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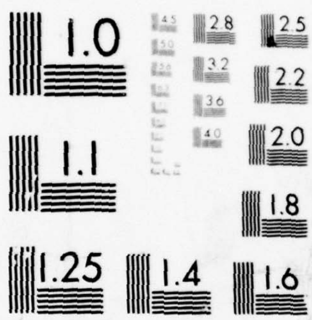

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Bistatic Scattering Reflection Asymmetry and Polarization Reversal Symmetry	G. E. Heath 17 April 1979
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This technical report has been reviewed and is approved for publication.

FOR THE COMMANDER

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BISTATIC SCATTERING REFLECTION ASYMMETRY AND POLARIZATION REVERSAL SYMMETRY

G. E. HEATH

Group 35

TECHNICAL NOTE 1979-4

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ABSTRACT

Circularly polarized bistatic scattering from plane-symmetric targets with otherwise arbitrary electrical properties exhibits reflection asymmetries and polarization reversal symmetries which are useful for bistatic radar data interpretation. In addition, the symmetries can be used to reduce the time and expense in obtaining bistatic scattering data from static range measurements and theoretical or computer calculations.

The reflection asymmetry and polarization reversal symmetry relations are derived in this note. The symmetry relations are then combined with the principle of electromagnetic reciprocity to determine the minimum number of measurements needed to completely characterize target scattering.

CONTENTS

I. INTRODUCTION

Consider a target with arbitrary electrical properties that exhibits symmetry about a plane. Without loss of generality, the symmetry plane can be taken to be the x-z plane of a right-handed x-y-z cartesian coordinate system whose origin is at the target center of mass. The direction of the z axis can be chosen in whatever manner is convenient, e.g., along a principal or unique axis of the target.

When the transmitter line of sight is in the symmetry plane, circularly polarized scattering is asymmetric about the plane, i.e., identical circularly polarized receivers at positions (x,y,z) and (x,-y,z) in the far field of the target will generally measure different cross sections and phases. When the relationship between the scattered far field components and the incident components is expressed via the scattering matrix relation

$$\begin{bmatrix} E_R^{\,s}(x,y,z) \\ E_L^{\,s}(x,y,z) \end{bmatrix} = \begin{bmatrix} a_{RR}(x,y,z) & a_{RL}(x,y,z) \\ a_{LR}(x,y,z) & a_{LL}(x,y,z) \end{bmatrix} \begin{bmatrix} E_R^{\,i} \\ E_L^{\,i} \end{bmatrix} , \tag{1}$$

the principle of reflection asymmetry implies that, in general,

$$a_{ij}(x,-y,z) \neq a_{ij}(x,y,z) \qquad y \neq 0 \quad . \tag{2}$$

A derivation will be provided in the Sections II - VII. It will also be shown that there is reflection symmetry under polarization reversal, i.e.,

$$a_{LR}(x,-y,z) = a_{RL}(x,y,z)$$

and (3)

$$a_{RR}(x,-y,z) = a_{LL}(x,y,z) \quad .$$

It follows from these results that

$$a_{RL}(x,o,z) = a_{LR}(x,o,z)$$

and (4)

$$a_{LL}(x,o,z) = a_{RR}(x,o,z)$$

but, in general,

$$a_{RL}(x,y,z) \neq a_{LR}(x,y,z)$$

and (5)

$$a_{LL}(x,y,z) \neq a_{RR}(x,y,z) \quad .$$

The relations in Equations 2 and 3 are derived in Sections II-VII for a plane-symmetric target with arbitrary electrical properties. Circularly polarized field components are decomposed into linearly polarized components in Section II. Symmetry properties of electric current components induced by linearly polarized incident field components are obtained in Section III by invoking heuristic symmetry arguments. However, a mathematical derivation is provided in the appendix for the important case of a perfectly conducting target. In Section IV symmetry properties of the scattered magnetic vector potential are obtained from those of the induced electric current. The concept of electromagnetic duality is introduced in Section V and used in

Section VI to obtain the symmetry properties of the induced magnetic current and scattered electric vector potential from those of the induced electric current and scattered magnetic vector potential. The symmetry properties of the scattered electric field are finally obtained in Section VII from the symmetry properties of the scattered magnetic and electric vector potentials. Additional symmetry relations are obtained in Section VIII for bodies of revolution by combining the principles of polarization reversal symmetry and electromagnetic reciprocity. In Section IX symmetry relations are used to determined the minimum number of measurements needed to completely characterize target scattering.

II. LINEAR POLARIZATION DECOMPOSITION

The proof of Equations 2 and 3 is most easily accomplished by decomposing the circularly polarized fields into a superposition of linearly polarized fields. The incident electric field is decomposed into components polarized either parallel or perpendicular to the incidence plane which is the symmetry plane of the target. The scattered electric field is decomposed into components polarized either parallel or perpendicular to the scattering plane containing the z axis and the receiver line of sight.

It will be convenient to use spherical coordinate notation for the incident and scattered fields. Accordingly, the terms "θ" and "ϕ" polarization will be used for horizontal and vertical polarization, respectively. The four spherical electric field components are illustrated in Fig. 1.

For $\exp(i\omega t)$ time variations the relationship between the circular and linear components is given by

$$\begin{bmatrix} E_\theta{}^i \\ E_\phi{}^i \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ i & -i \end{bmatrix} \begin{bmatrix} E_R{}^i \\ E_L{}^i \end{bmatrix} \tag{6}$$

and

$$\begin{bmatrix} E_R{}^s \\ E_L{}^s \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & i \\ 1 & -i \end{bmatrix} \begin{bmatrix} E_\theta{}^s \\ E_\phi{}^s \end{bmatrix} . \tag{7}$$

The relationship between the scattered and incident linearly polarized fields is expressed in terms of the linear polarization scattering matrix relation

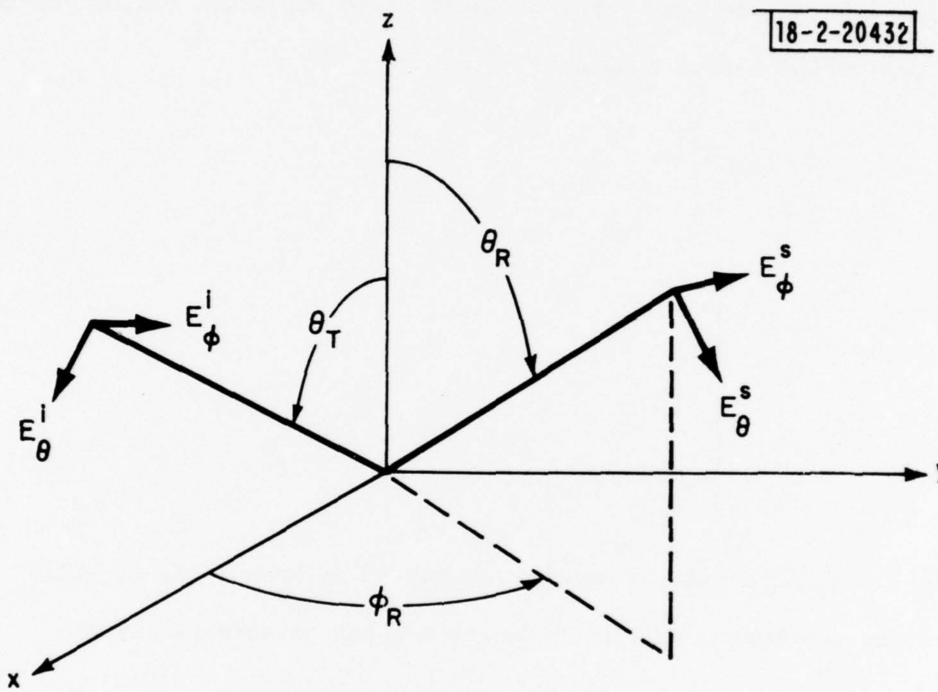

Fig. 1. Θ and ϕ polarized components of the incident and scattered electric fields.

$$\begin{bmatrix} E_\theta^s \\ \\ E_\phi^s \end{bmatrix} = \begin{bmatrix} a_{\theta\theta} & a_{\theta\phi} \\ \\ a_{\phi\theta} & a_{\phi\phi} \end{bmatrix} \begin{bmatrix} E_\theta^i \\ \\ E_\phi^i \end{bmatrix} \tag{8}$$

If the scattered field is known when the incident field is either θ or ϕ polarized, normalization by the incident field amplitude yields the corresponding scattering matrix component, e.g.,

$$a_{\theta\phi} = E_{\theta\phi}/E_\phi^i \tag{9}$$

where

$$E_{\theta\phi} = E_\theta^s \Bigg|_{\substack{E_\theta^i = 0 \\ E_\phi^i \neq 0}} \tag{10}$$

Once the linear polarization scattering matrix is known, the circular polarization scattering matrix in Equation 1 can be obtained from

$$\begin{bmatrix} a_{RR} & a_{RL} \\ a_{LR} & a_{LL} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & i \\ 1 & -i \end{bmatrix} \begin{bmatrix} a_{\theta\theta} & a_{\theta\phi} \\ a_{\phi\theta} & a_{\phi\phi} \end{bmatrix} \begin{bmatrix} 1 & 1 \\ i & -i \end{bmatrix} \tag{11}$$

Therefore,

$$a_{LR} = \frac{1}{2} \{a_{\theta\theta} + a_{\phi\phi} + i(a_{\theta\phi} - a_{\phi\theta})\} \quad ,$$

$$a_{RL} = \frac{1}{2} \{a_{\theta\theta} + a_{\phi\phi} - i(a_{\theta\phi} - a_{\phi\theta})\} \quad ,$$

$$a_{RR} = \frac{1}{2} \{a_{\theta\theta} - a_{\phi\phi} + i\,(a_{\theta\phi} + a_{\phi\theta})\} \quad ,$$

and (12)

$$a_{LL} = \frac{1}{2} \{a_{\theta\theta} - a_{\phi\phi} - i\,(a_{\theta\phi} + a_{\phi\theta})\} \quad .$$

Consequently, the circular polarization reflection properties in Equations 2 and 3 can be readily determined from the properties of the linearly polarized scattered fields.

III. INDUCED ELECTRIC CURRENT SYMMETRY

In this section, induced electric current symmetry is obtained from symmetry arguments.

Consider the y symmetric profile of the target as illustrated in Fig. 2. If the incident electric field is polarized either parallel or perpendicular to the x-z plane, the cartesian components of induced volume and surface currents must be either even or odd in y. If the incident electric field is ϕ (perpendicular) polarized, $\vec{E}^i = E_\phi^i \hat{y}$, $J_{y\phi}$ is even in y, and $J_{x\phi}$ and $J_{z\phi}$ are odd in y. For θ (parallel) polarization, $\vec{E}^i = E_\theta^i(\cos\theta_T \hat{x} - \sin\theta_T \hat{z})$, $J_{y\theta}$ is odd in y, and $J_{x\theta}$ and $J_{z\theta}$ are even in y. These symmetries are summarized in Table 1.

A mathematical derivation of the induced electric current symmetries for a perfectly conducting target is provided in the appendix.

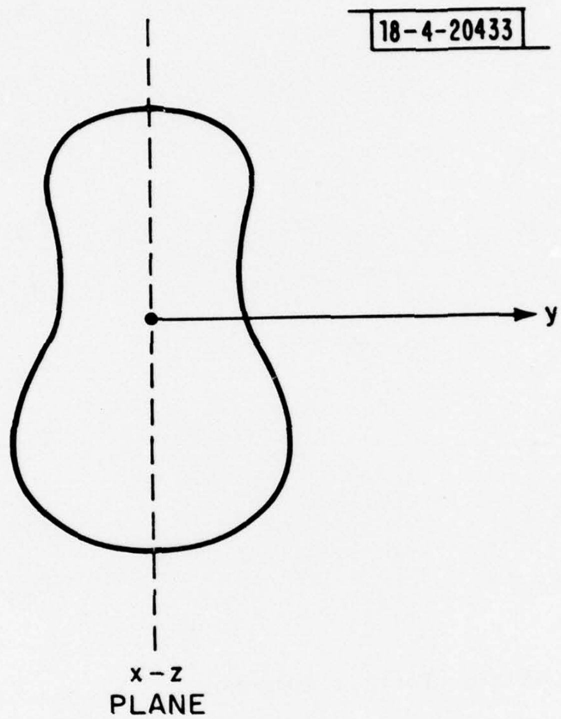

Fig. 2. y symmetric profile of a target that exhibits mirror symmetry about the x-z plane.

TABLE 1

LINEAR POLARIZATION y REFLECTION SYMMETRIES*

Even Reflection Symmetry	Odd Reflection Symmetry
$J_{x\theta}$, $A_{x\theta}$	$J_{x\phi}$, $A_{x\phi}$
$J_{y\phi}$, $A_{y\phi}$	$J_{y\theta}$, $A_{y\theta}$
$J_{z\theta}$, $A_{z\theta}$	$J_{z\phi}$, $A_{z\phi}$
$A_{\theta\theta}$, $A_{\phi\phi}$	$A_{\phi\theta}$, $A_{\theta\phi}$
$M_{x\phi}$, $F_{x\phi}$	$M_{x\theta}$, $F_{x\theta}$
$M_{y\theta}$, $F_{y\theta}$	$M_{y\phi}$, $F_{y\phi}$
$M_{z\phi}$, $F_{z\phi}$	$M_{z\theta}$, $F_{z\theta}$
$F_{\theta\phi}$, $F_{\phi\theta}$	$F_{\phi\phi}$, $F_{\theta\theta}$
$E_{\theta\theta}$, $E_{\phi\phi}$	$E_{\phi\theta}$, $E_{\theta\phi}$

J - induced electric current

A - scattered magnetic vector potential

M - induced magnetic current

F - scattered electric vector potential

E - scattered electric field

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*The second subscript denotes incident electric field polarization.
θ polarization - parallel to the x-z plane.
ϕ polarization - perpendicular to the x-z plane.

IV. MAGNETIC VECTOR POTENTIAL SYMMETRY

In this section induced electric current symmetry is used to deduce magnetic vector potential symmetry.

The volume current contribution to the i^{th} cartesian component of the scattered magnetic vector potential is given by

$$A_i(x,y,z) = \frac{\mu}{4\pi}\iint dx'dz' \int_{-f(x',z')}^{f(x',z')} dy' \; J_i(x',y',z')\frac{e^{-ikR}}{R} \tag{13}$$

where

$$R = \sqrt{(x-x')^2+(y-y')^2+(z-z')^2} \quad . \tag{14}$$

If $J_i(x',y',z')$ is even in y' only the even part of $R^{-1}\exp[-ikR]$ with respect to y' survives the y' integration. Since this part is also even in y, $A_i(x,y,z)$ will also be even in y. Similar arguments hold when $J_i(x',y',z')$ is odd and for surface current contributions to A. Therefore, as indicated in Table 1, the cartesian components of $\vec{A}(x,y,z)$ have the same y symmetry properties as the corresponding cartesian components of $\vec{J}(x,y,z)$.

The transverse spherical components of $\vec{A}$ are given by

$$A_\theta = \frac{zx}{r\rho}A_x + \frac{zy}{r\rho}A_y - \frac{\rho}{r}A_z$$

and (15)

$$A_\phi = -\frac{y}{\rho}A_x + \frac{x}{\rho}A_y$$

where $\rho = \sqrt{x^2+y^2}$ and $r = \sqrt{\rho^2+z^2}$. Combining these equations with the symmetry properties of the cartesian components in Table 1 yields

$$A_{\theta\theta} \text{ and } A_{\phi\phi} \text{ are even in } y$$

and (16)

$$A_{\theta\phi} \text{ and } A_{\phi\theta} \text{ are odd in } y$$

which are summarized in Table 1.

V. ELECTROMAGNETIC DUALITY

In this section the concept of electromagnetic duality is introduced so that the symmetry properties of the induced magnetic current and electric vector potential can be obtained from those of the induced electric current and magnetic vector potential.

Two equations of the same mathematical form are called <u>dual equations</u>. Quantities occupying the same position in dual equations are called <u>dual quantities</u> [Ref. 1, p. 98]. The duality of the electric and magnetic quantities in Table 2 follows from the symmetry of the Maxwell equations

$$\nabla\mathrm{x}\begin{pmatrix}\vec{E}\\ \vec{H}\end{pmatrix} = i\omega\begin{pmatrix}-\mu\vec{H}\\ \varepsilon\vec{E}\end{pmatrix} + \begin{pmatrix}-\vec{M}\\ \vec{J}\end{pmatrix} \tag{17}$$

with the solutions

$$\begin{pmatrix}\vec{E}\\ \vec{H}\end{pmatrix} = -i\omega\left[\overleftrightarrow{I} + \frac{1}{k^2}\nabla\nabla\right]\cdot\begin{pmatrix}\vec{A}\\ -\vec{F}\end{pmatrix} + \nabla\mathrm{x}\begin{pmatrix}\vec{F}/\varepsilon\\ \vec{A}/\mu\end{pmatrix} \quad , \quad k^2 = \omega^2\mu\varepsilon \tag{18}$$

where

$$\left[\nabla^2 + k^2\right]\begin{pmatrix}\vec{A}\\ \vec{F}\end{pmatrix} = \begin{pmatrix}-\mu\vec{J}\\ \varepsilon\vec{M}\end{pmatrix} \quad , \tag{19}$$

and $\overleftrightarrow{I}$ is the unit dyadic defined by

$$\overleftrightarrow{I} = \hat{x}\hat{x} + \hat{y}\hat{y} + \hat{z}\hat{z} \quad . \tag{20}$$

A systematic replacement of the electric and magnetic quantities in one column of Table 2 by the corresponding magnetic and electric quantities in the other column merely causes the Maxwell equations to be interchanged. Therefore a

TABLE 2

ELECTROMAGNETIC DUALITY RELATIONS *

$\vec{E}$		$\vec{H}$
$\vec{H}$		$-\vec{E}$
$\vec{J}$		$\vec{M}$
$\vec{M}$		$-\vec{J}$
ε	$\Leftrightarrow$	μ
μ		ε
$\vec{A}$		$-\vec{F}$
$\vec{F}$		$\vec{A}$

*Replacing all of the quantities in one column by the corresponding quantities in the other column causes the Maxwell curl equations to be interchanged.

known relationship between quantities in one column also exists between the dual quantities in the other column.

VI. SYMMETRY OF INDUCED MAGNETIC CURRENTS AND ELECTRIC VECTOR POTENTIAL

The symmetries of the induced magnetic currents and electric vector potential summarized in Table 1 are obtained from the duality relations in Table 2 and the fact that when $\vec{E}^i$ is θ polarized, $\vec{H}^i$ is ϕ polarized and vice versa.

VII. LINEAR POLARIZATION SCATTERED FIELD SYMMETRY

In the far field, the electric field expression in Equation 18 reduces to

$$\vec{E}^s = -i\omega\left\{(A_\theta - \eta F_\phi)\hat{\theta} + (A_\phi + \eta F_\theta)\hat{\phi}\right\} \tag{21}$$

where $\eta = \sqrt{\mu/\varepsilon}$. Therefore the transverse spherical components of $\vec{E}^s(x,y,z)$ have the same y reflection properties as the transverse spherical components of $\vec{A}(x,y,z)$. Consequently, as summarized in Table 1,

$E_{\theta\theta}(x,y,z)$ and $E_{\phi\phi}(x,y,z)$ are even in y

and (22)

$E_{\phi\theta}(x,y,z)$ and $E_{\theta\phi}(x,y,z)$ are odd in y .

Combining these results with equations of the type found in Equations 9 and 10 yields

$a_{\theta\theta} \pm a_{\phi\phi}$ are even in y

and (23)

$a_{\theta\phi} \pm a_{\phi\theta}$ are odd in y.

The proof of Equations 2 and 3 follows from combining Equations 12 and 23.

VIII. SYMMETRY RELATIONS FOR BODIES OF REVOLUTION

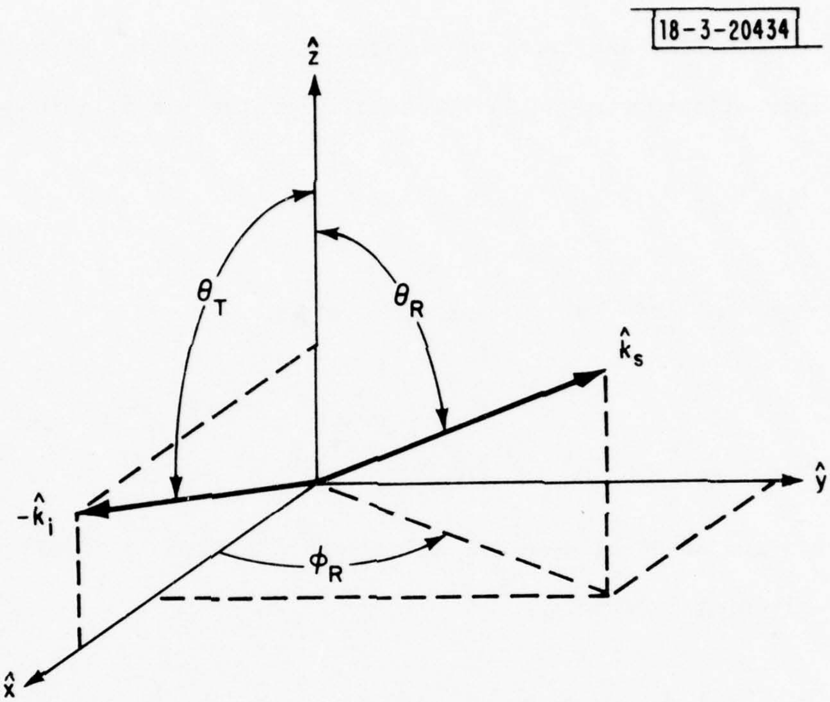
When applied to electromagnetic scattering, the principle of electromagnetic reciprocity requires that cross sections and phases remain unchanged when the roles of transmitter and receiver are interchanged [Ref. 2, p. 252]. In this section the principles of electromagnetic reciprocity and polarization reversal symmetry are combined to obtain additional symmetry relations for bodies of revolution.

When the target is a body of revolution, it is convenient to make the z axis the axis of revolution. For a transmitter and receiver in the spherical coordinate directions $(\theta_T, 0)$ and (θ_R, ϕ_R), respectively (see Figure 3), scattering matrix amplitudes have the form

$$a_{ij}\,(\theta_T,\, r,\, \theta_R,\, \phi_R) = \frac{e^{-ikr}}{\sqrt{4\,\pi r^2}} \sqrt{\sigma_{ij}}\; e^{i\rho_{ij}} \tag{24}$$

where σ_{ij} and ρ_{ij} are the corresponding cross sections and range independent phases. In the following the variation with r will be ignored and symmetry relations with respect to the scattering angle triplet $(\theta_T, \theta_R, \phi_R)$ will be obtained.

When the positions of the two antennas remain fixed but the roles of transmitting and receiving are interchanged, the new transmitter and receiver spherical coordinate directions are $(\theta_R, 0)$ and $(\theta_T, -\phi_R)$, respectively. The transmitter and receiver aspect angles are interchanged and the receiver azimuth angle changes sign, i.e.,



RADAR ASPECT ANGLE COORDINATES $(\theta_T, \theta_R, \phi_R)$

$\hat{z}$	forward directed principal axis unit vector of target	
$\hat{x}, \hat{y}, \hat{z}$	cartesian coordinate unit vectors	
$\hat{k}_i$	unit incident wave vector	
$\hat{k}_s$	unit scattered wave vector	
θ_T	transmitter (monostatic) aspect angle	$0 \le \theta_T \le \pi$
θ_R	receiver aspect angle	$0 \le \theta_R \le \pi$
ϕ_R	receiver to transmitter azimuthal angle	$-\pi < \phi_R \le \pi$

Fig. 3. The radar aspect angle coordinate system.

$$(\theta_T, \theta_R, \phi_R) \rightarrow (\theta_R, \theta_T, -\phi_R) \quad . \tag{25}$$

Since the transmitter and receiver polarizations have also been interchanged, the principle of electromagnetic reciprocity yields the relations

$$a_{LR}(\theta_R, \theta_T, -\phi_R) = a_{RL}(\theta_T, \theta_R, \phi_R)$$

$$a_{RR}(\theta_R, \theta_T, -\phi_R) = a_{RR}(\theta_T, \theta_R, \phi_R)$$

and (26)

$$a_{LL}(\theta_R, \theta_T, -\phi_R) = a_{LL}(\theta_T, \theta_R, \phi_R) \quad .$$

The polarization reversal symmetry relation in Equation 3 yields the receiver azimuth reflection relations

$$a_{LR}(\theta_T, \theta_R, -\phi_R) = a_{RL}(\theta_T, \theta_R, \phi_R)$$

and (27)

$$a_{RR}(\theta_T, \theta_R, -\phi_R) = a_{LL}(\theta_T, \theta_R, \phi_R) \quad .$$

Combining Equations 26 and 27 yields the aspect angle interchange relations

$$a_{LR}(\theta_R, \theta_T, \phi_R) = a_{LR}(\theta_T, \theta_R, \phi_R) \quad ,$$

$$a_{RR}(\theta_R, \theta_T, \phi_R) = a_{LL}(\theta_T, \theta_R, \phi_R) \quad ,$$

and (28)

$$a_{RL}(\theta_R, \theta_T, \phi_R) = a_{RL}(\theta_T, \theta_R, \phi_R) \quad ,$$

i.e., principally polarized scattering is invariant to an interchange of the transmitter aspect angle θ_T and receiver aspect angle θ_R but orthogonally polarized scattering is not. The radar aspect angle symmetry relations in Equations 26 to 28 are summarized in Table 3.

Bistatic static range measurements are typically obtained in terms of the bistatic measurement coordinates (τ, β, γ) [Ref. 3]. The coordinate system used to display the measurements is illustrated in Fig. 4. $\hat{k}_i$ and $\hat{k}_s$ are the incident and scattered wave unit normals, respectively. The fixed transmitter-line-of-sight (TLOS) and receiver-line-of-sight (RLOS) define the bistatic plane. The bistatic angle $\beta (0 \leq \beta \leq 180^{\circ})$ is the angle between the TLOS and RLOS. The target is mounted on a turntable in the bistatic plane which rotates 360° during each measurement run. The turntable angle $\tau (-180^{\circ} < \tau \leq 180^{\circ})$ is the angle between $\hat{b}$ (the unit bisector of the TLOS and RLOS) and the projection of $\hat{z}$ (the unit vector in the direction of the target principal axis) on the bistatic plane. The turntable angle is positive when the projection of $\hat{z}$ in the bistatic plane is on the receiver side of the bisector. The pitch angle $\gamma \ (-90^{\circ} \leq \gamma \leq 90^{\circ})$ is the angle between $\hat{z}$ and the bistatic plane. It is positive when the projection of $\hat{z}$ on $\hat{n}$ (the unit normal to the bistatic plane in the $\hat{k}_s \times \hat{k}_i$ direction) is positive.

The transformation between bistatic measurement coordinates and radar aspect coordinates is given in Table 4. Notice that

$$\phi_R \rightarrow -\phi_R \quad \Longleftrightarrow \quad \gamma \rightarrow -\gamma$$

TABLE 3

RADAR ASPECT COORDINATE SYMMETRY RELATIONS

Aspect Angle Interchange

$$a_{LR}(\theta_R, \theta_T, \phi_R) = a_{LR}(\theta_T, \theta_R, \phi_R)$$

$$a_{RR}(\theta_R, \theta_T, \phi_R) = a_{LL}(\theta_T, \theta_R, \phi_R)$$

$$a_{RL}(\theta_R, \theta_T, \phi_R) = a_{RL}(\theta_T, \theta_R, \phi_R)$$

Receiver Azimuth Reflection

$$a_{LR}(\theta_T, \theta_R, -\phi_R) = a_{RL}(\theta_T, \theta_R, \phi_R)$$

$$a_{RR}(\theta_T, \theta_R, -\phi_R) = a_{LL}(\theta_T, \theta_R, \phi_R)$$

Electromagnetic Reciprocity

$$a_{LR}(\theta_R, \theta_T, -\phi_R) = a_{RL}(\theta_T, \theta_R, \phi_R)$$

$$a_{RR}(\theta_R, \theta_T, -\phi_R) = a_{RR}(\theta_T, \theta_R, \phi_R)$$

$$a_{LL}(\theta_R, \theta_T, -\phi_R) = a_{LL}(\theta_T, \theta_R, \phi_R)$$

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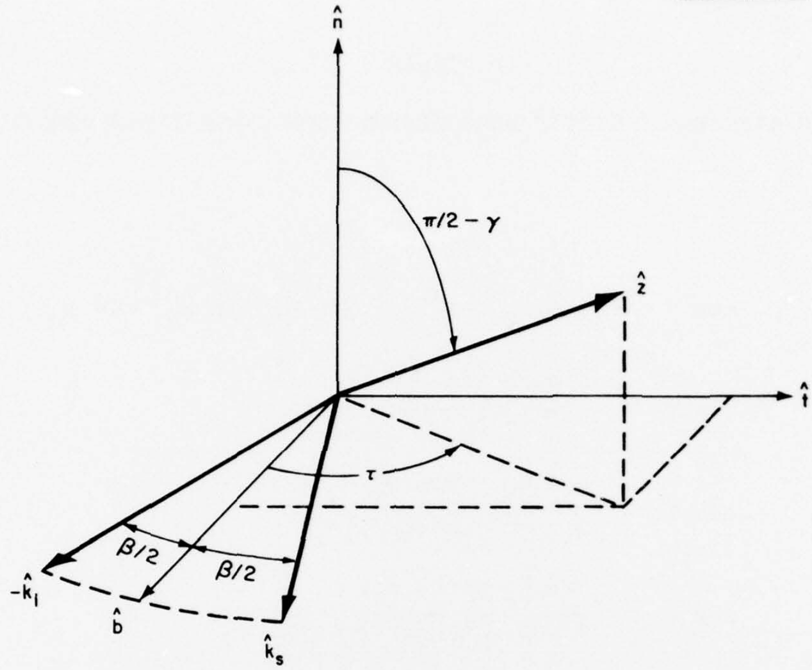

BISTATIC MEASUREMENT COORDINATES (τ, β, γ)

$\hat{z}$	forward directed principal axis unit vector of target	
$\hat{k}_i$	unit incident wave vector	
$\hat{k}_s$	unit scattered wave vector	
$\hat{b} = \dfrac{\hat{k}_s - \hat{k}_i}{\lvert \hat{k}_s - \hat{k}_i \rvert}$		bistatic unit bisector
$\hat{t} = \dfrac{\hat{k}_s + \hat{k}_i}{\lvert \hat{k}_s + \hat{k}_i \rvert}$		bistatic unit tangent
$\hat{n} = \hat{b} \times \hat{t}$		bistatic unit normal
τ	bistatic turntable angle	$-\pi < \tau \leq \pi$
β	bistatic angle	$0 \leq \beta \leq \pi$
γ	bistatic pitch angle	$-\pi/2 \leq \gamma \leq \pi/2$

Fig. 4. The bistatic measurement coordinate system.

TABLE 4

RADAR ASPECT - BISTATIC MEASUREMENT COORDINATE TRANSFORMATIONS

$$\cos \beta = \cos \theta_T \cos \theta_R + \sin \theta_T \sin \theta_R \cos \phi_R$$

$$\sin \gamma = \frac{\sin \theta_T \sin \theta_R}{\sin \beta} \sin \phi_R$$

$$\cos \tau = \frac{\cos \theta_R + \cos \theta_T}{2 \cos \gamma \cos(\beta/2)}$$

$$\sin \tau = \frac{\cos \theta_R - \cos \theta_T}{2 \cos \gamma \sin(\beta/2)}$$

Inverse Transformation

$$\cos \theta_T = \cos \gamma \cos(\tau+\beta/2)$$

$$\cos \theta_R = \cos \gamma \cos(\tau-\beta/2)$$

$$\cos \phi_R = \frac{\cos \beta - \cos \theta_T \cos \theta_R}{\sin \theta_T \sin \theta_R}$$

$$\sin \phi_R = \frac{\sin \beta}{\sin \theta_T \sin \theta_R} \sin \gamma$$

and (29)

$$\theta_T \leftrightarrow \theta_R \quad \Longleftrightarrow \quad \tau \rightarrow -\tau \quad , \tag{29}$$

i.e., a sign change in pitch angle is equivalent to a sign change in receiver azimuth angle and a sign change in turntable angle is equivalent to an interchange in transmitter and receiver aspect angle. The bistatic measurement symmetry relations summarized in Table 5 are obtained from the radar aspect symmetry relations in Table 3 by using Equation 29.

TABLE 5

BISTATIC MEASUREMENT COORDINATE SYMMETRY RELATIONS

Turntable Angle Reflection

$$a_{LR}(-\tau, \beta, \gamma) = a_{LR}(\tau, \beta, \gamma)$$

$$a_{RR}(-\tau, \beta, \gamma) = a_{LL}(\tau, \beta, \gamma)$$

$$a_{RL}(-\tau, \beta, \gamma) = a_{RL}(\tau, \beta, \gamma)$$

Pitch Angle Reflection

$$a_{LR}(\tau, \beta, -\gamma) = a_{RL}(\tau, \beta, \gamma)$$

$$a_{RR}(\tau, \beta, -\gamma) = a_{LL}(\tau, \beta, \gamma)$$

Electromagnetic Reciprocity

$$a_{LR}(-\tau, \beta, -\gamma) = a_{RL}(\tau, \beta, \gamma)$$

$$a_{RR}(-\tau, \beta, -\gamma) = a_{RR}(\tau, \beta, \gamma)$$

$$a_{LL}(-\tau, \beta, -\gamma) = a_{LL}(\tau, \beta, \gamma)$$

IX. INDEPENDENT CIRCULAR POLARIZATION MEASUREMENTS

In this section the principles of electromagnetic reciprocity and polarization reversal symmetry are used to determine the minimum number of measurements needed to completely characterize target scattering.

A. Bodies of Revolution

From the static measurement reflection relations in Table 5, it is seen that for fixed γ and β, LR and RL static range patterns in τ will always be symmetric about $\tau = 0$. In contrast, RR and LL patterns will be symmetric about $\tau = 0$ only when $\gamma = 0$. However, since the LL and RR patterns are always mirror images of each other about $\tau = 0$, only one of these measurements need be made. In addition, negative pitch patterns can be obtained from positive pitch patterns (or vice versa) by using the polarization reversal symmetry relations. Therefore, for fixed β, three sets of 360° static patterns (e.g., LR, RR and RL for $0 \leq \gamma \leq 90^\circ$, $-180^\circ < \tau \leq 180^\circ$) are sufficient to completely characterize the target. However, since the LR and RL patterns are symmetric about $\tau = 0$, measurement redundancy cannot be completely avoided.

The measurement redundancy inherent in 360° experimental static range patterns can be avoided when making theoretical or computer calculations. For fixed β four sets of 180° static patterns (e.g., LR, RR, RL and LL for $0 \leq \gamma \leq 90^\circ$, $0 \leq \tau \leq 180^\circ$) are sufficient to completely characterize the target. Negative pitch and turntable angle data can be obtained by using the reflection relations in Table 5.

B. Arbitrary Targets

If a target is not a body of revolution, none of the symmetry relations in Table 5 are applicable. For example, consider a target whose physical shape is a body of revolution but whose electrical properties vary in roll angle ρ (modulo 2π) about the physical symmetry axis. The circular polarization electromagnetic reciprocity relations are

$$a_{ij}(-\tau, \beta, -\gamma, \rho + \pi) = a_{ji}(\tau, \beta, \gamma, \rho) \quad . \tag{30}$$

However, as the target rotates on the turntable, the incidence plane will not always be a symmetry plane. Therefore, the static patterns will not exhibit polarization reversal symmetry. Consequently, unless the target has additional symmetries in roll angle, all four static patterns over a 360° range of roll angles are needed to completely characterize the target for a fixed $|\gamma|$ and β. Measurement redundancy is avoided by using reciprocity to obtain negative pitch patterns from positive pitch patterns (or vice versa).

REFERENCES

1. R. F. Harrington, Time Harmonic Electromagnetic Fields (McGraw-Hill, New York, 1961).

2. J. Van Bladel, Electromagnetic Fields (McGraw-Hill, New York, 1964).

3. P. E. Robillard and R. H. Herrett, "Circular Polarized Static RCS Patterns for Bistatic Scattering," Vols. I and II, Delco Electronics, Goleta, California, Technical Reports TR76-85 (August 1976) and TR77-30 (April 1977).

APPENDIX

POLARIZATION SYMMETRY OF INDUCED SURFACE CURRENT

The induced surface current $\vec{J}(r)$, on a perfectly conducting scatterer with unit outward pointing normal $\hat{n}(r)$, is a solution of the inhomogeneous integral equation [Ref. 2, p. 354]

$$2\hat{n}(\vec{r}) \times \vec{H}^i(r) = \vec{J}(\vec{r}) + 2\hat{n}(\vec{r}) \times \iint ds' \left[\vec{J}(r') \times \nabla G(R)\right] \tag{A1}$$

where

$$G(R) = \frac{e^{-ikR}}{4\pi R} \qquad R = |\vec{r} - \vec{r}\,'| \tag{A2}$$

is the free space scalar Green's function and $\vec{H}^i(r)$ is the incident magnetic surface field. The integral equation can be reduced to the form

$$2\hat{n}(\vec{r}) \times \vec{H}^i(\vec{r}) = \vec{J}(\vec{r}) + \iint ds' \overleftrightarrow{\Gamma}(\vec{r}, \vec{r}\,') \cdot \vec{J}(\vec{r}) \tag{A3}$$

where

$$\overleftrightarrow{\Gamma}(\vec{r}, \vec{r}\,') = \Psi(R) \overleftrightarrow{K}(\vec{r}, \vec{r}\,') \quad ,$$

$$\Psi(R) = -2(ik + 1/R) \frac{G(R)}{R} \quad ,$$

and (A4)

$$\overleftrightarrow{K}(\vec{r}, \vec{r}\,') = (\hat{n} \cdot \vec{R}) \overleftrightarrow{I} - \vec{R}\hat{n} \quad .$$

$\overleftrightarrow{I}$ is the unit dyadic $\hat{x}\hat{x} + \hat{y}\hat{y} + \hat{z}\hat{z}$ and $\vec{R} = \vec{r} - \vec{r}\,'$.

Let

$$\vec{J} = \vec{J}_e + \vec{J}_o \quad ,$$

$$\hat{n} \times \vec{H}^i = (\hat{n} \times \vec{H}^i)_e + (\hat{n} \times \vec{H}^i)_o \quad ,$$

$$\Psi = \Psi_e + \Psi_o \quad ,$$

and (A5)

$$\overleftrightarrow{K} = \overleftrightarrow{K}_{ee} + \overleftrightarrow{K}_{eo} + \overleftrightarrow{K}_{oe} + \overleftrightarrow{K}_{oo}$$

where the subscripts e and o denote evenness or oddness with respect to the y coordinate, e.g., $\overleftrightarrow{K}_{eo}$ is even in y and odd in y'. (Only one subscript is needed for Ψ because it is invariant with respect to an interchange of y and y'.) Then for a surface which is symmetric about the y = 0 plane,

$$2(\hat{n} \times \vec{H}^i)_e = \vec{J}_e + \iint ds' \left[\overleftrightarrow{\Gamma}_{ee} \cdot \vec{J}_e + \overleftrightarrow{\Gamma}_{eo} \cdot \vec{J}_o\right]$$

and (A6)

$$2(\hat{n} \times \vec{H}^i)_o = \vec{J}_o + \iint ds' \left[\overleftrightarrow{\Gamma}_{oe} \cdot \vec{J}_e + \overleftrightarrow{\Gamma}_{oo} \cdot \vec{J}_o\right]$$

where

$$\overleftrightarrow{\Gamma}_{ee} = \Psi_e \overleftrightarrow{K}_{ee} + \Psi_o \overleftrightarrow{K}_{oo} \quad ,$$

$$\overleftrightarrow{\Gamma}_{eo} = \Psi_e \overleftrightarrow{K}_{eo} + \Psi_o \overleftrightarrow{K}_{oe} \quad ,$$

$$\overleftrightarrow{\Gamma}_{oe} = \Psi_e \overleftrightarrow{K}_{oe} + \Psi_o \overleftrightarrow{K}_{eo} \quad ,$$

and (A7)

$$\overleftrightarrow{\Gamma}_{oo} = \Psi_e \overleftrightarrow{K}_{oo} + \Psi_o \overleftrightarrow{K}_{ee} \quad .$$

If

$$y^2 - f(x,z) = 0 \tag{A8}$$

is the equation of the surface, $\hat{n}$ is given by

$$\hat{n} = n_y\hat{y} + \vec{n}_\perp = \frac{\nabla[y^2 - f(x,z)]}{|\nabla[y^2 - f(x,z)]|}$$

$$= \frac{2y\hat{y} - f_x\hat{x} - f_z\hat{z}}{\sqrt{4y^2 + f_x^2 + f_z^2}} \quad . \tag{A9}$$

Therefore,

$$\hat{n}_e = \vec{n}_\perp$$

and (A10)

$$\hat{n}_o = n_y\hat{y} \quad .$$

For incident plane wave propagation perpendicular to the y axis

$$\vec{H}^i = H_y^{\ i}\ \hat{y} + \vec{H}_\perp^{\ i} \tag{A11}$$

is independent of y. Then since

$$\hat{n} \times \vec{H}^i = \vec{n}_\perp \times \vec{H}_\perp^{\ i} + n_y\hat{y} \times \vec{H}_\perp^{\ i} + H_y^{\ i}(\vec{n}_\perp \times \hat{y}) \quad , \tag{A12}$$

$$(\hat{n} \times \vec{H}^i)_e = \vec{n}_\perp \times \vec{H}_\perp^{\ i} + H_y^{\ i}\ (\vec{n}_\perp \times \hat{y})$$

and (A13)

$$(\hat{n} \times \vec{H}^i)_o = n_y\hat{y} \times \vec{H}_\perp^{\ i} \quad .$$

Notice that

$$\hat{y} \cdot (\hat{n} \times \vec{H}^i)_o = 0 \quad . \tag{A14}$$

Let

$$\vec{R} = (y - y')\hat{y} + \vec{R}_\perp \quad . \tag{A15}$$

Then since

$$\overleftrightarrow{K}_{ee} + \overleftrightarrow{K}_{eo} + \overleftrightarrow{K}_{oe} + \overleftrightarrow{K}_{oo} = (\vec{R} \cdot \hat{n}) \overleftrightarrow{I} - \vec{R}\hat{n} \quad , \tag{A16}$$

$$\overleftrightarrow{K}_{ee} = (\vec{n}_\perp \cdot \vec{R}_\perp + yn_y) \overleftrightarrow{I} - \vec{R}\,\vec{n}_\perp - yn_y\hat{y}\hat{y} \quad ,$$

$$\overleftrightarrow{K}_{eo} = y'\hat{y}\vec{n}_\perp \quad ,$$

$$\overleftrightarrow{K}_{oe} = -n_y\vec{R}_\perp\hat{y} - y\hat{y}\vec{n}_\perp \quad ,$$

and

$$\overleftrightarrow{K}_{oo} = y'n_y\hat{y}\hat{y} - y'n_y\overleftrightarrow{I} \quad . \tag{A17}$$

If $\hat{e}_\perp$ is a unit vector in the x-z plane, Equations A17 and A7 can be combined to yield

$$\hat{e}_\perp \cdot \overleftrightarrow{\Gamma}_{ee} \cdot \hat{e}_\perp = \Psi_e[\vec{n}_\perp \cdot \vec{R}_\perp + yn_y - (\hat{e}_\perp \cdot \vec{R}_\perp)(\vec{n}_\perp \cdot \hat{e}_\perp)] - \Psi_o y'n_{y'} \quad ,$$

$$\hat{y} \cdot \overleftrightarrow{\Gamma}_{ee} \cdot \hat{y} = \Psi_e\, \vec{n}_\perp \cdot \vec{R}_\perp \quad ,$$

$$\hat{y} \cdot \overleftrightarrow{\Gamma}_{eo} \cdot \hat{e}_{\perp} = \Psi_e \, y'(\vec{n} \cdot \hat{e}_{\perp}) - \Psi_o \, y(\vec{n}_{\perp} \cdot \hat{e}_{\perp}) \quad ,$$

$$\hat{e}_{\perp} \cdot \overleftrightarrow{\Gamma}_{oe} \cdot \hat{y} = -\Psi_e \, n_y(\hat{e}_{\perp} \cdot \vec{R}_{\perp}) \quad ,$$

$$\hat{y} \cdot \overleftrightarrow{\Gamma}_{oe} \cdot \hat{e}_{\perp} = -\Psi_e \, y(\vec{n}_{\perp} \cdot \hat{e}_{\perp}) + \Psi_o \, y'(\vec{n}_{\perp} \cdot \hat{e}_{\perp}) \quad ,$$

and (A18)

$$\hat{e}_{\perp} \cdot \overleftrightarrow{\Gamma}_{oo} \, \hat{e}_{\perp} = -\Psi_e \, y'n_y + \Psi_o \left[\vec{n}_{\perp} \cdot \vec{R}_{\perp} + yn_y - (\hat{e}_{\perp} \cdot \vec{R}_{\perp})(\vec{n}_{\perp} \cdot \hat{e}_{\perp})\right] \quad ,$$

all other combinations being zero.

By combining Equations A18, A13 and A6 it is found that J_{xo}, J_{zo} and J_{ye} are solutions of

$$2\hat{y} \cdot (\vec{n}_{\perp} \times \vec{H}_{\perp}^{i}) = J_{ye} + \iint ds' \, [(\hat{y} \cdot \overleftrightarrow{\Gamma}_{ee} \cdot \hat{y})J_{ye} + \hat{y} \cdot \overleftrightarrow{\Gamma}_{eo} \cdot \vec{J}_{\perp o}]$$

and (A19)

$$2n_y\hat{e}_{\perp} \cdot (\hat{y} \times H_{\perp}^{i}) = \hat{e}_{\perp} \cdot \vec{J}_o + \iint ds' \, [(\hat{e}_{\perp} \cdot \overleftrightarrow{\Gamma}_{oe} \cdot \hat{y})J_{ye} + \hat{e}_{\perp} \cdot \overleftrightarrow{\Gamma}_{oo} \cdot \vec{J}_{\perp o}]$$

whereas J_{xe}, J_{ze} and J_{yo} are solutions of

$$2H_y^{i}\hat{e}_{\perp} \cdot (\vec{n}_{\perp} \times \hat{y}) = \hat{e}_{\perp} \cdot \vec{J}_e + \iint ds' \; \hat{e}_{\perp} \cdot \overleftrightarrow{\Gamma}_{ee} \cdot \vec{J}_{\perp e}$$

and (A20)

$$0 = J_{yo} + \iint ds' \; \hat{y} \cdot \overleftrightarrow{\Gamma}_{oe} \cdot \vec{J}_{\perp e} \quad .$$

For incident parallel polarization the incident electric field is polarized in the x-z plane and the incident magnetic field only has a y component. Therefore, $\vec{H}_{\perp}^{i} = 0$ and a solution to Equation A19 is

$$\vec{J}_{\perp o} = 0$$

and (A21)

$$J_{ye} = 0 \quad .$$

These solutions are unique except when $\omega = ck$ is a resonant frequency of the cavity formed by the bounding surface of the target. At resonance cavity mode currents may exist. However, since cavity modes are nonradiating, they can be ignored when primary interest is in the properties of far field scattering. Therefore,

J_x and J_z are even in y

and (A22)

J_y is odd in y

for parallel polarization.

For perpendicular polarization $H_y^{i} = 0$ and the solution to Equation A20 is

$$\vec{J}_{\perp e} = 0$$

and (A23)

$$J_{yo} = 0 \quad .$$

Therefore,

$$J_x \text{ and } J_z \text{ are odd in } y$$

and (A24)

$$J_y \text{ is even in } y$$

for perpendicular polarization.

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Circularly polarized bistatic scattering from plane-symmetric targets with otherwise arbitrary electrical properties exhibits reflection asymmetries and polarization reversal symmetries which are useful for bistatic radar data interpretation. In addition, the symmetries can be used to reduce the time and expense in obtaining bistatic scattering data from static range measurements and theoretical or computer calculations.

The reflection asymmetry and polarization reversal symmetry relations are derived in this note. The symmetry relations are then combined with the principle of electromagnetic reciprocity to determine the minimum number of measurements needed to completely characterize target scattering.

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