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HIGH SPEED ROTOR BALANCING.(U)  
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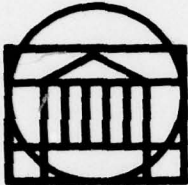
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SCHOOL OF ENGINEERING AND  
APPLIED SCIENCE

UNIVERSITY OF VIRGINIA

Charlottesville, Virginia 22901

15 DAAG 29-78-G-0019

Final Report

6 HIGH SPEED ROTOR BALANCING

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9 Final rept. 22 Oct 77 - 30 Nov 78

APPROVED FOR PUBLIC RELEASE:  
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12 14p

14 Report No. UVA/525088/MAE78/102

11 December 1978

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HIGH SPEED ROTOR BALANCING

FINAL REPORT

WALTER D. PILKEY

1978

U. S. ARMY RESEARCH OFFICE

GRANT NO. DAAG29 78 G 0019

UNIVERSITY OF VIRGINIA

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| 4. TITLE (and Subtitle)<br><br>HIGH SPEED ROTOR BALANCING                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |                       | 5. TYPE OF REPORT & PERIOD COVERED<br><br>Final             |
| 7. AUTHOR(s)<br><br>Walter D. Pilkey                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |                       | 6. PERFORMING ORG. REPORT NUMBER<br>DAAG29 78 G 0019        |
| 9. PERFORMING ORGANIZATION NAME AND ADDRESS                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |                       | 8. CONTRACT OR GRANT NUMBER(s)<br>↓                         |
| 11. CONTROLLING OFFICE NAME AND ADDRESS<br>U.S. Army Research Office<br>Post office Box 12211<br>Research Triangle Park NC 27709                                                                                                                                                                                                                                                                                                                                                                                                     |                       | 10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS |
| 14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)                                                                                                                                                                                                                                                                                                                                                                                                                                                          |                       | 12. REPORT DATE<br>1978                                     |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |                       | 13. NUMBER OF PAGES<br>9                                    |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |                       | 15. SECURITY CLASS. (of this report)<br>Unclassified        |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |                       | 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE<br>NA            |
| 16. DISTRIBUTION STATEMENT (of this Report)<br><br>Approved for public release; distribution unlimited.                                                                                                                                                                                                                                                                                                                                                                                                                              |                       |                                                             |
| 17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)<br><br>NA                                                                                                                                                                                                                                                                                                                                                                                                                                 |                       |                                                             |
| 18. SUPPLEMENTARY NOTES<br><br>The findings in this report are not to be construed as an official Department of the Army position, unless so designated by other authorized documents.                                                                                                                                                                                                                                                                                                                                               |                       |                                                             |
| 19. KEY WORDS (Continue on reverse side if necessary and identify by block number)<br><br>Rotating Shaft<br>Balancing<br>Rotor Dynamics<br>Dynamics                                                                                                                                                                                                                                                                                                                                                                                  |                       |                                                             |
| 20. ABSTRACT (Continue on reverse side if necessary and identify by block number)<br><br>This report summarizes the accomplishments of the final year of a study exploring new methods for balancing, analyzing and designing flexible rotating shafts. A technique for identifying rotor bearing parameters is proposed. Then a quadratic programming formulation is presented for influence coefficient balancing with constraints. Finally, a method is developed for determining the optimal axial location of balancing planes. |                       |                                                             |

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## PROBLEM STATEMENT AND ACCOMPLISHMENTS

The goal of the program is to explore new techniques for balancing, analyzing, and designing rotating shafts. This study began with the assumptions that the balancing problem is rightfully classified as an identification/optimization problem and that modern computational techniques hold great promise in aiding in the solution of the problem. Throughout the study emphasis is given to computational optimization formulations.

The achievements of the final year of this study, which was performed during the period 21 October 1977 - 30 November 1978, can be grouped into three categories:

- Estimation of Parameters in a Linear Rotor-Bearing System Model
- The Determination of Rotor Balance Weights by Quadratic Programming Models
- Study of the Sensitivity of the Balanced-Rotor Residual Deflections to Deletions of Balancing Weights

These efforts are summarized below.

### I. The Estimation of Parameters in a Linear Rotor-Bearing System Model

For a system of ordinary, linear differential equations with constant coefficients which models a rotor-bearing system, the problem of estimating viscous damping constants and other parameters which appear in the model is approached from an optimization standpoint. The magnitudes of the constants are adjusted to give the best agreement between some measured values of the system properties and the values computed from the model. An estimation method is developed whereby a steepest descent or creeping random search optimization algorithm is applied to the linear algebraic equations which govern the rotor's steady state response. The optimization is

conducted. The optimization is conducted at a single frequency and it requires that at least as many displacement measurements be obtained as there are unknown parameters to be estimated. Because the actual response characteristics of the rotor bearings are known to be nonlinear, the confinement of the optimization to a single frequency should result in a linear model which is useful in some range about that frequency. Of course, if the linear model is to apply over a wide range of frequencies, the optimization can be performed using measurements taken at a number of rotor speeds.

## II. The Determination of Rotor Balance Weights by Quadratic Programming Models

In the influence coefficient method of balancing, the rotor response is expressed as a linear function of applied unbalance weights. Let  $(x,y,z)$  represent a system of coordinates rotating at the rotor spin speed about the  $z$ -axis, which is parallel to the undeformed shaft axis. In an isotropic system, the orbit of the geometric centroid of the shaft cross section can be considered to be circular as it undergoes synchronous whirl due to unbalance inertia forces.

Let  $\overline{W}_X$ ,  $\overline{W}_Y$  represent the components of deflections at various locations along the rotor axis. Let  $\overline{U}_X$ ,  $\overline{U}_Y$  represent components of masses applied to the rotor. Let  $[A_X]$ ,  $[A_Y]$  be matrices of influence coefficients relating the in-phase and out-of-phase components of deflection to the applied unbalances ( $[A_X]$  and  $[A_Y]$  are independent of the orientation of the coordinates).

$$\overline{W} = \overline{W}_X + j \overline{W}_Y \quad (j = \sqrt{-1})$$

$$\overline{U} = \overline{U}_X + j \overline{U}_Y$$

$$[A] = [A_X] + j [A_Y]$$

$$\overline{W} = [A] \overline{U}$$

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Let  $\overline{WM}$  be a vector of displacements measured along the rotor which are caused by intrinsic (not applied) mass unbalance. A typical rotor balancing procedure computes correction weights by minimizing the sum of the squares of the residual rotor deflection:

$$E = (\overline{WM} + \overline{W})^T (\overline{WM} + \overline{W}) = (\overline{WM} + [A]\overline{U})^T (\overline{WM} + [A]\overline{U})$$

In the case in which the elements of  $\overline{U}$  are unconstrained, the balance weights are obtained by solving the set of linear equations  $\text{GRAD}(E) = \overline{0}$ , in which the gradient components are derivatives with respect to the elements of  $\overline{U}$ . Often, the balance weights computed by this method are too large to conveniently apply to the rotor. To overcome this problem, the expression for  $E$  can be minimized subject to upper bounds on the components of  $\overline{UX}$  and  $\overline{UY}$ . Let  $\overline{X}$  denote a vector the components of which are sets of pairs of real number  $(UX, UY)$  for each balance plane. Carrying out the multiplications in the expression for  $E$  results in:

$$E = W_0 + \overline{C}^T \overline{X} + \overline{X}^T [D] \overline{X}$$

$W_0$  is the sum of the squares of the measured deflections.

$\overline{C}$  is the cost vector (a function of  $\overline{WM}$  and  $[A]$ ).

$[D]$  is the quadratic matrix which is assumed to be positive definite ( $[D]$  is a function of  $[A]$ ).

Let  $\overline{UM}$  be a vector of the upper and lower bounds on the elements of  $\overline{X}$ .

$$- \overline{UM} \leq \overline{X} \leq \overline{UM}$$

By introducing a shift of coordinates, these constraints can be converted to the form  $\overline{0} \leq \overline{X}' \leq \overline{UM} + \overline{UM}$  in which  $\overline{X}' = \overline{X} + \overline{UM}$ .

In this study, a quadratic programming routine was employed to demonstrate the usefulness of the quadratic programming algorithm to rotor balancing.

### III The Study of the Sensitivity of the Balanced-Rotor Residual Deflections to Deletions of Balancing Weights

The third study deals with the problem of estimating which of  $N$  possible balance places can be deleted with the smallest increase in the rotor residual deflection. Because this process will involve the simultaneous, finite change of the variables of a quadratic function, a computationally attractive exact method of finding the correct answer does not appear to be possible. Suppose that the optimal choice of  $n$  of  $N$  possible planes ( $n < N$ ) is sought; then, there will be  $\frac{N!}{n!(N-n)!}$  possible choices. As  $N$  increases, an exhaustive search rapidly becomes impractical (e.g. for  $N=20$ , there are 184,756 possible choices of 10 planes). A random search algorithm could be a feasible approach; for example, the probability of finding a solution in the best 1% of all solutions is better than 99% with a random sample of 500 trials. However, running even 500 problems is a fairly unattractive alternative.

Because only  $n$ -plane solutions which do not too greatly increase the rotor residual deflection from the  $N$ -plane level are of interest, a region near the  $N$ -plane optimum may exist where variable interactions can be neglected in estimating the effects of balance plane deletions. Any technique which attempts to consider the interactions of deleted balance planes would entail examination of all or a great number of the  $N!/(N-n)!n!$  possibilities. By considering only independent effects, a small fraction of the great number of possibilities can be segregated for exact solution.

To estimate the effects of balance plane deletions, two studies were conducted. In the first, the unconstrained solution with  $N$ -planes was obtained; then  $N$  additional problems were solved in which the upper bounds

on one of the N-planes was reduced either to zero or to a very small value. From the change in the objective function and the value of the Lagrange multiplier for each deleted variable, the most insensitive balance plane can be estimated. These estimated selections are then compared with actual n-plane solutions determined by exhaustive or random searches.

In the second study, the constraint limits were allowed to approach zero simultaneously, and the order in which constraints become active, the change in the objective function as constraints decreased, and the changes in the Lagrange multipliers were studied. This information was used to estimate balance plane sensitivity, as in the first study. These results indicate that independent effects can be used to estimate the sensitivity of the rotor balance level to plane deletions.

## LIST OF PUBLICATIONS (1974-1978)

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Other papers are being considered for publication.

## PARTICIPATING PERSONNEL (During Final Year)

W. D. Pilkey, Principal Investigator

C. Thasanatorn, Received Ph.D. degree

F.H. Chu, Received Ph.D. degree

M. Taylor, Received Ph.D. degree

E. Woomer, Ph.D. Candidate

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