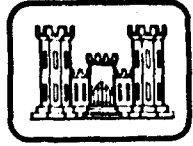


CRREL

12 B.S.



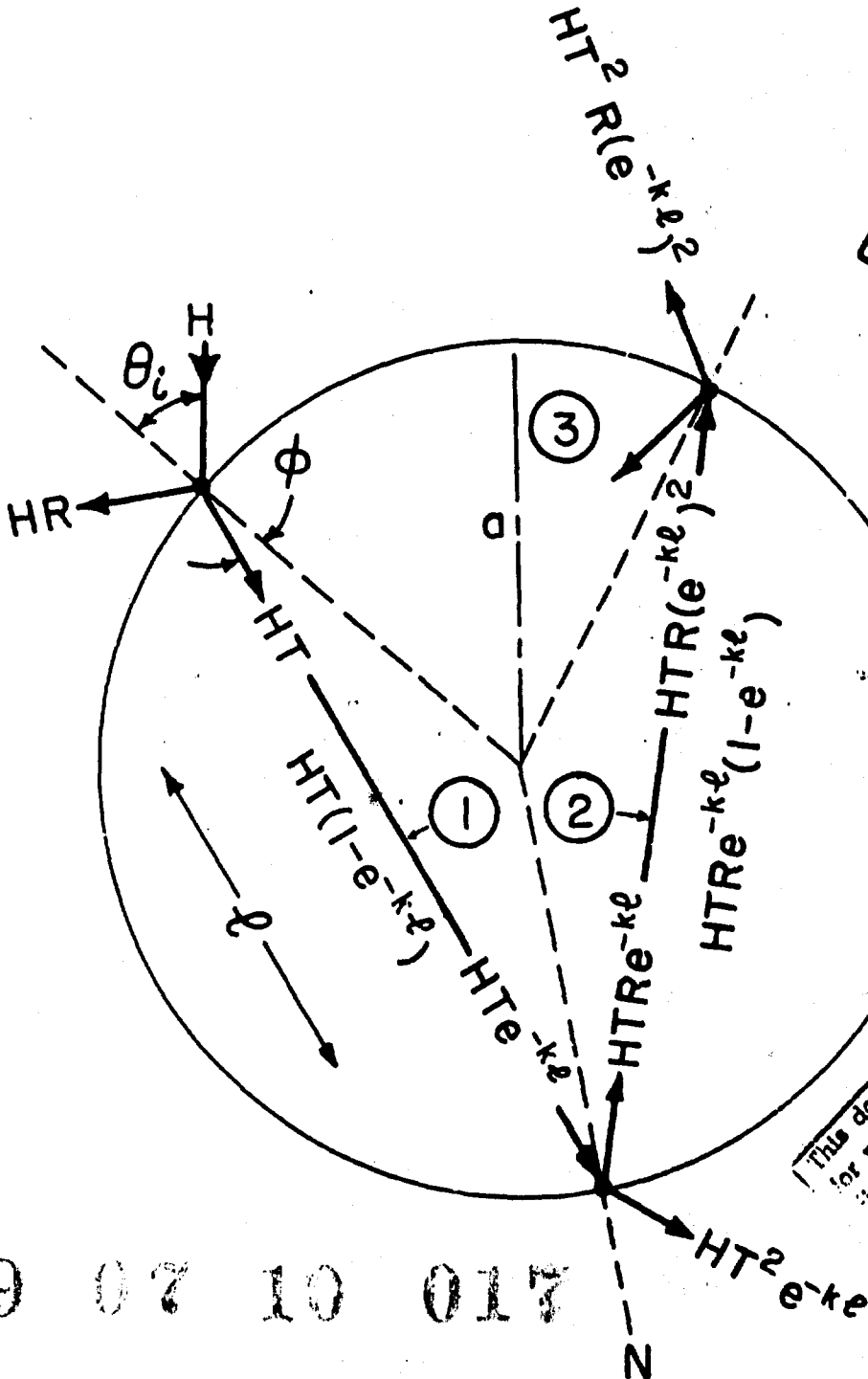
REPORT 79-11

LEVEL

ADA 071 004

nowpack optical properties in the infrared

D D C
DRA
JUL 10 1979
REGULATORY



This document has been approved for public release and sale; its distribution is unlimited.

DRC FILE COPY

79 07 10 017

For conversion of SI metric units to U.S./British customary units of measurement consult ASTM Standard E380, Metric Practice Guide, published by the American Society for Testing and Materials, 1916 Race St., Philadelphia, Pa. 19103.

14

CRREL Report-79-11



6

Snowpack optical properties in the infrared

10

Roger H. Berger

11

May 1979

12 16p.

16 4A 76273 ϕ AT 42

17 A 1

Prepared for
DIRECTORATE OF MILITARY PROGRAMS
OFFICE, CHIEF OF ENGINEERS
By
UNITED STATES ARMY
CORPS OF ENGINEERS
COLD REGIONS RESEARCH AND ENGINEERING LABORATORY
HANOVER, NEW HAMPSHIRE, U.S.A.

Approved for public release; distribution unlimited.

$\phi 37 100$

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER CRREL Report 79-11	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) SNOWPACK OPTICAL PROPERTIES IN THE INFRARED	5. TYPE OF REPORT & PERIOD COVERED	
	6. PERFORMING ORG. REPORT NUMBER	
7. AUTHOR(s) Roger H. Berger	8. CONTRACT OR GRANT NUMBER(s)	
9. PERFORMING ORGANIZATION NAME AND ADDRESS U.S. Army Cold Regions Research and Engineering Laboratory Hanover, New Hampshire 03755	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS DA Project 4A762730AT42 Technical Area A1, Work Unit 004	
11. CONTROLLING OFFICE NAME AND ADDRESS Directorate of Military Programs Office, Chief of Engineers Washington, D.C. 20314	12. REPORT DATE May 1979	
	13. NUMBER OF PAGES 12	
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	15. SECURITY CLASS. (of this report) Unclassified	
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE	
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Infrared radiation Optical properties Reflection Snow		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) A theory of the optical properties of snow in the 2-20 μm region of the infrared has been developed. Using this theory it is possible to predict the absorption and scattering coefficients and the emissivity of snow, as a function of the snow parameters of grain size and density, for densities between 0.17 and 0.4 g/cm^3 . The absorption and scattering coefficients are linearly related to the density and inversely related to the average grain size. The emissivity is independent of grain size and exhibits only a weak dependence upon density.		

micrometers

cu. cm

DD FORM 1 JAN 73 1473

EDITION OF 1 NOV 65 IS OBSOLETE

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

PREFACE

This report was prepared by Roger H. Berger, Research Physicist, of the Physical Sciences Branch, Research Division, U.S. Army Cold Regions Research and Engineering Laboratory. Funding for this project was provided by DA Project 4A762730AT42, *Design, Construction and Operations Technology for Cold Regions*, Technical area A1, *Ice and Snow Technology*, Work Unit 004, *Camouflage and Decoy Techniques in Winter Environments*.

The author gratefully acknowledges the interest and suggestions of Dr. Y.C. Yen and Dr. S.C. Colbeck, who technically reviewed the manuscript of this report.

Accession For	
NTIS GFA&I	<input checked="" type="checkbox"/>
DESC EAB	<input type="checkbox"/>
Unannounced	<input type="checkbox"/>
Justification	<input type="checkbox"/>
By	
Distribution/	
Availability Codes	
Dist	Avail and/or special

NOMENCLATURE

a	particle radius
C	single particle cross section
E	energy
f	fraction of the snowpack surface occupied by ice particles
H	irradiance
K	absorption coefficient
N	number of particles/unit volume
n	refractive index
Q	factor used to convert the transmission coefficient to the transmissivity and assure the conservation of radiant energy at an interface
R	reflectivity
T	transmissivity
X	extinction coefficient
γ	radiation phase change produced by transmission across a boundary
ϵ	emissivity
η	radiation phase change produced by reflection at a boundary
θ	angle of incidence or coaltitude
λ	wavelength
ρ	density
σ	scattering coefficient
\bar{r}	skin depth
ϕ	angle of refraction
ψ	azimuth angle

Subscripts

c	cavity, denotes the emissivity associated with the pores on the snowpack surface
i	ice, incident and imaginary used as ρ_i , θ_i and n_i , respectively
r	real
s	snow, denotes properties associated with the bulk snowpack
ab	absorption
ext	extinction
sc	scattering
tr	transmitted

SNOWPACK OPTICAL PROPERTIES IN THE INFRARED

Roger H. Berger

INTRODUCTION

The interaction of electromagnetic radiation with a natural snow cover is of considerable practical interest and scientific importance. The remote sensing community requires an improved understanding of the optical properties of snow in order to correctly interpret the imagery of snow-covered regions and to distinguish between snowpacks and cloud covers. Radiant energy transfer is important in the thermodynamics and metamorphosis of the snowpack and is therefore of interest to those who study the mechanical and hydrological properties of the snowpack. Therefore, the establishment of relationships between the optical properties of snow and frequently measured snowpack quantities, such as grain size and density, would be of considerable utility. A model is proposed in this report for estimating the infrared absorption, reflection

and transmission coefficients and the emissivity of a snowpack from its mean grain size and density.

As shown in Figure 1, the infrared region of the electromagnetic spectrum extends from about 0.8 to 1000 μm , but absorption by various atmospheric constituents limits most infrared remote sensing to two atmospheric windows at 3-5 and 8-14 μm . The theory of the infrared optical properties of snow developed in this report is valid for wavelengths between 2 and 20 μm , the region most useful for remote sensing.

THE MODEL

Radiation incident upon the snowpack surface undergoes either single or multiple interactions as illustrated in Figure 2. These multiple interactions have been treated by Dunkle and Gier (1955) and Bergen (1970)

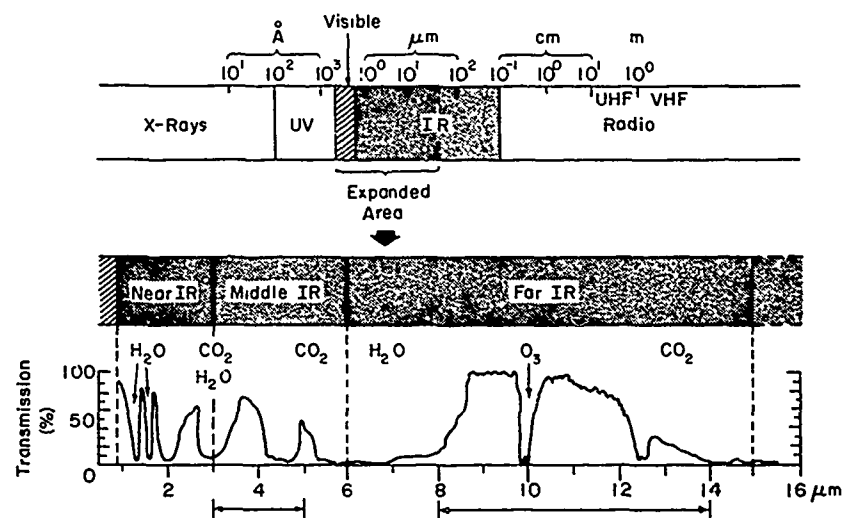


Figure 1. The electromagnetic spectrum.

a. The electromagnetic spectrum.

b. Designations within the infrared region of the electromagnetic spectrum.

c. Atmospheric transmission in the infrared showing the principal atmospheric constituents causing absorption. The horizontal arrows indicate the atmospheric 'windows' through which most remote sensing in the infrared is done.

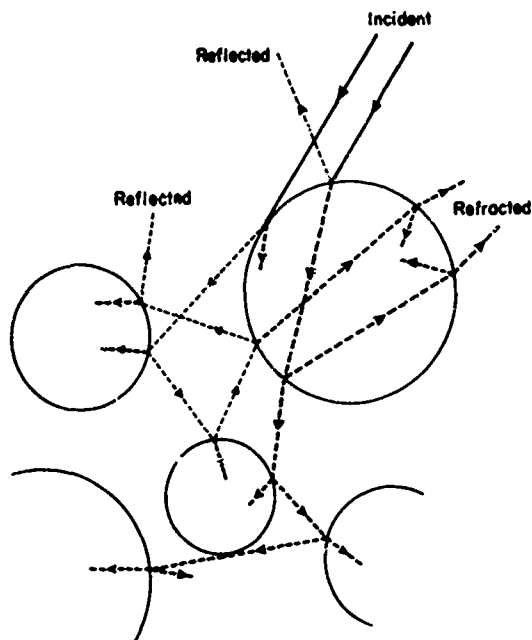


Figure 2. Interaction between electromagnetic radiation and the surface layer of ice particles in a snowpack. The solid lines represent the incident radiation, the dotted lines represent radiation that is specularly reflected either singly or multiply, and the bold dashed lines represent refracted and absorbed radiation.

and Giddings and LaChapelle (1961) as a radiative transfer or diffusion problem in a continuum. If the work of these authors is extended from the visible to infrared wavelengths, problems arise due to the large absorption coefficient of ice in the infrared.

A much simpler approach to this scattering problem and one which is easily adaptable to the longer infrared wavelengths is to look at the problem in terms of geometrical optics (Bohren and Barkstrom 1974). A rigorous treatment of the snowpack model proposed in this report would require Mie scattering theory, but the arbitrary shapes of the actual ice grains within the snowpack make this refinement unnecessary.

SNOWPACK DESCRIPTION

The snowpack model proposed is a random distribution of spherical ice grains immersed in a nonabsorbing medium. Below the plane which defines the surface of the snowpack, the grain distribution is homogeneous and isotropic and may be characterized by the mean grain radius a , the density of ice ρ_i and the snow density ρ_s . Since the density of ice is a slowly varying function of the temperature, ρ_i can be considered a constant. For geometrical optics to be

applicable to this model two conditions have to be satisfied. First, the particles must be large compared to the wavelength of the radiation. Second, the intergranular spacing must be large compared to the average grain dimensions. The first condition is easily satisfied, since for most snows, the mean grain diameter is more than an order of magnitude larger than $20 \mu\text{m}$, the maximum wavelength considered here. The second implies that the geometrical optics approach is valid only for low-density snow. The range of snow densities for which this assumption is valid will be examined more closely in relation to the results of the theory.

RADIANT INTERACTIONS

If we consider a single grain within the snowpack, it emits radiation isotropically and is immersed in the flux of radiation from the surrounding grains. Let us consider this external radiation flux as a plane wave and examine the interaction between it and an ice particle. This incident radiation of intensity H will either be absorbed or scattered and these processes can be described in terms of the single particle cross sections C_{ab} for absorption and C_{sc} for scattering. These cross sections are defined by the ratios of the energy absorbed or scattered to the incident flux:

$$C_{ab} \equiv E_{ab}/H \quad \text{for absorption} \quad (1a)$$

and

$$C_{sc} \equiv E_{sc}/H \quad \text{for scattering.} \quad (1b)$$

By the law of conservation of energy these cross sections are related to the extinction cross section:

$$C_{ext} = C_{ab} + C_{sc}. \quad (2)$$

With the initial assumption of a random distribution of grains spaced relatively far apart, these cross sections may be related to the bulk material absorption, scattering and extinction coefficients by

$$K_s = C_{ab}N \quad (3a)$$

$$\sigma_s = C_{sc}N \quad (3b)$$

$$X_s = C_{ext}N. \quad (3c)$$

The assumptions of large grain separation and random distribution are necessary so that the contributions from the individual grains may be added without considering phase coherence. The number of grains per

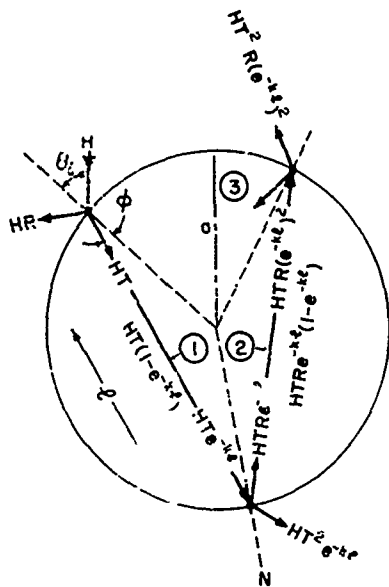
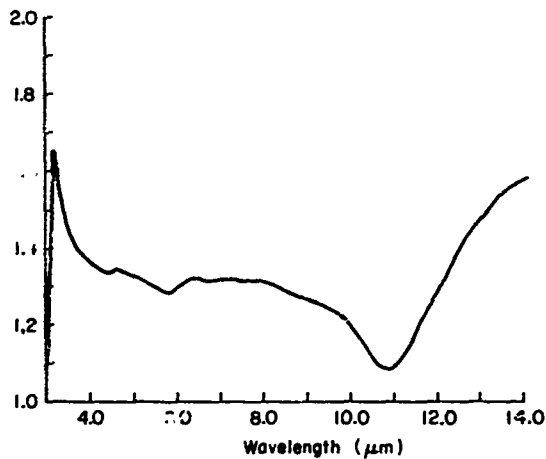
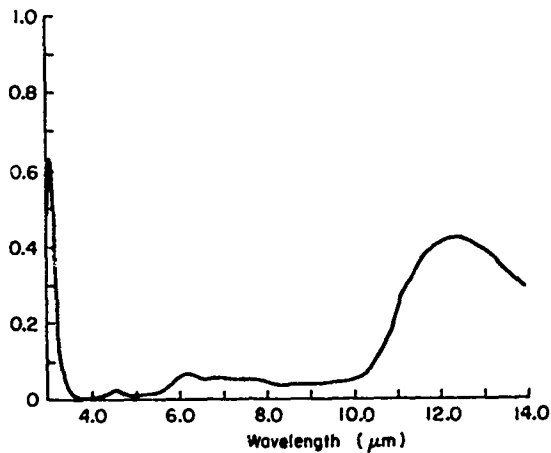


Figure 3. Ray paths illustrating how the incident radiation H interacts with an ice particle. The single particle absorption cross section is calculated by summing the energies $[HT(1-e^{-K\ell}), HTRe^{-K\ell}(1-e^{-K\ell}), \dots]$ which are absorbed along successive ray paths $[1, 2, \dots]$ of length ℓ within the sphere. The scattering cross section is calculated by summing the initially reflected energy HR and the transmitted energies $[HT^2 e^{-K\ell}, HT^2 R(e^{-K\ell})^2, \dots]$.



c. Real part.



b. Imaginary part.

Figure 4. Wavelength dependence of the complex refractive index of bulk ice. (Adapted from the measurements of Schaaf and Williams 1973).

volume N may be expressed as a function of the snow density and grain size:

$$N = 3\rho_s / 4\pi a^3 \rho_i \quad (4)$$

Therefore the absorption, scattering and extinction coefficients of the snow can be expressed as functions of the snow density and grain size if the single particle cross sections for these processes can be calculated.

CALCULATION OF CROSS SECTIONS

For calculation of the cross sections, consider a plane wave incident upon a spherical ice grain as shown

in Figure 3 and on the cover. The sphere is immersed in air with a refractive index of 1, and the ice has a complex refractive index, $\hat{n} = n_r - in_i$. The dependence of the refractive index upon wavelength is illustrated in Figure 4. In the succeeding development this wavelength dependence is implicit, so that all quantities depending upon the refractive index should be considered spectral quantities. The fractions of the incident energy reflected and transmitted at each air/ice interface are given by the Fresnel coefficients R and T respectively. The irradiance at a distance s along the first ray path inside the sphere (path 1, Figure 3) is

$$H_{1(s)} = HT e^{-Ks} \quad (5)$$

Therefore the total energy adsorbed by the sphere along path 1 of length l is

$$H_{1ab} = HT(1 - e^{-Kl}). \quad (6)$$

Similarly for path 2 (see Fig. 3)

$$H_{2ab} = (HT e^{-Kl})R(1 - e^{-Kl}) \quad (7)$$

and the total for all internal transits is

$$H_{ab} = \sum_{j=1}^{\infty} H_{j,ab} = HT(1 - e^{-Kl}) \sum_{j=0}^{\infty} [R e^{-Kl}]^j \quad (8)$$

where the sum is the addition of the effects of each transit. For the total energy absorbed by the sphere from the incident plane wave, this expression is integrated over all angles of incidence θ_i .

$$E_{ab} = \int_0^{\pi/2} d\theta_i \int_0^{2\pi} d\psi \sin\theta_i \cos\theta_i a^2 HT(1 - e^{-Kl}) \sum_{j=0}^{\infty} [R e^{-Kl}]^j \quad (9)$$

where ψ is the azimuthal angle.

The total radiation scattered by the sphere may be expressed similarly:

$$E_{sc} = \int_0^{\pi/2} d\theta_i \int_0^{2\pi} d\psi \sin\theta_i \cos\theta_i a^2 H [R + R e^{-Kl} \sum_{j=0}^{\infty} (R e^{-Kl})^j] \quad (10)$$

where the first term is due to the reflection of the incident beam and the other terms are due to the radiation that is transmitted after 1, 2, 3 ... transits of the sphere.

The reflectivity and transmissivity are derived from the Fresnel formulae (Born and Wolf 1965):

$$r_{\perp} = R_{\perp}^{1/2} e^{i\eta_{\perp}} = (\cos\theta - \hat{n}\cos\hat{\phi}) / (\cos\theta + \hat{n}\cos\hat{\phi}) \quad (11a)$$

$$t_{\perp} = T_{\perp}^{1/2} e^{i\gamma_{\perp}} = 2\cos\theta / (\cos\theta + \hat{n}\cos\hat{\phi}) \quad (11b)$$

$$r_{\parallel} = R_{\parallel}^{1/2} e^{i\eta_{\parallel}} = (\hat{n}\cos\theta - \cos\hat{\phi}) / (\hat{n}\cos\theta + \cos\hat{\phi}) \quad (11c)$$

$$t_{\parallel} = T_{\parallel}^{1/2} e^{i\gamma_{\parallel}} = 2\cos\theta / (\hat{n}\cos\theta + \cos\hat{\phi}) \quad (11d)$$

where the symbols \perp and \parallel refer respectively to the orientation of the electric vector perpendicular and parallel to the plane of incidence. In these formulae both n and $\hat{\phi}$ are complex, since they are related by Snell's law:

$$\sin\theta = \hat{n}\sin\hat{\phi}. \quad (12)$$

Substituting $\hat{n}\cos\hat{\phi} = u - iv$ and $\hat{n} = n_r - in_i$ into the Fresnel formulae and solving for the reflectivity and transmissivity components yields

$$R_{\perp} = [(\cos\theta - u)^2 + v^2] / [(\cos\theta + u)^2 + v^2] \quad (13a)$$

$$T_{\perp} = 4\cos^2\theta / [(\cos\theta + u)^2 + v^2] \quad (13b)$$

$$R_{\parallel} = \quad (13c)$$

$$\frac{(n_r^2 + n_i^2)^2 \cos^2\theta - 2[u(n_r^2 - n_i^2) + 2vn_r n_i] \cos\theta + u^2 + v^2}{(n_r^2 + n_i^2)^2 \cos^2\theta + 2[u(n_r^2 - n_i^2) + 2vn_r n_i] \cos\theta + u^2 + v^2}$$

$$T_{\parallel} = 4\cos^2\theta / \{ (n_r^2 + n_i^2)^2 \cos^2\theta + 2[u(n_r^2 - n_i^2) + 2vn_r n_i] \cos\theta + u^2 + v^2 \}. \quad (13d)$$

In order to satisfy the law of conservation of energy, at the boundary of the ice particle, the fraction of the incident energy which is reflected and the fraction transmitted must add up to unity. This condition, which must be satisfied by each polarization component, is achieved by the addition of factors Q_{\perp} and Q_{\parallel} :

$$R_{\perp} + Q_{\perp} T_{\perp} = 1 \text{ and } R_{\parallel} + Q_{\parallel} T_{\parallel} = 1 \quad (14)$$

where

$$Q_{\perp} = u / \cos\theta \quad (15a)$$

and

$$Q_{\parallel} = [u(n_r^2 - n_i^2) + 2vn_r n_i] / \cos\theta. \quad (15b)$$

For circularly polarized light, the polarization components may be combined to form the total transmissivity and reflectivity:

$$T = 1/2(Q_{\perp} T_{\perp} + Q_{\parallel} T_{\parallel}) \text{ and } R = 1/2(R_{\perp} + R_{\parallel}). \quad (16)$$

Combining $\hat{n} \cos \hat{\phi} = u - iv$ with Snell's law and solving for u and v yield:

$$u = 707 \left[n_r^2 - n_i^2 - \sin^2 \theta + \sqrt{(n_r^2 - n_i^2 - \sin^2 \theta)^2 + 4n_r^2 n_i^2} \right]^{1/2} \quad (17a)$$

$$v = 0.707 \left[-(n_r^2 - n_i^2 - \sin^2 \theta) + \sqrt{(n_r^2 - n_i^2 - \sin^2 \theta)^2 + 4n_r^2 n_i^2} \right]^{1/2} \quad (17b)$$

It will be noted that neither eq 13 nor 17 contain any dependence upon the integration variable ψ , so that eq 9 and 10 may be integrated. These integrals may then be simplified further by examining the exponential terms. The absorption coefficient of ice in the 3- to 100- μm wavelength range is between 100 and 14,000 cm^{-1} (Irvine and Pollock 1968, Schaaf and Williams 1973), and eq 9 and 10 become

$$E_{ab} = 2\pi a^2 H \int_0^{\pi/2} T(\theta, n_r, n_i) \sin \theta \cos \theta d\theta \quad (18)$$

and

$$E_{sc} = 2\pi a^2 H \int_0^{\pi/2} R(\theta, n_r, n_i) \sin \theta \cos \theta d\theta. \quad (19)$$

Equation 18 gives the total energy absorbed by a single ice particle of radius a , immersed in a radiation field of incident radiant intensity H . The remaining energy in the incident field is scattered by reflection with the energy distribution strongly peaked in the forward direction as shown in Figure 5.

The single particle cross sections for absorption and scattering can now be found by combining eq 1, 18 and 19.

RESULTS

As a result of the large absorption coefficient, all the scattering of the radiation incident upon the particle is due to reflection. Combining eq 1a, 3a, 4 and 18 yields the absorption coefficient for the ensemble of ice particles:

$$K_s = 1.5\rho_s/a\rho_i \int_0^{\pi/2} T(\theta, n_r, n_i) \sin \theta \cos \theta d\theta. \quad (20)$$

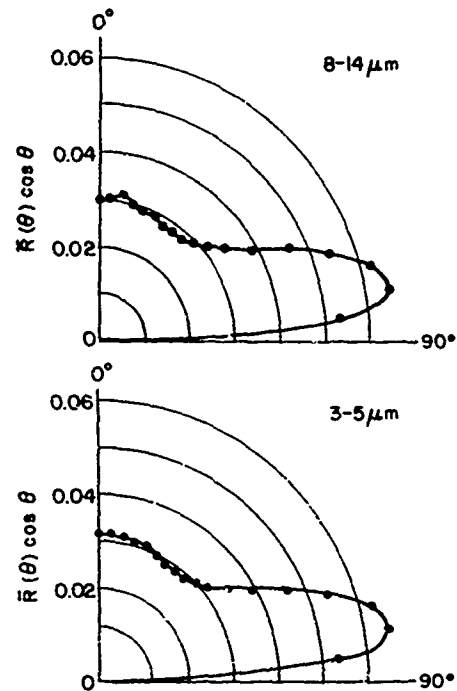


Figure 5. The reflected energy distribution as a function of the angle of incidence. The energy has been averaged over the wavelength interval for each of the atmospheric windows.

This absorption coefficient is dependent upon the ratio of the snow density to the particle size and wavelength. This wavelength dependence is illustrated in Figure 6 where the quantity $K_s a / \rho_s$ is plotted as a function of wavelength. Within the 3-5- and 8-14- μm wavelength bands for typical snowpack values for the grain size and density, the absorption coefficient is in the range of 2.6 to 12.7 cm^{-1} . Under the same circumstances the scattering due to reflection has a calculated range of 0.2 to 0.8 cm^{-1} .

In addition to the absorption and reflection due to the refractive index of the particles, the diffraction effects must be included when considering forward scattering. All the energy which is incident upon the particle is removed from the incident wave either by absorption or reflection, giving an effective cross section equal to the projected area of the sphere, πa^2 . By Babinet's principle the cross section for light diffracted by a large particle, i.e. $1 \ll x < 2\pi a / \lambda$, is equal to its projected area. Therefore the total extinction cross section is twice the geometrical cross section:

$$C_{ext} = 2\pi a^2. \quad (21)$$

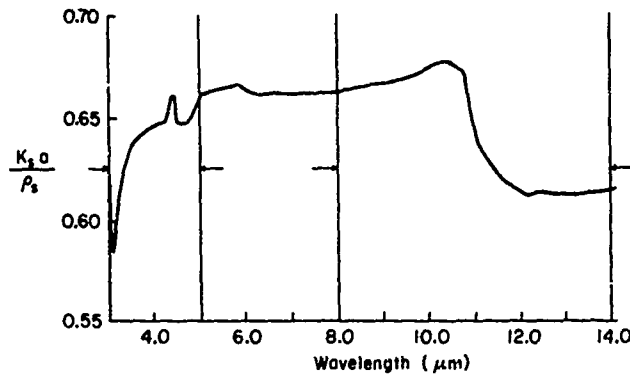


Figure 6. The normalized absorption coefficient as a function of wavelength. The arrows indicate the two atmospheric windows, 3-5 and 8-14 μm .

The extinction coefficient for snow is

$$X_s = K_s + \sigma_{ts} \quad (22)$$

where the subscript ts of the scattering coefficient σ denotes total scattering effects of the particle. Following the notation of Bohren and Barkstrom (1974) an effective extinction coefficient may be defined by

$$X_s^* = K_s + \sigma_r + \sigma_t \quad (23)$$

where the r and t stand for the reflected and transmitted parts of the scattering coefficients. Since in the derivation of eq 19 from eq 10 it was seen that the transmitted radiation was negligible, σ_t is essentially zero. From Babinet's principle and eq 3c the effective extinction coefficient is

$$X_s^* = 0.75\rho_s/\rho_i \quad (24)$$

and by writing the scattering and absorption coefficient in terms of this parameter we have

$$\sigma_r = 0.013X_s^* \text{ and } K_s = 0.99X_s^* \quad (25)$$

which shows that 99% of the effective extinction is due to absorption.

Defining an extinction and an absorption depth as the reciprocal of the coefficient, these depths are respectively

$$\tau_{\text{ext}} = 50 \text{ mm and } \tau K_s = 3.8 \text{ mm.}$$

These values are calculated for a typical aged snowpack with density of 350 kg/m^3 , mean crystal size of 2 mm and ice density of 917 kg/m^3 .

EMISSIVITY CALCULATION

This absorption length is of the order of a few particle diameters and indicates that no significant amount of radiation at these wavelengths penetrates beyond the first few ice particles. The snowpack emissivity can therefore be found by considering only the radiative interaction with these surface layers. To account for all the radiation that is absorbed within the surface layers, three absorption processes have to be considered. These processes are 1) absorption at initial incidence with the ice particles, 2) absorption after one or more reflections within the snowpack, and 3) cavity absorption in the pore spaces within the snowpack. These processes are expressed by the first two terms, the third and fourth terms, and the last term, respectively, in the following equation which is derived using eq 18 and 19:

$$\begin{aligned} \epsilon = & 2\pi a^2 HfA \int_0^{\pi/2} T(\theta, n_r, n_i) \sin\theta \cos\theta d\theta \\ & + 2\pi r^2 HPB \int_0^{\sin^{-1}(r/a)} T(\theta, n_r, n_i) \sin\theta \cos\theta d\theta \\ & + 2\pi a^2 HfA \int_{\theta(\rho)}^{\pi/2} R(\theta, n_r, n_i) \sin\theta \cos\theta d\theta \\ & + 2\pi r^2 HPB \int_{\theta'(\rho)}^{\sin^{-1}(r/a)} R(\theta, n_r, n_i) \sin\theta \cos\theta d\theta \\ & + \epsilon_c [1 - \pi N^{-2/3} (a^2 + r^2)] \quad (26) \end{aligned}$$

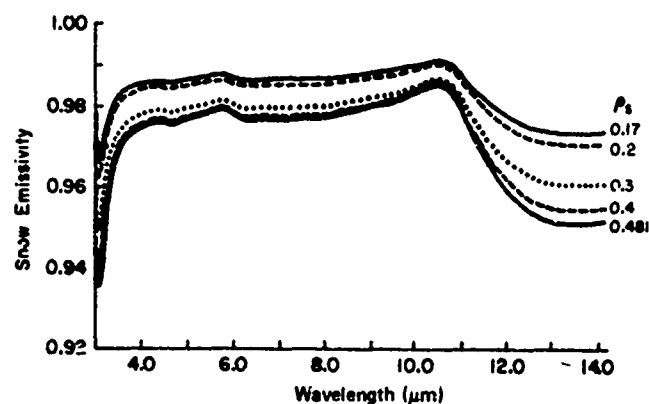


Figure 7. The spectral emissivity of snow illustrating the dependence upon the density, ρ_s in g/cm^3 .

where

$$1/A = 2\pi a^2 H \int_0^{\pi/2} [R(\theta, n_r, n_i) + T(\theta, n_r, n_i)]$$

$$\sin\theta \cos\theta d\theta$$

and

$$1/B = 2\pi r^2 H \int_0^{\sin^{-1}(r/a)} [R(\theta, n_r, n_i) + T(\theta, n_r, n_i)]$$

$$\sin\theta \cos\theta d\theta$$

and where f is the fraction of the surface area occupied by the projection of the surface layer of particles, and P is the fraction of the surface area occupied by the portion of the projection of the subsurface particles which is not obscured by the surface layer of particles. The two functions $\theta(\rho)$ and $\theta'(\rho)$ define the minimum angle of incidence at which total absorption takes place within the snowpack structure. ϵ_c is the apparent emissivity of a pore which opens onto the surface of the snowpack.

In order to evaluate these integrals it is necessary to make a further assumption about the snowpack. This assumption is that the average properties of the snowpack may be approximated by a regular array of particles. Since most of the radiation incident upon an individual ice particle is absorbed and the absorption length is short, the effects of phase coherence may be neglected. Under these conditions $\theta(\rho)$ is a nearly linear function of density and varies between 50.33° and 51.36° over the density range of 170 to 481 kg/m^3 . The second function $\theta'(\rho)$ is a constant

22.5° over this range. The two projected areas as functions of the density become

$$f = (M\rho_s)^{2/3} \quad (27)$$

$$P = \pi [1/2 - \sqrt{2} (M\rho_s)^{1/3} + (M\rho_s)^{2/3}] \quad (28)$$

where $M = 3/4\pi\rho_i$. Substituting eq 27 and 28 into eq 26, the expression for the emissivity becomes

$$\begin{aligned} \epsilon = & \pi(M\rho_s)^{2/3} \left\{ 1 - \sin^2\theta(\rho) + [1/\sqrt{2} - (M\rho_s)^{1/3}]^2 / \right. \\ & \left. (M\rho_s)^{2/3} - \sin^2\rho/8 \right\} \\ & + 2\pi(M\rho_s)^{2/3} \left[\int_0^{\pi/8} T(\theta, n_r, n_i) \sin\theta \cos\theta d\theta \right. \\ & \left. + \int_0^{\theta(\rho)} T(\theta, n_r, n_i) \sin\theta \cos\theta d\theta \right] \\ & + \left\{ 1 - \pi [1/2 - \sqrt{2} (M\rho_s)^{1/3} + 2(M\rho_s)^{2/3}] \right\} \epsilon_c. \end{aligned} \quad (29)$$

The pore emissivity ϵ_c is 0.95, based on the calculations of Chandos and Chandos (1974). The evaluation of eq 29 for the 3-14- μm spectral region is shown in Figure 7. It can be seen from this figure that the dependence of emissivity upon the snow density is very weak. Likewise, the emissivity is not a function of the mean grain size, a .

There have been very few emissivity measurements on snow in the 2-14- μm wavelength range and there are no published results known to this author which report its spectral emission or reflectance and characterize

the snow by grain size and density. Values of the emissivity appearing in the literature range between 0.81 to 0.99, with the higher value being most common, but generally no information is included on the spectral bandwidth or the snow properties. The most complete documentation is furnished by Dunkle and Gier (1955) who measured the emissivity of two natural and two artificial snows and obtained values of 0.82, 0.89, 0.81 and 0.95. The lower values apply to the finer-grained snow of each variety. The difference in emissivity between the fine- and coarse-grained snow was attributed to either particle size or surface roughness, but the change in density might have also been the cause since no density measurements were made. These results are not sufficiently detailed for a definitive comparison with the theory developed in this report.

For a definitive test of this theory it is necessary to have the snow samples well documented so that the effect of grain size and density variations may be known. The wavelength band of the measurement must also be limited to the region of validity of the theory. This is important because inclusion of radiation of wavelengths shorter than $2.5 \mu\text{m}$ will reduce the measured emissivity since reflectance measurements in this region indicate that the emissivity decreases rapidly. A series of emissivity measurements which can be compared with this theory is in progress. Preliminary results indicate, in agreement with the theory, that the emissivity does not depend upon the grain size, but the dependence on density is not clearly defined. These results will be presented in a subsequent report.

DISCUSSION OF RESULTS

The theory used to derive the optical constants of a snowpack contains no adjustable parameters and is based only upon the refractive index of pure bulk ice and the density and average grain size of the snowpack. In order to evaluate the validity of the results derived above, the implications of the initial assumption of spherical grains and large interparticle spacing must be examined.

If an arbitrarily shaped particle is compared with a spherical particle of the same volume, the sphere has a smaller surface area. Chylek (1977) has shown that the average geometrical cross section of the nonspherical particle is always larger than that of the corresponding spherical particle. It is uncertain what the net effect of this on the emissivity would be, since the last term in eq 29 would be decreased while the other terms would be increased. Since the absorption coef-

ficient of ice is large, the absorption length within an ice particle is much smaller than the particle dimensions and therefore the absorption coefficient is independent of the particle shape.

The limitations of the theory due to the assumption of large interparticle spacing are more difficult to assess. The development of the bulk material constants from the particle interaction cross sections is dependent upon the particle spacing being large enough so that phase coherence effects may be neglected. Under these conditions the contribution of each particle may be added to get the scattered field of the ensemble. The random distribution of the particle sizes (Bohren and Barkstrom 1974, Hodkinson and Greenleaves 1962) and the random shape and orientation (Bohren and Barkstrom 1974) of the particles within the snowpack may average out the phase coherence effects as the density of the snow increases. Experiments with pigments (Blevin and Brown 1961) show that the reflectance of an ensemble is independent of particle concentration over a wide range and then usually decreases at high concentrations, indicating that the theory developed for snow may be valid for densities up to 400 kg/m^3 .

Therefore the absorption and scattering coefficients and the emissivity values calculated using the theory developed here should be useful for densities between 170 and 400 kg/m^3 . The theory also shows that the absorption and scattering coefficients are linearly related to the snowpack density and inversely related to the grain size. The emissivity is independent of grain size and exhibits only a weak dependence upon the snowpack density.

LITERATURE CITED

- Bergen, J.D. (1970) A possible relation between grain size, density and light attenuation in natural snow cover. *Journal of Glaciology*, vol. 9, no. 55, p. 154-156.
- Blevin, W.R. and W.J. Brown (1961) Effect of particle separation on the reflectance of semi-infinite diffusers. *Journal of the Optical Society of America*, vol. 51, no. 2, p. 129-134.
- Bohren, C.F. and B.R. Barkstrom (1974) Theory of the optical properties of snow. *Journal of Geophysical Research*, vol. 79, no. 30, p. 4527-4535.
- Born, M. and E. Wolf (1965) *Principles of optics*. New York: Pergamon Press.
- Chandos, R.J. and R.E. Chandos (1974) Radiometric properties of isothermal, diffuse wall cavity sources. *Applied Optics*, vol. 13, no. 9, p. 2142-2152.
- Chylek, P. (1977) Extinction cross sections of arbitrarily shaped randomly oriented nonspherical particles. *Journal of the Optical Society of America*, vol. 67, no. 10, p. 1348-1350.
- Dunkle, R.V. and J.T. Gier (1955) Snow characteristics project: Final progress report. University of California, Institute of Engineering Research, Berkeley, California, series 63, issue 5.

- Giddings, J.C. and E. LaChapelle (1961) Diffusion theory applied to radiant energy distribution and albedo of snow. *Journal of Geophysical Research*, vol. 66, no. 1, p. 181-189.
- Hodkinson, J.R. and J. Greenleaves (1962) Computations of light-scattering and extinction by spheres according to diffraction and geometrical optics, and some comparisons with the Mie theory. *Journal of the Optical Society of America*, vol. 53, no. 5, p. 577-586.
- Irvine, W.M. and J.B. Pollock (1968) Infrared optical properties of water and ice spheres. *Icarus*, vol. 8, p. 324-360.
- Schaaf, J.W. and D. Williams (1973) Optical constants of ice in the infrared. *Journal of the Optical Society of America*, vol. 8, no. 6, p. 726-732.
- Van de Hulst, H.C. (1957) *Light scattering by small particles*. New York: John Wiley and Sons.

A facsimile catalog card in Library of Congress MARC format is reproduced below.

Berger, Roger H.

Snowpack optical properties in the infrared / by Roger H. Berger. Hanover, N.H.: U.S. Cold Regions Research and Engineering Laboratory; Springfield, Va.: available from National Technical Information Service, 1979.

iii, 12 p., illus., 27 cm. (CRREL Report 79-11.)
Prepared for Military Engineering Division, Office, Chief of Engineers by Corps of Engineers, U.S. Army Cold Regions Research and Engineering Laboratory, under DA Project 4A762630AT42.

Bibliography: p. 8.

1. Infrared radiation. 2. Optical properties.
3. Reflection. 4. Snow. I. United States. Army. Corps of Engineers. II. Cold Regions Research and Engineering Laboratory, Hanover, N.H. III. Series: CRREL Report 79-11.

U.S. GOVERNMENT PRINTING OFFICE: 1979-601-520/91