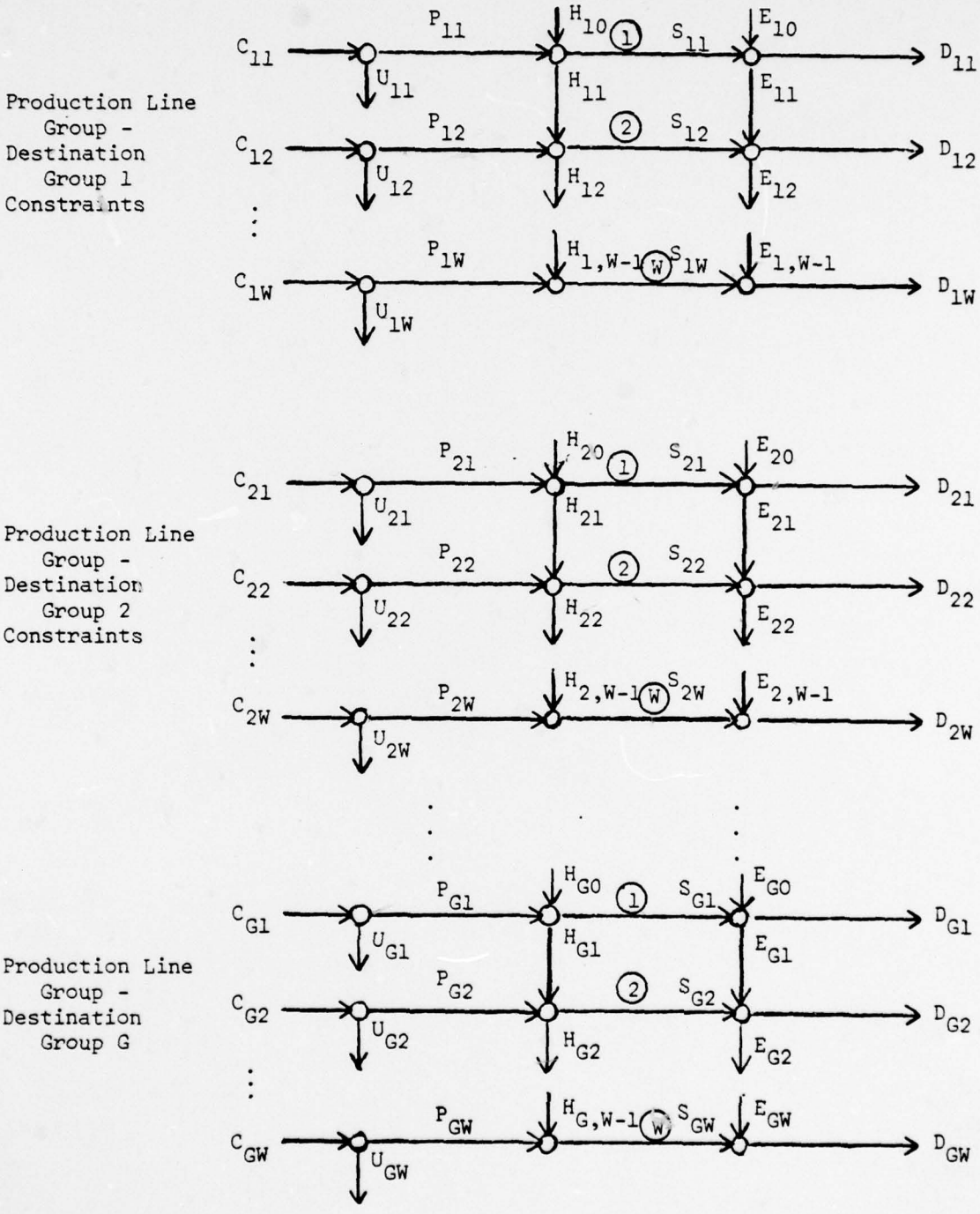


Figure 2

Graphical Representation of the Constraints for the Aggregate Production and Shipping Scheduling Problem

(5) (Rail car loading constraints at the component plants)

$$\sum_g S_{gw} + V_w = L_w$$

$P_{gw}, H_{gw}, E_{gw}, S_{gw}, U_{gw}, V_w \geq 0$, where $g=1, \ldots, G$ and $w=1, \ldots, W$ in all of the above cases.

The above problem is a near network problem. This can be seen by examining the graphical representation of the constraints given in Figure 2. First, we observe that by ignoring the rail car loading constraints, as we have done in the diagram in Figure 2, the problem decomposes into G independent network problems, one for each production line group-destination group combination. Next, observe that the rail car loading constraint for week 1 states that the sum of the flows over the arcs labelled (1) in Figure 2 cannot exceed L_1 . In general, the rail car loading constraint for week w states that the sum of the flows over arcs having label (w) (flow is $\sum_g S_{gw}$ over these arcs) cannot exceed L_w . Constraints that cut across arcs in this fashion are often called "bundle constraints." The presence of these bundle constraints cause the problem to have a structure that is not a network flow structure.

We now discuss an algorithm for finding a solution to the problem, which is based on the problem's near network structure. The complicating rail car loading constraints are first relaxed to take advantage of the simplicity of the structure of the remaining problem. As we have stated, the remainder of the problem has the form of G independent problems. These G problems are all network flow problems that have the form of a linear production-distribution problem with upper bounds on production in each period. This problem was first discussed by Bowman [1]. The solution to these problems is found by simply producing the destinations' requirements as late as possible. Thus it is easy to obtain an optimal solution to each of these G problems. Once these solutions have been found, the rail car loading constraints are systematically

considered. The rail car loading constraints that are violated by the solution obtained for the G independent problems are addressed one at a time. Inventory and production decisions are revised but remain as close to the solution found when solving the G independent problems as possible. Let us now formally state the algorithm.

Algorithm for Solving the Aggregate
Production and Shipping Scheduling Problem

Step 1: Obtain the least cost production schedule for each group that satisfies production capacity constraints and demand requirements ignoring the rail car loading constraints. In the solution, carry inventory only at the destinations. The algorithm used to determine the optimal production plan places production as close to the period in which it is consumed as possible. (A formal statement of this algorithm can be found in Wagner [2].)

Step 2: If all rail car loading constraints are satisfied by the solution found in Step 1, then that solution is optimal. Otherwise, beginning with period 1, and proceeding period-by-period, resolve rail car loading infeasibilities by following in order Step 2a and, if necessary, Step 2b.

Step 2a: If $V_w < 0$, w^1 is the earliest period following w having positive slack loading capacity ($V_{w1} > 0$), and there is a destination group g having $E_{gj} > 0$ for $j=w, w+1, \ldots, w^1-1$, then increase H_{gk} by $a=\min(-V_w, V_{w^1}, \min_{j=w,\ldots,w^1-1} E_{gj})$, $k=w, \ldots, w^1-1$. Next, adjust V_w, V_{w1}, S_{gw} and S_{gw^1} to reflect the fact that car loads previously shipped in period w are now shipped in period w^1; also, decrement E_{gj}, $j=w, \ldots, w^1-1$, by a. Examine additional destinations until either $V_w = 0$, there are no destinations for

which shipping can be moved into the future, or there is no slack car loading capacity in any future period.

Step 2b: If $V_w = 0$, then return to Step 2a and examine the next period for which the car loading constraint is violated (if there are no future periods for which the car loading constraint is violated, then the algorithm terminates). If $V_w < 0$, then beginning with period w-1, and moving back period-by-period as necessary, attempt to find a destination g for which $P_{gw} > 0$ (positive production at destination g in week w) and for which there exists a week $j < w$ for which $U_{gj} > 0$ (there is slack production capacity in week j) and a week k, $j \leq k < w$, in which $V_k > 0$ (excess car loading capacity exists in week k). Let

$$a = \min (U_{gj}, P_{gw}, -V_w, V_k) .$$

Then decrement U_{gj}, V_k, P_{gw}, S_{gw} by a and increment P_{gj}, S_{gk}, V_w, $H_{g\ell}$, $\ell=j, \ldots, k-1$, $E_{g\ell}$, $\ell=k, \ldots, w-1$ by a. Repeat until $V_w = 0$. If V_w cannot be increased to a value of zero, then no feasible solution exists.

The first step of the algorithm establishes the optimal production and shipping plan ignoring the rail car loading constraint. Step 2a adjusts the shipping schedule to eliminate infeasibilities in the rail car loading constraints without increasing the amount of inventory carried. If all infeasibilities are eliminated via these adjustments, then an optimal solution has clearly been obtained. If Step 2b is invoked for a given week, inventory carrying costs are increased by as small an amount as possible.

Although we have no proof that this algorithm reaches an optimal solution when Step 2b must be used, we do conjecture that an optimal solution is obtained. The initial solution of the linear programming problem is dual feasible, obeys complementary slackness but may be primal infeasible on one or more of the rail car loading constraints. The algorithm appears to maintain dual feasibility

and complementary slackness at each step, so that when primal feasibility is established the answer would constitute an optimal solution.

We now illustrate the algorithm on an example problem. Assume there are two production line-destination group combinations (G = 2) and the planning horizon is five weeks long (W = 5). The demand, production capacity, and rail car shipping capacity data are given in Table 1. We also assume the initial inventories are zero.

		Week				
	week, w	1	2	3	4	5
Group 1	D_{1w}, demand	4	6	8	10	10
	C_{1w}, production capacity	6	8	8	8	8
Group 2	D_{2w}, demand	3	5	7	9	9
	C_{2w}, production capacity	6	6	8	8	8
	rail car shipping capacity	10	15	13	16	17

Table 1

Demand and Capacity Data

The solution to the production and shipping problem ignoring the rail car loading constraint is given in Table 2; that is, the solution obtained from Step 1 of the algorithm. We see that this solution violates the rail car loading constraint in week 3. Therefore, we must invoke Step 2a of the algorithm. Week 3 is the first week, and in this example the only week, with a negative slack on rail car loading capacity. The first week following week 3 having positive slack rail car loading capacity is week 5 ($w^1 = 5$ and $V_{w1} = 1$). Destination 1 has positive inventory carried at the destination at the end of weeks 3 and 4 ($E_{13} = 4$ and $E_{14} = 2$). Instead of carrying one rail

		Week (w)				
		1	2	3	4	5
	P_{1w}, production	6	8	8	8	8
	S_{1w}, shipments	6	8	8	8	8
Group 1	H_{1w}, factory inventory	0	0	0	0	0
	E_{1w}, destination inventory	2	4	4	2	0
	U_{1w}, slack production	0	0	0	0	0
	P_{2w}, production	3	6	8	8	8
	S_{2w}, shipments	3	6	8	8	8
Group 2	H_{2w}, factory inventory	0	0	0	0	0
	E_{2w}, destination inventory	0	1	2	1	0
	U_{2w}, slack production	3	0	0	0	0
	V_w, slack rail car loading capacity	1	1	-3	0	1

Table 2

Initial Solution

car load of inventory at destination 1 we can carry that one rail car load of inventory at the component plant. Formally, the algorithm states that the maximum increase in H_{13} and H_{14} is $a = min\ \{-(-3), 1, min\ (4,2)\} = 1$. Then V_w and S_{15} are increased by 1, and V_{w1}, E_{13}, S_{13} and E_{14} are all decreased by 1. The results of these calculations are given in Table 3. Note that the total inventory carried is the same as it was at the end of Step 1. Consequently, if the solution found after making these adjustments yields a feasible solution (i.e. satisfies the rail car loading constraints), then that new solution is optimal.

	week, w	1	2	3	4	5
	P_{1w}, production	6	8	8	8	8
	S_{1w}, shipments	6	8	7	8	9
Group 1	H_{1w}, factory inventory	0	0	1	1	0
	E_{1w}, destination inventory	2	4	3	1	0
	U_{1w}, slack production	0	0	0	0	0
	P_{2w}, production	3	6	8	8	8
	S_{2w}, shipments	3	6	8	8	8
Group 2	H_{2w}, factory inventory	0	0	0	0	0
	E_{2w}, destination inventory	0	1	2	1	0
	U_{2w}, slack production	3	0	0	0	0
	V_w, slack rail cars	1	1	-2	0	0

Table 3

Results of First Iteration

Week 3 is still the first week with a negative slack on rail car loading capacity. Looking forward in time from week 3, it is not possible to delay any rail car shipping since there is no slack rail car shipping capacity in either week 4 or week 5. Therefore, we invoke Step 2b of the algorithm. Thus we will now look back in time to see what changes need to be made to the production and shipping schedule.

Production lines in group 1 have no excess capacity in weeks 1, 2, or 3 ($u_{11} = u_{12} = u_{13} = 0$) so that no changes will be made to the production or shipping schedule for the production lines in group 1. There is additional capacity on production lines in group 2, however, since $U_{21} = 3$. Furthermore, in week 1 there is available rail car loading capacity ($V_1 = 1$). Thus we can reduce the production on lines in group 2 in week 3 by $a = \min(3, 8, -(-2), 1) = 1$. Then $u_{21} = 2$, $V_1 = 0$, $P_{23} = 7$, $S_{23} = 7$, $P_{21} = 4$, $S_{21} = 4$, $V_3 = -1$, $E_{11} = 1$, and

$E_{12} = 2$. Since $V_3 = -1$, we repeat Step 2b. There is still additional production capacity for production lines in group 2 in week 1 ($U_{21} = 2$). We see that $V_2 = 1$, that is, there is excess car loading capacity in week 2. The production on lines in group 2 can therefore be reduced by $a = \min(2, 7, 1, 1) = 1$ in week three. The results of applying Step 2b are displayed in Table 4. Since all of the rail car loading constraints are now satisfied, the solution displayed in Table 4 is the final solution.

	week, w	1	2	3	4	5
	P_{1w}, production	6	8	8	8	8
	S_{1w}, shipments	6	8	7	8	9
Group 1	H_{1w}, factory inventory	0	0	1	1	0
	E_{1w}, destination inventory	2	4	3	1	0
	U_{1w}, slack production	0	0	0	0	0
	P_{2w}, production	5	6	6	8	8
	S_{2w}, shipments	4	7	6	8	8
Group 2	H_{2w}, factory inventory	1	0	0	0	0
	E_{2w}, destination inventory	1	3	2	1	0
	U_{2w}, slack production	1	0	2	0	0
	V_w, slack rail cars	0	0	0	0	0

Table 4

Final Solution

III. DETAILED PRODUCTION AND SHIPPING PLANNING

The solution of the aggregate production and shipping scheduling problem discussed in Section II is a plan that smooths weekly fluctuations in demand over the planning horizon subject to weekly constraints on rail car loadings and production line capacities. In particular, the solution determines what the total production should be for each group of production lines and what amounts should be shipped to each destination group in the first week considering various capacity constraints in future weeks. Thus the solution specifies the values of P_{g1}, the number of rail cars of products to produce on group g during week 1, and S_{g1}, the number of rail cars to be shipped to destinations in group g during week 1. This aggregate plan can be accomplished in week 1 since $P_{g1} \leq C_{g1}$ (the production does not exceed production capacity) and $\sum_g S_{g1} \leq L_1$ (the number of rail car loadings is not greater than loading capacity in week 1).

The aggregate planning information must be disaggregated to establish the detailed production and shipping plan for the first week. We will show how to disaggregate the quantities P_{g1} and S_{g1} in three senses. First, we disaggregate the first week's production and shipping plan by indicating production and shipping requirements for each of the T shifts during that week. At this stage no attempt is made to establish production and shipping goals by individual product or by specific destination location. All production quantities will be measured in rail car loads without regard for individual product requirements; the shipping requirements and plan will be stated in terms of the number of rail cars for each group of destinations.

The second level of disaggregation specifies how the first week's shipments should be allocated among the individual destinations within each group. Individual destination requirements, measured in rail car loads, are used to make the allocations. At this stage no attempt is made to disaggregate by

individual product types.

In the final level of disaggregation, the aggregate quantities of production and shipping determined in the first two levels of the disaggregation process are divided among the individual products made and shipped to each group.

The purpose of the three stage disaggregation process is to determine what products, measured in container loads, should be produced on each group of production lines during each shift, what portion of a shift's production and the inventory on-hand at the beginning of each shift at the component plant should be loaded into rail cars and shipped during that shift to each destination, and what portion of a shift's production should be carried in inventory at the component plant into the next shift.

First, we will show how the first week's aggregate production and shipping schedule can be disaggregated into a shift-by-shift production and shipping schedule for the first week. Recall that the solution of the aggregate model specifies the number of rail car loads of products produced on production line group g in week 1 (P_{g1}), the number of rail car loads shipped to group g during week 1 (S_{g1}), the number of rail car loads of products produced in group g in inventory at the component plant at the end of week 1 (H_{g1}), and the number of rail car loads of inventory at destinations in group g at the end of week 1 (E_{g1}).

In addition to the weekly production and shipping goals, we have other data that are used to determine shift-by-shift production and shipping decisions. The capacity of group g during shift t is c_{gt}, which is measured in rail car loads. Furthermore, we assume $\sum_{t=1}^{T} c_{gt} = C_{g1}$. We also have the shipping capacity for each shift, ℓ_t, which is also measured in rail cars. We also assume $\sum_{t=1}^{T} \ell_t = L_1$.

The decision variables we will have in this first level disaggregation problem are:

p_{gt} = the production on lines in group g during shift t measured in rail car loads,

s_{gt} = the number of rail cars shipped to destination group g during shift t,

h_{gt} = the number of rail car loads produced in production group g remaining in inventory at the component plant at the end of shift t, and

e_{gt} = the number of rail car loads carried by destinations in group g corresponding to shipments from the component plant made prior to the end of shift t.

Using these data we can state the first level disaggregation problem as:

find the aggregate (without regard to product types or destination within a group) production and shipping schedule that minimizes inventory holding cost at both the component plant and destination groups during week 1 subject to a) meeting end of the week inventory goals at both the component plant and each group of destinations, b) satisfying aggregate demand requirements for each group of destinations, and c) not exceeding shift-by-shift production and rail car loading capacities.

As before, we assume that the cost of holding a container load is the same for all products.

The mathematical statement of this first level disaggregation problem is

$$(6)\quad \min Z = \sum_g \sum_t h_{gt} + \sum_g \sum_t e_{gt}$$

subject to

$$(7)\quad h_{g,t-1} + p_{gt} = h_{gt} + s_{gt} ,$$

$$(8)\quad e_{g,t-1} + s_{gt} = e_{gt} + d_{gt} ,$$

$$(9)\quad p_{gt} + u_{gt} = c_{gt} ,$$

$$(10)\quad \sum_g s_{gt} + v_t = l_t ,$$

$$p_{gt},\ h_{gt},\ e_{gt},\ s_{gt},\ u_{gt},\ v_t \geq 0 , \quad g=1, \ldots, G \text{ and } t=1, \ldots, T ,$$

where u_{gt} is the slack production capacity for production line group g in shift t, v_t is the slack rail car shipping capacity for shift t, and $d_{gt} = 0$ for $t=1, \ldots, T-1$ and $d_{gT} = D_{g1}$, the aggregate demand for destination group g in week 1. The reason for defining the d_{gt} as we have is obvious once we examine the structure of this problem.

Observe that this disaggregation problem (6) - (10) is mathematically equivalent to the aggregate production and shipping scheduling problem (1) - (5) discussed in Section II. Since this problem's structure is identical to that of the aggregate production and shipping scheduling problem, we can use the special algorithm developed in Section II to find the optimal (or possibly near optimal) values for each of the decision variables.

The second level of disaggregation involves assigning the rail car quantities of shipping and production by shift obtained in the first level of disaggregation to each of the specific destinations within a group. For group g, suppose there are I destinations, indexed by $i=1, \ldots, I$. Let r_{iw} be the rail car demand at destination i in week w; the aggregate demand for group g in week w, D_{gw}, is $\sum_{i=1}^{I} r_{iw}$.

At the start of week 1 a total of E_{go} rail cars are at the destinations

of group g and a total of H_{go} rail cars of product are in inventory at the component plant. We will show shortly that the quantities E_{go} and H_{go} can be considered to be already disaggregated by destination by showing how the quantities E_{gl} and H_{gl} are disaggregated by destination.

Figure 3 shows the essential features of the procedure for deciding to which specific destination rail cars are shipped in each shift and for which specific destination rail cars of product are produced in each shift. The top scale has the specific destination demand in rail cars ordered by week and then by destination. The aggregate relations $D_{gl} + E_{gl} = E_{go} + S_{gl}$, and $S_{gl} + H_{gl} = H_{go} + P_{gl}$ are shown. The disaggregation of D_{gl} into specific destination demands, of S_{gl} into shift shipping, and of P_{gl} into shift production are also shown.

To find to which destination the k^{th} rail car of shift t (the k^{th} unit of s_{gt}) is to be shipped, one simply projects to the top scale and finds the destination index. Similarly, to find for which destination the k^{th} rail car of shift t (the k^{th} unit of P_{gt}) is produced, one also projects to the top scale and finds the destination index.

The final level of disaggregation is to determine the number of container loads of each of the J products to ship in each rail car or to produce for each rail car. Let q_{ijw} be the demand at destination i for product j in week w; the aggregate demand for destination i in week w, r_{iw}, is $\sum_{j=1}^{J} q_{ijw}/R_g$, where R_g is the number of container loads for products of group g that fill a rail car.

Figure 4 shows how this final level of disaggregation is accomplished. To find which product to put in the k^{th} container of a rail car, one simply projects to the top scale and finds the product index.

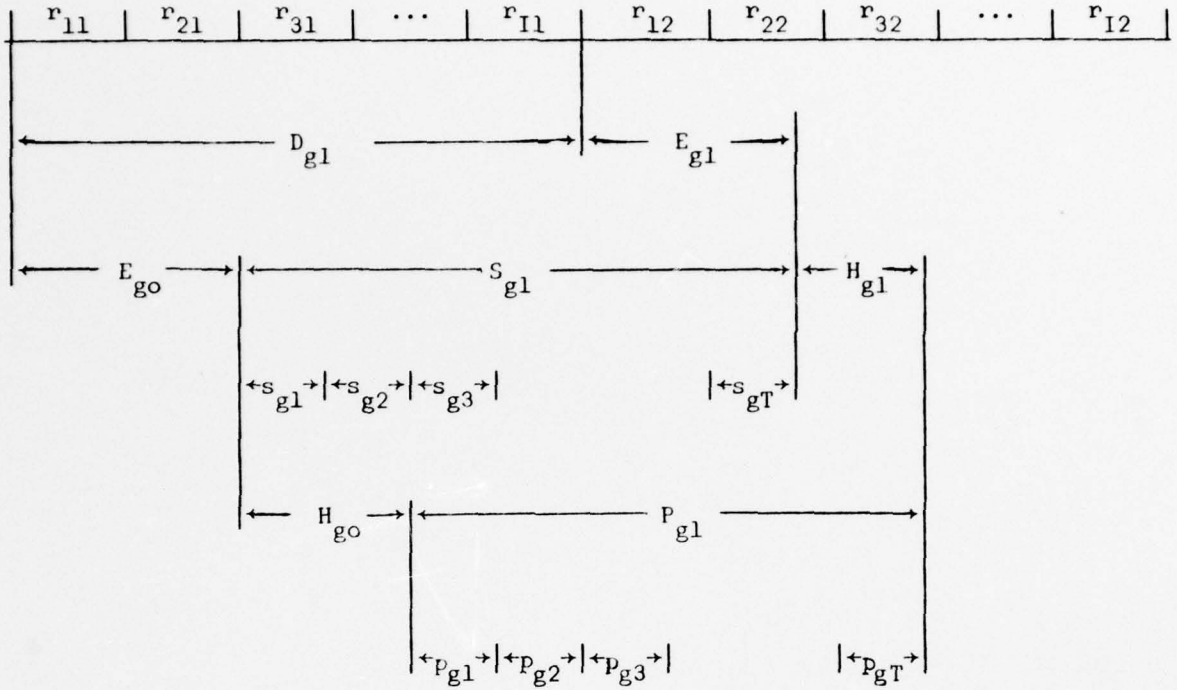


Figure 3

Disaggregation of Rail Cars by Destination

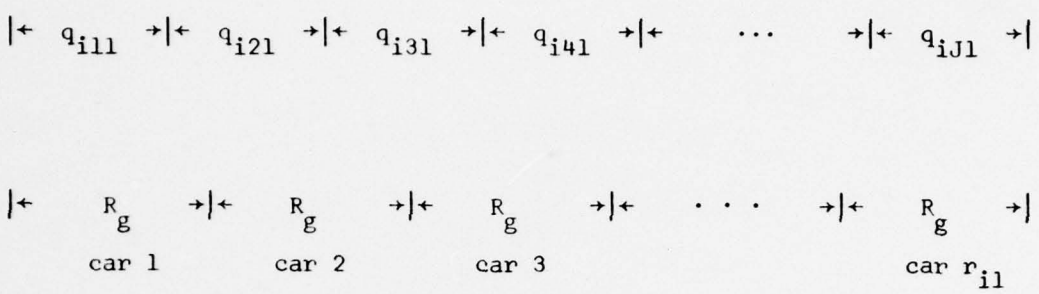


Figure 4

Disaggregation of Rail Car Loads into Products

IV. SUMMARY AND EXTENSIONS

In this paper we have shown how coordinated production and transportation decisions can be made in an actual multi-facility system operated by an automobile component manufacturer. We first developed a model that establishes a production and shipping schedule for each week in the horizon. This model was based on several key observations concerning the system's operation.

First, the production lines could be divided into mutually exclusive groups. The products produced within a group could be produced on any line within the group, but on no other group of production lines. Second, the assembly plants could be similarly divided into groups such that products produced on one group of production lines could only be used at assembly plants within the same group, and products used at a particular assembly plant could all be produced on one group of production lines. Third, because only full rail cars are shipped from the component plant to the destinations, the automobile schedule could be expressed in terms of the number of rail cars that have to be shipped by the end of each week to each destination. Fourth, the length of time required to produce a container load of any product produced within a group is the same for all products and lines within a group, and also there is no time required to change from production of one product to another. These four observations guaranteed that a feasible shift-by-shift production and shipping schedule for each product could be obtained from the solution of an aggregate model in which production requirements were aggregated over products and individual assembly plants for each of the destination groups for each week, and the production capacity for each group of lines in each week was expressed in terms of rail car loads of products produced on each group.

We also observed that the only cost that varies with the production and shipping schedule is the inventory carrying cost. Furthermore, this cost could be expressed in terms of rail car loads of product held since a car load

of any product had the same value as that of any other product, and the cost of carrying a car load of inventory at the component plant is the same as the cost of carrying that car load at a destination plant.

Based on these observations we developed a model that determines aggregate weekly production quantities measured in rail car loads for each group of production lines and an aggregate weekly shipping schedule to each group of destinations. We also presented a simple algorithm for finding the solution to this problem.

Next we showed how the aggregate solution can be disaggregated so that a shift-by-shift production and shipping schedule for each product and each specific destination can be established for the first week in the planning horizon.

The models and algorithms we have developed can be extended to other situations. If products produced on different groups of production lines cost different amounts to produce, then the objective function can be modified to reflect the difference in holding costs. Furthermore, the algorithms we presented can be modified so that adjustments to shipping and production decisions are made in order from highest to lowest holding cost for the production line group-destination group combinations.

Also, if holding costs are higher at destinations than at the component plant, the algorithm we presented for the aggregate scheduling problem can be modified so that initial inventory is all carried at the component plant rather than at the destinations. The shipping plan can be adjusted in the same general manner as described in the algorithm presented in Section II. The difference is that inventory is sent to the destinations as late as possible rather than as early as possible. Thus the roles of the destination and component plant in the algorithm stated in Section II would be essentially reversed.

The model we have presented can also be used as a capacity planning tool in this actual situation we have modelled. In this situation, production capacity is to a large extent dependent upon the number of workers assigned to particular groups of lines. Within limitation this labor can be shifted from one group of lines to another. Thus if the solution to the aggregate planning problem indicates that there is a shortage of capacity on one group of lines while there is excess capacity on other lines in certain weeks, the lines can be rebalanced. The model can be effectively employed to analyze the effects of changing production capacity on the amount of inventory carried throughout the horizon.

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