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NUMERICAL METHODS AND APPROXIMATION AND MODELLING PROBLEMS IN S--ETC(U)
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Research was done on diffusion processes depending on small parameters and on optimal stochastic control. Another area of research was in measure-valued diffusion processes. Work was completed in the area of stochastic control and in optimality conditions for stochastic systems under partial observations. Work was completed dealing with the approximation and computational problem for the average cost per unit time problem for a diffusion model. A monograph was published dealing with constrained and unconstrained problems of the stochastic approximation type.

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FINAL REPORT

NUMERICAL METHODS AND APPROXIMATION AND MODELLING PROBLEMS IN STOCHASTIC CONTROL THEORY

Contract Number AFOSR-AF-76-3063

July 1, 1977 - June 30, 1978

AIR FORCE OFFICE OF SCIENTIFIC RESEARCH

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W. H. Fleming continued his work on diffusion processes depending on small parameters and on optimal stochastic control. In joint work with C-P Tsai, differential games techniques were used to study minimum exit probabilities for controlled diffusions, when the noise entering the system has low intensity. The results are formally similar to the asymptotic estimates of Ventsel-Freidlin, but the method of proof is quite different. It depends heavily on ideas from control theory and game theory.

A separate line of work, by Fleming and Viot, concerns measure-valued diffusion processes. The particular class of distributed-parameter Markov stochastic processes they treated arose from a population model. However, the form of the generators of these Markov processes implies a certain useful structure to the partial differential equations for means and covariances of population numbers in different regions. A similar moment structure appears in interesting physical applications, for instance, in the study of wave propagation with a continuum of modes in a random medium. The Fleming-Viot method turns out to be of interest for such problems as well.

Papers published

W.H. Fleming, "Exit probabilities and optimal stochastic control", Applied Math. and Optimization, 4(1978), p.329-346.

W.H. Fleming and C-P Tsai, Minimum exit probabilities and differential games, Differential Games and Control Theory III, Eds. P-T. Liu and E. Roxin (Proc. Kingston Conference, June 1978), Lecture Notes in Pure and Applied Math 44, 1979, Marcel Dekker, pp. 67-76.

W.H. Fleming and M. Viot, Some measure-valued population processes, Stochastic Analysis, Eds. A. Friedman and M. Pinsky (Proc. Northwestern Conference, April 1978), Academic Press, pp. 97-108.

J. Ruzicka visited Brown during April - June 1978. His work dealt both with basic theoretical questions in stochastic control and with an aerospace application of the separation principle. The latter work is summarized in the report "Research in problems associated with flight control in turbulence".

R. J. Elliott visited Brown during May, June 1978. His work concerned optimality conditions for stochastic systems under partial observations. A paper by Elliott and P.P. Varaiya "A sufficient condition for the optimal control of a partially observed stochastic system" was written during this period. This is part of a substantial body of work by Professor Elliott in this area, done recently.

The work [1] was completed by Kushner, dealing with the approximation and computational problem for the average cost per unit time problem for a diffusion model. In our past work [2], we developed a fairly comprehensive approach to the computational problem for optimal control problems on diffusion and jump-diffusion models. Ref. [1] is a non-obvious extension of the ideas in [2] to the practically important average cost per unit time problem. Difficulties are caused by the fact that the average cost problem involves control over an infinite time interval; in particular, little is known concerning the relations between existence of invariant measures of the approximating chains and the diffusion. This is an important question, since it is these measures which determine the costs, and even if the diffusion is nicely behaved, the approximating chain may not be, and vice versa.

Stochastic approximation methods for constrained and unconstrained systems.

Kushner has published the monograph [3], which covers an enormous amount of material and new ideas and techniques for many constrained and unconstrained problems of the stochastic approximation type. It uses much better and more realistic conditions and easier and more intuitive methods, and gets better results than the more classical works in the area. Furthermore, it gets better rates of convergence.

Many problems in adaptive control and identification theory involve recursive equations such as (1)

$$X_{n+1} = X_n + a_n H(X_n, \xi_n), \tag{1}$$

where $\{\xi_n\}$ is a noise sequence, a_n positive and real, and

$\sum a_n = \infty$, but where H is not additive in the noise ξ. It is, of course, a stochastic approximation type of algorithm and convergence is treated in [3], for the case where $a_n \to 0$. But no work had been done on the rate of convergence. It turns out [6] that we can get results as extensive as those for the case where H is additive in ξ. This is particularly important since it is usually by a comparison of rates of convergence that one gets a useful comparison between competing algorithms. The conditions in [6] are quite reasonable for the types of applications that we have in mind.

References

[1] Kushner, H.J., "Approximation methods for the minimum average cost per unit time problem with a diffusion model", to appear in Approximation Methods in Probability, North Holland,, Bharucha-Reid, editor.

[2] Kushner, H.J., Probability Methods for Approximations in Stochastic Control and for Elliptic Equations, Academic Press, New York, 1977.

[3] Kushner, H.J., Clark, D.S., Stochastic Approximation Methods for Constrained and Unconstrained Systems, Springer-Verlag, New York, 1978.