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A LITERATURE SEARCH & REVIEW OF THE DYNAMICS OF AIRCRAFT-SURFACE INTERACTION

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This report presents a literature survey and review of the research concerned with the dynamics of aircraft-surface interaction. Of primary concern is the determination of surface roughness criteria based on how rough the surface can be without inducing structural damage in the aircraft. Attention is concentrated on analytical research in the form of computer simulation that deals with the aircraft response as a function of the input (external excitation), vehicle model, and the output. Basic conclusions include the fact that no

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20. ABSTRACT (CONCLUDED)

current computer code is satisfactory but that one could be formulated within the state-of-the-art to correlate with on-going actual aircraft tests. To facilitate correlation the roll, yaw, and lateral degrees of freedom should be included and a fatigue analysis for the test aircraft should be made.

PREFACE

This report was prepared by the Department of Civil Engineering, Engineering Mechanics and Materials, United States Air Force Academy, Colorado 80840 for the Civil and Environmental Engineering Development Office (CEEDO), Detachment 1, Armament Development and Test Center, Tyndall Air Force Base, Florida 32403. This work was accomplished during the period from 1 October 1977 to 31 May 1978.

Effective 1 March 1979 CEEDO was inactivated and became the Engineering and Services Laboratory (ESL) a directorate of the Air Force Engineering and Services Center located on Tyndall AFB, Florida 32403.

This report has been reviewed by the Information Office (OI) and is releasable to the National Technical Information Service (NTIS). At NTIS it will be available to the general public, including foreign nations.

This technical report has been reviewed and is approved for publication.

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TABLE OF CONTENTS

LIST OF FIGURES

SECTION 1

INTRODUCTION

Recent studies and analyses have established the need to improve rapid runway repair capability. Air Force efforts to solve the launch and recovery surface problem are described in a recent program management plan (Reference 1). The problem is of such complexity that the program is divided into four technical areas. One of these areas has to do with surface roughness. Here the object is to determine how rough the launch and recovery surface can be without incurring structural damage to the aircraft or its external stores.

The determination of how rough the surface can be implies the generation of a detailed set of criteria. The importance of resonable criteria is evidenced by the fact that the rougher the allowable operational surface, the less time it takes to repair it. In the long term, perhaps the criteria will not be so much runway oriented as aircraft oriented. That is, it is quite possible that new landing gear design criteria are needed.

Good judgment and financial considerations dictate that surface roughness criteria be obtained, to the maximum extent possible, by mathematical modeling, simulation, and associated analyses. This, in turn, leads to the development of a versatile computer code or codes. The use of a code allows flexibility, makes parameter studies possible, and significantly reduces costs.

Codes and simulations cannot, of course, stand alone. A validation of results is needed to obtain a feeling of confidence that the code represents the physical problem. The need for the tactical commander to have confidence in the simulated results cannot be overemphasized. If computer predictions and test results differ considerably, confidence will undoubtedly suffer. This indicates that considerable detail must be utilized in simulating the complex problem of an aircraft traversing a rough or repaired surface for the validation phase.

This literature review catalogs lessons learned and comments on past findings of the dynamics of aircraft surface interaction as they apply to runway roughness problems. Attention is given to the development of a means to simulate taxi, takeoff, and landing from a variety of surfaces.

The problem under investigation is depicted in a highly simplified block diagram fashion (Figure 1). The complexity and intrrelationships of the various blocks depend upon detail desired or required. In general, a simulation, be it analog, digital, or hybrid, has three key ingredients. These are the

- input,
- vehicle model,
- and output.

These ingredients are highly dependent upon one another, as changes in one area have significant impacts on the other areas.

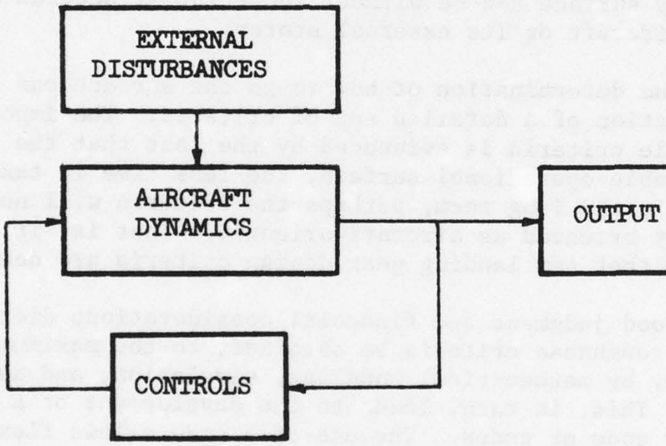

Figure 1. Simplified Block Diagram of Aircraft Response

Section II describes the sources and characteristics of the input. The type of input has a significant effect on the nature of the solution and on the mathematical model allowed for the aircraft. A part of this section is devoted to the pilot. He is difficult to model and has a profound effect on system response. Pilot effects are especially important when computer generated response time histories are compared with test results.

Section III discusses the vehicle model. In modeling the vehicle, the goal is to have the least complex model which accurately describes the aircraft's response. Factors that influence the complexity of the model include flexibility effects, linear versus nonlinear representation of suspension parameters, total number and placement of degrees of freedom, and the purpose of the model. The latter factor implies the idea that there should be different models for different purposes. For instance, a stability analysis implies a relatively simple model while stress predictions require considerable detail.

System output is discussed in Section IV. Output can be generated in a variety of forms, including displacements, velocities, accelerations, loads, and stresses. These are available on a statistical or discrete basis. System output is in itself no hurdle other than the fact that the desired form drives the input and the model.

Section V discusses solution techniques which have been successfully employed on a variety of dynamic analyses. Techniques are generally available to solve the system of equations with the choice of the technique depending on how the problem is cast.

Investigations of the automotive and rail industries are discussed in Section VI. These complementary studies offer considerable insight into the aircraft-rough runway problem and show promise worth further study.

Section VII deals with a review of available computer codes. In general, no code presently in being is adequate for the total surface roughness problem. However, there are really no state of the art advances necessary to obtain at least an adequate response time history code.

Section VIII concludes this review by offering some conclusions and recommendations.

SECTION II

INPUT

The input to the aircraft comes from a variety of sources. These sources can be as simple as gravity or as complicated as the pilot. Some of the disturbances are controllable (thrust) while others are not (wind). The nature of the input largely determines the nature of the output and has a profound effect upon the vehicle model. This section presents some of the important considerations concerning the modeling of the system input.

2.1 Contact Surface

The contact surface provides an obvious input to the dynamic system. Indeed, the undulations of the contacting surface provide the primary excitation to the model. What is not obvious is the exact nature of the input for a particular problem under consideration. The representation of the contact surface has far reaching implications in that it impacts the models of assorted subsystems such as the tires and landing gear as well as the solution technique employed. This subsection details some of the implications of contact surface representations regarding past efforts and the key concepts: statistical, deterministic, yielding, and nonyielding.

There have been a variety of programs to measure the profiles of runways and taxiways and of standard bomb damage repair (BDR) tests (References 2, 3, and 4). Thus, there is no lack of data to define the environment encountered in the past.

The character of the data leads to two primary concepts for defining runway or surface roughness models. The first is the deterministic model or "exact" representation of the surface. The term "exact" is loosely used in that discrete bumps with large wave lengths may be handled with (1-cos) representations (with or without actual waviness superimposed) or with the use of step or ramp functions. The main idea with the deterministic model is that runway elevation is provided as a specific function of the distance down the runway. The deterministic approach is ideally suited for direct integration of the equations of motion and, in its simplest form, for obtaining closed-form solutions. This approach appears to be a necessity for obtaining a proper representation of the discrete bumps encountered with patched runways, etc., or for determining the size of an obstacle which can be encountered without inducing a failure.

The second model uses a statistical approach. Here the surface roughness is assumed to be a stationary random process wherein obstacles are in some sense smoothed. Power spectrum analysis techniques are then used to obtain the frequency of occurrence of loads. This representation has been used in the past (References 5, 6, and 7) especially as related to fatigue studies. A significant drawback with this model is that it does not allow proper treatment of system nonlinearity, and the landing gear is an inherently nonlinear element. Methods attempted to circumvent this situation are detailed in Section V. The statistical approach to modeling the ground input may have merit in investigating the effects of cannon fire.

There are arguments in favor of both approaches as applied to vehicle dynamics. The deterministic approach is best when the concern is with possible catastrophic failures induced when a single obstacle is encountered. The statistical approach is best when no discrete surface can be chosen for analytical evaluation or for optimization of subsystem design. The model choice must be made on what the investigator feels is of primary importance.

The other primary choice that must be made is whether the contact surface should be considered yielding or nonyielding. While a prepared runway can normally be considered nonyielding, such is not the case with an unprepared strip (the use of which may be a viable option). The main problem with choosing a yielding representation is that it complicates the wheel-soil interaction model. Models exist (References 7, 8, and 9) for the yielding case, and many studies have been made of the kinetics of surface movement (References 10, 11, and 12). These models tend to be complex and require additional degrees of freedom over the nonyielding case. The investigator is simply forced into making a judgment as to how to best balance the limited number of degrees of freedom he has at his disposal.

In summary, data and models exist to describe the contact surface input. All that is necessary is to make a choice where the consequences of the choice are extremely important.

2.2 Aerodynamics

The aerodynamic forces acting on the vehicle are those forces caused by the interaction of the vehicle and surrounding fluid. Those forces can be further broken down by considering the particular part of the vehicle-fluid interaction which causes the force, that is:

- rigid body motion,
- control surface motion,
- or flexible body motion.

A complete discussion of all the aerodynamic effects caused by rigid aircraft motion may be found in any standard flight mechanics text (Reference 13). The usual approach is to express the aerodynamic forces and moments (acting at the aircraft center of gravity and referred to an aircraft body axis) in terms of nondimensional aircraft aerodynamic force and moment coefficients. These coefficients are, in turn, assumed to be linear functions of the rigid body degrees of freedom and control surface motion:

- angle of attack,
- sideslip angle,
- vehicle translation velocity,
- and control surface deflection.

Normally the coefficients are obtained from tests conducted at steady state conditions and the respective force or moment equation modified for instantaneous values of the variable, i.e., quasi-steady aerodynamics. No efforts in the literature have attempted to employ full unsteady aerodynamic theory, aeroelastic effects, or ground effects.

For the taxi, takeoff, and landing response problems, not all terms are of the same importance. A complete set of coefficients which are thought to be of importance for this class of problems is given in Reference 14. Control surface deflections can be assumed to be known as explicit functions of time or generated through a transfer function as the pilot's response to a set of input data. It is felt that great care should be taken in expressing wing lift and elevator position as accurately as possible since they have the most substantial effect on an internal aircraft load (Reference 5). If stability is considered, then stability derivative type terms must be included.

It appears that the area of takeoff and landing aerodynamics provides a fruitful area for basic research. This research should be directed toward establishing the validity and relative importance of the various terms in the approach as stated above.

The wind can be considered as a subsystem of aerodynamics. Wind effects are most significant during takeoff and landing phases of an operation. Crosswind landings require the pilot to use a wing low or crab technique, while takeoff requires nosewheel steering or rudder for directional control and aileron to keep the upwind wing down. In either case, the loads are affected through a complex interaction with pilot controls and aircraft dynamics.

An analytic description of the wind is most easily given in the ground axis system in terms of its magnitude, elevation angle, and azimuth angle. These quantities can be expressed either as deterministic explicit functions of time, solutions to another set of differential equations, i.e., a gust, or as a stochastic process. Again, if the purpose of the analysis is to determine the single worst case, a gust would provide a good initial wind disturbance model. As a final step, the wind, as expressed in the ground system, must be rotated into the aircraft body axis system.

2.3 Gravitational Force

The gravitational force acting on the aircraft is directed downward in the ground coordinate system. The magnitude is a constant given by the product of the total aircraft mass and the local acceleration due to gravity. To be used in the equation of motion, this vector must be rotated into the aircraft body axis using the Euler angle transformation matrix.

If the vehicle is treated as a rigid body, the gravitational force acts at a point coincident with the origin of the body axis coordinate system; this is, of course, the center of mass. If fuel burn-off and flexibility are taken into account, the instantaneous location of the center of mass will not coincide with the origin of the body axis system. These latter two effects are negligible and are not considered in the equations of motion.

2.4 Thrust

The thrust vector is a known explicit function of time or is known as a function of the variables describing the vehicle motion. The direction is usually stated in terms of its direction cosines relative to the body axis system. In a flexible aircraft the direction of the thrust would also depend on the deformation of the aircraft. This effect is certainly second order and is not considered in this report.

The moments due to the thrust vector about the body axes are calculated in a straightforward manner by taking the cross product of the distance from the center of mass to the line of action of the thrust. Once again, flexibility may be included as appropriate in the distance expression.

2.5 Pilot

As indicated in Section I, the pilot is central to the control portion of a representation of the total system. The human pilot sets up a variety of closed loops around an airplane. In so doing, he ideally allows the closed loop to accomplish tasks not otherwise achievable. The pilot has been described as a "complex beast" or more favorably as a "multi-input, multi-output device of enormous complexity" (Reference 15). He is an adaptive element in the control system and extremely versatile. There is a wealth of literature on man-machine dynamics (References 16, 17, and 18) which detail the complexity of trying to arrive at an adequate human describing function.

The majority of past efforts have led to a describing function that is applicable to simple, single-axis compensation systems as indicated by Figure 2. The resulting model contains a system gain, a transport delay, and lead and lag time constants (for details, see Reference 18). It has been found that the single-axis model works well for multi-axis systems when there is no significant cross-coupling between the axes. Unfortunately this is not always the case and predictions then become much more qualitative than quantitative. Additionally, the pilot's dynamics are governed by vehicle dynamics and other forces. These include the state of the environment as with vibrations and accelerations as well as less quantifiable variables such as training and motivation.

Of particular importance to this effort is the pilot's control of the elevator, the rudder-nosewheel steering combination, and the aileron in crosswind situations. That system coupling may occur for these functions is evident.

An obvious alternative to modeling the pilot mathematically is to place him in the system as an active participant, i.e., a

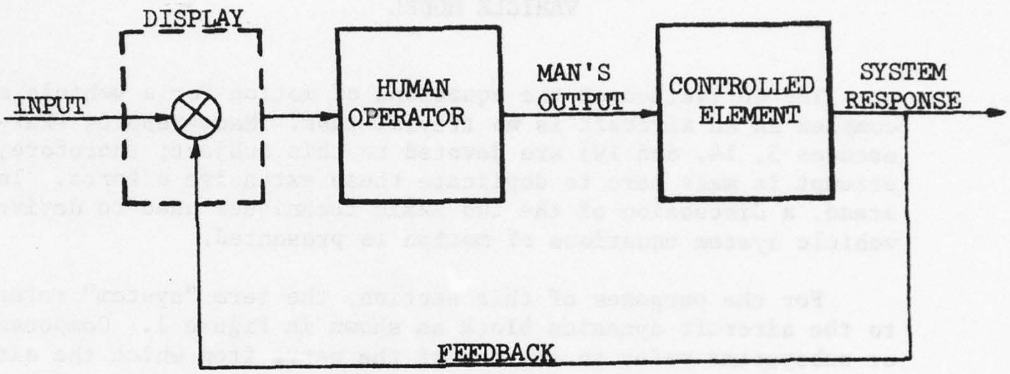

Figure 2. Single Axis Compensation System

simulator. This approach has definite benefits in a training mode in that it saves airframe life and may allow for more adequate predictions of both aircraft and pilot tolerances.

If it is decided to model the system in an open loop fashion, there is still much to be gained by comparing system requirements with known pilot capabilities. That is, if the system is unstable can a pilot do anything to stabilize it?

SECTION III

VEHICLE MODEL

The derivation of the equations of motion for a vehicle as complex as an aircraft is no trivial task. Many reports (References 5, 14, and 19) are devoted to this subject; therefore, no attempt is made here to duplicate these extensive efforts. Instead, a discussion of the two basic techniques used to derive the vehicle system equations of motion is presented.

For the purposes of this section, the term "system" refers to the aircraft dynamics block as shown in Figure 1. Components or subsystems refer to any one of the parts from which the aircraft dynamic system is assembled. Thus, the aircraft system can be thought of in terms of as few as four components:

- airframe (including fuselage, empennage, and wings),
- and three landing gears.

In this discussion the two techniques employed in deriving vehicle equations of motion are referred to as the "classical" and "modern" approaches. Both employ a building block approach as a fundamental concept. The main difference in the methods is that the classical technique considers the total system first, adding the details of the components as a final step, while the modern technique considers the components first and only as a final step assembles them into the system.

In the classical method, the motion of a given point is considered as being composed of:

- aircraft rigid body motion,
- aircraft flexible motion,
- landing gear rigid body motion relative to the aircraft,
- and landing gear flexible motion.

The equations of motion for the total system are then derived in terms of these variables. Aircraft rigid body motion is handled in the same manner as a free flight problem where the rigid aircraft is described in terms of overall properties such as the total mass and the various mass moments of inertia about the rigid body center of mass. Aircraft flexibility is most commonly introduced through the use of an arbitrary number of the unconstrained aircraft mode shapes. Landing gear details are introduced through force/motion (displacement/velocity) relationships for the various landing gear subsystems such as the tire, oleo strut, etc. Landing gear flexibility, while seldom included in the analyses reviewed, is included in the same manner as fuselage flexibility, i.e., through an arbitrary number of the unconstrained landing gear mode shapes.

Ferguson (Reference 14) gives the most comprehensive and general form of this type of derivation. His results are for a completely arbitrary vehicle using any type or number of surface contacting devices. As a consequence of this generality, the results are difficult to employ. Changes in aircraft or landing gear configuration require extensive modification or rederivation of the system equations of motion. A simple example illustrating the reduction of the general equations of motion to those for the specific case of a conventional aircraft landing is also given in Reference 14. These results require 14 pages to compute the equations of motion even though the system includes only 11 degrees of freedom.

The classical approach or modified forms of it is the most common technique employed at the present time by the aerospace industry. It is used in formulating the equations of motion for both Air Force Flight Dynamics Laboratory (AFFDL) and Boeing simulation codes.

The modern method uses a building block approach from the outset. Variations in this approach use either the component coordinates or component model coordinates as generalized coordinates describing component motion. The components are then assembled into larger systems, i.e., landing gear components into landing gear and airframe components into the airframe. These super components are then assembled to form the total aircraft dynamic system.

This technique is based on the proven ideas of the finite element method and offers all the advantages inherent in that method. These advantages are primarily in the ease of obtaining the system equations of motion, i.e., through assembly of component equations of motion and in the ease of system modification. Ease of modification is an important facet of the overall program, not only in changing aircraft components, but also in changing the program to simulate different aircraft. By far the greatest advantage is the ease of assembling the total vehicle since details are carried only at the component level, the computer assembles the total system.

Nonlinear elements or components cause no added complexity in terms of the assembly procedure if component coordinates are employed as long as their element equations of motion can be written in the form:

$$\underset{\sim}{m}\ddot{x} + \underset{\sim}{c}\dot{x} + \underset{\sim}{k}x = \underline{f}(t),$$

where $\underset{\sim}{m}$, $\underset{\sim}{c}$, and $\underset{\sim}{k}$ may be functions of $\underline{x}$ and $\dot{\underline{x}}$.

Nonlinear elements described in terms of their nonlinear mode shapes are only beginning to appear in the literature (Reference 20) but seem to provide a profitable area for further basic research.

3.1 Rigid Vehicle

If only the equations of motion for the rigid system are necessary, as in the case of a stability analysis, the classical approach can be applied in the most direct fashion. A special rigid body element must be used in the modern approach which is essentially the same as that obtained from the classical method; hence, all further discussion of rigid vehicle motion is restricted to the classical approach.

Three basic coordinate systems are used in the derivation of the rigid body equations of motion. They are the:

- ground or inertial coordinate system,
- body coordinate system,
- and component coordinate system.

The inertial coordinate system is fixed to the runway. The body system is fixed to the aircraft. Its origin is located at the mass center of the rigid vehicle and moves with it. The position of the mass center is given by its Cartesian coordinates relative to the inertial reference frame, and its angular orientation is given by three rotations relative to the body system.

Three types of body axes are in general usage. These are:

- principal axes,
- stability axes,
- and body axes.

Principal axes are chosen to coincide with the principal axes of the vehicle so that the products of inertia vanish. This greatly simplifies the angular equations of motion, but introduces the problem that the axes orientation must be recalculated each time a different asymmetrical external stores configuration is used.

Stability axes are chosen so that one of the axes points in the direction of motion of the vehicle in a reference condition of steady motion such that two of the reference velocities of the center of mass, i.e., vertical, and lateral, can be taken as zero. This simplifies the linearization of the equations of motion and is the system adopted in most flight mechanics texts.

When the axes are neither principal nor stability axes, they are commonly referred to as body axes. In this case one of the axes, usually the x axis, is fixed along a longitudinal reference line in the aircraft. This may be the most convenient system to obtain wind tunnel measured aerodynamic data.

The component coordinate systems are fixed relative to the body axes system and oriented such that the motion of the component is as simple to describe as possible.

These coordinate systems are related to each other through linear transformations. Experience, historical inertia, and the dominance of flight mechanics has led to the fact that the equations of motion are usually derived referenced to the body axes system. The equations are then solved in this system and the final results transformed to the ground system, i.e., the runway, for evaluation.

Thus, quantities most easily expressed in the terms of a ground system such as wind and gravity must be rotated into the body axes. Quantities most easily expressed in terms of a component system such as landing gear stroke and oleo forces must also be rotated into the body axes system. Other quantities such as aerodynamic forces and moments are most easily expressed in terms of the body axes system. They are usually introduced as concentrated loads acting at the vehicle center of mass.

3.2 Flexible Vehicle

A discussion of the derivation of the equations of motion for a rigid vehicle was presented in the previous subsection. It is known that the stability, control, and response of flexible aircraft may be significantly influenced by structural deformations under transient conditions. A discussion of the changes that must be made to the rigid-body equations as obtained by the classical approach to include flexibility is presented. Since the modern approach was omitted in the rigid vehicle section, it is included below.

Two basic techniques are used to alter the rigid vehicle equations of motion using the classical approach. They are:

- the method of quasi-static deflections,
- and the method of normal modes.

The method of quasi-static deflections accounts for flexibility simply by altering the aerodynamic derivatives. The assumption is made that the changes in aerodynamic loading take place so slowly that the structure is in static equilibrium at all times. (This assumption is equivalent to requiring that the natural frequencies of vibration of the structure are much higher than the frequencies of the rigid body motions.) Thus, a change in load produces a proportional change in the shape of the aircraft which, in turn, influences the load. This tool has been primarily employed in the field of flight mechanics but was not employed to the taxi or takeoff problem in any of the literature reviewed.

If the separation in frequency between the flexible motion and the rigid body motion is not large, then significant coupling can occur between the two. In that case an analysis which takes the flexible time dependent motion into account must be conducted.

In the classical method the equations of motion are rederived by adding the flexible motion of the airframe and/or component to the previous rigid body motions of the airframe and/or component. This procedure results in a system of differential equations in which the variables describing the rigid body and flexible motion are highly coupled. Note that this does not say that the motion is highly coupled but only that the equations of motion are coupled. The separation of the rigid body and flexible body natural frequencies determines the magnitude of the coupling. This point is considered in detail in Klosterman (Reference 21).

The modern approach handles flexibility through the component mass, damping, and stiffness matrices. Typical elements used in aircraft analysis to simulate structural components are:

- beams (axially, torsionally, or transversely loaded),
- shear panels,
- membranes,
- plates,
- shells,
- and three-dimensional solids.

Special elements describing springs, dampers, and concentrated masses are also easily incorporated to simulate the landing gear or any other idealized component.

Dynamical models are most easily derived using the natural coordinates of the elements, i.e., deflections and rotations. Computationally this may be very inefficient, since the degrees of freedom are not apportioned according to their importance.

Two approaches are commonly used to overcome this problem; both involve the mode shapes and are hence limited to linear systems. Further developments should extend a similar technique to nonlinear systems.

In the first technique, an eigenvalue economizer is employed to reduce the dynamic degrees of freedom but retain as much of the static detail as possible. Many schemes exist for accomplishing this task.

The second method, which offers considerably more promise in this area, was developed by Hurty (Reference 22). In this method, the dynamic analysis of complex structures is performed using the component mode shapes rather than the system mode shapes as is done in the first model method.

Displacements of the separate components are expressed in generalized coordinates defined by the displacement modes. These are generated in three categories:

- rigid body,
- constraint,
- and normal modes.

Rigid body modes are convenient for displacements defined in inertial space. Constraint modes are included to treat redundancies in the interconnection system. Normal modes define displacements relative to the connections. Generalized mass, stiffness, and damping matrices are determined for each component, as are generalized forces. The requirement of system continuity gives rise to equations of displacement compatibility at the connections. These serve as equations of constraint among the component coordinates and are used to construct a transformation relating component coordinates to system coordinates. This transformation is then used to obtain system properties and forces from component properties and forces. System equations of motion are formulated and solved to determine system response.

In many cases, economy or component complexity may dictate that the component natural frequencies and mode shapes be obtained experimentally. Klosterman (References 21, 23, and 24) has demonstrated that the modal synthesis ideas described above can be successfully combined with experimental techniques to yield excellent system models.

It must be reiterated that the modal synthesis techniques have only been proven on linear systems. Whether these techniques can be utilized on nonlinear systems remains to be seen. This does not mean to imply that this method cannot be employed on the problem at hand. Once the degree of nonlinearity is established, i.e., just how nonlinear and how important the nonlinearities are, then the manner in which modal synthesis is used can be determined.

3.3 Suspension System

The only suspension system considered in this report is standard type landing gear as employed on fighter aircraft. The parameters which quntify a given landing gear system are:

- geometry (tricycle, main gear mounted to fuselage or wing),
- construction (cantilever, articulated, semi-articulated),
- shock absorber characteristics,
- and surface contacting component (tires).

These items are discussed, in turn, in the following subsections.

3.3.1 Geometry

All fighter aircraft considered in the current study in the Rapid Runway Repair Program Management Plan utilize a tricycle gear arrangement. Transport aircraft under consideration, except for the C-5A and B-747, also use this type arrangement. Therefore, any general model should be built based on the tricycle configuration.

The nose gear on all aircraft considered are attached to the fuselage centerline with the exception of the A-10. Here the nose gear is offset from the fuselage centerline. The only other difference is in the number of wheels employed ranging from a single wheel to as many as four.

The main gear locations differ in whether the gear is attached to the wing or fuselage. Both the F-4 and A-10 are attached to the wings while the F-15, F-111, and F-16 are attached to the fuselage. The transport aircraft uses a fuselage attachment with the exception of the DC-10 and B-747. The DC-10 uses a wing location while the B-747 uses a combination of both.

3.3.2 Construction

The three basic types of landing gear found on fighter aircraft are:

- cantilevered,
- articulated,
- and semi-articulated.

A cantilevered landing gear is commonly defined as one in which the cylinder enclosing the stroking element has a fixed position and orientation relative to the rigid vehicle as shown in Figure 3. The gear is assumed to be pin supported at the vehicle. It is also supported by a lateral brace and drag link. Torsion is transmitted from the lower to upper oleo by a conventional scissors or torque link arrangement. The component equations of motion for this class of landing gear are given in Reference 14, Volume II, pages 202-231. Care must be exercised in using these results as some results consider a constant vehicle orientation.

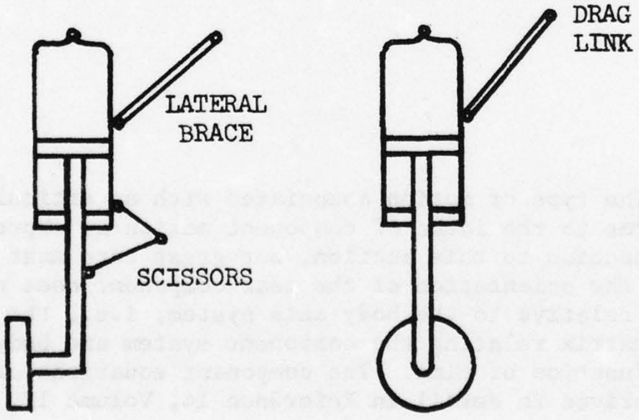

Figure 3. Cantilevered Landing Gear

An articulated landing gear is one in which the landing gear assembly changes orientation with respect to the body coordinate system. The gear trunk, which supports the wheel and tire, rotates with respect to an axis fixed in the vehicle body. The shock strut, a stroking member, is attached to the trunk and to the body. A third member, a drag link, is usually added to provide fore and aft support. The general configuration of an articulated gear is shown in Figure 4.

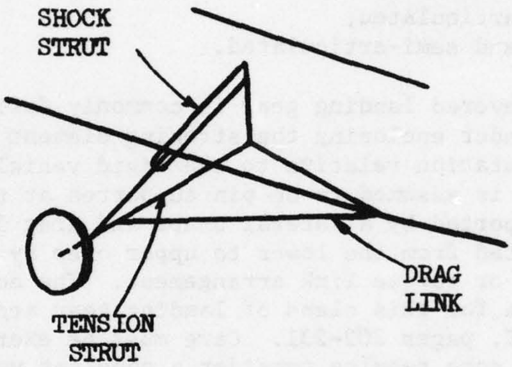

Figure 4. Articulated Landing Gear

The type of motion associated with an articulated strut conforms to the ideas of component motion as expressed in the introduction to this section, but great care must be exercised since the orientation of the gear component does not remain fixed relative to the body axis system, i.e., the transformation matrix relating the component system and body axis system is a function of time. The component equations of motion are derived in detail in Reference 14, Volume II, pages 232-241.

The semi-articulated landing gear configuration is shown in Figure 5. The outer cylinder is fastened to the fuselage with a fork-pin arrangement for forward retraction. A drag link is added so that the outer cylinder is like that of the cantilevered gear. The inner cylinder is allowed to rotate about the gear centerline and, thus, needs no lateral support strut other than the fork-pin arrangement.

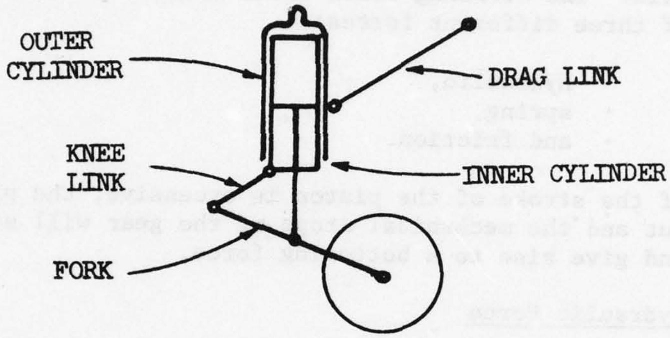

Figure 5. Semi-articulated Landing Gear

The semi-articulated gear problem may be formulated by either of two methods, depending on whether the fork between the knee link and the wheel is to be considered elastic. If fork elasticity is included, the formulation is similar to that of the articulated gear, in which the component coordinate system rotates with the fork. The elastic displacements are then measured relative to this rotating system. This procedure allows both elastic and component rigid body motion to occur in one direction.

The fork may be considered rigid since the spring rate of the tire is normally much lower than that of the fork. If this assumption is made, the fork rigid body rotation does not constitute a degree of freedom since it is geometrically related to the piston stroke. A single stroking equation then governs the motion. Elasticity of the cylinder and piston is included in the same manner as for a cantilevered gear. No detailed derivations for this type of gear were found.

If a bogey arrangement is used to support multiple wheels, the bogey can be included either as a rigid or flexible rotational element with the price of the additional degrees of freedom. However, Reference 19 shows how the additional rotational degree of freedom can be removed if a rigid bogey is assumed.

3.3.3 Characteristics of Shock Absorbing Mechanisms

The shock absorbing mechanisms are best described in terms of their contribution to the strut or stroking force. This force is the sum of all the forces acting on the piston along the stroking axis. The stroking force under normal operations is comprised of three different forces:

- hydraulic,
- spring,
- and friction.

If the stroke of the piston is excessive, the piston will bottom out and the mechanical stops of the gear will act like springs and give rise to a bottoming force.

Hydraulic Force

The hydraulic force is caused by the piston being displaced with a stroke velocity. This creates a pressure field in the lower chamber as shown in Figure 6.

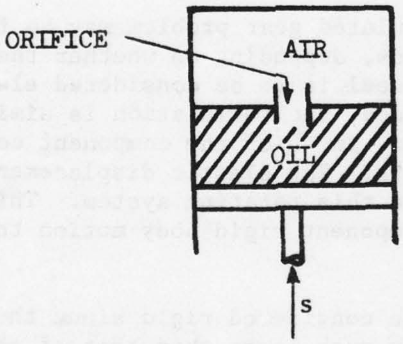

Figure 6. Strut Lower Chamber Model

If fluid viscosity, compressibility, and unsteady effects are included, the problem is very difficult to solve especially in a form useable in the larger problem. Satisfactory definitions of loads have been achieved by semi-empirical means. This analysis leads to the hydraulic force being given by:

$$F_H = \frac{\rho_H A_H^3 \dot{s}^2}{2(C_D A_N)^2} \quad (\dot{s}>0)$$

where

F_H = hydraulic force,

$\dot{s}$ = stroke velocity,

ρ_H = density of hydraulic fluid,

A_H = hydraulic area of piston,

C_D = orifice coefficient,

A_N = effective orifice area (actual orifice area minus metering area).

The value of the coefficient C_D is dependent on many parameters. However, there is general agreement the coefficient has minimum and maximum values ranging between 0.86 and 0.93 and that a constant average value of the coefficient can be used.

Compressibility effects do not appear to be important in the taxiing or takeoff problem. They are only important when the stroke velocity of the strut is high (greater than 15 feet per second).

In general, the hydraulic force of a self-positioning strut made up of a spring and hydraulic damper in series will be different during the extension and compression stroke. Extension characteristics are governed by two requirements:

- rapid return of oil to the lower chamber,
- and sufficient damping to reduce bottoming loads.

The energy available for the extension stroke is that stored in the spring mechanism of the gear which may be pneumatic, liquid, or mechanical.

The form of the hydraulic force for the extension stroke is the same as for the compression stroke except for different constants:

$$F_H = \frac{-\rho_H A_{Hs}^3 \dot{s}^2}{2(C_D A_N)_s^2} \quad (\dot{s}<0)$$

During taxi and takeoff these equations are representative of the hydraulic force. Since this force is proportional to the velocity squared, the hydraulic force is not a significant effect for low stroke velocities. This feature is primarily for absorbing landing shock.

The hydraulic force is highly nonlinear over its complete operating range. This is due not only to the fact that it is proportional to the square of the stroke velocity, but also to the fact that the effective orifice area is changing as a function of the stroke position. The importance of each of these nonlinear effects has not been established for the taxi/takeoff problem.

The metering device used to vary the effective orifice area with stroke can be:

- conventional metering pin,
- fluted metering pin,
- or metering tube.

The change of the effective orifice area with strut stroke for each type of device is accounted for in the models investigated.

Spring Force

The spring force is used to return the strut to its original position after being displaced. This restoring force is generally a function of the stroke displacement, s. If energy is dissipated by the spring in the compression-extension cycle, the force/deflection relationships will be different for each portion of the stroke. Generally, inertial effects of the spring are neglected.

Three types of springs are considered here:

- mechanical,
- pneumatic,
- and liquid.

Mechanical springs are probably the most common springs used in mechanical systems although they were not observed to be used in any of the landing gear analyses investigated. The characteristics of these type of springs can be expressed in the form:

$$F_s = f_{s1}(s) \quad s \geq 0$$

$$F_s = f_{s2}(s) \quad s < 0$$

where

F_s = spring force,

f_{s1} = function relating stroke displacement in compression stroke to spring force,

f_{s2} = function relating stroke displacement in extension stroke to spring force,

s = stroke displacement.

For springs in which hysteresis is negligible $f_{s1} = f_{s2}$. If the spring is linear, then $f_{s1} = f_{s2} = ks$.

Pneumatic springs depend upon the compressibility of a gas to generate the restoring force. This gas is usually the air contained in the upper portion of the strut as shown in Figure 6.

Two types of landing gear struts which affect the pneumatic spring force are in common usage. These are:

- single acting or conventional oleo - pneumatic strut,
- and double chambered.

Figure 7 shows a typical single acting strut. The double chambered strut is shown in Figure 8.

Reference 19 gives the single-acting strut force as:

$$F_A = \frac{P_1 V_1 A_1}{(V_1 - sA_1)}$$

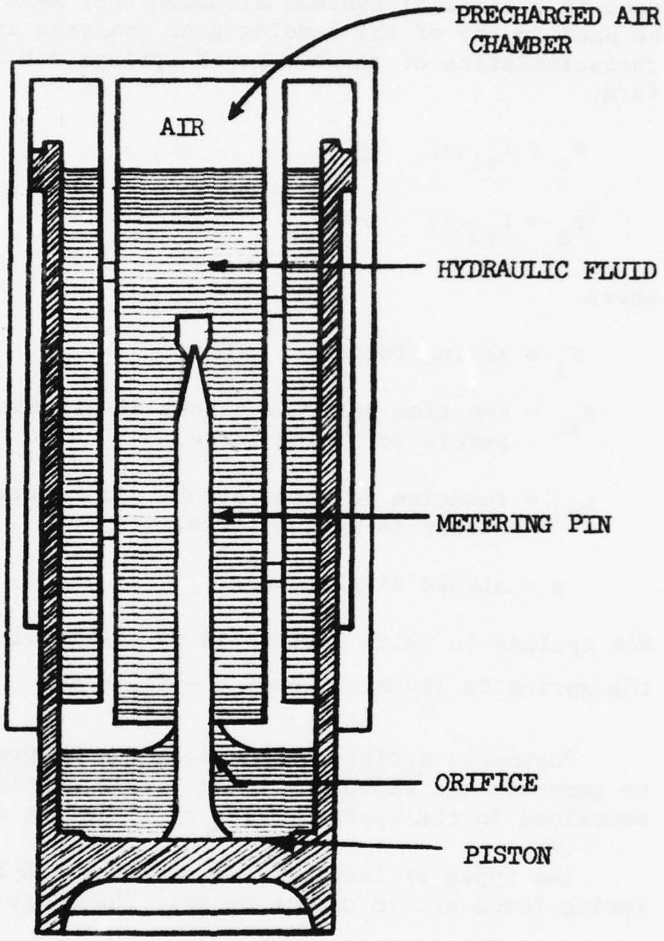

Figure 7. Single Acting Strut

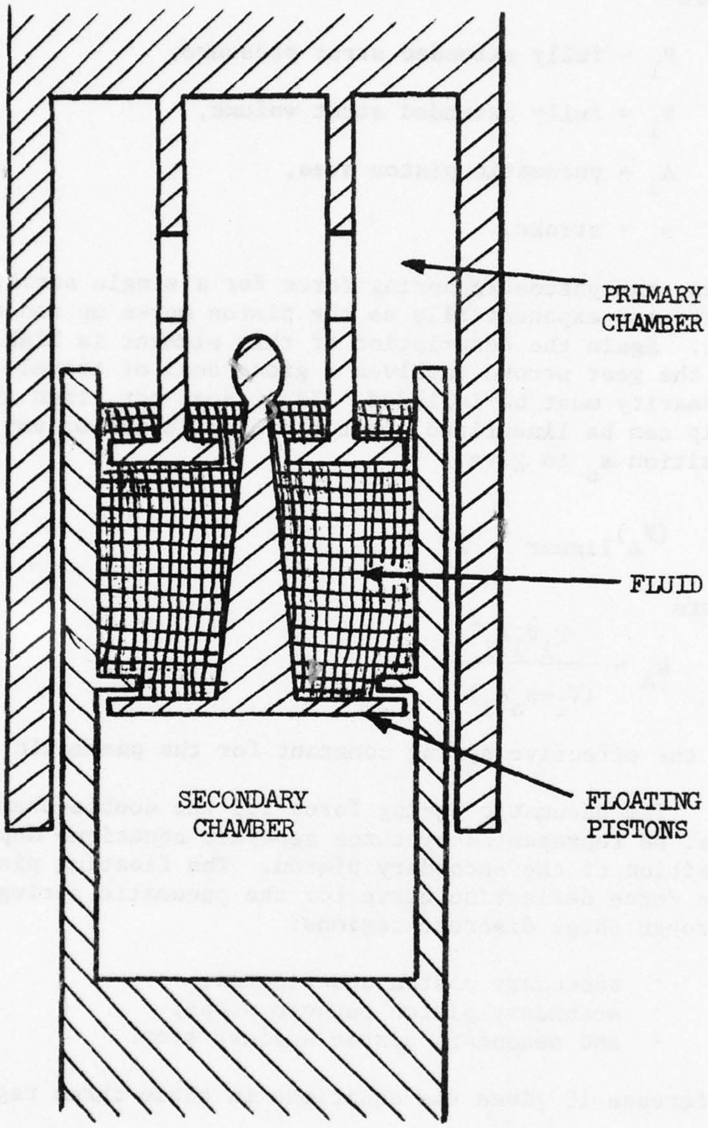

Figure 8. Double Acting Strut

where

P_1 = fully extended strut pressure,

V_1 = fully extended strut volume,

A_1 = pneumatic piston area,

s = stroke.

Thus, the pneumatic spring force for a single acting strut increases exponentially as the piston moves up and compresses the air. Again the description of this element is highly nonlinear. If the gear stroke involves a great deal of travel, this nonlinearity must be included. If it does not, then this relationship can be linearized about the equilibrium or any desired position s_o to give:

$$(F_A)_{linear} = k_A s$$

where

$$k_A = \frac{P_1 V_1 A_1^2}{(V_1 - s_o A_1)^2}$$

is the effective spring constant for the pneumatic spring.

The pneumatic spring force for the double-chambered strut must be represented by three separate equations depending on the position of the secondary piston. The floating piston causes the force deflection curve for the pneumatic spring to vary through three discrete regions:

- secondary piston undeflected,
- secondary piston between stops,
- and secondary piston against stop.

Reference 19 gives the equations in these three regions as:

$$F_{A1} = \frac{P_1 V_1 A_1}{(V_1 - sA_1)}$$

$$F_{A2} = \frac{A_1 (P_1 V_1 + P_2 V_2)}{V_1 + V_2 - A_1 s}$$

$$F_{A3} = \frac{P_1V_1A_1}{V_1+s_2A_2-A_1s}$$

where

A_1 = primary piston area,

P_1 = primary chamber unloaded strut pressure,

V_1 = primary chamber unloaded strut volume,

P_2 = secondary chamber unloaded strut pressure,

V_2 = secondary chamber unloaded strut volume,

s = stroke,

s_2 = maximum deflection of secondary piston.

Once more, these forces are highly nonlinear because of the discontinuities and because of the inverse relationship with s. As before, all three are susceptible to linearization about a given strut position. This force is by far the most important of the strut forces during the taxi and takeoff conditions.

In recent years a number of aircraft (Reference 14) have been designed using hydraulic principles where the function of the pneumatic spring has been replaced with a liquid spring. Relationships for this type of spring are summarized in Reference 14, pages 100-101.

Friction Force

When the piston strokes within the cylinder, energy is dissipated by friction acting at the bearing surfaces between the piston and cylinder. The frictional force opposing the motion of the piston is expressed as:

$$F_F = (\mu_{BL}|F_{BL}|+\mu_{BU}|F_{BU}|+F_{FO})\,\frac{\dot{s}}{|\dot{s}|}$$

where

F_{BU}, F_{BL} = bearing force at the upper and lower bearings respectively required to balance the lateral loading on the piston.

μ_{BU}, μ_{BL} = coefficients of sliding friction at the upper and lower bearings respectively

F_{FO} = frictional force at zero loading.

Details of the calculation of F_{BU} and F_{BL} as functions of the ground reactions for a cantilevered landing gear are given in Reference 14, pages 101-104.

To insure that the piston remains within the required stroke range, a bottoming force is introduced. For strokes less than s=0, the piston will contact the lower stop, while for strokes greater than s_{max}, the piston will contact the upper stop. The bottoming forces introduced to describe this are:

$$F_B = k_{BL} s \qquad s \leq 0$$

$$F_B = k_{BU}(s - s_{max}) \qquad s \geq s_{max}$$

where

F_B = bottoming force

k_{BU}, k_{BL} = spring constants of upper and lower ends of the cylinder.

The values of the spring constants are usually chosen to be quite large and may be arbitrarily assigned. Arbitrary damping may be introduced also to preclude oscillations caused by the bottoming and stroking forces. This is purely a mathematical artifice.

In summary, the description of the strut forces appear quite well known. The relative importance of each and the significance of their nonlinearities are yet to be determined and should be the subject of further studies.

3.3.4 Surface Contacting Components

The surface contacting components give rise to forces on the body due to interactions with the ground. While it is possible to consider a variety of such components such as skids, surface pods, etc., tires are of prime interest to this study and this discussion is limited to them. Additionally, this discussion is limited to nonyielding surfaces. The effects of surface yielding can be important. Examples of the added complexity due to yielding are found in References 7 and 8.

The physics involved with tires is intricate in the extreme and has never been fully described on a theoretical basis. In addition to providing the forces and associated kinematic constraints for both the rolling and slipping tire, failure modes such as tire cutting or blowout are of definite importance. Stability investigations have their own wide body of literature and the past has shown them to be important in certain areas.

This subsection presents some general comments and then discusses the vertical and in-plane ground forces. The stability problem is considered next, and finally a discussion of problems associated with failure modes is presented.

It should be reiterated that, in all areas, engineering judgment is required as to the proper degree of freedom balance and that models may change depending on the problem under consideration.

Numerous studies have been made concerning the mechanical properties of tires. Smiley and Horne (Reference 25) report on these and present an excellent tabulation of such properties as vertical force deflection characteristics, lateral, fore and aft, and torsional spring constants, etc. They also present a large number of empirical equations based upon a variety of research efforts. Of course, the optimum solution is to use experimental data furnished by various manufacturers, but this can lead to more complexity than the often cumbersome empirical equations.

As an example of this, consider the case of the vertical ground force. The variation in this force with the deflection is nonlinear and has been written in many forms. Ferguson, Mollik, and Kitts (Reference 14) present the force as a complex function of tire deflection, contact area, tire pressure, and other variables and additionally allow it to be zero (i.e., the tire is not touching the ground). Gerardi and Lohwasser (Reference 19) use a simple linear spring law which does not allow for tire bounce. Milwitzky and Cook (Reference 26) have found that a piecewise linear segment approach for the force deflection curve is adequate up to the instant of bottoming. They note, however, that there is a pronounced increase in landing gear load as a result of tire bottoming. Whether an empirical equation is used or a piecewise linear approximation is made, the authors feel that allowance should be made for the possibility of tire bounce.

The in-plane (as related to a ground coordinate system) ground forces contain special complexity due to the distinct regions into which they are divided. These are the pre-spinup and the post-spinup regions plus a transition region between the two.

In the pre-spinup phase the magnitude of the force is usually written as the product of the coefficient of sliding friction and the vertical ground force. The force is directed opposite the axle velocity parallel to the ground. Variations in the coefficient of sliding friction with pressure, velocity, temperature, etc. are discussed in a variety of areas including the work of Tomita (Reference 27).

In the post-spinup region it is usually assumed that rolling friction is negligible in the absence of braking. Thus, the only force of consequence is a side force which is the product of the tire cornering coefficient and the slip angle. The above approximation is valid for small slip angles. Steeds (Reference 28) indicates that an upper limit of slip angle less than 10 degrees appears adequate.

In the transition region the beginning and ending values for the in-plane forces are known in that they must match the pre- and post-spinup values. The transition path is another matter entirely. A quote from Reference 14 is worthy of repetition. "A search of the literature shows that no analytical procedures derivable from physical laws are in existance for this region and no experimental data is of sufficient accuracy to define any variation with tire parameters." Thus, for lack of a more accurate representation, MIL-A-8862 suggests a sinusoidal variation. This is an accepted and reasonable assumption.

The problem of shimmy has been studied by many investigators. The theories used to explain the shimmy of pneumatic tires lead to essentially two models, the point contact model of de Carbon (Reference 29), and Moreland (Reference 30), and the string model of Von Schlippe (Reference 31). Other models are available, but these appear to be too complicated for normal use. Collins (Reference 32) compares the two fundamental developments in these theories of tire mechanics and arrives at the conclusion that both are adequate for qualitative and, in most cases, quantitative studies, that is, provided that proper tire constants are known. Rogers and Brewer (Reference 33) and Rogers (Reference 34) provide an approximation to the string theory which leads to a system of linear constant coefficient differential equations. The prime loads of importance are the cornering force and self-realigning torque. On balance it appears that some variation of the string theory should be used when shimmy is felt to be an important consideration. In any case, it is noted that any shimmy theory requires a no slip condition. Once slip is encountered (including pure sideslip) there is no theory that is adequate for stability investigations.

While force transmittal is of obvious importance and offers significant complexity, perhaps the most important and least understood area is associated with the possibility of failure modes. That is, what criteria should be used to govern whether the surface contacting subsystem has failed? The obvious results of a tire blowout upon takeoff or landing give some indication of the importance of this area. Tomita (Reference 27) indicates that a one second full skid can cause excessive tire wear and possibly a blowout. Additionally, if the tire bottoms, the carcass will be cut by the rim. This indicates that a tire deflection criteria in terms of bottoming deflection is necessary. The possibility of carcass cutting due to contact with an unrepaired hole in a runway offers complexity not found in any of the literature that the authors have seen. The implications on what kind of damage to the runway may be, left unrepaired, are obvious.

SECTION IV

OUTPUT

The output of a simulation provides a measure of aircraft response to a given input. The nature and accuracy of the output are tied directly to the model and the input. The output applicable to this study is usually given in some combination of the following three forms:

- displacements, velocities, and accelerations,
- loads (forces),
- and stresses.

They can be represented in time history, discrete maximum, and minimum or statistical fashions. The nature of the output is of critical importance in determining whether failure criteria have been met. That is, if a limit is placed on any variable, then said variable must be part of the simulation output, either directly or indirectly. This section considers some of the important considerations of the form and choice of the system output.

The displacement, velocity, and acceleration of various points on a body are readily found from numerical integration of the equations of motion for a system subjected to a deterministic input. For the equations of motion expressed as

$$\underset{\sim}{m}\ddot{\underline{x}} + \underset{\sim}{c}\dot{\underline{x}} + \underset{\sim}{k}\underline{x} = \underline{f}(t),$$

it is seen that they are the natural outputs in that they represent the dynamic unknowns.

Displacement, velocity, and acceleration outputs are found in a wide variety of programs and are quite popular. Their utility in terms of being compared against failure criteria is limited but not inconsequential. Their main use is in the determination of ride quality (acceleration) limits at the pilot station, stroke limits for the landing gear, deflection limits for tires, and tire/runway contact. The acceleration output has great utility as a check on the adequacy of the simulation. As mentioned previously, the aircraft user is very much interested in the fact that the simulation represents the actual situation as much as possible. In this regard, it is relatively easy (as compared with stress measurements) to obtain the acceleration of various points during testing and then compare simulation versus test.

Similar output is also available for stochastic input. For statistical studies the output is in the form of root mean square values of the displacements, velocities, and accelerations. If the output is Gaussian, then approximations to maxima can be found by noting that the peak values will not exceed three times the root mean square values 99 percent of the time.

Loads type output is also readily obtainable from most simulations. In general, load information can be determined from a solution to the set of equations

$$\underline{L} = \underset{\sim}{A}\underline{\Phi},$$

where $\underline{L}$ is the load vector, $\underset{\sim}{A}$ is the linear or nonlinear matrix which relates loads to the state variables of the problem, and $\underline{\Phi}$ is the state vector. The generation of the matrix $\underset{\sim}{A}$ is, of course, a function of the detail of the model.

Loads are not easily found at arbitrary points after the fact. Thus, if a particular wing station is thought to be critical, it is best that it appear in the model. Interpolation schemes, for points intermediate to those modeled, can be used, however. Boeing uses a concentrated lift effect during the dynamic analysis and then reverts to a normal lift distribution when calculating wing loads.

Loads output is generally not very useful in providing a check on the validity of the simulation by comparing the computer output with test results since loads are not measured directly. It can be quite useful in terms of determining whether a particular part has failed. A comparison of simulated loads with information contained in the design loads document can provide a measure of failure. Some caution must be exercised when making comparisons for parts with multiple load paths. For these paths either the input loads must be in proportion to the design loads or the structure must have been analyzed for the design loads one at a time.

Stress information can be found from the load data. In equation form, the stresses, $\underline{\sigma}$, are given by

$$\underline{\sigma} = \underset{\sim}{B}\underline{L}$$

where $\underset{\sim}{B}$ is a linear matrix which relates stresses to loads. The generation of the matrix B requires significant structural detail. If area properties are smeared out, stresses have little or no meaning. There appears to be little reason to have the output in stress form for response time histories or discrete maxima or minima. Too little is gained at the cost of a significant increase in the detail of the simulation.

One area where stress information is of value as an output is in a fatigue analysis. Comparisons are made with stresses, and either the fatigue analyst must change his time proven methods or the output should be in stress form.

SECTION V

SOLUTION TECHNIQUES

A variety of solution techniques have been successfully employed in a wide range of dynamic analyses. These techniques tend to be input and model sensitive, and each has its share of primary applicability. This section describes some considerations of the main techniques of direct integration, stability analyses, and statistical solutions, including a frequency domain approach. Finally, some comments are made concerning the requirements of achieving some type of design optimization.

When the input is determinsitc, as with a measured profile or some discrete engineering approximation to the profile such as (1-cos) bump, the equations of motion must be directly integrated to give response time histories. This may be accomplished either by means of an analog route or by numerical integration. A variety of well proven integration schemes exist. The TAXI code (Reference 19) uses a three-term Taylor series numerical approximation, while Boeing (Reference 35) indicates that its program uses a fourth-order Runga-Kutta technique. More complicated integration routines such as Newmark-β or Wilson-Θ and other predictor-corrector methods are available (References 36, 37, and 38). However, past experience has apparently demonstrated that this extra sophistication is not needed to adequately determine the aircraft response. It is important to note that direct integration for the system with deterministic inputs allows for either linear or nonlinear equations of motion, whereas nonlinearities cannot be handled directly by other methods. The price paid for this flexibility is that integration is time intensive. That is, the number of runs required to investigate a spectrum of inputs can be prohibitive.

Much can be learned about system controllability from a stability analysis. This analysis requires that the equations of motion be cast in the matrix form

$$\underset{\sim}{m}\,\underline{\ddot{x}} + \underset{\sim}{c}\,\underline{\dot{x}} + \underset{\sim}{k}\,\underline{x} = \underline{0},$$

where the mass, damping, and stiffness matrices are constant at some instant in time. Thus, it is seen that the stability analysis can be used in conjuction with an integration scheme where the latter is used to provide the state about which stability is investigated. For the system of equations written as above (or alternately in state variable form), any of a number of matrix solution techniques are available to determine the system eigenvalues. Examples of the utility of a stability analysis, coupled with the proper model, are that it can be used to determine the "hands off" directional stability of the rigid body aircraft as influenced by flexibility or an investigation can be made into nose or main gear shimmy (References 39, 40, and 41).

The remaining solution techniques which are generally applied fall under the broad classification of statistical techniques. The application of standard statistical techniques requires two primary assumptions. These concern the mathematical nature of the input and a required linearization of the model. These points were mentioned in Section II but will be expanded upon here.

The statistical method assumes that the runway unevenness profile can be classified as a stationary random process. That is, its statistical properties (e.g., probability distribution) are independent of the position measured along the length of the runway. This assumption has been justified in the past in fatigue studies because aircraft have been expected to use a large number of similar runways. The unevenness associated with BDR patches is hardly uniform along the entire length of the runway, leaving this assumption at least suspect.

For the above assumption, the key relationship then employed in conjunction with a linear system is

$$P_o(\omega) = |T(j\,\omega)|^2 P_i(\omega)$$

where P_o and P_i are the power spectral densities of the output and input, $|T(j\omega)|$ is the absolute value of the complex frequency transfer function, and ω is the circular frequency. The root mean square value of the output can then be determined from

$$[o^2]^{1/2} = [\int_0^\infty P_o(\omega)\,d\omega]^{1/2}.$$

The use of power spectral densities is widely applied, including repair simulation (Reference 42).

The key phrase in the preceding paragraph is "linear system." For example, Van Deusen (Reference 7) approaches the n-wheeled vehicle problem by using superposition with the linearized differential equations of motion, expressed in the frequency domain, to obtain the system transfer function. This required linearity is a special problem due to the nonlinear spring forces, the velocity squared hydraulic damping, and the Coulomb friction associated with modern landing gear.

Efforts have been made by Tung, Penzien, and Horonjeff (Reference 6) and more recently by Kirk (Reference 43) to use a quasi-linear approach which can extend the use of statistical methods. Displacement and velocity nonlinearities are handled by expressing them in polynomial form. The damping is linearized by computing an equivalent linear damping coefficient which dissipates the same average energy as the nonlinear damping. These efforts involve some restriction such as the fact that elastic modes are assumed to uncouple the system. Nonetheless, the technique offers some promise worthy of further study.

Even though the statistical approach has the definite advantage of not requiring the generation of output time histories and of being useful in studying long term effects as with fatigue life, it has a large drawback. A statistical approach does not allow the aircraft response to be determined for a specific section of runway, say a single patch. Instead, it provides only average or root mean square values of a particular output variable. The worst case, occurring at some specific point on the runway, can be lost in the statistical definition of the surface. Thus, the catastropic failure may have been smoothed away. It makes little difference that a statistical investigation shows that it is safe, from a fatigue standpoint, to perform 20 take-offs and landings when in a worst-case sense, the wing may fail the first time down the runway. Thus, a complete investigation would include both analyses. Additionally, this tendency to smooth inputs may have a significant effect on something like damage caused by cannon fire because the disturbances resulting from such damage have a comparatively short wavelength.

The design engineer's ultimate goal is a design which is "optimum." Thus, in the ideal sense, a computer simulation should finally be utilized in a large optimization routine. Such a routine provides an automated method of investigating how changes in various parameters affect overall vehicle response. An optimum solution is found within the solution space boundaries generated by a selected set of constraints. Examples of constraints are maximum allowable "g" loading and stress in a particular area. The path leading to the optimum solution is neither obvious nor easy to obtain for any problem of engineering significance. Optimization procedures will always yield an answer providing one exists. The validity of the answer depends on the validity of the model and associated constraints. As one would expect, an oversimplified model will lead to erroneous results. An example of this is demonstrated in a recent investigation of railcar optimization (Reference 44).

Optimization techniques may be applied for either discrete or statistical input. In so doing, care must be exercised that the input adequately reflects the condition for which an optimum solution is sought. If proper attention is given in this area, then optimization procedures provide long-term promise when applied to the least complex model which adequately represents the aircraft's behavior.

SECTION VI

COMPLEMENTARY STUDIES

In attempting to investigate the dynamics of an aircraft on takeoff or landing, the initial tendency is to focus one's attention on aircraft literature alone. There is, however, a wide body of literature directed toward dynamic analysis of other vehicular systems. An investigation of the literature provides valuable insight into the problem at hand. Two areas, in particular, which offer the possibility of fruitful results are found in the efforts of the rail and automotive industries.

Over the last 15 years, an extensive effort has been devoted to railway vehicle research and development. One of the performance objectives of the rail vehicle suspension system design is to provide effective vibration isolation such that passenger comfort is maintained and freight is not damaged. A completely analogous situation occurs for the aircraft response to a rough and/or bomb-damaged runway. Here the aim is to provide effective vibration isolation such that the pilot can experience a ride that allows him to perform those duties necessary for a safe takeoff. Structural limitations are also imposed such that the loads on the aircraft do not exceed those that would damage the aircraft and its ordnance. Several implications from railcar suspension research are directly applicable to aircraft suspension research.

First, both the general rail vehicle system and the aircraft runway system are quite complex. Thus, for each system, there is major emphasis on reduced degree-of-freedom models for specific purposes. Various mathematical models using combinations of the six degrees-of-freedom for rigid car bodies plus body flexibility in bending and torsion have been derived for railcar vehicles. These models have been used to compute railcar response to various inputs using sophisticated computer algorithms. One report (Reference 45) has shown that a three rigid degree-of-freedom model for the railcar suspension system does not adequately predict correct suspension design parameters. In fact, it leads to design changes that are opposite of what is necessary for system design optimization. Instead, a 15-degree-of-freedom model is used to give realistic response. In the aircraft studies encountered, only pitch, vertical, and horizontal translation are considered. The railcar suspension analysis implies that roll, yaw, and lateral degrees-of-freedom should at least be considered and are essential for unsymmetrical inputs or asymmetrical aircraft configurations. Runways are presently repaired in a manner such that the AM-2 matting is always placed over the entire lateral dimension, thus eliminating unsymmetrical inputs. However, the loading of the aircraft may be asymmetrical. The four typical configurations for a combat F-4 given by USAFE to CEEDO for computer simulation were all asymmetrically loaded. Even if the runway repairs are placed over the entire lateral dimension, at high speed, the pilot will have difficulty hitting a bump with both main gears at the same time. Additionally, runway damage caused by 5 or 6 bombs could force the use of several repairs in which the available amount of AM-2 matting may not allow repairs over the entire lateral dimension of the runway, thus resulting in unsymmetrical inputs. An analysis using the roll, yaw, and lateral degrees-of-freedom will show if a takeoff over these unsymmetrical inputs is feasible and if the pilot can maintain control.

The second major implication from railcar research is found in the use of both deterministic and random inputs. The modeling of a bump by a (1-cos) function should be sufficient for aircraft response to AM-2 matting repairs. This corresponds to deterministic transient inputs of track displacement in a majority of railcar dynamic studies. A random input could be used to model a runway that had been damaged by a strafing pass with a 20 mm or 30 mm cannon. Use of the random input in the form of spectral densities (distribution of mean square amplitude as a function of frequency) would require a completely different solution technique than the integration of the equations of motion as is done in many simulations. One paper on passenger railcar response to a random rough track input (Reference 44) uses two techniques which are applicable to the aircraft response to strafe damaged runways.

A third implication that is analogous between railcar and aircraft modeling is the use of a rigid track or runway. The rigid input assumption for runways considers only displacement inputs while the interaction of the aircraft and runway are neglected. This assumption is more than adequate for hard paved runways. The aircraft response to a soft field takeoff, however, must include the interaction between the aircraft and soft field runway. A mathematical model describing this effect is complex, but initial efforts have been started in guideway studies for vehicle elevated transit systems.

The next area of railcar research that appears applicable to aircraft studies is the use of standard linear frequency analysis techniques. These techniques yield results in one of two forms.

In one, the amplitudes of the vehicle response variables, i.e., displacements, accelerations, forces across suspension elements and between the wheel and rail, are obtained as a function of the frequency of the track input. Alternately, the power special densities (PSD's) of the same vehicle response variables are obtained in response to the PSD for the input. Frequency domain computer programs for prediction and analysis of rail vehicle dynamics have been developed (Reference 46). These programs give responses that provide a measure of passenger vibration, component life, and reliability and safety associated with vehicle lateral and roll displacement amplitudes. For aircraft response, a frequency analysis can give significant insight into the design of suspension parameters based on the frequency content of various bumps as a result of runway repair.

The fifth implication from railcar research is found in the use of computer optimization algorithm that can aid the designer for the 1990 era follow-on fighter. That is, he can choose aircraft suspension parameters (tire pressure, strut stiffness and damping, etc.) to enable the aircraft to perform satisfactorily on rough or soft runways. An initial effort to solve similar problems by utilizing optimization procedures to design a high speed railway vehicle has recently been reported (Reference 52). In this paper, constraints are put on accelerations, adequate adhesion for traction, and guidance forces, and then the speed at which the lateral instability occurs maximized. Use of a similar program for aircraft suspension can have constraints on accelerations that the pilot could sustain, and stresses that the aircraft and pylons can endure without failure.

The automotive industry has been a leader in the field of vehicle dynamics for the last 40 years. Many of the techniques developed by the industry for the analysis of road vehicles (automobiles and trucks) are directly applicable to the aircraft taxi and takeoff problem. A number of methods and results achieved are similar to those discussed in the railcar section. The bulk of the literature dealing with automobile dynamics is restricted to linear systems. Because of their highly nonlinear suspension systems and large load variations, the dynamic analysis of trucks most closely follows that of the aircraft problem. Considerable progress has been made in the areas of:

- tire behavior (References 47, 48, 49, 50, and 51)
- stability and control (References 52, 53, 54, and 55)
- driver/vehicle performance and simulators (References 56, 57, 58, 59, 60, and 61)

- and flexible vehicle structural dynamics (References 24, 62, 63, 64, 65, 66, 67, 68, 69, and 70)

Reference 28 contains many fine pre-1960 references on general topics of road vehicle suspension systems while Reference 71 contains many good references on the effects of tires and aerodynamics on the stability and control of road vehicles.

Of particular interest to the current effort is the extensive work being conducted by the automotive industry in the use of modal analysis for large complex structures. The pioneering work of Klosterman (Reference 70) allows the analytical finite element model to be replaced with a model of fewer degrees of freedom generated from experimental data. This new model can be interfaced directly with large finite element programs such as NASTRAN for further studies of the complex structure's forced response. If the structure is physically too large to test, modal synthesis techniques can be used wherein smaller components are tested and the most significant modal data retained from each component and used to synthesize the assembled structure. This concept seems to warrant considerable attention in dealing with the problem at hand.

SECTION VII

EXISTING CODES

There are a number of codes reported in the literature that are aimed at simulating the dynamic effects of taxiing aircraft. The best of all worlds would be to find one of these that would completely solve the problem at hand. Unfortunately, the codes already in being were developed for special purposes, and each has its shortcomings in addition to its good points. All is not lost, however, in that with each code comes more information. Parts of the codes may be applicable in the larger study required. This section presents some details as to published computer codes. The main thrust of the information is in broad discussions of what the codes can and cannot do. Additionally, some of the philosophy first mentioned in Section I is expanded upon. No attempt is made at providing such details as program listings, etc. For this information, the reader is directed to the original documents referenced. Primary attention will be given to digital computer programs as opposed to those developed for hybrid or analog computers such as that reported by Drevet (Reference 72). It should be noted, however, that such programs can work equally as well as a digital program.

A goal in writing a program is to obtain the least complex program that provides correct answers. The amount and placement of the required refinement is dependent on the problem being investigated and past experience. In this regard, perhaps a family of related programs is necessary. Wignot (Reference 5) reports of efforts to develop a flexible aircraft digital computer program for determining dynamic taxi design procedures. In so doing, various other programs of lesser refinement were used for exploratory and support purposes. This approach has definite merit and is a reinforcement of the building block approach.

An early effort with some applicability to an understanding of BDR effects is the work of Boozer and Butterworth (References 73 and 74). This program was developed to determine the dynamic response of a C-141A to variations in runway roughness during taxi. It uses a flexible aircraft, nonlinear single chamber landing gear, and nonlinear tire springs. Rigid body coefficients are used in a discrete fashion. The output is aircraft specific and includes time histories of:

- wing root bending moment,

- main gear strut vertical force,
- vertical acceleration at the pilot's seat,
- vertical acceleration at the aircraft center of gravity,
- and aircraft pitch angle.

The program considers only symmetrical runway forcing and, as a result, considers only three of the six rigid body motions. It cannot handle variable braking or pilot action.

The above mentioned efforts served as a precursor to the work of Gerardi and Lohwasser (References 19 and 75), wherein an attempt was made to provide a versatile program for determining the dynamic response of an aircraft to runway roughness. This code has, as a stated goal, the development of a program capable of simulating any aircraft in a reasonable amount of computer time. It is a relatively simple and quick running code that has proven popular and has provided a tool for the BDR studies of Hokanson (Reference 76) and Rollings (Reference 77). The code uses discrete input as applied to a centerline model. It has the capability of handling fuselage flexibility, nonlinear strut characteristics, a variety of metering pins or metering tubes, and either articulated or nonarticulated landing gear. The tire is modeled as a simple spring, which must stay in contact with the runway. The aerodynamic characteristics are constant for any computer run. The output is limited to time histories of:

- acceleration of the pilot section, center of gravity, and tail section,
- strut stroke and total forces in the main and nose struts,
- and horizontal position of the center of gravity and distance down the runway.

The centerline model restriction means that the lateral, roll, and yaw degrees of freedom are not included. Pilot effects are not considered. Additionally, it is only possible to predict internal loads in the landing gear assembly.

The recent effort of Kilner (Reference 78) employs a digital time history simulation to predict aircraft component loads on an airframe structure. These loads are used, in turn, in an attempt to define acceptable BDR techniques in terms of aircraft component failures. The code is tailored for F-4C and F-111 fighter aircraft, and as a result, considerable detail has been given to their construction. The program uses a combination of FORTRAN and MIMIC. The model equations are not documented in detail. However, the input is deterministic, rigid body motion is limited to horizontal, vertical, and pitch, and the output is in the form of component load ratios (ratio of computed load to design load).

It is worthwhile to reiterate a few points that are considered to be important regarding the programs found in the literature. First and foremost is their failure to include the lateral, roll, and yaw degrees of freedom. The problem is simplified by using only the symmetric degrees of freedom of a centerline model. However, antisymmetric excitation and responses contribute significantly to structural loads on the wing, and practical situations dictate that antisymmetric loadings will occur. Secondly, the pilot appears to be the forgotten man. While an open loop investigation tells us many things, some way must be found to determine that the motion is either acceptable or controllable by the pilot. An example that illustrates both points is that of a failure induced by the aircraft veering off the side of a repaired surface due to an asymmetrical encounter. The centerline model allows no lateral movement, while without some feedback to the pilot it is merely a guess that he has the time and ability to react satisfactorily. The next point is best stated as follows: Takeoff and landing are not the same. Finally, the investigator must be willing to pay the price in terms of program complexity. If the effect of external stores on a particular pylon is the critical parameter for a run, one can hardly hope to find the result from a program that does not consider wing flexibility. Again the key is "the least complex program that provides good results."

SECTION VIII

CONCLUSIONS AND RECOMMENDATIONS

From the preceding discussion, a number of conclusions can be drawn concerning any attempt to simulate an aircraft traversing a rough surface. In generating a simulation or code, the areas of interest are the input, the vehicle model, and the output.

Surface input may be made in discrete or statistical terms. The former has merit for catastrophic failure investigations and response time history generation while the latter is best for fatigue analyses, future optimization studies, and possibly for the study of cannon fire effects. For models which include the roll, yaw, and lateral degrees of freedom, care must be used such that the correct surface effects are input for each ground track.

The pilot is difficult to model accurately. He can have a profound effect when computer solutions are compared to test results. Open loop (stick free) analyses remain a fruitful avenue both for criteria generation and stability studies.

The vehicle model can be as detailed or as simple as is required by that part of the problem being investigated. Considerable detail will probably be required for the computer runs which will be compared with test results. A building block approach is needed to provide the flexibility necessary to investigate a variety of airframes. The idea is that aircraft really do not differ in terms of principal components but rather they differ in terms of detail.

Desired output drives the input and the model in terms of detail and form required. Position, velocity, and acceleration output has criteria value and is most valuable in providing test versus computed response comparisons. Loads output is of considerable value in criteria generation. Structural output has its main use in fatigue studies.

Solution techniques for a simulation are available within the state of the art. Direct integration, stability analyses, and statistical solutions can be generated depending upon the analysis under consideration.

The literature associated with vehicle dynamics is extensive and broadly based. Perhaps the two most logical references are the works of Wignot, et al (Reference 5) and Ferguson, Mollick and Kitts (Reference 14). The former is an excellent reference on the development of dynamic design procedures, and the latter contains many detailed equations relative to the dynamic landing loads problem.

Current codes available in the literature are not adequate to handle the complete runway roughness problem. A prime limitation of available codes is their inability to handle asymmetrically loaded aircraft or asymmetrical input. The elimination of the limitations does not require any state of the art improvement but rather additional application of well developed techniques.

This report has focused on the response of the aircraft. In addition to this, the development of runway roughness criteria will inherently involve tracking the structural response of the runway surface. One factor of interest is the concurrent response and potential feedback to the aircraft from the pavement itself. A second consideration is the deterioration of a BDR repair system, resulting in an increasing surface roughness with each repetition of load from takeoff and landing operations. An existing computer code capable of addressing portions of the pavement response problem is AFPAV (Reference 79). AFPAV is a static nonlinear three-dimensional finite element code capable of treating single and multi-wheeled landing gears acting on multi-layered landing surfaces. This code has been recently modified to accept data from nondestructive pavement evaluation tests and to subsequently attempt to predict the life expectancy of a normal landing surface. To estimate the life expectancy of a BDR crater repair patch, a "bowl" of crater repair material would have to be inserted into the finite element grid and an appropriate material model developed (very difficult) for the repair material. Since static loading of a repair patch is its most critical loading mode, perhaps a static axisymmetric code would be adequate for this type of analysis, again assuming an available and accurate material model.

As for pavement response feedback to the aircraft, it is not deemed feasible to develop a single computer code to simultaneously analyze the aircraft and pavement/patch response. Core limitations and computation costs dictate that the two systems be analyzed separately. Sensitivity analyses of each system to varied input from the other appear to be the most promising avenue of approach.

The following recommendations are offered:

- The development of the primary code should be made in the modern sense. Here the equations of motion of the components are handled separately, and the system is then constructed from the components.
- Roll, yaw, and lateral degrees of freedom should be included in the computer code as soon as feasible. Determination of yaw rates due to asymmetrical loading or inputs is critical in determining if the pilot can maintain the aircraft on the runway.
- An investigation should be made to determine how important the nonlinearities are in the aircraft's response.
- Careful judgment for computer correlation should be made based on the elevator position, velocity, and distance down the runway.
- The computer code should be initially large to accurately determine the aircraft's response. It then should have the capability to reduce to a lower degree-of-freedom model if that reduced model still gives accurate results (within 5 to 10 percent). Then the large number of codes required for a numerical integration scheme to a deterministic input can be run. It is quite possible that the large degree-of-freedom code will show that the aircraft response is not as violent as is predicted by those presently in being.
- Fatigue analyses should be performed. While this may be of lesser importance in a wartime mode in that projected sorties or losses dictate only a few ground-air-ground cycles, it is of central importance to any test aircraft. Pilots may be forced to make several attempts to hit multiple repairs at an exact airspeed. The necessity of fatigue analyses increases as the loads approach the limit value.

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