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In-House Report
January 1979

RADIATION PATTERN SIDELOBES AND NULL FILLING PRODUCED BY AIRCRAFT VIBRATIONS

Ronald L. Fante

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ROME AIR DEVELOPMENT CENTER
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We have studied the effect of the temporal vibrations of an array of radiators on their radiation pattern. Results are obtained for both large and small vibration amplitudes in the limit when the vibrations are periodic. In general the vibration-induced sidelobe levels are found to depend on the size of the vibration in wavelengths, the spatial distance over which the vibrations are correlated, and the number of elements in the array.

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Radiation Pattern Sidelobes and Null Filling Produced by Aircraft Vibrations

1. INTRODUCTION

Presently there is considerable activity devoted to the development of very low sidelobe antennas and antennas with movable nulls. Such systems clearly can work well if ground-based but when mounted on aircraft there must be some concern about the effect of aircraft vibrations on the radiation pattern. In this report we have developed a general formalism to consider this problem by considering the radiation pattern of an arbitrary array of radiating elements which is subjected to vibrations. In Section 3 and Appendix A, we treat the case of small (compared with a wavelength) amplitude harmonic vibrations, and in Section 4 we study the situation when the vibration amplitudes are comparable with or greater than a wavelength.

2. ANALYTICAL PRELIMINARIES

The scalar field, $u(\underline{r}, t)$, produced at a point $\underline{r}$ by a source distribution $f(\underline{s}, t)$ can be written[1] as

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(Received for publication 23 January 1979)

1. Panofsky, W., and Phillips, M. (1955) Classical Electricity and Magnetism, Addison-Wesley, Reading.

$$u(\underline{r}, t) = \frac{1}{4\pi} \iiint \frac{f\left[\underline{s}, t - \frac{|\underline{r} - \underline{s}|}{c}\right]}{|\underline{r} - \underline{s}|} d^3s \quad , \tag{1}$$

where c is the speed of light and the integral is over all space. We now assume that the source is an arbitrary collection of $N_1 + N_2 + 1$ moving monochromatic point sources. We can then write

$$f(\underline{s}, t) = R_e \sum_{n=-N_1}^{N_2} a_n e^{i\omega_o t} \delta[\underline{s} - \underline{s}_n(t)] \quad , \tag{2}$$

where R_e denotes "real part of," a_n is the complex amplitude of the n^{th} point source, $\underline{s}_n(t)$ is its position and $\delta(\ldots)$ is the Dirac delta function. If we assume that the source motion is non-relativistic, we may approximate

$$\delta\left[\underline{s} - \underline{s}_n\left(t - \frac{|\underline{r} - \underline{s}|}{c}\right)\right] \simeq \delta[\underline{s} - \underline{s}_n(t)] \quad .$$

Then, upon inserting (2) into (1) we get

$$u(\underline{r}, t) \simeq R_e \sum_{n=-N_1}^{N_2} a_n \frac{\exp\left\{i\omega_o\left(t - \frac{|\underline{r} - \underline{s}_n|}{c}\right)\right\}}{4\pi|\underline{r} - \underline{s}_n(t)|} \quad . \tag{3}$$

If we now assume that the observation point is in the Fraunhofer zone of the array of sources, we can approximate (3) as

$$u(\underline{r}, t) \simeq R_e \left\{ \frac{\exp\left[i\omega_o\left(t - \frac{r_o}{c}\right)\right]}{4\pi r_o} \sum_{n=-N_1}^{N_2} a_n e^{i\underline{k}_o \cdot \underline{r}_n(t)} \right\} \quad , \tag{4}$$

where r_o is the distance from some reference point O (chosen roughly at the center of the non-vibrating array) to the observation point, as shown in Figure 1. Also $\underline{r}_n$ is the distance from 0 to the nth point source and $\underline{k}_o = \omega_o/c\ \hat{k}$ where $\hat{k}$ is a unit vector in the direction of the observer. We shall next decompose $\underline{r}_n$ as

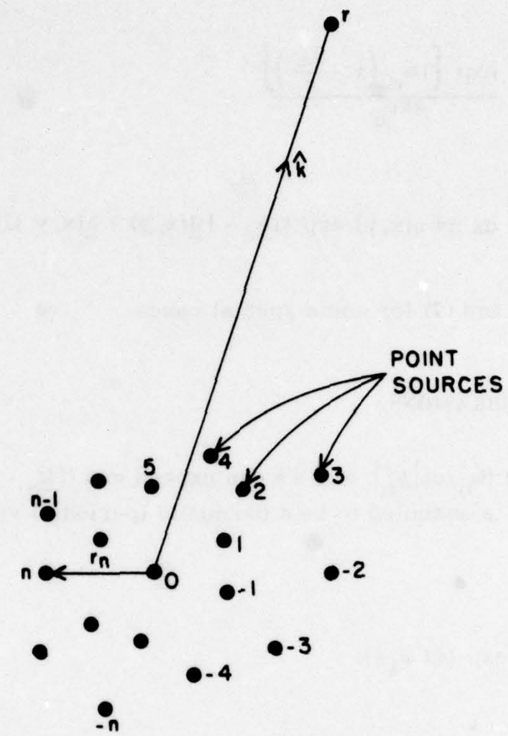

Figure 1. Three Dimensional Array of Vibrating Sources

$$\underline{r}_n = \underline{R}_n + \underline{\delta}_n(t) \tag{5}$$

where $\underline{R}_n$ is the location of the n^{th} point source in the absence of vibration and $\delta_n(t)$ is the perturbation in its position caused by vibrations. We can then write

$$u(\underline{r}, t) = R_e \frac{\exp\left[i\omega_o\left(t - \frac{r_o}{c}\right)\right]}{4\pi r_o} \sum_{n=-N_1}^{N_2} a_n \exp\left\{i\underline{k}_o \cdot \left[\underline{R}_n + \underline{\delta}_n(t)\right]\right\} \quad . \tag{6}$$

We can also obtain results for a continuous source distribution. For example, if the unperturbed antenna is planar we get

$$u(\underline{r}, t) = R_e \frac{\exp\left[i\omega_o\left(t - \frac{r_o}{c}\right)\right]}{4\pi r_o}$$

$$\iint_S dx\, dy\, a(x, y) \exp\{i\underline{k}_o \cdot [\underline{R}(x, y) + \underline{\delta}(x, y, t)]\} \quad . \tag{7}$$

We will now study (6) and (7) for some special cases.

3. SMALL HARMONIC VIBRATIONS

If we assume that $(\omega_o/c)|\underline{\delta}_n| \ll 1$ we can expand $\exp(i\underline{k}_o \cdot \underline{\delta}_n)$ in a Taylor series. Also if $\underline{\delta}_n(t)$ is assumed to be a harmonic (periodic) vibration we may write

$$\underline{\delta}_n(t) = \sum_{\ell=-\infty}^{\infty} \underline{\delta}_{n\ell} \exp\{i\ell\omega_1 t\} \tag{8}$$

where ω_1 is the lowest vibration harmonic, $\underline{\delta}_{n,-\ell} = \underline{\delta}^*_{n\ell}$ because $\underline{\delta}_n$ is real and $\underline{\delta}_{n0} = 0$ because $\underline{\delta}_n$ is assumed to be the displacement about the average position $\underline{R}_n$. If we expand $\exp(i\underline{k}_o \cdot \underline{\delta}_n)$ in a Taylor series, retaining only the first two terms, substitute (8) and then take the real part of the result we obtain from (6)

$$v(\underline{r}, t) = \sum_{n=-N_1}^{N_2} |a_n| \cos\left[\omega_o\left(t - \frac{r_o}{c}\right) + \underline{k}_o \cdot \underline{R}_n + \psi_n\right]$$

$$- \sum_{n=-N_1}^{N_2} \sum_{\ell=-\infty}^{\infty} |a_n|(\underline{k}_o \cdot |\underline{\delta}_{n\ell}|) \sin[(\omega_o + \ell\omega_1)t + \beta_{n\ell}] \quad , \tag{9}$$

where $v(\underline{r}, t) = 4\pi r_o\, u(\underline{r}, t)$, ψ_n is the phase of a_n, $\beta_{n\ell} = \psi_n + \underline{k}_o \cdot \underline{R}_n + \Phi_{n\ell} - \omega_o r_o/c$ and $\Phi_{n\ell}$ is the phase of $\underline{\delta}_{n\ell}$.

We next assume that the radiation is measured by the receiver system shown in Figure 2. For mathematical simplicity we shall assume that the filter passes all frequencies from $\omega_o - M_1\omega_1$ to $\omega_o + M_2\omega_1$ and completely rejects all others. In this case the output of the filter is given by (9) except with the summation on ℓ running from $-M_1$ to M_2 instead of from $-\infty$ to ∞. Also if the integration time T is such that $T \gg 1/\omega_1$, and $T \gg 1/\omega_o$ the output of the integrator (ignoring constants of proportionality) when the receiver is at (θ, ϕ) is

$$P(\theta,\phi) = \frac{1}{T}\int_0^T v^2\, dt = \frac{1}{2}\sum_{n=-N_1}^{N_2}\sum_{s=-N_1}^{N_2} |a_n|\,|a_s| \cos [\psi_n - \psi_s + \underline{k}_o \cdot \underline{R}_n - \underline{k}_o \cdot \underline{R}_s] + \frac{1}{2}\sum_{n=-N_1}^{N_2}\sum_{s=-N_1}^{N_2}\sum_{\ell=-M_1}^{M_2} |a_n|\,|a_s|(\underline{k}_o \cdot |\underline{\delta}_{n\ell}|)(\underline{k}_o \cdot |\underline{\delta}_{s\ell}|) \cos(\beta_{n\ell} - \beta_{s\ell}) \quad . \tag{10}$$

The result in (10) is, of course, the time-averaged power pattern. This quantity is more conveniently written as

$$P(\theta,\phi) = \frac{1}{2}\left|\sum_{n=-N_1}^{N_2} a_n e^{i\underline{k}_o \cdot \underline{R}_n}\right|^2 + \frac{1}{2}\sum_{\ell=-M_1}^{M_2}\left|\sum_{n=-N_1}^{N_2} a_n(\underline{k}_o \cdot \underline{\delta}_{n\ell})\, e^{i\underline{k}_o \cdot \underline{R}_n}\right|^2 . \tag{11}$$

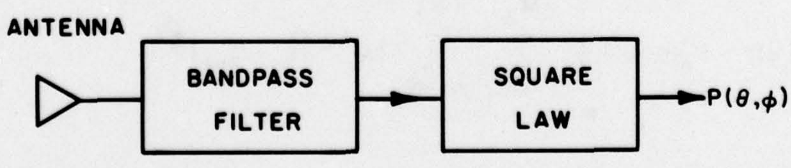

Figure 2. Assumed Receiver System (Located at θ, ϕ)

The first term in (11) is the time-average power $P_o(\theta, \phi)$ which would be received in the absence of vibration. Therefore, the second term in (11) represents the correction to the received power due to the vibration of the point sources.

It is interesting to examine (11) in some special limiting cases. Let us first assume that all the sources vibrate with the same amplitude and phase, so that $|\underline{\delta}_{n\ell}| = |\underline{\delta}_\ell|$ and $\Phi_{n\ell} = \Phi_\ell$. In this case (11) becomes

$$P(\theta, \phi) = P_o(\theta, \phi)[1 + g(\theta, \phi)] \quad , \tag{12}$$

where

$$P_o(\theta, \phi) = \frac{1}{2} \left| \sum_{n=-N_1}^{N_2} a_n e^{i\underline{k}_o \cdot \underline{R}_n} \right|^2 \quad , \tag{13}$$

$$g(\theta, \phi) = 1 + \sum_{\ell=-M_1}^{M_2} |\underline{k}_o \cdot \underline{\delta}_{n\ell}|^2 \quad . \tag{14}$$

The result in (12) means that the locations of the nulls in $P_o(\theta, \phi)$ are not affected by a uniform vibration in which all of the sources vibrate in phase.

A more interesting result occurs when the point sources do not vibrate in phase, but rather the phase of the vibration of each point source is random and uncorrelated. If the ℓ^{th} harmonic phase, $\Phi_{n\ell}$, of the vibration of the n^{th} source is independent of the phase, $\Phi_{s\ell}$, of the s^{th} source we have that the compound probability density is

$$p_2(\Phi_{n\ell}, \Phi_{s\ell}) = p_1(\Phi_{n\ell})\, p_2(\Phi_{s\ell}) \quad . \tag{15}$$

If we further assume that the phases are uniformly distributed, so that $p_1(\Phi) = 1/2\pi$ for $0 \le \Phi \le 2\pi$, we obtain from (11) for the ensemble averaged received power

$$\langle P(\theta, \phi) \rangle = P_o(\theta, \phi) + \frac{1}{2} \sum_{\ell=-M_1}^{M_2} \sum_{n=-N_1}^{N_2} |a_n|^2 \, |\underline{k}_o \cdot \underline{\delta}_{n\ell}|^2 \quad . \tag{16}$$

Finally, if in addition to the random phase, the direction* of the perturbation is also random we must average (16) over all directions. Upon using the result that

$$\frac{1}{4\pi}\int_0^{2\pi} d\phi' \int_0^{\pi} \sin\theta'\, d\theta'\, (\underline{k}\cdot\underline{\delta})^2 = \frac{1}{3}k^2\delta^2 \quad , \tag{17}$$

where θ' is the angle between $\underline{k}$ and $\underline{\delta}$ we find

$$\langle P(\theta,\phi)\rangle = P_o(\theta,\phi) + \frac{k_o^2}{6}\sum_{\ell=-M_1}^{M_2}\sum_{n=-N_1}^{N_2} |a_n|^2|\delta_{n\ell}|^2 \quad . \tag{18}$$

From (18) it is clear that if the peak of the radiation pattern is assumed to be at $\theta = \phi = 0$, the average sidelobe level produced by the antenna vibrations is

$$SL = \frac{k_o^2}{3}\,\frac{\sum_{n=-N_1}^{N_2}|a_n|^2\sum_{\ell=-M_1}^{M_2}|\delta_{n\ell}|^2}{\left(\sum_{n=-N_1}^{N_2}|a_n|\right)^2} \quad . \tag{19}$$

Therefore, when the phases are random and uncorrelated, the vibrations produce an average sidelobe level which can fill in the nulls in the radiation pattern $P_o(\theta,\phi)$, and also alter the relative sidelobe level for the case of a very low sidelobe antenna. As is clear from (19), the amount of null filling will depend on the aperture taper (that is, the distribution of a_n) and the amplitude in wavelengths of the vibrations.

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*We assume that all the sources vibrate along the same axis, but that the alignment of that axis is completely random with respect to the vector k_o.

We can also express (19) in terms of the time averaged vibration amplitude, provided the receiver bandwidth is large* enough to pass all of the vibration harmonics. Upon recalling that

$$\overline{\delta_n^2} \equiv \frac{1}{T} \int_0^T \delta_n^2(t)\, dt = \sum_{\ell=-\infty}^{\infty} |\delta_{n\ell}|^2 \tag{20}$$

where $\underline{\delta}_n(t)$ is the vibration amplitude of the n^{th} radiator, we may rewrite (19) as

$$SL = \frac{k_o^2}{3} \frac{\sum_{n=-N_1}^{N_2} \overline{\delta_n^2}\, |a_n|^2}{\left(\sum_{n=-N_1}^{N_2} |a_n| \right)^2} \quad . \tag{21}$$

As an example of the application of (21) suppose we have a linear array with uniform amplitude weighting, $|a_n| = 1$, and the vibration is monochromatic, so that

$$\delta_n(t) = (2)^{1/2}\, \delta_{RMS} \cos \omega_1 t$$

Then

$$SL = \frac{1}{3} \frac{(k_o \delta_{RMS})^2}{N_1 + N_2 + 1} \tag{22}$$

where $N_1 + N_2 + 1$ is the total number of elements in the array. For example, a 21 element array with a 1/20 wavelength rms deflection due to vibrations will have

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*That is, provided

$$\sum_{\ell=-M_1}^{M_2} |\delta_{n\ell}|^2 \simeq \sum_{\ell=-\infty}^{\infty} |\delta_{n\ell}|^2 \quad .$$

a -28 dB average sidelobe level due to the vibrations. For a 21 element symmetric ($N_1 = N_2$) array with taper $\cos^2(n\pi/20)$ the average* sidelobe level is -26 dB.

The results we have obtained here assume that the vibration of the n^{th} radiator is uncorrelated with that of the s^{th} radiator. The analysis for the case when they are correlated is given in Appendix A.

4. LARGE HARMONIC VIBRATIONS

The results in Section 3 are valid provided the deflections are very much smaller than a wavelength. When this condition is not satisfied it is not permissible to expand $\exp(i\underline{k}_o \cdot \underline{\delta}_n)$ in Eq. (6) in a Taylor series. In this section we will assume that the magnitude of the deflections are arbitrary, but we will require that the vibration be monochromatic, so that

$$\underline{\delta}_n(t) = \underline{\Delta}_n \sin(\omega_1 t + \Phi_n) \quad , \tag{23}$$

where $\underline{\Delta}_n$ is a real quantity. In this case (6) becomes

$$u(\underline{r}, t) = R_e \frac{\exp\left[i\omega_o\left(t - \frac{r_o}{c}\right)\right]}{4\pi r_o} \sum_{n=-N_1}^{N_2} a_n \exp\{i\underline{k}_o \cdot \underline{R}_n + i\xi_n \sin(\omega_1 t + \Phi_n)\} \tag{24}$$

where $\xi_n = \underline{k}_o \cdot \underline{\Delta}_n$. Upon using the well known expansion of $\exp(i\rho \sin\phi)$ in Bessel functions, we may rewrite (19) as

$$u(\underline{r}, t) = R_e \frac{\exp\left[i\omega_o\left(t - \frac{r_o}{c}\right)\right]}{4\pi r_o} \sum_{n=-N_1}^{N_2} a_n e^{i\underline{k}_o \cdot \underline{R}_n} \sum_{m=-\infty}^{\infty} J_m(\xi_n)\, e^{im(\omega_1 t + \Phi_n)} \quad . \tag{25}$$

---

*This result is obtained from (19) by using the results in Jolley[2] (pp 82-84) that

$$\sum_{n=-N}^{N} \cos^2\left(\frac{n\pi}{2N}\right) = N \quad ; \quad \sum_{n=-N}^{N} \cos^4\left(\frac{n\pi}{2N}\right) = \frac{3N}{4} \quad .$$

2. Jolley, L. (1961) Summation of Series, Dover, New York.

If we again assume that the signal is passed through a filter which passes only frequencies from $\omega_o - M_1\omega_1$ to $\omega_o + M_2\omega_1$ (and rejects all others), the time averaged power pattern is given by

$$P(\theta, \phi) = \frac{1}{T}\int_0^T u^2(r, t)\, dt = \frac{1}{2}\sum_{m=-M_1}^{M_2}\left|\sum_{n=-N_1}^{N_2} a_n J_m(\xi_n) \exp\left(im\phi_n + i\underline{k}_o \cdot \underline{R}_n\right)\right|^2 . \tag{26}$$

As was the case before, if all the sources vibrate with the same amplitude and phase the radiation pattern is simply the product of the nonvibrating pattern, P_o, and a correction pattern. That is

$$P(\theta, \phi) = \left[\sum_{m=-M_1}^{M_2} J_m^2(\xi)\right] P_o(\theta, \phi) \quad , \tag{27}$$

where $\xi = \underline{k}_o \cdot \underline{\Delta}$ is a function of θ and ϕ.

In the other limit when the phases Φ_n of the vibration of each radiator are random, uniformly distributed and statistically independent, we obtain* for the ensemble averaged power pattern

$$\langle P(\theta, \phi) \rangle = \frac{1}{2}\left|\sum_{n=-N_1}^{N_2} a_n J_0(\xi_n)\, e^{i\underline{k}_o \cdot \underline{R}_n}\right|^2 + \frac{1}{2}\sum_{m=-M_1}^{M_2}{}' \sum_{n=-N_1}^{N_2} |a_n|^2 J_m^2(\xi_n) \quad , \tag{28}$$

*We use the fact that for $n \neq s$

$$\langle e^{im(\phi_n - \phi_s)} \rangle = \left(\frac{1}{2\pi}\right)^2 \int_{-\pi}^{\pi} d\phi_n \int_{-\pi}^{\pi} d\phi_s\, e^{im(\phi_n - \phi_s)} = 0 \quad .$$

In obtaining (28), however, we have not averaged over all directions as we did in Section 3. Therefore (28) can be compared with (16) but not (18).

where the prime denotes that the m = 0 term is excluded from the summation. When $\xi_n \ll 1$, Eq. (28) reduces to (16), provided we retain only the $\ell = \pm 1$ terms in (16), so that δ_n has only a single vibration frequency.

If all the radiators vibrate in the same direction and with the same amplitude, so that $\xi_n = \xi_s = \xi = \underline{k}_o \cdot \underline{\Delta}$, we can approximate (28) by

$$\langle P(\theta, \phi) \rangle = J_o^2(\xi) \, P_o(\theta, \phi) + \frac{1}{2} \left[\sum_{m=-M_1}^{M_2}{}' J_m^2(\xi) \right] \left[\sum_{n=-N_1}^{N_2} |a_n|^2 \right] . \tag{29}$$

Therefore in this case the primary power pattern is altered by the amplitude factor $J_o^2(\xi) \leq 1$.

By using (29) and (13) we then see that the average vibration-produced sidelobe level relative to the peak power (at the center of the main beam) in the absence of vibrations is

$$SL = \left[\sum_{m=-M_1}^{M_2}{}' J_m^2(\xi) \right] \left\{ \frac{\sum_{n=-N_1}^{N_2} |a_n|^2}{\left(\sum_{n=-N_1}^{N_2} |a_n| \right)^2} \right\} . \tag{30}$$

For (30) we see that if $\xi \equiv \underline{k}_o \cdot \underline{\Delta}$ is of order unity the average sidelobe level can be quite large. For example if $\xi = 1$, and the array taper is uniform so that $|a_n| = 1$ for all n we get (provided $M_1, M_2 \gg 1$)

$$SL \simeq \frac{0.416}{N_1 + N_2 + 1} \tag{31}$$

Therefore, for a 21 element array ($N_1 + N_2 + 1 = 21$) we find that the vibrations produced an average sidelobe level of -17 dB. Consequently, any nulls in the radiation pattern would, on the average, be filled in to a depth of -17 dB if $\xi = 1$. For $\xi = 3$ we find the average sidelobe level have the even larger value of -13.5 dB.

5. DISCUSSION

Which formula should be used in calculating the sidelobe levels will depend upon the physical conditions. If the vibration amplitude is small compared with a wavelength ($k_o\delta \ll 1$) and we can actually estimate the lateral distance over which the vibrations are correlated then clearly Eqs. (A6) or (A12) in Appendix A are appropriate. However, if we are not sure what the correlation distance will be then Eq. (21) may be most appropriate for estimating the vibration induced sidelobe levels. If the vibration amplitude is of order or exceeds a wavelength then Eq. (30) is most useful.

Appendix A

Here we shall remove the restriction that the amplitude and phase of each radiator is statistically independent of those of all other radiators. We shall assume, however, that each radiator vibrates along the same axis in space (but that the orientation of that axis is completely random), so that $\underline{k}_o \cdot \underline{\delta}_{n\ell} = k_o \delta_{n\ell} \cos\theta'$, $\underline{k}_o \cdot \underline{\delta}_{s\ell} = k_o \delta_{s\ell} \cos\theta'$, etc., where θ' is the angle between $\underline{k}_o$ and $\underline{\delta}_{n\ell}$. In this case (11) becomes, after using (17) to average over all vibration directions

$$\langle P(\theta, \phi)\rangle = P_o(\theta, \phi) + \frac{k_o^2}{6} \sum_{\ell=-M_1}^{M_2} \sum_{n=-N_1}^{N_2} \sum_{s=-N_1}^{N_2} a_n a_s^* \langle \delta_{n\ell} \delta_{s\ell}^* \rangle \, e^{i\underline{k}_o \cdot (\underline{R}_n - \underline{R}_s)} \tag{A1}$$

If the variables $\delta_{n\ell}$ and $\delta_{s\ell}$ are assumed to be random phasors with a uniformly distributed phase, it can be shown[3] that $\langle \delta_{n\ell} \delta_{s\ell}^* \rangle = \rho_\ell(n, s)$ where the correlation function $\rho_\ell(n, s)$ is real. Consequently, we can rewrite (A1) as

1. Beckmann, P. (1967) Probability in Communication Engineering, Harcourt, Brace and World, New York.

$$\langle P(\theta, \phi)\rangle = P_o(\theta, \phi) + \frac{k_o^2}{6} \sum_{\ell=-M_1}^{M_2} \sum_{n=-N_1}^{N_2} \sum_{s=-N_1}^{N_2} a_n a_s^* \rho_\ell(n, s)$$

$$e^{i\underline{k}_o \cdot (\underline{R}_n - \underline{R}_s)} \quad . \tag{A2}$$

As an example of the application of (A2) consider a symmetric ($N_1 = N_2 = N$) linear array of radiators lying along the x-axis with each radiator separated from the adjacent one by a distance d. If we further assume that the deflections are a stationary random process, so that $\rho_\ell(n, s) = \rho_\ell(n - s)$ we can show that (A2) becomes

$$\langle P(\theta, \phi)\rangle = P_o(\theta, \phi) + \frac{k_o^2}{6} \sum_{\ell=-M_1}^{M_2} \sum_{n=-N}^{N} \sum_{s=-N}^{N} a_n a_s^* \rho_\ell(n - s)$$

$$\cdot \exp\{ik_o d(n - s) \sin\theta \cos\phi\} \quad . \tag{A3}$$

we next assume that

$$\rho_\ell(n - s) = \langle|\delta_{n\ell}|^2\rangle \exp\left\{-\frac{(n - s)^2}{N_o^2}\right\} , \tag{A4}$$

that the array weighting is uniform, so that $a_n = 1$ for all n, and that the mean square deflections $\langle|\delta_{n\ell}|^2\rangle$ are the same for all radiators, so that $\langle|\delta_{n\ell}|^2\rangle = \langle|\delta_\ell|^2\rangle$.

Then upon using these assumptions, substituting (A4) into (A3) and using* (20) we can show that if $N \gg N_o$ the quantity $\langle P(\theta, \phi)\rangle$ can be approximated as

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*We assume that the filter bandwidth in Figure 2 is sufficiently large that

$$\sum_{\ell=-M_1}^{M_2} |\delta_{n\ell}|^2 \simeq \sum_{\ell=-\infty}^{\infty} |\delta_{n\ell}|^2 \equiv \overline{\delta_n^2} \quad .$$

$$\langle P(\theta,\phi)\rangle \simeq P_o(\theta,\phi) + \frac{k_o^2\,\overline{\langle\delta^2\rangle}\,\sqrt{\pi}\,N\,N_o}{3}\,\exp\left\{-\frac{k_o^2\,N_o^2\,d^2\,\sin^2\theta\cos^2\phi}{4}\right\}, \tag{A5}$$

where

$$\overline{\langle\delta^2\rangle} = \frac{1}{T}\int_0^T dt\,\langle\delta^2(t)\rangle \quad .$$

Finally, if we assume that the peak of the unperturbed radiation pattern is at $\theta = \phi = 0$ we find that the average relative sidelobe is

$$SL \simeq \frac{k_o^2\,\overline{\langle\delta^2\rangle}\,(\pi)^{1/2}}{6}\left(\frac{2N_o}{2N+1}\right)\exp\left\{-\frac{k_o^2\,N_o^2\,d^2\,\sin^2\theta\cos^2\phi}{4}\right\} \quad . \tag{A6}$$

As an example of the application of (A6) suppose $\langle\delta^2\rangle = (2)^{1/2}\,\delta_{RMS}\cos\omega_1 t$, $d = \lambda/2$, λ = wavelength, $\delta_{RMS} = \lambda/20$, $2N + 1 = 21$ and $2N_o = 5$. Then (A6) gives for the average sidelobe level at $\theta = 45^o$, $\phi = 0^o$ the result $SL \simeq -55$ dB. However, for $\theta = 45^o$ and $\phi = 90^o$ the sidelobe level produced by the vibrations is -22 dB.

We can also specialize (A2) to the case of a continuously distributed planar antenna. In that case (A2) becomes

$$P(\theta,\phi) = P_o(\theta,\phi) + \frac{k_o^2}{6}\sum_{\ell=-M_1}^{M_2}\iint_A dx\,dy\iint_A dx'\,dy'\,a(x,y)\,a^*(x',y')$$
$$\cdot\,\rho_\ell(x,y;x',y')\exp\left[i\underline{k}_o\cdot(\underline{r}-\underline{r}')\right] \quad , \tag{A7}$$

where $\underline{r} = (x,y)$, $\underline{r}' = (x',y')$, A is the surface area of the antenna and $a(\underline{r})$ is the excitation. We can study (A7) for the case when the vibrations are a stationary random process, so that $\rho_\ell(\underline{r},\underline{r}') = \rho_\ell(\underline{r}-\underline{r}')$. In particular we shall assume that

$$\rho_\ell = \langle\delta_\ell{}^2\rangle\exp\left\{-\frac{(r-r')^2}{\beta^2}\right\} \quad . \tag{A8}$$

If $a(\underline{r})$ changes slowly in comparison with the variation in the correlation function $\rho_\ell(\underline{r})$ and $A \gg \beta^2$ we may approximate (A7) by

$$\langle P(\theta,\phi)\rangle \simeq P_o(\theta,\phi) + \frac{k_o^2}{6} \sum_{\ell=-M_1}^{M_2} \langle \delta_\ell{}^2 \rangle \iint_A dr^2 \, |a(\underline{r})|^2$$

$$\cdot \iint_{-\infty}^{\infty} d^2\xi \, \exp\left\{ -\frac{\xi^2}{\beta^2} + i\underline{k}_o \cdot \underline{\xi} \right\} \tag{A9}$$

where $\underline{\xi} = \hat{x}(x - x') + \hat{y}(y - y')$. The assumption that $A \gg \beta^2$ allows us to extend the limits on the ξ integration to infinity. The integral on ξ can be performed by writing $d^2\xi$ in cylindrical coordinates as $d^2\xi = \xi \, d\xi \, d\psi$ and $\underline{k}_o \cdot \underline{\xi} = k_o \xi \sin\theta \cos(\phi - \psi)$. Upon performing the integration on $d^2\xi$ we then get

$$\langle P(\theta,\phi)\rangle = P_o(\theta,\phi) + \frac{\pi k_o^2}{6} \sum_{\ell=-M_1}^{M_2} \langle \delta_\ell{}^2 \rangle \left[\iint_A |a(\underline{r})|^2 \, d^2r \right]$$

$$\exp\left(-\frac{k^2 \beta^2 \sin^2\theta}{4} \right) . \tag{A10}$$

If we assume that the filter passes nearly all the harmonics so that

$$\sum_{\ell=-M_1}^{M_2} \langle \delta_\ell{}^2 \rangle \simeq \sum_{\ell=-\infty}^{\infty} \langle \delta_\ell{}^2 \rangle = \frac{1}{T} \int_0^T \langle \delta^2(t) \rangle \, dt \equiv \overline{\langle \delta^2 \rangle} \tag{A11}$$

we can use (A10) to give for the average sidelobe level

$$SL \simeq \frac{\pi}{3} k_o^2 \, \overline{\langle \delta^2 \rangle} \, \beta^2 \left\{ \frac{\iint_A |a(\underline{r})|^2 \, d^2r}{\left| \iint_A |a(r)|^2 d^2r \right|^2} \right\} \exp\left(-\frac{k_o^2 \beta^2 \sin^2\theta}{4} \right) \tag{A12}$$

For a uniform excitation, $a(r) = 1$, and $\beta = 3\lambda$, $A = 100\lambda$, $\theta = 10^o$ and a single harmonic vibration, so that $\langle \delta^2 \rangle = 2\delta^2_{RMS} \cos^2 \omega_1 t$, $\overline{\langle \delta^2 \rangle} = \delta^2_{RMS}$, we find an average sidelobe level of -29.5 dB.

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