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FRACTURE ALONG PLANAR SLIP BANDS

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Abstract

The conditions for the propagation of a crack along an inhomogeneous, planar slip band is considered. Allowing slip to occur in the crack plane but restricting the shear displacement to the crack plane results in a relaxed "elastic-plastic" state of stress near the crack-tip. Calculations of the resulting crack-tip flow behavior indicates that once a crack with a co-planar shear band has formed, it becomes difficult to activate secondary slip with a shear displacement inclined to the crack plane and inhomogeneous slip prevails. The difficulty of activating non-coplanar secondary slip results in large hydrostatic and normal stress near the crack-tip. The combination of large shear strain due to the inhomogeneous shear band and large normal or hydrostatic stresses present near the crack-tip leads to easy crack propagation along such bands. Fracture along slip bands in Ni-, Al-, and Ti-base alloys is briefly discussed in terms of the model.

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INTRODUCTION

Fracture along planar, inhomogeneous slip bands is a common occurrence in high strength alloys. Such fracture behavior can occur under both tensile and cyclic loading conditions and has been reported in fcc, bcc, as well as hcp alloys. For example, tensile fracture along planar slip bands has been observed in Al-,[1] Ni-,[2,3] β(bcc) Ti-,[4] and α(hcp)-β Ti[5,6] alloys. Crack growth along planar slip bands under cyclic loading, e.g., crystallographic Stage I fatigue, is very common to a large number of high strength, structural alloys: Al-base,[7-9] Ni-base,[10-12] brass,[13] and α-β Ti[14-19] alloys. In all cases, the crack path occurs along planes which are subject to large shear stresses and shear strains. At the same time, this "slip band decohesion" process is usually characterized by a cleavage-like or a substantially brittle appearance. Thus, many investigators have concluded that normal stresses are also important in slip-band fracture processes. The purpose of this paper is to examine the conditions associated with the propagation of a crack along an inhomogeneous, co-planar slip band. Under such conditions, the resulting state of stress near the crack-tip is conducive both for continued localization of planar slip ahead of the crack and for the development of large normal stresses near the crack-tip. Many of the features in the fracture/fatigue studies noted above can be readily understood in terms of these concepts.

THE MODEL

Crack growth along shear bands usually occurs on planes inclined to the stress axis. Such a crack involves all three modes of crack opening (modes I, II, and III), and the stress components σ_{ij} ahead of the crack-tip is described by the well known relations of the form (see Appendix)

$$\sigma_{ij} = \sum_{\ell=1}^{\ell=3} \frac{K_\ell}{\sqrt{2\pi r}} f_{ij,\ell}(\theta) \tag{1}$$

in which K_ℓ is the stress intensity appropriate to Mode I, II, or III; r is the radial distance from the crack-tip and θ is the angular rotation around the crack-tip. Knowing the stress components of the stress tensor $\vec{\vec{\sigma}}$, the shear stress τ on a slip plane whose normal is $\vec{n}$ and the slip direction $\vec{b}$ is:[20]

$$\tau = \left(\frac{1}{b}\right) \vec{b} \cdot \vec{\vec{\sigma}} \cdot \vec{n}. \tag{2}$$

For slip which is co-planar with the crack such that the slip vector lies in the crack plane ($\theta = 0°$), eqn. (2) reduces to

$$\tau = \sigma_{xy} \sin\theta + \sigma_{yz} \cos\phi. \tag{3}$$

where the slip direction lies in the crack plane but is inclined at an angle ϕ to the crack tip (i.e., the Z axis of a crack), as shown in Fig. 1.

In the elastic region, σ_{xy} and σ_{yz} increase with decreasing r according to eqn. (1). However, once a slip band is initiated, eqn. (3) obeys $\tau = \tau_f$ where τ_f is the flow stress of the material. The presence of the co-planar slip band thus roughly fixes the values of $\sigma_{xy} (= \sigma^*_{xy})$ and $\sigma_{yz} (= \sigma^*_{yz})$ as flow occurs (i.e., $r \leq r_p$). This is shown in Fig. 1 for a material of negligibly small work hardening.

a. <u>On the Continuation of Slip Localization</u>

Once a crack with a co-planar slip band has formed, its very presence acts to restrict the possibility of secondary slip. Thus slip ahead of the crack tends to remain localized on the plane of the crack. This can be seen by considering the following. At $r \leq r_p$ and $\theta = 0$ the stress tensor for secondary slip in a direction inclined to the primary slip band is:

$$\vec{\vec{\sigma}} = \begin{bmatrix} \sigma_{xx} & \sigma^*_{xy} & 0 \\ \sigma^*_{xy} & \sigma_{yy} & \sigma^*_{yz} \\ 0 & \sigma^*_{yz} & \sigma_{zz} \end{bmatrix} \tag{4}$$

Since plastic relaxation has roughly fixed the values of σ^*_{xy} and σ^*_{yz} and because $\sigma_{xz} = 0$ at $\theta = 0°$, the only stress components which can increase at $r < r_p$ are

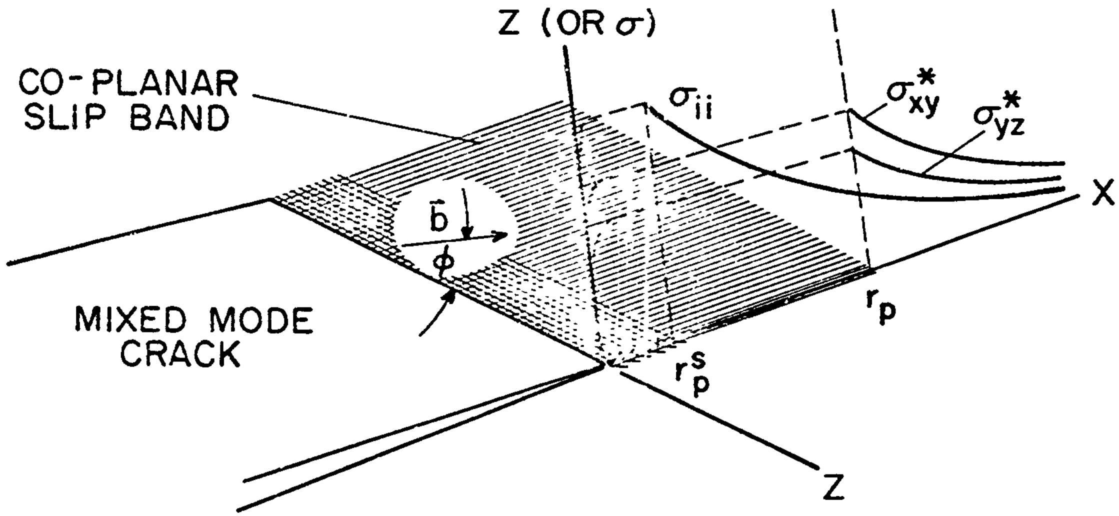

Fig. 1. *A schematic illustration of the stress state ahead of a mixed mode crack with a co-planar slip band characterized by a slip vector $\vec{b}$ at an angle ϕ to the crack front. The co-planar slip extends to r_p while secondary slip is activated to r_p^s.*

σ_{xx}, σ_{yy}, and σ_{zz}. Thus, the increase in shear stress necessary for secondary out-of-plane slip must come from the normal stress components. Furthermore, if slip is confined to a band co-planar with the crack with a slip vector in the crack plane, the normal stresses remain elastic and continue to obey the stress singularity given by eqn. (1)*. At $\theta = 0°$ and under conditions of plane strain, eqn. (1) reduces to: $\sigma_{xx} = \sigma_{yy} = 3/2\ \sigma_{zz}$, where $\sigma_{ii} \propto 1/\sqrt{r}$. This state of nearly balanced triaxial stress is obviously not conducive for easy activation of secondary slip ahead of the crack-tip. On the other hand, it is well suited for the development of large elastic normal or hydrostatic stress before secondary slip and plastic relaxation out-of-the-crack plane can occur. This is also shown schematically in terms of the component in Fig. 1.

The above arguments can be quantified somewhat by considering the equivalent stress $\bar{\sigma}$. For secondary slip to be activated in the plane of the crack ($\theta = 0°$) by the state of stress given by eqn. (4) under plane strain conditions we have:

$$\bar{\sigma} \simeq \left[\frac{\sigma_{yy}^2}{9} + 3[(\sigma_{xy}^*)^2 + (\sigma_{yz}^*)^2]\right]^{\frac{1}{2}}. \tag{5}$$

As discussed, plastic relaxation determines σ_{xy}^* and σ_{yz}^* at $r \lesssim r_p$. In eqn. (5) only the increase of σ_{yy} with decreasing r acts to increase to activate secondary slip at r_p^s (see Fig. 1). Thus, the presence of an inhomogeneous slip band co-planar with a crack hinders the activation of secondary slip, making homogeneous flow even more difficult.

The plastic zone size in the plane of the crack due to secondary slip r_p^s can be calculated by providing a yield criterion for secondary slip and substituting the appropriate stresses into eqn. (5). For example, the simple case of multiple

*In many instances, there is bound to be a limited degree of plastic relaxation of the σ_{ii} stress components due to some secondary slip within the inhomogeneous shear bands activated by work hardening within the band. In this case the σ_{ii} would not obey eqn. (1) but rather an elastic-plastic solution probably typical of a material of very high and exhaustion-type work hardening. The normal stresses would still be expected to increase to a substantial level before profuse secondary slip is activated near the crack-tip, and thus the basis of our discussion still applies although the scale of the effects will not be as great.

secondary slip based on the Von Mises yield criteria will be considered. Setting the equivalent stress $\bar{\sigma}$ *equal to the flow stress for homogeneous yielding* σ_{ys} *(Von Mises Yield Criteria) and substituting* $\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r_p^S}}$ *(at* $\theta = 0°$*) into eqn. (5), an expression for* r_p^S *is obtained:*

$$r_p^S = \frac{\sigma^2 \;\; a \cos^4\psi}{18 \;[\sigma_{ys}^2 - 3(\sigma_{xy}^{*2} + \sigma_{zy}^{*2})]} \tag{6}$$

where ψ *is the angle between the stress axis and the y axis of the crack (see Fig. A-1). The plastic zone size for the inhomogeneous co-planar slip band is given by*

$$r_p = \frac{\sigma^2 \;\; a \cos^2\psi \;(\cos\eta\sin\phi + \cos\xi\cos\phi)^2}{2 \;\; \tau_f^2} \tag{7}$$

This equation is obtained by substituting τ_f, $K_{II}/\sqrt{2\pi r_p}$ *and* $K_{III}/\sqrt{2\pi r_p}$ *for* τ, σ_{xy}^* *and* σ_{yz}^* *respectively into eqn. (3). The angles* η *and* ξ *refer to the angles between the stress axis and the x and z axes, respectively, of the crack (see Fig. A-1). The plastic zone size ratio defined as* r_p^S/r_p *can then be obtained by dividing eqn. (6) by eqn. (7).*

$$\frac{r_p^S}{r_p} = \left(\frac{\tau_f}{\sigma_{ys}}\right)^2 \frac{\cos^2\psi}{9\left[1 - \frac{3(\sigma_{xy}^{*2} + \sigma_{yz}^{*2})}{\sigma_{ys}^2}\right]\left[\cos\eta\sin\phi + \cos\xi\cos\phi\right]^2} \tag{8}$$

Since the relaxed shear stresses σ_{xy}^* *and* σ_{yz}^* *are governed by* τ_f *through eqn. (3), the plastic zone size ratio is therefore a function of the flow stress/yield stress ratio* τ_f/σ_{ys} *as well as the crack orientation.*

The magnitude of the plastic zone size ratio (eqn. (8)) is an indication of whether homogeneous or inhomogeneous slip should occur at the tip of a moving crack. When $r_p^S/r_p \ll 1$*, inhomogeneous slip is expected and vice versa. Thus once a crack with a co-planar, inhomogeneous slip band is initiated, according to eqn. (8), a transition to homogeneous crack-tip deformation can only occur if: (1) the flow stress* τ_f *within the co-planar slip band increases rapidly with strain due to work hardening (note: this would also serve to increase* σ_{xy}^* *and* σ_{yz}^**) and/or (2)* *if the crack changes orientations causing the angular terms in eqn. (8) to change. However, the high strength alloys (Ti-, Ni-, or Al-base alloys) which are susceptible to cracking along slip bands are known to have very small work hardening rates. The crystallography of deformation in some hcp alloys also makes* τ_f *considerably smaller than* σ_{ys} *(contrast the ease of basal slip versus the difficulty of extension along the C axis if twinning is difficult). Finally, the mode of stressing can in some cases cause deformation-induced softening (for example, cyclic softening which is often observed in the low cycle fatigue of high strength alloys) which will also cause a decrease of the* (τ_f/σ_{ys}) *ratio and result in a continuation of inhomogeneous slip co-planar with a crack and propagation above that slip band.*

An illustration of the dependence of continued slip localization (in terms of the plastic zone size ratio, eqn. (8)) on the flow/yield stress ratio is shown in Fig. 2. Two mixed mode I and II cracks are considered, one inclined at 45° and the other at 75° to the stress axis. Fig. 2 depicts the conditions for homogeneous and inhomogeneous crack-tip deformation as well as the existence of a critical flow/yield stress ratio $(\tau_f/\sigma_{ys})_c$ *necessary for homogeneous slip. Above* $(\tau_f/\sigma_{ys})_c$*, the size of the multiple slip plastic zone* r_p^s *exceeds that of the inhomogeneous slip band and* $r_p^s/r_p > 1.0$ *and homogeneous crack-tip deformation dominates. However, when the shear stress component acting on a crack is large (for example, when a crack is inclined* $\sim$*45° to the stress axis),* $(\tau_f/\sigma_{ys})_c$ *is also large (see Fig. 2), making homogeneous slip difficult. This tendency for continued flow localization for a 45° crack is especially pronounced when* $\phi = 90°$*; e.g., when the slip vector in the inhomogeneous slip band is perpendicular to the crack tip in Fig. 1.*

The preceding discussion is based on a Mises yield condition for homogeneous slip. While such a criteria lends itself to an easy illustration of the flow localization concept, it is a rather rigorous requirement for homogeneous slip.

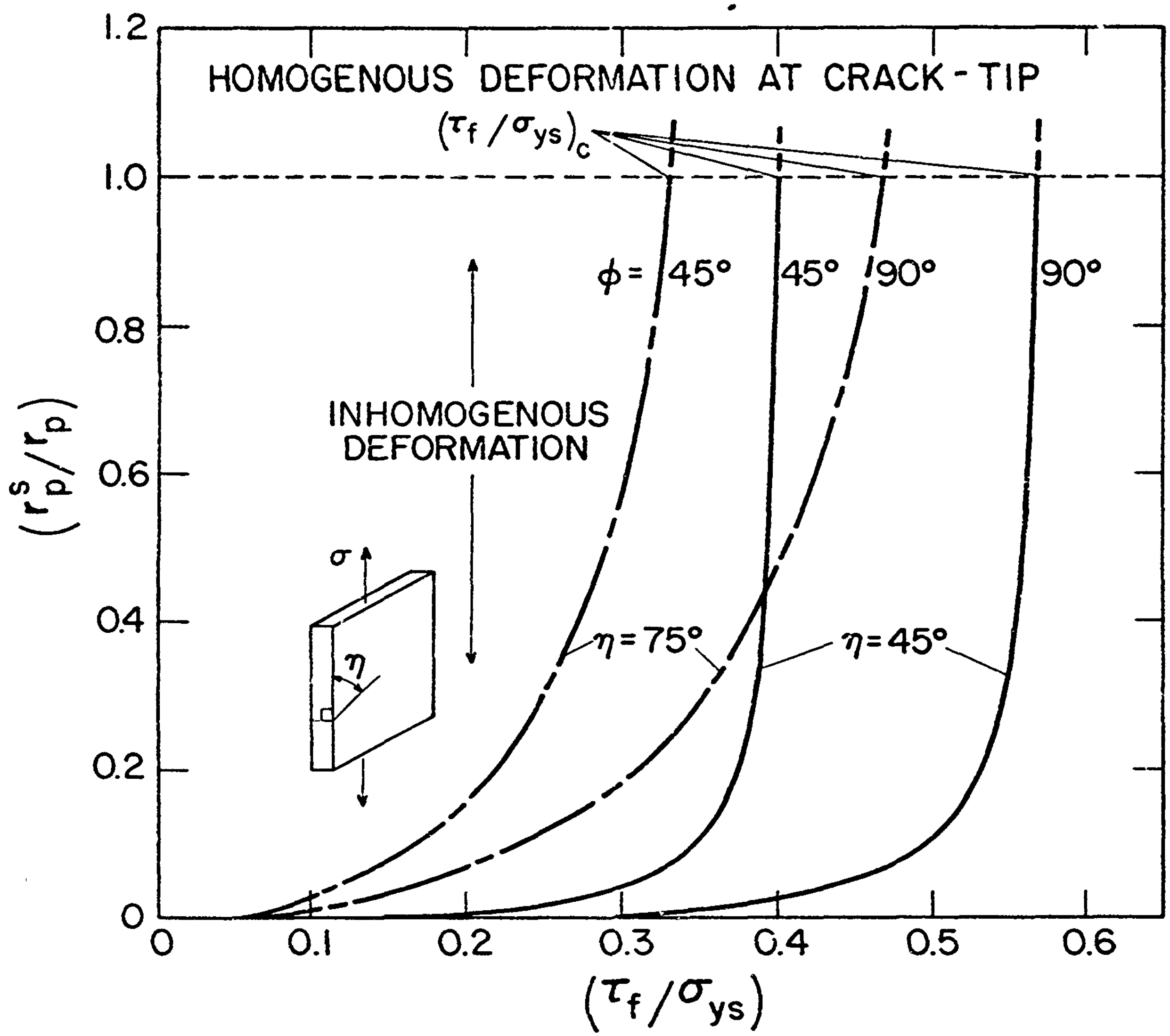

Fig. 2. The cxtent of secondary, non-coplanar slip, as expressed in terms of the ratio r_p^s/r_p, as a function of the ratio of the flow stress τ_f for primary, co-planar slip to the Mises yield stress σ_{ys} for secondary slip.

In many cases, a sufficient criteria may be the activation of a second deformation system with a shear vector inclined to the crack plane. However, to treat this case the specific orientation of the shear plane and direction must be known. Conceptually the results will take the same form as those in Fig. 2 and eqns. (6) and (8), and the predicted trends will be the same.

b. On the Development of Large Hydrostatic and Normal Stresses

Confining the slip vector of the shear band to a plane which is co-planar with the crack results in the state of stress ahead of the crack-tip given by eqn. (4). The hydrostatic stress component σ_H *remains elastic until secondary slip is activated at* r_p^s. *Thus, in the crack plane:*

$$\sigma_H = \frac{1}{3}(\sigma_{xx} + \sigma_{yy} + \sigma_{zz}) \simeq \frac{8}{9}\frac{K_I}{\sqrt{2\pi r_p^s}} \quad , \tag{9}$$

where K_I *is the mode I stress intensity factor (see Appendix). It should be noted that the stress normal to the slip band is also quite large because of the previously discussed constraints acting on the stress tensor* $\vec{\vec{\sigma}}$ *in eqn. (4). The value of* r_p^s *can be quite small compared to* r_p *for the primary slip band. Thus the very elastic-plastic conditions which aid in localizing flow into a band co-planar with the crack also result in a large hydrostatic (or normal) stress near the crack-tip.*

The influence of the elastic-plastic state of stress on the development of large normal stresses near the crack-tip is illustrated in Fig. 3. In this case, the maximum principal stress σ_1 *is calculated in terms of the yield stress for three different slip situations governing a 45° mixed mode I and II crack. In this case, solving the three dimensional state of stress gives* $\sigma_1 = \sigma\sqrt{a/2r}$ *under conditions of plane strain. Since work hardening is assumed to be negligibly small, the maximum value of* σ_1 *occurs near* r_p^s. *Fig. 3 shows that with the slip vector confined to the co-planar slip band at* $r > r_p^s$, *there is a large difference*

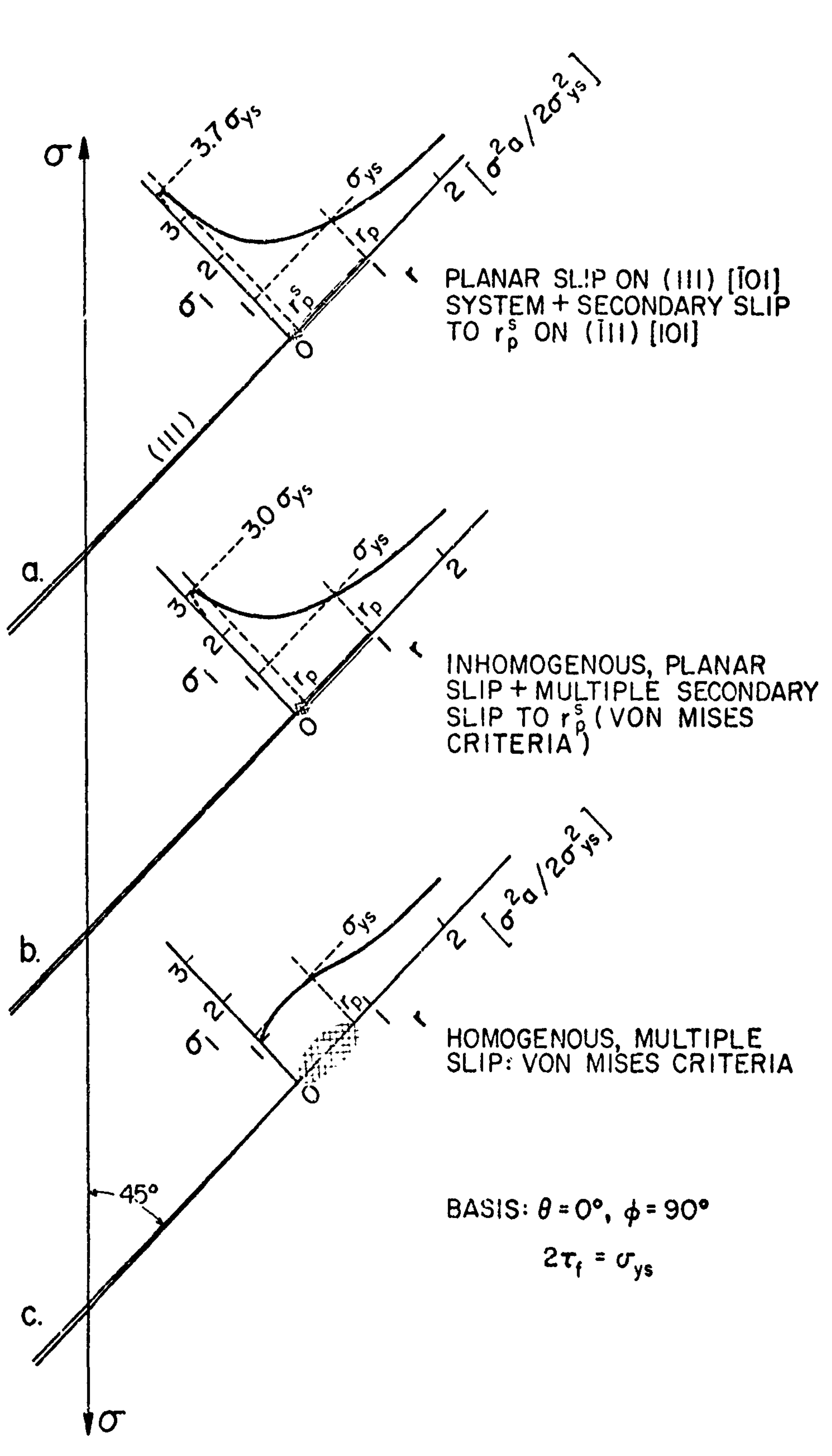

Fig. 3. The approximate distribution of the maximum principal stress σ_1 ahead of a mixed mode I and II crack assuming the relaxed elastic-plastic state of stress. Three different conditions for crack-tip plasticity are considered.

in stress states between multiple and planar slip conditions. For homogeneous, multiple slip (Fig. 3c), the plastic constraint factor σ_1/σ_{ys} is only 1.13, and resistance to crack propagation is expected to be high.

Inhomogeneous, planar slip results in very high plastic constraint ratios, and two examples are illustrated in Figs. 3a and 3b. Fig. 3b shows a generalized case where multiple secondary slip obeys a Mises yield condition. To reduce the extremity of the criteria, the stress requirement for secondary slip has been minimized by taking $\bar{\sigma} = \sigma_{ys} = 2\tau_f$. Nonetheless, $\sigma_1/\sigma_{ys} = 3$, showing considerable normal stress intensification due to planar slip. A more rigorous requirement for multiple secondary slip would be to recognize that the Taylor factor for tensile polycrystalline deformation of cubic metals is about 3.[20] Taking $\sigma_{ys} = 3\tau_f$, this would lead to $\sigma_1/\sigma_{ys} = 7.3$ in Fig. 3b. We believe this criteria for r_p^s is too restrictive and overestimates the magnitude of the normal stresses.

Fig. 3a depicts the fcc single crystal case of a crack propagating along the (111) plane in the $[\bar{1}01]$ direction, which is a good description of Stage I fatigue fracture in Ni-[10-12] and Al-[4] base alloys. In this case calculations based on eqns. (3) and (4) [assuming equal yield stress on all {111} slip systems but negligibly small work hardening and no plastic relaxation effects] show that: $r_p^s/r_p \simeq .07$. Thus secondary slip is very difficult, and the resulting plastic constraint ratio is quite high (3.67). It should be no surprise that such a shear-induced fracture process should have cleavage-like fracture appearance characteristic of large normal stresses even in alloys not susceptible to cleavage fracture in tension. This is precisely what appears to happen in Stage I fatigue behavior of Ni-base superalloys.[10-12]

The model is obviously very sensitive to the ease of slip on secondary slip systems and to the planar nature of the slip ahead of an advancing crack. Thus taking into account thermally activated slip processes, comparisons of Fig. 3a to Fig. 3c clearly indicate why Ni-base superalloys when tested at high strain rates and low temperatures are conducive to tensile behavior described by planar slip and crystallographic cracking along inhomogeneous slip bands.[3,12] The presence of "45° fracture" occurring on slip planes in a substantially brittle manner has also been reported in high strength Al-Zn-Mg-Cu alloy.[1] Unstable shear and fracture along intense slip bands is well known to occur in α-β Ti alloys.[5,6,14-19] The previous discussion briefly explained why such cracking should be expected along the basal slip planes. This is discussed more extensively in a separate paper examining Stage I fatigue crack propagation in Ti and other alloys.[22]

SUMMARY

On a very simplified basis, we have shown that the propagation of a crack along a co-planar shear band results in conditions which aid continued shear localization and, at the same time, create large normal or hydrostatic stresses near a crack-tip. While the analysis presented is strictly not correct for a work hardening material, it establishes guidelines for understanding crack propagation along intense shear bands. For example:

1. The large shear strain and hydrostatic/normal stress accompanying crack growth along an inhomogeneous slip band produces strain/stress conditions favorable to crack advance. Thus crack propagation along a shear band should be easy and local fracture toughness low. This is observed in Ti alloys. Such fracture is sometimes characterized by a cleavage-like fracture appearance suggesting the presence of large normal stresses.

2. As discussed previously, certain important features describing the tensile or fatigue fracture of high strength Ni-, Al-, and Ti-base alloys along slip bands appear to be a natural consequence of the relaxed "elastic-plastic" state of stress described herein. There is no need to invoke specific dislocation arrangements

to create a stress state conducive for fracture along inhomogeneous slip bands or for "slip band decohesion."

3. Any slip band softening which might occur as in the shearing of penetrable precipitates, will serve to lower σ^*_{xy} *and* σ^*_{yz} *and will therefore accentuate the effects discussed. This is expected to occur in low cycle fatigue of high strength alloys.*

4. The initial conditions for flow localization ahead of a crack depend on the state of stress ahead of the crack-tip and therefore on the orientation of the crack. For example in single grain of an hcp alloy in which crack propagation occurs on the basal plane, co-planar basal slip is preferred by the state of stress for a mixed mode crack in Fig. 3, but non co-planar pyramidal slip (or twinning) occurs when the basal plane is perpendicular to the stress axis. Therefore, the fracture resistance of individual grains should be orientation dependent. Using the hcp alloy example, if basal cleavage is difficult, propagation of a mode I basal crack (i.e., crack is perpendicular to the stress axis) should also be relatively difficult, which has been observed.

5. On the basis of the above discussion with regard to inhomogeneous co-planar vs. multiple slip behavior characterized by Fig. 3, shear fracture along slip bands should always be easier than that fracture characterized by mode I cracking and homogeneous crack-tip plasticity. However, tortuous crack paths, crack branching, and renucleation of shear band cracks at boundaries can increase the overall fracture resistance of an alloy susceptible to slip band fracture. It should be recognized that the conditions of the development of inhomogeneous shear bands with a crack initiating along such a band are not the subject of this discussion.

In conclusion, crack propagation along an inhomogeneous shear band which is co-planar with the crack has been considered. A simple analysis of the interaction of an "elastic-plastic" crack-tip stress field with slip ahead of the crack-tip *shows that: (a) once a crack with a co-planar slip band has formed, activating secondary slip (with a shear displacement component perpendicular to the crack plane) is difficult, and (b) the difficulty of activating secondary slip in turn results in large hydrostatic and normal stresses near the crack-tip. The combination of inhomogeneous shear and large normal stresses results in easy crack propagation and low toughness usually, but not always, associated with inhomogeneous shear band cracking. Many experimental characteristics of fracture along slip bands in high strength Ni-, Al-, and Ti-base alloys are readily interpreted in terms of the analysis.*

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APPENDIX

Figure A1 shows a mixed-mode I, II and III crack inclined to the stress axis Y' in a manner such that Y' makes angles η, ψ and ξ with the X, Y and Z axes of the crack respectively. Let σ be the tensile stress in the Y' direction, then the stress transformation of σ into the X Y Z coordinate system would be:

$$\sigma_{yy} = \sigma \cos^2\psi$$
$$\sigma_{yx} = \sigma \cos\psi\cos\eta \qquad \text{Eq. (A1)}$$
$$\sigma_{yz} = \sigma \cos\psi\cos\xi$$

Hence:

$$K_I = \sigma \cos^2\psi \sqrt{\pi a}$$
$$K_{II} = \sigma \cos\psi\cos\eta \sqrt{\pi a} \qquad \text{Eq. (A2)}$$
$$K_{III} = \sigma \cos\psi\cos\xi \sqrt{\pi a}$$

The stress field for the mixed-mode (I, II and III) crack under plane strain is:[23]

$$\sigma_{xx} = \frac{\sigma\sqrt{\pi a}}{\sqrt{2\pi r}}\{\cos^2\psi\cos\frac{\theta}{2}(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}) - \cos\eta\cos\psi\sin(\frac{\theta}{2})(2+\cos\frac{\theta}{2}\cos\frac{3\theta}{2})\}$$

$$\sigma_{yy} = \frac{\sigma\sqrt{\pi a}}{\sqrt{2\pi r}}\{\cos^2\psi\cos\frac{\theta}{2}(1+\sin\frac{\theta}{2}\sin\frac{3\theta}{2}) + \cos\eta\cos\psi(\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2})\}$$

$$\sigma_{zz} = \nu(\sigma_{xx} + \sigma_{yy}) \qquad \text{Eq. (A3)}$$

$$\sigma_{xy} = \frac{\sigma\sqrt{\pi a}}{\sqrt{2\pi r}}\{\cos^2\psi\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2} + \cos\eta\cos\psi\cos\frac{\theta}{2}(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2})\}$$

$$\sigma_{xz} = -\frac{\sigma\sqrt{\pi a}}{\sqrt{2\pi r}}\{\cos\psi\cos\xi\sin\frac{\theta}{2}\}$$

$$\sigma_{yz} = +\frac{\sigma\sqrt{\pi a}}{\sqrt{2\pi r}}\{\cos\psi\cos\xi\cos\frac{\theta}{2}\}$$

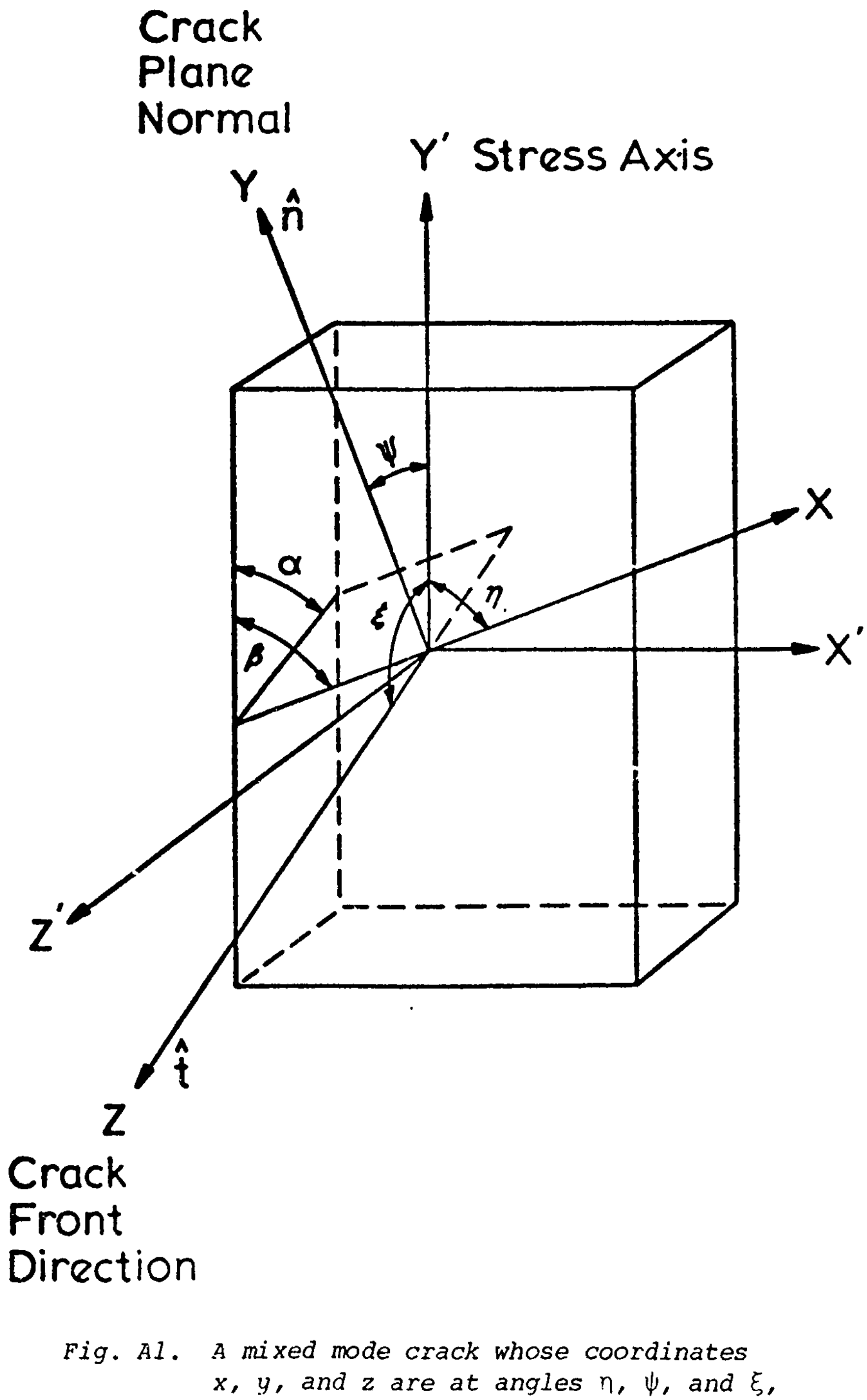

Fig. A1. A mixed mode crack whose coordinates x, y, and z are at angles η, ψ, and ξ, resp., to the stress axis of the specimen.

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The conditions for propagation of a crack along an inhomogeneous slip band is considered. Allowing slip to occur in the crack plane but restricting the shear displacement to the crack plane results in a relaxed "elastic-plastic" state of stress near the crack-tip. Calculations of the resulting crack-tip flow behavior indicate that once a crack with a co-planar shear band has formed, it becomes difficult to activate secondary slip with a shear displacement inclined to the crack plane and inhomogeneous (cont'd)

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slip prevails. The difficulty of activating non-coplanar secondary slip results in large hydrostatic and normal stress near the crack-tip. The combination of large shear strain due to the inhomogeneous shear band and large normal or hydrostatic stresses present near the crack-tip leads to easy crack propagation along such bands. Fracture along slip bands in Ni-, Al-, and Ti-base alloys is briefly discussed in terms of the model.