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by

O.A. Yevdokimova, V.D. Zimin

EDITED TRANSLATION

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Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ë in Russian, transliterate as yë or ë.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	sinh^{-1}
cos	cos	ch	cosh	arc ch	cosh^{-1}
tg	tan	th	tanh	arc th	tanh^{-1}
ctg	cot	cth	coth	arc cth	coth^{-1}
sec	sec	sch	sech	arc sch	sech^{-1}
cosec	csc	csch	csch	arc csch	csch^{-1}

Russian	English
rot	curl
lg	log

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Advective Flow from a Linear Heat Source Located on a Liquid Surface

O. A. Yevdokimova, V. D. Zimin

Our experiments on the natural convection from a fine heated wire located, on a horizontal liquid surface, were published earlier [1]. The temperature fields and the rates for different values of output, of heat liberated by a source, were measured in the work. Similarity transformations, characteristic for self-similar solutions of boundary layer equations were found empirically. This problem is examined analytically in the present work.

The free convection equations for boundary layer approximation, written for the function of flow ψ and temperature T, in the present case take the following form

$$\frac{\partial}{\partial y}\left(\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}-\nu\frac{\partial^3\psi}{\partial y^3}\right)=g\beta\frac{\partial T}{\partial x}$$

$$\frac{\partial\psi}{\partial y}\frac{\partial T}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial T}{\partial y}=\chi\frac{\partial^2 T}{\partial y^2}. \qquad (1)$$

The origin of the Cartesian coordinate system was selected on the liquid surface, the y-axis was directed vertically downward , the x-axis was directed along the surface. The linear heat source was located along the z-axis.

In accordance with the conditions of the experiments, described in work [1], the boundary conditions when y = 0 we will write in the following form

$$y = 0, \quad \psi = 0, \frac{\partial \psi}{\partial y} = 0, \frac{\partial T}{\partial y} = 0.$$

When $y = \infty$ we require the vanishing of temperature T and the rate components $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$. This requirement denotes, that the perturbation, created by the heat source, is concentrated close to the surface and does not penetrate deeply into the liquid. Employing the symmetry of the problem, it is easy to show, that

$$\psi(x) = -\psi(-x), \qquad T(x) = T(-x). \tag{2}$$

The conditions $\frac{\partial \psi}{\partial y} = \frac{\partial \psi}{\partial x} = 0$ are equivalent to the condition ψ = const, since taking into account (2) the boundary conditions at infinity can be written in the following form

$\psi = 0$ T = 0 when y = ∞.

By integrating the second equation from (1) with respect to y within the limits from 0 to ∞ and taking the boundary conditions into account, it is possible to obtain the integral condition

$$\int_0^\infty \frac{\partial \psi}{\partial y} T dy = \frac{Q}{\rho c},$$

which expresses the constancy of the heat flow, passing through any vertical section. The magnitude of 2Q is the output of the heat source per unit of length.

Assuming

$$\psi = \nu \left(\frac{x}{l}\right)^{\frac{1}{2}} f(\eta), \quad T = \frac{Q}{\rho c \nu} \left(\frac{x}{l}\right)^{-\frac{1}{2}} t(\eta),$$

$$\eta = \left(\frac{x}{l}\right)^{-\frac{1}{2}} \frac{y}{l}, \quad l = \left(\frac{Q g \beta}{\rho c \nu^3}\right)^{-\frac{1}{3}},$$

we obtain the equations for the dimensionless functions f and t:

$$(ff'' + 2f''')' = t + \eta t',$$
$$-\frac{1}{2}\left(tf' + t'f\right) = \frac{1}{P_r} t'' \qquad (3)$$

with the boundary conditions $\eta = 0, \quad f = f' = 0,$

$$\eta = \infty, \quad f = t = 0, \qquad (4)$$

to which the integral condition of the constancy of heat flow is connected

$$\int_0^\infty tf' d\eta = 1.$$

By integrating the equations of (3) once and by determining from the boundary conditions the integration constant in the heat conduction equation, we find

$$ff'' + 2f''' = \eta t - 2c, \qquad (5)$$

$$t' = -\frac{Pr}{2} tf. \qquad (6)$$

Condition t' = 0 when η = 0 is now a consequence of (6) and should be discarded. For determining c it is necessary to know the law of the decrease in t with large η. If t decreases more rapidly than $\frac{1}{\eta}$, then c = 0. However, when c = 0 there are no solutions for (5) and (6), which satisfy the boundary condition f = 0 when $\eta = \infty$, thus, $c \neq 0$, and $t \sim \frac{1}{\eta}$ when $\eta \to \infty$.

Equations (5) and (6) with boundary conditions (4) are invariant relative to the following transformation:

$$f = I^{-\frac{1}{6}} \tilde{f} \quad . \quad \eta = I^{\frac{1}{6}} \tilde{\eta}$$

$$t = I^{-\frac{5}{6}} \tilde{t} \qquad c = I^{-\frac{4}{6}} \tilde{c} \qquad I = \int_0^\infty \frac{d\tilde{f}}{d\tilde{\eta}} \tilde{t}\, d\tilde{\eta}. \qquad (7)$$

By employing (7), it is possible to transform a solution with an arbitrary value I into a solution with I = 1. In connection with this when finding solutions it was assumed that c = 1, and the integral condition was temporarily not taken into account.

An approximate solution of the boundary value problem for equation system (5) was obtained by the method of approximating functions. In the interval $0<\eta<\eta_1$ the desired functions were in the form of polynomials f_1 and t_1, and f and t with $\eta>\eta_1$ were replaced by asymptotic solutions f_2 and t_2, valid with large η:

$$f_2 = \frac{2}{P_r}\frac{1}{\eta} - \frac{32}{P_r^3}(3P_r - 2)\frac{1}{\eta^5} + \ldots$$
$$t_2 = 2\frac{1}{\eta} - \frac{8}{P_r^2}(3P_r - 2)\frac{1}{\eta^5} + \ldots$$

From the structure of these expansions it is evident, that a sufficiently good approximation is already ensured by terms, proportional to η^{-1}. When $\eta=\eta_1$ both concepts coalesce. The following expressions were obtained for f_1 and t_1 :

$$f_1 = \frac{\eta_1^3}{18} z^2 (1 - z)^3 + \frac{2}{P_r \eta_1} z^2 (5 - 9z^2 + 5z^3); \; z = \frac{\eta}{\eta_1}$$
$$t_1 = t_0(1 - 3z^2 + 2z^3) + \frac{2}{\eta_1}(4z^2 - 3z^3); \; t_0 = t|_{\eta=0}.$$

The form parameters t_o and η_1 were determined from the condition of equality to zero of the average values of the discrepancies, which arise with the substitution of approximate expressiosn for f and t in equation (5).

$$\int_0^\infty (f')^2 d\eta + (f'')_{\eta=0} + \int_0^\infty \left(\frac{1}{2} t\eta - 1\right) d\eta = 0$$

$$t_0 = \frac{P_r}{2}\int_0^\infty tf d\eta.$$

After carrying out the integration a system of algebraic equations is obtained for the values η_1 and t_o. The solution of this system was found numerically for P_r = 7. (The Prandtl number for distilled water at a temperature of

20°C is equal to 7.06). The values $\eta_1 = 2.01$, $t_0 = 2.04$ were found for the values η_1 and t_0.

Now by calculating the value of I and carrying out the transformation of (7), we obtain expressions for functions f and t, which already do not contain any indeterminate constants.

For comparison with the experimental results [1] we write the expressions for the horizontal rate component u and the vertical temperature gradient $\frac{\partial T}{\partial y}$.

$$u = \left(\frac{g\beta Q}{\rho c}\right)^{\frac{1}{3}} f'(\eta), \quad \frac{\partial T}{\partial y} = \frac{Q}{\rho c\nu} |x|^{-1} t'(\eta), \quad \eta = \left(\frac{g\beta}{\nu^2} \frac{Q}{\rho c\nu}\right)^{\frac{1}{6}} |x|^{-\frac{1}{2}} y.$$

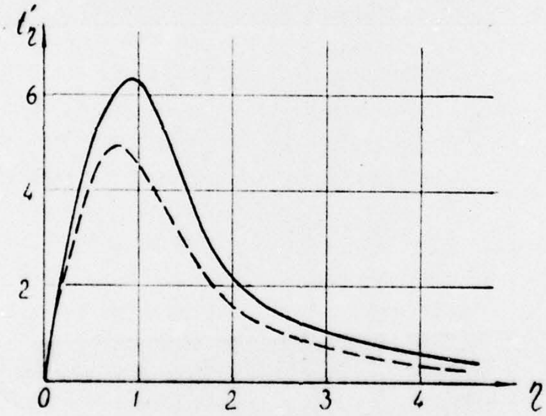

Fig. 1. Dimensionless profile of the vertical temperature gradient. The broken line represents the experimental results of work [1], the solid line represents the calculations.

A comparison with [1] reveals complete coincidence of the functional dependences of the values u, $\frac{\partial T}{\partial y}$ and η on x and Q. The universal profiles t' and f' are given in Fig. 1 and 2 (the solid line represents the calculations, the broken line represents the average of the experimental results [1]).

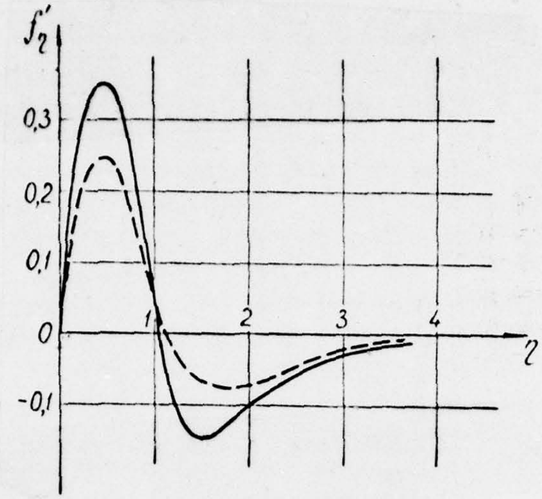

Fig. 2. Dimensionless profile of the horizontal rate component. The broken line is the experiment, the solid line is the calculation.

As is evident from the figures, the shapes of the curves t' (η) and f' (η) are found in satisfactory agreement with the experiment, the maximum values of the magnitudes f' and t' are considerably higher than the experimental values. The quantitative differences are connected, apparently, with the limitedness of the volume of liquid along the direction of motion.

Experiments with a broader vessel, than in work [1], were carried out to clarify the effect of the horizontal dimension. The experiments gave a more accurate coincidence of the law of decrease $\frac{\partial T}{\partial y}$ with theoretical $\frac{\partial T}{\partial y} \sim \frac{1}{y^2}$ (with large y), however, in other respects the increase in the horizontal dimension did not lead to a more accurate realization of the conditions, convenient for analytical investigation.

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