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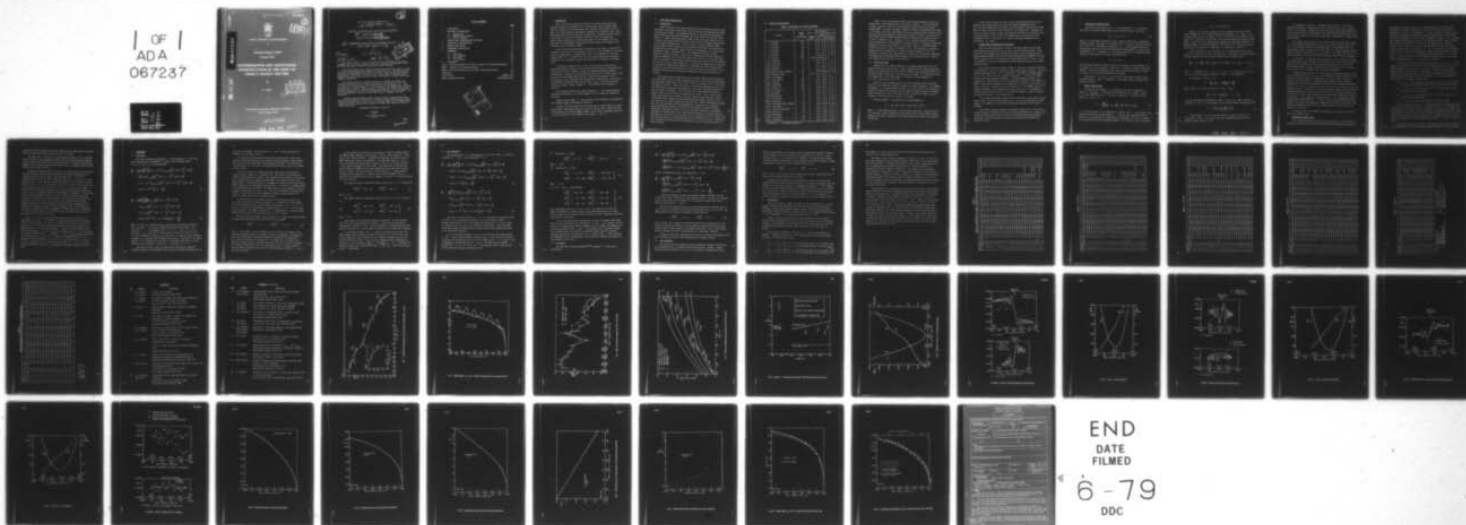
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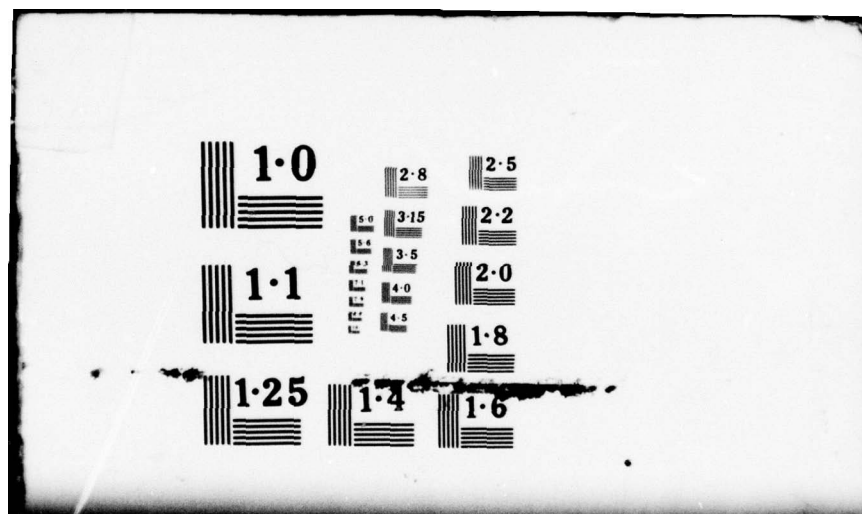
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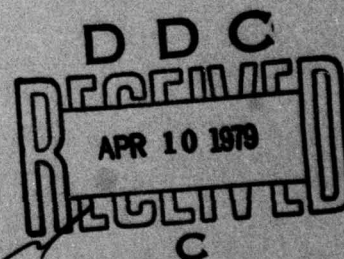
August 1978

**DETERMINATION AND GEOPHYSICAL
INTERPRETATION OF THE ORBIT OF
CHINA 2 ROCKET (1971-18B)**

by

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SUMMARY

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The orbit of China 2 rocket, 1971-18B, has been determined at 114 epochs throughout its 5-year life, using the RAE orbit refinement program PROP 6, with more than 7000 radar and optical observations from 83 stations.

The rocket passed slowly enough through the resonances 14:1, 29:2, 15:1 and 31:2 to allow lumped geopotential harmonic coefficients to be calculated for each resonance, by least-squares fittings of theoretical curves to the perturbation-free values of inclination and eccentricity. These lumped coefficients can be combined with values from satellites at other inclinations, to obtain individual harmonic coefficients.

The rotation rate of the upper atmosphere, at heights near 300 km, was estimated from the decrease in orbital inclination, and values of 1.15, 1.05, 1.10 and 1.05 rev/day were obtained between April 1971 and January 1976. From the variation in perigee height, 25 values of density scale height were calculated, from April 1971 to decay. Comparison with values from the COSPAR International Reference Atmosphere 1972 shows good agreement between April 1971 and October 1975, but the observational values are 10 per cent lower, on average, than CIRA thereafter.

A further 1400 observations, made during the final 15 days before decay, were used to determine 15 daily orbits. Analysis of these orbits reveals a very strong west-to-east wind, of 240 ± 40 m/s, at a mean height of 195 km under winter evening conditions, and gives daily values of density scale height in the last 7 days before decay.

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1 INTRODUCTION

China 2 rocket was launched on 3 March 1971 into an orbit having the following elements: inclination 69.9° , perigee height 265 km, apogee height 1825 km, eccentricity 0.105 and period 106.1 minutes. After nearly 5 years in orbit, the rocket decayed in the Earth's atmosphere on 16 February 1976. Over 7000 observations have been used with the RAE orbit refinement program¹, PROP 6, to determine 114 orbits, 40 of which contained at least one sequence of Hewitt-camera observations. Orbits between July 1971 and January 1972 have previously been determined² by Brookes and Ryland, mostly with slightly lower accuracies, for air-density studies.

As the orbit contracted under the influence of air drag, it passed first through the tail-end of 27:2 resonance in April 1971, then through 14:1 resonance (when the ground track is repeated daily, every 14 revolutions of the orbit) during October 1972. In March 1974, the orbit passed through 29:2 resonance, followed by 15:1 resonance in March 1975, and then 31:2 resonance in November 1975.

An analysis was made of each of the four resonances specified above, using least-squares fittings to the perturbation-free values of inclination and eccentricity for each resonance, to give the lumped geopotential harmonic coefficients. Although the coefficients obtained are not of high accuracy, they may still be good enough, when used with values from satellites at other inclinations, to give improved individual harmonic coefficients. The variation of inclination during the whole life is shown in Fig 1, with the variations during the resonant regions 'mapped in'. The resonances are discussed in section 7.

The rotational speed of the upper atmosphere, Λ , has been determined between the resonances, for various heights and local times, as discussed in section 6.

Density scale height, H , was calculated from variations in perturbation-free perigee height, as discussed in section 5.

Finally, when this study was near completion, a further 3000 or so radar observations, made during the final 15 days of orbital life, arrived from the North American Air Defense Command, NORAD. Daily orbits, determined from 1400 of these observations, were analysed to determine Λ and H .

2 MAIN ORBIT DETERMINATION

2.1 Observations

About 7500 optical and radar observations were available initially, from which about 450 US Navy observations, with elevations below 20° , were discarded as being less reliable. The remaining 7000 or so were distributed among 114 orbits which were determined throughout the 5-year orbital life of the rocket (Table 1). Almost 1500 observations (about 20 per cent) were rejected as ill-fitting, leaving about 5560 observation (49 per orbit on average) actually used in the orbital determinations. Forty of these 114 orbits contained highly accurate Hewitt-camera sequences of observations (with about four observations selected from each sequence), made at Malvern or Edinburgh: 31 of these orbits had Malvern only; 7, Edinburgh only. The remaining two orbits included sequences from both stations. Nine of the 31 Malvern-only orbits contained at least two sequences, one of which was rejected for orbits 31, 32 and 38. Only orbit 29 contained a single Malvern sequence which was rejected but, fortunately, the orbit was fairly accurate without it. This rejected sequence was at the end of the group of observations being used to determine the orbit and, in such a situation, rejection occasionally does occur, not because of any defects in the observations but because of irregular variations in air drag that spoil the fitting.

The largest group of observations used was about 2800 from the US Navy, of which only about 8 per cent were ultimately rejected. British radar observations were available almost daily from 22 April 1975 to 15 February 1976 (*ie* the final 300 days of the life). Since there were on average four sequences per day, only one observation per sequence was selected so that these observations would not have too great an influence. This resulted in 1225 observations distributed among 34 orbits, giving about 36 observations per orbit. About 15 per cent of these observations were ultimately rejected. Malvern radar supplied a further 51 sequences, with a mean of three observations per sequence, during 1971-73, but nearly half of these observations were rejected. Other observations available were 136 from the South African kinetheodolite (43 rejected); 115 sequences comprising 380 observations (23 rejected) from Jokioinen, Finland; and about 2180 visual observations (about 25 per cent rejected) supplied by the Appleton Laboratory at Slough. These visual observations were made by volunteer observers, predominantly in Britain, with stations 2414 (D.J. Hopkins), 2420 (R.D. Eberst) and 2421 (D.M. Brierley) all contributing more than 250 observations each. There were also observations from South Africa, Australia, Cyprus, the Netherlands and ten other countries.

2.2 Observational accuracyTable 2 - Residuals for selected stations

Station		Number of observations	Rms residuals			
			Range km	Minutes of arc		
				RA	Dec	Total
1	US Navy	243		1.6	1.9	2.5
2	US Navy	63		1.4	1.8	2.3
3	US Navy	48		1.7	1.4	2.2
4	US Navy	62		1.8	1.4	2.3
5	US Navy	175		1.7	1.7	2.4
6	US Navy	195		1.6	1.7	2.3
29	US Navy	1775	0.6*	0.25*	0.4*	
414	Capetown	74		2.9	3.0	4.1
616	Adelaide 5	13		0.9	1.4	1.7
2152	Akrotiri 2	11		1.5	2.5	2.9
2265	Farnham	43		2.2	2.4	3.3
2303	Malvern Hewitt Camera	134		0.03	0.04	0.05
2304	Malvern radar	71	0.9	2.4	2.4	
2402	Stevenage 2	16		1.8	1.9	2.6
2414	Bournemouth	367		4.1	3.6	5.5
2419	Tremadoc	96		2.8	2.4	3.7
2420	Willowbrae	353		2.1	2.2	3.0
2421	Malvern 4	278		2.0	1.9	2.7
2430	Stevenage 4	16		1.0	1.7	2.0
2432	Horsham	15		2.0	1.7	2.7
2437**	Warrington	10		7.6	2.1	7.9
2513	Colchester	17		4.0	2.4	4.6
2528	Aldershot	12		1.4	1.4	1.9
2534	Edinburgh Hewitt Camera	36		0.04	0.03	0.05
2539	Dymchurch	13		1.4	1.4	2.0
2577	Cape Kinetheodolite	93		0.7	0.8	1.1
2596	Akrotiri	20		2.7	3.8	4.7
4126	Gröningen	14		2.9	3.3	4.4
6702	Jokioinen	357		3.4	3.4	4.8
8597	Adelaide 4	19		2.4	3.9	4.5

* Geocentric ** includes station 47

Table 2 lists the observing stations which have contributed at least ten observations to the determination of the orbits in Table 1. The observational accuracy is given in the form of rms residuals, obtained using the computer program ORES³. As usual, the Hewitt cameras at Malvern and Edinburgh (the latter is no longer in use) have the highest accuracy, averaging around 3 seconds of arc. The South African kinetheodolite had a mean residual of about 1.1 minutes of arc. The US Navy stations showed remarkably uniform topocentric accuracies, between 2.2 and 2.5 minutes of arc. (Station 29 is a fictitious geocentric station whose residuals must be multiplied by a factor of about 5 to obtain an equivalent topocentric accuracy, which comes to 2.3 minutes of arc.) The Malvern radar yielded an accuracy of about 3.4 minutes of arc. The theodolite at Jokioinen, Finland was up to standard with a little less than 5 minutes of arc. The remaining 18 stations, all visual observers reporting to the Appleton Laboratory at Slough, have rms residuals mostly between 2 and 5 minutes of arc. Lists of the residuals have been sent to the observers.

2.3 Orbital accuracy

Computed sets of orbital elements at 114 epochs between April 1971 and February 1976 are listed in Table 1, with standard deviations where appropriate. The sd in inclination varies from 0.0003 to 0.0026°, the rms being 0.0012°; for the Hewitt-camera orbits only, the rms value was 0.0008°. For eccentricity, the sd varies from 3×10^{-6} (equivalent to 22 metres in perigee height) to 44×10^{-6} , with an rms of 15×10^{-6} .

The sd in right ascension of the node varies between 0.001° and 0.002°, except for the final orbit (0.004°), showing very good consistency. The argument of perigee, ω , and mean anomaly at epoch, M_0 , have similar sd, varying from 0.003° to 0.016° for the first 64 orbits; then increasing, generally to between 0.02° and 0.04° up to orbit 102; then to 0.05° to 0.08° up to orbit 112; and finally to 0.13° and 0.34° for the last two orbits.

The mean anomaly M is modelled in the PROP program⁴ as

$$M = M_0 + M_1 t + M_2 t^2 + M_3 t^3 + M_4 t^4 + M_5 t^5,$$

where t is the time from epoch. It can be seen from Table 1 that during 1971 the M_5 -term is rarely required; by 1974 about half the orbits require the M_5 -term; for the final 12 orbits (in the three months before decay), ten required the M_5 -term.

Fifteen orbits contained at least one day during which there was a geomagnetic storm (defined here as having the planetary geomagnetic index $A_p \geq 50$), even though (some successful) steps were taken to exclude such stormy days by careful selection of the time-span of the observations. Although the accuracy of some of these 15 orbits may have been affected by the density variations associated with the storms, they compared well in accuracy with the 'stormless' orbits. (The effect of a storm on the upper atmosphere is not indicated by A_p alone.)

3 FURTHER ORBIT DETERMINATIONS NEAR DECAY

After the orbits described in section 2 had been computed, about 3000 observations, made during the last 15 days of the life, were received. These were made by the assigned and contributing sensors of the North American Air Defense Command (NORAD) Space Detection and Tracking System (SPADATS), and about 1400 of them were used to determine daily orbits during these 15 days. It was not possible to use all the observations because PROP only accepts 100 observations per epoch.

The 15 daily orbits are listed in Table 3. They are of excellent accuracy and consistency. The sd in inclination varies from 0.0007 to 0.0011°. The sd in eccentricity varies from 7×10^{-6} (equivalent to 50 metres in perigee height) to 17×10^{-6} . The sd in right ascension of the node Ω varies from 0.0006 to 0.0015°, while the argument of perigee ω and mean anomaly at epoch, M_0 , (generally of similar accuracy) have sd between 0.04 and 0.40°. The elements in Tables 1 and 3 can be compared for the same epoch, *ie* orbit 113 with orbit C and orbit 114 with K. On orbits 113 and 114, the sd in e is 0.00004, and this is reduced to 0.00001 on the one-day orbits, while the sd in i is reduced by a factor of about 2½ and the sd in Ω by a factor of about 3. These illustrate the improved accuracy of the one-day orbits.

It is worth noting that the epoch of the final orbit is only about 2 hours before decay. This is why the values of $M_2 - M_4$ are so exceptionally large. However, the imminence of decay did not affect the accuracy of the orbital elements.

The orbital elements of Table 3 are plotted in Figs 14-18. These orbits are more accurate and frequent than any previously published, for a time so near decay, and their analysis gives values of scale height and upper-atmosphere winds with a finer time resolution than has previously been possible (see sections 5 and 6).

4 VARIATION IN PERIGEE HEIGHT

The values of semi major axis, a , and eccentricity, e , from Table 1 were used to calculate perigee height over a spherical Earth, h_p , from

$$h_p = a(1 - e) - R, \quad (1)$$

where R is the Earth's equatorial radius, taken as 6378.16 km, as assumed in the PROP program. The values are plotted in Fig 2 as circles. The main oscillation is due to the odd-harmonic perturbation in e , which, combined with the lunisolar perturbation, was calculated using the PROD program⁴, to give a total perturbation Δe . A parameter Q , the perigee height over a spherical Earth, cleared of perturbations, can now be determined from

$$Q = h_p + a\Delta e,$$

and the values obtained are shown in Fig 2 by triangles. The perturbation Δe is assumed to be zero initially, so that the Q values are relative rather than absolute. This is not significant here, since only the slope dQ/dt is used (in section 5) for calculating density scale height.

The values of h_p and Q , in the final 15 days before satellite decay, calculated using a and e from Table 3, are plotted in Fig 19, which is an extension of Fig 2.

5 DENSITY SCALE HEIGHT

The density scale height H is a measure of the rate of decrease of density ρ as height y increases, and is defined by $\frac{1}{H} = -\frac{1}{\rho} \frac{d\rho}{dy}$. Values of H were calculated from $\dot{Q}$, the rate of change of Q due to air drag, where⁵, for $ae/H > 3$,

$$\dot{Q} = -\frac{2HM_2}{3M_1e} \left(1 - 2e + \frac{H}{4ae} - \frac{2\epsilon'}{e} \sin^2 i \cos 2\omega \right), \quad (2)$$

ϵ' is the ellipticity of the atmosphere ($=0.00335$) and H is at a height $1.5 H_p$ above the satellite's perigee height y_p . Ignoring small perturbations, we may take y_p as given by the right-hand side of equation (1) with R replaced by the local Earth radius at perigee latitude. For 1971-18B, at 70° inclination,

$$y_p = h_p + 18.8 \sin^2 \omega .$$

Values of $\dot{Q}$ for use in equation (2) are obtained in the form $\Delta Q/\Delta t$, from the change ΔQ in Q over a suitable time interval Δt . Since Q has an accuracy of about 0.1 km, values of ΔQ of 3 km or more are required to give values of $\dot{Q}$ accurate to 3 per cent, and the time intervals were chosen on this basis. Values of H were obtained from equation (2) using these values of $\dot{Q}$ and mean values of the other parameters. The values of H are plotted against time in Fig 3 as circles, up to MJD 42770. At this point, $ae/H \simeq 3$.

In the remainder of the calculations for H , from MJD 42770 to the end of the life, the value of ae/H was less than 3. The 'phase 2' regime discussed in Ref 6 (page 88) can then be used: equation (5.35) of Ref 6 gives

$$\frac{da}{dx} = y_0 + \frac{1}{2}e \left(4 - 3y_0^2 - y_0 y_2 \right) - \frac{1}{2}c \cos 2\omega (y_0 - 2y_2 + y_0 y_3) = \beta, \text{ say,} \quad \dots\dots(3)$$

where $c = \frac{\epsilon' a(1-e)}{2H} \sin^2 i$, $y_r = I_r/I_1$, and I_r is the Bessel function of the first kind and imaginary argument, of order r (*ibid*, page 36).

To calculate H from $\dot{Q}$, put

$$\dot{Q} = \frac{dQ}{da} \cdot \frac{da}{dt} = - \frac{d(a-x)}{da} \cdot \frac{2na}{3n},$$

where $n(=M_1)$ is the mean motion and $\dot{n} = 2M_2$. Hence

$$\frac{da}{dx} = 1/(1 + 3M_1 \dot{Q}/4M_2 a) = \alpha, \text{ say.} \quad (4)$$

H is now determined by guessing two values, H_1 and H_2 , which are used to calculate β_1 and β_2 from equation (3), and then by linear interpolation from

$$H = \frac{H_1(\beta_2 - \alpha) + H_2(\alpha - \beta_1)}{\beta_2 - \beta_1}. \quad (5)$$

These values of H have also been plotted in Fig 3 as circles, for $42770 < \text{MJD} < 42824$. The sd shown have been estimated from the errors in perigee height (usually 0.1 km).

For comparison, values of H obtained⁷ from *CIRA 1972*, for the same heights and exospheric temperatures as the calculated values, are also plotted in Fig 3, as triangles. Consecutive points have been joined (where possible) to give an overall view of the difference. To avoid confusion between some of the points, only one half of the sd-bar is shown where overlap with a triangle would have occurred.

Fig 3 shows that from April 1971 to October 1975 the observational values of H , for heights between 250 and 350 km, are generally within 10 per cent of the *CIRA 1972* values and have the same mean value. From November 1975, the observational values are mostly lower than *CIRA 1972*. Since atmospheric density varies so widely - by a factor up to 5 at 300 km - Fig 3 shows that *CIRA 1972*, which incorporates Jacchia's 1971 model⁸, provides a good reference atmosphere.

Ten values of H obtained from equations (3) to (5), using the 15 daily orbits near decay (given in Table 3), are also plotted in Fig 3 for MJD between 42810 and 42824; during this time the height falls from 260 to 205 km. The observational values are mostly lower than those from *CIRA 1972*, on average by about 10 per cent, a difference that is probably significant.

The observational values of H have been plotted against height in Fig 4; they are shown as crosses or circles, and comparative curves from *CIRA 1972* are given, for various exospheric temperatures. The observational values are numbered 1-25, with points 1-15 having bracketed numbers giving exospheric temperatures, corrected for semi-annual variation⁹, which have been calculated⁷ using *CIRA 1972* with the appropriate values of solar 10.7 cm radiation energy and geomagnetic index. For points 1-7, the local time at perigee is used in calculating the temperature; but for points 8-15, local time is averaged, because the eccentricity is lower, and the position on the orbit where drag has most effect in altering Q (at a height 1.5 H above perigee) is at a considerable angular distance from perigee (60° when $e = 0.02$), so that the appropriate local time required is not that at perigee but averaged over a wide arc of the orbit. The bracketed temperatures given in Fig 4 are those used in calculating the *CIRA* values of H in Fig 3.

For points 16-25, the exospheric temperatures are between 770 and 810 K and are not shown on Fig 4.

6 ATMOSPHERIC ROTATION RATE

The 114 derived values of inclination, given in Table 1, were cleared of lunisolar and geopotential perturbations using the PROD program⁴ with numerical

integration at one-day intervals. The modified values are plotted in Fig 1. The theoretical change in inclination was calculated for several values of atmospheric rotation rate (expressed as Λ times the Earth's rotation rate), using oblate-atmosphere theory¹⁰, with numerical integration at about 18-day intervals (corresponding to 22.5° steps in argument of perigee). The 114 points were considered in groups separated by the resonance regions. (These regions were filled, as explained in section 7, by the resonance oscillations.) Orbits 1-37 were taken as the first group (MJD 41068 to 41547) and the best fitting, shown in Fig 1, is given by $\Lambda = 1.15 \pm 0.05$ rev/day, at a mean height $\bar{y} = \bar{y}_p + 0.75H = 315$ km; the second group, orbits 42-61 (MJD 41665 to 42028), gives $\Lambda = 1.05 \pm 0.05$ at $y = 310$ km; the third group gives $\Lambda = 1.10 \pm 0.05$ for $y = 295$ km, for orbits 65-76 (MJD 42181-42413); and fourthly, for orbits 82-112 (MJD 42533-42798), $\Lambda = 1.05 \pm 0.05$ at $y = 270$ km.

It is known¹¹ that the zonal wind speed depends on local time, Λ being greater in the evening (18-24 h) than in the morning (04-12 h): so the local time at perigee is plotted at the top of Fig 1. The fitting of the first group of points reveals no bias in local time, *ie* evening and morning were sampled equally (mean conditions). The second fitting shows a slight morning bias; the third and fourth fittings are unbiased. So the results from the four fittings can be summarised as follows:

- (1) $\Lambda = 1.15 \pm 0.05$, implying a west-to-east wind of 60 ± 20 m/s, under mean (evening and morning) conditions, at a mean height of 315 km.
- (2) $\Lambda = 1.05 \pm 0.05$ (west-to-east wind of 20 ± 20 m/s), with a slight morning bias, at 310 km height.
- (3) $\Lambda = 1.10 \pm 0.05$ (west-to-east wind of 40 ± 20 m/s), under mean conditions, at 295 km height.
- (4) $\Lambda = 1.05 \pm 0.05$, under mean conditions, at 270 km height.

The sd in the above values of Λ are obtained from the estimated accuracy of fitting the curve to the values of inclination in Fig 1. For example, the first group are fitted with an accuracy estimated as slightly better than 0.001° , say 0.0008° (of the 36 points, none is more than 0.0025° from the curve). This error is just under 5 per cent of the decrease in inclination over the first group of points, giving a similar proportional error in Λ , that is an error of 0.05.

In calculating the curves of Fig 1, meridional winds have been ignored: they are unlikely to have an appreciable effect because (a) the local time is

averaged over day and night, and (b) the satellite has a high orbital inclination (70°), for which meridional wind effects tend to be small.

The values of Λ , with sd, are plotted against height in Fig 5, together with the curves of mean atmospheric rotation rate, and morning and evening wind speeds, derived from analysis of 31 previous orbits¹¹. The values obtained here suggest that the average curve should be rather lower at heights between 260 and 300 km, for the years 1971-1975.

In the inset diagram on the left of Fig 1, the final two values of inclination from the main curve have been replaced by the values obtained from the 15 daily orbits just before decay. The curve shown has $\Lambda = 1.05$ up to 5 February 1976: but for the last 11 days the value is $\Lambda = 1.6 \pm 0.1$, at a mean height of 195 km. This implies a west-to-east wind of 240 ± 40 m/s at a local time of 19-23 h. It is well established (see Ref 11) that the west-to-east winds are strongest during the evening hours, 18-24 h, but 240 m/s is the highest reliable value of wind speed that has been obtained from analysis of satellite orbits. This is presumably because 1971-18B happened to sample the evening winds at their strongest during its last few days in orbit, and because the daily orbits allow an analysis over a much shorter time span than has generally been possible in the past. West-to-east winds of up to 300 m/s at the evening maximum in winter have been recorded by radar back-scatter measurements¹² at heights near 200 km at latitude 47° north in February 1970. These conditions are closely comparable with those experienced by the perigee of 1971-18B during its last 11 days in orbit (6 to 16 February 1976), when the perigee latitude increased from 15° north to 50° north.

The $\Lambda = 1.6$ curve has been plotted (unbroken line) in Fig 20, with a comparative curve for $\Lambda = 1.4$ (dashed). To assess the influence of meridional winds¹³ (south-to-north), the effect of a 200 m/s south-to-north wind (a rotation rate $\mu = 0.4$ rev/day) has been calculated and is shown in Fig 20 by the dash-dot curve. It is seen that changing μ from 0 to 0.4 has slightly less effect than changing Λ from 1.4 to 1.6; so a change of 0.1 in Λ is equivalent to a change of about 0.3 in μ . Thus the $\Lambda = 1.6$; $\mu = 0$ curve might be replaced by the alternative $\Lambda = 1.5$; $\mu = 0.3$. However, a meridional wind of $\mu = 0.3$ (150 m/s) is most unlikely at local time 19-23 h, when such winds are usually slight¹⁴, averaging less than 50 m/s: so the estimate of $\Lambda = 1.6 \pm 0.1$ rev/day can stand unaltered, any effects of meridional winds being absorbed in the 0.1 error.

7 RESONANCES

7.1 14th order

The rates of change of inclination i and eccentricity e near 14th-order resonance may be expressed¹⁵, in terms of the resonance angle $\phi = \omega + M + 14(\Omega - \nu)$, by the equations:

$$\begin{aligned} \frac{di}{dt} = & \frac{n}{\sin i} \left(\frac{R}{a} \right)^{14} \left[\frac{R}{a} (14 - \cos i) \bar{F}_{15,14,7} \left\{ \bar{S}_{14}^{0,1} \sin \phi + \bar{C}_{14}^{0,1} \cos \phi \right\} \right. \\ & + \frac{15e}{2} (14) \bar{F}_{14,14,7} \left\{ \bar{C}_{14}^{1,0} \sin(\phi - \omega) - \bar{S}_{14}^{1,0} \cos(\phi - \omega) \right\} \\ & + 11e(7 - \cos i) \bar{F}_{14,14,6} \left\{ \bar{C}_{14}^{-1,2} \sin(\phi + \omega) - \bar{S}_{14}^{-1,2} \cos(\phi + \omega) \right\} \\ & \left. + \text{terms in } e^{|q|} \frac{\cos}{\sin} (\gamma\phi - q\omega) \right] ; \end{aligned} \quad (6)$$

$$\begin{aligned} \frac{de}{dt} = & \frac{n}{2} \left(\frac{R}{a} \right)^{14} \left[e \left(\frac{R}{a} \right) \bar{F}_{15,14,7} \left(\bar{S}_{14}^{0,1} \sin \phi + \bar{C}_{14}^{0,1} \cos \phi \right) \right. \\ & - 15 \bar{F}_{14,14,7} \left\{ \bar{C}_{14}^{1,0} \sin(\phi - \omega) - \bar{S}_{14}^{1,0} \cos(\phi - \omega) \right\} \\ & + 11 \bar{F}_{14,14,6} \left\{ \bar{C}_{14}^{-1,2} \sin(\phi + \omega) - \bar{S}_{14}^{-1,2} \cos(\phi + \omega) \right\} \\ & \left. + \text{terms in } \left[e^{|q|-1} \left\{ q - \frac{1}{2}(k+q)e^2 \right\} \frac{\cos}{\sin} (\gamma\phi - q\omega) \right] \right] . \end{aligned} \quad (7)$$

$\bar{C}_{14}^{q,k}$ and $\bar{S}_{14}^{q,k}$ are lumped 14th-order geopotential coefficients, defined in Ref 15, and the $\bar{F}$ are Allan's normalized inclination functions¹⁶. The quantities γ , q are integers, with $\gamma = 1, 2, 3 \dots$ and $q = 0, \pm 1, \pm 2 \dots$. The value $\gamma = 2$ is associated with geopotential harmonics of order 28, and $\gamma = 3$ with harmonics of order 42, etc; only the $\gamma = 1$ terms have been used here, although $\gamma = 2$ terms were tried (without much success). The value $q = 2$ leads to terms in e^2 in equation (6), and since $e = 0.08$ here, only the $q = 0$ and ± 1 terms have been considered. The suffix k is given by $k = \gamma - q$.

The orbit of China 2 rocket was appreciably perturbed by 14th-order resonance between July and December 1972, and exact 14th-order resonance ($\dot{\phi} = 0$)

occurred on 14 October. The variations of ϕ and $\dot{\phi}$ between May 1972 and February 1973 are shown in Fig 6.

The variations in inclination and eccentricity during this time, after removal of other perturbations, were analysed using the computer program THROE¹⁷, which fits the values with numerically-integrated versions of the equations of the form (6) or (7). The orbits used include US Navy orbits as well as the PROP orbits of Table 1.

The final inclination fittings utilized 7 PROP values and 10 US Navy values between 11 August and 14 December 1972. The values of inclination were cleared of perturbations as follows. The lunisolar and odd harmonic perturbations were calculated using the computer program PROD⁴, the combined maximum value being 0.0052° . The perturbation due to atmospheric rotation was calculated within THROE, assuming a mean atmospheric rotation rate of 1.1 rev/day, the maximum value of the perturbation being 0.0068° . The PROP values were also cleared of tesseral harmonic perturbations due to the $J_{2,2}$ term in the geopotential (maximum value 0.0016°). The US Navy values were assigned a uniform standard deviation of 0.003° ; the PROP values had the sd given in Table 1.

In the analysis of the resonant variation in inclination, various (γ, q) pairs were tried, and the number of values of i included was also the subject of a trial-and-error process. After several trials with $(\gamma, q) = (1, 0), (1, 1)$ and $(1, -1)$ in various combinations, it appeared that $(\gamma, q) = (1, 0)$ alone gave the best results. The number of values of i included was not a critical factor. Closely similar results were obtained with 35, 20 or 17 values.

The most favoured fitting, to the 17 values of i , gave the following values of lumped 14th-order geopotential coefficients of odd degree:

$$10^9 \bar{C}_{14}^{0,1} = 32 \pm 11 ; \quad 10^9 \bar{S}_{14}^{0,1} = -2 \pm 20 \quad (8)$$

with $\epsilon = 0.76$, where ϵ is the measure of fit ($\epsilon^2 = \text{sum of the squares of the weighted residuals divided by the number of degrees of freedom}$). The values (8) were used in determining individual 14th-order coefficients of odd degree¹⁵.

Fig 7a shows the fitting for 35 values of i (14 PROP plus 21 US Navy): this diagram is shown because it indicates (more fully than the 17-value fitting) how the resonant perturbations fall off on going away from the central resonance, and also shows conclusively that there is a mean decrease of 0.006° in i due to 14th-order resonance.

In the analysis of eccentricity, the effects of drag were removed within the THROE program, assuming a scale height $H = 50$ km. Other values of H were found to give inferior results. Lunisolar perturbations, calculated using the PROD program, were found to be negligible. The nine PROP values of e were given the standard deviations shown in Table 1, while the 11 US Navy values of e were initially assigned an sd of 0.0001. Two other values of sd were also tried (0.0002 and 0.0004) and the value 0.0002 was finally selected as giving the best fitting. This best fitting of the 20 values of e , made using the integrated form of equation (7), between 18 July and 24 December 1972, was shared by two alternative sets of (γ, q) , namely $(1, 1)$ and $(1, \pm 1)$, and the fittings are shown in Fig 7b.

The lumped 14th-order coefficients given by the $(\gamma, q) = (1, 1)$ fitting are:-

$$10^9 \bar{C}_{14}^{1,0} = 790 \pm 110 ; \quad 10^9 \bar{S}_{14}^{1,0} = 400 \pm 20 \quad (9)$$

with $\epsilon = 1.21$.

The lumped 14th-order coefficients given by the $(\gamma, q) = (1, \pm 1)$ fitting are:

$$\text{and } \left. \begin{aligned} 10^9 \bar{C}_{14}^{1,0} &= 850 \pm 150 ; & 10^9 \bar{S}_{14}^{1,0} &= 580 \pm 270 \\ 10^9 \bar{C}_{14}^{-1,2} &= 130 \pm 250 ; & 10^9 \bar{S}_{14}^{-1,2} &= 40 \pm 170 \end{aligned} \right\} \quad (10)$$

with $\epsilon = 1.27$.

These values are considerably larger than expected¹⁵, and the fittings (Fig 7b) are not satisfactory near resonance. In view of this deficiency, a simultaneous fitting of i and e was not attempted. No explanation has been found for the poor fit; at first sight the values in Fig 7b seem quite promising.

It was found necessary, for e , to make a correction in the use of the THROE program, which integrates assuming the value of M_2 for the first orbit of two consecutive orbits, whereas the M_2 should be averaged over the time interval between the orbits. This correction, which has been accomplished by taking M_2 at the n th epoch as $\left[(M_1)_{n+1} - (M_1)_n \right] / 2(t_{n+1} - t_n)$, represents the integrated effect of air drag between times t_n and t_{n+1} . This error does not affect i appreciably.

7.2 29:2 resonance

The resonance angle ϕ is now given by $\phi = 2(\omega + M) + 29(\Omega - \nu)$ and the variations of i and e may be expressed¹⁹ as:

$$\begin{aligned} \frac{di}{dt} = & \frac{n}{\sin i} \left(\frac{R}{a} \right)^{29} \left[\frac{R}{a} (29 - 2 \cos i) \bar{F}_{30,29,14} \left\{ \bar{S}_{29}^{0,2} \sin \phi + \bar{C}_{29}^{0,2} \cos \phi \right\} \right. \\ & + 16e(29 - \cos i) \bar{F}_{29,29,14} \left\{ \bar{C}_{29}^{1,1} \sin(\phi - \omega) - \bar{S}_{29}^{1,1} \cos(\phi - \omega) \right\} \\ & + 12e(29 - 3 \cos i) \bar{F}_{29,29,13} \left\{ \bar{C}_{29}^{-1,3} \sin(\phi + \omega) - \bar{S}_{29}^{-1,3} \cos(\phi + \omega) \right\} \\ & \left. + \text{terms in } e \left| q \right| \frac{\cos(\gamma\phi - q\omega)}{\sin(\gamma\phi - q\omega)} \right] ; \end{aligned} \quad (11)$$

$$\begin{aligned} \frac{de}{dt} = & n \left(\frac{R}{a} \right)^{29} \left[- \frac{R}{a} \bar{F}_{30,29,14} e \left(\bar{S}_{29}^{0,2} \sin \phi + \bar{C}_{29}^{0,2} \cos \phi \right) \right. \\ & - 16 \bar{F}_{29,29,14} \left\{ \bar{C}_{29}^{1,1} \sin(\phi - \omega) - \bar{S}_{29}^{1,1} \cos(\phi - \omega) \right\} \\ & + 12 \bar{F}_{29,29,13} \left\{ \bar{C}_{29}^{-1,3} \sin(\phi + \omega) - \bar{S}_{29}^{-1,3} \cos(\phi + \omega) \right\} \\ & \left. + \text{terms in } e \left| q \right|^{-1} \left\{ q - \frac{1}{2}(k + q)e^2 \right\} \frac{\cos(\gamma\phi - q\omega)}{\sin(\gamma\phi - q\omega)} \right] . \end{aligned} \quad (12)$$

This time, 16 sets of values of i or e were fitted by integrated forms of (11) or (12), as described in section 7.1 (for 14th-order), between December 1973 and April 1974. Fig 8 shows the variations of ϕ and $\dot{\phi}$: exact 29:2 resonance ($\dot{\phi} = 0$) occurred on 2 March 1974 (MJD 42108). Only three PROP values were available, combined with 13 US Navy values (with standard deviations as for 14th-order resonance).

As for 14th-order resonance, $(\gamma, q) = (1, 0)$ gave the best least-squares fit for i alone, shown in Fig 9a. However, for e alone, the best fit, given in Fig 9b, was obtained with $(\gamma, q) = (1, \pm 1)$, where $[q, k] = [1, 1]$ and $[-1, 3]$. A simultaneous least-squares fit to i and e was made using the SIMRES program¹⁸ for $(\gamma, q) = (1, 0), (1, \pm 1)$. The resulting sets of lumped values for the 29th-order harmonic coefficients are:

(1) Inclination i alone:

$$10^9 \bar{C}_{29}^{0,2} = 50 \pm 20, \quad 10^9 \bar{S}_{29}^{0,2} = -160 \pm 70, \quad (13)$$

where $\epsilon = 0.63$.

(2) Eccentricity e alone:

$$\left. \begin{aligned} 10^9 \bar{C}_{29}^{1,1} &= 40 \pm 610, & 10^9 \bar{S}_{29}^{1,1} &= -2750 \pm 860, \\ 10^9 \bar{C}_{29}^{-1,3} &= -980 \pm 590, & 10^9 \bar{S}_{29}^{-1,3} &= -1950 \pm 700, \end{aligned} \right\} \quad (14)$$

where $\epsilon = 0.72$.

(3) For i and e simultaneously:

$$\left. \begin{aligned} 10^9 \bar{C}_{29}^{0,2} &= 340 \pm 230, & 10^9 \bar{S}_{29}^{0,2} &= -360 \pm 200 \\ 10^9 \bar{C}_{29}^{1,1} &= -240 \pm 280, & 10^9 \bar{S}_{29}^{1,1} &= -510 \pm 340 \\ 10^9 \bar{C}_{29}^{-1,3} &= -620 \pm 300, & 10^9 \bar{S}_{29}^{-1,3} &= -260 \pm 250, \end{aligned} \right\} \quad (15)$$

with a weighting factor of 1.41 and $\epsilon = 0.61$. The values in (14) and (15) cannot be regarded as reliable because the standard deviations are so large, but the values (13) are better and may be useful in future in deriving individual coefficients of 29th order.

It is to be expected that the numerical values of the lumped coefficients would be poor because the total variations in i and e are no greater than the errors in the observational values. However, Fig 9 is valuable in showing that the overall changes in both i and e are virtually zero in passing through 29:2 resonance, due to the effects of resonance itself, and the fitting at resonance provided the end points for the Λ curves in Fig 1. After restoring the atmospheric-rotation perturbation, the inclination variation was mapped into the appropriate gap in Fig 1.

7.3 15th order

The 15th-order resonance equations^{18,20}, ignoring e^2 terms, may be expressed as:

$$\begin{aligned} \frac{di}{dt} = & \frac{n}{\sin i} \left(\frac{R}{a} \right)^{15} \left[(15 - \cos i) \bar{F}_{15,15,7} \left\{ \bar{C}_{15}^{0,1} \sin \phi - \bar{S}_{15}^{0,1} \cos \phi \right\} \right. \\ & + \left(\frac{R}{a} \right) \frac{17e}{2} \bar{F}_{16,15,8}^{(15)} \left\{ \bar{S}_{15}^{1,0} \sin(\phi - \omega) + \bar{C}_{15}^{1,0} \cos(\phi - \omega) \right\} \\ & \left. + \left(\frac{R}{a} \right) \frac{13e}{2} (15 - 2 \cos i) \bar{F}_{16,15,7} \left\{ \bar{S}_{15}^{-1,2} \sin(\phi + \omega) + \bar{C}_{15}^{-1,2} \cos(\phi + \omega) \right\} \right] \quad (16) \end{aligned}$$

and the corresponding terms in the equation for e are:

$$\begin{aligned} \frac{de}{dt} = & n \left(\frac{R}{a} \right)^{15} \left[\frac{e}{2} \bar{F}_{15,15,7} \left(\bar{C}_{15}^{0,1} \sin \phi - \bar{S}_{15}^{0,1} \cos \phi \right) \right. \\ & - \left(\frac{R}{a} \right) \frac{17}{2} \bar{F}_{16,15,8} \left\{ \bar{S}_{15}^{1,0} \sin(\phi - \omega) + \bar{C}_{15}^{1,0} \cos(\phi - \omega) \right\} \\ & \left. + \left(\frac{R}{a} \right) \frac{13}{2} \bar{F}_{16,15,7} \left\{ \bar{S}_{15}^{-1,2} \sin(\phi + \omega) + \bar{C}_{15}^{-1,2} \cos(\phi + \omega) \right\} \right] . \quad (17) \end{aligned}$$

15th-order resonance has been investigated between 4 December 1974 and 13 June 1975, where 13 PROP orbits were available (Table 1) as well as 10 US Navy orbits. Exact 15th-order resonance ($\dot{\phi} = 0$) occurred on 9 March 1975: Fig 10 shows the variations of ϕ and $\dot{\phi}$.

The best least-squares fitting of the integrated form of equation (16) to the 23 values of perturbation-free inclination was given by $(\gamma, q) = (1, 0)$. The lumped values for 15th-order odd harmonic coefficients are:-

$$10^9 \bar{C}_{15}^{0,1} = -37 \pm 6, \quad 10^9 \bar{S}_{15}^{0,1} = 10 \pm 6, \quad (18)$$

with $\epsilon = 0.58$. Fig 11 shows the inclination values and the fitted curve. The fitting is entirely satisfactory and the change in inclination should be large enough to give reliable values of the lumped coefficients. Fig 11 also clearly shows an increase of 0.004° in inclination due to 15th-order resonance.

Various fittings were attempted with eccentricity values, using equation (17), but no satisfactory results were obtained and so no graph is shown.

7.4 31:2 resonance

No reliable values of lumped 31st-order geopotential harmonic coefficients can be expected here, as resonance passed rather rapidly. However, a least-squares fitting was made for $(\gamma, q) = (1, 0)$ in order to determine the overall

change in inclination. The resonance region considered was between 14 October and 25 December 1975 and Fig 12 shows the variations of ϕ and $\dot{\phi}$. Exact resonance, where $\dot{\phi} = 0$, occurred on 19 November. Ten PROP and nine US Navy inclination values were fitted by a least-squares curve after removing the usual perturbations, to give 31st-order lumped coefficients as:

$${}_{10}^{9-0,2}\bar{C}_{31} = 24 \pm 18, \quad {}_{10}^{9-0,2}\bar{S}_{31} = 30 \pm 23, \quad (20)$$

with $\epsilon = 0.99$. The fitting is quite satisfactory, and the coefficients are as good as can be expected when drag is so high.

Atmospheric rotation has a strong influence at this late stage in the satellite's life and Fig 13a shows the values of inclination originally and after removal of the perturbation due to an atmosphere rotating at $\Lambda = 1.05$ rev/day. It can be seen that this perturbation can change the inclination by up to 0.01° . Fig 13b gives the inclination values with the perturbation removed, showing the best least-squares fitting. The overall change in i due to resonance, in passing through the resonance, is virtually zero and so can be ignored in Fig 1.

8 CONCLUSIONS

The orbit of China 2 rocket, 1971-18B, has been determined at 114 epochs between 1971 and 1976, using more than 7000 radar and optical observations, including 170 observations made by Hewitt cameras. The standard deviations in inclination varied from 0.0003 to 0.0026° , and the sd in eccentricity was between 3×10^{-6} (equivalent to 22 m in perigee height) and 44×10^{-6} .

Values of density scale height have been determined from the change in perigee height and compared with values calculated from the CIRA 1972 reference atmosphere: the agreement is good between April 1971 and October 1975, but the ensuing observational values (near decay) are about 10 per cent lower, on average.

Four values of atmospheric rotation rate Λ have been determined for heights y between 270 and 315 km. All four give west-to-east winds (super-rotation). The results (Fig 1) are:

- (1) $\Lambda = 1.15 \pm 0.05$ at $y = 315$ km between April 1971 and August 1972;
- (2) $\Lambda = 1.05 \pm 0.05$ at $y = 310$ km between December 1972 and December 1973;
- (3) $\Lambda = 1.10 \pm 0.05$ at $y = 295$ km between April 1974 and January 1975;
- (4) $\Lambda = 1.05 \pm 0.05$ at $y = 270$ km between May 1975 and February 1976.

The values (1), (3) and (4) are for mean (evening and morning) conditions; the value (2) has a slight morning bias.

The effects of four orbital resonances, of order 14:1, 29:2, 15:1 and 31:2, have been investigated, and the lumped harmonic coefficients in the geopotential have been determined. Although not of great accuracy, these values should be of use, in combination with values from satellites at other inclinations, in determining individual harmonic coefficients. The analyses of the resonances were necessary to determine the overall change in inclination and eccentricity on passing resonance, and they served this purpose well. However, the difficulties encountered suggest that accurate values of harmonic coefficients for the 29:2 and 31:2 resonances cannot be obtained from a high-drag satellite unless a number of very accurate orbits from Hewitt camera observations are available close to the time of resonance.

Further orbits were determined for each day during the final 15 days before decay in February 1976 from 1400 NORAD observations: the accuracy of these daily orbits was very consistent, and slightly better than the average of the original 114 orbits. Analysis of the change in inclination in the last 12 days before decay revealed a very strong west-to-east wind, of 240 ± 40 m/s, at a mean height of 195 km, at a local time of 19-23 h and at a latitude of $15-50^\circ$ N in winter (5-16 February 1976). This is the highest reliable wind speed obtained from analysis of satellite orbits: it arises because the satellite's perigee was at the evening maximum of the zonal wind, and because of the fine time resolution and excellent accuracy of the daily orbits. Analysis of the perigee height gave a further 10 values of density scale height H with a sharper time resolution than has previously been possible, and an accuracy of about ± 3 per cent. These values of H , like those in the preceding three months, are about 10 per cent lower than indicated by *CIRA 1972*.

Table 1
ORBITAL PARAMETERS FOR CHINA 2 ROCKET, WITH STANDARD DEVIATIONS

Orbit	MJD	Date	a	e	i	Ω	ω	M_0	M_1	M_2	M_3	M_4	M_5	ϵ	D	N
1	41068	1971 Apr 27	7403.9249	0.102908	69.9039	169.196	124.898	279.280	4906.8071	0.1516	-0.00156	0.00015	-	0.64	7.7	62
2	076	May 5	7401.6121	0.102578	69.9006	152.597	114.993	303.023	4909.1075	0.1379	0.00062	0.00032	-	0.68	8.4	41
3*	096	May 25	7395.5575	0.101997	69.9005	111.023	90.308	265.754	4915.1386	0.1185	-0.00065	-	-	0.71	9.0	45
4*	105	Jun 3	7393.2521	0.101730	69.9023	92.282	79.132	231.596	4917.4384	0.1592	0.00006	-0.00041	-	0.54	8.8	38
5	137	Jul 5	7382.7167	0.100327	69.9018	25.472	39.443	72.745	4927.9705	0.1534	-0.0032	-	-	0.66	8.0	40
6	145	Jul 13	7380.7358	0.099986	69.8995	8.717	29.469	264.917	4929.9549	0.1094	0.00021	-	-	0.62	9.9	35
7	180	Aug 17	7373.2557	0.098516	69.9043	295.272	345.605	149.246	4937.4607	0.0919	0.00083	0.00008	-	0.50	9.4	57
8*	189	Aug 26	7371.2001	0.098145	69.9008	276.345	334.267	315.058	4939.5265	0.1320	0.00060	-	-	0.48	7.9	70
9*	197	Sep 3	7369.0098	0.097793	69.8986	259.505	324.174	240.070	4941.7293	0.1263	-0.0020	0.00012	-	0.51	6.8	83
10	204	Sep 10	7367.1547	0.097508	69.8984	244.754	315.313	278.618	4943.5964	0.1263	-0.0018	0.00029	-	0.52	6.1	69
11*	212	Sep 18	7364.9801	0.097177	69.8981	227.881	305.231	235.839	4945.7864	0.1585	0.0069	0.0009	-	0.64	6.1	79
12*	220	Sep 26	7362.3797	0.096781	69.9010	210.992	295.037	212.894	4948.4078	0.1746	0.00009	-0.00077	-	0.53	9.0	62
13*	229	Oct 5	7359.9665	0.096500	69.9039	191.973	283.568	120.407	4950.8424	0.1151	-0.0011	-	-	0.61	6.5	71
14*	232	Oct 8	7359.3034	0.096345	69.9023	185.631	279.732	213.996	4951.5116	0.1119	0.0119	-	-	0.48	2.0	56
15*	235	Oct 11	7358.6202	0.096282	69.9019	179.286	275.905	309.638	4952.2013	0.1022	-0.0017	0.00077	-	0.49	6.5	60
16*	240	Oct 16	7357.5530	0.096111	69.9031	168.704	269.537	233.389	4953.2791	0.0921	-0.0048	0.0007	-	0.54	5.7	60
17*	246	Oct 22	7356.5665	0.096017	69.9005	156.003	261.894	76.125	4954.2756	0.0836	0.0020	0.0012	-	0.56	4.1	55
18	252	Oct 28	7355.4360	0.095923	69.8985	143.293	254.239	285.343	4955.4180	0.1032	0.0083	0.00035	-0.00025	0.45	8.4	52
19	270	Nov 15	7350.4727	0.095459	69.8990	105.114	231.311	245.480	4960.4392	0.1604	0.00020	0.00021	-	0.42	9.3	48
20	281	Nov 26	7346.2436	0.095100	69.9007	81.739	217.283	112.297	4964.7244	0.2229	-0.0020	-0.00038	-	0.46	8.9	47
21	294	Dec 9	7340.7119	0.094651	69.9029	54.058	200.723	250.467	4970.3387	0.2128	0.0027	-0.00003	-0.000048	0.47	9.4	45
22	306	Dec 21	7335.5136	0.094166	69.8986	28.444	185.381	166.352	4975.6241	0.1983	-0.0021	0.00036	-0.00003	0.77	9.4	37
23	329	1972 Jan 13	7328.4579	0.093664	69.8946	339.222	156.032	214.076	4982.8130	0.1407	-0.00044	-	-	0.69	9.0	43
24	340	Jan 24	7325.0952	0.093440	69.8956	315.633	142.021	323.039	4986.2455	0.1818	-0.0009	-0.00034	0.000035	0.54	10.1	37
25	351	Feb 4	7321.4994	0.093062	69.8940	292.010	127.939	112.535	4989.9201	0.1555	-0.0022	-	-	0.74	5.9	28

Table 1 (continued)

Orbit	MJD	Date	a	e	i	Ω	ω	M_0	M_1	M_2	M_3	M_4	M_5	ϵ	D	N
26	41361	1972 Feb 14	7318.5070 ⁸	0.092809 ⁷	69.8949 ⁷	270.504 ¹	115.235 ⁸	346.581 ⁷	4992.9816 ⁸	0.1755 ⁴	0.0012 ¹	-0.00018 ²	0.000039 ⁵	0.66	9.9	56
27	391	Mar 15	7306.4464 ¹²	0.091380 ¹⁰	69.8911 ¹¹	205.754 ¹	76.769 ¹³	206.949 ¹¹	5005.3522 ¹³	0.1879 ⁵	0.0054 ²	0.00012 ²	-0.000097 ⁸	0.71	9.4	39
28*	422	Apr 15	7294.2885 ⁹	0.089656 ⁹	69.8897 ⁶	138.516 ¹	36.881 ⁷	41.082 ⁶	5017.8743 ⁹	0.2731 ³	0.00206 ⁶	-0.00014 ²	-	0.74	9.3	43
29*	433	Apr 26	7288.2153 ⁷	0.088834 ¹⁵	69.8886 ⁹	114.550 ¹	22.654 ⁶	192.506 ⁷	5024.1491 ⁷	0.2698 ⁴	0.00140 ⁵	0.00037 ¹	-	0.45	10.6	43
30*	445	May 8	7281.7835 ⁴	0.087846 ⁹	69.8902 ⁴	88.336 ¹	7.056 ³	43.179 ³	5030.8087 ⁴	0.2687 ⁴	0.00356 ⁴	0.00030 ³	-	0.28	7.1	43
31*	450	May 13	7278.9615 ¹⁸	0.087425 ²⁴	69.8901 ⁹	77.394 ²	0.535 ⁹	4.413 ⁹	5033.7353 ¹⁹	0.3027 ⁷	0.00166 ³⁶	-	-	0.53	4.9	41
32*	456	May 19	7275.3874 ¹⁷	0.086929 ¹⁴	69.8927 ⁸	64.240 ¹	352.685 ⁷	338.155 ⁹	5037.4461 ¹⁸	0.2693 ³	-0.00425 ³²	-	-	0.59	4.9	33
33	481	Jun 13	7266.1517 ⁷	0.085419 ¹⁴	69.8898 ¹¹	9.262 ¹	319.790 ⁸	43.979 ⁸	5047.0555 ⁸	0.1375 ⁴	-0.00141 ⁴	0.00030 ²	-	0.60	9.9	55
34*	516	Jul 18	7258.7068 ⁵	0.084178 ⁸	69.8932 ⁵	292.003 ¹	273.350 ⁹	80.634 ¹⁰	5054.8242 ⁵	0.0971 ⁵	0.00023 ⁴	-0.00014 ²	-	0.50	9.5	54
35	529	Jul 31	7255.9683 ⁹	0.083859 ⁸	69.8916 ¹⁰	263.249 ¹	256.024 ⁹	291.238 ¹⁰	5057.6867 ¹⁰	0.1092 ⁵	0.00077 ⁸	0.00037 ³	-	0.43	8.2	62
36	540	Aug 11	7252.9148 ¹¹	0.083570 ¹⁰	69.8854 ⁹	238.881 ¹	241.371 ⁷	143.204 ⁸	5060.8815 ¹²	0.1161 ¹⁵	-0.0073 ²	0.0008 ²	-	0.48	5.3	50
37	547	Aug 18	7251.6709 ¹¹	0.083485 ¹¹	69.8838 ¹¹	223.363 ¹	232.047 ⁸	294.125 ⁸	5062.1840 ¹²	0.0968 ⁶	0.0053 ³	-	-	0.33	5.0	41
38	554	Aug 25	7249.9816 ¹⁴	0.083410 ¹³	69.8844 ¹³	207.833 ¹	222.706 ⁸	95.524 ⁸	5063.9539 ¹⁵	0.1355 ¹³	0.0055 ²	0.0011 ¹	-	0.41	5.9	40
39	561	Sep 1	7247.4379 ¹²	0.083207 ²⁴	69.8878 ¹⁸	192.290 ²	213.424 ¹¹	272.087 ¹¹	5066.6210 ¹⁵	0.2055 ¹⁷	0.0012 ⁶	-	-	0.37	4.1	34
40	574	Sep 14	7242.7158 ¹⁴	0.082760 ²³	69.8882 ¹⁵	163.382 ¹	196.133 ⁹	291.631 ⁹	5071.5781 ¹⁴	0.1861 ⁸	0.0025 ²	-	-	0.84	7.0	51
41	582	Sep 22	7240.1223 ¹⁴	0.082621 ¹⁴	69.8891 ²³	145.561 ²	185.542 ¹¹	195.584 ¹¹	5074.3041 ¹⁵	0.1622 ⁵	0.0047 ²	0.00083 ⁶	-	0.66	7.7	43
42*	665	Dec 14	7204.3373 ⁴	0.078991 ⁶	69.8753 ⁶	318.796 ¹	74.915 ⁹	191.224 ⁸	5112.1670 ⁴	0.1513 ¹	0.00423 ³	-0.00005 ¹	-	0.41	9.6	32
43*	698	1973 Jan 16	7192.1928 ¹¹	0.077191 ¹³	69.8750 ⁸	243.843 ¹	30.615 ⁸	259.314 ⁹	5125.1240 ¹¹	0.1639 ⁹	-0.0049 ¹	0.00022 ⁷	-	0.81	7.5	44
44	706	Jan 24	7189.6570 ¹⁴	0.076838 ²⁰	69.8706 ¹⁷	225.606 ²	19.831 ¹⁰	230.467 ¹⁰	5127.8360 ¹⁵	0.1730 ⁷	-0.0014 ¹	0.00053 ⁴	-	0.57	8.9	28
45*	727	Feb 14	7182.7159 ⁵	0.075588 ¹⁶	69.8720 ⁵	177.632 ¹	351.345 ⁶	353.805 ⁶	5135.2727 ⁵	0.1683 ⁴	0.00054 ³	0.00025 ²	-	0.49	9.7	41
46	741	Feb 28	7177.6835 ¹³	0.074729 ¹²	69.8743 ¹⁰	145.562 ¹	332.203 ⁷	284.470 ⁸	5140.6757 ¹⁴	0.1796 ⁵	-0.0008 ²	0.00016 ²	-0.000034 ⁷	0.43	9.6	52
47*	757	Mar 16	7172.5672 ⁴	0.073887 ¹⁰	69.8709 ⁴	108.820 ¹	310.227 ⁸	138.977 ⁹	5146.1782 ⁵	0.1830 ²	0.00090 ⁴	-	-	0.52	9.7	80
48*	764	Mar 23	7169.7492 ¹²	0.073504 ²⁶	69.8692 ⁶	92.717 ¹	300.561 ¹²	172.487 ¹⁵	5149.2132 ¹³	0.2169 ¹⁷	-0.0049 ¹⁶	-	-	0.69	2.1	73
49*	768	Mar 27	7168.3791 ¹⁰	0.073245 ¹²	69.8680 ⁵	83.506 ¹	295.013 ¹¹	252.512 ¹³	5150.6998 ¹¹	0.1628 ⁸	-0.00041 ¹⁶	0.0008 ¹	-	0.37	5.9	53
50*	775	Apr 3	7165.8266 ¹⁶	0.072925 ¹⁰	69.8699 ⁸	67.369 ¹	285.312 ¹²	316.275 ¹⁴	5153.4429 ¹⁷	0.2270 ⁵	-0.0101 ³	-0.00121 ³	0.00027 ¹	0.50	8.9	73

Table 1 (continued)

Orbit	MJD	Date	a	e	i	Ω	ω	M_0	M_1	M_2	M_3	M_4	M_5	ϵ	D	N
51	41789	1973 Apr 17	7161.4938	0.072348	69.8735	35.050	265.877	138.065	5158.1218	0.1428	-0.0024	0.00017	-	0.50	7.9	47
52	798	Apr 26	7159.4807	0.072123	69.8763	14.247	253.399	131.340	5160.2982	0.1296	0.00135	-0.00017	-	0.52	8.6	44
53*	847	Jun 14	7143.9945	0.070910	69.8723	260.556	185.452	774.486	5177.0906	0.2237	-0.00117	-0.00012	-	0.53	9.7	48
54*	857	Jun 24	7139.8580	0.070486	69.8708	237.246	171.618	229.815	5181.5914	0.2369	0.0093	-0.0011	-0.00018	0.61	7.3	45
55*	867	Jul 4	7136.1398	0.070319	69.8644	213.891	157.824	227.418	5185.6423	0.1363	-0.00478	0.00032	-	0.47	9.5	49
56*	902	Aug 8	7128.2351	0.069620	69.8697	131.949	109.442	71.256	5194.2731	0.1364	0.00123	-0.00035	-0.000033	0.51	9.6	42
57*	972	Oct 17	7105.6723	0.066293	69.8654	326.930	12.201	177.632	5219.0389	0.2069	0.0098	-0.00005	-0.00015	0.71	9.5	42
58*	993	Nov 7	7094.8762	0.064643	69.8611	277.027	342.573	97.267	5230.9588	0.2682	-0.00271	-0.00009	-	0.46	9.9	38
59*	42004	Nov 18	7090.3162	0.063872	69.8625	250.789	326.904	66.501	5236.0073	0.2377	0.0039	-	-	0.89	4.9	38
60	020	Dec 4	7084.1030	0.062896	69.8613	212.526	303.987	21.520	5242.8989	0.1706	0.0016	-0.00008	-0.00011	0.60	8.7	72
61	028	Dec 12	7082.1265	0.062571	69.8621	193.358	292.485	214.506	5245.0944	0.1017	-0.0028	0.00038	-	0.49	7.9	67
62	064	1974 Jan 17	7072.8926	0.061444	69.8673	106.893	240.633	207.048	5255.3723	0.2186	0.0014	-0.00026	-	0.45	9.4	41
63*	076	Jan 29	7068.5678	0.061076	69.8631	77.975	223.313	301.469	5260.1972	0.1617	-0.0032	0.00069	-0.000062	0.63	9.1	69
64	091	Feb 13	7064.5193	0.060784	69.8594	41.754	201.693	38.171	5264.7206	0.1778	-0.0006	-0.00067	-	0.53	9.4	60
65*	181	May 14	7026.7305	0.056870	69.8549	182.422	72.340	120.591	5307.2583	0.2027	0.0029	0.0014	0.00012	0.68	6.6	35
66*	185	May 18	7025.0333	0.056606	69.8567	172.582	66.573	113.275	5309.1823	0.2306	-0.0081	-	-	0.69	2.8	44
67*	210	Jun 12	7013.9480	0.054987	69.8552	110.880	30.454	143.866	5321.7769	0.3487	-0.0017	-0.00093	0.00009	0.62	9.6	32
68*	220	Jun 22	7008.7949	0.054124	69.8599	86.105	15.778	112.571	5327.6490	0.2440	-0.0073	-	-	0.62	4.9	38
69	278	Aug 19	6990.2930	0.050736	69.8560	301.447	289.469	226.761	5348.8201	0.1638	0.0082	-0.00014	-0.00012	0.68	9.6	52
70	289	Aug 30	6987.1839	0.050338	69.8519	273.889	272.919	44.779	5352.3914	0.1574	0.00129	-0.00012	-	0.49	9.8	79
71	300	Sep 10	6984.1411	0.050027	69.8491	246.283	256.287	260.928	5355.8904	0.1475	-0.00047	-	-	0.53	8.3	68
72	315	Sep 25	6979.1325	0.049504	69.8523	208.571	233.614	1.669	5361.6587	0.1825	0.0032	0.00028	-0.00009	0.62	9.5	48
73	329	Oct 9	6973.6422	0.049000	69.8529	173.291	212.609	226.330	5367.9936	0.3050	0.0066	-0.00011	-	0.80	9.6	35
74	364	Nov 13	6949.3004	0.046596	69.8439	84.472	159.955	290.541	5396.2298	0.4897	-0.0028	-0.00093	-0.00012	0.72	9.0	33
75	385	Dec 4	6936.5215	0.045218	69.8455	30.676	128.177	22.512	5411.1529	0.2656	-0.0081	-0.00083	0.00014	0.71	9.0	34

Table 1 (continued)

Orbit	MJD	Date	a	e	i	Ω	ω	M_0	M_1	M_2	M_3	M_4	M_5	ϵ	D	N
76	42413	1975 Jan 1	6926.5787	0.044081	69.8444	318.559	86.148	138.688	5422.8115	0.1880	-0.0014	0.00047	0.000073	0.46	9.5	32
77	437	Jan 25	6916.5660	0.042591	69.8426	256.453	50.137	105.392	5434.5943	0.2100	-0.0019	-0.00004	-	0.85	10.8	54
78	449	Feb 6	6911.9235	0.041929	69.8416	225.283	31.836	192.120	5440.0720	0.2379	0.00099	0.00031	-	0.62	10.1	44
79	498	Mar 27	6889.9236	0.038112	69.8427	97.179	315.722	265.214	5466.1558	0.2985	0.0035	-0.00028	-0.00007	0.63	9.9	43
80	510	Apr 8	6884.1121	0.037250	69.8401	65.587	296.605	21.922	5473.0808	0.3222	0.0095	-0.00043	-0.00020	0.58	9.8	37
81	523	Apr 21	6878.8483	0.036481	69.8458	31.268	275.784	300.031	5479.3662	0.1869	0.0014	-0.00064	-	0.48	9.0	58
82*	533	May 1	6876.3922	0.036184	69.8456	4.825	259.772	30.096	5482.3030	0.1366	0.0064	-0.00061	-0.00004	0.54	8.6	68
83	542	May 10	6873.5381	0.035899	69.8446	340.994	245.410	66.077	5485.7189	0.1766	-0.0028	-	-	0.54	8.2	55
84	550	May 18	6871.1651	0.035677	69.8410	319.781	232.744	42.691	5488.5616	0.2033	-0.0029	0.00048	0.00034	0.40	7.2	57
85	558	May 26	6868.0726	0.035376	69.8384	298.535	219.968	45.875	5492.2698	0.2627	0.0062	-0.00039	-0.00011	0.52	8.7	57
86	568	Jun 5	6863.3892	0.035011	69.8421	271.936	204.169	275.766	5497.8944	0.3184	-0.0026	-0.00076	-	0.45	8.0	58
87	576	Jun 13	6859.6454	0.034726	69.8426	250.609	191.532	356.977	5502.3973	0.3286	0.0025	-0.00196	-0.00017	0.51	7.9	46
88	585	Jun 22	6855.1892	0.034275	69.8391	226.571	177.365	223.342	5507.7647	0.2649	-0.0033	-0.00024	-	0.47	8.0	56
89	592	Jun 29	6852.3611	0.034053	69.8340	207.841	166.367	269.700	5511.1755	0.2485	0.0104	0.0012	-0.00039	0.50	8.0	50
90	603	Jul 10	6847.4964	0.033637	69.8328	178.347	148.903	84.319	5517.0511	0.3339	-0.0114	-0.00192	0.00033	0.68	8.5	37
91	615	Jul 22	6842.3234	0.033146	69.8358	146.094	130.260	86.755	5523.3110	0.2276	-0.0033	0.00073	0.00012	0.57	9.1	52
92*	623	Jul 30	6838.8524	0.032730	69.8388	124.551	117.756	9.490	5527.5178	0.2311	-0.0018	0.00154	-	0.50	6.6	32
93	631	Aug 7	6834.9051	0.032278	69.8364	102.963	105.338	326.777	5532.3082	0.3544	-0.0029	0.00039	0.00029	0.62	7.5	33
94	641	Aug 17	6828.1936	0.031479	69.8349	75.910	89.632	248.898	5540.4692	0.4061	-0.0156	-0.00037	0.00035	0.55	9.2	26
95	651	Aug 27	6822.3357	0.030692	69.8312	48.764	73.878	249.622	5547.6084	0.3446	0.00508	0.00073	-	0.55	9.9	32
96	663	Sep 8	6814.5967	0.029617	69.8359	16.085	55.029	276.628	5557.0644	0.4703	0.0020	-0.00096	0.00051	0.54	7.0	28
97	670	Sep 15	6809.2461	0.028881	69.8354	356.955	43.935	319.945	5563.6175	0.3584	-0.0067	0.00117	-	0.49	7.9	33
98	679	Sep 24	6803.8560	0.027960	69.8337	332.300	29.726	21.496	5570.2319	0.3652	-0.0027	0.00095	0.00030	0.50	7.6	47
99	687	Oct 2	6797.8403	0.027093	69.8315	310.317	16.808	331.415	5577.6296	0.4830	0.0035	0.00143	0.00015	0.47	7.9	57
100*	699	Oct 14	6785.5796	0.025278	69.8252	277.185	357.600	29.300	5592.7576	0.6680	0.0063	-0.00015	-0.00011	0.43	8.0	67
101	707	Oct 22	6776.6029	0.023950	69.8278	254.973	344.387	175.779	5603.8775	0.6908	-0.0022	-0.00031	-	0.48	7.9	62

Table 1 (concluded)

Orbit	MJD	Date	a	e	i	Ω	ω	M_0	H_1	M_2	M_3	M_4	M_5	ϵ	D	N
102	42715	1975 Oct 30	6768.1576 8	0.022706 12	69.8264 11	232.668 1	331.077 23	49.068 23	5614.3727 10	0.6920 7	0.00045 8	-0.00139 4	-	0.53	7.6	62
103	723	Nov 7	6758.9860 25	0.021399 18	69.8286 15	210.268 2	317.157 53	10.222 55	5625.8078 31	0.5868 13	-0.0220 9	0.0033 1	0.00070 6	0.54	7.1	45
104	731	Nov 15	6751.1880 21	0.020312 17	69.8252 15	187.761 2	303.390 53	56.994 54	5635.5604 26	0.5041 11	-0.0071 10	0.00197 9	0.00030 7	0.57	7.2	48
105*	739	Nov 23	6743.7770 22	0.019365 11	69.8225 8	165.166 1	289.380 42	177.900 42	5644.8553 28	0.6885 15	0.0042 9	-0.0052 1	-0.00041 6	0.67	7.3	56
106	747	Dec 1	6736.9873 10	0.018533 11	69.8197 12	142.485 2	275.454 67	12.524 69	5653.3935 13	0.6310 8	0.0111 1	-0.00307 6	-	0.50	7.6	58
107	755	Dec 9	6729.4549 23	0.017636 12	69.8194 13	119.720 2	261.679 65	278.457 68	5662.8909 29	0.6628 10	0.0160 6	-0.00169 6	-0.00041 3	0.59	7.7	64
108	763	Dec 17	6721.0092 21	0.016740 10	69.8206 12	96.862 1	248.096 62	263.898 64	5673.5717 27	0.7441 9	-0.0049 7	-0.00259 7	0.00027 4	0.46	7.1	61
109	771	Dec 25	6712.2880 19	0.015743 11	69.8228 12	73.912 1	234.561 42	336.686 42	5684.6363 24	0.7724 8	0.0404 6	0.00285 5	-0.00086 3	0.50	7.5	56
110	778	1976 Jan 1	6702.0751 19	0.014723 13	69.8201 13	53.721 1	222.489 49	215.247 51	5697.6387 25	0.8126 15	-0.0174 7	0.00208 15	0.00044 6	0.60	6.8	48
111	786	Jan 9	6691.1186 27	0.013519 22	69.8167 16	30.533 2	208.417 68	131.043 69	5711.6431 35	0.8505 29	-0.0051 21	0.0072 7	0.0011 3	0.53	5.2	40
112	798	Jan 21	6670.6838 46	0.011435 33	69.8160 18	355.435 2	187.598 76	68.742 79	5737.9168 60	1.1515 9	0.0222 8	0.00240 6	-0.00035 3	0.76	8.8	34
113	812	Feb 4	6638.4948 9	0.008400 44	69.8110 20	313.965 2	161.588 126	24.201 125	5779.7137 12	2.0322 10	0.0161 2	0.00246 8	-	0.47	7.2	24
114	820	Feb 12	6599.7955 49	0.005271 41	69.8014 26	289.869 4	146.240 335	352.358 332	5830.6397 65	4.7950 95	0.2546 50	0.08067 40	0.0057 16	0.64	4.0	18

Key: MJD Modified Julian Day
a semi major axis (km)
e eccentricity
i inclination (deg)
 Ω right ascension of node (deg)
 ω argument of perigee (deg)
* orbits with Hewitt camera observations

M_0 mean anomaly at epoch (deg)
 M_1 mean motion, n (deg/day)
 M_2-M_5 additional coefficients in polynomial for M
 ϵ measure of fit
D time coverage of observations (days)
N number of observations used

Table 3
ORBITAL PARAMETERS FOR THE FINAL 15 DAYS BEFORE DECAY

Orbit	MJD	Date	a	e	i	Ω	ω	M_0	M_1	M_2	ϵ	D	N
A	42810.0	1976 Feb 2	6644.603 3	0.009014 17	69.8127 11	319.9555 15	165.17 7	352.76 7	5771.744 4	1.99 2	0.89	0.88	99
B	11.0	Feb 3	6641.597 2	0.008791 11	69.8136 7	316.9723 7	163.30 4	6.56 4	5775.664 2	2.06 1	0.63	0.96	93
C	12.0	Feb 4	6638.494 2	0.008536 12	69.8118 10	313.9894 10	161.50 5	24.30 5	5779.715 3	2.01 1	0.64	0.89	83
D	13.0	Feb 5	6635.322 3	0.008208 11	69.8133 10	310.9916 10	159.74 5	46.04 5	5783.861 3	2.23 1	0.60	0.89	80
E	14.0	Feb 6	6632.051 4	0.007925 9	69.8126 9	307.9947 8	157.83 5	72.18 5	5788.142 5	2.25 2	0.59	0.69	79
F	15.0	Feb 7	6628.653 1	0.007731 16	69.8108 9	304.9943 12	156.27 7	102.33 7	5792.594 2	2.29 1	0.82	1.36	77
G	16.0	Feb 8	6624.508 3	0.007283 8	69.8110 7	301.9843 6	154.32 5	137.55 5	5798.034 4	3.41 1	0.70	0.74	83
H	17.0	Feb 9	6618.978 2	0.006825 9	69.8089 8	298.9691 8	152.26 6	179.37 6	5805.304 3	3.93 1	0.83	0.77	89
I	18.0	Feb 10	6613.099 2	0.006331 8	69.8068 7	295.9435 8	150.15 6	228.80 6	5813.049 2	4.11 1	0.66	1.00	82
J	19.0	Feb 11	6606.671 2	0.005841 9	69.8041 8	292.9097 10	147.96 9	286.43 9	5821.537 2	4.40 1	0.65	0.93	78
K	20.0	Feb 12	6599.745 2	0.005268 8	69.8028 9	289.8660 10	145.66 9	352.94 9	5830.707 3	4.81 1	0.64	0.93	72
L	21.0	Feb 13	6591.639 3	0.004680 8	69.7996 8	286.8060 9	143.10 11	69.48 11	5841.469 3	6.16 1	0.60	0.80	66
M	22.0	Feb 14	6580.692 3	0.003940 10	69.7966 11	283.7306 12	139.29 21	159.69 21	5856.055 4	8.88 2	0.98	0.81	78
N	23.0	Feb 15	6564.396 6	0.003048 7	69.7925 9	280.6310 6	136.08 16	267.03 16	5877.881 8	13.25 3	0.66	0.53	60
P	24.0	Feb 16	6522.207 17	0.001380 9	69.7831 11	277.4893 12	125.94 39	53.43 39	5935.024 23	79.94** 25	1.00	0.56	79

See Table 1 for key.

$$\left. \begin{array}{l} M_3 = 90.1 \pm 1.0 \\ M_4 = 56.7 \pm 1.3 \end{array} \right\} **$$

REFERENCES

- | <u>No.</u> | <u>Author</u> | <u>Title, etc</u> |
|------------|---------------------------------|---|
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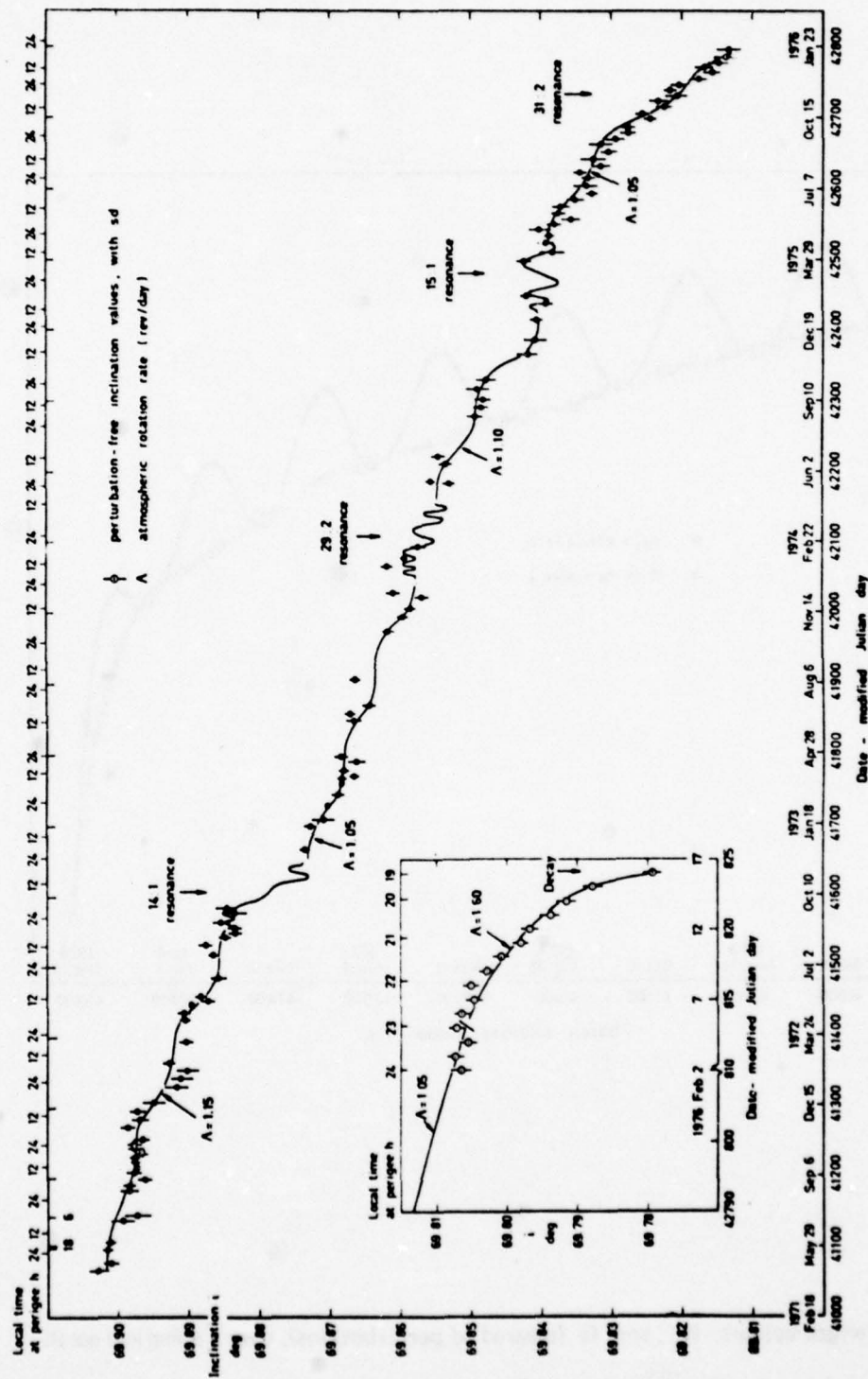


Fig 1 Perturbation-free inclination for China 2 rocket, showing fitted Λ -curves

Fig 2

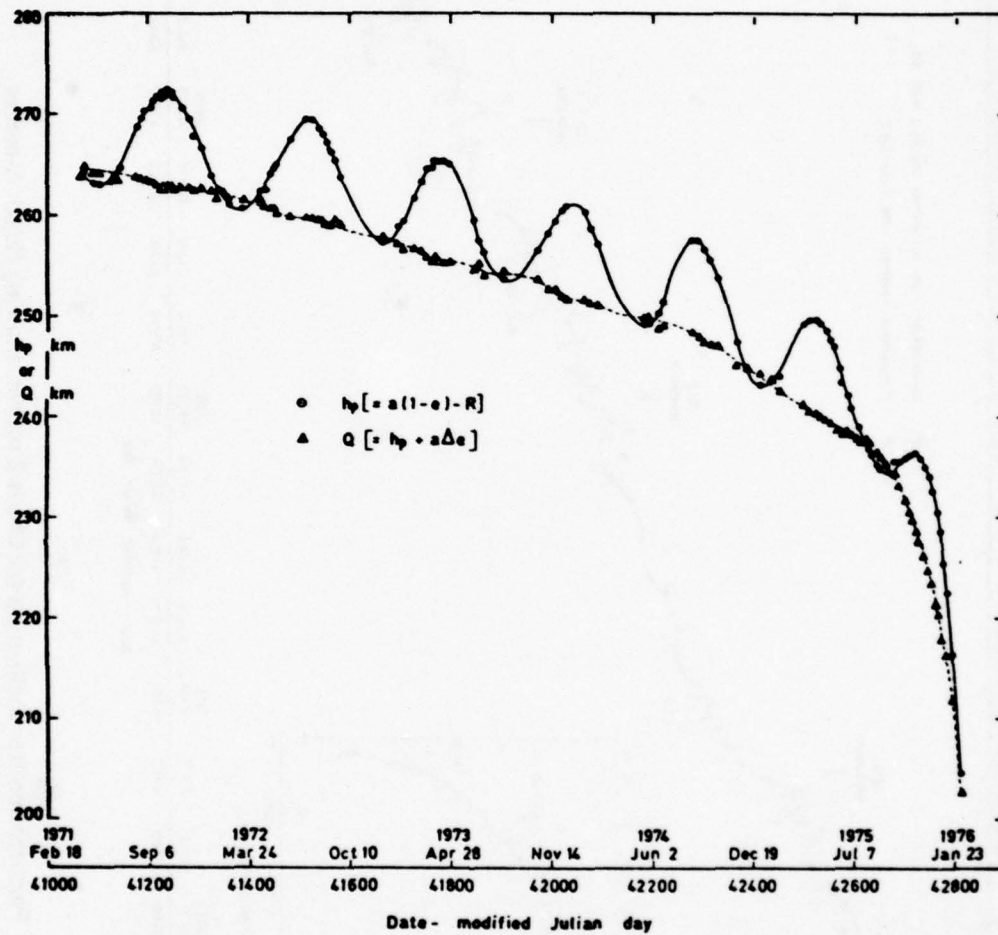


Fig 2 Perigee heights, h_p , and Q (cleared of perturbations), over a spherical earth

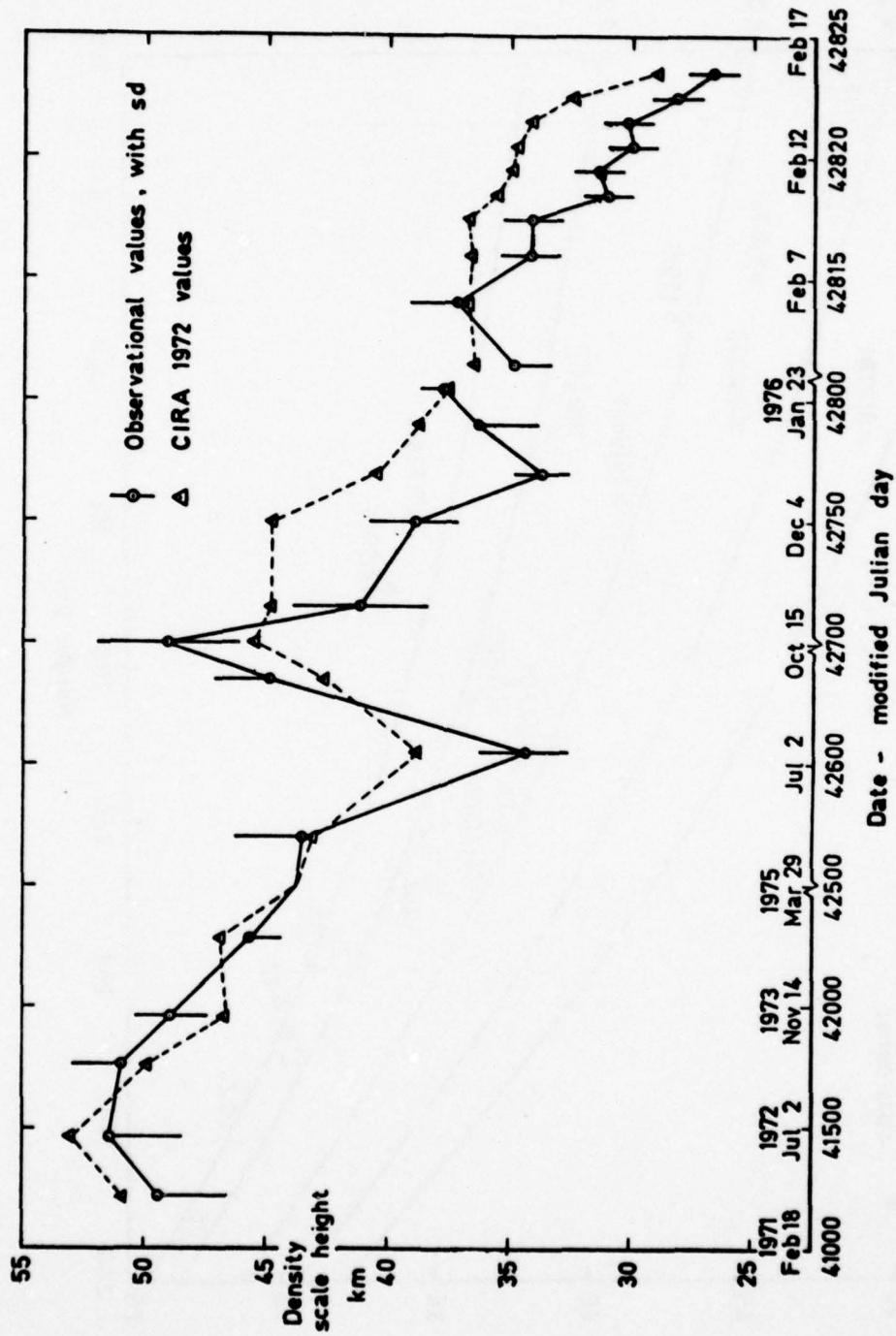


Fig 3 Density scale height compared with CIRA 1972 values

Fig 4

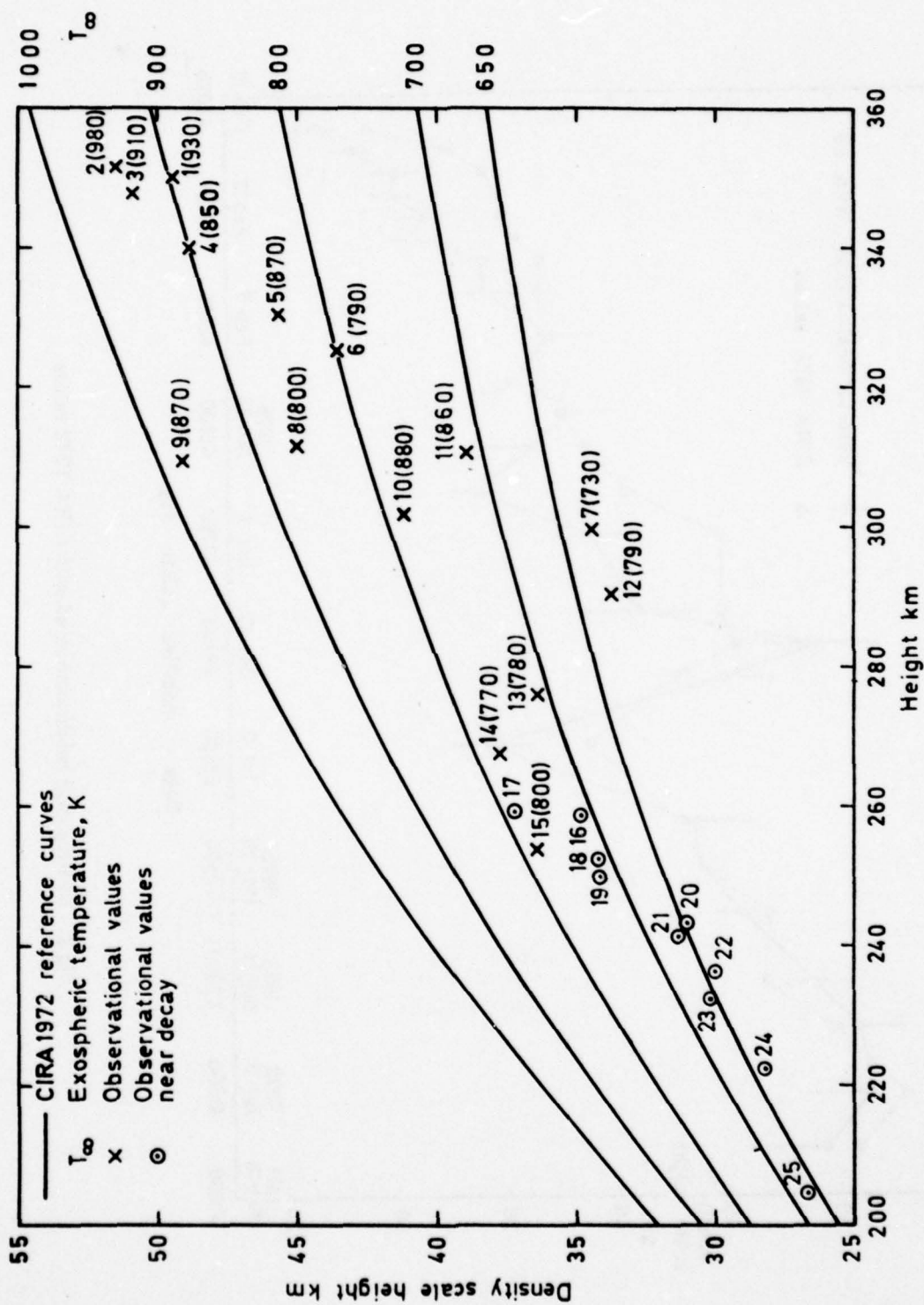


Fig 4 Density scale height values compared with CIRA 1972 curves

Fig 5

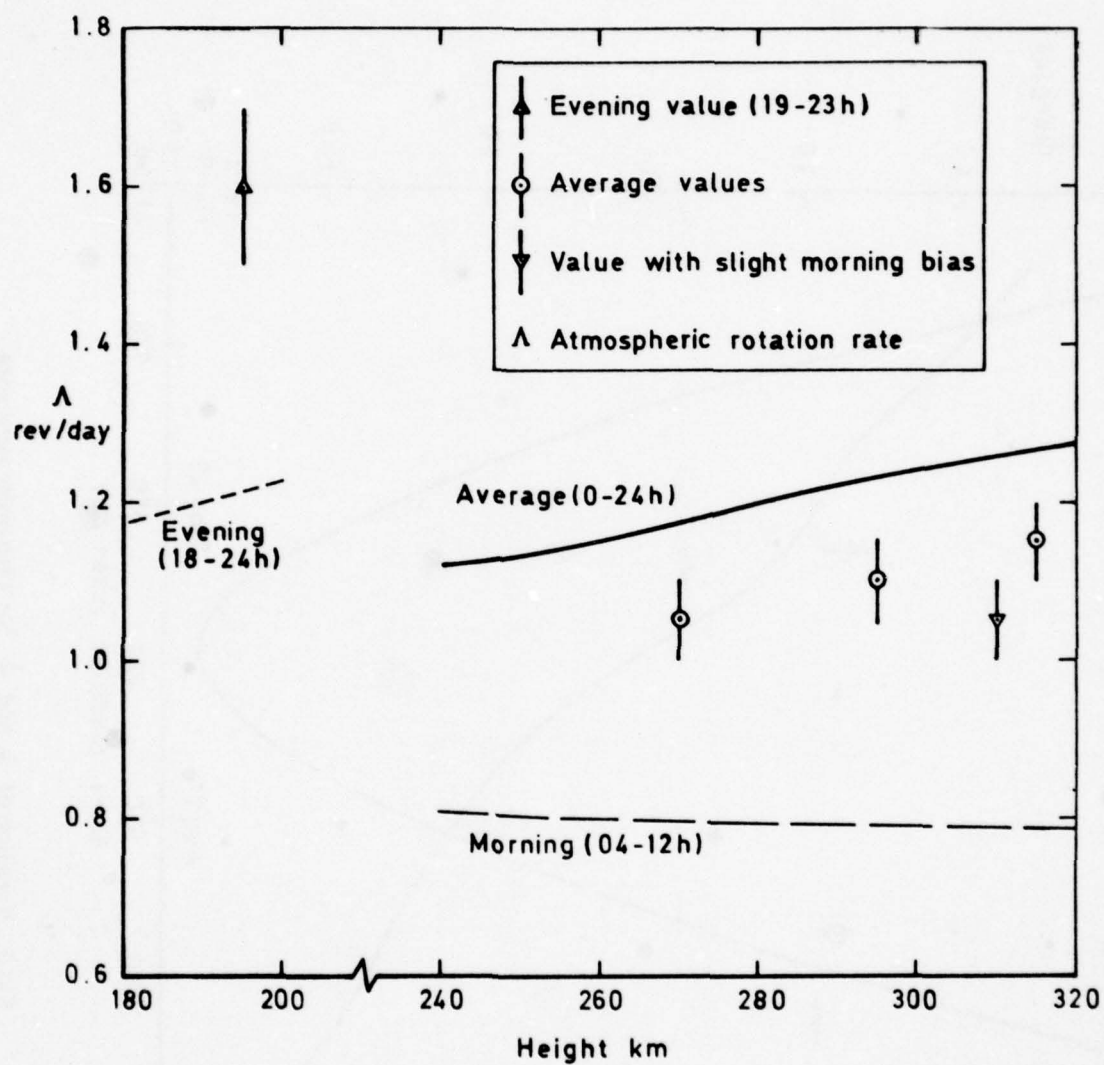
Fig 5 Values of Λ obtained from the orbit of 1971-18B, with curves from Ref 11

Fig 6

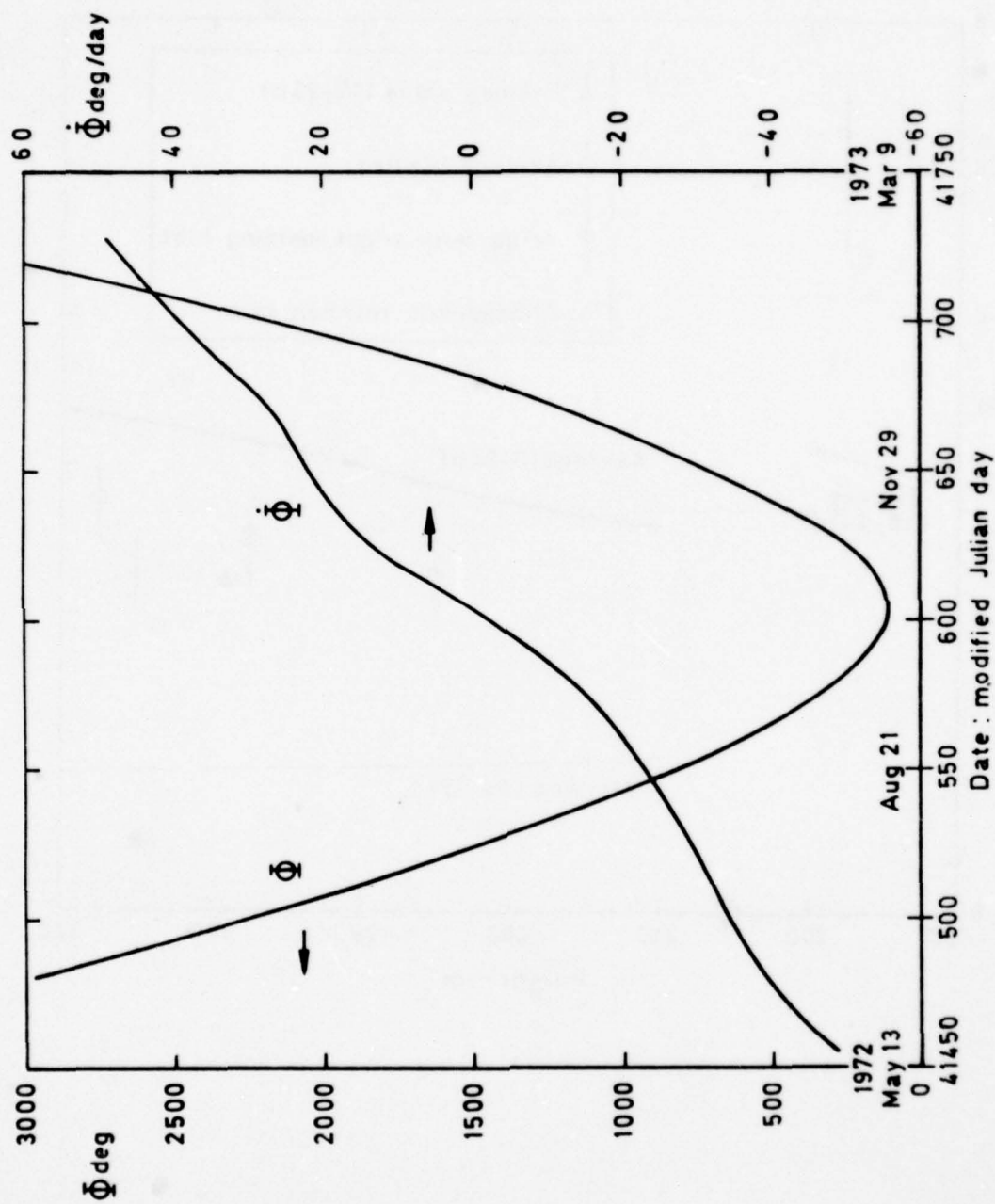
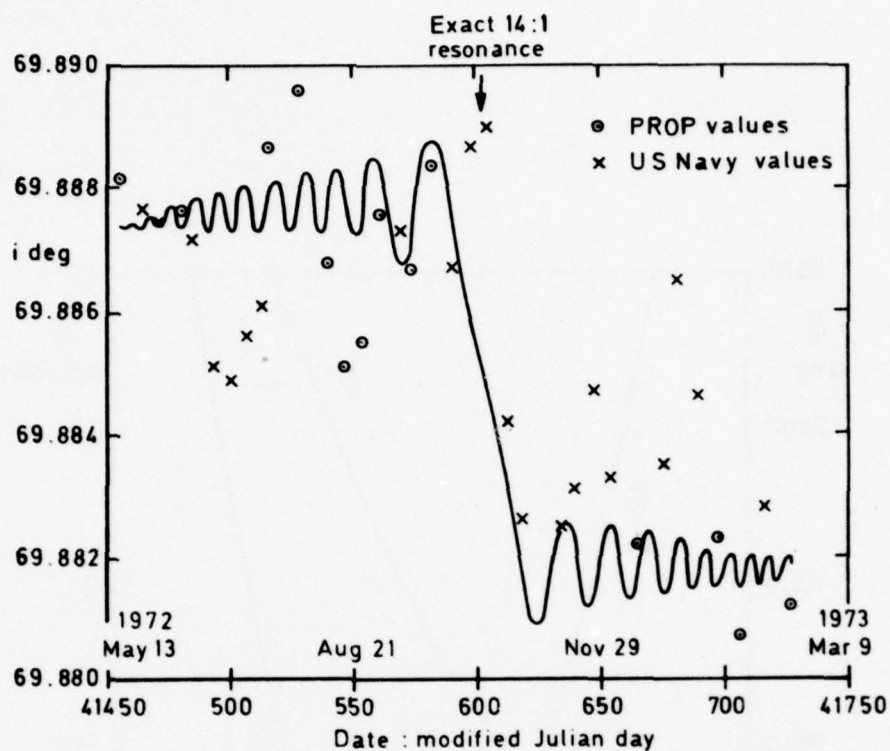
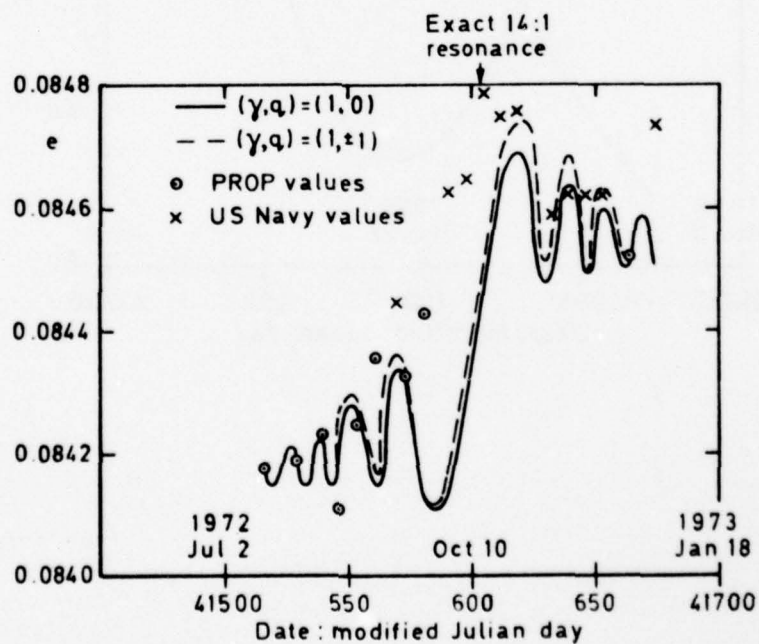


Fig 6 Variation of Φ and $\dot{\Phi}$, for 14th-order resonance

Fig 7a&b



a Inclination



b Eccentricity

Fig 7a&b Values for 14th-order resonance, with fitted curves

Fig 8

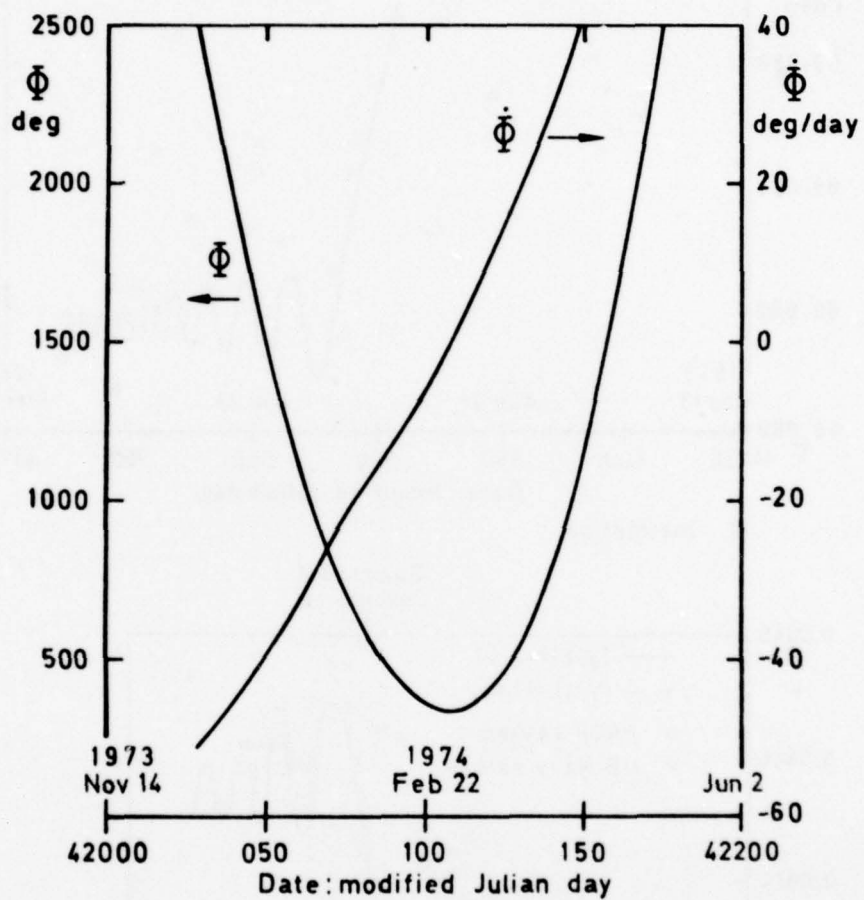
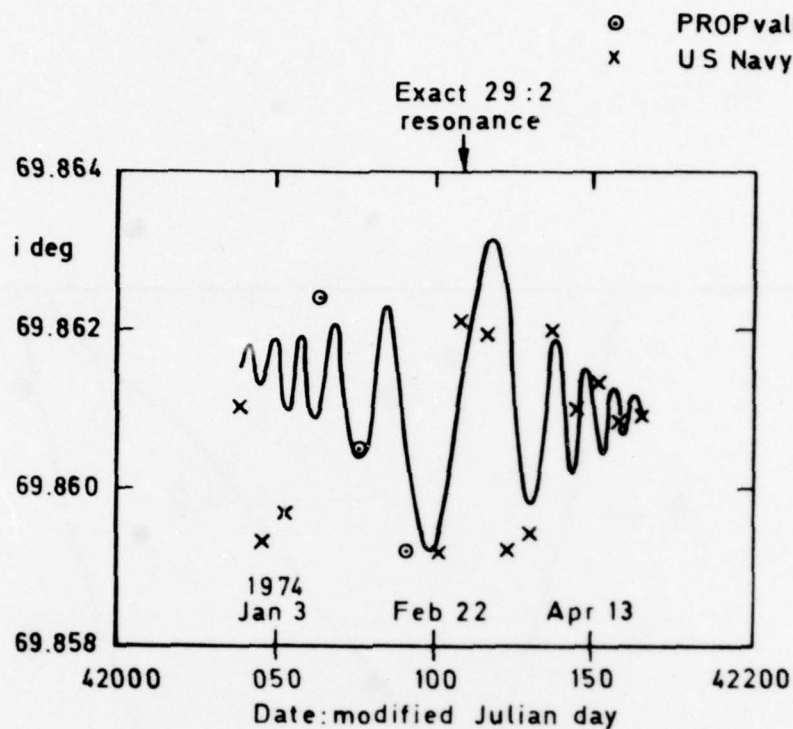
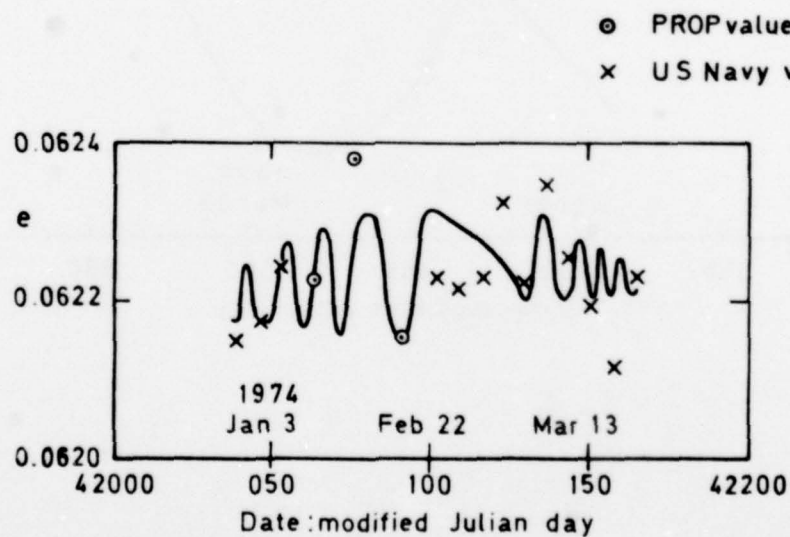


Fig 8 Φ and $\dot{\Phi}$, for 29:2 resonance



a Inclination



b Eccentricity

Fig 9a&b Values for 29:2 resonance, with fitted curves

Fig 10

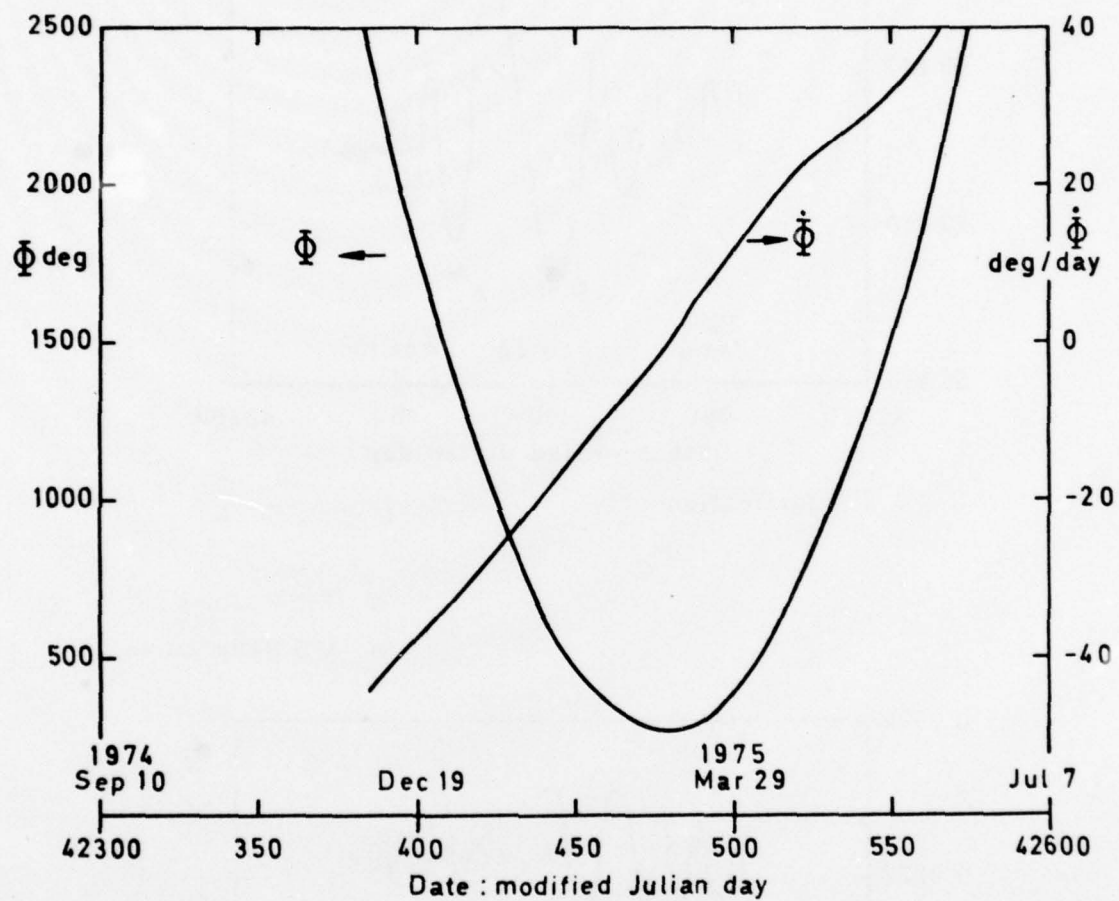


Fig 10 Φ and $\dot{\Phi}$, for 15th-order resonance

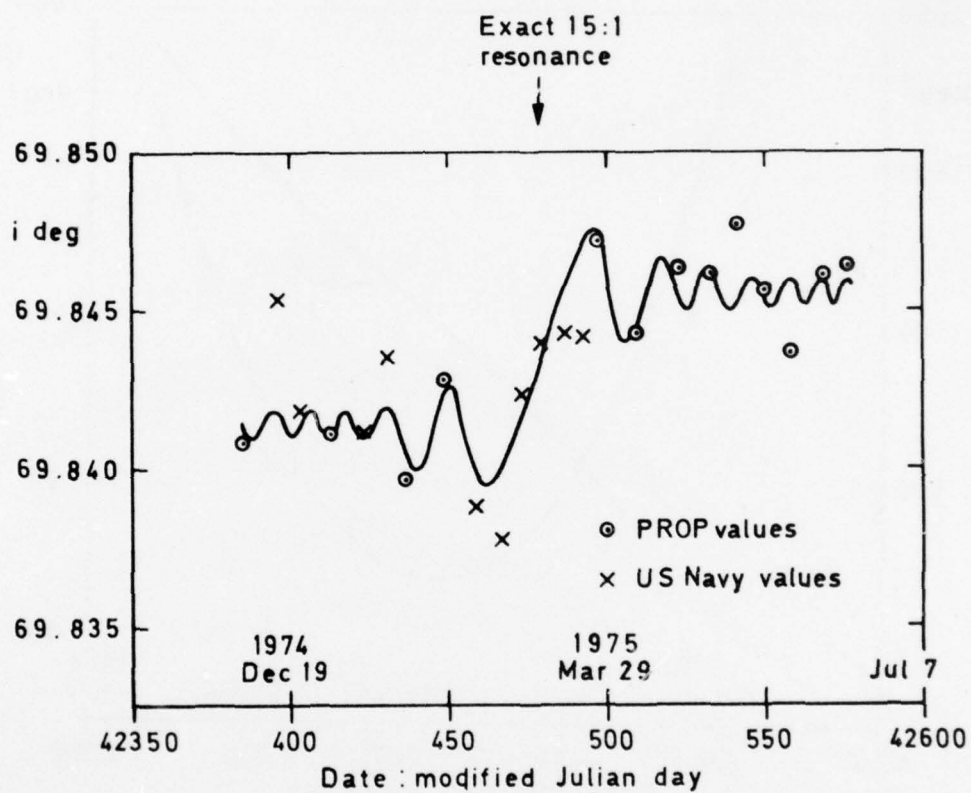


Fig 11 Inclination values for 15th-order resonance, with fitted curve

Fig 12

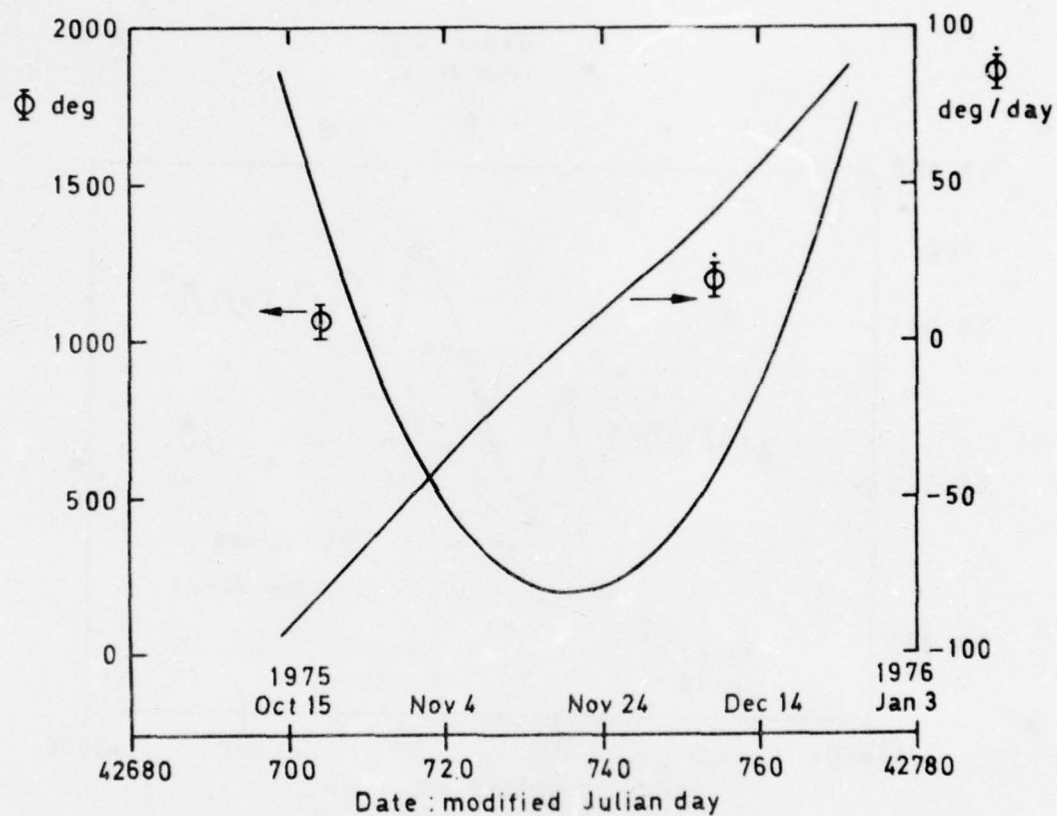
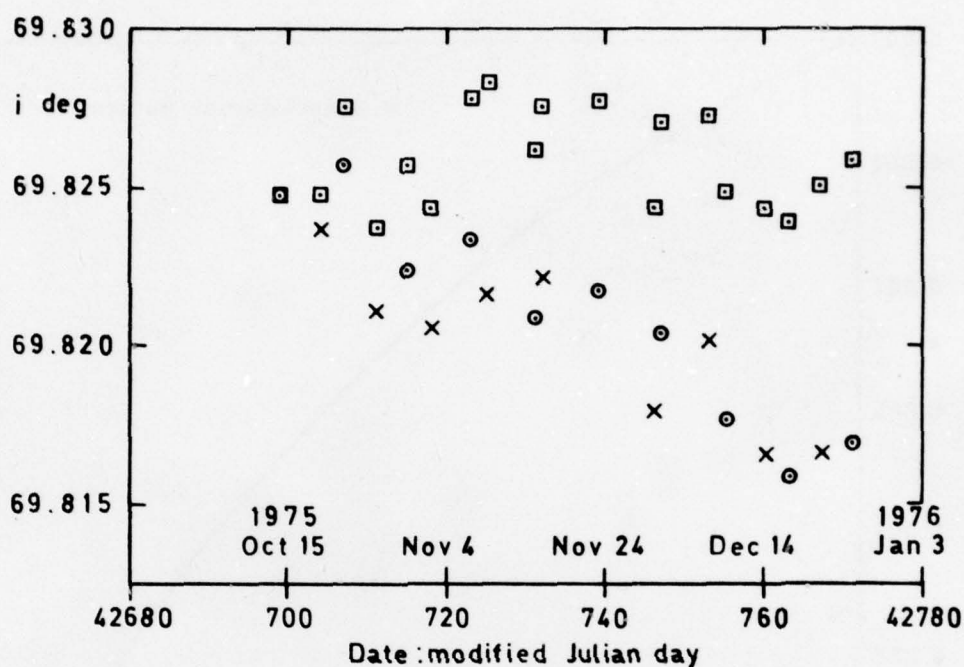
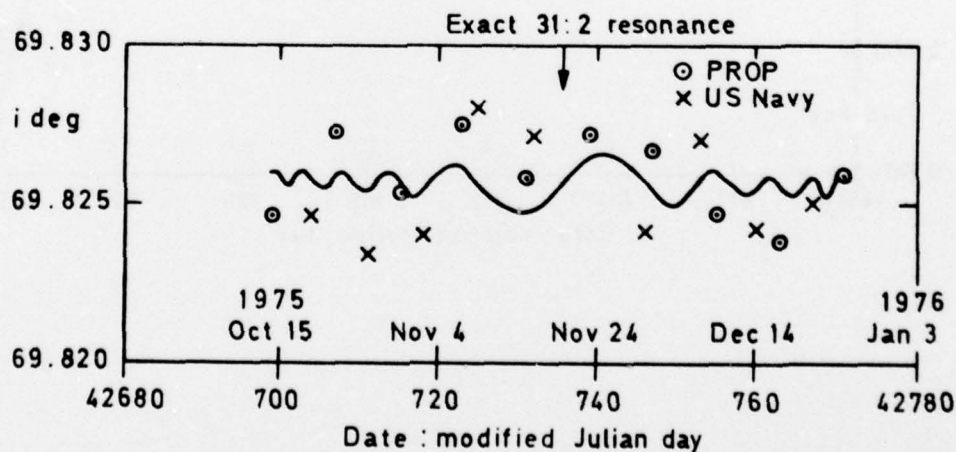


Fig 12 Φ and $\dot{\Phi}$, for 31:2 resonance

- PROP observational values
- × US Navy observational values
- Values cleared of atmospheric rotation perturbations ($\Lambda = 1.05$ rev/day)



a The removal of atmospheric-rotation perturbations



b Fitting of curve to perturbation - free values

Fig 13a&b Values of inclination for 31:2 resonance

Fig 14

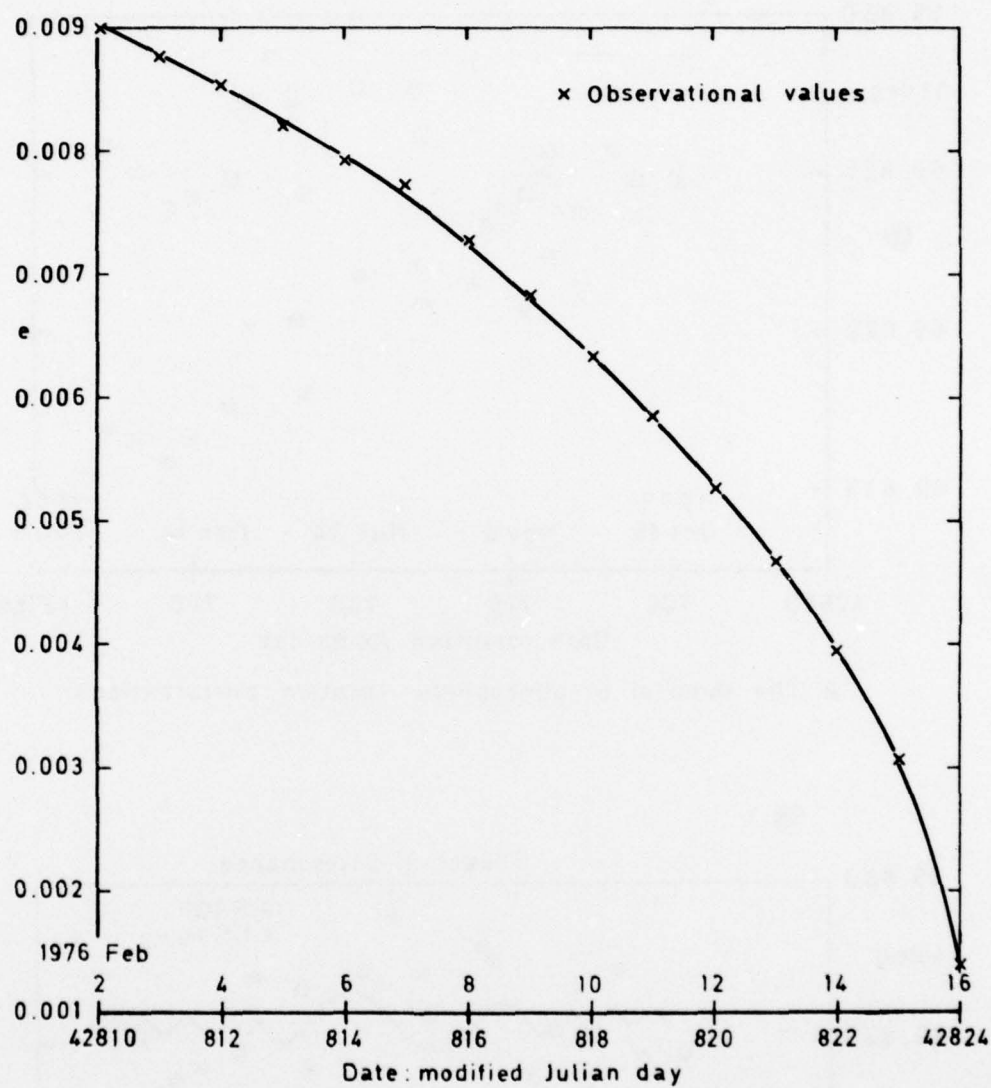


Fig 14 Observational values of eccentricity, near decay

Fig 15

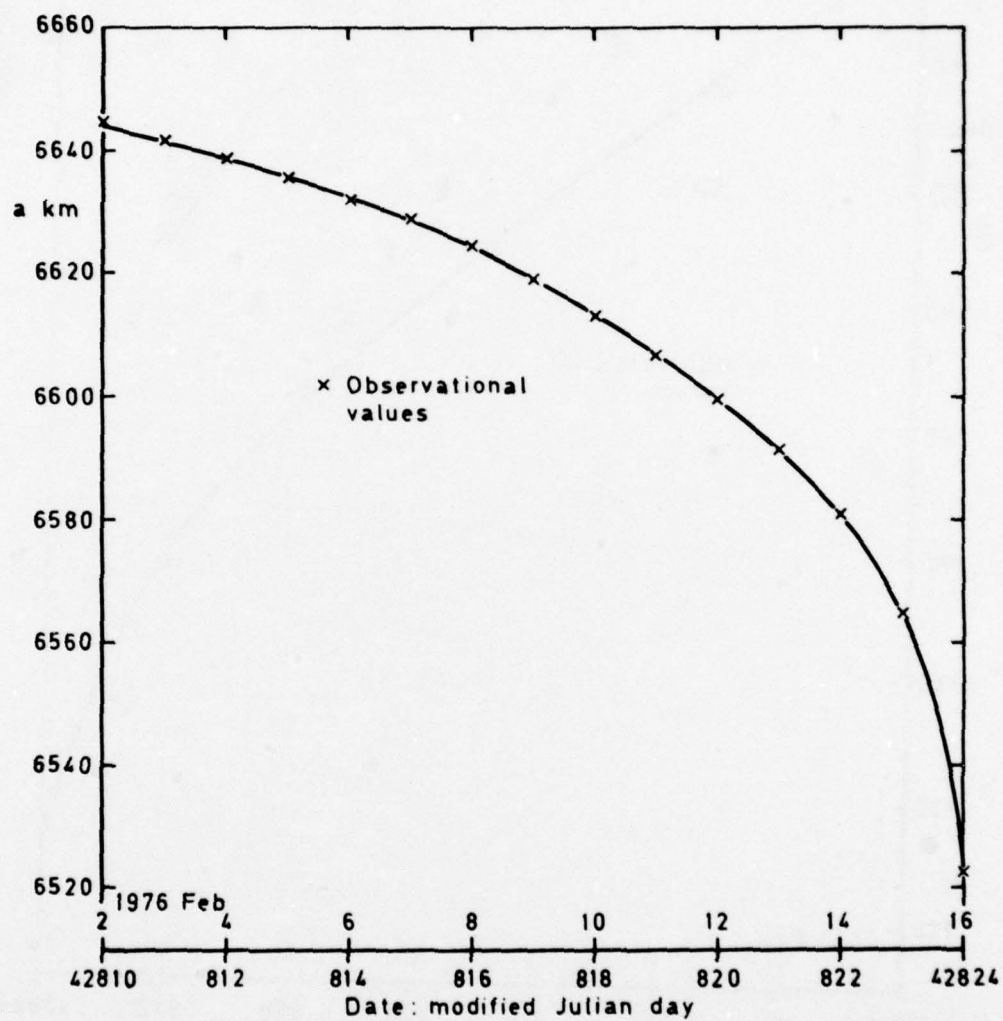


Fig 15 Observational values of semi major axis, near decay

Fig 16

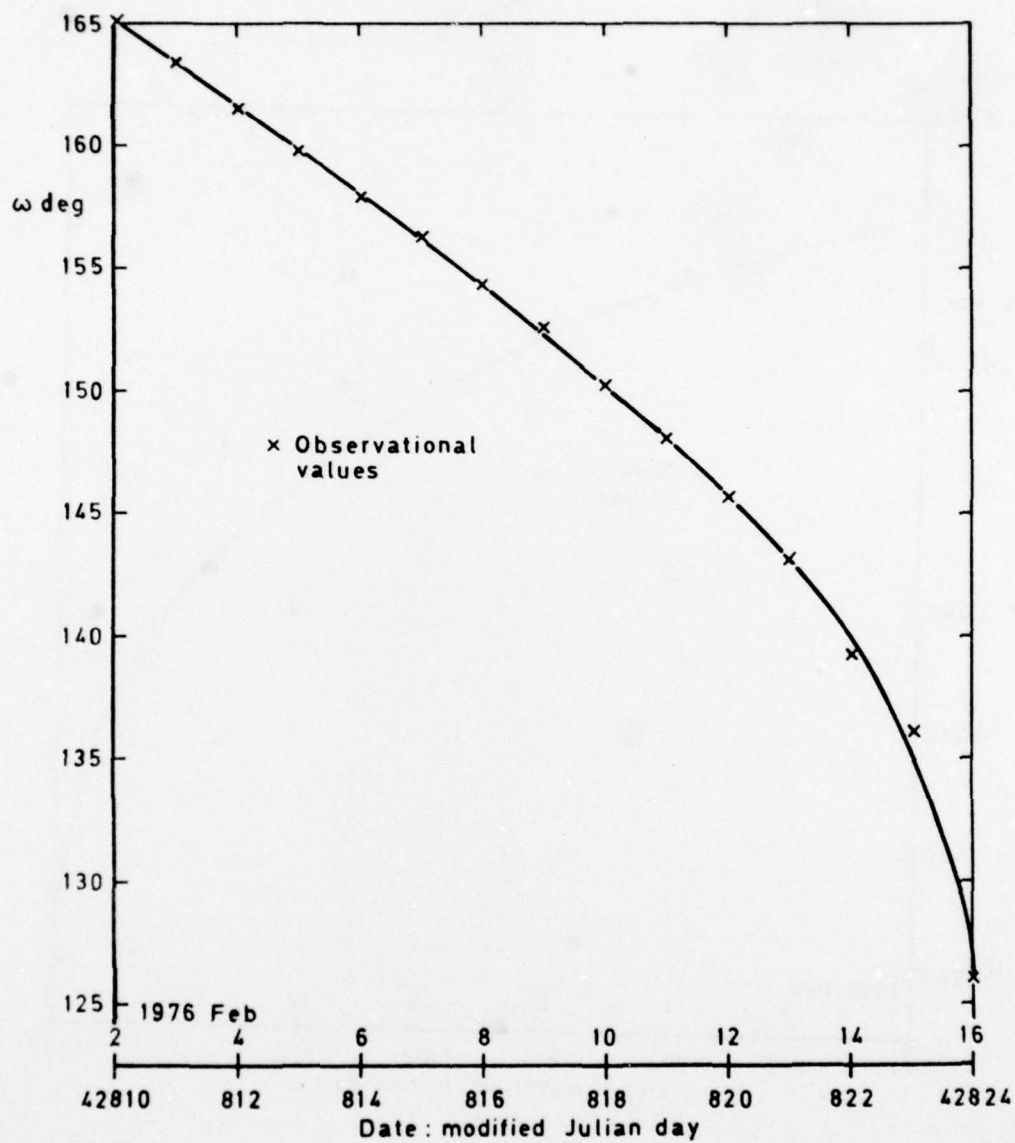


Fig 16 Observational values of argument of perigee, near decay

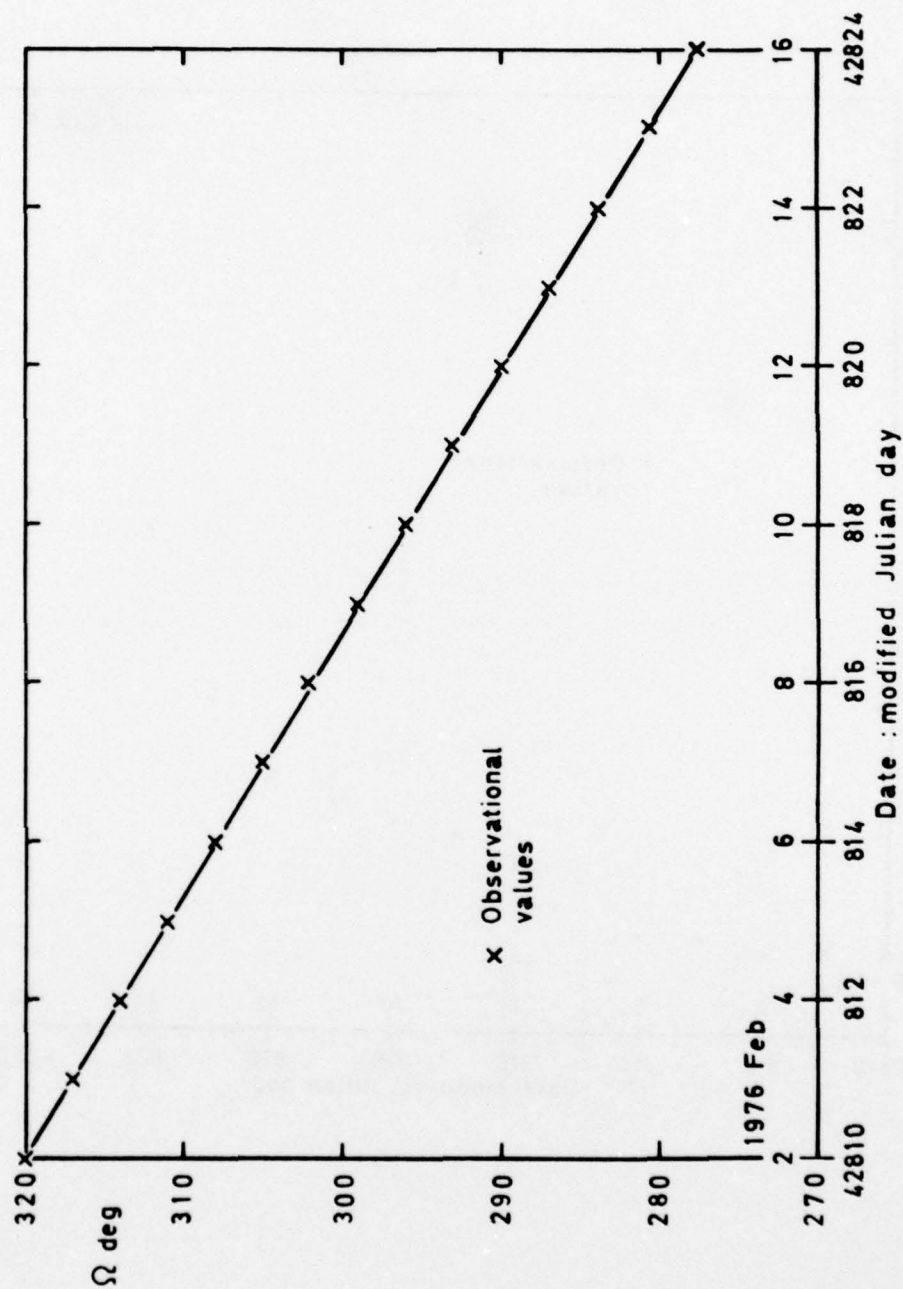


Fig 17 Observational values of right ascension of the ascending node, near decay

Fig 18

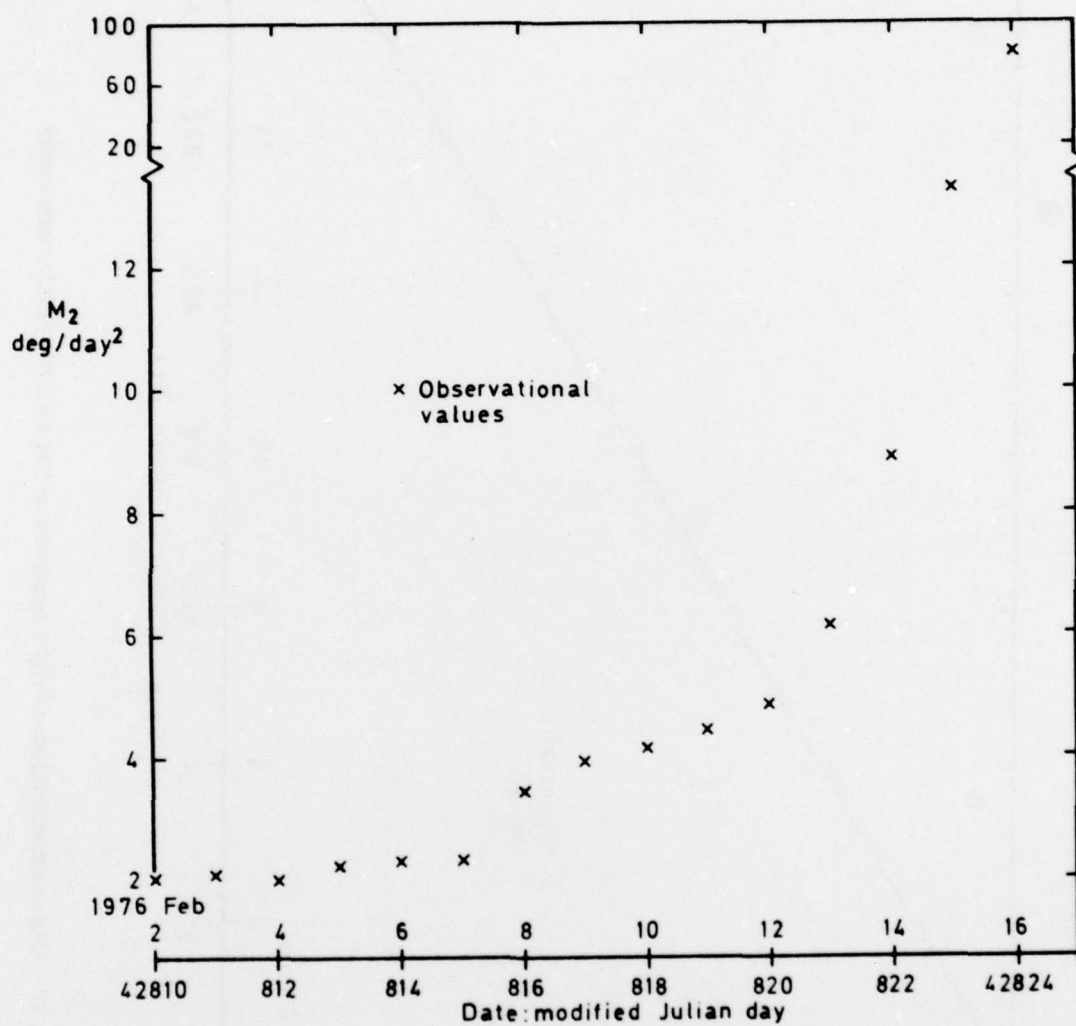


Fig 18 Observational values of orbital decay rate, M_2 , near decay

Fig 19

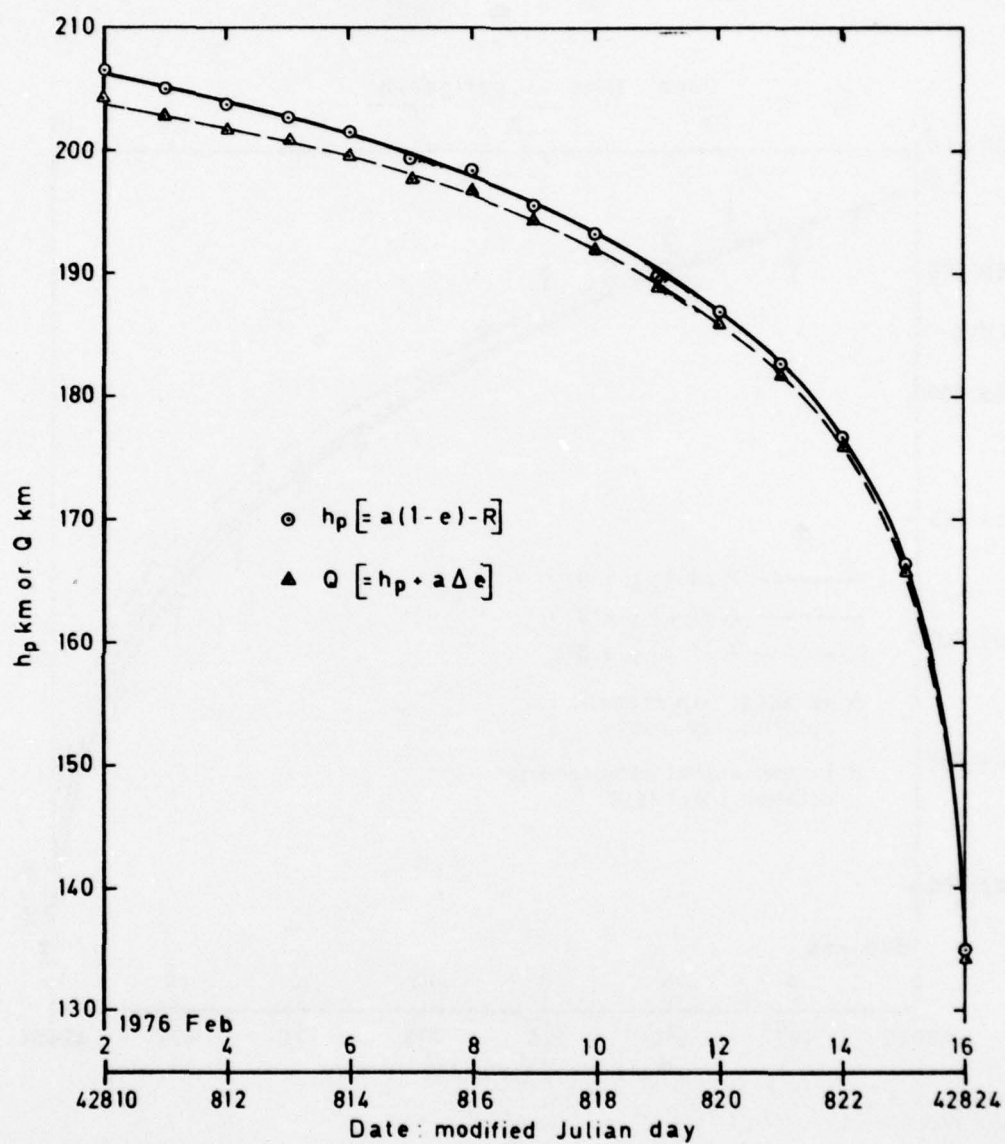


Fig 19 Perigee heights, h_p , and Q (cleared of perturbations), near decay

Fig 20

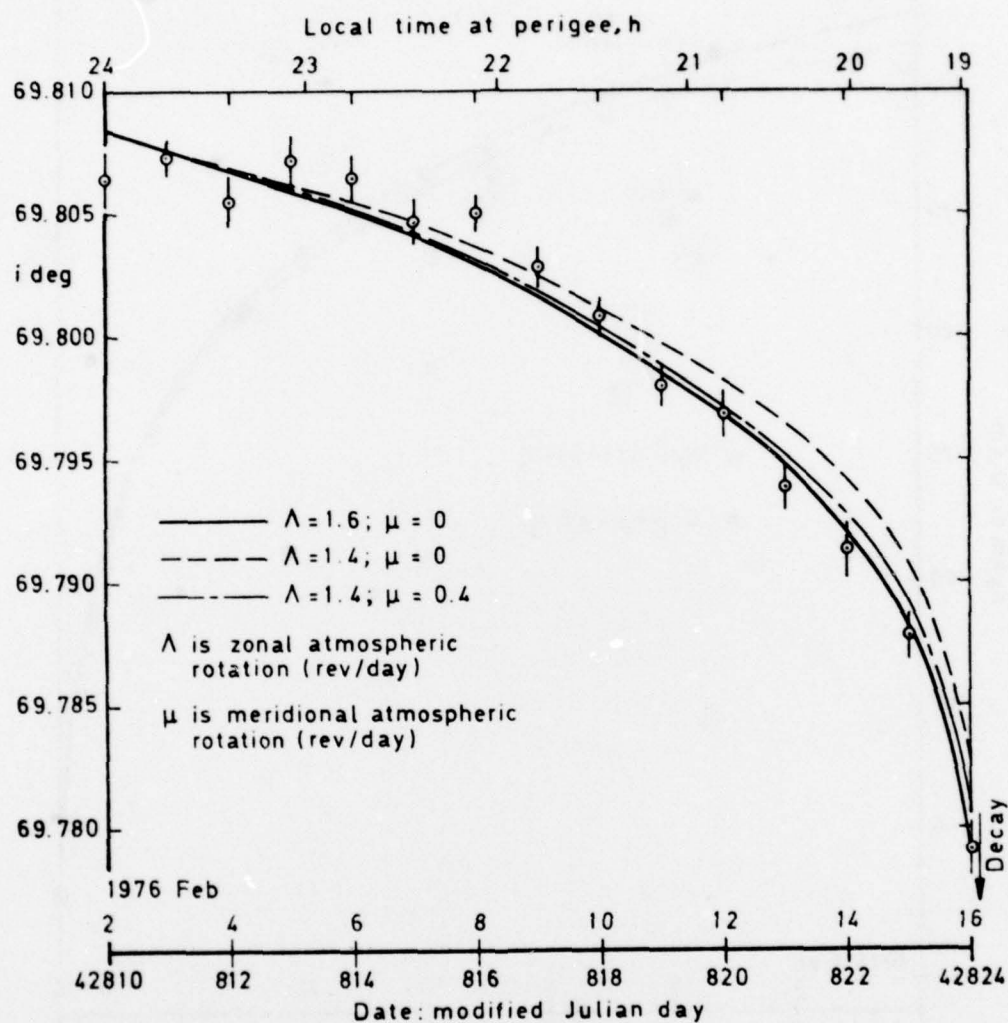


Fig 20 Perturbation-free inclination, with sd, and fitted rotation curves, near decay

REPORT DOCUMENTATION PAGE

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16. Descriptors (Keywords) (Descriptors marked * are selected from TEST) Orbits*. Orbital analysis. China 2 rocket. Rotation rate. Density scale height. Geopotential resonance.					
17. Abstract The orbit of China 2 rocket, 1971-18B, has been determined at 114 epochs throughout its 5-year life, using the RAE orbit refinement program PROP 6, with more than 7000 radar and optical observations from 83 stations. The rocket passed slowly enough through the resonances 14:1, 29:2, 15:1 and 31:2 to allow lumped geopotential harmonic coefficients to be calculated for each resonance, by least-squares fittings of theoretical curves to the perturbation-free values of inclination and eccentricity. The rotation rate of the upper atmosphere, at heights near 300 km, was estimated from the decrease in orbital inclination, and values of 1.15, 1.05, 1.10 and 1.05 rev/day were obtained between April 1971 and January 1976. From the variation in perigee height, 25 values of density scale height were calculated, from April 1971 to decay. A further 1400 observations, made during the final 15 days before decay, were used to determine 15 daily orbits. Analysis of these orbits reveals a very strong west-to-east wind, of 240 ± 40 m/s, at a mean height of 195 km under winter evening conditions.					

