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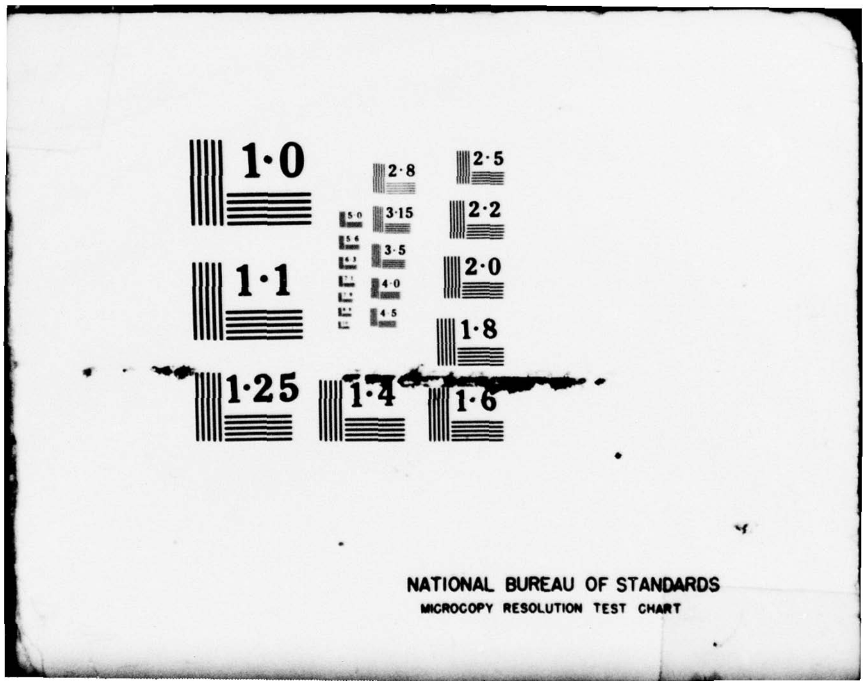
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NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

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RUSH DISTANCE FOR AN INFANTRY SQUAD
IN THE FRONTAL ATTACK

by

Cortez DeLeon Stephens

March 1979

Thesis Advisor: Glenn F. Lindsay

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An Analytic Approach to Determining Rush Distance for an Infantry Squad in the Frontal Attack

by

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Submitted in partial fulfillment of the requirements for the degree of

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from the

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March 1979

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ABSTRACT

A differential equation attrition model is used to deterministically simulate an infantry squad frontally attacking a four-man defensive position. The simulation results are used to determine the optimum rush distance.

SUMMARY

The rush distance an infantry squad uses in the frontal attack could be critical. If the rush distance is too short, it may take the attackers too long to close with the defenders. If the rush distance is too long, the attackers could possibly receive heavier casualties because they will be exposed to the defenders' fire for a longer time while rushing the longer rush distances.

This study sought the optimal rush distance by first modifying a differential equation attrition model which had been developed by Donald E. Christy in 1969. The modified model was then used in a deterministic computer simulation of a frontal attack involving a twelve-man infantry squad against a four-man defensive position. Several engagements were simulated with the varying parameters being rush distance, attacking squad organization and opening range.

The results of the simulations indicated a direct relationship between optimal rush distance and range. At ranges greater than 175 meters the numerical superiority of the attackers did not outweigh the more accurate fire of the defenders. However, at ranges less than 175 meters the accuracy of the attackers' fire was much closer to the accuracy of the defenders' fire and the attackers could take advantage of their superior numbers to inflict damage upon the defenders. This suggests that, at greater ranges the attackers should try to get closer to the defenders as quickly as possible, using longer rush distances (10-40 meters). At closer ranges the attackers should perform better with shorter rush distances (5-10 meters) to reduce their casualties.

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I. INTRODUCTION

The problem addressed in this paper is that of searching for the optimal rush distance for a small infantry force that is using fire and movement tactics in the frontal attack. The size of the force considered is twelve combatants, each armed with a semi-automatic rifle. This is approximately the size of a squad in most infantry organizations. The size of the defense force considered is four combatants, each armed with a semi-automatic rifle. The frontal attack is probably the form of maneuver most frequently used by infantry squads.[1]

The fire and movement tactic in the frontal attack has the entire attacking force closing frontally with the defenders. A designated portion of the attacking force rushes forward a short distance while the rest of the attackers fire at the defenders. The rushing attackers then take cover and fire at the defenders while another designated portion of the attacking force rushes forward. (In this paper those designated portions of the attacking force will be referred to as <u>rush teams</u>.) In such a manner the entire attacking force moves forward to an assault position. The distance covered during any of the rushes is called the <u>rush distance</u>. The assault position is normally just out of hand grenade range of the defenders' position, about 50 meters. It is a position where the entire attacking force can come on line and move forward as a single unit, non-stop, in an effort to overrun the defender's position. Long rush distances maintain the momentum of the attack and total time duration of the attack is relatively short. However, the attackers are completely exposed to the defenders' fire for long periods of time while rushing. Short rush distances expose the attackers for a shorter period of time for each rush but slow down the attack. Total time exposed to fire is the same, regardless of rush distance.

Rush distance depends in a large part on the terrain and the training the attackers have received. Other factors which influence rush distance include the physical condition of the attackers, the weight of the equipment they are carrying, and the nature of the weapons against which they are engaged. Nevertheless, terrain and training are often the overriding factors. When a combatant receives the command to rush forward, he searches the terrain to his front for a position which offers both cover from the enemy's fire and concealment from the enemy's observation. If his training has stressed that the rush distance should be about 20 meters, he will probably look for a position approximately 20 meters to his front. If there is a covered and concealed position in that vicinity, that is where he will probably go, even if there are similar positions located 10 meters to his front and 30 meters to his front.

Terrain encountered in ground combat may never present good positions consistently at the designated rush distances but "trained" attackers should seek positions in that general area and will attack using rush distances as close to that value as the terrain will allow. Therefore, a good reason for seeking the most effective rush distance is to improve the performance of small infantry units in which may be their most frequently used maneuver.

In 1969, Donald E. Christy, then a Captain in the United States Marine Corps, while studying certain tactics of infantry small units in the frontal attack, developed a differential equation attrition model for homogeneous forces which was an adaptation of Lanchester's Square Law.[2] The study in this paper approached the problem of optimal rush distance by first modifying Christy's attrition model, then using the modified model in a computer simulation of ground combat and analyzing the results.

Chapter II describes Christy's attrition model, his computer simulations and the results from those simulations. Chapter III describes the modifications made to Christy's model. Chapter IV describes the computer simulation used in this study and Chapter V presents the results of the simulated engagements. Conclusions drawn from those engagements and recommendations for further study are found in Chapter VI. The appendix contains a more detailed description of the single-shot kill probability function developed in this study.

II. CHRISTY'S ATTRITION MODEL

As was noted in the introduction, Donald E. Christy, in 1969, developed a differential equation attrition model for homogeneous forces. He did this by adapting Lanchester's Square Law to better reflect the dynamics of ground combat in fire and movement. In this chapter we will examine the basic structure of Christy's model, the scenario he used and the results of his computer simulation.

A. MODEL STRUCTURE

Christy's attrition model consists of a set of differential equations which express the sizes of the opposing forces as decreasing functions of time, implying continuous attrition without replacement. Let $A(t)$ and $D(t)$, respectively, be the size of the attacking force and the size of the defending force at time t, where $t = 0$ represents the beginning of the engagement. Let P_A and V_A , respectively, be the single-shot kill probability and rate of fire for an individual attacker. Let P_D and r_D be similarly defined for the defenders. The product of kill probability and rate of fire is the rate at which an individual combatant attrits the opposing force. Using this notation, the two simultaneous differential equations of Brackney's version of Lanchester's Square Law can be expressed in the following form:[3]

$$\frac{d\,A(t)}{dt} = -\,P_D\,r_D\,D(t)$$

and

$$\frac{d\,D(t)}{dt} = -\,P_A\,r_A\,A(t).$$

This model thus describes the opposing force sizes as decreasing functions of time. Reinforcements are not considered because the engagements investigated were of such short duration so as to preclude their use. The rate at which a force is attrited at any time t depends upon the size of the opposing force at that time and the ability of each element of the opposing force to inflict casualties. The theory of differential equations allows one to use these equations to determine the number of survivors in each force at any time t.

There are two aspects of ground combat which are not considered in Brackney's equations. The first is that the intensity of the engagement tends to increase as the attackers close with the defenders. The second is that some of the combatants are not firing their weapons at any given time during the engagement. Christy addressed this shortcoming by treating both kill probability and rate of fire as functions of force separation and considering suppression effects.

Since the closer a combatant is to the target the better is his chance of hitting it, Christy's kill probability was a decreasing function of range, or distance between the opposing forces. Since in a frontal attack the range decreases as time increases (if retreats are not considered), kill probability can be expressed as an increasing function of time. As the attackers move closer to the defenders, the accuracy of fire on both sides increases. Christy did not give both forces the same kill probability function. His kill probability functions reflect the facts that the defenders will probably present a more difficult target to the attackers than the attackers present to the defenders, and that the attackers present different target characteristics to the defenders when they are rushing than they do when they are not.

The rates of fire in Christy's model can also be expressed as increasing functions of time. The attackers had a single rate of fire function and the defenders had two rate of fire functions. The defenders had a rate of fire function for firing at those attackers who were actually rushing forward and a second function which gave higher rates for firing at those attackers who were not rushing. Both of the defenders' rate of fire functions approached the same limiting value as the attackers approached the offenders' position. In Christy's model, then, as time advances and the attackers move closer to the defenders' position, the intensity of the engagement increases in that both sides are firing more accurately and more rapidly.

Christy's model included consideration of suppression effects since some combatants do not fire their weapons all of the time due to reloading of weapons, confusion, fatigue and fear. Christy also assumed that if a combatant is not firing his weapon, he is not exposing himself to the enemy's fire.

Before we can examine a general form of Christy's attrition model some additional notations must be defined. Let,

$A_1(t)$ = number of attackers rushing,

$A_2(t)$ = number of attackers not rushing,

$D(t)$ = number of defenders,

$D_1(t)$ = number of defenders firing at rushing attackers,

$D_2(t)$ = number of defenders firing at non-rushing attackers

$r_x(t)$ = rate of fire for an element of force x,

$P_x(t)$ = single-shot kill probability for an element of force x

and

$S(t)$ = suppression factor (a number between zero and one).

Using this notation a general form of Christy's model is as follows:

The attrition rate for rushing attackers is

$$\frac{dA_1(t)}{dt} = -P_{D_1}(t)\ r_{D_1}(t)\ s(t)\ D_1(t)$$

and the attrition rate for non-rushing attackers is

$$\frac{dA_2(t)}{dt} = -P_{D_2}(t)\ r_{D_2}(t)\ s(t)\ D_2(t).$$

For defenders the attrition rate is

$$\frac{d\ D(t)}{dt} = -P_{A_2}(t)\ r_{A_2}(t)\ s(t)\ A_2(t).$$

Note that in this portrayal the defenders are not attrited by the rushing attackers, only by those attackers who are not rushing. In Christy's model the attackers do not fire while rushing.

B. CHRISTY'S SCENARIO

In the scenario used in Christy's study, both sides were armed with the M14 rifle and fired it semi-automatically. The attackers carried 300 rounds of ammunition per man whereas the defenders were given an unlimited ammunition supply. The range at the outset of the engagement was 600 meters. The terrain presented no obstacles to the attackers and offered them uniform cover from the defenders' fire and concealment from the defenders' observation.

Christy wrote a deterministic computer simulation which used his attrition model. He simulated several engagements and in each engagement a constant rush distance was used which was 5, 10, 20, 30 or 40 meters. Christy also varied defensive fire distribution, attacking force size, and number of rush units. The percentage of the defensive force which fired at those attackers who were rushing was 90%, 50% or 10% in any given engagement. The defensive force started with four combatants in every engagement while the attackers started with either 12 or 24 combatants. In any given engagement the number of rush units was 2, 3 or 5. Thus, by varying five rush distances, three defensive fire distributions, two attacking force sizes and three rush unit sizes, Christy simulated a totla of 90 different engagements.

The measure of effectiveness Christy used was victory or defeat. Victory went to that side which had the larger force size when the attackers had closed the range to 50 meters. In those cases where one of the forces had been annihilated before the 50 meter range had been attained, victory went to the force which still had survivors.

C. CHRISTY'S RESULTS

The results of Christy's simulations are quite interesting. In 84 out of the 90 simulated engagements one or the other of the two forces was annihilated before the 50 meter range was attained. The simulated engagements indicated that using five rush teams was relatively ineffective for the attackers. The coordination and movement required for an attack with five rush teams took too much time. There did not appear to be any significant difference between using two or three rush teams. The defenders fared better when they directed a greater percentage of their fire towards the rushing attackers.

Christy's simulated engagements also indicated that rush distances of 30 or 40 meters were more effective than smaller distances. Christy speculated that this result was due to the long distance traversed by the attackers (550 meters). He also suggested making the rush distance a function of time. That is, the rush distances would become shorter as the attackers moved closer to the defenders' position.

III. THE MODIFIED ATTRITION MODEL

The attrition model used in this study has the same general form as Christy's model, which was described in the previous chapter. However, the kill probability functions, the rate of fire functions, and the suppression functions are not the same ones used by Christy. In this study single-shot kill probability is defined to be the product of aiming point probability and target probability. These two probabilities are defined and explained in this chapter. This chapter also includes a description of the rate of fire functions and the suppression functions developed for this study.

A. AIMING POINT PROBABILITY

The development of the single-shot kill probability function used in this study begins with the assumption that a combatant will rarely encounter a visible, stationary target due to the fact that his enemy is usually either concealed or in motion. Nevertheless, it is asserted that more than likely a combatant will aim his weapon at a point where he thinks the enemy is located. If his enemy is running, it is asserted that a combatant will aim his weapon at a point where he thinks the enemy will be by the time a round gets there. Such a point, which will be referred to as an aiming point, can be anything from a bush or clump of grass to a spot on the ground. It will very seldom be a stationary, visible enemy combatant. An example of a good aiming point would be a bush which an enemy combatant is hiding behind or a tree which an enemy combatant will run in front of at the same time a round arrives there. If we visualize a silhouette target centered at a good aiming point, it can be assumed that if a combatant places a round close enough to that aiming point to hit the silhouette, he will hit the enemy combatant. Thus, the probability that a combatant hits an opponent is the product of the probability that he places a round close enough to a good aiming point given that he has selected a good aiming point and the probability that he selects a good aiming point.

When defending against fire and movement in the frontal attack, a defender is normally able to see the attackers when they are running but has to select an aiming point ahead of an attacker and "lead" that attacker as he is running. When the attackers are not running, they normally conceal themselves. In that case a defender may not know their exact positions but will have a good idea of their approximate positions by the sound of the attackers' fire and by having observed where they took cover after running their latest rush distance.

Since defenders can be expected in most situations to take camouflage and concealment measures, an attacker may not see a defender until he is within 50 meters of the defenders' position. Until then the attacker has estimated the defenders' position by the sound of the defenders' firing, an occasional muzzle flash, the attacker's ability to evaluate the terrain for likely defensive positions, or a combination of these and other factors, including military intelligence.

In both cases, the ability to select a good aiming point should become better as range decreases. Also, it should be easier to select a good aiming point if the opposing force is relatively large than it would be if the opposing force were smaller since there would b4 more good aiming points available with a larger opposing force. Thus aiming point probability, the probability that a good aiming point is selected, can be expressed as a function of range and size of the opposing force, varying inversely with range and directly with the size of the opposing force.

B. TARGET PROBABILITY

It has been stated that a combatant's kill probability is the probability that he hits what he is shooting at, given he is shooting at the right thing, multiplied by the probability that he is shooting at the right thing. We have discussed the probability that he is shooting at the right thing, or aiming point probability. Now let us turn our attention to the probability that, given a good aiming point, a combatant places a round close enough to that aiming point to hit an enemy combatant. This probability will be referred to as target probability.

Target probability is taken from rifle range data for a target of certain size, a combatant with a certain level of marksmanship ability with a certain rifle, firing from a certain firing position at a range of so many meters; the probability that the target is hit is well tabulated. Assuming a uniform level of marksmanship skill, target probability is a function of target size, range and firing position.

Since only infantry engagements are being considered here, it is reasonable to assume that a hit is equivalent to a kill. Therefore, a combatant's single-shot kill probability at any given time t is equal to his aiming point probability at time t multiplied by his target probability at time t. The exact form of the single-shot kill probability function as well as function values in certain situations can be found in the appendix.

C. THE RATE OF FIRE FUNCTIONS

Two rate of fire functions are used, one for the attackers and one for the defenders. Both are functions of time. The defenders' rate of fire is expressed as a linearly increasing function of time, since normal policy for defensive forces is to increase the rate of fire as the attackers move closer.[4] No distinction will be made in the defenders' rate of fire function for firing at rushing or non-rushing attackers.

Attackers, however, normally should not move forward in the attack until they have gained fire superiority.[1] A U.S. Army study defines fire superiority as "attaining a greater magnitude of target effects."[5] It isn't enough that the attackers fire more rounds than the enemy; the rounds they fire must be <u>effective</u> rounds. Effective rounds are rounds which either hit an enemy combatant or are placed close enough to have a suppressive effect on him. We know from an earlier discussion that every round fired is not an effective round, due to the firer either not being able to see the enemy or trying to hit a rapidly moving enemy. However, it is asserted that if a combatant has chosen good aiming point as defined earlier, the rounds he fires will be effective rounds. Thus, an operational definition of "magnitude of target effects" was chosen to be the product of rate of fire and aiming point probability, expressed in terms of effective rounds per minute.

As was stated earlier, the attackers should not move forward until they have gained fire superiority. In order to gain fire superiority, the attackers must fire at a rate which will attain a greater magnitude of target effects than the defenders. Since the defenders' magnitude of target effects is a function of time, so is the attackers' rate of fire.

D. THE SUPPRESSION FUNCTION

As we noted earlier, at any given time during the engagement a proportion of the combatants will not be firing their weapons because of fatigue, confusion, fear and other suppression factors. It is important to note that, while they are not firing their weapons, they are not inflicting any damage upon their opponents and they are not incurring any damage from their opponents since it was assumed that a combatant who is neither rushing nor firing is not exposed to fire. Therefore, at any time during the engagement, the ability of the attackers to attrit the defenders is reduced by the proportion of the attackers who are not firing. It is also reduced by the proportion of defenders who are not firing since those defenders are not exposed to the attackers' fire. The same could be said for the ability of the defenders to attrit the attackers.

In this study the proportion of combatants not firing their weapons (and thereby not exposed to fire) is taken to be a constant for both sides except for when an attacking rush team is rushing. When this happens the entire rush team is exposed to the fire of those defenders who are firing and the rest of the attackers have a higher proportion of firing combatants due to the added incentive of providing covering fire for their totally exposed comrades. Thus the suppression function used is a function of time and gave, at any time t, the product of the proportion of firing defenders and the proportion of firing attackers.

IV. THE SIMULATION

This chapter sets the stage by describing the scenario used in this study, the measure of effectiveness, and the computer program which simulated several engagements in that scenario by using the attrition model described in the previous chapter. Since we are already familiar with the scenario used by Christy, the scenario used in this study will be described by pointing out where it differs from Christy's scenario.

A. THE SCENARIO

The major departure made in this scenario from Christy's scenario was range. The opening range in each of Christy's simulated engagements was 600 meters. Such a range may be too large for engagements of the type modelled.[1] Much smaller ranges appear to be more appropriate.[6] In this study opening ranges were varied among engagements and ranges of 250, 200, 150 and 100 meters were used. Both defenders and attackers were armed with the M16 A1 rifle, fired semi-automatically. All combatants were given an unlimited supply of ammunition and the computer simulation kept track of expended ammunition.

The initial defender force size was always four and the initial attacker force size was always twelve. The attackers were organized into three rush teams of four combatants each or two rush teams of six combatants each. This was done as an interesting sidelight to explore, inasmuch as U.S. Army infantry squads are organized into two rush teams whereas U.S. Marine Corps infantry squads are organized into three rush teams. Defensive fire was distributed uniformly over the entire attacking force, both rushing and non-rushing.

This study used the same constant rush distances that Christy used; 5, 10, 20, 30 and 40 meters. The parameter values for the times used to rush these distances were, respectively, 1.8, 2.8, 4.6, 6.0 and 7.7 seconds. The coordination time used by the attackers between rushes was ten seconds. These time values were taken from Christy's study.[2]

Values used in determining single-shot kill probabilities can be found in the appendix. The constant proportion of combatants who were not firing due to suppression was taken to be 0.5 for both sides. When an attacking rush team was rushing, the proportion of non-firing combatants for the rest of the attackers was taken to be 0.25. When a rush team was rushing, fire superiority was considered achieved by the rest of the attackers when they had a magnitude of target effects of two effective rounds per minute greater than the defenders' magnitude of target effects. When there were no teams rushing, the attackers only needed an edge of one effective round per minute to achieve fire superiority.

B. MEASURE OF EFFECTIVENESS

The measure of effectiveness used was victory or defeat. The attackers would be victorious if they reached the 50 meter mark with a force size greater than the defenders or if they annihilated the defenders. Any other outcome would result in victory for the defenders. An attacker victory was considered more effective than another attacker victory if it resulted in a larger attacker force size at the end of the engagement.

C. THE PROGRAM

An event-step, deterministic computer simulation program was written which had two events, a rush event and a coordination event. A rush event consisted of one rush team rushing one rush distance. A coordination event was the time between successive rushes. Figure 1 shows a flow chart of the simulation program.

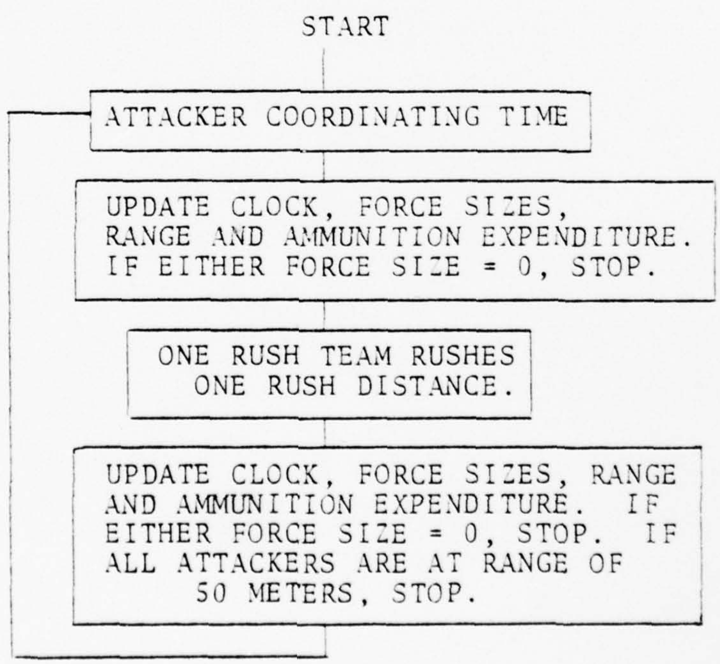

Fig. 1. Simulation Flow Chart for an Infantry Squad in the Attack Using Fire and Movement.

V. RESULTS

There were two sets of results in this study. The first set of results was from the initial simulated engagements, which used constant rush distances. Since these results suggested that variable rush distances may be more effective, additional engagements with variable rush distances were simulated.

A. RESULTS WITH CONSTANT RUSH DISTANCES

In the initial computer simulations the attacking squad was organized into either two or three rush teams. The opening range was either 250, 200, 150 or 100 meters. The rush distance (constant throughout any single engagement) was either 40, 30, 20, 10 or 5 meters. Thus, 40 engagements were initially simulated. In each engagement the attackers were victorious, that is, in each engagement the defenders were annihilated before the attackers had advanced to a range of 50 meters. Table I displays the constant rush distance results in numbers of surviving attackers.

One thing immediately apparent from the results shown in Table I is that the closer the attackers can come to the defenders before starting the attack, the better off they are. There does not appear to be any correlation between rush distance effectiveness and whether two or three rush teams were used. However, it does appear that at greater ranges long rush

TABLE I

CONSTANT RUSH DISTANCE SIMULATION RESULTS
IN TERMS OF SURVIVING ATTACKERS

(meters) Rush Distance	(meters) Opening Range	(Surviving Attackers) Three Rush Teams	(Surviving Attackers) Two Rush Teams
5	250	4.86	5.08
10	250	5.43	5.57
20	250	5.86	5.84
30	250	6.03	5.94
40	250	6.12	5.92
5	200	6.77	6.66
10	200	6.81	6.67
20	200	6.83	6.57
30	200	6.80	6.53
40	200	6.77	6.64
5	150	7.43	7.26
10	150	7.33	7.12
20	150	7.20	6.92
30	150	7.13	6.84
40	150	7.09	7.04
5	100	7.64	7.52
10	100	7.62	7.46
20	100	7.58	7.32
30	100	7.55	7.22
40	100	7.54	7.15

distances were more effective for both two and three rush team squads whereas at lesser ranges short rush distances were more effective. This suggests that perhaps rush distances which vary directly with range would yield better results.

B. VARIABLE RUSH DISTANCES

It was decided to simulate engagements in which variable rush distances were used. Two methods for varying rush distance were derived from the initial results; an exponential method and a linear method.

The exponential method was derived by examining the most effective rush distance for various opening ranges. Table II, which was derived from Table I, shows the most effective (in terms of surviving attackers) rush distance for a given opening range. Applying curve fitting techniques to the data in Table II suggested an exponential relationship between opening range and rush distance of the form:

$$Y = b\, e^{mx},$$

where Y is rush distance, x is opening range and b and m are constants. Attempts to fit such a curve to the data in Table II yeilded b = 1.11 and m = 0.012 for two rush teams and b = 0.82 and m = 0.015 for three rush teams.

In the variable rush distance simulations, rush distance was taken to be a function of whatever the range was at the start of a particular rush event instead of opening range. Thus, rush distance decreased as range decreased. Also, the

TABLE II

MOST EFFECTIVE CONSTANT RUSH DISTANCE
FOR A GIVEN OPENING RANGE

	Rush Distance	
Opening Range	Three Rush Teams	Two Rush Teams
250 meters	40 meters	30 meters
200 meters	20 meters	10 meters
150 meters	10 meters	5 meters
100 meters	5 meters	5 meters

variable rush distance simulations only considered rush distances between 5 and 40 meters.

By averaging the constant values from the curves which were fitted to the data in Table II, mean constant values of $b = 0.97$ and $m = 0.14$ were obtained. These were used in the variable rush distance simulations for both the two-rush team and the three-rush team case. Figure 2 shows the graph of the exponential rush distance formula which was used.

The linear method of varying rush distance was derived by examining, for each 50 meter range interval in the initial simulations, the most effective rush distance in terms of fewest attackers killed in that 50 meter interval. This was done by re-simulating the initial engagements after revising the program to extract the needed data. Table III presents these results.

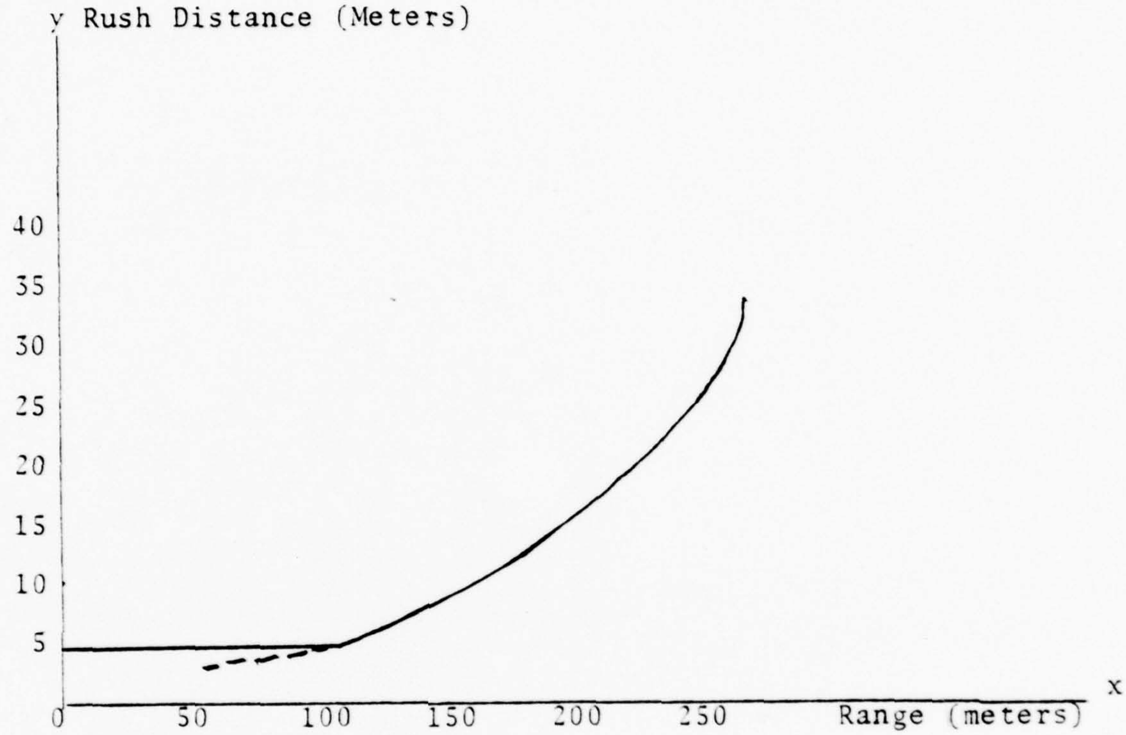

Fig. 2. Rush Distance as a Constrained Exponential Function of Range

TABLE III

MOST EFFECTIVE CONSTANT RUSH DISTANCE
FOR A GIVEN RANGE INCREMENT

Range Increment		Rush Distance	
From	To	Three Rush Teams	Two Rush Teams
250 meters	200 meters	40 meters	40 meters
200 meters	150 meters	40 meters	40 meters
150 meters	100 meters	40 meters	30 meters
100 meters	----	5 meters	5 meters

An attempt was made to fit both exponential and linear curves to the data in Table III. Linear curves gave the best fit as measured by the sum of the squared residuals. Expressing rush distance as a linear functionof the range at the start of a 50 meter increment, a slope of 0.21 and an intercept of -5.5 fitted the two-rush team data whereas a slope of 0.23 and an intercept of -11.5 fitted the three-rush team data. For the variable rush distance simulations a slope of 0.22 and an intercept of -8.5 was used for both two and three rush teams and rush distance was taken to be a function of the range at the start of a rush event. Figure 3 represents the graph of the linear function which was used.

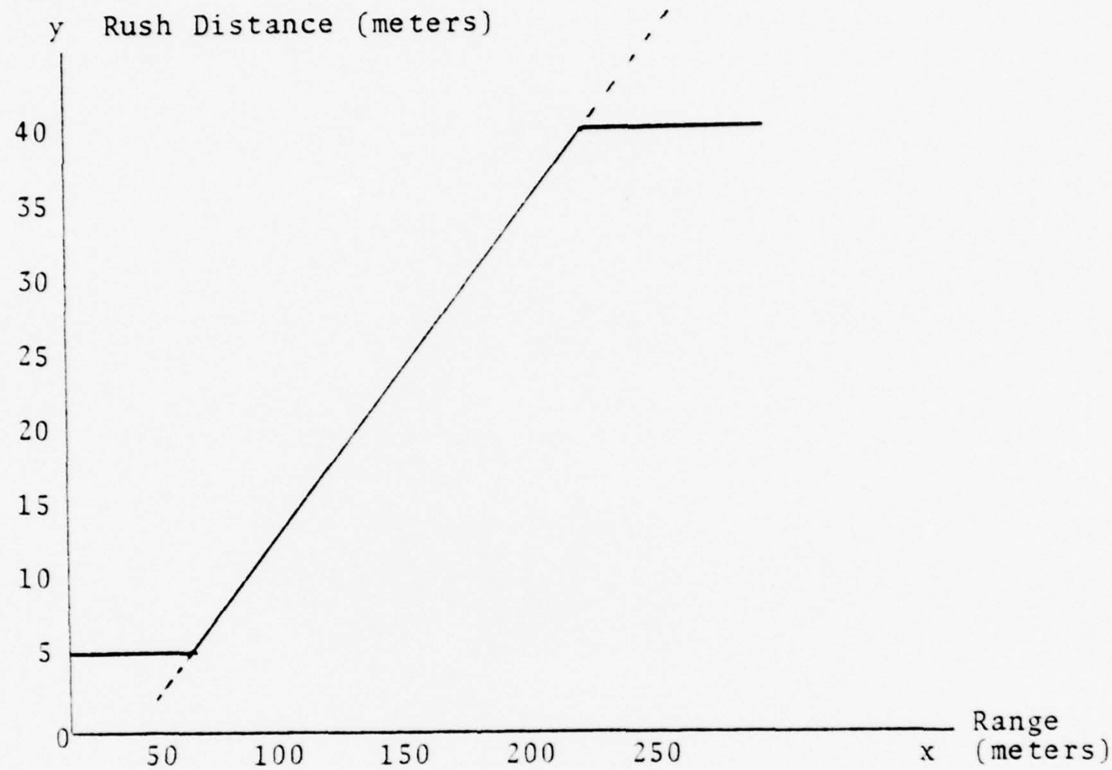

Fig. 3. Rush Distance as a Constrained Linear Function of Range

C. RESULTS WITH VARIABLE RUSH DISTANCES

Sixteen additional engagements were simulated using variable rush distances. As before, attacker organizations of two rush teams or three rush teams were used, as well as opening ranges of 250, 200, 150 or 100 meters. Rush distance was either a linear function or an exponential function of range. The results are presented in Table IV. The best results which had been obtained using constant rush distances are presented in Table V. The results in Table IV seem to compare favorably with the results in Table V.

Results in Table IV suggest, as did the values in Table I, that shorter opening ranges favor the attacker. Also, rush distances which are exponential functions of range seem to be more effective than rush distances which are linear functions of range.

TABLE IV

VARIABLE RUSH DISTANCE SIMULATION RESULTS
IN TERMS OF SURVIVING ATTACKERS

Opening Range (meters)	Linear Variation: Three Rush Teams (attacking survivors)	Linear Variation: Two Rush Teams (attacking survivors)	Exponential Variation: Three Rush Teams (attacking survivors)	Exponential Variation: Two Rush Teams (attacking survivors)
250	6.15	6.00	5.99	5.99
200	6.82	6.51	6.82	6.66
150	7.16	6.88	7.38	7.19
100	7.58	7.38	7.65	7.53

TABLE V

BEST CONSTANT RUSH DISTANCE SIMULATION RESULTS
IN TERMS OF SURVIVING ATTACKERS

Opening Range	# Survivors	Rush Distance	# Survivors	Rush Distance
250	6.12	40	5.94	30
200	6.83	20	6.67	10
150	7.43	10	7.26	5
100	7.64	5	7.52	5

VI. CONCLUSIONS AND SUGGESTIONS

This study offers two conclusions from the results presented in the previous chapter. In this chapter those conclusions and a possible explanation for them is presented, as well as suggestions for further study.

A. CONCLUSIONS

The first of the two conclusions offered by this study is that the attackers should get as close as possible to the defenders before beginning their attack. The second conclusion is that, for the scenario considered, rush distances of from five to ten meters should be used at ranges less than 175 meters, whereas rush distances of from 10 to 35 meters should be used at ranges greater than 175 meters. These are the values given by the exponential rush distance formula depicted in Figure 2, but it must be stressed that these are, at best, rough guidelines.

Perhaps an explanation for these conclusions can be given by an examination of Figure 4, which depicts single-shot kill probability as a function of range. Data points are not shown because they are not needed. It should be safe to assume that the defenders' single-shot kill probability is always greater than the attackers' single-shot kill probability and that both probabilities approach the same limiting value of one as the range approaches zero. Thus, at greater ranges the

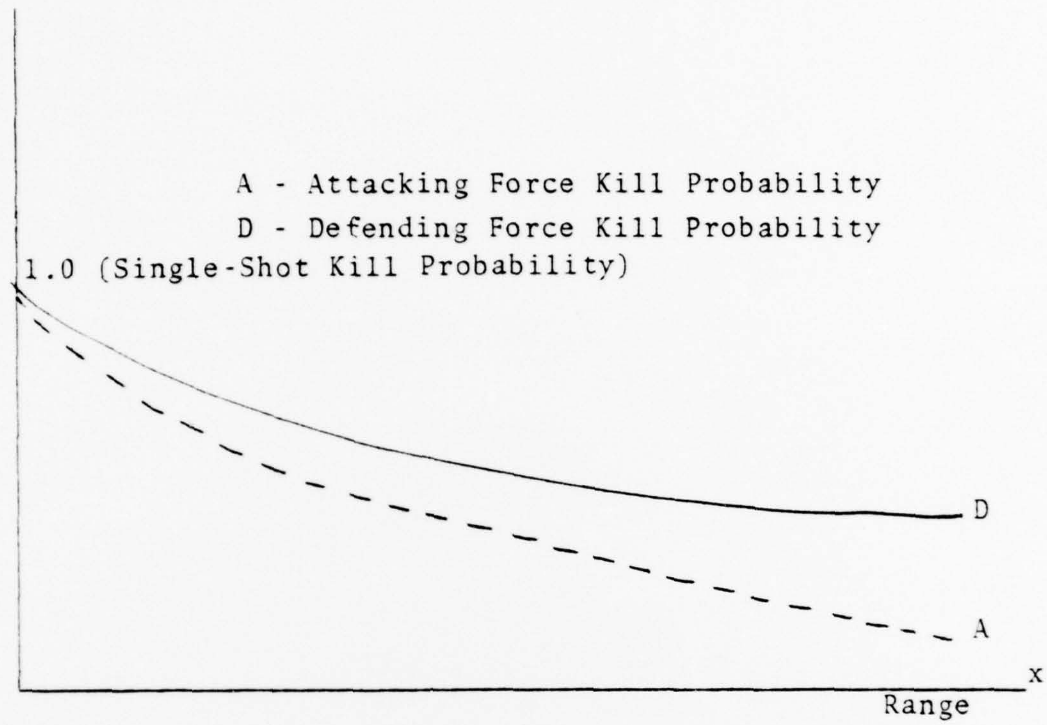

Fig. 4. Single-Shot Kill Probability As a Function of Range

defenders have a distinct advantage in single-shot kill probability. It is in the best interests of the attackers to reduce the range as quickly as they can because a reduction in range means a reduction in the single-shot kill probability advantage the defenders have. The attackers should reduce the range by moving as close as possible to the defenders before beginning the attack and by using long rush distances at greater ranges after the attack has begun. At lesser ranges the single-shot kill probability of the attackers is close enough to the defenders' single-shot kill probability that the attackers should have the advantage due to their superiority in numbers. Moreover, at shorter ranges the single-shot kill probability of both forces could become so high that exposure to fire during a rush would certainly be fatal if the rush were too long, hence short rush distances.

B. SUGGESTIONS

Perhaps these results or something similar could be stressed when training infantry units, but certainly not without further investigation into this area, to include field exercises. It would be interesting to extend this model to consider automatic weapons and to use a stationary base of fire for the attackers. Another interesting study would be looking into the relative efficiency between using two or three rush teams for the infantry squad in the frontal attack.

APPENDIX

SINGLE-SHOT KILL PROBABILITY

This appendix gives a more detailed description of the single-shot kill probability function used in this study as well as kill probability values at certain ranges.

BASIC ASSUMPTIONS

Consider a single combatant firing a single shot at a single opponent. The combatant will rarely encounter a visible stationary opponent. His opponent will either be running or camouflaged and concealed. The combatant will nevertheless select an aiming point where he surmises his enemy is located (or will be located, in the case of moving opponents). Let the universe set U be the set of outcomes when a combatant fires a shot at an opponent. The result is the situation depicted in Fig. 5. Referring to Fig. 5, let

A = event the opponent is hit

and

B = event the combatant has selected a good aiming point.

Assuming a hit is equivalent to a kill, the single-shot kill probability is defined as the probability that the combatant has a good aiming point and hits his opponent; i.e., $P\{A \cap B\}$. It is assumed that the probability that he does not have a good aiming point but hits his opponent anyway, $P\{A \cap \bar{B}\}$, is

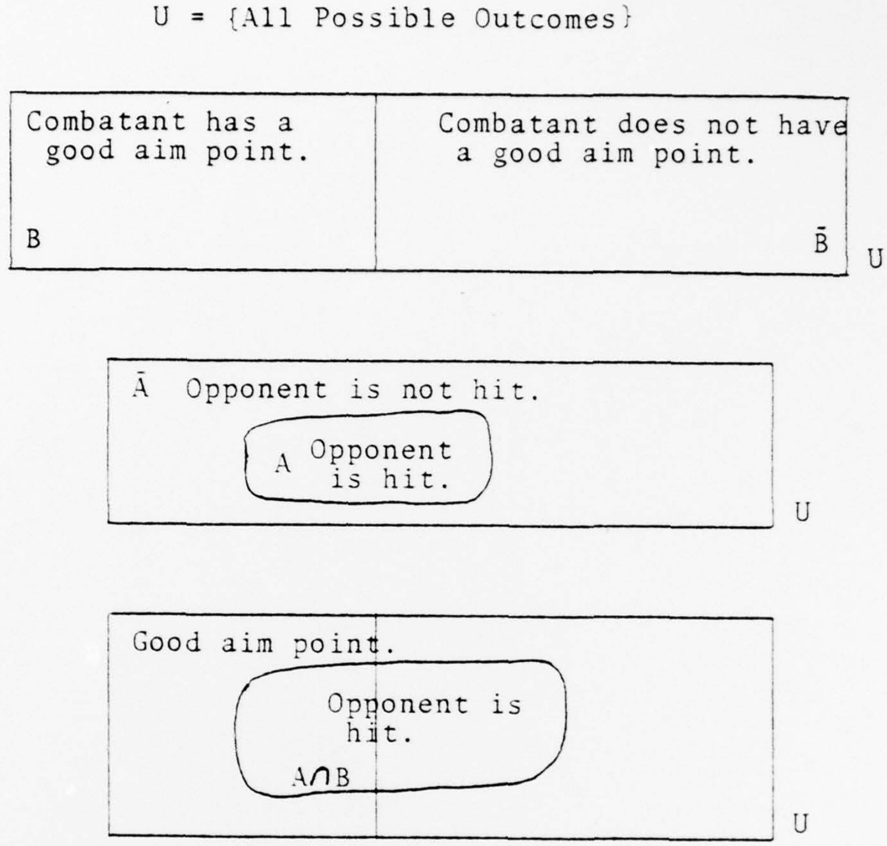

Fig. 5. Possible Outcomes When a Combatant Fires a Single Round at an Opponent.

negligible, since,

$$P\{A|B\} = \frac{P\{A \cap B\}}{P\{B\}}.$$

It follows that,

$$P\{A \cap B\} = P\{A|B\} \; P\{B\} = \text{single-shot kill probability.}$$

Thus, single-shot kill probability may be computed as the probability that a combatant hits what he is shooting at, given he is shooting at the right thing, multiplied by the probability that he is shooting at the right thing. The probability that he hits what he is shooting at given he is shooting at the right thing may be called Target Probability, $P\{A|B\}$, and is a function of target size, range and firing position. The probability that he is shooting at the right thing may be called aiming point probability, $P\{B\}$, and is a function of range and size of the opposing force.

AIMING POINT PROBABILITY

Aiming point probability is the probability that the combatant has chosen a good aiming point. It is reasonable to expect this probability to vary inversely with range and directly with the size of the opposing force. It is assumed that the target area is a rectangular area within which a combatant knows his opponent is located. Then target angle may be defined as that angle subtended by the front sight of a rifle as it is swung along the diagonal of the target area (with the rear sight held stationary). Since target angle varies inversely with range and directly with opposing force size, aiming point probability can be defined as follows:

$$\text{Aiming point probability at time } t = \frac{G(t)}{z},$$

where

$$G(t) = \begin{cases} \text{target angle at time } t, \\ \text{if target angle} \leq z \\ z, \text{ if target angle} > z \end{cases}$$

The actual computation of aiming point probability was not difficult. The target area used for a four-man defensive position had dimensions of 20 meters by 2-5 meters. This was taken from the ground combat confrontation model (G C C).[7] The value of z used for the attackers' aiming point probability was 17°. This value was chosen so that the aiming point probability for an attacker 150 meters away from a four-man defensive position would be approximately 0.5, a subjectively chosen value which had intuitive appeal. Table VI contains the attackers' aiming point probabilities against a four-man defensive position. Table VII contains the target area dimensions of a twelve-man attacking force. These were the target areas used to compute the defenders' aiming point probability and were proportional adjustments of the dimensions used in the G C C model.[7] The value of z used when computing aiming point probabilities for the defenders was 22°. This value was chosen so that the aiming point probability for the defenders against a twelve-man attacking squad 200 meters away would be approximately 0.5. Table VIII contains the aiming point probabilities for the defenders.

TABLE VI

ATTACKERS' AIMING POINT PROBABILITIES AGAINST
A FOUR-MAN DEFENSIVE POSITION

Range	Target Angle	Aim Point Probability
250 meters	4.8456°	0.2850
200 meters	6.0549°	0.3562
150 meters	8.0675°	0.4746
100 meters	12.0764°	0.7104
50 meters	23.8903°	1.0

TABLE VII

TARGET AREA DIMENSIONS FOR A TWELVE-
MAN ATTACKING FORCE

	Target Area Dimensions	
	Three 4-Man Teams	Two 6-Man Teams
Entire Squad (Not Rushing)	35 x 5.25 meters	35 x 5.25 meters
One Rush Team (Rushing)	12 x 7.34 meters	18 x 7.34 meters
One Rush Team (Not (Rushing)	12 x 5.25 meters	18 x 5.25 meters
Two Rush Teams (Not (Rushing)	24 x 5.25 meters	---------

TABLE VIII

DEFENDERS' AIMING POINT PROBABILITIES
AGAINST A TWELVE-MAN ATTACKING FORCE

	(meters) Range	Aim Point, Three Four-Man Teams	Probability, Two Six-Man Teams
Entire Squad (Not Rushing)	250	0.4005	0.4005
	200	0.5001	0.5001
	150	0.6655	0.6655
	100	0.9925	0.9925
	50	1.0	1.0
One Rush Team (Rushing)	250	0.1594	0.2227
	200	0.1993	0.2782
	150	0.2656	0.3708
	100	0.3980	0.5552
	50	0.7922	1.0
One Rush Team (Not Rushing)	250	0.1484	0.2148
	200	0.1855	0.2684
	150	0.2473	0.3576
	100	0.3707	0.5356
	50	0.7382	1.0
Two Rush Teams (Not Rushing)	250	0.2783	
	200	0.3472	
	150	0.4631	
	100	0.6927	
	50	1.0	

TARGET PROBABILITY

In deriving the target probabilities, a set of single-shot hit probabilities were obtained from the infantry weapons branch of the Army Materiel Systems Analysis Agency (AMSAA) in Aberdeen, Maryland. These probabilities were for a soldier of average marksmanship ability firing in M16 rifle from the prone position.

In a study on human error and firing positions, Arima found no difference in marksmanship error between the prone and supported foxhole positions.[8] Thus the AMSAA probabilities, unmodified, were used for the defenders' target probabilities. The attackers, however, are expected to use the standing or kneeling position 65-80 percent of the time.[6] Arima estimated a 55 percent error increase from the prone to the kneeling position and a 60 percent increase from the prone to the standing position.[8] Thus for the attackers' target probabilities, the AMSAA probabilities with a 42 percent error increase were used. Table IX contains the target probabilities.

SINGLE-SHOT KILL PROBABILITY VALUES

Tables X, XI and XII contain the single-shot kill probabilities used in the computer simulations. Linear interpolation was used to obtain kill probabilities for those ranges not given in the tables.

TABLE IX

TARGET PROBABILITIES

Range	Target Probability Defender	Attacker
250 meters	0.370	0.106
200 meters	0.508	0.301
150 meters	0.616	0.455
100 meters	0.637	0.485
50 meters	0.765	0.666

TABLE X

SINGLE-SHOT KILL PROBABILITIES
(ATTACKERS AGAINST FOUR DEFENDERS)

Range	Single-Shot Kill Probability	Expected Number of Shots to Kill
250 meters	0.03	33.3
200 meters	0.107	9.3
150 meters	0.216	4.6
100 meters	0.345	2.9
50 meters	0.666	1.5

TABLE XI

SINGLE-SHOT KILL PROBABILITIES
(DEFENDERS AGAINST THREE FOUR-MAN TEAMS)

	(meters) Range	Single-Shot Kill Probability	Expected Number of Shots to Kill
Entire Squad	250	0.1482	6.7
(Not Rushing)	200	0.2541	4.0
	150	0.4094	2.4
	100	0.6322	1.6
	50	0.7650	1.3
One Rush Team	250	0.0590	17.0
(Rushing)	200	0.1012	10.0
	150	0.1636	6.1
	100	0.2535	4.0
	50	0.6060	1.7
One Rush Team	250	0.0549	18.2
(Not Rushing)	200	0.0942	10.6
	150	0.1523	6.6
	100	0.2361	4.2
	50	0.5647	1.8
Two Rush Teams	250	0.1030	9.7
(Not Rushing)	200	0.1764	5.7
	150	0.2853	3.5
	100	0.4412	2.3
	50	0.7650	1.3

TABLE XII

SINGLE-SHOT KILL PROBABILITIES
(DEFENDERS AGAINST TWO SIX-MAN TEAMS)

	(Meters) Range	Single-Shot Kill Probability	Expected Number of Shots to Kill
Entire Squad	250	0.1482	6.7
(Not Rushing)	200	0.2541	4.0
	150	0.4094	2.4
	100	0.6322	1.6
	50	0.7650	1.3
One Rush Team	250	0.0824	12.1
(Rushing)	200	0.1413	7.1
	150	0.2284	4.4
	100	0.3537	2.8
	50	0.7650	1.3
One Rush Team	250	0.0795	12.6
(Not Rushing)	200	0.1363	7.3
	150	0.2203	4.5
	100	0.3412	2.9
	50	0.7650	1.3

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