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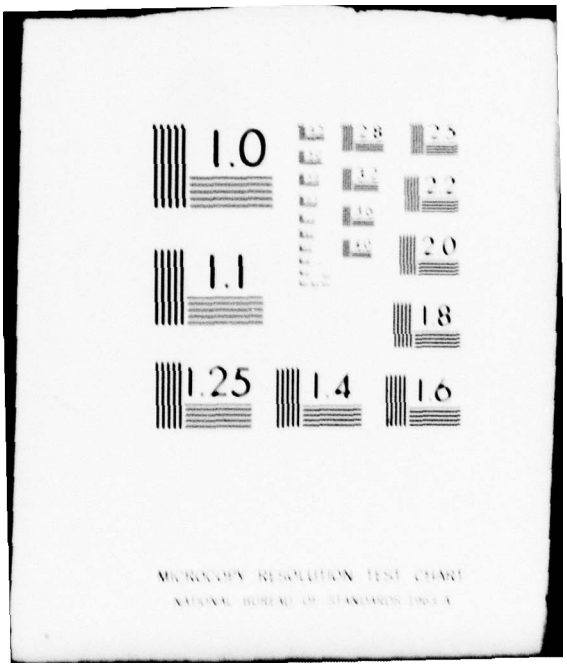
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CONSTANT INFORMATION MODEL: A NEW, PROMISING ITEM CHARACTERISTIC FUNCTION

FUMIKO SAMEJIMA

DEPARTMENT OF PSYCHOLOGY

UNIVERSITY OF TENNESSEE

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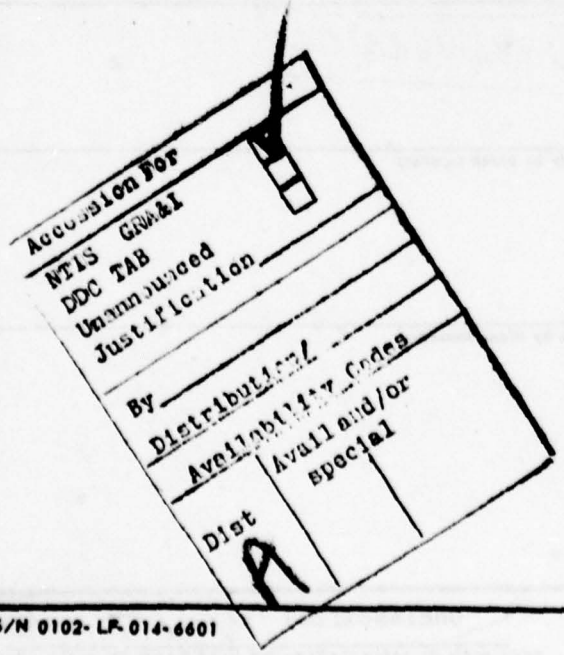
The methods and approaches introduced so far for estimating the operating characteristics of item response categories require the Old Test, or a set of items whose operating characteristics are known. To generalize these methods to apply for the situation where we start to develop a new item pool, i.e., there is no "Old Test," an approach is made by assuming that the tentative item pool has a substantial number of equivalent items, even though their common item characteristic function is not known yet. It is observed that, within the type of item characteristic function which is strictly increasing in the latent trait θ with zero and unity as its two asymptotes, the area under the square root of the item information function is a constant value, π. The item characteristic function which provides a constant item information is searched and discovered, and is named the constant information model. Using this model, it is observed that the subset of equivalent binary items can be used as a substitute for the Old Test, and these methods and approaches are generalized in the present situation. It is discovered that, for once, items with low discrimination power have a significant role.



CONSTANT INFORMATION MODEL, A NEW, PROMISING ITEM CHARACTERISTIC FUNCTION

ABSTRACT

The methods and approaches introduced so far for estimating the operating characteristics of item response categories require the Old Test, or a set of items, whose operating characteristics are known. To generalize these methods to apply for the situation where we start to develop a new item pool, i.e., there is no "Old Test," an approach is made by assuming that the tentative item pool has a substantial number of equivalent items, even though their common item characteristic function is not known yet. It is observed that, within the type of item characteristic function which is strictly increasing in the latent trait, θ with zero and unity as its two asymptotes, the area under the square root of the item information function is a constant value, π. The item characteristic function which provides a constant item information is searched and discovered, and is named the constant information model. Using this model, it is observed that the subset of equivalent binary items can be used as a substitute for the Old Test, and those methods and approaches are generalized in the present situation. It is discovered that, for once, items with low discrimination power have a significant role.

The research was conducted at the principal investigator's laboratory, 409 Austin Peay Hall, Department of Psychology, University of Tennessee, Knoxville, Tennessee. Those who worked for her as assistants include Robert L. Trestman, Philip S. Livingston and Paul S. Changas.

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I Introduction

Estimation of the operating characteristics of the item response categories of a test item has been investigated (Samejima, 1977b, 1977d, 1978a, 1978b, 1978c, 1978d, 1978e) without assuming any prior mathematical forms, and by using a relatively small number of examinees in the process of estimation. One common restriction in these methods and approaches presented so far is that we need the Old Test (cf. Appendix I) consisting of the items whose operating characteristics are known, and which provides us with a constant test information function for the interval of latent trait, or ability, θ of our interest. With this setting, each examinee's ability level is estimated from his response pattern by the maximum likelihood estimation, and the set of these maximum likelihood estimates is the main information source of the estimation procedures. The methods are useful, therefore, in such a situation that we already have an item pool, and we wish to add more items to it.

A question arises as to whether it is possible to overcome this restriction, and to make these methods and approaches useful in a more general situation, in which there is no Old Test, and, consequently, the maximum likelihood estimate of each examinee's ability is not a priori given. The fact is that it is not difficult to do so, if we are in such a situation that a large number of test items are administered to the group of examinees, which include a set, or sets, of equivalent items, i.e., the items whose operating characteristics are identical. In practice, we need to identify these equivalent items without knowing their operating characteristics, and in so doing several conditions should be satisfied.

In the present paper, theory and rationale behind the process of generalizing the estimation methods in the above situation will be presented and discussed, leaving the Monte Carlo study to observe the actual process and results to a later paper. Since, in practice, many researchers and testers use binary items in both tailored testing and paper-and-pencil testing, here we solely use binary items, although graded items are more informative and efficient to use (Samejima, 1969, 1977a, 1977c, 1977e). This restriction will give us a practical advantage, however, since it is much less likely that we find a subset of graded items, each of which has the same number of item response categories with the same set of operating characteristics as each other, among the test items in an actual test or a tentative item pool.

II Some Properties of the Information Functions

Throughout this paper, we only deal with the uni-dimensional latent space. The range of the latent trait θ is not necessarily the set of all real numbers, however, as will be exemplified later in following sections.

Let x_g be the graded item score, or graded response category, of item g, which assumes integers, 0 through m_g, and $P_{x_g}(\theta)$ be its operating characteristic, or the conditional probability, given ability θ, that the examinee's response to item g falls into category x_g. The item response information function (Samejima, 1969), $I_{x_g}(\theta)$, is defined by

$$(2.1) \qquad I_{x_g}(\theta) = -\frac{\partial^2}{\partial\theta^2} \log P_{x_g}(\theta) \quad .$$

The item information function, $I_g(\theta)$, is the conditional expectation of the item response information function, given θ, so that

$$(2.2) \qquad I_g(\theta) = E[I_{x_g}(\theta)|\theta] = \sum_{x_g=0}^{m_g} I_{x_g}(\theta)\, P_{x_g}(\theta)$$

$$= \sum_{x_g=0}^{m_g} [\frac{\partial}{\partial\theta} P_{x_g}(\theta)]^2 [P_{x_g}(\theta)]^{-1}$$

(cf. Samejima, 1969, Chapter 6). Let $P_V(\theta)$ be the operating characteristic of the response pattern V. By virtue of the local independence (Lord and Novick, 1968, Chapter 16, Samejima, 1969), this operating characteristic is the product of the operating characteristics of the participating item response categories, i.e.,

$$(2.3) \qquad P_V(\theta) = \prod_{x_g \in V} P_{x_g}(\theta) \quad .$$

The response pattern information function, $I_V(\theta)$, which is defined for every possible response pattern of a test, or a set of n items, can be written for a specified response pattern V such that

$$(2.4) \qquad I_V(\theta) = -\frac{\partial^2}{\partial\theta^2} \log P_V(\theta) = \sum_{x_g \in V} I_{x_g}(\theta) \ .$$

The test information function, $I(\theta)$, is defined as the conditional expectation, given θ, of the response pattern information function, and after some manipulations we can write

$$(2.5) \qquad I(\theta) = \sum_{V} I_V(\theta) P_V(\theta) = \sum_{g=1}^{n} I_g(\theta) \ .$$

Suppose we are in the position to transform the ability scale θ to another, by a strictly increasing function such that

$$(2.6) \qquad \tau = \tau(\theta) \ ,$$

We can easily see from the definitions of the operating characteristics that they are unchanged, or

$$(2.7) \qquad P^*_{x_g}(\tau) = P_{x_g}(\theta)$$

and

$$(2.8) \qquad P^*_V(\tau) = P_V(\theta) \ .$$

The information functions change, however, and we obtain the following.

$$(2.9) \qquad I^*_{x_g}(\tau) = I_{x_g}(\theta) \left[\frac{d\theta}{d\tau}\right]^2 - \frac{\partial}{\partial\theta} \log P_{x_g}(\theta) \cdot \frac{d^2\theta}{d\tau^2} \ .$$

$$(2.10) \qquad I_g^*(\tau) = I_g(\theta) \left[\frac{d\theta}{d\tau}\right]^2 .$$

$$(2.11) \qquad I_V^*(\tau) = I_V(\theta) \left[\frac{d\theta}{d\tau}\right]^2 - \frac{\partial}{\partial\theta} \log P_V(\theta) \cdot \frac{d^2\theta}{d\tau^2} .$$

$$(2.12) \qquad I^*(\tau) = I(\theta) \left[\frac{d\theta}{d\tau}\right]^2 .$$

The above results come from the relationship, which is directly derived from (2.7), such that

$$(2.13) \qquad \frac{\partial}{\partial\tau} P_{x_g}^*(\tau) = \frac{\partial}{\partial\theta} P_{x_g}(\theta) \frac{d\theta}{d\tau} ,$$

and the definition of the item response information function, and those of the other three types of information functions. It should be noted that the area under the curve of the item information function, and that of the test information function, do change with the transformation of the ability scale, since we have the relationships

$$(2.14) \qquad \int_{\underline{\tau}}^{\bar{\tau}} I_g^*(\tau)\, d\tau = \int_{\underline{\theta}}^{\bar{\theta}} I_g(\theta) \frac{d\theta}{d\tau} d\theta ,$$

and

$$(2.15) \qquad \int_{\underline{\tau}}^{\bar{\tau}} I^*(\tau)\, d\tau = \int_{\underline{\theta}}^{\bar{\theta}} I(\theta) \frac{d\theta}{d\tau} d\theta ,$$

where $\underline{\theta}$ and $\bar{\theta}$ are the lower and upper endpoints of the range of θ and

$$(2.16) \qquad \begin{cases} \underline{\tau} = \tau(\underline{\theta}) \\ \bar{\tau} = \tau(\bar{\theta}) \end{cases}$$

are those of the range of the transformed variable τ. If we integrate the square root of each of the two information functions, however,

we obtain

$$\int_{\underline{\tau}}^{\bar{\tau}} [I_g^*(\tau)]^{1/2} \, d\tau = \int_{\underline{\theta}}^{\bar{\theta}} [I_g(\theta)]^{1/2} \, d\theta \quad , \tag{2.17}$$

and

$$\int_{\underline{\tau}}^{\bar{\tau}} [I^*(\tau)]^{1/2} \, d\tau = \int_{\underline{\theta}}^{\bar{\theta}} [I(\theta)]^{1/2} \, d\theta \quad . \tag{2.18}$$

Thus the area under the curve of the square root of the item information function, and that of the test information function, are unchanged throughout the transformation of the latent trait by any strictly increasing function, $\tau(\theta)$. This fact has many important implications, as will be described in the next section.

On the dichotomous response level (Samejima, 1972), which is considered as a special case of the graded response level with $m_g = 1$ for each item g , the item information function is simplified to the form

$$I_g(\theta) = [\frac{\partial}{\partial\theta} P_g(\theta)]^2 \, [P_g(\theta) \, Q_g(\theta)]^{-1} \quad , \tag{2.19}$$

where $P_g(\theta)$ is the item characteristic function, defined by

$$P_g(\theta) = \text{prob.}[u_g=1|\theta] \tag{2.20}$$

with u_g (=0,1) denoting the binary item score, and

$$Q_g(\theta) = \text{prob.}[u_g=0|\theta] = 1 - P_g(\theta) \quad . \tag{2.21}$$

(2.19) is derived directly from (2,2), and also is equal to the item information function used by Birnbaum (Birnbaum, 1968).

It should be noted that, by definition, the item information function is non-negative, regardless of the values of the item response information functions, as is clear from (2.2) and (2.19). When we consider this function as a measure of the local accuracy of the estimation of ability θ, however, the function will be meaningless and misleading, if for some range of θ one, or more, of the item response information functions assumes negative values. This happens to the <u>three parameter models</u>, i.e.,

$$P_g(\theta) = c_g + (1 - c_g)\,\Psi_g(\theta) \quad , \tag{2.22}$$

where c_g is the guessing parameter and $\Psi_g(\theta)$ is some strictly increasing function of θ with 0 and 1 as its two asymptotes. The three-parameter normal ogive model and the three-parameter logistic model (Birnbaum, 1968) are two typical examples of (2.22). As was pointed out before (Samejima, 1972, 1973), any item characteristic function formulated by (2.22) has some range of θ in which the item response information function for the item score 1 assumes negative values, and, therefore, the unique local maximum is not assured for every possible response pattern, provided that $\Psi_g(\theta)$ satisfies the unique maximum condition (Samejima, 1969, 1972).

III Some Implications of the Constancy of the Totality of the Square Root of Item Information Function under the Transformation of the Latent Trait

It can be observed easily that any specified item characteristic function can be transformed to another, which belongs to the same model, through the transformation of the latent trait. To give an example, suppose that item g has an item characteristic function in the normal ogive model, such that

$$(3.1) \qquad P_g(\theta) = [2\pi]^{-1/2} \int_{-\infty}^{a_g(\theta - b_g)} \exp[-t^2/2]\, dt \ .$$

To change this to another item characteristic function which also follows the normal ogive model but with different item discrimination and difficulty parameters, a_g^* and b_g^*, all we need is such a linear transformation of θ to τ that

$$(3.2) \qquad \tau = a_g(\theta - b_g)\, a_g^{*-1} + b_g^* \ .$$

Thus we can write for the resultant item characteristic function $P_g^*(\tau)$

$$(3.3) \qquad P_g^*(\tau) = [2\pi]^{-1/2} \int_{-\infty}^{a_g^*(\tau - b_g^*)} \exp[-t^2/2]\, dt \ .$$

An important implication of this fact, combined with (2.17), is that the area under the curve of the square root of the item information function is constant for every item characteristic function which belongs to the same model. For the purpose of illustration, Figures 3-1 and 3-2 present the item information functions and their square roots for three items, all of which belong to the normal ogive model with $a_1 = 1.0$, $a_2 = 2.0$ and $a_3 = 3.0$, and $b_1 = b_2 = b_3 = 0.0$,

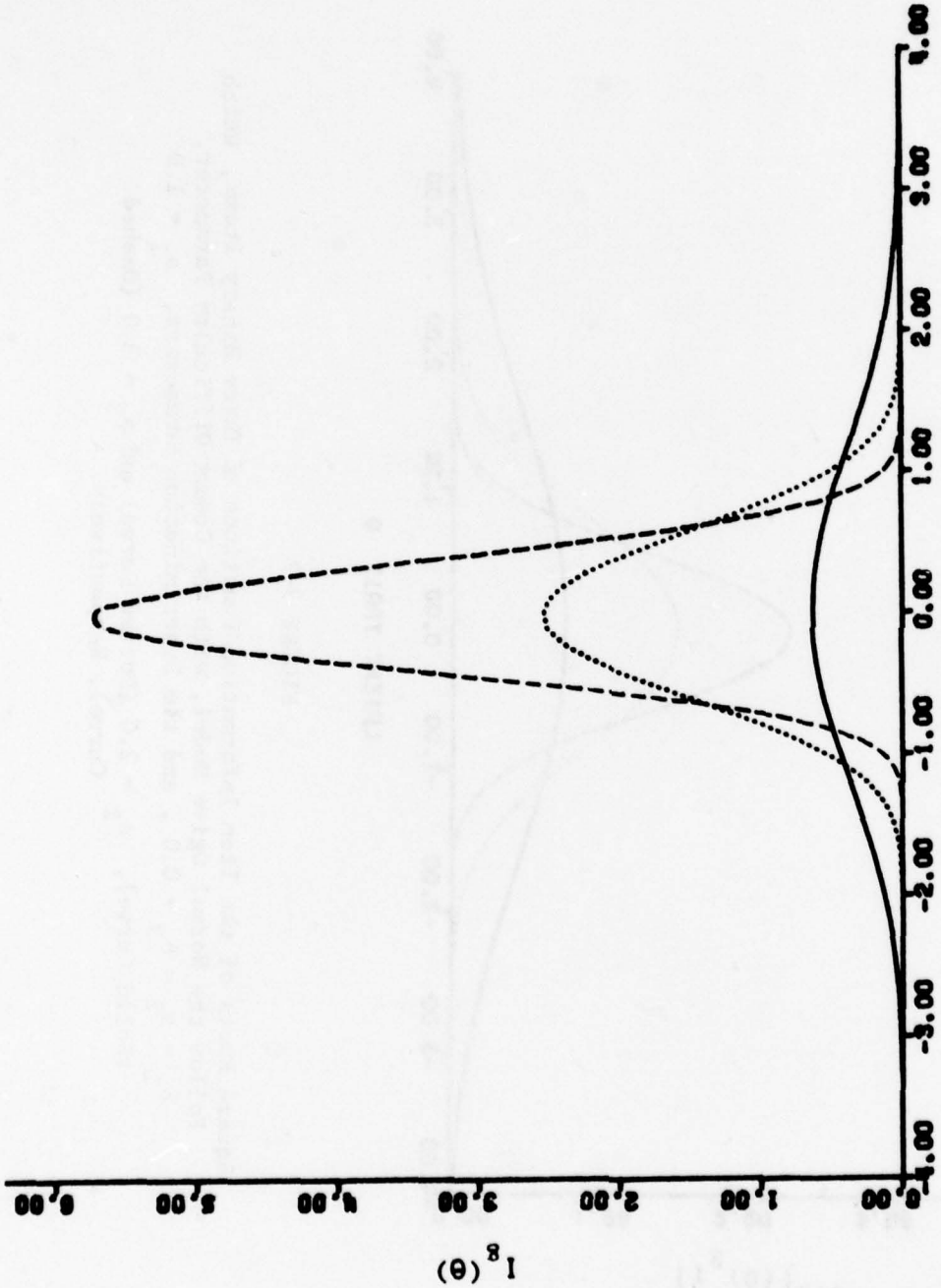

FIGURE 3-1

Item Information Functions of Three Binary Items, Which Follow the Normal Ogive Model, with the Common Difficulty Parameter, $b_1 = b_2 = b_3 = 0.0$, and the Discrimination Parameters, $a_1 = 1.0$ (Solid Curve), $a_2 = 2.0$ (Dotted Curve) and $a_3 = 3.0$ (Dashed Curve), Respectively.

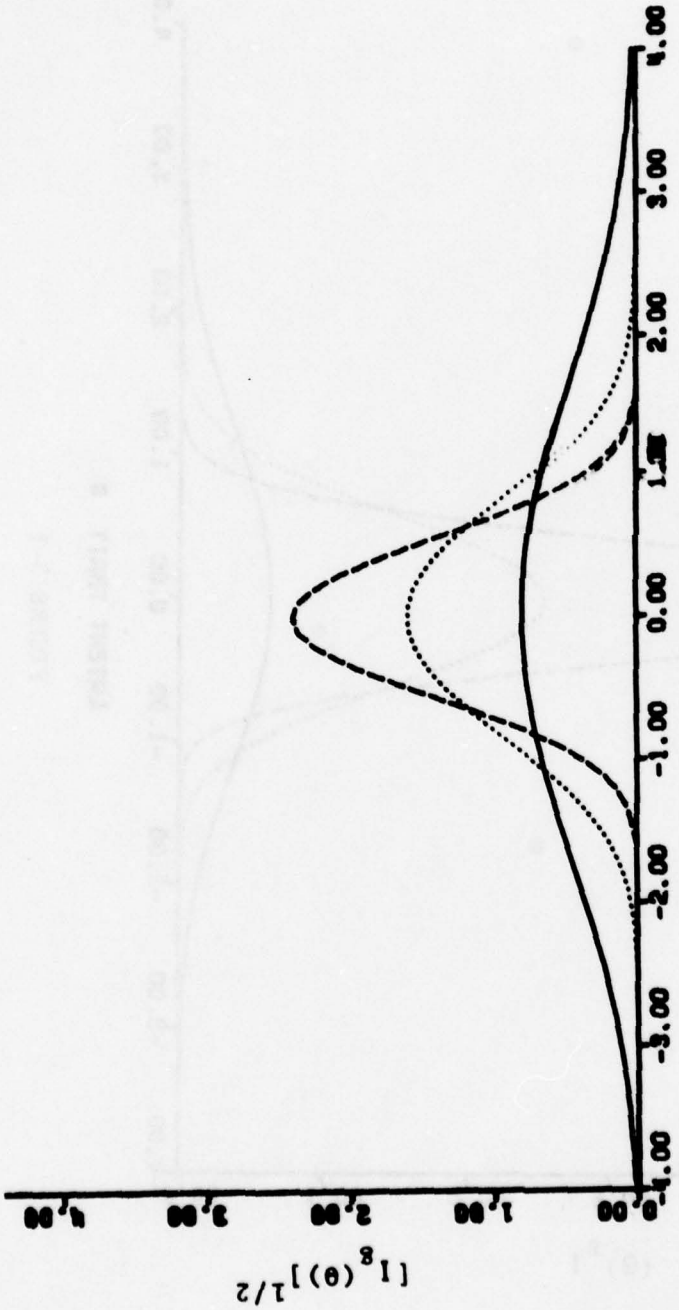

FIGURE 3-2

Square Roots of the Item Information Functions of Three Binary Items, Which Follow the Normal Ogive Model, with the Common Difficulty Parameter, $b_1 = b_2 = b_3 = 0.0$, and the Discrimination Parameters, $a_1 = 1.0$ (Solid Curve), $a_2 = 2.0$ (Dotted Curve) and $a_3 = 3.0$ (Dashed Curve), Respectively.

respectively. We can see that in Figure 3-1 the three areas are substantially different from one another, just like the case of the logistic model with the same parameter values (cf, Birnbaum, page 461, Figure 20.4.1). On the other hand, in Figure 3-2, these areas under the three curves are the same, just as was expected.

This fact can be generalized further, to include item characteristic functions of different models. Suppose that item g has some strictly increasing item characteristic function, with

$$(3.4) \qquad \begin{cases} \lim_{\theta \to \underline{\theta}} P_g(\theta) = 0 \\ \lim_{\theta \to \bar{\theta}} P_g(\theta) = 1 \quad . \end{cases}$$

We can see that, regardless of the model to which $P_g(\theta)$ belongs, it can be transformed to any other item characteristic function, which is strictly increasing and has the two asymptotes, 0 and 1, through the transformation of the latent trait θ to τ. To be more precise, it should be noted that, for any pair of models which belong to the type described above, and provide us with the item characteristic functions $P_g(\theta)$ and $P^*_g(\theta)$ respectively, there is one and only one pair of values of θ which corresponds to any specific value of probability between zero and unity exclusive. Let us call the second value of θ τ, and consider it as a function of the first value of θ. Thus we have

$$(3.5) \qquad \tau = \tau(\theta) \ .$$

Since both $P_g(\theta)$ and $P^*_g(\theta)$ are strictly increasing in θ, the functional relationship in (3.5) must be strictly increasing, and,

therefore, we can also write

$$\theta = \theta(\tau) \quad , \tag{3.6}$$

which is also a strictly increasing function of τ . Thus we have

$$\begin{cases} P_g(\theta) = P^*_g[\tau(\theta)] \\ P^*_g(\tau) = P_g[\theta(\tau)] \end{cases} \quad . \tag{3.7}$$

The general form of the transformation of θ to τ is, therefore,

$$\tau = P^{*-1}_g[P_g(\theta)] \quad , \tag{3.8}$$

and that of τ to θ is

$$\theta = P_g^{-1}[P^*_g(\tau)] \quad . \tag{3.9}$$

For the purpose of illustration, let us consider two binary items, one of which has a linear item characteristic function such that

$$P_g(\theta) = (\theta - \alpha_g)(\beta_g - \alpha_g)^{-1} \qquad \alpha_g < \theta < \beta_g \quad , \tag{3.10}$$

and the other has an item characteristic function in the logistic model, such that

$$P_h(\theta) = [1 + \exp\{-Da_h(\theta - b_h)\}]^{-1} \qquad -\infty < \theta < \infty \quad , \tag{3.11}$$

where a_h is the discrimination parameter, b_h is the difficulty parameter, and D is the scaling factor. It is well known that (3.11) is very close to the item characteristic function in the normal ogive model of the same parameter values, if we set $D = 1.7$

(Birnbaum, 1968). These two item characteristic functions are illustrated in Figure 3-3, with $\alpha_g = -2.5$ and $\beta_g = 2.5$ for item g, and with $D = 1.7$, $a_h = 1.0$ and $b_h = 0.0$ for item h. Suppose that our original item characteristic function is linear, as is defined by (3.10). We shall try to transform θ to τ so that the resultant item characteristic function is written as

$$P_g^*(\tau) = [1 + \exp\{-Da_g(\tau - b_g)\}]^{-1} \quad , \tag{3.12}$$

the identical form as (3.11). Since we have from (3.7), (3.10) and (3.12)

$$(\theta - \alpha_g)(\beta_g - \alpha_g)^{-1} = [1 + \exp\{-Da_g(\tau - b_g)\}]^{-1} \quad , \tag{3.13}$$

a direct expansion of the above provides us with

$$\tau = (Da_g)^{-1} \log [(\theta - \alpha_g)(\beta_g - \theta)^{-1}] + b_g \quad . \tag{3.14}$$

Figure 3-4 presents the relationship between the original latent trait θ and the transformed latent trait τ with $\alpha_g = -2.5$, $\beta_g = 2.5$, $D = 1.7$, $a_g = 1.0$ and $b_g = 0.0$.

If the situation is reversed, and the transformation is from τ to θ, then we obtain in the same way

$$\begin{aligned} \theta &= [\beta_g + \alpha_g \exp\{-Da_g(\tau - b_g)\}][1 + \exp\{-Da_g(\tau - b_g)\}]^{-1} \\ &= \alpha_g + (\beta_g - \alpha_g)\, P_g^*(\tau) \quad . \end{aligned} \tag{3.15}$$

It is obvious that the range of θ resulting from the above transformation is

$$\alpha_g < \theta < \beta_g \quad . \tag{3.16}$$

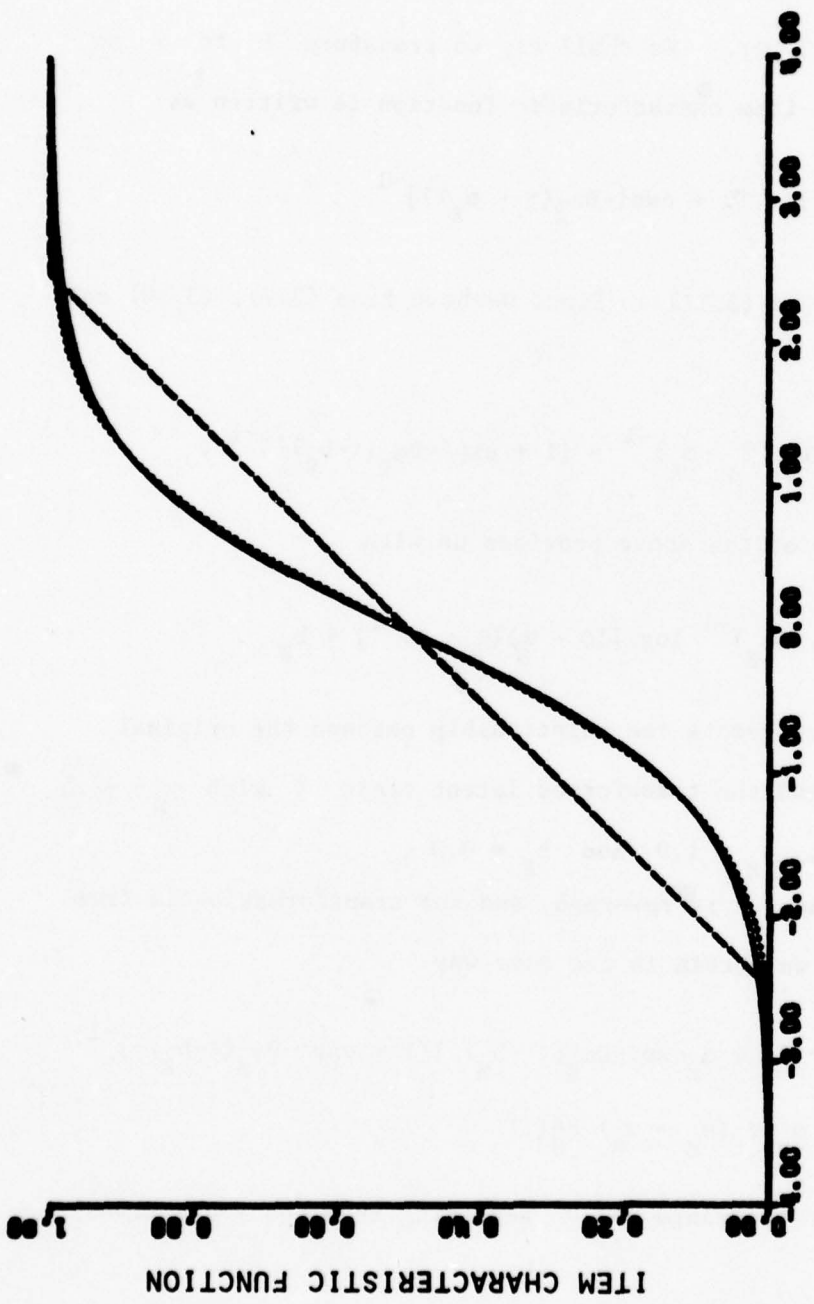

FIGURE 3-3

Item Characteristic Functions of Two Items, One of Which, Item g (Dashed Curve) Follows the Linear Model with $\alpha_g = -2.5$ and $\beta_g = 2.5$, and the Other of Which, Item h (Solid Curve), Follows the Logistic Model with D = 1.7, $a_h = 1.0$ and $b_h = 0.0$. For Comparison, the Item Characteristic Function of an Item Following the Normal Ogive Model with the Same Parameters as Item h Is Also Drawn by a Dotted Curve.

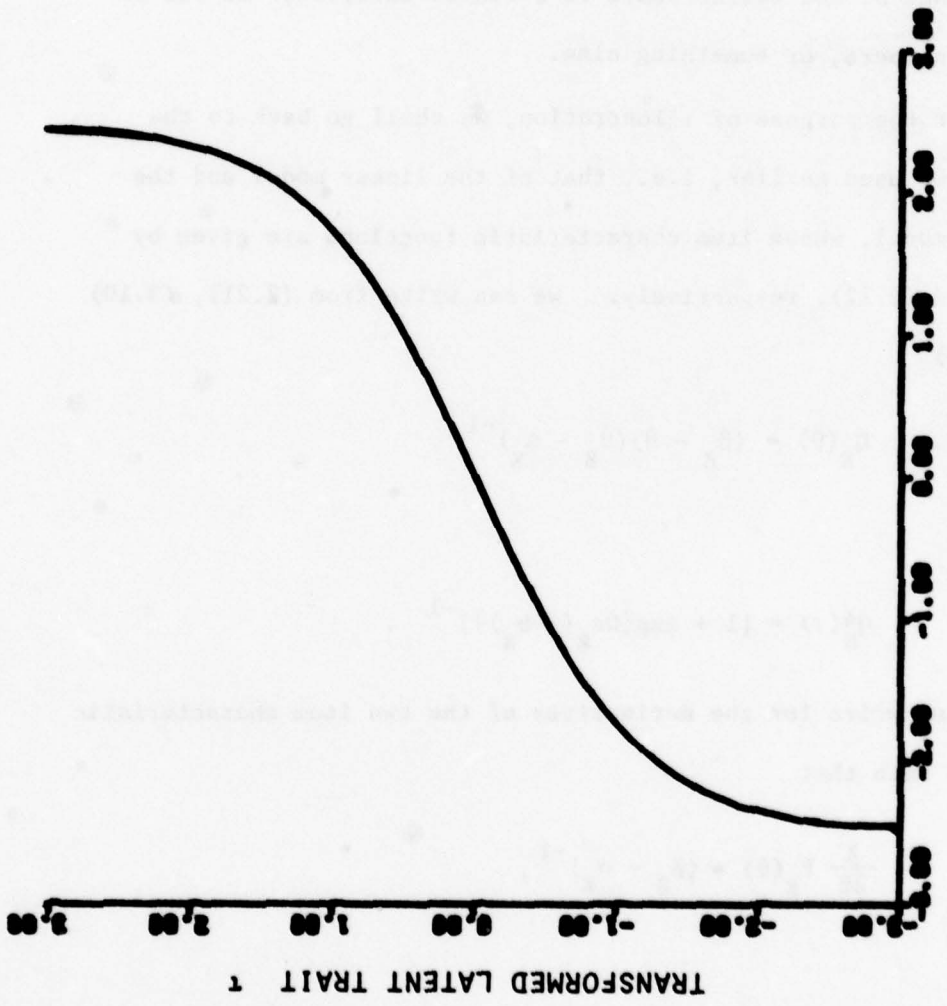

FIGURE 3-4

Functional Relationship between the Original Latent Trait θ and the Transformed Latent Trait τ, When the Transformation Is Made by Formula (3.14), with $a_g = -2.5$, $\beta_g = 2.5$, $D = 1.7$, $a_g = 1.0$ and $b_g = 0.0$.

The above fact implies that the area under the curve of the square root of the item information function derived from any item characteristic function, which is strictly increasing in θ and satisfies (3,4), is constant, regardless of the model to which it belongs. As we have seen in the above example, it does not matter if the range of the latent trait is a finite interval, the set of all real numbers, or something else.

For the purpose of illustration, we shall go back to the example we used earlier, i.e., that of the linear model and the logistic model, whose item characteristic functions are given by (3.10) and (3.12), respectively. We can write from (2.21), (3.10) and (3.12)

$$Q_g(\theta) = (\beta_g - \theta)(\beta_g - \alpha_g)^{-1} \tag{3.17}$$

and

$$Q_g^*(\tau) = [1 + \exp\{Da_g(\tau - b_g)\}]^{-1} \quad . \tag{3.18}$$

We can also write for the derivatives of the two item characteristic functions such that

$$\frac{\partial}{\partial\theta} P_g(\theta) = (\beta_g - \alpha_g)^{-1}, \tag{3.19}$$

and

$$\begin{aligned} \frac{\partial}{\partial\tau} P_g^*(\tau) &= Da_g \exp\{-Da_g(\tau - b_g)\}[1 + \exp\{-Da_g(\tau - b_g)\}]^{-2} \\ &= Da_g P_g^*(\tau) Q_g^*(\tau) \quad . \end{aligned} \tag{3.20}$$

Thus the item information function, $I_g(\theta)$, in the linear model is given by

$$(3.21) \qquad I_g(\theta) = [(\theta - \alpha_g)(\beta_g - \theta)]^{-1} \quad .$$

We can also write for the item information function $I^*_g(\tau)$ after the transformation of the latent trait θ to τ

$$(3.22) \qquad \begin{aligned} I^*_g(\tau) &= D^2 a_g^2 P^*_g(\tau) Q^*_g(\tau) \\ &= D^2 a_g^2 \exp[-Da_g(\tau - b_g)][1 + \exp\{-Da_g(\tau - b_g)\}]^{-2} \\ &= D^2 a_g^2 P_g(\theta) Q_g(\theta) \\ &= D^2 a_g^2 (\theta - \alpha_g)(\beta_g - \theta)(\beta_g - \alpha_g)^{-2} \quad . \end{aligned}$$

The two item information functions given by (3.21) and (3.22) with the same set of item parameters and a scaling factor are shown in Figure 3-5, and their square roots are presented as Figure 3-6. It is interesting to note that, within each figure, the two curves are substantially different.

It can be observed easily that the constancy of the area under the curve of the square root of the item information function holds for the set of item characteristic functions which take the form given by (2.22), if the value of c_g is common. The meaningfulness of the item information function is, however, doubtful for this type of item characteristic function, as was pointed out earlier, so we will not pursue this type in the present paper.

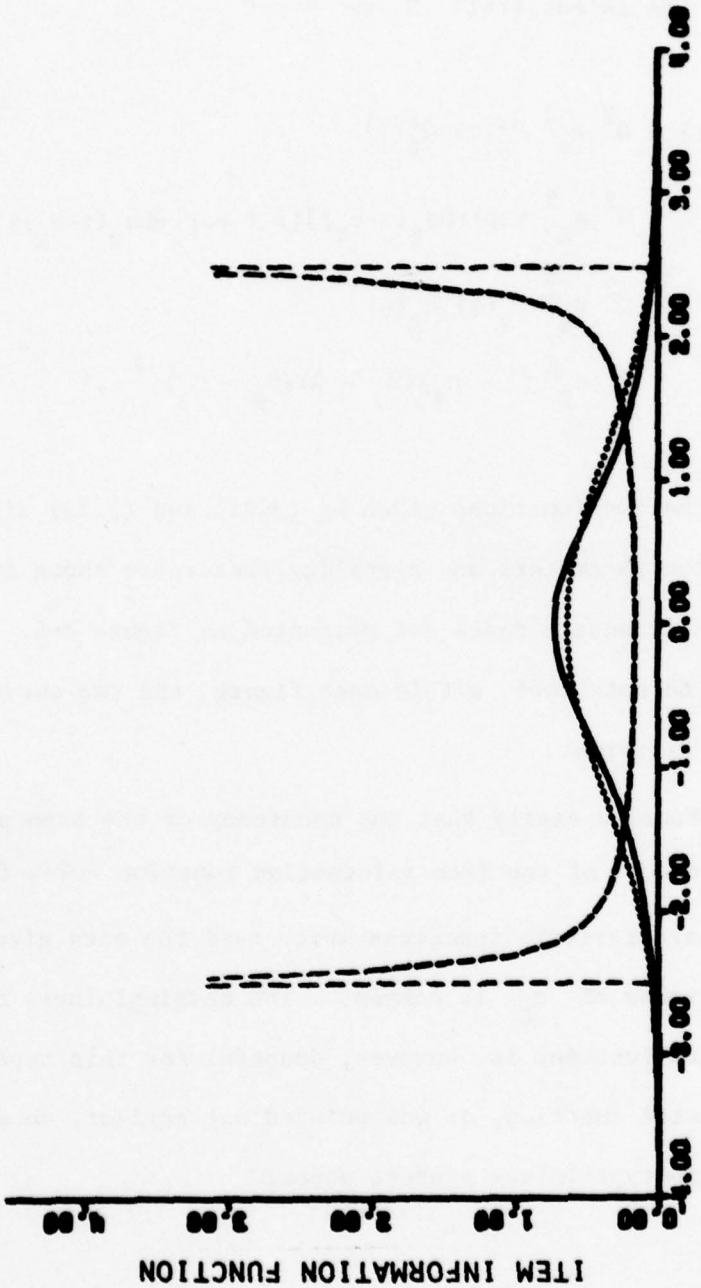

FIGURE 3-5

Item Information Function of Item g , Which Follows the Linear Model with $a_g = -2.5$ and $\beta_g = 2.5$ (Dashed Curve), and That of Item h , Which Follows the Logistic Model with D = 1.7, $a_h = 1.0$ and $b_h = 0.0$ (Solid Curve).

For Comparison, the Item Information Function of an Item, Which Follows the Normal Ogive Model with the Same Parameters as Item h , Is Drawn by a Dotted Curve.

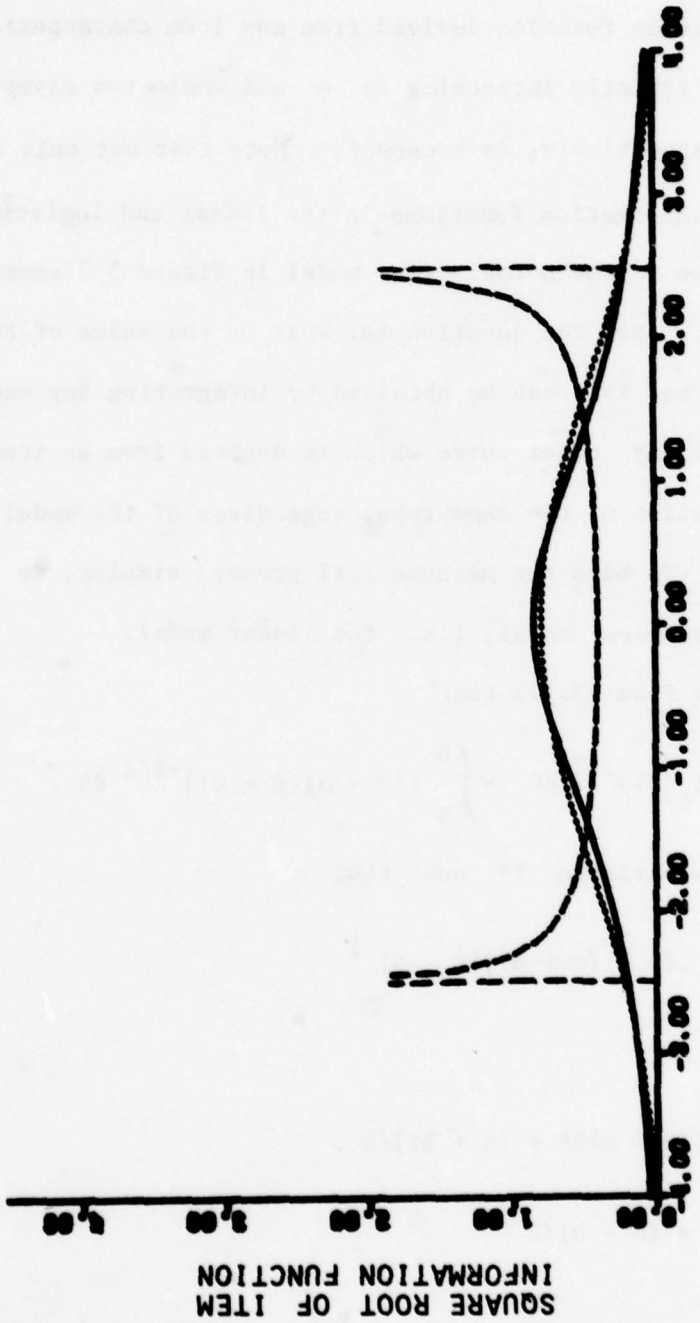

FIGURE 3-6

Square Root of the Item Information Function of Item g, Which Follows the Linear Model with $a_g = -2.5$ and $\beta_g = 2.5$ (Dashed Curve), and That of Item h, Which Follows the Logistic Model with D = 1.7, $a_h = 1.0$ and $b_h = 0.0$ (Solid Curve).

For Comparison, the Square Root of the Item Information Function of an Item, Which Follows the Normal Ogive Model with the Same Parameters as Item h, Is Drawn by a Dotted Curve.

IV Area under the Curve of the Square Root of the Item Information Function

We have seen that the area under the curve of the square root of the item information function derived from any item characteristic function, which is strictly increasing in θ and whose two asymptotes are 0 and 1, respectively, is constant. Note that not only the square roots of the item information functions in the linear and logistic models in Figure 3-6 but those in the normal ogive model in Figure 3-2 exemplify this constant area. Now the question is: What is the value of this area? We notice that this can be obtained by integrating any one of these curves, or any other curve which is derived from an item characteristic function of the same type, regardless of the model to which it belongs. To make the mathematical process simpler, we shall take up the simplest model, i.e., the linear model.

We can write from (3.21) that[§]

$$\int_{\alpha}^{\beta} [I_g(\theta)]^{1/2}\, d\theta = \int_{\alpha}^{\beta} [(\theta - \alpha)(\beta - \theta)]^{-1/2}\, d\theta \,. \tag{4.1}$$

Let us define a new variable θ^* such that

$$\theta^* = [2\theta - (\alpha + \beta)][\beta - \alpha]^{-1} \,. \tag{4.2}$$

Then we obtain

$$\theta = [(\beta - \alpha)\theta^* + (\alpha + \beta)]/2 \,, \tag{4.3}$$

$$\frac{d\theta}{d\theta^*} = (\beta - \alpha)/2 \tag{4.4}$$

and

$$-1 < \theta^* < 1 \,. \tag{4.5}$$

---

§ Since we deal only with one item in this section and in Section 5, for simplicity, we use α and β in place of α_g and β_g.

Using this new variable θ^* , (4,1) can be rewritten as

$$(4,6) \qquad \int_{\alpha}^{\beta} [I_g(\theta)]^{1/2}\, d\theta = \int_{-1}^{1} [(\beta - \alpha)/2]^{-1} (1 - \theta^{*2})^{-1/2} [(\beta - \alpha)/2]\, d\theta^*$$

$$= \int_{-1}^{1} (1 - \theta^{*2})^{-1/2}\, d\theta^*$$

$$= \sin^{-1}\theta^* \Big|_{-1}^{1}$$

$$= \pi .$$

Thus we have found out that the area under the curve of the square root of the item information function, which is derived from any strictly increasing item characteristic function with 0 and 1 as its two asymptotoes, is π .

The same result can be obtained just as easily, if we use the logistic model instead of the linear model. We can write from (3.22)

$$(4.7) \qquad \int_{-\infty}^{\infty} [I_g^*(\tau)]^{1/2}\, d\tau = Da_g \int_{-\infty}^{\infty} [\exp\{Da_g(\tau - b_g)\}]^{1/2} [1 + \exp\{Da_g(\tau - b_g)\}]^{-1}\, d\tau .$$

Defining τ^* such that

$$(4.8) \qquad \tau^* = [\exp\{Da_g(\tau - b_g)\}]^{1/2} ,$$

we obtain

$$(4.9) \qquad \frac{d\tau}{d\tau^*} = 2\,(Da_g)^{-1}\,\tau^{*-1} ,$$

and, substituting this result into (4.7), we can write

$$(4.10) \qquad \int_{-\infty}^{\infty} [I_g^*(\tau)]^{1/2}\, d\tau = Da_g \int_{0}^{\infty} \tau^*(1+\tau^{*2})^{-1}\, 2(Da_g)^{-1}\tau^{*-1}\, d\tau^*$$

$$= 2\int_{0}^{\infty} (1 + \tau^{*2})^{-1}\, d\tau^* = 2\tan^{-1}\tau^* \Big|_{0}^{\infty}$$

$$= \pi .$$

V Search for the Item Characteristic Function Which Provides a Constant Item Information Function

We are to find out the model for the item characteristic functions which provide us with constant item information functions, within the type of item characteristic function which is strictly increasing in θ with 0 and 1 as its lower and upper asymptotes. It is observed from (2,19) that such a model is unique, since the derivative of the item characteristic function is non-negative for any model which belongs to this type, and it is up to the numerator of the right hand side of (2,19) to make the item information function constant. It is also observed that the range of θ for such an item characteristic function is a finite interval, since the area under the square root of the item information function is a finite value, π, as was found out in the preceding section.

Let C be this constant amount of item information for $\underline{\theta} < \theta < \bar{\theta}$. Then we have

$$\bar{\theta} - \underline{\theta} = \pi C^{-1/2} . \tag{5.1}$$

Thus the length of the interval of θ depends upon the constant item information C.

We find that the model described by

$$P_g(\theta) = \sin^2[a_g(\theta - b_g) + (\pi/4)] \tag{5.2}$$

is the one we have looked for, if we set the parameter a_g such that

$$a_g = C^{1/2}/2 , \tag{5.3}$$

with the range of θ such that

$$(5.4) \qquad [-\pi a_g^{-1}/4] + b_g < \theta < [\pi a_g^{-1}/4] + b_g$$

To confirm this, we can write from (5,2) and (5,3)

$$(5.5) \qquad Q_g(\theta) = \cos^2[a_g(\theta - b_g) + (\pi/4)] ,$$

and

$$(5.6) \qquad \frac{\partial}{\partial\theta} P_g(\theta) = 2 \sin [a_g(\theta - b_g) + (\pi/4)] \cdot \cos [a_g(\theta - b_g) + (\pi/4)] \cdot a_g$$
$$= 2 a_g [P_g(\theta)Q_g(\theta)]^{1/2}$$
$$= C^{1/2} [P_g(\theta)Q_g(\theta)]^{1/2} ,$$

and substituting (5.2), (5,5) and (5.6) into (2.19) we obtain

$$(5.7) \qquad I_g(\theta) = C .$$

We can see from (5.2) that this model provides us with point symmetric item characteristic functions with $(b_g, 0.5)$ as the point of symmetry, just like the normal ogive model, the logistic model and the linear model. The parameter b_g can be called, therefore, difficulty parameter, just as in the normal ogive and logistic models. It is obvious from (5.6) that the parameter a_g is (proportional to) the slope of the line tangent to $P_g(\theta)$ at $\theta = b_g$, just as in these two models, so it can be called discrimination parameter. The meaning of this parameter is more obvious in (5.3), i.e., the fact that the amount of item information solely depends upon the parameter a_g.

We shall call this model, which is presented by (5.2), the constant information model. This model has an important role in the estimation of the operating characteristics of item response categories, which will be described in the following section.

It should be noted that this model can be derived from any other model of the present type, such as the normal ogive, logistic and linear models, by an appropriate transformation of the latent trait. For the purpose of illustration, we shall follow the process starting from the linear model, which is given by (3.10) . Let us define a new variable θ^{**} such that

$$\theta^{**} = (\theta - \alpha)(\beta - \alpha)^{-1} \quad . \tag{5.8}$$

From (5.8) we obtain for the item characteristic function

$$P_g^{**}(\theta^{**}) = \theta^{**} \quad , \tag{5.9}$$

and then

$$Q_g^{**}(\theta^{**}) = 1 - \theta^{**} \quad , \tag{5.10}$$

$$\frac{\partial}{\partial \theta^{**}} P_g^{**}(\theta^{**}) = 1 \tag{5.11}$$

and

$$0 < \theta^{**} < 1 \quad . \tag{5.12}$$

Then the item information function is derived from (5.9), (5.10), (5.11) and (2.19) such that

$$I_g^{**}(\theta^{**}) = [\theta^{**}(1 - \theta^{**})]^{-1} \quad . \tag{5.13}$$

Using the relationship

$$I_g^*(\tau) = I_g^{**}(\theta^{**}) \left[\frac{d\theta^{**}}{d\tau}\right]^2 \tag{5.14}$$

and setting

$$I_g^*(\tau) = C \ , \tag{5.15}$$

we obtain

$$\frac{d\tau}{d\theta^{**}} = [C\theta^{**}(1 - \theta^{**})]^{-1/2} \ . \tag{5.16}$$

From (5.16), we can write

$$\tau = (C)^{-1/2} \int [\theta^{**}(1 - \theta^{**})]^{-1/2} \, d\theta^{**} + d_1 , \tag{5.17}$$

where d_1 is an arbitrary constant. Let us define a variable λ such that

$$\lambda = \theta^{**1/2} \ . \tag{5.18}$$

Then we have

$$\theta^{**} = \lambda^2 \ , \tag{5.19}$$

$$\frac{d\theta^{**}}{d\lambda} = 2\lambda \tag{5.20}$$

and

$$0 < \lambda < 1 \ . \tag{5.21}$$

Using (5.19), (5.20) and (5.21), the integral in the right hand side of (5.17) can be rewritten as

$$(5.22) \qquad \int [\theta^{**}(1 - \theta^{**})]^{-1/2} \, d\theta^{**} = 2 \int [1 - \lambda^2]^{-1/2} \, d\lambda + d_2$$

$$= 2 \sin^{-1}\lambda + d_3$$

$$= 2 \sin^{-1}(\theta^{**})^{1/2} + d_3 \ ,$$

where d_3 is an arbitrary constant. Substituting (5.22) into (5.17), we obtain

$$(5.23) \qquad \tau = 2(C)^{-1/2} \sin^{-1}(\theta^{**})^{1/2} + d_4 \ ,$$

where d_4 is an arbitrary constant, and then

$$(5.24) \qquad \theta^{**} = \sin^2[(C)^{1/2}(\tau - d_4)/2] \ .$$

We can write from (5.9) and (5.24)

$$(5.25) \qquad P_g^*(\tau) = P_g^{**}(\theta^*) = \sin^2[(C)^{1/2}(\tau - d_4)/2] \ .$$

Defining the parameter a_g by (5.3) and setting

$$(5.26) \qquad d_4 = b_g - \pi C^{-1/2}/2 \ ,$$

we obtain (5.2) and (5.4).

Figure 5-1 presents a few examples of the item characteristic function of the constant information model, together with the corresponding item information functions.

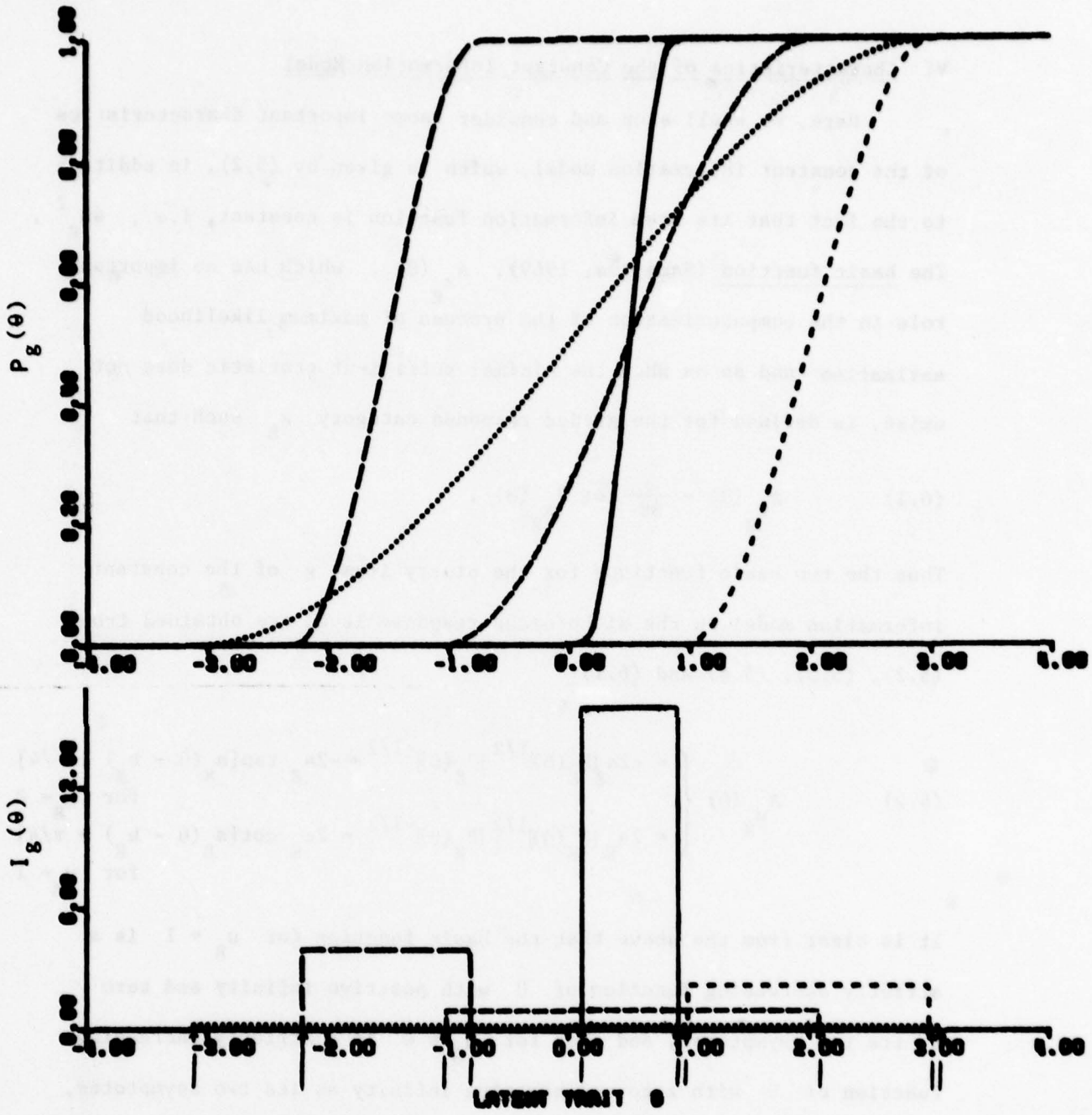

FIGURE 5-1

Item Characteristic Functions (Upper Graph) and the Item Information Functions (Lower Graph) of Five Binary Items Following the Constant Information Model. The Item Parameters Are: $a_1 = 0.25$ **and** $b_1 = 0.00$ **(Smaller Dots),** $a_2 = 0.50$ **and** $b_2 = 0.50$ **(Shorter Dashes),** $a_3 = 0.75$ **and** $b_3 = 2.00$ **(Larger Dots),** $a_4 = 1.0$ **and** $b_4 = -1.5$ **(Longer Dashes), and** $a_5 = 2.00$ **and** $b_5 = 0.50$ **(Solid Line).**

VI Characteristics of the Constant Information Model

Here, we shall stop and consider some important characteristics of the constant information model, which is given by (5.2), in addition to the fact that its item information function is constant, i.e., $4a_g^2$. The basic function (Samejima, 1969), $A_{x_g}(\theta)$, which has an important role in the computerization of the process of maximum likelihood estimation and so on when the minimal sufficient statistic does not exist, is defined for the graded response category x_g such that

$$A_{x_g}(\theta) = \frac{\partial}{\partial\theta} \log P_{x_g}(\theta) . \tag{6.1}$$

Thus the two basic functions for the binary item g of the constant information model on the dichotomous response level are obtained from (5.2), (5.5), (5.6) and (6.1)

$$A_{u_g}(\theta) \begin{cases} = -2a_g [P_g(\theta)]^{1/2} [Q_g(\theta)]^{-1/2} = -2a_g \tan[a_g(\theta - b_g) + \pi/4] \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{for } u_g = 0 \\ = 2a_g [Q_g(\theta)]^{1/2} [P_g(\theta)]^{-1/2} = 2a_g \cot[a_g(\theta - b_g) + \pi/4] \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{for } u_g = 1 . \end{cases} \tag{6.2}$$

It is clear from the above that the basic function for $u_g = 1$ is a strictly decreasing function of θ with positive infinity and zero as its two asymptotes, and that for $u_g = 0$ is a strictly decreasing function of θ with zero and negative infinity as its two asymptotes, respectively. Figure 6-1 illustrates a few examples of these basic functions with different sets of item parameters.

The item response information function, which can be written as

$$I_{x_g}(\theta) = -\frac{\partial}{\partial\theta} A_{x_g}(\theta) \tag{6.3}$$

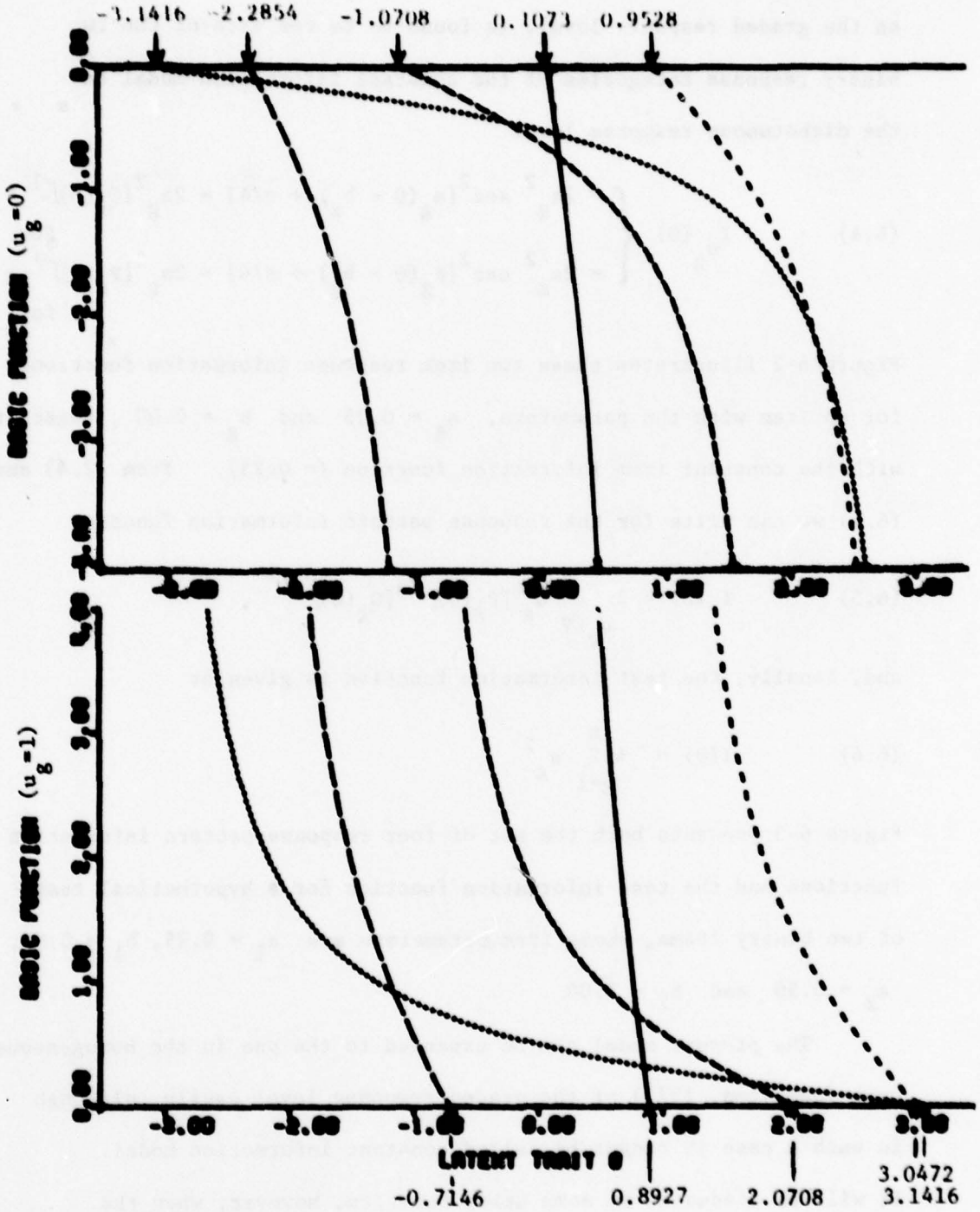

FIGURE 6-1

Basic Functions of Five Items Following the Constant Information Model, with the Parameters, $a_1 = 0.25$ and $b_1 = 0.00$ (Smaller Dots), $a_2 = 0.50$ and $b_2 = 0.50$ (Shorter Dashes), $a_3 = 0.75$ and $b_3 = 2.00$ (Larger Dots), $a_4 = 1.00$ and $b_4 = -1.50$ (Longer Dashes) and $a_5 = 2.00$ and $b_5 = 0.50$ (Solid Curve), for $u_g = 0$ (Upper Graph) and for $u_g = 1$ (Lower Graph).

on the graded response level, is found to be for each of the two binary response categories of the constant information model on the dichotomous response level

$$(6.4) \qquad I_{u_g}(\theta) \begin{cases} = 2a_g^2 \sec^2[a_g(\theta - b_g) + \pi/4] = 2a_g^2[Q_g(\theta)]^{-1} > 0 \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{for } u_g = 0 \\ = 2a_g^2 \csc^2[a_g(\theta - b_g) + \pi/4] = 2a_g^2[P_g(\theta)]^{-1} > 0 \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{for } u_g = 1 \end{cases}.$$

Figure 6-2 illustrates these two item response information functions for an item with the parameters, $a_g = 0.25$ and $b_g = 0.00$, together with the constant item information function (= 0.25). From (2.4) and (6.4) we can write for the response pattern information function

$$(6.5) \qquad I_V(\theta) = 2 \sum_{u_g \in V} a_g^2 [P_g(\theta)]^{-u_g} [Q_g(\theta)]^{u_g - 1},$$

and, finally, the test information function is given by

$$(6.6) \qquad I(\theta) = 4 \sum_{g=1}^{n} a_g^2 .$$

Figure 6-3 presents both the set of four response pattern information functions and the test information function for a hypothetical test of two binary items, whose item parameters are $a_1 = 0.25$, $b_1 = 0.00$, $a_2 = 0.50$ and $b_2 = 1.00$.

The present model can be expanded to the one in the homogeneous case (Samejima, 1972) of the graded response level easily, although in such a case it cannot be called constant information model. It will be discussed on some other occasion, however, when the necessity has come.

We have seen in an earlier section that the area under the curve of the square root of the item information is the same, regardless

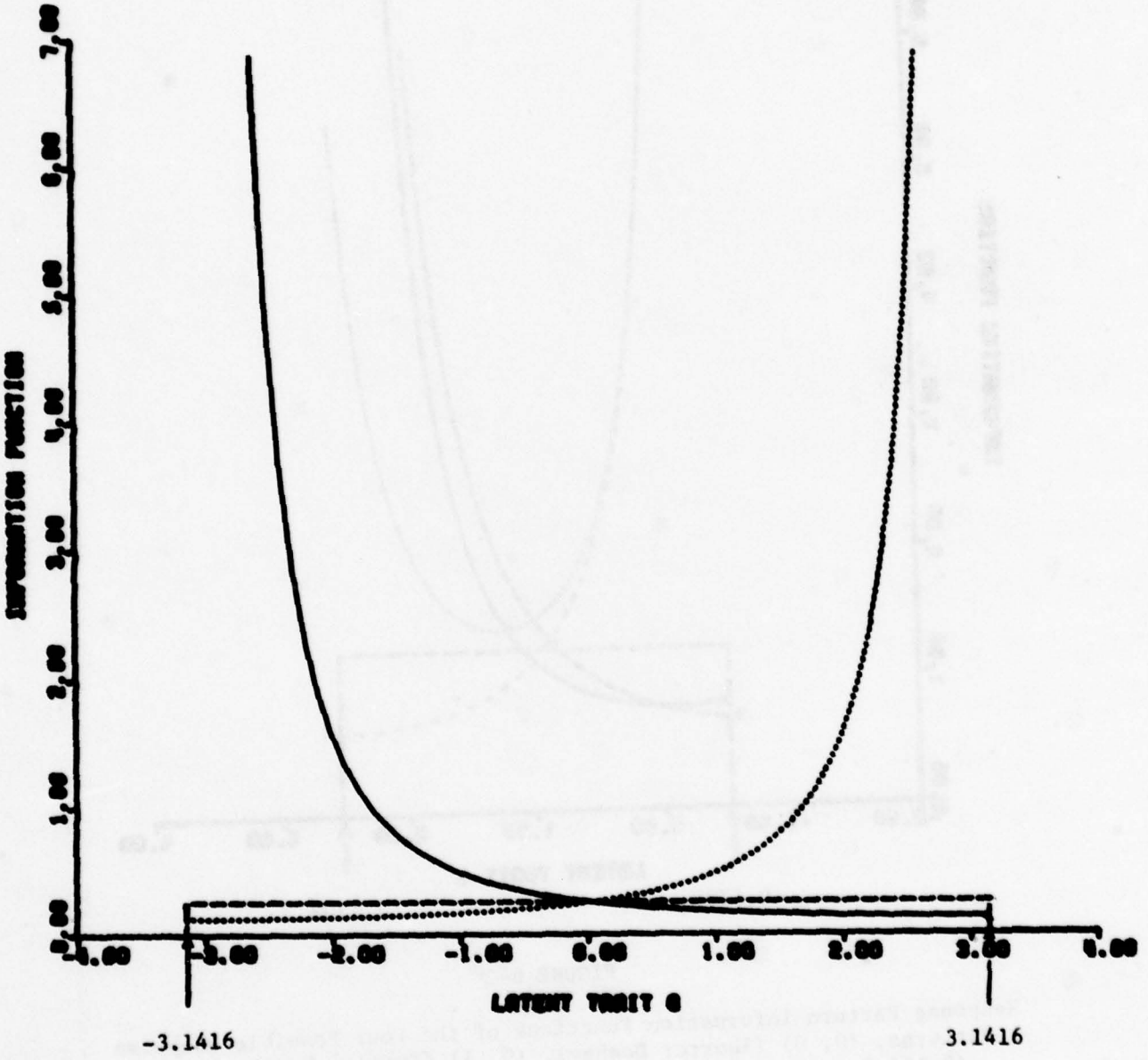

FIGURE 6-2

Item Response Information Functions of an Item Following the Constant Information Model, with the Parameters, $a_g = 0.25$ and $b_g = 0.00$, for $u_g = 0$ (Dotted Curve) and for $u_g = 1$ (Solid Curve), Together with the Constant Item Information Function (Dashed Curve).

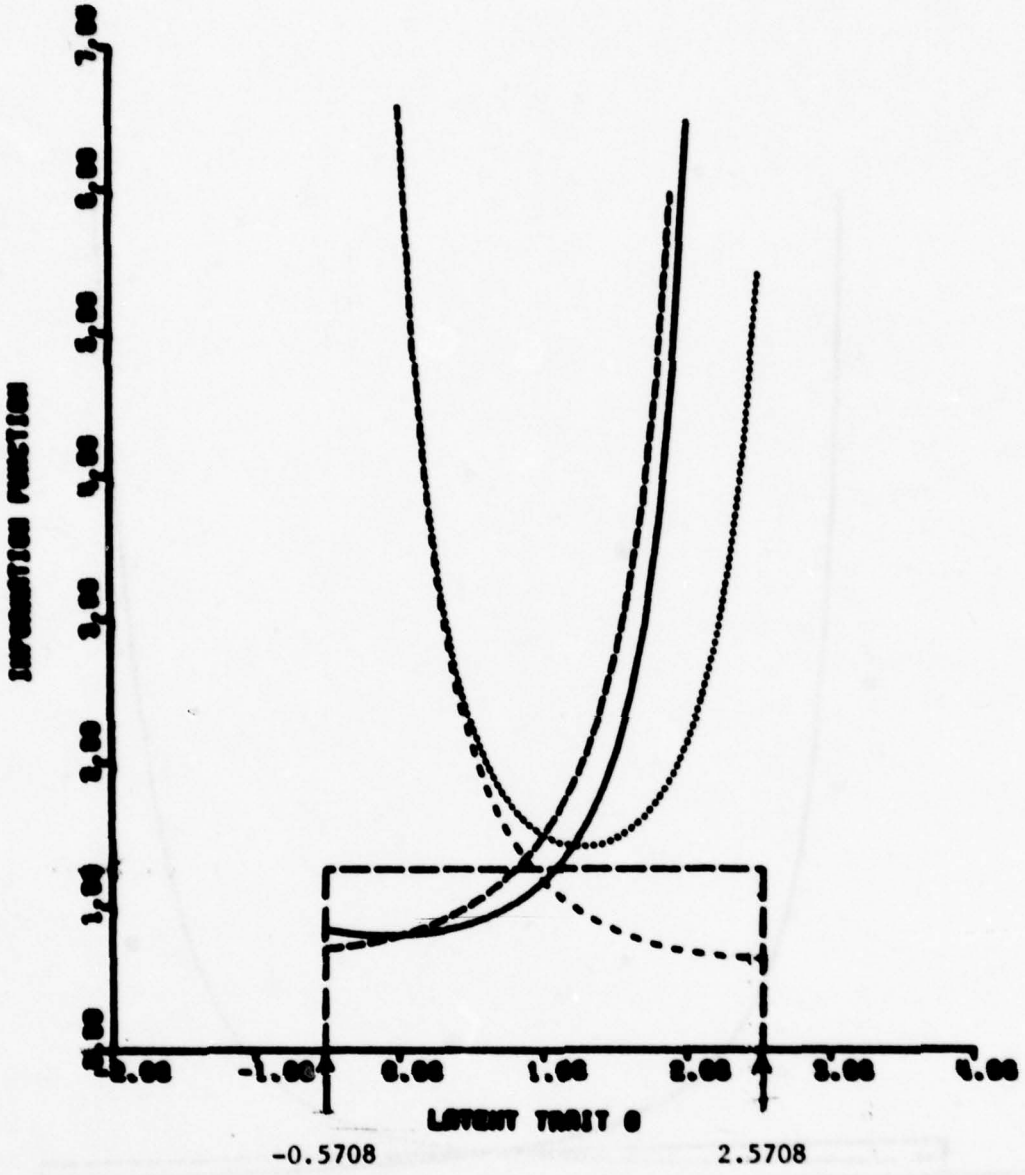

FIGURE 6-3

Response Pattern Information Functions of the Four Possible Response Patterns, (0, 0) (Shorter Dashes), (0, 1) (Smaller Dots), (1, 0) (Solid Curve) and (1, 1) (Larger Dots), of Two Binary Items Following the Constant Information Model with the Parameters, $a_1 = 0.25$, $b_1 = 0.00$, $a_2 = 0.50$ and $b_2 = 1.00$. The Test Information Function Is Also Given by Longer Dashes.

of the specific model, provided that it belongs to the present type which has a strictly increasing item characteristic function with zero and unity as its two asymptotes. We should note, however, that the important index is the reciprocal of the square root of the test information function, which is considered as the standard error of estimation specified as a function of θ. For this reason, we shall make some observations upon the area under the curve of the reciprocal of the square root of the item information function of the constant information model, in comparison with those of other models of the same type.

It can be seen that, if the range of θ is the set of all real numbers, then the asymptote of the item information function when θ approaches either positive or negative infinity should be zero regardless of the specific model which belongs to the present type, since the area under the curve of the item information function is finite. From this fact, it is obvious that the area under the curve of the reciprocal of the square root of the item information function is positive infinity, since the function tends to positive infinity as θ approaches either positive or negative infinity. This subset of models includes both the normal ogive and logistic models. We can say, therefore, that the accuracy of estimation is low on the average, when we consider the whole range of θ, even though these models may provide a high accuracy of estimation locally. When the range of θ is a finite interval, the area under the curve of the reciprocal of the square root of the item information function can be finite, and, therefore, the accuracy of estimation can be higher on the average, when we consider the whole range of

θ . Following a similar mathematical process as we take in the proof of the theorem that the harmonic mean can never be greater than the arithmetic mean when all the observations are positive, it can be shown that the area under the curve of the reciprocal of the square root of the item information function is minimal for the constant information model, among those models of the present type which have the same interval of θ as the range of θ . It can be said, therefore, the constant information model has the best accuracy of estimation on the average, when we consider the whole range of θ .

For the purpose of illustration, Figure 6-4 presents two graphs, one of which provides us with the square root of the item information function of the constant information model, with $a_g = 1/4 = 0.25$ and $b_g = 0.00$, and that for another model, which increases from 0.1 to 0.9 in the first half of the interval of θ and then decreases from 0.9 to 0.1 in the second half. It is obvious that the areas under these two curves are both π . The other graph in Figure 6-4 presents the reciprocals of these two functions in the first graph. It is clear that the area under the curve for the constant information model is much less than that for the other model. It should be noted that, when we consider the corresponding reciprocals of the square roots of the test information functions for the set of n equivalent items following these models, the configuration in the second graph is still the same, with all the ordinate values divided by the square root of n .

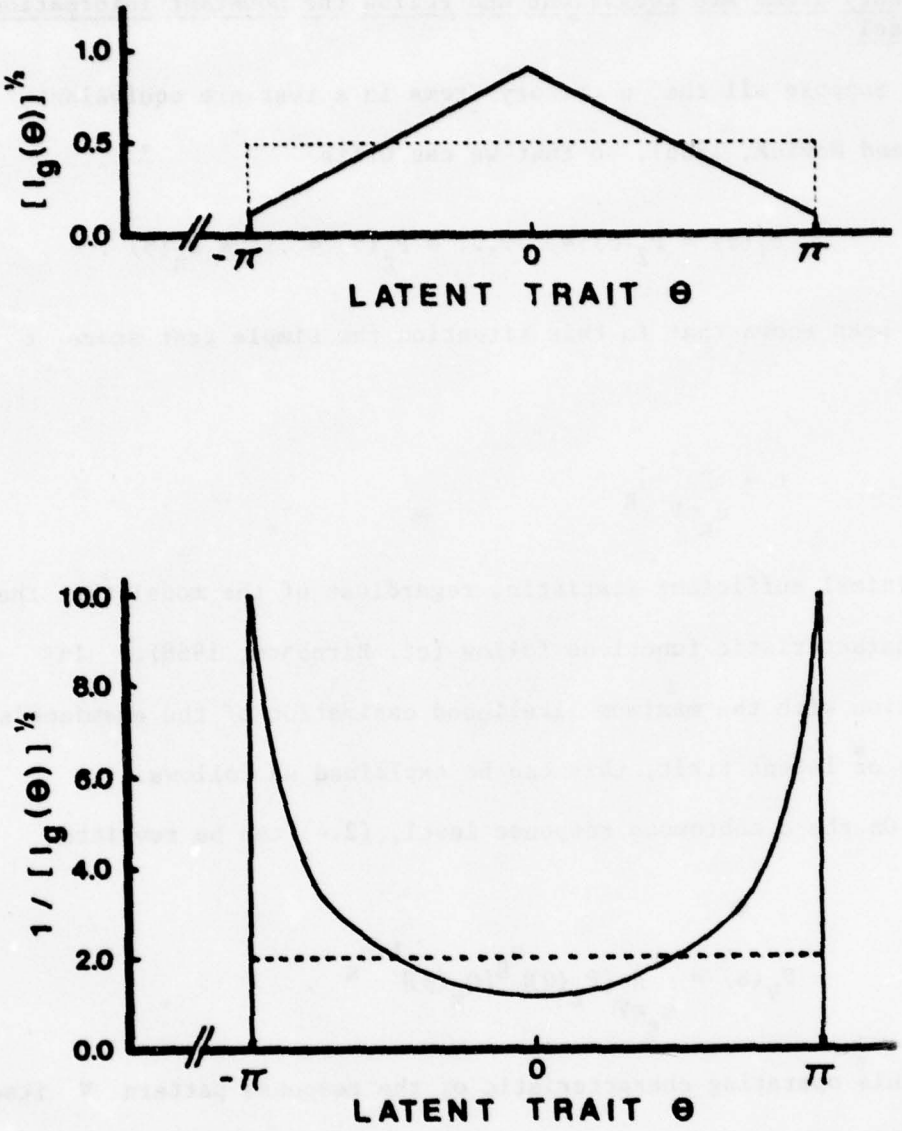

FIGURE 6-4

Square Roots of the Item Information Functions of the Constant Information Model (Dotted Curve) and Another Model (Solid Curve), Which Have the Same Area, Are Shown in the Upper Graph, While in the Lower Graph the Reciprocals of the Corresponding Functions in the Upper Graph Are Drawn. The Item Parameters in the Constant Information Model Are:

$a_g = 0.25$ and $b_g = 0.00$.

VII Maximum Likelihood Estimation of Ability When the Set of Binary Items Are Equivalent and Follow the Constant Information Model

Suppose all the n binary items in a test are equivalent (Lord and Novick, 1968), so that we can write

$$P_1(\theta) = P_2(\theta) = \ldots\ldots = P_g(\theta) = \ldots = P_n(\theta) \quad . \tag{7.1}$$

It has been shown that in this situation the simple test score t such that

$$t = \sum_{u_g \varepsilon V} u_g \tag{7.2}$$

is a minimal sufficient statistic, regardless of the model that the item characteristic functions follow (cf. Birnbaum, 1968). In connection with the maximum likelihood estimation of the examinee's ability or latent trait, this can be explained as follows.

On the dichotomous response level, (2.4) can be rewritten as

$$P_V(\theta) = \prod_{u_g \varepsilon V} [P_g(\theta)]^{u_g} [Q_g(\theta)]^{1-u_g} \quad . \tag{7.3}$$

Since this operating characteristic of the response pattern V itself is the likelihood function in estimating the examinee's ability, we are to use the symbol $L_V(\theta)$ for this function in the present section. When all the items are equivalent, we can rewrite (7.3) in the form

$$L_V(\theta) = [P_g(\theta)]^t [Q_g(\theta)]^{n-t} \quad . \tag{7.4}$$

Thus we have for the likelihood equation

$$\frac{\partial}{\partial \theta} \log L_V(\theta) = [\frac{\partial}{\partial \theta} P_g(\theta)][t - nP_g(\theta)][P_g(\theta)Q_g(\theta)]^{-1} = 0 \quad . \tag{7.5}$$

It should be noted that, although we used the operating characteristic of the specific response pattern as the likelihood function, on the right hand side of (7.4) the response pattern V does not show itself, and all the information given by that particular response pattern is summarized in the form of the test score t. Thus all we need in the estimation process is this simplified, sufficient statistic t, instead of the original response pattern V. From (7.5) we obtain

$$t = nP_g(\theta) , \tag{7.6}$$

and the maximum likelihood estimate $\hat{\theta}$ is given by

$$\hat{\theta} = P_g^{-1}(t/n) . \tag{7.7}$$

When this common item charactreristic function follows the constant information model, we obtain from (5.2) and (7.7)

$$\hat{\theta} = a_g^{-1} [\sin^{-1}(t/n)^{1/2} - \pi/4] + b_g . \tag{7.8}$$

It is obvious from (5.4) and (7.8) that the range of $\hat{\theta}$ is given by

$$[-\pi a_g^{-1}/4] + b_g \leq \hat{\theta} \leq [\pi a_g^{-1}/4] + b_g . \tag{7.9}$$

VIII Estimation of the Operating Characteristic without Using the Old Test

It has been mentioned that the family of methods and approaches developed for estimating the operating characteristics of item response categories (Samejima, 1977b, 1977d, 1978a, 1978b, 1978c, 1978d, 1978e) preassumes the Old Test, or a set of items whose operating characteristics are known and which provides us with a constant test information function over the range of θ of our interest. In this section, we are to see how these methods can be generalized to the situation in which we are to develop a new item pool, and, naturally, cannot depend upon any "Old Test." Suppose, for developing the new item pool, a substantial number of test items are administered to a substantial number of examinees, and there exists a subset of equivalent binary items among these items. In this situation, we can use this subset of items as the substitute for the Old Test.

We assume that these equivalent items have a strictly increasing item characteristic function with 0 and 1 as its two asymptotes. As we have seen in previous sections, we can adjust the latent trait scale in such a way that the resulting common item characteristic function for these equivalent items follow the constant information model, which is given by (5.2). Then the response pattern of each examinee with respect to this subset of equivalent binary items is specified, and is summarized in the form of test score. The origin and unit of the latent trait are set more or less arbitrarily, say, $a_g = 0.25$ and $b_g = 0.00$. From the test score of the subset of equivalent binary items, the maximum likelihood estimate of the examinee's ability is obtained through (7.8). The resulting set

of the maximum likelihood estimates for all the examinees can be used in the same way as we use the set of maximum likelihood estimates obtained from the results of the Old Test. The operating characteristics of each of the other items can be estimated in the same way as we do when we use the Old Test. After this has been done, we can transform the latent trait in whatever way we wish.

In using the generalized method, we should be aware of a few problems. First of all, the constant test information provided by the subset of equivalent binary items with the constant information model should be substantially large, so that the normal approximation for the conditional distribution of $\hat{\theta}$, given θ, should be acceptable. On the other hand, we need a substantially wide range of ability θ for which the test information is constant, in order to make the estimation of the operating characteristics of the other items meaningful. These two are opposing factors, as is obvious from (5.3) and (5.4). The solution for this problem is to use a substantially large number of equivalent binary items, whose common discrimination parameter is low.

Another problem is the effect of the range of $\hat{\theta}$ on the speed of convergence of the conditional distribution of $\hat{\theta}$, given θ, to the normal distribution, $N(\theta, (nC)^{-1/2})$. Since the range of $\hat{\theta}$ is a finite interval which is given by (7.9), it should be expected that the truncation of the conditional distribution makes the convergence slow around the values of θ close to $(-\pi a_g^{-1}/4)+b_g$ and $(\pi a_g^{-1}/4)+b_g$, as is illustrated in Figure 8-1. We must turn to Monte Carlo studies to investigate this problem further and in more detail, which will be done in the near future. A solution for this problem is again to use a

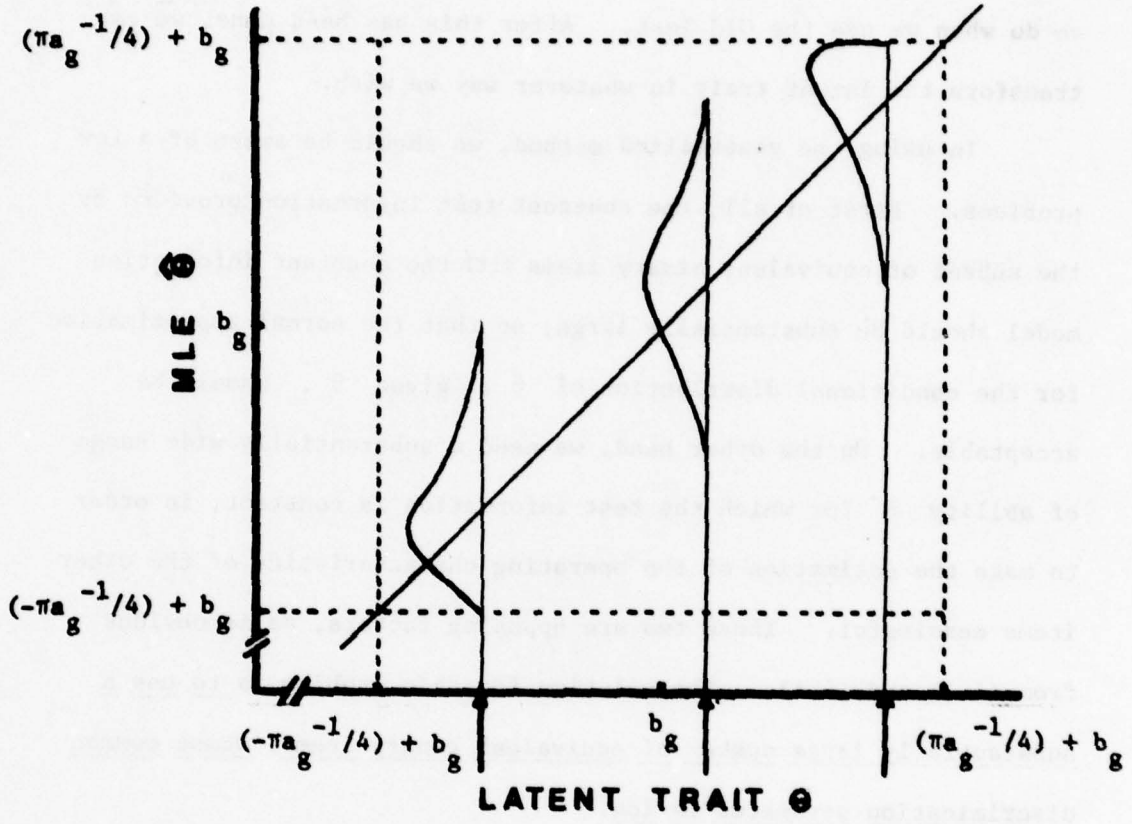

FIGURE 8-1

Graphical Illustration of the Conditional Density Functions of the Maximum Likelihood Estimate $\hat{\theta}$, Given the Latent Trait θ.

set of equivalent binary items whose common discrimination parameter is low, so that the range of θ is wide enough to include all the examinees far inside of the two endpoints of the interval of θ. An alternative for the above solution is to exclude examinees whose $\hat{\theta}$'s are close to $[-\pi a_g^{-1}/4]+b_g$ or $[\pi a_g^{-1}/4]+b_g$. In the second solution, however, the number of examinees will be decreased and this may affect the accuracy of the estimation of the operating characteristics. It is worth noting that the solution for the first problem, which is underscored in the preceding paragraph, is also the solution for the second problem.

If there exist more than one subset of equivalent binary items within the tentative item pool, we can make a full use of all the subsets. We follow the process described earlier for each subset of equivalent binary items, and the resultant estimated operating characteristics can be equated by appropriate transformations of the separately defined latent traits, using, say, the least squares principle, to integrate all of them into one scale.

IX How to Detect a Subset of Equivalent Binary Items

A natural question is how to detect a subset of equivalent binary items out of the tentative item pool. In empirical sciences, it is often difficult to obtain a sufficient evidence. The second best way will be, therefore, to formulate a set of necessary evidences, and to check our data with respect to each criterion. If we find out that our data satisfy all the necessary conditions thus formulated, then we can assume that we have obtained what we wanted, until another necessary criterion becomes available and our data contradict it.

In our situation, first of all, it is necessary, though not sufficient, that the proportions correct should be the same value for all the equivalent binary items, within the range of sampling fluctuations. This can be checked easily, and we can find out a group of binary items which satisfy this condition, if there is any. Next, it is necessary that the 2 x 2 contingency tables of the bivariate frequency distributions should be symmetric and identical among all the pairs of equivalent binary items, within the range of sampling fluctuations. This can be checked for every pair of binary items which have passed the first selection, and, possibly, some items have to be dropped. We can go ahead to the 2^3 contingency tables after this step, to the 2^4 contingency tables, etc., if we wish.

As was pointed out in the preceding section, it is desirable that these equivalent items have a low common discrimination power, in addition to being substantial in number. A necessary condition for this is that the two frequencies for the response patterns (0,1) and (1,0), which are, theoretically, the same value if the two items are equivalent, should be large, or compatible to the other two. This can be checked

therefore, in the same process for checking the equivalency of the binary items. Figure 9-1 illustrates two typical 2 x 2 contingency tables, one of which is for a pair of equivalent binary items which have a common low discrimination parameter, and the other is for a pair of those which have a common high discrimination parameter.

The above are just several examples of the necessary conditions. It is desirable that researchers work on this problem and eventually produce an appropriate set of necessary conditions which contains more varieties of conditions and yet is useful from the practical point of view also.

Low Discrimination Parameter

Item h / Item g	$u_h = 0$	$u_h = 1$	Total
$u_g = 0$	110	243	353
$u_g = 1$	248	399	647
Total	358	642	1000

High Discrimination Parameter

Item h / Item g	$u_h = 0$	$u_h = 1$	Total
$u_g = 0$	300	53	353
$u_g = 1$	58	589	647
Total	358	642	1000

FIGURE 9-1

Two Typical 2 x 2 Contingency Tables for a Pair of Equivalent Items with a Common Low Discrimination Parameter, and for Those with a Common High Discrimination Parameter, Respectively

X Discussion and Conclusions

It has been pointed out that the area under the square root of the item information function is a constant value, π, for any strictly increasing item characteristic function having zero and unity as its two asymptotes. The item characteristic function of this type, which provides a constant item information function, has been searched and discovered, and the model is named constant information model. It has been observed that the amount of information, which such a binary item provides for a specified value of ability θ, and the range of θ, for which the item has a constant information, have a "trading off" relationship. The characteristics of the new model, such as the basic function, the item response information function, the response pattern information function, etc., have been investigated. The process of obtaining the maximum likelihood estimate of the examinee's ability or latent trait when the n items are binary and equivalent has been shown, using the constant information model and the sufficient statistic, t, i.e., the simple test score. It has been shown that, in the methods and approaches of estimating the operating characteristics of item response categories, which have been introduced earlier, a set of equivalent binary items in the tentative item pool can be used as a substitute for the Old Test, and, therefore, those methods are usable even in the situation where we start from the very beginning of developing an item pool without the Old Test. In so doing, attention has been called upon some problems and, for them, possible solutions have been suggested. Some practical considerations as to how we can detect a subset of equivalent items out

of all the binary items in the tentative item pool, if there is any, have been given.

It is interesting to note that, for once, items with low discrimination power have an important role, in preference to those with high discrimination power, which are usually considered to be better items. This fact leads to a more general conviction, which is related with the attenuation paradox (Tucker, 1946, Loevinger, 1954). The important point is to distinguish two different situations, i.e., 1) the situation in which the local accuracy of estimation by an item at a certain level of ability or latent trait is important, and 2) the one in which the overall accuracy of estimation by an item for a certain interval of ability or latent trait is important. As an example of the first situation, we can name tailored testing, or the computerized adaptive testing, and in such an occasion binary items with high discrimination power are more useful. The second situation is exemplified by the estimation of the operating characteristics of graded item response categories, which has been pursued and developed without assuming any mathematical forms for the operating characteristics. In this situation, items with high discrimination power are not necessarily useful, and, as we have seen in the preceding sections, low discrimination items are more useful if we use a subset of equivalent binary items in the tentative item pool as a substitute for the Old Test.

It has been pointed out earlier (Samejima, 1969, Chapter 2) that there is a philosophical difficulty in determining which scale of latent trait is the best one out of those which are strictly increasing transformations of one another. Rescaling of the latent trait on the

final stage of the generalized methods and approaches of estimating the operating characteristics of graded item response categories, therefore, may have to depend solely upon convenience.

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APPENDIX I

A-I Simulated Data

The simulated data used in the present study are characterized as follows.

(1) There are 500 hypothetical examinees.

(2) Their ability, or latent trait, distributes uniformly for the interval of θ, (-2.5, 2.5). Actually, we use 100 discrete points of θ, such as -2.475, -2.425, -2.375, -2.325, ..., 2.375, 2.425 and 2.475, i.e., the midpoints of the 100 subintervals with the width of 0.05, and at each point five examinees are located.

(3) There is a hypothetical test of 35 graded items, each of which has four item score categories, and which provides us with an approximately constant test information function, 21.63, for the interval of θ, [-3.0, 3.0], following the normal ogive model of the graded response level (Samejima, 1969, 1972). The test is called the Old Test, to distinguish from the New Test, which will be described later.

(4) Each of the 500 examinees is assumed to have taken the Old Test, and his response pattern on the 35 graded items has been calibrated by the Monte Carlo method. The score categories of each item are 0, 1, 2 and 3, and a typical response pattern looks like: (3,3,3,2,3,3,2,2,2,2,2,2,1,2,2,2,1,2,1,1,1,0,1,1, 1,0,1,0,1,1,0,0,0,0,0,0).

(5) From each response pattern, the maximum likelihood estimate of the examinee's ability has been obtained, using a computer program written for this purpose. In this process, out of 140 basic functions (Samejima, 1969, 1972), an appropriate set of 35 basic

functions are chosen depending upon the item scores in the response pattern, and, using the Newton-Raphson procedure, the point of θ at which the sum total of these 35 basic functions equals zero is searched.

(6) There is another hypothetical test of 10 binary items, each of which follows the normal ogive model of the dichotomous response level. This is called the New Test.

(7) Each of the 500 examinees is assumed to have taken the New Test also, and his response pattern on the New Test has been calibrated by the Monte Carlo method. A typical response pattern looks like: (0,0,0,1,0,0,1,0,1,1).

(8) The item characteristic functions of the test items of the New Test are assumed to be unknown, and they are the target of estimation. Each method of estimation is evaluated by the the "closeness" of the resultant estimated item characteristic functions to the true item characteristic functions,

$$P_g(\theta) = (2\pi)^{-1/2} \int_{-\infty}^{a_g(\theta-b_g)} e^{-\frac{t^2}{2}} dt \quad .$$

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