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APPROXIMATION WITH EXPONENTIAL SUMS.(U)
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APPROXIMATION WITH EXPONENTIAL SUMS

(USAF Grant No. 74-2653)

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Information Science
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INTRODUCTION

In the study of many physical and biological phenomena we encounter situations where we wish to approximate a given function F by a sum of exponentials

$$(1) \qquad Y(t) = a_1 e^{-\lambda_1 t} + \dots + a_n e^{-\lambda_n t}$$

or by a more general exponential sum Y which satisfies some differential equation

$$(2) \quad Y^{(n)}(t) + b_n Y^{(n-1)}(t) + \dots + b_2 Y^{(1)}(t) + b_1 Y(t) \equiv 0$$

along with certain initial conditions

$$(3) \qquad Y^{(i-1)}(0) = c_i \,, \quad i = 1, \dots, n.$$

The objective is to find Y such that

$$\|F - Y\| = \min$$

where $\| \ \|$ is the L_p-norm associated with some given measure $d\mu(t)$ on $[0, \infty)$. The goal of this project has been to investigate this important problem in nonlinear approximation theory, i.e., to consider questions of existence, unicity, characterization, etc., and to devise and test numerical methods for finding a best approximation Y when F is specified by means of a formula or through sampled data.

An enormous simplification in the characterization and construction of best approximations occurs when the class of functions F is suitably restricted. It is well known that certain naturally occurring growth and decay processes can be modeled exceptionally well with a sum of exponentials. One might try to characterize an "ideal growth function" G(t) by requiring it to be very smooth (i.e., to have derivatives of all orders) and to have the property that G, G', G", ... never take negative values. Likewise, an "ideal decay function" F(t) would be the time reversal of an "ideal growth function" and thus have the properties that

$$F \in C^{\infty}(0, \infty)$$

$$(-1)^k F^{(k)}(t) \geq 0, \; 0 < t < \infty, \; k = 0, 1, 2, \ldots \; .$$

Such an "ideal decay function" F is said to be completely monotonic. A detailed study of the approximation of completely monotonic functions by sums of exponentials thus is of immediate use in the modeling of both decay and growth processes.

During the course of the grant a total of 13 research papers, one technical report, and one doctoral dissertation were produced by the principal investigator, Dr. David W. Kammler, and by his graduate student Dr. Robert J. McGlinn who was supported in part with grant funds. A listing of these research works and a brief nontechnical discussion of their significance is given below.

RESEARCH PAPERS PRODUCED UNDER THE GRANT

[1] D. W. Kammler, An alternation characterization of best uniform approximations on non compact intervals, J. Approximation Theory, 16(1976), pp. 97-104.

[2] D. W. Kammler, Approximation with sums of exponentials in $L_p[0,\infty)$, J. Approximation Theory, 16(1976), pp. 384-408.

[3] D. W. Kammler, Chebyshev approximation of completely monotonic functions by sums of exponentials, SIAM J. Numer. Anal., 13(1976), pp. 761-774.

[4] D. W. Kammler, Prony's method for completely monotonic functions, J. Math. Anal. Appl., 57(1977), pp. 560-570.

[5] D. W. Kammler, Existence of good and best approximations on unbounded domains by exponential sums in several independent variables, J. Approximation Theory, 21 (1977), pp. 215-223.

[6] D. W. Kammler and R. J. McGlinn, A bibliography for approximation with exponential sums, J. Comp. Appl. Math., 4(1978), pp. 167-173.

[7] R. J. McGlinn, Uniform approximation of completely monotone functions by exponential sums, J. Math. Anal. Appl. 65(1978), pp. 211-218.

[8] D. W. Kammler, Numerical evaluation of exp(tA) when A is a companion matrix, SIAM J. Numer. Anal., 15(1978), pp. 1077-1102.

[9] D. W. Kammler, L_1-Approximation of completely monotonic functions by sums of exponentials, SIAM J. Numer. Anal., 16(1979), pp. 30-45.

[10] R. J. McGlinn, Approximation by exponential sums on discrete and continuous domains, J. Approximation Theory, 25(1979), pp. 65-88.

[11] R. J. McGlinn and D. W. Kammler, Local conditioning of parametric forms used to approximate continuous functions, Amer. Math. Monthly, 86(1979), to appear.

[12] D. W. Kammler, Least squares approximation of completely monotonic functions by sums of exponentials, SIAM J. Numer. Anal. 16(1979), to appear.

[13] D. W. Kammler and R. J. McGlinn, An alternative to Prony's method, in preparation.

TECHNICAL REPORT PRODUCED UNDER THE GRANT

[14] D. W. Kammler and R. J. McGlinn, A Family of FORTRAN Programs for Finding Best Chebyshev Approximations with Applications to Exponential Sums, Southern Illinois University, Carbondale, Illinois, 1975, 187 pp.

DISSERTATION PRODUCED UNDER THE GRANT

[15] R. J. McGlinn, Approximation by Exponential Sums, Ph.D. Dissertation, Southern Illinois University, Carbondale, Illinois, 1976.

MAJOR ACCOMPLISHMENTS

One major accomplishment of the study has been the devising of a very general theory for establishing the existence of a best approximation to an arbitrary function F by an exponential sum of a given order. In [2] the theory was extended from the case where du(t) = dt on [0,1] to where du(t) = dt on $[0,\infty)$, and in [10] a further extension was made to the case of an arbitrary measure du(t) on $[0,\infty)$ (so as to allow for ℓ_p-approximation on discrete subsets). In this latter case it was necessary to augment the set of exponential sums with certain pseudo exponential sums to avoid the irregular endpoint conditions associated with some measures du(t). A related extension of [2] to establish the existence of best L_p-approximations to a given F by exponential sums in several independent variables is given in [5].

A second major accomplishment has been the development of an efficient numerical scheme for evaluating an exponential sum $Y(\underline{b}, \underline{c}, t)$ when the parameters $\underline{b} = (b_1, \ldots, b_n)$, $\underline{c} = (c_1, \ldots, c_n)$ and t of (2)-(3) are given. If Y is real valued, the parameters b_i, c_i are all real even when the characteristic roots $\lambda_1, \ldots, \lambda_n$ of (2) are complex, and (unlike (1)) this parametric form does not change when the characteristic roots coalesce. The algorithm uses a preprocessing step, a series evaluation, and repeated squarring to numerically evaluate the matrix exponential exp(tA) where A is the companion matrix associated with (2). Each step exploits the companion matrix structure of A so as to make the algorithm both fast and accurate. The algorithm is presented in detail in [8] along with a detailed error analysis and the results of extensive numerical testing on carefully chosen model problems. A numerical study of the conditioning of this parametric form is given in [14].

A third (and perhaps the most important) accomplishment has been the creation of a mathematical theory and workable numerical methods for constructing best Chebyshev, least mean, and least squares approximations for a given completely monotonic function F by sums of exponentials. Such an optimal approximation Y always exists (even when du(t) is discrete) and has the form (1) with $0 < \lambda_1 < \ldots < \lambda_n$ and with $a_i > 0$, $i = 1, \ldots, n$ (except when F has the form (1) with some $a_i = 0$ or when the support of du(t) has less than

2n points). The characterization of a best approximation leads to interesting interpolation theorems which involve exponential sums in the case of Chebyshev and least mean approximation and which involve the transform

$$\mathcal{Y}(s) = \int_0^\infty Y(t)e^{-st}du(t)$$

for least squares approximation. A Remez type algorithm together with a homotopy process for solving the associated nonlinear interpolation problem have been coded and used to construct best Chebychev approximations on finite and semi-infinite intervals and on discrete point sets. Quadratically convergent descent algorithms have been devised, studied, coded, and tested for constructing best least mean and least squares approximations. The results are presented in detail in [3], [4], and [7] (for Chebyshev approximations), in [9] (for least mean approximations) and in [12] (for least squares approximation).

A fourth major accomplishment has been the compilation and publication [6] of a bibliography for approximation with exponential sums. This will undoubtedly be of enormous value to others working in the area.

ORAL PRESENTATIONS

Most of the research findings published in the above papers were first announced by Dr. Kammler during talks given at various professional meetings. A listing of these oral presentations is given below.

[1] "Approximation with exponential sums in $L_p[0,\infty]$," Society of Industrial and Applied Mathematics National Meeting, California Inst. of Technology, June, 1974. Abstract: SIAM Review, 16, (Oct., 1974), p. 583.

[2] "Chebychev approximation of completely monotonic functions by sums of exponentials," American Mathematical Society Winter Meeting, Washington, D.C., Jan., 1975. Abstract: Notices of A.M.S. 22 (1975), pp. A161-162.

[3] Invited participant, Air Force Office of Scientific Research--Rome Air Development Command Joint Conference on Temporal Mode Techniques for Target Classification, Griffiss Air Force Base, New York, October, 1975.

[4] "Numerical evaluation of exp(tA) when A is a companion matrix," Society of Industrial and Applied Mathematics National Meeting, San Francisco, California, Dec., 1975. Abstract: SIAM Review 18, (1976)- p. 811.

[5] "An error analysis for the improved squaring method for computing exp(tA) when A is a companion matrix," Society of Industrial and Applied Mathematics National Meeting, Atlanta, Georgia, Oct., 1976.

[6] "L_1-approximation of completely monotonic functions by sums of exponentials," Society of Industrial and Applied Mathematics National Meeting, Albuquerque, New Mexico, November, 1977. Abstract: SIAM Review 20, (1978), p. 628.

[7] "Least squares approximation of completely monotonic functions by sums of exponentials," Society of Industrial and Applied Mathematics National Meeting, Knoxville, Tenn., October, 1978. Abstract: SIAM Review 21, (1979), to appear.

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During the course of the grant a total of 13 research papers, one technical report, and one doctoral dissertation were produced by the prinicipal investigator and his graduate student. One major accomplishment has been the devising of a very general theory for establishing the existence of a best approximation to an arbitrary function by an exponential sum of a given order. A second result was the creation of mathematical theory and workable numerical methods for constructing best Chebyshev, least mean, and least squares approximations for a given completely monotonic functon by sums of exponentials. Finally, the compilation

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20. Abstract (continued)

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