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FOREIGN TECHNOLOGY DIVISION

EXTERNAL BALLISTICS

Ву

A. A. Dmitriyevskiy



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Chapter VIII

STABILITY OF MOTION AND THE STABILIZATION OF FOCKETS AND OF PROJECTILES.

§1. Common/general/total concepts of stability of motion and of the stabilization of rockets and of projectiles.

The problem of statility of motion of kodies was solved for the first time by noted Russian scientists by N. Ye. Joukowski in work "The strength of motion" (1882) and by A. K. Lyapunov in work "common/general/total problem of statility of motion" (1892). At present stability theory is widely developed and improved, it examines not only questions of stability of motion of mechanical systems (aircraft, rockets, projectiles), but also questions of the stability of the systems of control, automatic control systems, etc.

During investigation the motion of the rockets and projectiles usually is subdivided into disturbed and that not disturbed. The undisturbed motion is called similar, which they would complete the DCC = 78107111

rocket or projectile in standard atmosphere or in the vacuum under the action previously provided, that were being subordinated to the specific laws, forces. The corresponding to the undisturbed motion trajectory is also called not disturbed or calculated (nominal).

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However, under actual conditions the motion of rockets and projectiles occurs with the supplementary, random factors which usually during the calculation of rominal trajectories are not taken into account (deviation of temperature of air from normal law, wind and wind gusts, the pulsation of the engine thrust, nonprogrammed control forces, etc.). The actions of these factors, called disturbing, lead to the fact that the rocket and projectile will move not over nominal (calculated) trajectory, fut differing from it more or less considerably depending on value and direction of disturbance/perturbations. The motion, the reflection effect of the perturbation factors, calls the disturbed soction, and the corresponding to it trajectory - by the disturbed trajectory.

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With the concept of the disturbed motion of rockets and projectiles, is organically-bound the concept of the stability of their motion. COC = 78107111

In accordance with classical stability theory, the motion of solid body can be named stable in such a case, when it possesses property to return after the break-down of the perturbation factors to the interrupted by them not disturted motion.

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Let us explain this position with the aid of Fig. 8.1. On figure by solid line is depicted the trajectory, which corresponds to the undisturbed motion. Let on section AE of trajectory on rocket act some disturbance/perturbations, which will force it to move over the disturbed trajectory AE*, which differs from the nominal. The motion of rocket will be stable, if after the break-down of disturbance/perturbations at point E* the disturbed trajectory will converge from nominal and it will coincide from the latter at certain point V.

But if this does not occur and rockets it will fly along the trajectory B*C*, then its action must be described as unstable. It is completely obvious that stability of motion in the sense, which corresponds to the given above definition, wirtually it cannot have provided not only the urguided projectiles and the rockets, but also of rockets with the complex control system, since no real control system due to the which are inherent in it errors not in state to just as ideally parry disturbance/perturbations and consequences of their effect, in order always to accurately return rocket to nominal DCC = 78107111

trajectory. Based on this, we subsequently will understand under the stable motion of rockets similar, during which the deviation of real trajectory from the nominal under the action of short-term or prolonged disturbance/perturbations does not exceed the established/installed limits.

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In Fig. 8.1 zones of the stable motion of the relatively nominal trajectory CC is isclated by dot-dash lines.

During the study of stability of motion of rockets and projectiles, are usually examined separately stability of motion of the center of mass along trajectory and the stability in their rotary motion relative to the center of mass. The provision for the latter is the necessary stability condition of the motion of the center of mass and it is inseparably connected with the concept of the angular stabilization of rockets and projectiles. Under stabilization is understood the totality of the measures which provide preservation/retention/maintaining by rocket or projectile in the trajectory of correct position relative to direction of motion.

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Fig. 8.1. Diagram to the determination of stability of motion of rockets and projectiles.

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Other conditions being equal, the drag will be smallest in the case when the axis of projectile coincides with direction of motion. However, in the process of flight, the velocity vector of the center of mass $\overline{\mathbf{v}}$ is turned. Because of this stable rocket or projectile they must continuously change its angular position, being turned relative to the center of mass following by vector $\overline{\mathbf{v}}$ in the manner that it is shown on Fig. 8.2.

If the longitudinal axis of the housing of projectile or finless rocket is deflected from direction of motion at least in insignificant angle, then the resulting aerodynamic force \overline{F} , which acts on rocket or projectile, will be applied in the center of

pressure (ts.d.) which is located between the apex/vertex of nocket and its center of mass (Fig. 8.3). This leads to appearance relative to the center of mass of the tilting moment \overline{M} , which during the motion of the nonstabilized nocket along trajectory will cause random movement relative to the center of mass and, as a result, is considerable the distortion of trajectory. Thus, the position of the housing of the projectile at which the axis of masses and apex/vertex, can be described as position of unstable equilibrium, since the least misalignment of nocket from vector \overline{v} will cause the irreversible increase in this deviation. For preventing of this phenomenon and provision for a correct position in flight (by nose section forward) the nocket and projectile must be stabilized. In connection with the unguided nockets and projectiles, are utilized two different of the "passive" method of stabilization - spin stabilization and stabilization by tail assetly.

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The stabilization of rockets and projectiles by tail assembly consists in the fact that on the tail section of the oblong housing are fastened the diverse in their structural/design forms and size/dimensions stabilizers. This leads to the fact that during the flow around housing of airflow at argle of attack of the character of pressure distribution along the length of rocket will change, as a result of which ts.d. shifts relative to the center of mass to the side of stabilizers. During the appropriate selection of the DOC = 78107111

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size/dimensions of stabilizers, it is possible to attain this displacement/movement of ts.d. that it with respect to the apex/vertex of rocket or projectile will render/show benind the center of mass. In this case with o#C the resulting aerodynamic force $\bar{\mathbf{F}}$ will act in the manner that shown on Fig. 8.4 (for motion in vertical plane).

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Fig. 8.2. Diagram of the consecutive positions of the fin-stabilized projectile in trajectory during correct flight.

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It is obvious that caused by force of F torque/moment \overline{M}_2 will attempt to decrease the angle a, give housing to such position in which the axis $0x_1$ coincides with velocity vector \overline{v} , and torque/moment itself \overline{M}_2 will become equal to zero, i.e., it will turn out that unlike the preceding/previous case, the stabilizing (or restoring) effect. The position of housing, which is characterized by the value a=0, relative to which torque/moment \overline{M}_2 stabilizes housing, will be the position of stable equilibrium.

If the phenomenon indicated is is examined only in statics, then for torque/moment M_2 it is possible (with low a) to accept

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dependence $M_i = Sqlm_i^{\alpha}$ (see Chapter II, §4). Based on this expression, we find the sign/critericn, which shows the character of action \overline{M}_i on rocket or projectile.

For the tilting moment whose sign coincides with angle α , we have $m_s^* > 0$; for stabilizing moment $m_s^* < 0$. Derivative m_s^* is connected with lift coefficient by the relationship/ratio

 $m_z^* \approx c_y^* \frac{l_{u,u} - l_{u,1}}{l}, \qquad (8.1)$ where 1 - a overall length of rocket.

 $l_{u.M}$ and $l_{u.\pi}$ - respectively distance from the apex/vertex of rocket to the center of masses and center of pressure (see Fig. 8.3 and 8.4).

Since $c_{\mu}^{*} > 0$, then, obviously, the sign of derivative coincides with the sign of difference $l_{n,\mu} - l_{n,n}$. Based on this, the conditions of the static stability of rocket and projectile can be formulated as follows:

 $l_{n,n} - l_{n,n} < 0$ - rocket or the projectile "statically" stable;

 $l_{n,n} - l_{u,n} > 0$ - rocket or projectile are unstable;

 $l_{u,u} - l_{u,u} = 0$ - rocket or projectile they are located in the state of neutral equilibrium.

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FIGE 20601



Fig. 8.3. Pattern of the action of aerodynamic moment on finless rocket (projectile).

Fig. 8.4. Pattern of action of aerodynamic moment on fin-stabilized rocket (projectile).

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During the motion of projectile along trajectory, the velocity of its flight and crientation relative to velocity vector continuously change, which leads to a change in the position of ts.d. relative to housing. Furthermore, on powered flight trajectory as a result of the high fuel consumption in the engine operation the center of mass of the rocket also is displaced from its initial position. These reasons can cause a considerable change in value $l_{n,m} - l_{n,n}$ and, therefore, in the derivative $m_{j_1}^*$ that determines the stability level of rocket. Fased on this, it follows that for

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providing the angular statilization of rocket in flight it is necessary that along an entire trajectory would be fulfilled condition $m_i^* < 0$ inequality $|m_i^*| \ge |m_{tann}^*|$.

In other words, stabilized by rocket fin or projectile must possess the so-called factor of "static" stability. The factor of "static" stability they usually characterize by expressed in percentages value $|m_{e}'|$, which is located from the relationship/ratio

 $\left| m_{x}^{c} r \right| = \frac{\left| m_{x}^{a} \right|}{c_{\mu}^{a}} = \frac{\left| l_{u,u} - l_{u,u} \right|}{l} \quad 100^{\circ} i = \left| c_{u,u} - c_{u,u} \right| \quad 100^{\circ} i, \quad (8, 2)$ where $c_{u,u}$ - a center-ot-pressure coefficient;

Cum - the coefficient of the center of mass.

It is customary to assume that the unguided fin-stabilized rockets and projectiles are well stabilized, if they possess the factor of "static" stability, equal to $|m_{x}^{e}| = (10 - 15)$ %.

We investigate the character of the motion of the unguided "statically" stable projectile of its relatively center of mass, assuming that it is completed only in range plane. For simplicity let us examine the trajectory phase, during motion along which it is possible to consider constant m;, and also 6=const. In this case without the account of damping, the equation of motion relative to the center of mass will be written in the form

(8.3)

 $\frac{n^2a}{da} + n^2a = 0,$

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where $n^{0} = \frac{M_{z}}{J_{z}a} = \frac{Sql}{J_{z}} |m_{z}^{*}|$ - the coefficient which can be designed previously along the known trajectory of the center of mass of the fin-stabilized projectile.

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Set/assuming within the limits of the small phase of trajectory $n^2=ccnst$, for initial conditions when $t_{\rm H}=0, a=a_{\rm H}$ and $a=a_{\rm H}^2$ we will obtain the solution of equation (8.3) in the form

$$\alpha = \alpha_{\rm H} \cos nt + \frac{\dot{\alpha}_{\rm H}}{n} \sin nt. \qquad (8.4)$$

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By simple conversions this solution car be given to the more convenient form

$$a = a_m \sin(mt + \epsilon), \qquad (8.5)$$

where phase shift

$$\mathbf{s} = \operatorname{arctg} \frac{n \mathbf{a}_{\mathrm{H}}}{\mathbf{a}_{\mathrm{H}}} \,. \tag{8.6}$$

Solution shows that with the adopted assumptions the motion "statically" stable fin-statilized projectile relative to the center of mass represents by itself the flat/plane harmonic oscillations, which are characterized by the amplitude

$$\alpha_m = \sqrt{\left(\frac{\dot{\alpha}_n}{n}\right)^2 + \alpha_n^2} \tag{8.7}$$

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and by period T=2*/n.

The curve/graphs of a change in the angle a are in connection with the obtained solution represented in Fig. 8.5.

During the analysis of the cscillatory motion of finned "statically" stable rockets and projectiles, we did not consider damping moment \overline{M}_{01} . The rature of this torque/moment was examined earlier (chapter II, §4) and let us here note just action \overline{M}_{01} leads to the rapid attenuation of the oscillatory motion, caused by initial disturtances and torque/moment \overline{M}_{11} . The character of a change in angle of a upon consideration of camping shows in Fig. 8.5 curve/graph, carried out by fine/thin line.

Thus, in the dense layers of the atmosphere rin-stabilized rockets and projectiles move over trajectory so that their longitudinal axis smoothly "follows" vector \vec{v} until any disturbance/perturbations again excite the oscillations of rockets.

Let us examine attitude control by rotation. During motion along the trajectory of the projectile, which rapidly rotates relative to their longitudinal axis, aerodynamic forces, creating torque/moment, attempt to invert projectile, but it, as gyroscope, it is not inverted, but it moves stable. The longitudinal axis of projectile, COC = 78107111

PAGE 1.05

"following" the tangent to trajectory, cscillates in the process of soving the relatively dynamic axis of equilibrium. Artillery projectile obtains rotation in bore during the motion of the driving tand of projectile along screw-shared threads. The rockets, which are stabilized by rotation, are called the spin-stabilized missiles (TRS); they notate because of the cutflow of gas behind oblique nozzles.

The angular rate of rotation of artillery shell or TRS must be calculated so that during the motion along trajectory its longitudinal axis continuously "would follow" the direction of the motion of the center of mass of projectile, differing from the latter within the limits of the permissible angles.

Faç€ 307.

The stabilization of the guided missiles and projectiles is realized because of the active control forces, capable, besides the conduct of rocket along programmed trajectory of the trajectory of induction, to parry the complex system of the disturbance/perturbations, which act on rocket in flight. For producing of control forces, flight vehicles are supplied with controls (chapter II, §6). The guided missile can not possess the necessary steady-state stability factor, but because of the operation CC = 78107111

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cf the complex of the control system is well stabilized and execute the predetermined trajectory of motion. The composite examination of the stability conditions of the motion of rocket and control system is the object/subject of the study of the dynamics of the control systems by unmanned flight vehicles [19, 36, 37].

Buring ballistic calculations the study of stability of motion of rockets and projectiles is conducted in two ways.

First, it is possible to comprise and to solve the complete system of differential equations, which describes the flight of flight vehicle taking into account the action of all forces, including disturbing, which can cause the incorrect flight of flight vehicle. After its solution by the obtained motion characteristics, they judge the stability of flight vehicle in flight. This path, although it is theoretically strict however not always is utilized in practice due to the impossibility to have the comprehensive data about all perturbation factors, for example, action of wind gusts, eccentricity of thrust, etc.

In the second place, an investigation for stability of motion can be carried out, after comprising the differential equations of the deviations of trajectory elements from the calculated and performing analysis of these equations, without examining directly DCC = 78107111 EIGE 26

the action of the perturbation factors.

In the theory of the analysis of stability of flight vehicles wide distribution obtained the method of the slight disturbances. In this the method of the deviation of the parameters of the disturbed motion from that not disturbed they are accepted as so low that in the equations of the disturbed motion these parameters can be represented in the form of the sums, which contain the deviations of the parameter only to the first degree.

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Fig. 8.5. Curve/graphs of a change in the argle of attack α (t) (or δ (t)): a) without damping: 1) during damping of angular motion.

Fage 308.

For example, the velocity, angle of attack and fitch angle in the disturted motion can be represented thes:

 $v = v_* + \Delta v_*$ $a = a_* + \Delta a_*$ $\vartheta = \vartheta_* + \Delta \vartheta_*$ (8.8)

where $\mathbf{v}_{\mathbf{v}}$ $\mathbf{v}_{\mathbf{v}}$ and $\mathbf{\delta}_{\mathbf{x}}$ - velocity, angle of attack and pitch angle in the undisturbed motion whose characteristics are determined without the account of the action of the perturbation factors, and $\Delta \mathbf{v}$, $\Delta \alpha$ and $\Delta \mathbf{\vartheta}$ - deviation of these trajectory elements, citaired as a result of acting the perturbation factors.

Thus, perturbing forces themselves and torque/moments and the mechanism of their action on flight vehicle are not examined. Is studied only a change in the deviations of motion characteristics already after the action of the perturbation factors on the

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assumption that these deviations are low. The method of the slight disturbances makes it possible the equations of the disturbed motion of rocket to reduce to the linear differential equations, solved relatively simply.

§2. Linearization of the equations of motion of rockets and of projectiles.

The mathematical sense of linearization lies in the fact that the unknown deviation of cell/element is located by the way of expansion of the corresponding to it function in Taylor series in terms of the degrees of deviation of cell/element. Recall the formula of expansion in Taylor series for functioning many arguments $f(\xi_1, \xi_2, ..., \xi_n)$, since to of this type to functions are related the trajectory elements of rockets. During the writing of the results of expansion, we will use the conventional designations of total differentials of the function of many variables

 $d^{m}A = \left(\frac{\partial}{\partial \xi_{1}} d\xi_{1} + \frac{\partial}{\partial \xi_{2}} d\xi_{2} + \ldots + \frac{\partial}{\partial \xi_{n}} d\xi_{n}\right)^{m} f(\xi_{1}, \xi_{2}, \ldots, \xi_{n}).$

The formula of expansion takes the form

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 $f(\xi_{1}, \xi_{2}, \dots, \xi_{n}) = f(\xi_{10} + \delta\xi_{1}, \xi_{20} + \delta\xi_{2}; \dots \xi_{n0} + \delta\xi_{n}) =$ $= f(\xi_{10}, \xi_{20}, \dots, \xi_{n0}) + \frac{1}{1!} \left(\frac{\partial}{\partial \xi_{10}} \delta\xi_{2} + \frac{\partial}{\partial \xi_{20}} \delta\xi_{2} + \dots + \frac{\partial}{\partial \xi_{n0}} \delta\xi_{n} \right) \times$ $\times f(\xi_{10}, \xi_{20}, \dots, \xi_{n0}) + \frac{1}{2!} \left(\frac{\partial}{\partial \xi_{10}} \delta\xi_{1} + \frac{\partial}{\partial \xi_{20}} \delta\xi_{2} + \dots + \frac{\partial}{\partial \xi_{n0}} \delta\xi_{n} \right)^{2} \times$ $\times f(\xi_{10}, \xi_{20}, \dots, \xi_{n0}) + \frac{1}{3!} \left(\frac{\partial}{\partial \xi_{10}} \delta\xi_{1} + \frac{\partial}{\partial \xi_{20}} \delta\xi_{2} + \dots + \frac{\partial}{\partial \xi_{n0}} \delta\xi_{n} \right)^{3} \times$ $\times f(\xi_{10}, \xi_{20}, \dots, \xi_{n0}) + R, \qquad (8.9)$

where &. - calculated (nominal) values of the determining parameters.

The deviation of function $I(\xi_1, \xi_2, ..., \xi_n)$, caused by the deviations of the parameters from computed values $\delta \xi_1, \delta \xi_2, ..., \delta \xi_n$, will be equal

$$\delta f(\xi_1, \xi_2, ...; \xi_n) = f(\xi_{10} + \delta \xi_1, \xi_{20} + \delta \xi_2 + ... + \xi_{n0} + \delta \xi_n) - - f(\xi_{10}, \xi_{20}, \xi_{20}, ..., \xi_{n0}).$$
(8.10)

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The first term of expansion (E.9) and the second (d.10) are equal, have different signs and will be shortered; therefore common/general/total formula for the deviation of function $f(\xi_1, \xi_2, ..., \xi_n)$ can be obtained in the following form:

 $\delta f(\xi_{3}\xi_{2},\ldots,\xi_{n}) = \frac{1}{1!} \left(\frac{\partial}{\partial\xi_{10}} \delta\xi_{2} + \frac{\partial}{\partial\xi_{20}} \delta\xi_{2} + \ldots + \frac{\partial}{\partial\xi_{n0}} \delta\xi_{n} \right) \times .$ $\times f(\xi_{10},\xi_{20},\ldots,\xi_{n0}) + \frac{1}{2!} \left(\frac{\partial}{\partial\xi_{10}} \delta\xi_{1} + \frac{\partial}{\partial\xi_{20}} \delta\xi_{2} + \ldots + \frac{\partial}{\partial\xi_{n0}} \delta\xi_{n} \right)^{2} \times f(\xi_{10},\xi_{20},\ldots,\xi_{n0}) + \frac{1}{3!} \left(\frac{\partial}{\partial\xi_{10}} \delta\xi_{1} + \frac{\partial}{\partial\xi_{20}} \delta\xi_{2} + \ldots + \frac{\partial}{\partial\xi_{n0}} \delta\xi_{n} \right)^{3} \times f(\xi_{10},\xi_{20},\ldots,\xi_{n0}) + R. \qquad (8.11)$

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The number of terms of expansion, held in calculations, depends on the required accuracy of the determination of deviation. Most frequently during the solution of the practical problems of external hallistics, hold only linear terms of expansion. In this case formula (8.11) will take the form

$$\delta f(\xi_1, \xi_2, \dots, \xi_n) = \left(\frac{\partial f}{\partial \xi_1}\right)_{\bullet} \delta \xi_1 + \left(\frac{\partial f}{\partial \xi_2}\right)_{\bullet} \delta \xi_2 + \dots + \left(\frac{\partial f}{\partial \xi_n}\right)_{\bullet} \delta \xi_n.$$
(8.12)

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Let us find expression for the deviation of the derivative of form $\delta d\epsilon/dt$. Since

$$\delta \frac{d\xi}{dt} = \frac{d\xi}{dt} - \left(\frac{d\xi}{dt}\right)_{\bullet}, \ a \ \xi = \xi_{\bullet} + \delta\xi,$$

that we will obtain

$$\delta\left(\frac{d\xi}{dt}\right) = \frac{d}{dt}(\xi_* + d\xi) - \left(\frac{d\xi}{dt}\right)_* = \frac{d}{dt}\,\xi.$$
 (8.13)

Thus, if we have a system of differential equations of the disturbed motion, comprised of n of the equations of the form

$$\frac{d\xi_1}{dt} = f_1(\xi_1, \xi_2, \xi_3, \dots);$$

$$\frac{d\xi_2}{dt} = f_1(\xi_1, \xi_3, \xi_3, \dots);$$
(8.14)

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then on the basis (8.12) and (8.13) it easily is reduced to system of equations in the deviations

$$\frac{d}{dt} \delta\xi_1 = \left(\frac{\partial f_1}{\partial \xi_1}\right)_{\bullet} \delta\xi_1 + \left(\frac{\partial f_1}{\partial \xi_2}\right)_{\bullet} \delta\xi_2 + \left(\frac{\partial f_1}{\partial \xi_3}\right)_{\bullet} \delta\xi_3 + \cdots$$

$$\frac{d}{dt} \delta\xi_2 = \left(\frac{\partial f_2}{\partial \xi_1}\right)_{\bullet} \delta\xi_1 + \left(\frac{\partial f_2}{\partial \xi_2}\right)_{\bullet} \delta\xi_2 + \left(\frac{\partial f_2}{\partial \xi_3}\right)_{\bullet} \delta\xi_3 + \cdots$$
(8.15)

Fage 310.

If the undisturbed motion is known, i.e., cell/elements $\xi_{1,\mu}(t)$; $\xi_{2,\mu}(t)$; $\xi_{3,\mu}(t)$ and others are assigned, then they will be also known in the function of time and the partial derivatives of form $\left(\frac{\partial f_i}{\partial \xi_i}\right)_{e^*}$. which stand in system (8.15) during deviations $\delta\xi_i$ of cell/elements. In this case system (8.15) will represent by itself the system of linear differential equations, since the new variables enter in equations only to the first degree and their cross products of the type $\delta\xi_1\delta\xi_2$ they are absent, while the initial equations of undisturbed and disturbed actions (8.14) linear they are not. ICC = 78107111

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Utilizing a method presented, let us context the linearization of the system of the differential equations, which describe the simple case of the axial motion of the unguided fin-stabilized missile. Let us take the known to us system of differential equations (3.51). Considering that thrust it is directed along the axis of rocket (i.e. after placing angle $\xi=0$) and after accepting $X_{pl}=Y_{pl}=0$, after conversions from (3.51) and (3.52) we will obtain

$$m \frac{dv}{dt} = P \cos a - X - Q \sin \theta;$$

$$mv \frac{d\theta}{dt} = P \sin a + Y - Q \cos \theta;$$

$$J_s \frac{d^{2\theta}}{dt^{2}} = M_s.$$
(8.16)

During linearization we will not consider effect on the disturbed motion characteristics of a change in the mass δm and in the moment of inertia δJ_z , let us consider that the mass and the moment of inertia for the undisturbed and disturbed motions change on time equally: $m(t) = m_+(t)$ and $J_z(t) = J_{z+}(t)$.

Furthermore, let us disregard the effect of the deviation of height/altitude of aerodynamic characteristics and thrust. For low values δy , this effect is unessential, since functions H(y), $\rho(y)$, $\pi(y)$ and a(y), through which it it is exhibited, change slowly.

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During the adopted simplifications in the deviation of aerodynamic forces and torque/moment, they will depend only on two values - deviation of the flight speed by and of the deviation of angle of attack ba. If we designate

 $\begin{aligned} X_* &= f_1(v_*, a_*) & X = f_1(v, a); \\ Y_* &= f_2(v_*, a_*) & \text{and} & Y = f_2(v, a); \\ M_{**} &= f_4(v_*, a_*) & M_s = f_4(v, a), \end{aligned}$

that, expanding last/latter dependences in ϵ series on formula (8.9), we will obtain taking into account only linear terms:

$$\delta X = X - X_{*} - \left(\frac{\partial X}{\partial v}\right)_{*} \delta v + \left(\frac{\partial X}{\partial a}\right)_{*} \delta a;$$

$$\delta Y = Y - Y_{*} = \left(\frac{\partial Y}{\partial v}\right)_{*} \delta v + \left(\frac{\partial Y}{\partial a}\right)_{*} \delta a;$$

$$\delta M_{s} = M_{s} - M_{s*} - \left(\frac{\partial M_{s}}{\partial v}\right)_{*} \delta v + \left(\frac{\partial M_{s}}{\partial a}\right)_{*} \delta a.$$
(8.17)

Mark * shows that the catum is related to unperturbed motion at the torque/moment, which corresponds to the beginning of disturbance/perturbation. Let us introduce the abbreviated recording of the partial derivatives

$$\left(\frac{\partial A}{\partial \mathbf{t}_l}\right)_{\bullet} = A^{\mathbf{t}_l}$$

and let us rewrite formulas (8.17) in this form

 $\delta X = X^* \delta v + X^* \delta a; \quad \delta Y = Y^* \delta v + Y^* \delta a; \quad \delta M_s = M_s^* \delta v + M_s^* \delta a.$

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Let us similarly find deviations for the terms, which contain thrust P:

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 $\delta(P \sin \alpha) = P \cos \alpha_* \delta \alpha; \quad \delta(P \cos \alpha) = -P \sin \alpha_* \delta \alpha.$

Counting the weight of rocket ζ on the section of disturbance/perturbation with constant, we will obtain

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 $\delta(Q\sin\theta) = Q\cos\theta_{\bullet} \delta\theta; \quad \delta(Q\cos\theta) = -Q\sin\theta_{\bullet} \delta\theta.$

Taking into account the obtained expressions for the deviations of forces and torque/moments, it is possible to write:

$$m\left(\frac{dv}{dt} - \frac{dv_{\bullet}}{dt}\right) = \delta(P\cos a) - \delta X - \delta(Q\sin \theta);$$

$$m\left(v\frac{d\theta}{dt} - v_{\bullet}\frac{d\theta_{\bullet}}{dt}\right) = \delta(P\sin a) + \delta Y - \delta(Q\cos \theta);$$

$$J_{z}\left(\frac{d2\theta}{dt^{2}} - \frac{d2\theta_{\bullet}}{dt^{2}}\right) = \delta M_{z}.$$

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Eearing in mind that

$$\frac{\frac{dv}{dt} - \frac{dv_*}{dt}}{\frac{dt}{dt}} = \frac{d}{dt} \delta v;$$

$$v \frac{d\theta}{dt} - v_* \frac{d\theta_*}{dt} = v_* \frac{d}{dt} \delta \theta + \delta v \frac{d\theta_*}{dt} + \delta v \frac{d}{dt} \delta \theta \approx v_* \frac{d}{dt} \delta \theta$$

(without the account of the members of the second order of smallness),

(8.3)

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$$\frac{d^{2\vartheta}}{dt^2} - \frac{d^{2\vartheta}}{dt^2} = \frac{d^2}{dt^2} \delta \vartheta \quad \text{and} \quad \delta \theta = \delta \theta - \delta a,$$

and also accepting in view of the smallness of angle α_{μ} that $\sin \alpha_{\mu} \approx \alpha_{\mu}$ and $\cos \alpha_{\mu} \approx 1$, we will obtain the following system of the linearized equations:

$$m \frac{d \delta v}{dt} = -X^{v} \delta v - (Pa_{*} + X^{\bullet} - Q \cos \theta_{*}) \delta a - Q \cos \theta_{*} \delta \theta;$$

$$m v_{*} \left(\frac{d \delta \theta}{dt} - \frac{d \delta a}{dt}\right) = Y^{v} \delta v + (P + Y^{*} - Q \sin \theta_{*}) \delta a + Q \sin \theta_{*} \delta v;$$

$$J_{z} \frac{d^{2} \delta \theta}{dt^{2}} = M_{z}^{v} \delta v + M_{z}^{\bullet} \delta a.$$
(8.18)

This system consists of linear homogeneous differential equations with the coefficients, which are the known functions of time.

Let us now conduct the linearization of the more complex system of the differential equations, which describe the spatial motion of the rocket of class "surface - surface", controlled in flight by aerodynamic controls. Let us take system (3.25) and let us lead it to the form, more convenient for linearization. Let us assume that the rocket is axisymmetric and thrust is directed along longitudinal axis C_{X_1} ; for an axisymmetric rocket it is possible to also take $J_{\mu_1} = J_{\mu_1}$. DOC = 78107111

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Since the control forces are considerably lesser than the thrust and the aerodynamic forces, it is possible to assume at small angles a and f that

$X_{p1} \cos a \cos \beta \approx X_{p1}$; $Y_{p1} \cos a \approx Y_{p}$; $Z_{p1} \cos \beta \approx Z_{p}$.

Let us introduce into system of equations perturbing forces and the torque/moments, which act on rocket in flight and calling its deviation from the undisturked trajectory - X_y , Y_y , Z_y , $M_{y,v}$ and $M_{z,v}$. Subsequently let us consider them the knowr functions of time. Furthermore, in system (3.29) the trigonometric relationship/ratios between angles θ , ϑ , α and ψ , Ψ and f let us replace more convenient for linearization.

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After conversions the system of equations will take the form

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$$\begin{aligned}
\vec{v} &= \frac{P \cos a \cos \beta}{m} - \frac{X + X_p}{m} - g \sin \theta + \frac{X_s}{m}; \\
\dot{\theta} &= \frac{P \sin a}{mv} + \frac{Y + Y_v}{mv} - \frac{g \cos \theta}{v} + \frac{Y_s}{mv}; \\
\dot{\Psi} &= \frac{P \cos a \sin \beta}{mv \cos \theta} - \frac{Z + Z_p}{mv \cos \theta} + \frac{Z_s}{mv \cos \theta}; \\
\dot{x}_3 &= v \cos \theta \cos \Psi; \quad \dot{y}_3 &= v \sin \theta; \quad \dot{z}_3 &= -v \cos \theta \sin \Psi; \\
\dot{w}_{y_1} &= \frac{M_{y_1} + M_{pv_1}}{J_{y_1}} + \frac{J_{z_1} - J_{x_1}}{J_{y_1}} \omega_{x_1} \omega_{z_1} + \frac{M_{y_1}n}{J_{y_1}}; \\
\dot{w}_{z_1} &= \frac{M_{z_1} + M_{pz_1}}{J_{z_1}} - \frac{J_{y_1} - J_{x_1}}{J_{z_1}} \omega_{x_1} \omega_{y_1} + \frac{M_{z_1}n}{J_{z_1}}; \\
\dot{\theta} &= \omega_{z_1}; \quad \dot{\psi} &= \omega_{y_1} \frac{1}{\cos \theta}; \quad \dot{\omega}_{x_1} &= \omega_{y_1} \operatorname{tg} \theta; \\
\sin \theta &= \cos a \cos \beta \sin \theta - \sin a \cos \beta \cos \theta; \\
\sin \Psi \cos \theta &= \cos a \cos \beta \sin \phi \cos \theta + \\
+ \sin a \cos \beta \sin \phi \sin \theta - \sin \beta \cos \phi.
\end{aligned}$$

During linearization, as before let us accept $\mathbf{m}(t) = \mathbf{m}_{\mathbf{w}}(t)$, $J_{w_1}(t) = J_{w_{10}}(t)$, $J_{x_1}(t) = J_{x_{10}}(t)$, we will not consider effect by on a change in the aerodynamic forces and thrust, let us introduce the abbreviated recording of derivatives and will drop/omit for simplicity of writing mark of the coefficients during deviations. During the writing of members, who contain the characteristics of aerodynamic control forces and torque/moments, we will use dependences (2.133), (2.134), (2.135) and the abbreviated form of the recording of derivatives, the angles of deflection of controls let us designate $\Delta \delta_{x_1}, \Delta \delta_{y_1}$ and $\Delta \delta_{x_1}$.

(8.19)

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The linearization of the written system of equations is

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conducted on common/general/total formula (E.15).

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Taking into account the large volume of the systematic recording of the process of linearization, let us give here without conclusion/derivation the result of the linearizations of system (8.19), obtained by Yu. F. Kol*tsov [18]:

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$$\frac{d}{dt} \delta v = -\frac{X^{t} + X_{p}^{v}}{m} \delta v - g \cos \theta \delta \theta - \frac{P \sin a \cos \beta + X^{a}}{m} \delta a - \frac{P \cos a \sin \beta}{m} \delta \beta - \frac{X_{p}^{\delta_{x_{1}}}}{m} \Delta \delta_{x_{1}} + \frac{X_{s}}{m};$$

$$\frac{d}{dt} \delta \theta = \frac{1}{mv} \left(Y^{v} + Y_{p}^{v} - \frac{P \sin a + Y + Y_{p} - mg \cos \theta}{v} \right) \delta v + \frac{g \sin \theta}{v} \delta \theta + \frac{P \cos a + Y^{a}}{mv} \delta a + \frac{Y_{p}^{\delta_{x_{1}}}}{mv} \Delta \delta_{x_{1}} + \frac{Y_{s}}{mv};$$
(8.20)

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$$= \int \frac{1}{4\pi} V = \int \frac{1}{\pi r \cos 1} \left(z^{2} + z^{2}_{3} + \frac{P \cos \alpha \sin \beta - z - z^{2}_{3}}{\pi r \cos \theta} \right) \Delta v + \\ + \nabla V_{12} \log \frac{P \sin \alpha \sin \beta}{\pi r \cos \theta} \log + \frac{P \cos \alpha \cos \beta - z^{2} - z^{2}_{3}}{\pi r \cos \theta} \log \frac{1}{2} \log \frac{P \cos \alpha \cos \beta}{\pi r \cos \theta} \log \frac{1}{2} \log$$

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The analysis of the system of the linearized equations, carried out by Yu. P. Kol'tsov, showed that system (8.20) can be considerably simplified. Simplifications these are based on what for the rockets cf the class in question in the undisturbed motion lateral kinematic cell/elements $\Psi_{\bullet}, \Psi_{\bullet}, \Psi_{\bullet}, \Psi_{\bullet}, \bullet, \bullet_{\nu}, \bullet$ deflectice $\delta_{\nu_1} \bullet$ of rudders, governing yawing motion, the value of the angles of attack α_{ν} and of angular velocity w2, ., and time derivatives of the cell/elements of the lcngitudinal and yawing action of rocket $\Psi_{*}, \psi_{*}, \phi_{*}, \phi_{*}, \delta_{z_{1}}$ are also so low that it is possible to disregard the products of these cell/elements and other low values. Sc, in equation $\frac{d}{dt} \delta \Psi$, it is ressible for this reason to disregard terms Ψ to the and $\frac{P \sin \alpha \sin \beta}{m \nu \cos \theta}$ ba. in equation $\frac{d}{dt}\delta x_3$ - by term $v\cos\theta\sin\Psi\delta\Psi$, in equation $\frac{d}{dt}\delta z_3$ - by terms costsin Wav and vsintsin W60, in equation $\frac{d}{dt} \delta \omega_{\nu_1}$ - by terms $\frac{M_{\nu_1}^o + M_{\nu_2}^o}{J_{\nu_1}} \partial v, \frac{J_{x_1} - J_{x_1}}{J_{\nu_1}} \oplus_{x_1} \partial w_{x_1} \text{ and so forth. Furthermore, during}$ simplifications on the basis of smallness q_{μ} , β_{μ} , Ψ_{\bullet} , Ψ_{\bullet} , was conducted the replacement of trigcncmetric functions from these angles by their approximate values, i.e.,

 $\sin \alpha_{\phi} = \alpha_{\phi} = \alpha; \quad \sin \beta_{\phi} = \beta_{\phi} = \beta,$ $\cos^{2}_{\alpha_{\phi}} = \cos \beta_{\phi} = \cos \Psi_{\phi} = 1 \text{ and } \text{ sc forth.}$ EOC = 78107111

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Euring simplification in the construct relationship/ratios between the deviations of angles θ and θ , Ψ and ψ it was also accepted that cos0#cos0.

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As a result of the analysis indicated which we here do not bring in detail, in the equations of system (8.20) can be reject/thrown a series of the terms the order of smallness of which higher than first, after which system will take the following simplified form:

$$\frac{d}{dt} \delta \theta = \frac{Y^{t} + Y_{p}^{b}}{mv} \delta v + \frac{g \sin \theta}{v} \delta \theta \div \frac{P + Y^{a}}{mv} \delta a + \\
+ \frac{Y_{p}^{b_{z_{1}}}}{mv} \Delta \delta_{z_{1}} + \frac{Y_{a}}{mv}; \\
\frac{d}{dt} \delta x_{3} = \cos \theta \delta v - v \sin \theta \delta \theta; \quad \frac{d}{dt} \delta y_{3} = \sin \theta \delta v \div v \cos \theta \delta \theta; \\
\frac{d}{dt} \delta w_{z_{1}} = \frac{M_{z_{1}}^{t} + M_{p}^{b}z_{1}}{J_{z_{1}}} \delta v \div \frac{M_{z_{1}}^{a}}{J_{z_{1}}} \delta a + \frac{M_{pz_{1}}^{b_{z_{1}}}}{J_{z_{1}}} \Delta \delta_{z_{1}} + \frac{M_{z_{1}}^{w}}{J_{z_{1}}} \delta w_{z_{1}} + \\
- \frac{M_{z_{1}}^{b}}{J_{z_{1}}} \delta a + \frac{M_{pz_{1}}^{b}}{J_{z_{1}}} \Delta \delta_{z_{1}} + \frac{M_{z_{1}}^{b}}{J_{z_{1}}}; \\
\frac{d}{dt} \delta \theta = \delta w_{z_{1}}; \quad \delta a = \delta \theta - \delta \theta; \\
\frac{d}{dt} \delta \Psi = \frac{P - Z^{\beta}}{mv \cos \theta} \delta \beta - \frac{Z^{b} v}{mv \cos \theta} \Delta \delta_{v_{1}} + \frac{Z_{n}}{mv \cos \theta}; \\
\frac{d}{dt} \delta z_{3} = -v \cos \theta \delta \Psi; \\
\frac{d}{dt} \delta z_{3} = -v \cos \theta \delta \Psi; \\
\frac{d}{dt} \delta w_{v_{1}} = \frac{M_{v_{1}}^{b}}{J_{v_{1}}} \delta \beta + \frac{M_{p}^{b} v}{J_{v_{1}}} \Delta \delta_{v_{1}} + \frac{M_{v_{1}}^{w}}{J_{v_{1}}} \Delta w_{x_{1}} + \frac{M_{v_{1}}^{w}}{J_{v_{1}}} \delta w_{v_{1}} + \\
+ \frac{M_{v_{1}}^{b}}{J_{v_{1}}} \delta \beta + \frac{M_{p}^{b} v}{J_{v_{1}}} \Delta \delta_{v_{1}} + \frac{M_{v_{1}}^{w}}{J_{v_{1}}} \delta w_{v_{1}} + \\
\frac{d}{dt} \delta \phi = \frac{1}{\cos \theta} \delta w_{v_{1}}; \quad \delta w_{x_{1}} = tg \delta \delta w_{v_{1}}; \quad \delta \beta = \cos \theta \delta \phi - \cos \theta \delta \Psi.
\end{cases}$$
(8.22)

The common/general/total system of the nonlinear differential equations of motion of the class of rockets in guestion as a result of its linearization and simplifications decomposes into two independent systems of linear differential equations of motion in

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deviations, moreover one of these systems (t.21) describes the longitudinal disturbed motion of rocket, which cocurs in plane $0x_3y_3$, and another (8.22) - the lateral disturbed motion in inclined plane $0xz_3$.

It is obvious that two independent systems of linear equations can be solved considerably simpler than single reference system from nonlinear differential equations and geometric relationship/ratios, especially if will be used the mathematical calculating and analog computers. However, it is clear that simplicity of solution is reached to the detriment its accuracy.

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Consequently, during the use of a method of equations of motion at tasis of which lie/rests the condition of sufficient smallness of disturbance/perturbations, it is necessary to know, which the accuracy of those obtained by the calculation of results, or, otherwise, by which limits must be limited the disturbance/perturbations being investigated, so that the errors for calculation according to by equations in deviations would not exceed permissible. The comprehensive answer/response to this question can be obtained by the comparison of results the approximate and exact solutions, however, taking into account the already noted labor DCC = 78107111

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expense the latter, this method of evaluating the accuracy it does not have extensive application.

Therefore are frequently utilized less strict, but simpler indirect, or those approximated, the methods of estimation of error. One Of them is the evaluation of sum R of terms of expansion, rejected during the linearization of initial equations, with the aid of which it is possible to tentatively establish/install the permissible region of disturbance/perturbations even prior to accomplishing of calculations.

Let us find, for example, which error we allow/assume during the linearization of the equation θ of system (8.19) in it component Y/mv. Let us preserve in the common/general/total formula of expansion (8.9) only linear terms, after designating through R permanent. Then

$$R = f(\xi_{1*} + \partial \xi_{1}; \quad \xi_{2*} + \partial \xi_{2} \dots, \xi_{n*} + \partial \xi_{n}) - - f(\xi_{1*}, \xi_{2*}, \dots, \xi_{n*}) - \left(\frac{\partial f}{\partial \xi_{1}}\right)_{\bullet} \partial \xi_{1} - - \left(\frac{\partial f}{\partial \xi_{2}}\right)_{\bullet} \partial \xi_{2} - \dots - \left(\frac{\partial f}{\partial \xi_{n}}\right)_{\bullet} \partial \xi_{n}.$$
(8.23)

Assuming that Y changes only because of a change in velocity and angle of attack, we will obtain

$$R_{\gamma} = \frac{Y}{mv} - \frac{Y_{\bullet}}{mv_{\bullet}} - \left(\frac{Y}{mv}\right)_{\bullet}^{\bullet} \delta v - \left(\frac{Y}{mv}\right)_{\bullet}^{\bullet} \delta a =$$
$$= \frac{Sqe_{\mu}^{\bullet}}{2m} \left\{ \left[(a_{\bullet} + \delta a) \delta v + v_{\bullet} \delta a \right] - \left[a_{\bullet} \delta v + v_{\bullet} \delta a \right] \right\}$$

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Hence

$$R_{\gamma} = \frac{Sqc_{\gamma}^{a}}{2m} b \sigma b a = \frac{Y_{\bullet}}{mv_{\bullet}} \frac{bv}{v_{\bullet}} \frac{ba}{a_{\bullet}}.$$



Fig. 8.6.

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Fig. 8.6. Range of the allowed values of the ratios $\delta \alpha / \alpha_{*}$ and $\delta v / y_{*}$ which corresponds to relative error e=0.03

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Consequently, during linearization we allow/assume relative error e_i equal to

$$\varepsilon = \frac{R_Y}{Y_{\bullet} m v_{\bullet}} = \frac{bv}{v_{\bullet}} \frac{ba}{a_{\bullet}} . \qquad (8.24)$$

Even g assigned by the values of permissible error ε , it is possible to construct on dependence (8.24) the curve/graphs which will restrict the range of the allowed values of deviations δv and δa . This range for $\varepsilon = 0.03$ is given to Fig. 8.6 and it shows that the allowed values $\delta \alpha / \alpha_x$ are great with low $\varepsilon v / v_x$ and decrease with increase $\delta v / v_x$.

After conducting this type of analysis and for other terms of initial equations, it is possible to establish/install the generalized limiting values of the disturbance/perturbations within which the accuracy of calculation according to the linearized equations will be satisfactory. CCC = 78107111

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§3. Methods of the solution of also of the study of the linearized equations of the disturked motion of rockets and of projectiles.

The systems of equations of the longitudinal (8.21) and lateral (8.22) disturbed motion of lockets for their aralysis and solutions it is accepted to record/write in a simpler form, introducing the abbreviations of those stand during the deviations of the ccefficients which are called dynamic. The cynamic coefficients, entering the equations of axial motion, let us designate a_{ij} ; those entering the equations of yawing motion - b_{ij} , utilizing in this case not the digital indices, but literal, since they are more demonstrative. The first index (i) designates the equation which includes the coefficient: the second (j) - the deviation, during which it stands. Thus, for instance, coefficient a_{r+1} is related to the system of equations of axial motion and stands in equation for the calculation of the deviation of the velocity of the motion of the rocket δv during the deviation of the flight path angle $\delta \theta$. For the terms, which reflect in equations the perturbation factors, let us accept designations $\frac{X_{s}}{m} = f_{\varphi}(t); \frac{M_{z_{1}s}}{J_{z_{1}}} = f_{w_{z_{1}}}(t)$ and so forth. With the even number of this, the examine/considered by us systems of equations accept following form.

1. Equations of longitudinal disturbed motion

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$$\frac{d}{dt} \delta v = a_{yy} \delta v + a_{yi} \delta \theta + a_{yz} \delta a + a_{yz} \Delta \delta_{z_1} + f_y(t);$$

$$\frac{d}{dt} \delta \theta = a_{\theta v} \delta v + a_{\theta \theta} \delta \theta + a_{\theta z} \delta a + a_{b\delta z_1} \Delta \delta_{z_1} + f_y(t);$$

$$\frac{d}{dt} \delta x_3 = a_{xv} \delta v + a_{x\theta} \delta \theta; \quad \frac{d}{dt} \delta y_3 = a_{yv} \delta v + a_{y\theta} \delta \theta;$$

$$\frac{d}{dt} \delta w_{z_1} = a_{wz1} \delta v + a_{wz1} \delta a + a_{wz1} \delta_{z_1} + a_{wz1} \delta_{z_1} + a_{wz1} \delta a_{z_1} + a_{wz1} \delta a + a_{wz1} \delta a_{z_1} + f_{wz1}(t);$$

$$\frac{d}{dt} \delta \theta = \delta w_{z_1}; \quad \delta a = \delta \theta - \delta \theta.$$
(8.25)
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The entering these equations dynamic coefficients are respectively equal to

$$a_{vv} = -\frac{X^{v} + X_{p}^{v}}{m}; \qquad a_{vi} = -y \cos \theta;$$

$$a_{va} = -\frac{Pa + X^{a}}{m}; \qquad a_{vi_{21}} = -\frac{X_{p}^{i_{21}}}{m};$$

$$a_{iv} = \frac{Y^{v} + Y_{p}^{v}}{mv}; \qquad a_{iv} = \frac{y \sin \theta}{v};$$

$$a_{ia} = \frac{P + Y^{a}}{mv}; \qquad a_{ik_{21}} = \frac{Y_{p}^{i_{21}}}{mv};$$

$$a_{va} = \cos \theta; \qquad a_{xb} = -v \sin \theta;$$

$$a_{yv} = \sin \theta; \qquad a_{yv} = v \cos \theta;$$

$$a_{w_{21}v} = \frac{M_{z_{1}}^{v} + M_{p,z_{1}}^{v}}{J_{z_{1}}}; \qquad a_{w_{21}a} = \frac{M_{z_{1}}^{i_{21}}}{J_{z_{1}}}; \qquad a_{w_{21}b_{21}} = \frac{M_{p,z_{1}}^{i_{21}}}{J_{z_{1}}};$$

$$a_{w_{21}v} = \frac{M_{z_{1}}^{v} + M_{p,z_{1}}^{v}}{J_{z_{1}}}; \qquad a_{w_{21}a} = \frac{M_{z_{1}}^{i_{21}}}{J_{z_{1}}}; \qquad a_{w_{21}b_{21}} = \frac{M_{p,z_{1}}^{i_{21}}}{J_{z_{1}}}.$$
(8.26)

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2. Equations of lateral disturted motion

$$\frac{d}{dt} \partial \Psi = b_{\Psi y} \partial \beta + b_{\Psi i y 1} \Delta \delta_{y_1} + f_{\Psi}(t);$$

$$\frac{d}{dt} \partial z_3 = b_z \psi \partial \Psi;$$

$$\frac{d}{dt} \partial \omega_{y_1} = b_{w y 1} \partial \beta + b_{w y 1} b_{y 1} \Delta \delta_{y_1} + b_{w y 1} w_{x 1} \partial w_{x_1} + b_{w y 1} w_{y 1} \partial w_{y_1} + b_{w y 1} \partial \delta_{y_1} + f_{w y 1}(t);$$

$$\frac{d}{dt} \partial \Psi = b_{\psi w y 1} \partial w_{y_1}; \quad \partial w_{x_1} = tg \partial \partial w_{y_1};$$

$$\partial \beta = \cos \partial \partial \phi - \cos \partial \partial \Psi.$$
(8.27)

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Dynamic coefficients in these equations have the values

$$b_{\Psi\beta} = \frac{P - Z^{\beta}}{mv\cos\theta}; \qquad b_{\Psi\delta_{y1}} = -\frac{Z_{y}^{\delta_{y1}}}{mv\cos\theta}; \\ b_{z} = -v\cos\theta; \qquad b_{\psi\omega_{y1}} = \frac{1}{\cos\theta}; \\ b_{\omega_{y1}} = \frac{M_{y_{1}}^{\beta}}{J_{u_{1}}}; \qquad b_{\omega_{y1}\delta_{y1}} = \frac{M_{p,u_{1}}^{\delta_{y_{1}}}}{J_{u_{1}}}; \qquad b_{\omega_{y1}\omega_{x1}} = \frac{M_{p,u_{1}}^{\omega_{x_{1}}}}{J_{u_{1}}}; \\ b_{\omega_{y1}\omega_{y1}} = \frac{M_{y_{1}}^{\omega_{y_{1}}}}{J_{u_{1}}}; \qquad b_{\omega_{y1}\delta_{y1}} = \frac{M_{p,u_{1}}^{\beta}}{J_{u_{1}}}; \qquad b_{\omega_{y1}\omega_{y1}} = \frac{M_{p,u_{1}}^{\delta_{y_{1}}}}{J_{u_{1}}}; \\ b_{\omega_{y1}\omega_{y1}} = \frac{M_{y_{1}}^{\omega_{y_{1}}}}{J_{u_{1}}}; \qquad b_{\omega_{y1}\delta_{y1}} = \frac{M_{p,u_{1}}^{\beta}}{J_{u_{1}}}; \qquad b_{\omega_{y1}\delta_{y1}} = \frac{M_{p,u_{1}}^{\delta_{y_{1}}}}{J_{u_{1}}}. \end{cases}$$

Subsequently to avoid repetitions whole presentation let us conduct in connection with the system of equations of axial motion, since the methods of its solution and analysis as the results of this analysis, in essence the same as for the system, which describes the

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lateral disturbed mcticr.

From the examination of system of equations (8.25) it follows that in it the together solved equations are the dynamic and geometric equations, i.e.,

$$\frac{d}{dt} \delta v = a_{vv} \delta v + a_{vi} \delta \theta + a_{vv} \delta a + a_{vv\delta_{21}} \Delta \delta_{z_1} + f_v(t);$$

$$\frac{d}{dt} \delta \theta = a_{\theta v} \delta v + a_{i\theta} \delta \theta + a_{iu} \delta a + a_{\theta \delta_{21}} \Delta \delta_{z_1} + f_{\theta}(t);$$

$$\frac{d}{dt} \delta \omega_{z_1} = a_{\omega_{21}v} \delta v + a_{\omega_{21}} \delta a + a_{\omega_{21}\delta_{21}} \Delta \delta_{z_1} + a_{\omega_{21}\omega_{21}} \delta \omega_{z_1} + a_{\omega_{21}v} \delta a + a_{\omega_{21}\delta_{21}} \delta \delta_{z_1} + f_{\omega_{21}}(t);$$

$$\frac{d}{dt} \delta \theta = \delta \omega_{z_1}; \quad \delta a = \delta \theta - \delta \theta.$$
(8.29)

The supplementary dependence, necessary for the calculation of value δa , entering the equation for determination $\delta \omega_{s_1}$, is obtained by differentiation with respect to the time of geometric relationship/ratio $\delta a = \delta \delta - \delta \delta$.

After making this process/operation, we will obtain

$$\frac{d}{dt} \, \delta a = \frac{d}{dt} \, \delta \vartheta - \frac{d}{dt} \, \delta \vartheta,$$

whence finally it follows

$$\frac{d}{dt}\mathbf{b}\mathbf{a} = \mathbf{b}\frac{d\mathbf{a}}{dt} = \mathbf{b}\mathbf{a} = \frac{1}{dt}\mathbf{b}\mathbf{b} - \frac{d}{dt}\mathbf{b}\mathbf{b}.$$
 (8.30)

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The kinematic relationship/ratios

and

$$\frac{d}{dt} \delta y_3 = a_{yy} \delta v + a_{yy} \delta v$$

with the made by us assumptions in the course of the solution of system of equations (8.29) are not utilized; they are integrated after the solution of this system.

The initial space of the solution of system of equations (8.29) is the determination of entering in them dynamic coefficients $a_{i,i}$. Above has already been noted that for this preliminarily must be designed the cell/elements of the undisturbed motion of rocket $v_{x_i}(t)$, $\delta_{x_i}(t)$, $y_{x_i}(t)$, $\alpha_{x_i}(t)$, $\delta_{x^{1+}}(t)$ etc. Knowing these cell/elements as functions from the flight time of motion data also its weight m(t) and inertial characteristics $J_{x_i}(t), J_{y_i}(t)$ and $J_{x_i}(t)$, on dependences (8.26) it is possible to calculate the unknown dynamic coefficients of motion (8.29). Giving overall qualitative evaluation to dynamic coefficients, it is necessary to note that many of them strongly change along the trajectory of the motion of motion (6.27) gives the curve/graphs of a change in four dynamic coefficients

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and any any any any any for the projectile "Cerlikor" during its flight in trajectory [36]. • • • • • • •

- - Frcm curve/graphs it is evident that _ Sugin - approximately 12 times, but coefficient auge - is more than 50 times.

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Fig. 8.7. Character of a change in scae dynamic coefficients of the recket during its action along trajectory.

Key: (1). $1/s^2$. (2). $1/\epsilon$.

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Taking into account the above, and also that that deviations $\Delta \delta_{x_i}(t), \Delta \delta_{y_i}(t)$ and so forth are the assigned functions of time, the examine/considered by us system of equations (8.29) can be described as system of linear nonhomogeneous equations with variable coefficients, the dependence on time $a_{ij}(t)$ having the most diverse, that not yielding to simple mathematical description, form.

It is known that in the general case of this type of system of

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equations they cannot be solved analytically. The integration of system (8.29) can be realized by numerical methods with the use of digital computers and the methods of electronic simulation with the aid of the analog continuous computers. Since the method of sigulation for the solution of problems regarding the dynamic properties of rockets and projectiles, stability of their motion and stability of the work of the control system finds at present the widest application, let us give the rattern of the solution of system (8.29) in the simulating electronic computer, comprised by Yu. F. Kcl'tscv for case $f_{\bullet}(t) = f_{\bullet}(t) = 0$ [18]. Fattern is comprised taking into account the designations, described in Section 3.1 chapters VI, on it for clarity, are shown real physical quartities - cell/elements of the disturbed motion of rocket (Fig. 8.8). In working pattern must figure as the propertieral to them veltage/stresses for which the scale during the translaticr/conversion of initial equations into machine is accepted similar so that during entire process of the solution of problem in any of the blocks voltage/stress would not exceed permissible (3, charter VI). Therefore with the reading of block diagram, it is necessary to remember that under designations (for example, $bv, a_{ve}, \frac{d}{dt} bw_{t}$, and so forth) hide themselves not same these values, but proportional to them voltages.

Furthermore, for a larger graphic clarity pattern is shown on figure in cpen-circuited form, i.e., are not carried out the lines,

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which connect the cutput points of circuits with intake points. Instead of this of the points of circuits with intake points. Instead of this of the point of input/introduction and output of the values, utilized during the solution of equations, are numbered. With the reading of circuit, it is necessary to lear in mind, that exit points and input/introduction of values, which have identical rumbers, in working pattern are connected with each other, and the electrical circuits between them are locked. Thus, for instance, in the circuit of the solution of equation d/dtov for the formation of the derived at the entry of integrating block point of input/introduction are supplied the following significance of a deviation:

into point 1 - deviation ov from cutput 1;

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to point 4 - deviation $\delta\theta$ from cutput 4;

tc point 10 - deviation do from cutput 10.

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In turn, values $\delta v (-\delta v)$ from cutputs 2(1) are supplied in the circuit of the integration of other equations of the system in guestion, etc. In other respects diagram accurately reflects the corrse of solution of problem.

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During training/preparation for solution on the basis of the previously known undisturted motion of rocket about which it is necessary to examine the unknown disturbed action, they are designed, are scaled and are solderred on the blocks of the variable coefficients of the functions, which approximate the curve/graphs of the dynamic coefficients of equations $a_{ij}(t)$; furthermore, are calculated and with the aid of the blocks of constant coefficients are introduced into diagram the constant scale factors, for example, coefficients $\mu_{0,i}$, $\mu_{i,j}$, $\mu_{i,j}$, and so forth.

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Lig. 8.8. Schematic of solution in analog computer of problem on study of dynamic properties of rocket in its intrinsic disturbed axial motion.

Façe 324.

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We examine an example, when the disturbed action is caused only by any deviation of steering organ/controls (in our case of elevators) from their position in the trimmed/steading-state itself reference-flight conditions - zero or kalarcing, i.e., by values $\Delta \delta_{z_1}(t)$ and $\Delta \delta_{z_1}(t)$. (It is obvious that if the nominal trajectory corresponds to the zero position of controls, then it is possible in that case to count deviations $\Delta \delta_{z_1} = \delta_{z_1}$ and $\Delta \delta_{z_1} = \delta_{z_1}$. The form of dependences $\Delta \delta_{z_1}(t)$ and $\Delta \delta_{z_1}(t)$ can be different depending on the target/purposes of investigation. Figures 8.9, for example, shows different forms of the deviations of angle $\Delta \delta_{z_1}$, introduced to machine for the study of the dynamic properties of the rocket motion by which is simulated.

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As can be seen from Fig. 8.9a, these deviations can be assigned in the form of square-waves signal, which differ in terms of amount of deflection $\Delta \hat{s}_{i,i}$ and in terms of the duration of its actions $\underline{\gamma}_{i,j}$ with the aid of which is imitated the action on the rocket of stepped ratios of steering organ/controls.

Deviations can be also assigned in the form of the harmonic functions (see Fig. 8.91), whose amplitude $\Delta b_{i,i}$, period T_i and the dvration of action τ_i can be changed. The introduction of the perturbation signals of this form makes it possible to investigate the ability of rocket to follow the control displacement.

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Fig. 8.9. Diagram of the forcing functions, utilized during the study of the dynamic properties of rocket and its control system.

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Finally, changing time t_m it took effect ci disturtance/perturbation, we can investigate dynamic properties and the stability of nocket on the most unfavorable phases of trajectory of its motion. As for coefficients a_{ij} , the approximated curve/graphs of forcing functions $\Delta \delta_{z_1}(t)$ and $\Delta \dot{\delta}_{z_1}(t)$ are solderred to the blocks of the variable coefficients, by means of which they are introduced into diagram after the starting/launching of machine. If $t_m > 0$, then in the period of time $0 - t_{ii}$ machine will issue zero solution $\delta v(t) =$ $\delta \delta(t) = \delta \alpha(t) = \delta y_3(t) = 0$, showing thereby, that in the absence of disturbance/perturbations the nocket moves over nominal trajectory.

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From the torque/moment of time $t=t_{\rm H}$ programmed disturbance/perturbations $\Delta \delta_{t_1}(t)$ and $\Delta \delta_{t_1}(t)$ will begin to enter the appropriate blocks of variable coefficients, assigning dynamic coefficients a_{ij} , where there occurs the multiplication of these values.

In passing by through the scaling blocks of constant coefficients, the voltages, proportional to forcing functions, will hit the integrating operational amplifiers by which they will be integrated. Obtained as a result of this deviation δv , $\delta \theta$, δw_{cl} , ... with those signs with which they enter in the equations of the disturbed motion (for the report/communication to them of the necessary signs in diagram are provided the inverting units), they are transferred by the appropriate channels to the lead-in points of the units of variable coefficients for the formation of the new values of the derived or component at entries units of operational amplifiers. This process of the unceasing formation of input values and their integration (or summation) in analog computers occurs on closed cycle continuously before the termination of the interval of the process of the disturbed motion of the disturbed motion relation along trajectory.

The unknown values, which characterize the disturbed motion of rocket $\delta v(t)$, $\delta \theta(t)$, $\delta a(t)$, $\delta w_{r_i}(t)$ and of so forth, can be written

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with the aid of multichannel loop or electron-radiation, equipped with special photo attachment, the oscillograph to which they must be given from the appropriate output of operational amplifiers. The example of the recording of the results of the solution of a similar problem for a wingless rocket is shown on Fig. 8.10, to which are given the curve/graphs of changes in the deviations $\delta \alpha(t)$, $\Delta \theta(t)$, $\delta \theta(t)$ and $\delta v(t)$, that correspond to stepped forcing function $\Delta b_{r_1}(t)$ wher $\Delta \dot{b}_{r_1}(t)=0$.

If we look the curve/graphs of deviation indicated change for a statically stable rocket in prolonged time interval, then it is not difficult to establish that they all reflect the presence of cscillating processes, moreover very different. As can be seen from Fig. 8.10, dependence $\delta \alpha(t)$ is characterized by the presence of rapidly damping oscillations with their short period, which correspond to transient process from one halance angle α_6 to another. So at this time changes the deviation $\delta \delta$ of pitch angle. This oscillation they are called short-period.

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Changes in kinematic cell/elements δv_1 , $\delta \theta_2$, δx_3 , δy_3 etc. are also accompanied by oscillations, but the period of these oscillations is already ten times more (it can ever exceed the flight time of rocket

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on active phase of trajectory) and attenuate they slower.

The oscillations indicated are called long-period or phugoid; most distinctly they are exhibited at winged missiles and cruise missiles. For rockets and projectiles, whose lift (i.e. coefficient c_y^*) is comparatively small or little flight time, phugoid oscillation are also low and into calculation usually do not enter.

After the interpretation of cscillograms on the nature of the change in the deviations of the cell/elements of the motion of rocket, it is possible to judge the stability of its motion, about the quality of transient process from one conditions/mode of the steady flight to another, the correctness of selection during the aerodynamic design of the steady-state stability factor, damping characteristics, etc.

The method of the solution of the equations of the disturbed sction of rockets presented with the aid of the linear electronic analog computers, as we already noted above, has very wide acceptance, but nevertheless it very laboricus. In connection with this for conducting the analytical solution of the questions, connected with the selection of many design, weight, aerodynamic and other parameters of rocket, in the process of its initial design they gc for even more considerable simplifications in the problem. DCC = 78107111

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The essence of this simplification consists in the fact that the dynamic properties of the design/projected icoket are estimated not according to the results of the solution of the single system of equations of disturbed motion (8.29) with variable coefficients $a_{ij}(t)$, but by the analytical investigation of the totality of similaries systems of equations, dynamic coefficients in which consider constant values a_{ij}^* .

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Fig. 8.10. Example of the recording of solution in the analog computer of problems on the study of a charge in the deviations of the trajectory elements of rocket in its intrinsic disturbed motion in the case of the step deflection of control.

Key: (1). Time scale.

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The number of such together investigated systems must be selected depending on the number of characteristic points of the undisturbed trajectory of the motion of rocket. To such characteristic points in the trajectory, can be attributed the points of inclusion and engine cutoff, in which occurs a considerable charge in the thrust, the point of the booster ejection and stages of rocket, it is characterized by an abrupt change in mass and torque/moments of DQC = 78107111

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inertia, point with the greatest and smallest values of velocity head, etc.

For established/installed characteristic points in the trajectory, must be calculated the concrete/specific/actual values of dynamic coefficients $a_{ij}^*(t_1), a_{ij}^*(t_2), \dots, a_{ij}^*(t_n)$, which will enter into the systems of equations being investigated. During similar investigations, furthermore, they make still one simplification they set/assume in the reference system of equations (8.29) $\Delta t_{ii} = \Delta t_{ii} = 0$ (this corresponds to the attacked elevators) and $f_{\mathbf{v}}(t) = f_{\mathbf{b}}(t) = \dots = 0$, this system is converted into the system of homogeneous linear differential first-order equations with constant coefficients. Taking into account the aforesaid and uniting in (8.29) the third and fourth equations, let us write it for concrete/specific/actual point in the trajectory in question in the following form:

$$\frac{d}{dt} \delta v = a_{vv}^{*} \delta v + a_{vs}^{*} \delta \theta + a_{tv}^{*} \delta a;$$

$$\frac{d}{dt} \delta \theta = a_{vv}^{*} \delta v + a_{\theta \delta}^{*} \delta \theta + a_{\delta a}^{*} \delta a;$$

$$\frac{d^{2}}{dt^{2}} \delta \theta = a_{\omega_{21}v}^{*} \delta v + a_{\omega_{21}a}^{*} \delta a + a_{\omega_{21}w_{21}}^{*} \frac{d}{dt} \delta \theta + a_{\omega_{21}a}^{*} \frac{d}{dt} \delta a;$$

$$\delta a = \delta \theta - \delta \theta.$$
(8.31)

In this system we have four equations with four unknown values $\delta v(t) \downarrow \delta \theta(t)$, $\delta \delta(t)$ and $\delta o(t)$. Consequently, system is locked and can be integrated.

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The general solution of the system of homogeneous linear equations (8.31) is located as sum of its perticular solutions which, as is known from mathematics, for similar equations they take the form

$$bv = Ae^{\lambda t}; \quad b0 = Be^{\lambda t}; \\ bb = Ce^{\lambda t}; \quad ba = De^{\lambda t}. \end{cases}$$
(8.32)

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FIGE 64

After substituting expressions (8.32) into system of equations (8.31), we will obtain

$$\lambda A e^{\lambda t} = a_{vv}^{*} A e^{\lambda t} + a_{vv}^{*} B e^{\lambda t} + a_{vv}^{*} D e^{\lambda t};$$

$$\lambda B e^{\lambda t} = a_{vv}^{*} A e^{\lambda t} + a_{vv}^{*} B e^{\lambda t} + a_{vv}^{*} D e^{\lambda t};$$

$$\lambda^{*} C e^{\lambda t} = a_{ust}^{*} A e^{\lambda t} + a_{ust}^{*} D e^{\lambda t} + a_{ust}^{*} a_{ust}^{*} \lambda C e^{\lambda t} + a_{ust}^{*} \lambda D e^{\lambda t};$$

$$D e^{\lambda t} = C e^{\lambda t} - B e^{\lambda t}.$$

(8.33)

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In all terms of the chtained equations enters common factor $e^{\lambda t}$. After contraction to this factor and the transfer of all terms of equations into left side, we will obtain the following system:

$$(a_{vv}^{*} - \lambda) A + a_{vv}^{*} B + a_{vv}^{*} D = 0;$$

$$a_{vv}^{*} A + (a_{vv}^{*} - \lambda) B + a_{vv}^{*} D = 0;$$

$$a_{vz1}^{*} A + \lambda (a_{vz1}^{*} - \lambda) C + (a_{vz1}^{*} + \lambda a_{vz1}^{*}) D = 0;$$

$$-B + C - D = 0.$$

(8. 34)

This system includes no longer differential, but linear

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algebraic equations, moreover unknewn values in it they are the parameter λ and coefficients A, E, C and D. The procedure of the solution of systems of type (8.34) is known. At first their characteristic equation, which in the case in question represents by itself the equation of the fourth degree

$$k^{4} + k_{1}\lambda^{3} + k_{3}\lambda^{3} + k_{3}\lambda + k_{4} = 0,$$
 (8.35)

they find the values of its roots. Such rocts will be four: λ_1 , λ_2 , λ_3 , λ_{\bullet} . Each root is substituted in algebraic system (8.34) and from it unarbiguously find values of coefficients of A, B, C and D, which correspond to this root:

for $\lambda_1 = A_1$, E_1 , C_1 , D_2 .

for $\lambda_2 = A_2$, B_2 , C_2 , D_2 .

and so forth.

After this the general solution of system (8.31) can be written in the form

$$\delta v = A_{g} e^{\lambda_{1} t} + A_{g} e^{\lambda_{0} t} + A_{g} e^{\lambda_{0} t} + A_{g} e^{\lambda_{0} t};$$

$$\delta v = B_{g} e^{\lambda_{1} t} + B_{g} e^{\lambda_{0} t} + B_{g} e^{\lambda_{0} t} + B_{g} e^{\lambda_{0} t};$$

$$\delta v = C_{g} e^{\lambda_{1} t} + C_{g} e^{\lambda_{0} t} + C_{g} e^{\lambda_{0} t} + C_{g} e^{\lambda_{0} t};$$

$$\delta u = D_{1} e^{\lambda_{1} t} + D_{s} e^{\lambda_{0} t} + D_{s} e^{\lambda_{0} t} + D_{s} e^{\lambda_{0} t}.$$

(8.36)

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The coefficients of general solution (E.36) A_i , B_i , C_i and D_i , and also the coefficients of characteristic equation k_i are the real values which depend on the dynamic coefficients of the equations of disturbed motion (8.31) and for each specific case are different. In this case, roots λ_i of characteristic equation (8.35) can be both the real and those conjugate/costined composite, moreover in the case (for quartic equation) in question are possible only three following combinations of their values.

1. All four roots of characteristic equation are real. In this case changes of each of the deviations of the trajectory elements of rocket δv , $\delta \vartheta$, ... will be be defined as result of the summation of four aperiodic functions and record/written in the form (8.36).

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2. Two Roots of characteristic equation real, other two roots - conjugated/combined conjugated.

 $e^{i\pi t} - e^{-i\pi t} = 2l \sin \pi t$

In this case, utilizing Euler's known relationship/ratios $e^{iw} + e^{-iw} = 2\cos w$

(8.37)

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equations (8.36) can be converted as follows:

 $\begin{aligned} \delta v &= A_2 e^{\lambda_1 t} + A_3 e^{\lambda_0 t} + A_3, e^{\xi t} \sin(v_t + \gamma_1); \\ \delta \theta &= B_1 e^{\lambda_1 t} + B_3 e^{\lambda_1 t} + B_3, e^{\xi t} \sin(v_t + \gamma_2); \\ \delta \vartheta &= C_1 e^{\lambda_1 t} + C_2 e^{\lambda_1 t} + C_3, e^{\xi t} \sin(v_t + \gamma_3); \\ \delta a &= D_1 e^{\lambda_1 t} + D_2 e^{\lambda_2 t} + D_3, e^{\xi t} \sin(v_t + \gamma_4). \end{aligned}$ (8.38)

The entering in them new constant coefficients $A_{3,4}$, $B_{5,4}$, $C_{3,4}$, $C_{3,4}$, $C_{3,4}$, P_{i} , find in the process of transformation. From expressions (8.38) it follows that in the case in question a change of each deviation will be determined by the summation of two aperiodic functions with the function, which describes the oscillating process which is characterized, for example, for velocity with an amplitude of $A_{i,e}^{i_{i}}$, by angular frequency ν and by phase shift γ_{i} .

3. Finally, let us examine solution when all four roots of characteristic equation form two pairs of conjugated/combined composite roots:

$\lambda_{1,2} = \zeta \pm lm; \quad \lambda_{3,4} = \xi \pm lv.$

In this case a change of each of the cell/elements will be determined as a result of the summation of two oscillatory/vibratory functions, and decisions for them make the following form:

 $\begin{aligned} &\delta \sigma = A_{1,g} e^{\xi t} \sin (\eta t + \eta_2) + A_{3,g} e^{\xi t} \sin (\eta t + \eta_2); \\ &\delta \sigma = B_{1,g} e^{\xi t} \sin (\eta t + \eta_3) + B_{3,g} e^{\xi t} \sin (\eta t + \eta_3); \\ &\delta \sigma = C_{1,g} e^{\xi t} \sin (\eta t + \eta_3) + C_{3,g} e^{\xi t} \sin (\eta t + \eta_3); \\ &\delta \alpha = D_{1,g} e^{\xi t} \sin (\eta t + \eta_3) + D_{3,g} e^{\xi t} \sin (\eta t + \eta_4). \end{aligned}$ $\end{aligned}$ $\end{aligned}$

The given three types of the solutions of system of equations

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(8.31) show, as they would change with respect to the time of amount of deflection δv , $\delta \theta$, $\delta \vartheta$ and $\delta \alpha$, if the initial undisturbed motion of rocket, beginning with characteristic point in the trajectory in question, it represented by itself straight-and-level flight with the constant velocity, i.e., under conditions, clearly different from the real.

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Therefore the obtained analytical solutions are utilized, as a rule, only for approximate qualitative evaluation of the dynamic properties of rocket, especially as this evaluation can be made on the basis of the analysis of the roots of characteristic equation.

If real roots or the real parts of the composite roots of characteristic equation (8.35) (whele or part of them) will be gesitive, then the entering all the solutions (8.39) factors of the type e^{tr} will grow/rise in the course of time. In this case, the deviations being investigated from the undisturbed motion δv , $\delta \theta$, $\delta \vartheta$, δa will also after the treak-down of disturbance/perturbation grow/rise with larger or lower speed, what is the sign/oriterion of the instability of rocket or projectile in this state of motion and testifies to the incorrect selection of the dynamic coefficients, which determine, as noted alove, the values of the roots of EQC = 78107111

characteristic equation. If during the design of value m(t), J(t), m_{z}^{*} , $m_{\partial z}$, c_{y}^{*} etc., entering the dynamic coefficients, undertaken correctly, then all real roots and the real parts of the composite roots will be negative or their part with the others negative will obtain zero value. In this case, the deviations δv , $\delta \theta$, $\delta \vartheta$, $\delta \alpha$ will respectively or decrease (attenuate) or not attenuate, but also not increase. In the first case the rocket calls stable in axial motion, in the second - neutral.

FIGE

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Investigations show that the rockets and the fin-stabilized projectiles unguided or with the attached controls are neutral dynamic systems, since their deviation from the trajectory of the undisturbed motion are not removed by themselves after the break-down of their caused disturbance/perturbation. Eynamically stable rocket becomes only when the acting automatic control system is present, of flight with the correctly selected equation of control (by law of regulation).

Qualitative answer/response to a question concerning the stability of flight vehicle can be obtained and without determining the roots of characteristic equation (8.35). For obtaining the negative values of real roots and real parts of the composite roots, it is necessary and it suffices to satisfy the condition

> $k_1 > 0; k_3 > 0; k_3 > 0; k_4 > 0;$ $k_1k_2k_3 - k_1^2k_4 - k_3^2 > 0.$

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With the observance of these conditions, the motion will be stable.

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The results of testing the statility of rocket with respect to the roots of characteristic equation cannot be considered sufficient. They are suitable only for the preliminary selection of the aerodynamic, weight, inertia and other parameters of the rocket during its design and they must be confirmed by the more precise methods of the study of the dynamics of the cortrol systems.

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3.1. Transient processes during the motion of rockets and projectiles.

The motion of rockets and projectiles are subdivided into steady and that being unsteady. Under steady motion is understood such motion, during which its determining parameters (velocity of the motion of the center of mass v, angles a, f, γ , angular rates of rotation w_{x_1}, w_{y_1} and w_{x_1}) in the course of time do not change. It is consistent that under the actual conditions of this steady motion the rockets and the projectiles does not happen to have. The concept of DCC = 78107111

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steady motion (or flight equilibrium) is most frequently introduced conditionally in the examination of motion relative to the center of mass (balancing flight conditions). In actuality, however, a change in the angles of rotation of controls on the appearance of any perturbation factors always disrupt the steady moment balance relative to the center of mass of rocket and they force it to transfer/convert from one conditions/mode of the steady flight to another.

Let the initial steady motion of a rocket is characterized by value $\delta_{z_1} = 0$, and therefore $\alpha_0 = 0$. Let us assume that at the moment of time t = t₁ the elevators were deflected by angle $\delta_{z_1} < 0$, to which will correspond under the assumption $\hat{\theta} = 0$ trim angle α_0 .

Further angular motion of the rocket with retention of angle δ_{11} by constant illustrate the curve/graphs, given to Fig. 8.11. In time interval $t_1 - t_2$ under the action of total torque/moment $M_{332} = M_{322} - M_{23} > 0$ the rocket legins to be turned relative to the center of mass, in this case, it will appear and will grow/rise angle of attack $\alpha > 0$ and angular velocity $\dot{\alpha}$. At the moment of time t_2 , the angle of attack α will become equal to trim angle α_0 , and $M_{342} - zero$. But, possessing at the sement of time in question angular velocity $\dot{\alpha}_2 > 0$, rocket as inert body will continue its rotation in previous direction until torque/moment M_{342} inhibits it in position 3 ($\dot{\alpha}_3 = 0$) and will force to move again to the position of trim equilibrium (position 4).

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Fig. 8.11. Change of the cell/elements of the angular motion of rocket (in the absence of damping) in the case of the instantaneous step deflection of control.

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Eage 332.

In the absence of camping, this escillating process will be consecutively repeated, i.e., it will be hemopic, moreover frequency will be determined by the moment of the inertia of rocket and by its steady-state stability factor. because of comping the process in question bears the damped character, in consequence of which during a certain time is establish/installed the trim equilibrium of rocket and it begins to move in the new trimmed/steading-state itself conditions/mode at value $\delta_{i} \neq 0$.

The process of transiting the rocket (or any other system) from one steady state to another calls transient process.

The form of transient process depends substantially on the damping characteristics of rocket and on their relationship/ratio with the reserve of its static stability.

For the illustration of this, let us describe mathematically transient process taking into account damping and will examine sclution. Let us introduce into equation (8.3) of harmonic

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cscillations instead of the angle o angle $a_m = a_{--a_0}$, which characterizes the oscillation of the axis of rocket of its relatively balancing position, and term $2ka_m$, which considers dauging moment $M_{o_{r_1}}$. After this the equation will take the form

 $\ddot{a}_{u} = 2\hbar\dot{a}_{u} + \pi^{0}a_{u} = 0,$ (8.40)

where

$$k = \frac{Sql}{J_{s_1}} \frac{l}{v} \left[m_{s_1}^{\omega_{s1}} \right].$$

with conditions for point $t=t_1=0$; $a_{0,n}=-a_0$; $a_{n,n}=0$, which characterize the beginning of transient process during the sudden deflection of elevators, we obtain the following solutions of equation (8.40):

a) if $n^2-k^2>0$, then

$$a_n = a_{n,m} e^{-M} \sin(\sqrt{n^2 - k^2} + \epsilon),$$
 (8.41)

where

$$a_{n,m} = \frac{na_{n,m}}{\sqrt{n^2 - k^2}}, \quad s = \operatorname{arctg} \frac{\sqrt{n^2 - k^2}}{k}$$

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Fig. 8.12. Different forms of the transient processes of a change of angle of attack during the instantaneous step deflection of control in the case of oscillation damping.

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As can be seen from solution (8.41), a change in angle a_n bears the character of oscillations with period $T = \frac{2\pi}{1 n^2 - k^2}$, moreover the amplitude of oscillations in the course of time decreases, approaching zero at the end of the transient process when $t = t_n$.

It is not difficult to see that, changing the damping characteristics $m_{x_i}^{w_{x_i}}$ of rocket and, consequently, also coefficient k, it is possible to regulate transit time t_{α} desirably. For example, with an increase in rotary derivative $m_{x_i}^{w}x_i$ of the fluctuation of angle a_{α} they will attenuate more intense, and ICC = 78107112

transit time will decrease. In Fig. 6.12 to examined solution (under identical initial conditions) correspond the curves 1 and 2, moreover $(m_{z_1}^{\omega_{z_1}})_1 < (m_{z_1}^{\omega_{z_1}})_2;$

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b) if $n^2-k^2<0$, then the scluticn of equation (8.40) is expressed by the formula

$$a_{n} = a_{n,n} e^{-kt} \left(\operatorname{ch} \left\{ \frac{k^{2} - n^{2}t}{k^{2} - n^{2}} + \frac{k}{\sqrt{k^{2} - n^{2}}} \operatorname{sh} \sqrt{k^{2} - n^{2}t} \right\}, \quad (8.42)$$

showing that in this case angle a_n changes aperiodically, approaching zero at the end of the transient process, moreover here with increase of k (i.e. and $m_{x_i}^{w_{x_i}}$) with the constant n transit time already grow/rises (curves 3 and 4).

The relationship/ration between coefficients of k and n +o which correspond the curves, given to Fig. E.12, were undertaken in an example following:

Maringa SC	1	2	3	4
k/n	0,3	0,9	1,1	2,5

Key: (1). curved.

Best are considered such transient processes when oscillations in system or do not appear (aperiodic process, curve 3), or are
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weakly expressed (curve 2), but transit time is small. An increase of transient period and the presence in it of considerable oscillations decrease the accuracy of the operation of the systems of rocket control and, as a result, the accuracy of the incidence/impingement by them into target/purpose.

§4. Conditions of the stable flight of the rotating rockets and of projectiles.

Artillery shells and TES, that are statilized in flight because of high-spin motion, intended for a firing to the comparatively small distances with which the rotation of the Earth and its curvature do not have virtually effect on the stability characteristics of motion.

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If we approximately consider the rotational effect and curvature of the Earth by the introduction of acceleration $\overline{g}=\overline{g}_0$, then complete system of equations can be written on the basis of equations (1.16) and (1.21).

In the right side of equations (1.16) they must be introduced to the projection of all forces, which act on projectile, on the axis of the driving/moving i coordinate system. For the rotating rocket

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 FIGE 6 [ab]

projectiles these forces are external aerodynamic forces [(2.109), (2.110), (2.111)], the thrust of all engines (2.120), Magnus' force (2.112) and the secondary forces (see Chapter II, Section 5.2). To secondary forces can be attributed the forces, caused by aerodynamic and gas-dynamic eccentricities, and the forces, caused by the lack of talance of masses.

In the right side of equation (1.21) ore should write the moments of the named forces and the torque/moment of surfice friction (2.113). For the guided missiles, it is logical, must be added the control forces and the moments of these forces; for the artillery shells of constant mass, reactive and variation forces must be excluded.

Has independent value the examination of effect on stability of motion of the conditions of firing from the fast-moving carrier, for example, of aircraft, and the account of the effect of the wind effect (see Chapter XI).

The compilation of system of equations taking into account all acting forces in the forg, suitable for numerical solution, represents complex and laborious problem. For the projectiles of constant mass, this problem was for the first time solved by V. S. Fugachev upon consideration of drag, normal force and Magnus' force.

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During the writing of the equations of rotary motion, were considered the torque/moments: inverted, damping, the moment of the Magnus force and the torque/moment of surfice friction. For TRS fundamental works on this question were carried out by Ya. M. Shapiro, Yu. A. Kochetkov, et al.

As showed many well known examinations, the motion of projectiles and rockets of their relatively centers of mass can be examined separately from forward motion along trajectory, counting that all the characteristics of last/latter are known functions of time [v(t), $\theta(t)$, $y_3(t)$ and so forth]. In this case, for the engineering calculations of the stability of the rotating projectile, it suffices in the first approximation, to consider only tilting scuent $M_z = M$ (2.111), which is the basic factor, which determines the character of rotary motion [9]. Missile attitude in its rotary motion is determined, as usual, by three argles: two angles determine the position of the axis of projectile relative to velocity vector and cse angle - rotation of projectile of relatively longitudinal rctational axis. Position of velocity vector of relatively earth-tased coordinate system of definition by two angles - θ and Ψ.

Face 335.

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Depending on the formulation of the problem, missile attitude relative to velocity vector can be determined by angle 6 between the axis of projectile and tangent to trajectory (Fig. 8.13). This angle, in accordance with the terminology of the theory of gyroscopic devices, is called nutation angle. In the plane of angle 6, by the called plane of resistance, acts tilting moment M_{i} . The position of the plane of the resistance of relatively vertical plane is determined by dihedral angle v_i whose fin/edge coincides with velocity vector v. Angle v_i is called precession angle. The rotation of projectile relative to the spin axis is determined by the angle *whose plane is perpendicular to the longitudinal axis of projectile Cx_1 .

The position of the longitudinal axis of projectile relative to velocity vector can be determined by the angles δ_1 and δ_2 , which replace angles 6 and v. Communication/connection between the named pairs of angles is determined from spherical right triangle with legs δ_1 and δ_2 . With the low values of angles δ_1 and δ_2 , which corresponds to stable projectiles, it is possible to write

 $\delta = \sqrt{\delta_1^2 + \delta_2^2}; \quad \delta_1 = \delta \sin v; \quad \delta_2 = \delta \cos v; \quad tg = \frac{\delta_1}{\delta_2} . \quad (8.43)$

Buring the solution of problem for flat/plane curved path taking into account a decrease in the tangent to trajectory into examination, are introduced the angles θ , ψ , δ_1 and δ_2 . For straight

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 PAGE 8 1064

Fath the solution proves to be more simply, since it suffices to introduce only angles ϕ , v and δ . Euring the study of the yawing motion of the rotating projectile, additionally is introduced angle Ψ .

Let us examine the rotary motion of projectile in connection with curved path taking into account a decrease in the tangent. The equations of rotary motion let us comprise in the form of the differential equations of Lagrange of 2 kirds (1.33).

We plan comprising of angular velocity to the principal axes of inertia of projectile, which let us designate in the manner that it is accepted in ballistics of the projectiles of the barrel systems: the longitudinal axis of projectile Cx_1 - through $O\zeta$, the equatorial axes of rectangular coordinate system - through $O\zeta$ and $O\eta$:





Fig. 8.13. Schematic of the angles, which determine the position of the rotating projectile in trajectory.

Fage 336.

The origin of coordinates it is consistent with the center of wass of projectile. The orientation of axes relative to velocity vector is shown on Fig. to 8.13. For use (1.33) it is necessary to have derivatives $\frac{\partial T}{\partial q_i}$ and $\frac{\partial T}{\partial \dot{q}_i}$, where T - kinetic energy of projectile in rotary motion; q_i - generalized coordinate. Kinetic energy is defined by the expression

 $T = \frac{1}{2} (Ap^{\mathbf{e}} + Bq^{\mathbf{e}} + Cr^{\mathbf{e}}),$

where C = axial moment of inertia, A and B = tergue/moments of inertia of relatively equatorial axes $C \in arc O_{n}$; p, g and r =

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 FIGE -11 [obje]

projection of the instantaneous angular velocity of projectile on axis 0ξ , 0η and 0ζ .

For the axisymmetric projectiles A = E and, therefore,

$$T = \frac{1}{2} [A(p^{a} + q^{a}) + Cr^{a}].$$

As the generalized coordinates, which cetermine missile attitude during rotation relative to the center of wass, let us take angles ϵ_1 , δ_2 and ϕ . Angle ϕ is not shown not in order not to complicate drawing, to simply show the angular velocity vector ϕ , necessary for the abandonment of equations. Angle ϕ and it derivatives $\dot{\theta}$ and $\ddot{\theta}$ are accepted as the known functions of time.

The vector of the instantaneous angular rate of rotation of **projectile** is determined by the sum

$$\bar{\mathbf{Q}} = \bar{\mathbf{\delta}}_1 + \bar{\mathbf{\delta}}_2 + \bar{\mathbf{\phi}} + \bar{\mathbf{0}}.$$

The sense of the vector angular velocities is selected according to the appropriate rule of mechanics. Vector $\overline{\delta}_1$ is perpendicular the plane of angle δ_1 and coincides with the negative direction of axis OE. Angular velocity vectors $\overline{\delta}_2$ and $\overline{\theta}$ are perpendicular to the single plane of angles δ_2 and θ and are directed along the positive direction of axis C2. Projecting vector $\overline{\Omega}$ to the coordinate axes, connected with projectile, let us have

$$p = -\dot{\mathbf{b}}_{1}; \quad q = (\dot{\mathbf{0}} + \dot{\mathbf{b}}_{0})\cos{\mathbf{b}}_{1}; \quad r = \dot{\mathbf{v}} + (\dot{\mathbf{0}} + \dot{\mathbf{b}}_{0})\sin{\mathbf{b}}_{1}.$$

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 FIGE $\frac{12}{100}$

At the low values of argles δ_1 and δ_2 , it is possible to accept sin $\delta_2 = \delta_1$, cos $\delta_1 = 1$. Then expression for kinetic energy of projectile in rotary motion we obtain in this form:

$$\mathbf{T} = \frac{1}{2} \left\{ A \left[\dot{\mathbf{b}}_{1}^{2} + (\dot{\mathbf{\theta}} + \dot{\mathbf{b}}_{2})^{2} \right] + C \left[\dot{\mathbf{\varphi}} + (\dot{\mathbf{\theta}} + \dot{\mathbf{b}}_{2}) \mathbf{b}_{1} \right]^{2} \right\}.$$
(8.44)

On the basis of the general consideration of mechanics, the generalized force for the angular parameter is equal to the projection of the acting torque/moment on the axis of elementary rotation. In the case in question generalized forces will be the projections of vector N on the directions of angular rates of rotation δ_1 , δ_2 and $\dot{\phi}$

$$Q_{b_1} = M_{b_1} = M \sin v \cos b_{g}; \quad Q_{b_2} = M_{b_3} = M \cos v; \quad Q_{\phi} = M_{\phi} = 0.$$
(8.45)

Eage 337.

After using (2.111), let us write

$$M = A\beta\delta, \qquad (8.46)$$

where

$$\beta = \frac{d^2 l}{Ag} 10^6 H(y) \, \boldsymbol{v}^8 K_M\left(\frac{v}{a}\right). \tag{8.47}$$

COC = 78107112 FACE -13 (68)

Then

$$Q_{\delta_1} = A$$
 is sin v cos δ_2 ; $Q_{\delta_1} = A$ is cos v; $Q_{\phi} = 0$.

Utilizing (8.43) and taking into account the smallness of angles δ_1 and δ_2 , we will obtain

$$Q_{b_1} = A_{\beta b_1}; \quad Q_{b_2} = A_{\beta b_2}; \quad Q_{\phi} = 0.$$
 (8.48)

Let us pass to the compilation of the equations of rotary motion in the form of equation (1.33). For the compilation of equation for generalized coordinate ε_1 , we differentiate (8.44) on $\dot{\delta}_1$ and δ_1 we find the following values of derivatives:

$$\frac{\partial T}{\partial \dot{\mathbf{b}}_1} = A\dot{\mathbf{b}}_1; \quad \frac{d}{dt} \left(\frac{\partial T}{\partial \dot{\mathbf{b}}_1} \right) = A\dot{\mathbf{b}}_1;$$
$$\frac{\partial T}{\partial \mathbf{b}_1} = C \left[\dot{\mathbf{\phi}} + (\dot{\mathbf{\theta}} + \dot{\mathbf{b}}_2) \, \mathbf{b}_1 \right] (\dot{\mathbf{\theta}} + \dot{\mathbf{b}}_2). \tag{8.49}$$

For the projectiles of constant mass, usually they accept

 $r = r_0 = \text{const},$

i.e. dc not consider a change in the angular rate of rotation relatively longitudinal axis (for example, the attenuation of the rotational speed). After using (1.33) and value Q_{k_1} , we will obtain

$$A\dot{\delta}_1 - Cr_0(\dot{\delta}_2 + \dot{\theta}) = A\beta\delta_1. \qquad (8.50)$$

For the compilation of equation for generalized coordinate δ_2 , we differentiate (8.44) on $\dot{\epsilon}_2$ and $\dot{\epsilon}_2$ we find derivatives

$$\frac{\partial T}{\partial \dot{\mathbf{i}}_2} = A(\dot{\theta} + \dot{\mathbf{\delta}}_2) + Cr_0 \dot{\mathbf{\delta}}_1;$$
$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{\mathbf{b}}_2}\right) = A(\ddot{\theta} + \ddot{\mathbf{\delta}}_2) + Cr_0 \dot{\mathbf{\delta}}_1; \quad \frac{\partial T}{\partial \mathbf{b}_2} = 0.$$

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 FIGE -14 1069

After using (1.33) and value Q_{4_0} , we will citain

$$A\dot{b}_{2} + A\dot{\theta} + Cr_{0}\dot{b}_{1} = A\beta b_{2}. \qquad (8.51)$$

For simplification in the writing, usually is introduced the designation

$$a = \frac{Cr}{2A} \approx \frac{Cr_0}{2A} \,. \tag{8.52}$$

Fage 338.

Carrying out replacement in (8.50) and (8.51), let us write

$$\ddot{\mathbf{\delta}}_{1} - 2\alpha (\dot{\mathbf{\delta}}_{1} + \dot{\boldsymbol{\theta}}) = \beta \mathbf{\delta}_{1}; \quad \ddot{\mathbf{\delta}}_{2} + \ddot{\boldsymbol{\theta}} + 2\alpha \dot{\mathbf{\delta}}_{1} = \beta \mathbf{\delta}_{2} \qquad (8.53)$$

cr, after transferring into the right side of the value obtained during the separate solution of the equations of motion of the center of mass of projectile, -

$$\ddot{\mathbf{\delta}}_1 - 2\mathbf{a}\dot{\mathbf{\delta}}_2 - 3\mathbf{\delta}_1 = 2\mathbf{a}\dot{\mathbf{\theta}}; \quad \ddot{\mathbf{\delta}}_2 - 2\mathbf{a}\dot{\mathbf{\delta}}_1 - 3\mathbf{\delta}_2 = -\ddot{\mathbf{\theta}}.$$
 (8.54)

In the general case under the alternating/variable α and β and initial conditions t=0; $\dot{\epsilon}_1 = \dot{\epsilon}_{10}$; $\dot{\delta}_1 = \dot{\epsilon}_{10}$; $\delta_2 = \dot{\epsilon}_{20}$; $\dot{\delta}_2 = \dot{\delta}_{20}$ system (8.54) can be solved only numerically.

Let us multiply the first equation by i and let us add with the

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 FAGE 15 670

second

$$\ddot{\delta}_2 + l\ddot{\delta}_1 + 2a(\dot{\delta}_1 - l\dot{\delta}_2) - \beta(\delta_2 + l\delta_1) = 2la\theta - \theta.$$

It is possible to replace $i\dot{\epsilon}_2 - \dot{\delta}_1 = i(\dot{\epsilon}_2 + i\dot{\epsilon}_1)$ and then

$$\ddot{\delta}_{2} + i\ddot{\delta}_{1} - 2ia(\dot{\delta}_{2} + i\dot{\delta}_{1}) - \beta(\delta_{2} + i\delta_{1}) = 2ia\dot{\theta} - \dot{\theta}.$$

After introduction to complex variable

$$z = \delta_2 + i\delta_1 \tag{8.55}$$

we will obtain one the equation, equivalent to system (8.54),

$$z - 2iaz - \beta z = 2ia\theta - \theta. \qquad (8.56)$$

last/latter equation - linear nonhomogeneous differential equation of the second order with variable coefficients and alternating/variable right side.

In conformity with the second equation of system (5.3)

$$\dot{\theta} = -\frac{g\cos\theta}{v}$$
.

Taking the second derivative and converting, we will obtain

$$\ddot{\theta} = \frac{\dot{\theta}}{v} (g \sin \theta - \dot{v}).$$
 (8.57)

To solve the equations, similar (8.56), is possible only by rumerical methods. initial conditions will be written as follows:

$$t=0; z=z_0=\delta_{10}+l\delta_{10}; z=z_0=\delta_{10}+l\delta_{10}.$$

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Introducing the supplementary simplifications, in particular, accepting β =const, it is possible equation (8.56) to solve analytically. The complete integral of differential equation (8.56) represents by itself the sum of the general solution of homogeneous equation and the particular integral of noncomplete equation. The homogeneous equation

$$\ddot{z} - 2ia\dot{z} - \beta z = 0$$
 (8.58)

by the substitution

$$z = \mu e^{i\alpha t} \tag{8.59}$$

is led to a simpler form. FICM (8.59) it follows

 $z = ue^{iat} + iaue^{iat}; \quad z = ue^{iat} + 2iaue^{iat} - a^{2}ue^{iat}.$

Faç€ 339.

Substituting z, \dot{z} and \ddot{z} in (8.58) and by reducing on $e^{i\alpha t}$, we will obtain

$$u - (a^2 - \beta) u = 0.$$

Eesignating

$$s=1-\frac{\beta}{a^2}, \qquad (8.60)$$

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let us lead last/latter equation to the form

$$u - \alpha^{2} \tau u = 0. \tag{8.61}$$

With variables α and σ_{e} equation (8.61) analytically is not sclued. On the assumption that α =const and ϵ =const, the integral of equation (8.61) takes the form

$$\boldsymbol{u} = \boldsymbol{C}_{1} e^{i \mathbf{a} \cdot \mathbf{j}' \cdot \mathbf{j}} + \boldsymbol{C}_{2} e^{-i \mathbf{a} \cdot \mathbf{j} \cdot \mathbf{j}}. \tag{8.62}$$

Substituting u in (8.59), we will obtain

$$z = C_1 e^{i \alpha (1 - 1^{-5}) t} + C_2 e^{i \alpha (1 - 1^{-5}) t}.$$
 (8.63)

Integration constant in general form are represented by the complex numbers

 $C_1 = \varrho_1 e^{i \iota_1}; \quad C_2 = \varrho_2 e^{i \iota_2},$

after which

$$z = c_{1} e^{i \left[e^{(1+1)} \right] t + c_{1} } \pm c_{2} e^{i \left[e^{(1-1)} \right] t + c_{2} }$$
(8.64)

Eesignating

$$\omega_1 = \alpha \left(1 + 1 \overline{\sigma}\right); \quad \omega_2 = \alpha \left(1 - 1 \overline{\sigma}\right) \qquad (8.65)$$

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and remembering that $e^{i\psi} = \cos \psi + i \sin \psi$, let us write instead of (8.64)

$$z = \varrho_1 \cos(\omega_1 t + \varepsilon_1) + \varrho_2 \cos(\omega_2 t + \varepsilon_2) +$$

+ $t [\varrho_1 \sin(\omega_1 t + \varepsilon_1) + \varrho_2 \sin(\omega_2 t + \varepsilon_2)].$ (8.66)

Comparing (8.55) and (8.66), we will obtain expressions for angles δ_1 and δ_2

$$\delta_1 = \varrho_1 \sin(\omega_1 t + \varepsilon_1) + \varrho_2 \sin(\omega_2 t + \varepsilon_2); \qquad (8.67)$$

$$\delta_2 = \varrho_1 \cos(\omega_1 t + \varepsilon_1) + \varrho_2 \cos(\omega_2 t + \varepsilon_2). \qquad (8.68)$$

Fig. 8.14 depicts the motion of the longitudinal axis of projectile in coordinates δ_1 and δ_2 . Foint K represents by itself the projection of the point of intersection of the longitudinal axis of projectile with the sphere of the unit radius, carried out from the center of mass of projectile, to coordinate plane $\delta_1 0\delta_2$. The velocity vector of the center of mass of projectile for the solution of homogeneous equation (8.58) in question is projected into the point, which corresponds to the origin of rectilinear coordinates δ_1 and δ_2 .

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The meticn of point M on coordinate image plane can be represented as vector sum of two motions: the rotation of point M along circle with a radius ρ_1 of the relatively instantaneous center of rotation and movable circular motion of instantaneous center of rotation along circumference with a radius ρ_2 . The angular velocities of rotary motion ω_1 and ω_2 are determined by expressions (8.56). It is obvious

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that $\mu_1 > \omega_2$.

To the projection of radius-vectors or coordinate axes δ_1 and δ_2 respectively equal to

$$\delta_1 = \delta_{13} + \delta_{11}; \quad \delta_3 = \delta_{33} + \delta_{31},$$

where $\delta_{11} = c_1 \sin(\omega_1 t + \varepsilon_1); \quad \delta_{12} = c_2 \sin(\omega_2 t + \varepsilon_2); \quad (8.69)$ $\delta_{21} = c_1 \cos(\omega_1 t + \varepsilon_1); \quad \delta_{22} = c_2 \cos(\omega_2 t + \varepsilon_2).$

the trajectory of point M in total motion on coordinate plane $\delta_1 0 \delta_2$ will be the epicycloid, constructed or circle with radius of $\rho_2 - \rho_1$. The value of radius-vectors ρ_1 and ρ_2 , their relationship/ratio and the constants e_1 and e_2 , on which depend the initial positions of radius-vectors, are determined by the initial corditions.

If we consider damping oscillations, then of curve/graphs, that characterizes a change of the nutation angle δ in the function of time, will take the form, shown on Fig. 8.15. $\mathbf{CC} = \mathbf{78107112}$



Fig. 8.14. Graph/diagram of dependence $\delta_2 = f(\delta_1)$ for the straight pertipe of the trajectory of the fast-turning projectile with $\delta_0 \neq 0$ and $\delta_0 \neq 0$.

Face 341.

Let us find now the particular integral of nonhomogeneous equation (8.56). Solution they usually search for in the form of the series:

$$z_{p} = z_{0} + \frac{1}{a} z_{1} + \frac{1}{a^{2}} z_{2} + \dots$$
 (8.70)

Investigations show that a - the high parameter and the terms of $a series rapidly they decrease. Functions from time <math>z_0$, z_1 , z_2 are subject to determination. Let us differentiate z_p and z_p and

together with z let us substitute in (8.56) $z_0 - 2laz_0 - \alpha^2 \left(\frac{\beta}{\alpha^2}\right) z_0 + \frac{1}{\alpha} z_1 - 2lz_1 - \alpha \left(\frac{\beta}{\alpha^2}\right) z_1 + \frac{1}{\alpha^2} z_2 - 2l \frac{1}{\alpha} z_2 - \left(\frac{\beta}{\alpha^2}\right) z_3 + \ldots = 2lad - 0.$

In the obtained equation usually the terms $\frac{1}{a}\ddot{z}_1$, $\frac{1}{a^2}\ddot{z}_3$ and $2l\frac{1}{a}\dot{z}_3$ are the members of the second order of smallness and by them it is possible to disregard.

Let us equate the coefficients when α with identical degree in the right and left sides of the last/latter equation.

Coefficients of α^2 :

$$\left(\frac{\beta}{\alpha^2}\right)z_0=0.$$

Ccefficients when α

$$-2i\dot{z}_0-\left(\frac{\beta}{\alpha^2}\right)z_1=2i\dot{\theta}.$$

Ccefficients of α^0

$$\ddot{z}_0 - 2\delta \dot{z}_1 - \left(\frac{\beta}{\alpha^2}\right) z_1 = -\ddot{\theta}.$$

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Fig. 8.15. Decrease of nutation angle δ from damping of medium depending on time.

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Of three last/latter equations we find

$$z_0 = 0; \quad z_1 = -\frac{2i\alpha^2}{\beta} \dot{\theta}; \quad z_2 = \frac{\alpha^2}{\beta} \ddot{\theta} - 2i \frac{\alpha^2}{\beta} \dot{z}_1.$$
 (8.71)

Let us differentiate cr t of the second equation of system (E.71) and let us substitute \dot{z}_1 into the third equation. After conversion let us have

$$z_2 = \frac{a^2}{\beta} \theta - 4 \frac{a^4}{\beta^2} \dot{\theta} \left(\frac{\theta}{\dot{\theta}} - \frac{\dot{\beta}}{\dot{\beta}} \right).$$

Substituting in (8.70) z_0 , z_1 and z_2 , we will obtain

$$z_{p} = -2i\frac{\alpha}{\beta}\dot{\theta} + \frac{\ddot{\theta}}{\beta} - 4\frac{\alpha^{2}}{\beta^{2}}\left(\frac{\ddot{\theta}}{\dot{\theta}} - \frac{\dot{\beta}}{\beta}\right). \qquad (8.72)$$

Taking into account the general view c1 ccmplex variable (8.55), let us write

$$z_p = \delta_{p} + i \delta_{1p}. \tag{8.73}$$

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Comparing (8.72) and (8.73), we will obtain $\delta_{1p} = -2 \frac{\alpha}{5} \dot{\theta}. \qquad (8.74)$ $\delta_{2p} = \frac{\ddot{\theta}}{\beta} - 4 \frac{\alpha^2}{\beta^2} \left(\frac{\ddot{\theta}}{\dot{\theta}} - \frac{\dot{\beta}}{\beta} \right). \qquad (8.75)$

Calculations show that angle δ_{ip} is considerably greater than angle δ_{2p} and it is chief constituent of argle δ_{p} . The solution of the problem of the motion of the rotating projectile on the curvilinear trajectory phase shows that in this case the epicycloid is oriented relative to the dynamic axis of equilibrium (relative to pcint D in Fig. 8.16). Ecint D represents by itself projection on image plane of the point of intersection of the dynamic axis of equilibrium with the sphere of a unit radius. For each moment of time, the position of point D on coordinate plane is determined by values δ_{1p} and δ_{2p} , which are located from formulas (8.66) and (8.67). Angles δ_{1p} and δ_{2p} are counted of i from the origin of coordinates 0, which represents by themselves projection on image glame of point of intersection with tangent to trajectory with the sphere of a unit radius. With gradual to trajectory with the sphere cf unit radius. During the gradual deviation of the dynamic axis of equilibrium from velocity vector (print D from point 0) the curve, described by a radius ρ_2 , obtains the form of spiral (unlike circumference in Fig. 8.14).

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Thus, are obtained expressions for the angles, which characterize the position of the axis of dynamic equilibrium. Since $\delta_{1p} \gg \delta_{2p}$, then in the first approximation, it is possible to count that

$$\delta_{p} \approx \delta_{ip} = 2 \frac{\alpha}{\mu} |\dot{\theta}|. \qquad (8.76)$$

Eage 343.

Substituting in (8.76) the second equation of system (5.3) and converting it, we will obtain

$$\delta_p \approx k \frac{\cos \theta}{H(y) v^3 K_M\left(\frac{v}{a}\right)}.$$
(8.77)

Angle δ_p determines the position of the dynamic axis of equilibrium, relative to which cours the periodic oscillatory motion of the longitudinal axis of the rotating projectile. If the trajectory of the motion of projectile approaches straight line and $\delta_p=0$ and the oscillations of the longitudinal axis of projectile will be completed relative to the velocity vector of the motion of the center of mass.

For the straight perticn of trajectory in the case θ =const and

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 PAGE $\frac{-25}{0}$

6=0 tc conveniently use angles ϕ , v and δ (Fig. 8.17). Then the angular rate of rotation of projectile is ϵ gual to

$\vec{\Omega} = \vec{\varphi} + \vec{v} + \vec{\delta}.$

Let us first comprise the system of differential equations for the more general case - for the rotary motion of the spin-stabilized missile. Let us consider the action of tasic torque/moments - turning remert $M_{\rm sp}$ and the aerodynamic tilting moment - M. The rotations of TBS of relatively longitudinal axis it is provided by the special construction of the nozzle unit in which the nozzles are fulfilled with slope/inclination toward by generatrix missile bodies at angle γ (Fig. 8.18). The plane of the action of turning moment is perpendicular to the longitudinal axis of projectile. The vector of torque/moment $M_{\rm sp}$ is directed along the axis $Cr_1(O\zeta)$ (8.17). The tilting moment acts in the plane of resistance, and its vector is directed along the axis Cz_1 , perpendicular to the plane of the resistance (see Fig. 8.17).

In the case of expression in question for generalized forces on the appropriate angles of rotation, they will take the form

$$Q_{p} = M \cos(z_{1}x_{1}) + M_{pp} = M_{pp};$$

$$Q_{s} = M \cos(z_{1}v) + M_{pp} \cos(x_{1}v) = M_{pp} \cos \delta;$$

$$Q_{s} = M + M_{pp} \cos(x_{1}z) = M.$$
(8.78)

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Fig. 8.16. Graph/diagram of dependence $\delta_2 = f(\delta_1)$ for the curvilinear phase of trajectory of the center of mass of the fast-turning projectile with $\delta_0 \neq 0$ and $\dot{\delta}_0 \neq 0$.

Fage 344.

Let us comprise the differential equations of Lagrange for generalized coordinates ϕ , v and δ , substituting alternately in (1.33) derivatives $\frac{\partial T}{\partial \dot{\phi}}$ and $\frac{\partial T}{\partial \psi}$; $\frac{\partial T}{\partial \dot{v}}$ and $\frac{\partial T}{\partial \dot{v}}$; $\frac{\partial T}{\partial \dot{b}}$ and $\frac{\partial T}{\partial b}$. After substitution we will obtain:

for a generalized coordinate #

$$C \frac{d}{dt} (\dot{\varphi} + \dot{\nu}_{s} \cos \delta) = M_{sp}; \qquad (8.79)$$

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fcr generalized cocrdinate v

$$\frac{d}{dt} [A\dot{v}\sin^2\delta + C(\dot{\varphi} + \dot{v}\cos\delta)\cos\delta] = M_{ep}\cos\delta; \quad (8.80)$$

for a generalized coordinate &

$$A\delta - A\dot{v}^{2}\sin\delta\cos\delta + C(\dot{\varphi} + \dot{v}\cos\delta)\dot{v}\sin\delta = M.$$
 (8.81)

The angular rate of rotation of TRS of relatively longitudinal axis ir powered flight trajectory approximately will be determined from (8.79). Let us write

$$\dot{\varphi} + \dot{v}\cos\delta = r = \int_{t_0}^{t} \frac{M_{sp}}{C} dt.$$

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 FIGE $-28 - 1083$



Fig. 8_17_

Fig. 8.18.

Fig. 8.17. Projections of the vector of instantaneous angular velocity and torque/moments, which act on TES, on rotational axis.

Fig. 8.18. Circuit of action of forces, which rotate spin-stabilized missile.

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Since, as show calculations, $v \ll \dot{\phi}$, term $v_{\cos \delta}$ in the first approximation, can be disregarded.

Taking M_{np} and C constant average values, let us determine the angular rate of rotation relative to the longitudinal axis:

$$\dot{r} \approx r \approx \frac{M_{\rm sp}}{C} \left(\frac{v}{a} - \frac{v_0}{a_0} \right),$$

where a_0 and a - constant longitudinal accelerations of the center of mass of TRS respectively on guides and on powered flight trajectory. If we count $a_0=0$ and $v_0=0$, then, by designating $k=\frac{M_{BP}}{Ca}$, we will obtain the approximate well known equality

(8.82)

where the velocity of the center of mass of projectile v is determined during the solution of the basic problem of external ballistics for the projectile of point variable mass.

Equations (8.80) and (E.81) without sufflementary simplifications are solved only by numerical methods. Their analytical solutions, instituted with a series of assumptions, examined in work [60].

The easily foreseeable analytical solution can be obtained for the artillery shell of constant mass, after assuming in (8.78) $\dot{M}_{\rm Sp}=0$. In this case system of three equations (8.79), (8.80) and (8.81) can be rewritten thus:

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fcr a generalized cocrdinate ¢

$$C \frac{d}{dt} (\dot{\varphi} + \dot{v} \cos \theta) = 0; \qquad (8.83)$$

for generalized coordinate .

$$\frac{d}{dt} [A\dot{v}\sin^{2}b + C(\dot{v} + \dot{v}\cos b)\cos b] = 0; \qquad (8.84)$$

for a generalized coordinate &

 $A^{3} - A^{2} \sin \delta \cos \delta + C(\varphi + \cos \delta)^{2} \sin \delta = M.$ (8.85)

System of equations (8.83-8.65) most frequently is solved under the most real initial conditions, which correspond to the torque/moment of the loss of tight coupling of projectile with the shaft of the artillery instrument: t=0; $\delta_0=0$; $\delta=\delta_0$ and $v=v_0$, is assumed that at zero time there is a precession angle v_0 and the engular velocity of nutation δ_0 , but nutation angle is equal to zero. Under the initial conditions accepted from (8.83) we will obtain

$\dot{\mathbf{v}} + \dot{\mathbf{v}}\cos \mathbf{v} = r_{\mathbf{v}} = \operatorname{const.}$

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Brom equation (8.44) let us have

 $Av \sin^2 \delta + Cr_0 \cos \delta = D.$

Under the initial conditions

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$$D = Cr_{\bullet},$$

$$(1 - \cos^{2} \delta) \dot{v} = \frac{Cr_{\bullet}}{A} (1 - \cos \delta),$$

accepted whence

$$v = \frac{Cr_0}{A(1 + \cos b)} = a.$$
 (8.86)

For stable projectiles with the smallness of angle δ_r it is possible to accept

and

$$\sin \delta \approx \delta \times \cos \delta = 1$$
,

then $p=Cr_0/2\lambda$ and - after integration (8.86) - we will obtain formula for determining the precession angle

$$\mathbf{v} = \mathbf{v}_{\mathbf{0}} + \mathbf{a}t. \tag{8.87}$$

Using already known to us substitution $M = A\beta\delta_0$ we will obtain from (8.71) with the smallness of angle δ

$$\ddot{\delta} + \alpha^{4}\delta - \beta\delta = 0. \tag{8.88}$$

It is accepted to designate:

$$\sigma = 1 - \frac{\beta}{\sigma^2} \tag{6.89}$$

and then

$$\ddot{a} + a^{a} \sigma \vec{a} = 0.$$
 (8.90)

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As it follows from (8.69) and expressions for α and β , coefficient σ - alternating/variable. More detailed investigations show that for small time intervals σ it is possible to accept as the fixed value, after assuming

v = const, $\beta = const$, e = const.

The characteristic equation, which corresponds (8.90), takes the following form:

 $k^a + a^{a_s} = 0.$

With .> 0 we will obtain the known solution

B=Ciele Val + Cee-le Val.

After determining arbitrary constants, we will obtain formula for a nutation angle

$$\delta = \frac{\delta_0}{e \sqrt{\sigma}} \sin \alpha \sqrt{\delta t}, \qquad (8.91)$$

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Thus, with $\sigma > 0$ nutation angle is expressed as trigonometric sine and the motion of the longitudinal axis of projectile bears the character of harmonic oscillations with period of $T = 2\pi/\alpha/\sigma$ and with ECC = 78107112 FICE 33 688

limited amplitude $\delta_{max} = \frac{\delta_0}{\alpha L^2}$. Precessional action in accordance with (8.87) represents by itself the rctation of the plane of resistance arcund taggent to trajectory with an almost constant angular velocity of $v \approx \alpha \approx \frac{Cr_0}{24}$. A change in the nutation argle is determined by equation (8.91) and occurs in the plane of resistance. On the trajectory phases, close to rectilinear, the complex spatial action of the longitudinal axis of the rotating projectile is accomplished relative to vector v and for initial conditions $v_0=0$, $\delta_0=0$, $\delta_0\neq 0$ can be visually illustrated by the curve/graph, constructed in polar coordinates h=f(v) (Fig. 8.19). Angle δ is depicted as the radius-vector whose positicr is determined by angle v. If we examine the motion of the axis of projectile depending on time, then plotted functions $\delta(t)$ and v(t) will take the form, presented in Fig. 8.20a. During processing of experimental data to conveniently examine only the absclute value of nutation angle δ ; therefore curve/graphs $\delta(t)$ they frequently construct crly in resitive half-plane; in this case curve/graph v(t) will obtain stepped form, since with transition δ through zero angle v undergoes the explosion, equal to $\pm \pi$. Constructed similarly curve/graphs $\delta(t)$ and v(t) with the explosion, undertaken equal to -w, are represented in Fig. 8.20b. During a change in the initial conditions of the graph $\delta = f(v), \delta(t)$ and v(t)they will obtain the form, different from that presented in Fig. 8.19 and 8.20.

The integration of equation (8.89) with a>0 led us to formula (8.90), determining the harmonic oscillations of the longitudinal axis of projectile with the limited amplitude. Integration (8.89) with a<0 will lead us to the dependence

 $\delta = -C \left(e^{\alpha \sqrt{|\sigma|}t} + e^{-\alpha \sqrt{|\sigma|}t} \right).$

After the determination of integratics constant, we will obtain

$$= \frac{b}{e \sqrt{|\sigma|}} \operatorname{sh}(e \sqrt{|\sigma|} t).$$

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Fig. 8.19. Curve/graph $\delta = f(v)$ for the straight pertion of the trajectory of the fast-turning projectile with $\delta_0 = 0$, $\delta_0 \neq 0$.

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Thus, with <<0 nutation angle is determined by the function, which unlimitedly increases with an increase in the time, what is the sign/criterion of the instability of projectile. Consequently, the condimination of the correct motion of the rotating projectiles can be expressed by the inequality

$$\sigma = 1 - \frac{\beta}{\sigma^2} > 0, \qquad (8.92)$$

in which coefficient σ it is called the criterion of the gyroscopic stability of projectile on the initial straight portion of the trajectory. Stability condition, on the basis of $\sigma>0$, can be written thus:

 $\frac{\beta}{\alpha^2} < 1. \tag{8.93}$

The values on which depends f, are determined in many respects by the size/dimensions of projectile, by its designation/purpose and the mption characteristics the centers of mess (0.44):

$$\beta = f\left[d^{\alpha}, \ \Phi^{\alpha}, A, l, H(y), K_{M}\left(\frac{v}{a}\right)\right].$$

Therefore to vary by value β for providing inequality (8.93) is virtually impossible. It is considerably simpler, with given one β , it is possible to accomplish of the condition of the gyroscopic statility of projectile by the appropriate selection of the parameter a cr, in accordance with formula (8.86), to angular rate of rotation Ta-

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Fig. 8.20. Curve/graphs of a change of the angles $\delta(t)$ and v(t): a) in the plane of resistance; b) in positive balf-plane.

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Buring motion along threaded/cut shaft, the angular velocity of frejectile at the sement of flight is equal to

$$r_0 = \varphi_0 = \frac{2w_0\pi}{d\gamma}, \qquad (8.94)$$

where η - length of the course of threads in fores.

The axial moment of the inertia of projectile can be expressed as follows:

$$C = \mu \frac{Q}{g} \frac{d^2}{4}, \qquad (8,95)$$

where μ - the coefficient, which characterizes the distribution of

the mass of projectile of its relatively longitudinal axis. The weight of projectile can be expressed in terms of the coefficient of weight C_0 , which has the close of value for the one-type projectiles

 $Q = C_0 d^*$.

where d - a fore in dm, C - weight in kg.

Carrying out replacement a and f, we will obtain from expression (8.93)

$$\eta < \frac{\pi}{2} \sqrt{\frac{\frac{\mu C_Q}{l}}{\frac{l}{d} \frac{A}{C} K_M(\frac{v}{a})}}.$$
 (8.96)

For obtaining the unique solution inequality (8.96) they replace by equality, introducing into it the margin of safety of gyroscopic stability - a.

In this case, it obtains the form

$$\eta = a \frac{\pi}{2} \sqrt{\frac{\frac{l}{l}}{\frac{l}{d} \frac{A}{C} K_M \left(\frac{\bullet}{a}\right)}}.$$
(8.97)

Equality (8.97) was called the formula of Zabudskiy-Venttsel*. Value a<1 concrete/specific/actually is set during design depending on the construction of projectile. In the general case for estimate calculations, it is possible to take a = 0.75 [59]. In practice value n oscillates within limits ~(20-30).

We will now obtain dependence for determining the necessary value of the nozzle cant angle of the nozzle unit of TRS, ensuring

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stable notion projectile (see Fig. 8.16). The value of angle γ can be designed in terms of assigned value $\frac{1}{2} - \frac{1}{2} = r_{\mu}$ as follows.

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The equation of the rotary motion of TRS relative to axis Ox_1 takes the form

$$J_{x_1} \frac{d\hat{\mathbf{v}}}{dt} = P_{\mathbf{y}\mathbf{E}} \frac{d_{\mathbf{x}}}{2} \sin \mathbf{y}, \qquad (8.98)$$

where $P_1 n = |\vec{m}| w_{orn}$ - the reaction force, developed with engine; n number of nozzles; $\frac{d_c}{2}$ - arm of force P_1 relative to the axis of 185.

In this case, composing thrusts $S_{a}(p_{a}-p)$ it is not considered, since it is directed in the first approximatics, along longitudinal axis of TRS and in the formation/education of the torsional moment participation does not accept.

As a result of integrating equation (8.58) under the assumption of constancy $J_{x_1} = J_{x_1, cp}$ we will obtain for the arbitrary point of preserved flight trajectory

$$r = \varphi \approx \varphi_0 + \frac{\Psi_{\text{org}} d_c |\mathbf{m}| (t - t_0)}{2J_{x_1}} \sin \gamma. \qquad (8.99)$$

Hence follows relationship/ratic for the calculation of the

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unknown angle of the slope of the axis of rozzle γ

$$\gamma = \arcsin \frac{2J_{x_1}(\bar{\tau}_k - \bar{\tau}_0)}{\mathbf{w}_{orm} d_c |m|(t_k - t_0)}.$$
 (8.100)

By index "k" are noted the values, which correspond to the enc/lead of powered flight trajectory.

Utilizing the found in a similar approximate manner value of angle γ , one should produce the refined verifying calculation of the angular rate of notation of TRS 4 by the integration of equation (8.98) in parts of active section. The length of each part must be such that within its limits the averaging of the kinematic parameters of the trajectory of TRS, values of its torque/moments of the inertia and other parameters would not introduce large errors into calculation. If necessary according to the results of verifying calculation, it is possible to find correction in the value of nozzle cant angle γ . The given above condition of the gyroscopic stability of TRS (8.92) is obtained for the beginning of the passive (or, that equivalently, for the end/lead of the active) trajectory phase.

Let us examine the special feature/peculiarities of the motion of TRS on powered flight trajectory. Within the limits of this section, continuously grow/rises the velocity v of the center of mass of TRS and in essence on it the coefficient & depending, which characterizes the tilting moment N. But also continuously in
accordance with (8.82) grow/rises the angular rate of rotation of TRS r = 4 and the determined by it angular velocity of precession $\sqrt{4}$. As a result of this, it proves to be that the criterion of stability of TRS $e=1-\beta/a^2$ on active section is changed insignificantly, having weakly grow/risen toward the end of the section. In this case, occurs the decrease of period and amplitude of nutsticmal oscillations of TBS, caused by the initial disturbances. Thus, with sufficient accuracy it is possible to count that if is provided the stability of TBS at the end of powered flight trajectory, then it will be stable also on all section.

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The numerical value of stability criterion and other motion characteristics of TRS on the active part of the trajectory can be approximately designed on the individual sections of trajectory with the use of formulas (8.44), (8.86), (8.69), (8.67) and (8.97) [17].

The stability condition of rotating projectile (8.93) is obtained for motion on the straight portion of trajectory. With an increase in the path curvature, the dynamic axis of equilibrium will all more move away from velocity vector by angle δ_{μ} . Angle δ_{μ} is defined by formula (8.77). EOC = 78107112 FAGE -42 697

The analysis of the values, entering formula (8.77), shows that angle ϕ_p along the curvilirear trajectory phase changes, moreover in area of peak of the trajectory, where value $\cos \theta$ is close to greatest, and $H(y) v^3 - to$ the smallest, it reaches the greatest value. During the incorrect selection of the rate of spin of projectile, angle ϕ_p in peak of the trajectory can become so large that it will lead to the loss by the projectile of stability. Consequently, angle ϕ_p must be considered as independent criterion, evaluating the stability of the rotating projectiles on the curvilinear section of the trajectory. The comparison of dependences (8.77) and (8.92) for criteria of stability e and ϕ_p shows the discrepancy of the latter.

It is real/actual, for the increase of the stability of the retating projectiles in the initial rectilizear cut of trajectory the speed of its rotation $r=\hat{\bullet}$ sust be increased. This will cause appropriate increase \hat{v} and angle δ_p and, therefore, it will lead to the decrease of the stability of projectile in peak of the trajectory.

To combine both stability conditions for the projectiles whose the length of 5-6 bores, accomplishes at angles of departure to 60-65°.

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In this case for the stabilization of projectile along an entire trajectory it suffices to select the speed of its rotation in the beginning of passive section, in order to obtain criterion of stability ϵ equal to $\sqrt[3]{a_1}=0.60-0.66$.

Let us note one additional special feature/peculiarity of the trajectories of the projectiles, which are stabilized by rotation. The existence of angle δ_p leads to the fact that the longitudinal axis of projectile the large part of the flight time is located through one of the sides of vector \overline{v} and projectile is deflect/diverted sideways from plane of reference of firing. The systematic lateral deviation of the fast-turning projectiles is called derivation. During right spin of projectile, the dynamic axis of equilibrium is deflect/diverted to the right from vector (if we lock in the direction of motion) and this leads to right derivation. During counterclockwise rotation projectile it will depart to the left from range plane, i.e., derivation will be left. The derivation of projectile can be determined and experimentally and by calculation.

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From theoretical formulas (in connection with the projectiles of constant mass) are known the formulas of A. N. Brylov, V. I.

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Cstapovich, and P. P. Eersnev, formula of E. A. Venttsel' and the calculation method, projected by V. N. Persbin. One Of the most widely known formulas is A. N. Krylov's formula

$$z = f \frac{g}{Q} \frac{2nC}{\lambda R \psi t} \Phi_0 \int (\varphi - \varphi_0) dt, \qquad (8.101)$$

where (besides well-known values) R = semi-caliter of projectile; $\phi = angle$ between the tangent to trajectory and the vertical line.

Permula contains the values, determined from the experiment: the unknown it is accurate value h, prepertional to distance between centers of mass and the resultant pressure of projectile, and the coefficient of agreement with experiment f. For one-type projectiles and trajectories, the formula gives satisfactory results. More compon/general/total is the formula of V. N. Pershina, who considers the phenomenon of the attenuation of the rate of spin of projectile for flight time.

Let us examine the calculated determination of derivation. The equation of motion of the center of mass of projectile along the axis z_r , directed perpendicular to range plane, will take the form

$$\frac{Q}{z} = R_{z} - X \cos(v, z), \qquad (8.102)$$

where R_{p} - lateral force, caused by angle i_{p} : $X_{\cos(v,z)}$ - the projection of drag on z axis.

It is possible to replace ccs (v, z) = z/v and then

$$z = \frac{g}{Q}R_z - \frac{g}{Q}X\frac{\dot{s}}{v}.$$
 (6. 103)

Let us replace also $(g/\zeta) X = c E(y) F(v)$ and, remembering that G(v) = F(v) / v, we will obtain

$$\ddot{z} = \frac{g}{Q} R_z - cH(y)Q(v)\dot{z}.$$
 (8.104)

Taking into account the relatively small rath curvature, determined by derivation, it is possible to use the dependence, valid for a planar trajectory,

$$-cH(y)Q(v)=\frac{i}{s}$$

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Then from (8.104) it is possible to write

$$\ddot{z} - \frac{\dot{a}\dot{z}}{u} = \frac{g}{Q} R_{z} \qquad (8.106)$$

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$$\frac{d}{dt}\left(\frac{\dot{z}}{u}\right) = \frac{g}{Q}R_{z}.$$
 (8.106)

Integrating, we will citain

$$:= \frac{R}{Q} \int_{0}^{1} u \, dt \int_{0}^{1} \frac{R_{i}}{u} \, dt.$$
 (8. 107)

Let us discover content R_p . It is usually assumed that the force, which deflects the rotating projectile from the plane of casting, is proportional to angle δ_p : then in accordance with the overall theory of the aerodynamic similarity

$$R_{s} = \frac{dl}{s} \ 10^{s} H(y) \, v^{s} K_{N} \delta_{p}. \tag{8.108}$$

Let us introduce in (8.108) value δ_p , using (8.76), and let us discover values α and β . Argular rate of rotation and the axial scenet of the inertia of artillery shell let us define according to (8.94). and (8.95); value $|\hat{\theta}|$ we defire as usually. Furthermore, let us remember that $\cos \theta = u/v$. Then, without taking into account the attenuation of rotation under the action of the torque/moment of surfice friction, we obtain for the calculation of derivation the following expression:

$$z = \frac{\pi g}{2} \frac{\mu l}{\eta k} v_0 \int u \, dt \int \frac{K_N}{K_M} \frac{dt}{v^2} \,. \qquad (8.109)$$

Having experimental aerodynamic characteristics $K_N\left(\frac{v}{a}\right)$ and $K_M\left(\frac{v}{a}\right)$ and obtained by calculation for the planar trajectory of value $v=f_1(t)$ and $u=f_2(t)$, it is possible to numerically calculate the integral of right side (8.109) and to calculate derivation.

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Let us consider the attenuation of rotation. Decrease of the angular rate of rotation of projectile relative to longitudinal axis in the process free flight is explained on the action of the torque/moment of surfice friction (2.113).

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The most widely known formulas, which determine the angular velocity of the projectile in the process of motion, they are: formula Roggla

$$\boldsymbol{\omega}_{t} = \boldsymbol{\omega}_{t} \boldsymbol{\varepsilon}^{t}, \qquad (8.110)$$

where ω_0 - initial angular velocity of projectile; e - Naperian base; v - length of projectile in hores; d - bore; C - axial moment of the inertia of projectile; t - missile flight time, and L. A. Slezkin's theoretical formula

$$w = w_0 e^{-0.598 \frac{dt'_{10}}{2Q} \int v'_{10} dt}, \qquad (8.111)$$

where, besides designations indicated above, 1 - length of projectile and v - velocity of the center of mass.

Both formulas can be given to the form:

where k - the calculated cr experimental ccefficient, which corresponds to the selected corcrete/specific/actual projectile. Since the initial angular rate cf rotation $\omega_0 = r_0$ by us is already taken into account in (8.105), the attenuation cf rotation let us consider by introduction into the right side of factor e^{-kt} . Then

$$z = \frac{\pi g}{2} \frac{\mu l}{\tau_{h} t} v_{0} \sqrt{\frac{\mu}{0}} u \, dt \sqrt{\frac{K_{N}}{K_{M}}} \frac{e^{-bt}}{v^{2}} \, dt. \qquad (8.113)$$

The obtained formula can be given to a simpler form. Taking into account attenuation for lateral force, it is possible to accept the dependence

$$R_{z} = \frac{2\pi C}{\pi d} v_{0} \frac{i}{kd} \frac{K_{N}}{K_{M}} |\dot{0}| e^{-kt}. \qquad (8.114)$$

Compiling an equation of yawing motion along type (8.106) and designating constants through B, we will obtain

$$u \frac{d}{dt} \left(\frac{\dot{s}}{u} \right) = B \dot{|\mathbf{b}|} e^{-bt}.$$
 (8.115)

Bellowing V. N. Pershir, after a series of simplifications in the process of double integration, it is presible to obtain

$$z = m_a \frac{k_2}{k_1} v_0 \int_0^t |0 - 0_0| dt.$$
 (8.116)

The variable coefficients, confronting the integral in (8.116), have mames, determined by their physical context.

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The coefficient of the derivation

$$m_{a} = \frac{g}{Q} \frac{2\pi C}{vd} \frac{l}{hd} \left(\frac{K_{N}}{K_{M}}\right)_{cp}, \qquad (8.117)$$

where $\left(\frac{K_N}{K_M}\right)_{cp}$ - average value of relation for the interval of integration.

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Curvature of the trajectory

$$k_1 = \frac{1}{2} \left(1 - \frac{u_0}{u_0 - u} \ln \frac{u}{u_0} \right). \tag{8.118}$$

Acderating ratio of the rotation

$$k_{g} = \left(\frac{2e^{-kt}}{k^{2}t^{2}} - \frac{2}{k^{2}t^{2}} + \frac{2}{kt}\right). \tag{8.119}$$

Calculation is simplified during the application/use of special tables for an integral and coefficients $k_1 \in \mathbb{R}$ and k_2 .

The calculation of the derivation of the spin-stabilized missiles on the passive section of the trajectory can be carried out recently by the described method. After initial velocity v_0 , the angle of departure θ_0 and initial angular rate of rotation $r_0=w_0$ should take the values of these quantities at the end of powered flight trajectory. For short powered flight trajectories, the derivation is insignificant and can be disregarded. If necessary for

calculation into the right side of equation (8.103) one should introduce term, that considers thrust.

§5. Conditions of the stable flight of the enguided specific rockets and of projectiles.

The finned unquided projectiles of constant and variable mass (mine, aircraft bomb, rccket, etc.) for providing the angular stabilization must possess the necessary steady-state stability factor, determined by dependence (8.2). During the solution of two-dimensional problem, the maximum angle of deflection of the loggitudinal axis of the fin-stabilized projectile from the velocity vector of the center of mass can be determined from formula (8.7). The stable flight of the fir-stabilized projectile with the smallest ressible for given construction scatter of scirts of an incidence/drop is assured when the maximum amplitude of angle, determined (8.7), will not be it exceeds certain assigned magnitude. It is necessary to keep in sind that dependence (8.7) is obtained for a planar trajectory with essential assumptions. By works of V. S. Fugachev, V. D. Kiricherko and other authors is shown, that the fin-stabilized projectiles complete three-dimensional/space cscillations. Of finned tc introduce essential features into the character of their motion relative to the center of mass. Therefore let us examine separately the action of the fir-stabilized

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projectiles of constant mass and nockets.

The characteristics of the spatial motion of fin-stabilized rockets and projectiles can be obtained by the numerical solution of the matching systems of the differential equations, given in chapter III and V. For the purpose of obtaining some generalizing conclusion/derivations, let us examine here analytical solutions.

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5.1. Notion of the fin-stabilized projectile of constant mass relative to the center of mass.

The characteristics of the forward motion of the center of mass let us assume known. From the acting factors let us consider stabilizing and damping torque/moments. The position of longitudinal axis let us determine by two angles δ_1 and δ_2 (Fig. 8.21). The position of the velocity vector of the center of mass let us determine by angle 0. Figures 8.21 depicts also angular velocity vectors $\dot{\theta}$, $\dot{\delta}_1$, $\dot{\delta}_2$ and the vector of statilizing moment M. Let us comprise differential equations in the form of the equations of Lagrange.

The vector of instantaneous angular rate cf rotation is

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determined by the sum

 $\overline{Q} = \overline{\delta_1} - \overline{\delta_2} + \overline{0}.$

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Since the senses of the vector angular velocities δ_1 , δ_2 and θ the same as in Fig. 8.13, the expression for kiretic rotational energy let us write, after using (E.44), after assuming in it $\dot{\phi}=0$

$$T = \frac{1}{2} \left| A \left[\dot{\mathbf{s}}_{1}^{2} + \left(\dot{\mathbf{0}} + \dot{\mathbf{s}}_{2} \right)^{2} \right] + C \left(\dot{\mathbf{s}}_{2} - \dot{\mathbf{0}} \right)^{2} \dot{\mathbf{s}}_{1}^{2} \right|. \qquad (8.120)$$

Let us bear in mind, that here, just as during obtaining (8.44), accepted with the smallness of angle & that six & does &

$$\mathcal{M} = -\frac{\varrho v^2}{2} Slc_m^{i} \delta, \qquad (8.121)$$

Basping moment -

$$\overline{D} = -\rho v S \overline{P}_{C_D} \overline{Q}. \tag{8.122}$$

Generalized forces in the case in question will be the projections of vectors M and D on the directions of angular rates of rotation δ_1 and δ_2 .

For the stabilizing scsent

$$Q_{ML} = M \sin v \cos k_{\rm s}; \quad Q_{ML} = M \cos v. \tag{8.123}$$

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Fig. 8.21. Schematic of cccrdirates and angles, which determine the position of the longitudinal axis of the fir-stabilized projectile.

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Equalizing $\cos \delta_2 = 1$, $\delta_1 = \delta \sin v$, $\delta_2 = \delta \cos v$, we can write

$$Q_{M_{\ell_1}} = -\frac{\varrho v^2}{2} S/c_m^{t} \delta_1 \ \text{i} \ Q_{M_{\ell_2}} = -\frac{\varrho v^2}{2} S/c_m^{t} \delta_2. \qquad (8.124)$$

The damping moment decreases angular flutter speed of the fin-stabilized projectile. its projections on the axis of elementary rotation will be proportional to the projection of the angular velocity of rotary motion or the appropriate axes

 $Q_{Di_1} = -\varrho v S \ell^2 c_D \dot{\mathbf{s}}_1; \quad Q_{Di_2} = -\varrho v S \ell^2 c_D (\dot{\mathbf{s}}_2 + \dot{\mathbf{0}}), \quad (8.125)$

Bet us pass to the compilation of the equations of rotary motion in the form of equation (1.33).

For the compilation of equation for generalized coordinate δ_1 , we differentiate (8.120) on $\dot{\epsilon}_1$ and $\dot{\epsilon}_1$. The conducted investigations for min of usual form showed that in equation (8.120) second term of right side comprises not more than 1c/c of the first and it subsequently can be disregarded. Then

$$T = \frac{A}{2} \left[\dot{\delta}_{1}^{2} + (\dot{\theta} + \dot{\delta}_{2})^{2} \right]. \qquad (8. 126)$$

After differentiation we will obtain

$$\frac{\partial T}{\partial \mathbf{b}_1} = 0; \quad \frac{\partial T}{\partial \mathbf{b}_1} = \mathbf{A}\mathbf{b}_1.$$

Let us introduce the designations

$$\mathbf{a} = \frac{\varrho S/e_m^2}{2A}; \quad \beta = \frac{\varrho S/2e_D}{A}. \quad (8.127)$$

After substitution in (1.33), let us have

$$\ddot{\mathbf{\delta}}_1 + \beta \boldsymbol{v} \boldsymbol{\delta}_1 + \boldsymbol{a} \boldsymbol{v}^2 \boldsymbol{\delta}_1 = 0. \tag{8.128}$$

Bet us comprise equation for generalized coordinate δ_2 . We differentiate (8.126) or δ_2 and $\dot{\delta}_2$

$$\frac{\partial T}{\partial b_2} = 0; \quad \frac{\partial T}{\partial \dot{b}_2} = A(\dot{b}_2 + \dot{\theta}).$$

Substituting the results of differentiation and expression for the generalized forces, which act with respect to coordinate δ_2 , in

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(1.33), after conversion we will obtain

$$\ddot{\mathbf{s}}_{s} + \beta \boldsymbol{v} \dot{\mathbf{s}}_{s} + \boldsymbol{a} \boldsymbol{v}^{\mathbf{a}} = -\ddot{\boldsymbol{v}} - \beta \boldsymbol{v} \dot{\boldsymbol{v}}. \qquad (8.129)$$

Equations (8.128) and (8.129) include variable coefficients and without supplementary simplifications are solved only numerically.

Let us give analytical sclution, after specifying the necessary assumptions. equation (8.129) is a nonhomogeneous linear second order equation with variable coefficients.

Fage 358.

The form of equation shows that in vertical plane the free cscillations nims are placed on forced cscillations, determined by the right side of equation (8.129). For the solution of equation, let us find $\dot{\theta}$ and $\ddot{\theta}$. During the definition of these values, it is possible to consider the fir-stabilized projectile as material point, then θ is determined by the second equation of system (5.3), and $\ddot{\theta}$ ty dependence (8.57).

Earge interest represents the case of the structurally developed tail assembly, when during determination θ and $\tilde{\theta}$ it is necessary to consider bouyancy effect Y. For the half-speed system of coordinates, for example, it is possible to write

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$$\dot{\theta} = -\frac{g\cos\theta}{v} + \frac{g}{Qv}Y^{*} \qquad (8.130)$$

[see, for example, equation (3.8) with F=0 and $Y_p=0$]. Here Y*=Y**a.

Transfer/converting to angle δ_{20} we will obtain

$$)^{*} =)^{*'} \delta_{\mathbf{g}} = \frac{\rho v^2}{2} S c_{y}^{i_1} \delta_{\mathbf{g}}$$

and

$$\dot{\theta} = -\frac{\rho \cos \theta}{v} + \gamma \sigma \delta_{\mu}, \qquad (8.131)$$

(8. 130)

where $\gamma = \frac{g}{Q} \frac{QS}{2} c_{\mu}^{i_{\mu}}$.

Differentiating (8.131), let us have

$$\ddot{\theta} = \frac{\varepsilon \sin \theta}{v} \dot{\theta} + \frac{\varepsilon \cos \theta}{v^2} \dot{v} + \gamma \delta_3 \dot{v} + \gamma v \dot{\delta}_3. \qquad (8.132)$$

Since v and, consequently, also v, depend on aerodynamic forces, the, obviously, further analytical sclution is possible only during introduction for the air resistance of analytical dependence. Let us assume that

$$v = -bv^2 - g\sin\theta, \qquad (8.133)$$

where for the relatively short trajectory place it is possible to ccunt

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$$b = \frac{e_s}{20} c_x(M, Re) = const.$$

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After substitution (8.131) and (8.133) in (8.129) let us have

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$$\ddot{\theta} = \gamma v (\dot{\theta}_g - b v \theta_g) - \frac{g \cos \theta}{c^2} (2g \sin \theta + b v^2). \qquad (8.134)$$

Utilizing (8.131) and (8.134), let us write expression for the right side of equation (8.129)

$$-\ddot{\theta} - \varrho \, \upsilon \dot{\theta} = \gamma (b - \beta) \, \upsilon^{\theta_{\theta_{g}}} - \gamma \upsilon \dot{\theta}_{g} + \rho \, \cos \theta \, \left(b + \beta + \frac{2g \sin \theta}{\sigma^{2}} \right). \quad (8. \ 135)$$

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After substituting (8.135) into equation (8.129), after conversions we will obtain:

$$\ddot{\mathbf{a}}_{s} + (3+\gamma) \, \boldsymbol{v} \dot{\mathbf{b}}_{s} + [\boldsymbol{a} - \gamma(\boldsymbol{b} - \beta)] \, \boldsymbol{v}^{s} \boldsymbol{b}_{s} = \boldsymbol{g} \cos \theta \left(\frac{3+\boldsymbol{b} + \frac{2\boldsymbol{g} \sin \theta}{\boldsymbol{v}^{2}}}{\boldsymbol{v}^{2}} \right).$$
(8.136)

Term $y(b-\beta)$ does not exceed 1070 of a and it it is possible not to consider; then

$$\delta_{2} \div (3+\gamma) v \delta_{2} \div a v^{\theta} \delta_{2} = g \cos \theta \left(3 + b \div \frac{2g \sin \theta}{v^{2}} \right). \quad (8.137)$$

Thus, the spatial motion of the fin-stabilized projectile of constant mass is described by two differential equations - (8.128)

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and (8.137). Since the cross aerodynamic and inertial couplings by us were not considered that equations (8.128) and (8.137) were not connected and can be solved separately. Recall that with variable coefficients with the minimum number of assumptions equation (8.137) is solved only numerically.

The analytical solution of nonhomogeneous linear differential equation (8.137) with constant coefficients is equal to the sum of solutions - general solution of the homogeneous equations (without right side) and of the particular solution of complete equation. Let us accept for the analytical solution v=corst, $c_x(M, Re)$ =const, $c_{y^*}^{L}$ =const, c_D =const. It is obvious, for providing sufficient accuracy of practical solution, it is necessary the numerical values of the named quantities to determine for the separate small trajectory phases.

Henogeneous differential equations - $(\pounds, 12\pounds)$ and $(\vartheta, 137)$ (written without right side), take the form of equation $(\vartheta, 40)$.

For equation (8.128) $2k_1 = fv_r$, $n^2_1 = ev_2$;

for equation (8.137)

 $2k_2 = (\beta + \gamma)v; \quad n_2^2 = av^2.$

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If we consider side component aerodynamic drag, then the equation of the transverse cscillations of mine it will take the form, similar to equation (8.137)

$$\vec{b}_1 + (\beta + \gamma) v \vec{b}_1 + \alpha v^2 \vec{b}_1 = 0;$$
 (8.138)

then in both cases

The relatively weak oscillation damping the unguided fin-stabilized projectiles in air redium almost always leads to case $r^{2}-k^{2}>0$.

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Then the solution of equations (8.128), (8.138) will take the form of sclution (8.41)

$$\mathbf{a}_{1} = A_{1}e^{-\frac{(\beta+\gamma)\nu}{2}t} \sin\left(\frac{\nu}{2}\sqrt{4\alpha-(\beta+\gamma)^{2}}t + \epsilon_{1}\right); \quad (8.139)$$
$$\mathbf{a}_{2} = A_{2}e^{-\frac{(\beta+\gamma)\nu}{2}t} \sin\left(\frac{\nu}{2}\sqrt{4\alpha-(\beta+\gamma)^{2}}t + \epsilon_{2}\right). \quad (8.140)$$

In the general case under initial conditions $\varepsilon_{10}\neq 0$, $\delta_{20}\neq 0$, $\dot{\delta}_{10}\neq 0$, $\dot{\delta}_{20}\neq 0$, we will obtain following dependences for determination of the maximum amplitudes:

$$A_{1} = \sqrt{\frac{\delta_{10}^{2} + \frac{[2\delta_{10} + (\beta + \gamma) \upsilon \vartheta_{10}]^{2}}{\upsilon^{2} [4\alpha - (\beta + \gamma)^{2}]}}; \quad (8. 141)$$

$$A_{s} = \sqrt{\frac{\delta_{20}^{2} + \frac{[2b_{30} + (\beta + \gamma) v^{2} s_{30}]^{2}}{v^{2} [4\alpha - (\beta + \gamma)^{2}]}}.$$
 (8. 142)

Phase shifts are determined from the formulas

$$s_1 = \operatorname{arctg} \frac{b_{10}v \sqrt{4a - (b + \gamma)^2}}{2b_{10} + (b + \gamma)vb_{10}};$$
 (8. 143)

$$s_{2} = \arctan \frac{b_{20}v \sqrt{4a - (\beta + \gamma)^{2}}}{2b_{20} + (\beta + \gamma) v b_{20}}.$$
 (8. 144)

In the particular case, with $\delta_{10} \neq 0$; $\delta_{2c} \neq 0$; $\delta_{10} = \delta_{20} = 0$, we will citain

$$A_{1} = \frac{b_{10}}{\sqrt{1 - \frac{(\beta + \gamma)^{2}}{4\alpha}}}; \quad A_{n} = \frac{b_{20}}{\sqrt{1 - \frac{(\beta + \gamma)^{2}}{4\alpha}}}.$$

Eut if $\delta_{10} \neq \delta_{20} = 0$; $\dot{c}_{10} \neq 0$; $\dot{c}_{20} \neq 0$, then

$$A_{1} = \frac{b_{10}}{v \sqrt{4a - (\beta + \gamma)^{2}}}; \qquad A_{0} = \frac{b_{00}}{v \sqrt{4a - (\beta + \gamma)^{2}}}.$$

In the general case the total angle

$$\delta = \sqrt{\frac{2}{3} + \frac{2}{3}} = \frac{2}{3} \sqrt{\frac{1}{3} \sin^2(\omega + e_1) + A_3^2 \sin^2(\omega + e_2)}} \qquad (8.145)$$

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where $=\frac{v}{2}\sqrt{4\varepsilon-(l+\gamma)^2}$.

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Angle, which determines the position of instantaneous plane of vitration

$$v = \operatorname{arctg} \frac{b_1}{b_2} = \operatorname{arctg} \frac{A_1 \sin\left(\frac{v}{2}\right) \overline{4a - (\beta + \gamma)^2 t} + \epsilon_1}{A_2 \sin\left(\frac{v}{2}\right) \overline{4a - (\beta + \gamma)^2 t} + \epsilon_2}$$

Let us determine form curved, by the described longitudinal axis of the fin-stabilized projectile, on image plane of unit radius in coordinates δ_1 and δ_2 . In equations (8.139) and (d.140) second terms of radicands considerably less the first and for simplicity of the writing of formulas and calculations can be lowered. Also we will not consider identical factors $e^{-\frac{(\theta+1)er}{2}}$, which characterize damping cscillations. Taking into account the made of characteristics of equation (8.139) and (8.140) they can be converted thus:

$$\frac{b_1}{A_1} = \sin v \sqrt{at} \cos \varepsilon_1 + \cos v \sqrt{at} \sin \varepsilon_1; \qquad (8. 146)$$

$$\frac{b_2}{A_2} = \sin v \sqrt{at} \cos \varepsilon_2 + \cos v \sqrt{at} \sin \varepsilon_2. \qquad (8. 147)$$

Let us multiply (8.146) cp $\sin s_1$, (8.147) - on $\sin s_1$, and let us deduct from the first equation the second. After this we will obtain

$$\frac{b_1}{A_1}\sin\varepsilon_2 - \frac{b_2}{A_2}\sin\varepsilon_3 = \sin \tau \sqrt{a} \sin (\varepsilon_2 - \varepsilon_3), \qquad (8.148)$$

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Further (8.146) let us multiply CP coses. (8.147) - on cos \mathcal{E}_{j} , let us deduct from the first equation the second and we will obtain:

$$\frac{b_1}{A_1}\cos\epsilon_3 - \frac{b_2}{A_2}\cos\epsilon_3 = \cos v \,\sqrt{e} t \sin(\epsilon_3 - \epsilon_3) = \\ = -\cos v \,\sqrt{e} t \sin(\epsilon_3 - \epsilon_3). \quad (8. 149)$$

Let us square of equation (8.148) and (8.149) and it is added them, after which let us have

$$\frac{b_1^2}{A_1^2} + \frac{b_2^2}{A_2^2} - \frac{2b_1b_2}{A_1A_2} \cos{(a_1 - a_2)} = \sin^2{(a_2 - a_3)}.$$
 (8. 150)

The obtained equation is an equation of ellipse. Using (8.139) and (8.140), it is possible to construct curve, described by the longitudinal axis of the fir-stabilized projectile on image plane in coordinates δ_1 and δ_2 . Curve will take the form, presented in Fig. 8.22.

The maximum amplitudes A_1 and A_2 are determined by the aerodynamic diagram of the fin-stabilized projectile, by its size/dimensions and initial conditions. Prequently the maximum values A_1 and A_2 are selected as the criteria of the stable flight of the fin-stabilized projectile on the initial trajectory phase.

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Their value sust not exceed the saxisus persissible values of angles $\delta_{1,max}$ and $\delta_{2,max}$. establish/installed from knowr, good itself

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recensended specimen/samples.

Scr the newly design/projected fin-statilized projectiles is necessary the observance of the conditions

$$A_1 < \delta_{1max}; A_2 < \delta_{2max}.$$
 (8. 151)

These inequalities are called of the conditions of the limitedness of amplitude on the initial trajectory phase.

The oscillation of the longitudinal axis of the fin-stabilized projectile on the curvilinear trajectory phase occurs relative to the dynamic axis of equilibrium whose position can be determined the particular solution of equation (8-137). By the numerical solutions it is shown that in the majority of the cases in (8.137) it is possible to accept $\delta_2=0$ and $\delta_2=0$. Then from (8.137) we obtain the particular solution, which determines the argle of the dynamic equilibrium

$$\mathbf{\delta_{sp}} = \frac{g\cos\theta}{av^2} \left(\beta + b + \frac{2g\sin\theta}{v^2}\right). \quad (8.152)$$

The oscillatory motion of the longitudinal axis of the fin-stabilized projectile (with angle δ_2) is realized relative to the axis dynamic equilibrium whose positics relative to velocity vector v, in turn, is determined by angle δ_{2p} .

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On the initial trajectory phase, angle δ_{2p} is close to zero. Great value angle δ_{2p} will have in aper/verter area of high-angle tajectories, however, as a rule, and there it is small. Therefore the fin-stabilized projectile whose maximum amplitudes on angles δ_1 and δ_2 answer conditions (8.151), will be stable also on the initial trajectory phase and in its aper/vertex.

In the study of the problems of the stability of the real specimen/samples of the fin-stabilized projectiles, it is necessary to keep in mind, that even so their low asymmetry, caused by technological errors, can lead to the resonance increase of the angles of attack and slip. In connection with this statically stable projectile, in the presence even of low asymmetry, can render/show virtually unstable. For the study of this question, one should turn to periodic publications and monographic literature, see, for example, [7]. DOC = 78107112

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Fig. 8.22. Graph/diagram of dependence $\delta_2 = f(\delta_1)$ for the fin-stabilized projectile.

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5.2. Bateral motion of the fin-stabilized projectiles of variable mass.

Let us examine the yawing motion of the finned unguided rockets on powered flight trajectory. Accepting, that for similar rockets aerodynamic drag R<<P and gravitational force the yawing motion affect weakly, during the compilation of equations of motion, let us consider only thrust P, the aerodynamic statilizing moment - M_{ν_i} and the moment of thrust $M_{\tau\nu_i}$, determined by its eccentricity e:

 $M_{\tau_{H_1}} = Pe$.

in this case we consider that torque/scmeat $M_{\tau_{F}}$, acts only in the plane of yawing motion. (The factors indicated in essence

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determine scattering trajectories are unguided rockets in side direction [13]).

Bor determining the position of the axis of rocket and velocity vector the center of mass in the plane of motion, let us introduce angles $\Psi_{\rm R}$, $\psi_{\rm R}$ and δ (Fig. 8.23).

The system of differential equations, which describes the motion of the rocket with the adopted assumptions, takes the following form:

$$m \frac{dv}{dt} = P \cos \delta; \quad mv \frac{d\Psi_{\tau}}{dt} = P \sin \delta;$$

$$J_{y_1} \frac{d\Psi_{x}}{dt^2} = M_{y_1} + M_{\tau y_1}.$$
(8. 153)

Cn the smallness of angle δ , let us replace sir $\delta \approx \delta$ and cos $\delta \approx 1$, after which will rewrite last/latter system that:

$$\frac{dv}{dt} = \frac{P}{m}; \quad v \frac{d\Psi_{\lambda}}{dt} = \frac{P}{m} \delta;$$

$$\frac{d^2\psi_2}{dt^2} = \frac{1}{J_{y_1}} \left(M_{y_1} + M_{\tau y_1} ;; \quad \psi_{\lambda} = \Psi_{\lambda} + \delta. \right)$$
(8. 154)

If, following work [13], to designate $\frac{M_{y_1}}{J_{y_1}} = k^2 v^2 \delta$, to present $v\delta = u$ and as argument to select path of s_v then system of equations (8.154) can be brought to one equation

$$\frac{d^2u}{ds^2} + h^2 u = \frac{M_{\tau y_1}}{J_{y_1} v}.$$
 (8. 155)

This equation is irregrated by well known method under the following initial conditions

$$(\mathbf{z})_{s-s_0} = \mathbf{z}_0 \underbrace{\mathbf{x}}_{\mathbf{0}} \left(\frac{d\mathbf{z}}{ds} \right)_{s-s_0} = \mathbf{z}_{0s}'.$$

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Fig. 8.23. Schematic of the angles, which determine the position of longitudinal axis and velocity vector the center of mass of rocket in the plane of motion.

Key: (1). Direction of the axis of launcher. (2). Trajectory of center of mass.

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It is obvious, with fixed launch vehicle at zero time

$$\boldsymbol{u}_{\boldsymbol{\theta}} = \boldsymbol{v}_{\boldsymbol{\theta}} \boldsymbol{\delta}_{\boldsymbol{\theta}} = \boldsymbol{v}_{\boldsymbol{\theta}} \boldsymbol{\psi}_{\boldsymbol{x}\boldsymbol{\theta}}. \tag{8.156}$$

Furthermore, from the first, second and fourth equations of system (8.154) it is possible obtain

$$\frac{d\psi_{\lambda}}{dt} = \frac{du}{ds},$$

whence for initial conditions one should

$$u_{0s}' = \phi_{s0}.$$
 (8. 157)

The general solution of differential equation (8.155) takes the form

$$u = u_{0} \cos k (s - s_{0}) + \frac{u_{0s}}{k} \sin k (s - s_{0}) + \frac{1}{k} \int_{s_{0}}^{s} \sin k (s - \sigma) \frac{M_{W_{1}}(\sigma)}{J_{y_{1}}(\sigma) v(\sigma)} d\sigma. \qquad (8.158)$$

Beglacing respectively u_0 and u'_{00} , we will citain comprising of the complete value of variable $u=v^8$

$$u_{N_{\tau}} = \frac{1}{k} \int_{s_{0}}^{s} \sin k(s-s) \frac{M_{y_{1}}(\sigma)}{J_{y_{1}}(\sigma) v(\sigma)} d\sigma;$$

$$u_{\phi_{x0}} = \phi_{x0} v_{0} \cos k(s-s_{0});$$

$$u_{\psi_{x0}} = \frac{\psi_{x0}}{k} \sin k(s-s_{0}).$$
(8.159)

After replacement the complete value of u can be written thus:

$$u = u_{M_T} + u_{\mu_{a0}} + u_{\mu_{a0}}. \tag{8.160}$$

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Bespectively let us have

$$\delta_{M_{\tau}} = \frac{u_{M_{\tau}}}{v}; \quad \delta_{\psi_{A0}} = \frac{u_{\psi_{A0}}}{v}; \quad \delta_{\psi_{A0}} = \frac{u_{\psi_{A0}}}{v} \text{ and } \quad \delta = \frac{u}{v}. \quad (8.161)$$

For determining the angle Ψ_{π} we take the second equation of system (8.153) let us pass to independent by the variable s

$$\frac{d\Psi_{\chi}}{dS} = \frac{Pb}{mv^2}.$$
 (8. 162)

Integrating, we will obtain

$$\Psi_{a} = \int_{a}^{s} \frac{\rho_{b}}{mv^{2}} ds. \qquad (8.163)$$

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If we assume the absence of initial angular disturbances, i.e., $\psi_{x0}=0$ and $\dot{\psi}_{x0}=0$, then of (8.159) and (8.161) we can write

$$\delta = \delta_{M_{\tau}} = \frac{1}{kv(s)} \int_{s_{\tau}}^{s} \sin k(s-\tau) \frac{M_{\tau,y_{\tau}}(a)}{J_{y_{\tau}}(a)v(a)} d\tau. \qquad (8.164)$$

Accepting the motion of the center of mass of the rocket on powered flight trajectory uniformly accelerated, i.e., considering that $v(s) = \sqrt{\frac{2}{\frac{P}{m}s}}$ and $v(s) = \sqrt{\frac{2}{\frac{P}{m}s}}$, accepting $J_{\mu_1}(s) = J_{\mu_2} = \text{const}$ and $M_{\tau,\mu_2}(s) = \frac{P}{m}e = \text{const}$, we will obtain:

$$\delta = \frac{em}{2kJ_{\nu,1}} \int_{s_{\nu}}^{s} \frac{\sin k \left(s-s\right)}{1 \cdot s} ds. \qquad (8.165)$$

Substituting last/latter equality in (E.163), we will obtain

$$\Psi_{k} = \frac{em}{4kJ_{y_{1}}} \int_{s_{0}}^{s} \frac{ds}{\sqrt{s^{3}}} \int_{s_{0}}^{s} \frac{\sin k(s-s)}{\sqrt{s}} ds. \qquad (8.166)$$

Let us discover the sine of difference under integral sign in equality (8.165), remembering that s - constant value for concrete/specific/actual solution. Then we obtain

$$\delta = \frac{em}{2kJ_{\nu_1}} \left(\frac{\sin ks}{\sqrt{s}} \int_{s_0}^{s} \frac{\cos ks}{\sqrt{s}} ds - \frac{\cos ks}{\sqrt{s}} \int_{s_0}^{s} \frac{\sin ks}{\sqrt{s}} ds \right). \quad (8. 167)$$

Let us rewrite last/latter formula, after replacing ks=z and ks= ζ , then

$$\delta = \frac{sm}{2kJ_{y_1}} \left(\frac{\sin x}{\sqrt{z}} \int_{z_0}^{z} \frac{\cos \zeta}{\sqrt{\zeta}} d\zeta - \frac{\cos x}{\sqrt{z}} \int_{z_0}^{z} \frac{\sin \zeta}{\sqrt{\zeta}} d\zeta \right). \quad (8.168)$$

Let us introduce into examination the integrals of Fresnel

$$C(z) = \frac{1}{\sqrt{2\pi}} \int_{0}^{z} \frac{\cos \zeta}{\sqrt{\zeta}} d\zeta; \quad S(z) = \frac{1}{\sqrt{2\pi}} \int_{0}^{z} \frac{\sin \zeta}{\sqrt{\zeta}} d\zeta \quad (8.169)$$

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$$J_{\frac{1}{2}}(z) = \sqrt{\frac{2}{\pi}} \frac{\sin z}{\sqrt{z}}; \quad J_{-\frac{1}{2}}(z) = \sqrt{\frac{2}{\pi}} \frac{\cos z}{\sqrt{z}}. \quad (8.170)$$

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Eetween these functions there is following communication/connection;

$$C(z) = \frac{1}{2} \int_{0}^{z} J_{-\frac{1}{2}}(\zeta) d\zeta; \quad S(z) = \frac{1}{2} \int_{0}^{z} J_{\frac{1}{2}}(\zeta) d\zeta. \quad (8.171)$$

Functions C(z), S(z), $J_{\frac{1}{2}}(z) = J_{-\frac{1}{2}}(z)$, as is known, are tabulated. Through the tabulated functions the angle \tilde{e} is determined as follows:

$$\delta = \left[\frac{\sqrt{\frac{\pi}{2}} \frac{em}{kJ_{y_1}}}{\frac{1}{2} \frac{1}{kJ_{y_2}}} \frac{\sin z}{1 \frac{z}{z}} \left[C(z) - C(z_0) \right] - \frac{\cos z}{1 \frac{z}{z}} \left[S(z) - S(z_0) \right] \right],$$
(8.172)

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$$8 = \frac{\pi em}{2kJ_{y_1}} \left\{ J_{\frac{1}{2}}(z) \left[C(z) - C(z_0) \right] - J_{-\frac{1}{2}}(z) \left[S(z) - S(z_0) \right] \right\}.$$
 (8.173)

For determining the angle Ψ_{π} it is necessary to express first Ψ_{π} through the tabulated functions, and then to take the difference

$$\Psi_{a} = \psi_{a} - \delta. \qquad (8.174)$$

Usitting fairly complicated detailed conclusion/derivation, let us give here final formulae for angles Ψ_A and Ψ_A [13]

$$\psi_{a} = \frac{nem}{2kJ_{\mu_{1}}} \left\{ [C_{1}z] - C(z_{0})]^{a} + [S(z) - S(z_{0})]^{a} \right\}. \quad (8. 175)$$

Otilizing (8.173) and (8.175), we will obtain

$$\Psi_{z} = \frac{\pi em}{2hJ_{y_{1}}} \left[(C_{z_{0}}^{z})^{2} + (S_{z_{0}}^{z})^{2} - J_{1}(z) C_{z_{0}}^{z} + J_{-\frac{1}{2}}(z) S_{z_{0}}^{z} \right], \quad (8.176)$$

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$$C_{z_{\bullet}}^{z} = C(z) - C(z_{\bullet}); \quad S_{z_{\bullet}}^{z} = S(z) - S(z_{\bullet}).$$

The determination of angles δ, ψ_{π} and ψ_{π} according to the given formulas is conducted with the aid of the tables of the Bessel functions $\int_{-\frac{1}{T}} \frac{1}{T} \frac{(z)}{T} \frac{1}{T} \frac{(z)}{T}$ and of Fresnel's integrals C(z) and S(z), as the entry into which serves argument z=ks. The tables of the named functions are placed in work [13].

Equations (8.173) and (8.176) can be written in abbreviated form:

 $\delta = e\delta^{\bullet}(s)$ and $\Psi_{\bullet} = e\Psi^{\bullet}_{\bullet}(s)$.

where the functions $\delta^*(s)$ and $\Psi^*_{\lambda}(s)$ correspond to eccentricity e=1.

Typical plotted function $\delta^*(s)$ and $\Psi^*(s)$ is given to Fig. 8.24. The initial part of the active section, on which occurs the growth/build-up of angle Ψ_{R} calls the critical trajectory phase. The length of the critical trajectory phase is equal to the length of the first half-period of the fluctuation of angle 6 on path of s.

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At the end of the critical trajectory phase, the angle δ the first time after start turns into zero. After the critical trajectory phase, the angle Ψ_{α} retains approximately constant value.

The given by us examination tears qualitative character, since are not considered all forces, which act or rocket in the period of its motion on powered flight trajectory. The account of drag, hoisting and lateral aerodynamic forces will change the quantitative value of functions $\delta(s)$ and $\Psi_{\pi}(s)$, however, the general view of these functions, presented in Fig. 8-24, will be preserved.

The solution of the problem in question in more complete setting, upon consideration of all acting forces, will come to the numerical solution of system of equations (5.44). In the right sides of three last/latter equations of this system, must be introduced the projections of the moment of thrust, determined by its eccentricity, and the damping moments. In the first three equations of system (3.44) of the projection of force vector of thrust P'_{x_1} , P'_{y_2} and P_{x_1} they must consider misalignment of the line of its action and eccentricity. The numerical solution of system (3.44) with the use of the missing equations from system (3.26) is possible only during use of ETSVM [digital computer]. Depending on the formulation of the problem, system (3.44) can be simplified.





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Chapter IX.

FLIGHT CONTBOL OF BOCKETS AND OF FROJECTILES.

The system of steering of flight vehicle in flight is the continuation of the devices, which ensure the displacement/movement of the certer of mass of rocket in space over the specific law. The law of motion is set either previously - by flight program, or it is form/shaped in the process of flight with induction to target/purpose depending on the characteristics of relative motion of target/purpose and rocket. And in both cases steering the center of mass is realized by the appropriate change in normal and forces of periphery. During the solution of ballistic protlems, is usually examined only the mechanism of the action of control forces of the motion of rocket relative to the center of mass and motion of rocket

The methods of producing control forces and the instrument realization of the control system are examined in special courses. Determining the motion characteristics of guided missiles with guidance to moving target/purposes is giver in chapter IV and Section

⁻ 3.4 chapters VII. Let us here examine the flight control of the rockets of class "surface - surface" and range control of the firing the projectiles of the constant mass of terrestrial artillery.

§1. Change in the firing distance the artillery shells of constant mass and by the unguided reckets.

Buring the trajectory calculation of the motion of artillery shells and unguided rockets, the initial conditions of free flight determine appropriate the trajectory of the center of mass and motion characteristic. In accordance with materials chapter V the flying range of the projectile of constant mass is determined by the parameters θ_0 , v_0 and c. Changing each of them, it is possible to coefficient of determine trajectory with the assigned ballistic coefficient of c.

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Changing ballistic coefficient because of a change in the aerodynamic shape of projectile by the installation of ganels, brake flaps or rings, it is possible to regulate firing distance. The complexities of structural/design formulation limit the applicability of this method.
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In multiply-charged artillery pieces (lowitzers, mortars, mortars) with firing at different charges (i.e. with different v_0) and the identical angle of departure θ_0 of cistance they change with jump during transfer/transition from one charge to the next (Fig. 9:1). At the constant initial velocity, which corresponds to one of the charges, the firing distance changes succthly because of a gradual change in the angle of departure θ_0 (Fig. 9.2). The combination of both methods provides the corriguity of a change of the distance in the assigned range from $z_{\rm cman}$. This is achieved by the selection of the necessary sumber of charges and by certain overlap of distances during transfer/transition from one charge to the next. Depending on the type of artillery piece, the number of charges changes from 2 to 12:

Complete missile-firing distance is composed of the distance, which corresponds to the powered flight trajectory, and the distance of power-off flight. Almost always the rockets of class have "surface - surface" inactive leg considerably greater than active and in essence determines complete firing distance. For the assigned rocket or its nose section, the distance, which corresponds to inactive leg, depends on the parameters of trajectory at the end of the active section. If we do not consider the possibility of changing the ECC = 78107113 FAGE -++ 733

ballistic coefficient of rocket on passive section, then the factors, which estimate distance of firing, will be: velocity $v_{\rm H}$, angle $\theta_{\rm H}$ and operdinates $x_{\rm H}$ and $y_{\rm H}$.

Bigures 9.3 shows a change in the firing distance during change $v_{\rm K}$ at the end of the active section. Figures 9.4 gives the trajectories of the motion of rocket to the passive phase of flight in the case of changing only angle of the slope of velocity vector at the end of the active section $\theta_{\rm K}$ to the horizon.

Figures 9.5 and 9.6 shows the trajectories of the motion of the rocket during a change of the coordinates $x_{\rm K}$ and $y_{\rm K}$ at the end of prevered flight trajectory. A change in the firing distance during change $x_{\rm K}$ and $y_{\rm K}$ insignificantly tears the character of the correction (see Chapter II). It is sufficiently effectively it is possible to control/guide distance, only changing velocity vector in scdule/modulus and directior.



Fig. 9.1. Trajectories of an artillery shell during firing with various initial velocities and a constant angle of departure.

Fig. 9.2. Trajectories of an artillery shell during firing with various angles of departure and a constant initial velocity.



Fig. 9.3.



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Fig. 9.3. Missile trajectories at different velocities at the end of engine operation.

Fig. 9.4. Missile trajectories during change of flight path angle at the end of engine operation.



Fig. 9.5. Missile trajectories during change in ordinate of cutoff found.

Fig. 9.6. Missile trajectories during change in abscissa of cutoff feint.

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§2. Change in the firing distance with the sutchangous method of recket control. \mathcal{H}_{A} change in the distance of the guided missiles of class "surface, surface," can be realized changing the parameters

cf motion at the end of powered flight trajectory and by rocket control in an entire trajectory, including active and passive sections. In the first case of the characteristic of powered flight, they are designed so as to ensure the most advantageous (for a specific problem) flight conditions on inactive leg. Considering that the complete distance is determined by inactive leg, it is possible to write the functional defendence

 $L = F(\boldsymbol{v}_{\kappa}, \boldsymbol{\theta}_{\kappa}, \boldsymbol{c}_{11}, \boldsymbol{y}_{\kappa}, \boldsymbol{x}_{\kappa}, \boldsymbol{a}_{1}, \boldsymbol{a}_{2}, \dots, \boldsymbol{a}_{n}), \qquad (9, 1)$

where, besides known values, $c_n - a$ tallistic coefficient on inactive leg:

 $a_1, a_2, ..., a_n$ - secondary factors, which estimate distance (to them can be attributed, for example, change in the characteristics of the state of the atmosphere).

Control of all factors, which affect the distance, or at least their account in the process of flight, is the problem whose solution is conjugate/combined with great difficulties. However, this and not is always necessary. For example, of long range ballistic missiles the large part of the trajectory passes in the rarefied layers of the atmosphere; therefore the effect of weather factors proves to be insignificant and it is possible not to consider them. Of the rockets the firing distance of which is relatively small and the large part

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cf the trajectory passes in the dense layers of the atmosphere, the effect of weather factors is considered by the introduction of corrections under the initial conditions of firing. Most frequently the range control realizes by changing the parameters of the motion of rocket at the end of powered flight trajectory.

Bor obtaining the assigned distance, it is necessary to switch cff an engine of rocket at the torque/moment when are reached the necessary values $v_{\kappa}, \theta_{\kappa}, y_{\kappa}$ and x_{κ} .

 $v_{\rm R}, \theta_{\rm R}, y_{\rm R}$ and $x_{\rm R}$ - rating values at the end of the active section, which determine the assigned distance, a $v_{\rm K,R}, \theta_{\rm K,R}, y_{\rm K,R}$ and $x_{\rm K,R}$ - actual values of these parameters.

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Bocket must be equipped with equipment which would make it possible to measure during flight the actual values of all parameters, estimating distance, and to compare them with calculated.

Only with a strict clservance of all design characteristics of prwered flight it is possible to obtain at one and the same moment of time the equality

 $v_{k,k} = v_{k}; \quad \theta_{k,k} = \theta_{k}; \quad x_{k,k} = x_{k}; \quad y_{k,k} = y_{k}.$ (9.2)

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Apparently, to attain design conditions of the motion more easily in all with the command methods of control with the constant sighting of rocket by radio aids. Similar control systems can be applied in the rockets of class (surface - air" and "air - surface". However, the rockets of class have "surface - surface" these methods for a number of reasons do not find wide application. In the autonomous systems of control which widely are stilized for the rockets of class "surface - surface", a strict observance of design conditions of motion along all parameters represents great difficulties and, therefore, so is difficult to attain the observance of all equalities (9.2).

Since the complete distance depends (ir essence) on four parameters, asserts itself throught, that deviation of one of them from computed value it would be possible to compensate for by the appropriate change in other parameters, but, so as to obtain calculated firing distance. For this, it is necessary during flight the real distance, defined as function from the combination of the parameters v_{R} , θ_{R} , x_{R} and y_{R} . to compare from calculated. In this case, besides devices for measuring all parameters in the process of flight, it is necessary to still have on heard rocket the computing mechanism which according to the obtained information would calculate

the expected real distance, it compared it firm calculated and supplied the appropriate command/crews to controls. The coordinates of position of rocket in this case not obligatorily must be determined relative to the Earth. For example, in the astronavigational system of control of the coordinate of the rocket they are determined relative to any large star.

In order to consider effect or the distance of each of the parameters, it is necessary to determine the deviation of distance from the calculated, caused by the deviation of each of the parameters from its computed value. Let

$$\boldsymbol{L} = \boldsymbol{F} \left(\boldsymbol{v}_{\mathbf{k}}, \ \boldsymbol{\theta}_{\mathbf{k}}, \ \boldsymbol{x}_{\mathbf{k}}, \ \boldsymbol{y}_{\mathbf{k}} \right). \tag{9.3}$$

Let us take total differential from this function

$$dL = \frac{\partial L}{\partial v_{\kappa}} dv_{\kappa} + \frac{\partial L}{\partial \theta_{\kappa}} d\theta_{\kappa} + \frac{\partial L}{\partial x_{\kappa}} dx_{\kappa} + \frac{\partial L}{\partial y_{\kappa}} dy_{\kappa}. \qquad (9.4)$$

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Partial derivatives $\frac{\partial L}{\partial v_{k}}$, $\frac{\partial L}{\partial \theta_{k}}$, $\frac{\partial L}{\partial x_{k}}$, $\frac{\partial L}{\partial y_{k}}$ determine a change in the complete distance during a change of each of the parameters individually to infinitesimal value.

It is possible with certain error to replace infinitesimal values final and to accept the lirear depercence of a change in the

distance on a change in each of the parameters.

Then from formula (9.4) we fird

$$\delta L = \frac{\partial L}{\partial v_{\kappa}} \, \delta v_{\kappa} + \frac{\partial L}{\partial \theta_{\kappa}} \, \delta \theta_{\kappa} - \frac{\partial L}{\partial x_{\kappa}} \, \delta x_{\kappa} + \frac{\partial L}{\partial y_{\kappa}} \, \delta y_{\kappa}. \tag{9.5}$$

Let us designate

$$\mu_{\phi} = \frac{\partial L}{\partial v_{\kappa}}, \quad \mu_{\phi} = \frac{\partial L}{\partial \theta_{\kappa}}, \quad \mu_{x} = \frac{\partial L}{\partial x_{\kappa}}, \quad \mu_{y} = \frac{\partial L}{\partial y_{\kappa}}$$

and represent δv_{μ} , $\delta \theta_{\mu}$, δx_{μ} and δy_{μ} as differences between the calculated and actual values of the corresponding parameters

$$\delta v_{\kappa} = v_{\lambda}(t) - v_{\kappa}; \quad \delta \theta_{\kappa} = \theta_{\lambda}(t) - \theta_{\kappa}; \quad \delta x_{\kappa} = x_{\lambda}(t) - x_{\kappa};$$
$$\delta y_{\kappa} = y_{\lambda}(t) - y_{\kappa}.$$

Then from formula (9.5) we find

$$\delta \mathcal{L} = \mu_{\varphi} \left[\boldsymbol{v}_{\mathbf{x}}(t) - \boldsymbol{v}_{\mathbf{x}} \right] + \mu_{\theta} \left[\boldsymbol{\theta}_{\mathbf{x}}(t) - \boldsymbol{\theta}_{\mathbf{x}} \right] + \\ + \mu_{x} \left[\boldsymbol{x}_{\mathbf{x}}(t) - \boldsymbol{x}_{\mathbf{x}} \right] + \mu_{y} \left[\boldsymbol{y}_{\mathbf{x}}(t) - \boldsymbol{y}_{\mathbf{x}} \right].$$
(9.6)

It is obvious that with $\delta I = 0$ the real distance coincides with calculated: in this case

$$\mu_{\varphi} v_{\kappa} + \mu_{b} \theta_{\kappa} + \mu_{x} x_{\kappa} + \mu_{y} y_{\kappa} = \mu_{\varphi} v_{A}(t) + \mu_{s} \theta_{A}(t) + \mu_{x} x_{A}(t) + \mu_{y} y_{A}(t), \qquad (9.7)$$

where $t=t_{n,n}$ - the time, which corresponds to the cutoff of engine.

For a concrete/specific/actual rocket, on the basis of the assigned distance, can be calculated the left side of equality (9.7), which is called linear four-membered steering function or linear

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four-membered functional. The method of control in question requires measurement in flight $v_{\rm R}(t)$, $\theta_{\rm R}(t)$, $x_{\rm R}(t)$, $y_{\rm R}(t)$, the calculation of the right side of equality (9.7) and of its comparison with the precomputed value of steering function.

At the moment of the creat of equality (9.7) the engine of rocket must be switched off. Since the value and the sense of the vector of speed at the cutoff of engine car be determined not only through v_{μ} and θ_{μ} , but also through the projections of velocity vector $v_{x\mu}$ and $v_{y\mu}$ on rectargular coordinate axes, then controlling functional can be comprised, on the basis of the functional derendence

$$L = F(\boldsymbol{v}_{\boldsymbol{x}\,\boldsymbol{x}}, \, \boldsymbol{v}_{\boldsymbol{y}\,\boldsymbol{x}}, \, \boldsymbol{x}_{\boldsymbol{x}}, \, \boldsymbol{y}_{\boldsymbol{x}}). \tag{9.8}$$

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In this case the rocket must be equipped by the instruments, which measure $v_{x,y}$, $v_{y,y}$, x_{x} , y_{y} .

The cutoff of engine when $t=t_{\kappa,\mu}$ will be determined by the equality

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$$\frac{\partial L}{\partial v_{x_{K}}} v_{x_{K}} + \frac{\partial L}{\partial v_{y_{K}}} v_{y_{K}} + \frac{\partial L}{\partial x_{K}} x_{K} + \frac{\partial L}{\partial y_{K}} y_{K} = \frac{\partial L}{\partial v_{x_{K}}} v_{x_{A}}(t) + \frac{\partial L}{\partial v_{y_{K}}} v_{y_{A}}(t) + \frac{\partial L}{\partial x_{K}} x_{A}(t) + \frac{\partial L}{\partial y_{K}} y_{A}(t), \qquad (9.9)$$

which, transfer/converting to the form of designations, taken in (9.7), can be written in the form

$$\mu_{v_{x}} v_{x \kappa} + \mu_{v_{y}} v_{y \kappa} + \mu_{x} x_{\kappa} + \mu_{y} y_{\kappa} = \mu_{v_{x}} v_{x \kappa}(t) + \mu_{v_{y}} v_{y \kappa}(t) + \mu_{x} x_{\kappa}(t) + \mu_{y} y_{\kappa}(t).$$
(9.10)

The noncoincidence of the left and right sides of equalities (9.7) or (9.8) at the cutoff of engine will give the range error

$$\delta \mathcal{L} = \Phi_{\mathbf{x},\mathbf{z}} - \Phi_{\mathbf{x}}, \qquad (9.11)$$

where through $\Phi_{K,R}$ and Φ_{K} are respectively designated the value of steering function, calculated in the process of moving the rocket, and the value of steering function, determined by flight program and chtained by calculation.

The value Φ_{μ} , which corresponds to the forthcoming concrete/specific/actual realization of precetermined program, is calculated previously and is stored in the memory of on-board computer. Function $\Phi_{\mu,\mu}$ is calculated or as they speak, "is accumulated" in the process of flight and is the function of time. For example, in connection with (9.10) it is possible to write

 $\Phi_{A}(t) = \mu_{v_{x}} v_{xA}(t) + \mu_{v_{y}} v_{yA}(t) + \mu_{x} x_{A}(t) + \mu_{y} y_{A}(t). \qquad (9.12)$

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Then in the process of the flight

 $\delta \mathcal{L}(t) = \mu_{\Psi_{x}} \nabla_{x,k}(t) + \mu_{\Psi_{y}} \nabla_{\Psi_{x}}(t) + \mu_{x} x_{k}(t) + \mu_{y} y_{k}(t) - \Phi_{k}, \quad (9.13)$

where Φ_R is determined by the left side of equality (9.10).

The selection of one or the other method of the compilation of steering function depends or specific conditions. Of the difficulty of realizing the control system, instituted on the use of four-membered steering functions, are obvious. At present for the rockets of class "surface - surface" having large inactive leg, is applied the method of the range control of firing, instituted on the account to different degree of the effect of the enumerated above parameters by firing distance.

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If we investigate the effect of the low deviations of each of the parameters $(\delta \sigma_m, \delta \theta_N, \delta x_N$ and $\delta y_N)$ of the deviation of complete distance, then it will seem that δx_N and δy_N barely they affect δL . Effect $\delta \theta_N$ depends on value itself θ_N . Figures 9.7 shows the curve/graphs, which illustrate the dependence of the distance,

which corresponds to inactive leg, cn v_{K} and $\theta_{K'}$. From curve/graph it is evident that the distance is less sensitive to a change of the angle of departure in the range of the angles, close to the angle of maximum range. Therefore, if we disconnect engine at tilt angle, close to the angle of maximum range, then effect $\delta \theta_{K}$ on 6L will be small.

From formula (9.6) we will obtain with certain error

$$\delta L = \mu_{\nabla} [v_{A}(t) - v_{\kappa}].$$

With &L=0 let us have the one-term linear functional

 $\mu_{\mathbf{v}} \boldsymbol{v}_{\mathbf{K}} = \mu_{\mathbf{v}} \boldsymbol{v}_{\mathbf{A}}(t),$

which corresponds to speed control. In this case during flight, is measured only the velocity and upper reaching of equality $v_{R}=v_{R,R}$ the engine is disconnect/turned off. Velocity must be measured with the largest possible accuracy. Contemporary onleard equipment for velocity measurement gives the errors, placed in very principles of measurements; however, the accuracy of firing during the range control on velocity proves to be acceptable.





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Fig. 9.7. Dependence of distance or velocity ve and angle of departure θ_{R} : 1 - for velocity v_{R1} : 2 - for velocity v_{R2} : $v_{R2} > v_{R1}$: (when $\Delta^{i}_{\mu_1} = \Delta^{i}_{\mu_2}$ the deviation of distance $\Delta x_{\mu_1} > \Delta x_{\mu_2}$

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2.1. Steering function on apparent velocity.

In the system of preset control of rocket as steering function, there can be use the so-called apparent velocity or the rseudcvelocity. The direct reasurement of velocity on board rocket is nct possible, but it can be calculated by the established/installed cn rocket cuboard equipment by integrating the dependence of acceleration on time.

For measuring the accelerations, are applied the accelerometers

FIGE 23 746 DOC = 78107113

- instruments, instituted on the use of principle of inertia. The accelerometers, establish/installed on board rocket, are measured not the absolute, but apparent acceleration, under which is understood the difference between the acceleration of relatively fixed coordinate system and the acceleration of gravity. The meters of the apparent accelerations are sometimes called Newton meters [23].

Let us examine the schematic diagram of the work of accelerometer and the principle of control on the apparent velocity.

The small load, spring-mounted, has the capability to be moved along guides (Fig. 9.8). Weight shifting is proportional to acceleration in the direction of the motion of displacement/movement is called the axis of the sensitivity of accelerometer. Assuming that the axis of the sensitivity of accelerometer coincides in the direction with axis of rocket and angle of attack $\alpha=0$ ($\theta=3$), then the acceleration a_{n} measured by accelerometer, it will be equal to the difference longitudinal acceleration of the motion of rocket $\frac{dv}{dt}$ and of the projection of the acceleration of gravity on the axis of rocket, i.e., it is possible to write

$$\boldsymbol{a}_{n} = \frac{d\boldsymbol{v}}{dt} - (-g\sin\theta) = \frac{d\boldsymbol{v}}{dt} + g\sin\theta. \tag{9.14}$$

Acceleration a_n is called of pseudoacceleration (apparent

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acceleration), since according to formula (5.14), it differs from the true longitudinal acceleration of rocket to the value of the projection of the acceleration of gravity indicated. Thus, the accelerometer, establish/installed from axis of rocket, always measures the pseudoacceleration of rocket. The value of pseudoacceleration in the form of voltage is fed to the entry of the integrator which it integrates. As a result of integration, we obtain the apparent velocity of the rocket v_n :

$$v_n = \int_0^t a_n dt = \int_0^t \frac{dv}{dt} dt + \int_0^t g \sin \Theta dt. \qquad (9.15)$$

Virst term in equation (9.15) is true eirspeed of the motion of rocket. Consequently, it is possible to write

$$\boldsymbol{v}_{n} = \boldsymbol{v} + \int_{\boldsymbol{\delta}}^{t} g \sin \boldsymbol{\theta} dt. \qquad (9.16)$$

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Fig. 9.8. Schematic of accelercmeter for measuring the rseudpacceleration of rccket.

Key: (1). Axis of rocket.

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Thus, pseudovelocity (the apparent velocity) at the moment of time t differs from true airspeed to value $\int_{0}^{t} g \sin \theta dt$. If ballistic missile flies accurately according to the program (at each moment of time pitch angle corresponds to programmed value), then in the value of pseudovelocity it is possible sufficiently accurately to judge the value of true airspeed.

At the end of the powered flight trajectory of rocket, we have $v_{n,\kappa} = v_{\kappa} + \int_{\delta}^{t} g \sin \theta dt.$ (9.17)

Consequently, during the sufficiently precise motion of

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ballistic missile along pitch angle to any value of true airspeed (v_{κ}) at the end of powered flight trajectory corresponds the completely specific value of pseudovelecity $(v_{n,\kappa})$. In this case, the engine cutoff of the rocket must be conducted in accordance with (9.9) at the torque/moment when

$$\boldsymbol{v}_{\mathbf{n},\mathbf{a}} - \boldsymbol{v}_{\mathbf{n},\mathbf{a}} = 0, \qquad (9.18)$$

where $v_{n,n}$ - value of pseudovelocity at current point in the trajectory, obtained on board rocket (steering function);

 $v_{\text{M}.\text{K}}$ - value of pseudovelocity, designed previously for this flight trajectory and corresponding to the recessary velocity (v_{K}) in the cutoff point.

In the process of flight on board rocket, continuously is determined the instantaneous value of the apparent velocity and it is compared with its computed value for the end/lead of powered flight trajectory. Upon reaching of their equality, is supplied the command/crew to engine cutoff. The accuracy of the operation of the cm-toard inertial control system with one integrating accelerometer can be raised, if with the aid of gyroscope-statilized platform the axis of the sensitivity of accelerometer to direct along the axis Of is perpendicular to the velocity vector of nocket $\overline{v_c}$ in the point of the passage of the calculated trajectory through target/purpose [23].

Then the projections of speed on new cccrdirate axes 0η and 0ξ (Fig. 9.5), turned relative to the axes starting coordinate system to angle of $90^{\circ} - |\theta_c|$, are equal to

$$\boldsymbol{v}_t = \boldsymbol{v}_x \sin |\boldsymbol{\theta}_C| + \boldsymbol{v}_y \cos |\boldsymbol{\theta}_C|; \quad \boldsymbol{v}_\eta = -\boldsymbol{v}_x \cos |\boldsymbol{\theta}_C| + \boldsymbol{v}_y \sin |\boldsymbol{\theta}_C|.$$
(9.19)

It is possible also to write:

$$\xi = x \sin |\theta_c| + y \cos |\theta_c|, \quad \eta = -x \cos |\theta_c| + y \sin |\theta_c|. \quad (9.20)$$

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Euring the measurement of accelerations along axes 0ξ and 0η it is possible to write

$$L = F(\boldsymbol{v}_{\boldsymbol{\xi},\boldsymbol{\kappa}}, \boldsymbol{v}_{\boldsymbol{\eta},\boldsymbol{\kappa}}, \boldsymbol{\xi}_{\boldsymbol{\kappa}}, \boldsymbol{\eta}_{\boldsymbol{\kappa}}) \qquad (9.21)$$

and, correspondingly,

$$\delta L = \frac{\partial L}{\partial v_{\xi \kappa}} \, \delta v_{\xi} + \frac{\partial L}{\partial v_{\eta \kappa}} \, \delta v_{\eta} + \frac{\partial L}{\partial \xi_{\kappa}} \, \delta \xi + \frac{\partial L}{\partial \eta_{\kappa}} \, \delta \eta. \qquad (9.22)$$

According to the investigations of work [23] with an accuracy to low second order, it is possible to assume

$$\frac{\partial L}{\partial \eta_{\kappa}} \approx \frac{\partial L}{\partial v_{\eta_{\kappa}}} \approx 0, \qquad (9.23)$$

and then

$$\delta L = \frac{\partial L}{\partial t_{x}} \delta \xi + \frac{\partial L}{\partial v_{\xi x}} \delta v_{\xi}. \qquad (9.24)$$

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Consequently, to this method of control corresponds the linear tinomial steering function, which has the form

$$\frac{\partial L}{\partial \xi_{\kappa}} \xi(t) + \frac{\partial L}{\partial v_{\xi \kappa}} v_{\xi}(t) = \frac{\partial L}{\partial \xi_{\kappa}} \xi_{\kappa} + \frac{\partial L}{\partial v_{\xi \kappa}} v_{\xi \kappa}. \qquad (9.25)$$

It is obvious, engine sust be switched off upon reaching of equality left and right side (9.25).

Is known the application/use of an autonomous inertial control system with two integrating accelerometers. In this case on board rocket, is establish/installed the gyroscope-stabilized platform, which is oriented along the axes of the starting coordinate system and does not change its position during rocket flight. On gyrosdope-stabilized platform are establish/installed two accelerometers. The axis of sensitivity of one of them is directed in parallel to axis 'Ox₃, another - in parallel to axis 'Oy₃. The schematic of gyroscope-stabilized platform with accelerometers is shown on Fig. 9.10.

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Fig. 9.9. Schematic of coordinates with the inclined location of the axis of the sensitivity of integrating accelercmeter.

Key: (1). Axis of the sensitivity of accelercmeter.



Fig. 9.10. Schematic of gyrcscope-statilized platform with two accelercmeters.

Key: (1). Axis of rccket.

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The accelerometers indicated will, correspondingly, measure the comprising accelerations

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As a result of integrating of dependences $v_{x_3}(t)$, $v_{y_{3n}}(t)$ and the geometric surmation of the projections of the velocity of rocket $v_{x_3}(t)$, $v_{y_{3n}}(t)$ in computer, we determine the value of the apparent velocity

$$v_{n}(t) = \sqrt{v_{x_{3}}^{2} + v_{y_{3}}^{2}}.$$
 (9.26)

The obtained instartanecus value of velocity is compared with value $v_{n.K}$, and upon reaching of equality occurs engine cutoff.

Bor the increase of the accuracy of the operation of the inertial control system, which contains two integrating accelerometers, direction of the axes of their sensitivity they must te selected on the basis of detailed tallistic investigation [23]. The direction of the axes of sensitivity depends on the flight program, motion characteristics and form of trajectory. For rockets with considerable firing distance, must be considered the rotational effect of the Earth and its aspnericity.

§3. Programmed equation of fitch argle.

The programmed equation of pitch angle usually establishes the dependence of tilt angle on time or the over-all payload ratio of ECC = 78107113 PAGE $\frac{31}{754}$

rocket $\mu = \frac{m}{m_0}$. The form of programmed equation depends on the designation/purpose of the rocket, its design-technical parameters and form of start (vertical, inclined, etc.). In programmed equation must be taken into account the character of the expected trajectory - its form, distribution of velocity and accelerations (normal and tangential) according to time and according to the path of the center of mass of rocket. It is necessary to consider the special feature/peculiarities of control - control only on active section or in an entire trajectory, the possible structural/design realization of the control system - autorements in command, etc.

With the correctly comprised program and conformity to its pessibilities of the system of control (limitedness of the deflection of controls) of the characteristic of angular motion $\vartheta(t)$, $\vartheta(t)$ and $\vartheta(t)$ they must smoothly change in the process of flight so that the moments of control forces and the longitudinal serodynamic torque/moment M_z would be located within the assigned limits, determined by the rigidity of construction and by the strength of rocket (i.e. and by its weight).

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The degree of curvature of the trajectory, adjustable within reasonable limits for this type of rockets, manifests itself, in

essence, the normal (transverse) accelerations of rocket and barely affects the tangential (axial) accelerations. Normal accelerations are determined by the value of pitching moment.

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 $M_{\mathbf{z}} = qSIm_{\mathbf{z}}^{*} \mathbf{a} \approx qS(l_{u,\mathbf{y}} - l_{u,\mathbf{z}})c_{\mathbf{z}}^{*}\mathbf{a}.$

Distance between centers of masses and the center of pressure $l_{u,u} = l_{u,u}$ barely depends on flight program; also are little affected with flight program (for a defined class of rockets) velocity head q and coefficients m_z^e and c_{y}^e . The determining value on M_z has an angle of attack $\alpha = \vartheta - \vartheta$.

Therefore in order to decrease M_r and, therefore, to maximally facilitate the construction of rocket, it is mecessary to approach possibly smaller α , especially at vital importance of q_r .

The trajectory phases with large g must be passed with zero (or minimum, close to zero) angles of attack.

The form of dependence $\vartheta(t)$ or u(t) must consider the effectiveness of the work of controls. In the area of transonic speed (B = 0.8-1.2) occurs an abrupt change in the aerodynamic coefficients m_{ν}^{*} and c_{ν}^{*} , which negatively affects the operation of the control system. For decreasing the effect of these charges m_{ν}^{*} and c_{ν}^{*} it is

necessary that the rocket would pass the number domain M indicated with fero angles of attack [2].

The noted conditions satisfies well following program for an angle a during motion in the dense layers of the atmosphere:

$$a = ak(k-2),$$
 (9.27)

where

 $k = 2e^{a(t_1-t)};$

 \overline{a} - a limiting value of angle of attack cr the subsonic trajectory phase:

 t_1 - time from start to the erd/lead of the vertical phase of flight;

a - certain constant coefficient, usually selected for this class of rockets so as upon reaching of numbers Maxa M=0.7-0.8, to obtain angle of attack 0=0.

Important requirement for programmed equation is the need of providing the conduct of firing for the complete range of distances from $x_{c,min}$ to $x_{c,max}$. If range $x_{c,min} - x_{c,max}$ is narrow or distances x_c are small, then program 4(t) can be one; with wide-band range, it is necessary to have several programs (it is desirable - possibly less).

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The instrument realization of the control system must allow the rapid connection/inclusion of the necessary for the assigned distance program in the process of the preparation of firing.

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Buring the compilation of the program equation, it is assumed that the axis of rocket is ideally fulfilled cutlined by program angular rotations. Typical curve/graphs $\vartheta_{mp}(t)$ and $a_{mp}(t)$ for a powered flight trajectory with the vertical launching of single-stage long range hallistic missile are represented in Fig. 9.11[2].

On first section $(0 < t < t_1)$ of rocket flight $\theta_{mp} = 90^\circ = const$. On the second section of program $(t_1 < t < t_3)$ the pitch angle smoothly changes from 90° to the value $\theta_{mp. N}$, which corresponds to the assigned distance x_{c} , moreover angle a here changes accordincy (9.27); time t_2 characterizes the torque/moment of achieving the numbers M=0.7-0.8. The third section of program $(t_3 < t < t_0) - this$ is the section of the motion of the rocket in the relatively rarefied layers of the atmosphere with small q, when it is possible to take $\alpha > 0$, necessary for providing the program of motion with $\theta_{mp.N}=const$. In time interval $t_3'=-t_3'''$ occurs the transition of program to straight portions with the aegles $\theta_{N,0}$, which ensure the range of distances from x_{cmin} to x_{cmas} . EOC = 78107113 FIGE 35 758

Program $\vartheta_{up}(t)$ for the curvilinear trajectory phase is described well by the equation

$$\theta_{np} = 90^{\circ} + (90^{\circ} - \theta_{np,k})(\vec{r} - 2\vec{i}), \qquad (9.28)$$

where $i = \frac{t - t_1}{t_{11} - t_1}$ - relative flight time of rocket on the second section of program whose value changes from zero to unity:

 t_1 - flight time of rccket from start to the end/lead of the first section of program;

 r_{II} - flight time of rccket from start to the end/lead of the second section;

t - current time of rccket flight, calculated off the tcrgue/mcment of start.

For designed calculations the program can be comprised not in the form $\vartheta(t)$, but as function $\theta(\mu)$. Let to the torque/moment ℓ_1 of the end/lead of the rectilinear vertical flight correspond mass ratio of rocket μ_1 , and to the torque/moment of the beginning of rectilinear inclined flight - μ_2 . The program of a change of the angle θ in the process of rocket flight on the active curvilinear CC = 78107113 PAGE -30 759

trajectory phase can be presented in the form

$$\mathbf{0} = A(\mu - \mu_{0})^{2} + B(\mu - \mu_{0}) + C. \tag{9.92}$$

For solution would be being concrete/specific/actual, it is necessary to assign values μ_1 and μ_2 . It is possible to accept $\mu_1=0.95$ and to count that the curvilinear section of program coacludes with $\mu_2=0.4-0.5$. Angle θ_n must correspond to the angle of maximum range. Taking into account the said program of a change in the angle θ will take form of $\theta=90^\circ$ with $1.(C \ge \mu \ge 0.95$;

$$= 4 \left(\frac{\pi}{2} - \theta_{\kappa}\right) (\mu - 0.45)^{\mu} + \theta_{\kappa} \text{ npu } 0.95 \ge \mu \ge 0.45; \qquad (9.30)$$

$$= \theta_{\kappa} \text{ npu } 0.45 \ge \mu \ge \mu_{\kappa}.$$

Key: (1) with.

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For a two-stage rocket the exemplary/approximate program of pitch angle takes the form, presented in Fig. 9.12 [2]. The form of the flight trajectory in the work of first-stage engine is selected according to the program of obtaining maximum angle of attack not more in subsonic range and obtaining o=0 in range of numbers H>0.7-0.8. The programmed equation, which determines the motion of the second stage of rocket, most frequently car be undertaken in the form

$$\hat{\boldsymbol{\vartheta}}_{jj} = \hat{\boldsymbol{\vartheta}}_{jjs} - \hat{\boldsymbol{\vartheta}} \left(\boldsymbol{t} - \boldsymbol{t}^{m} \right), \tag{9.31}$$

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where $\vartheta_{\Pi 0}$ - an initial programmed value of pitch angle for the second step/stage.

Depending on the basic design parameters of the rocket of value θ_{110} and $\dot{\theta}$ for providing the maximum of distance, they can take different values.

In a series of the cases when selecting of the most advantageous program, computed values θ_{1n} and θ_{10} can not coincide. In this case a change of the derivative $\vartheta(t)$ in the work of the second step/stage must provide smooth transition (section t¹-tⁿ in Fig. 9.12) from the program, establish/installed for first stage, to the curvilinear section, selected for the second step/stage, during gradual transition to the straight portion, on which is completed the work of second-stage engine. The straight portion of trajectory with the optimum angle of the slope of velocity vector at the end of the work of second-stage engine provides obtaining the distance, close to the greatest.



Fig. 9.11. Flow chart of the program of pitch argle for a sirgle-stage rocket and a change in the angle of attack.

Fig. 9.12. Flow chart of program of pitch angle for two-stage rocket.

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§4. Maneuverability and g-fcrces.

Buring completing of the assigned flight program or with missile targeting to the moved target/purpose (i.e. during maneuver accomplishment) change the value and the direction of the velocity of the motion of the center of mass of rocket. In accordance with this under the maneuverability of rockets, is understood the speed of a

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change in the flight speed in value and direction. Is estimated the maneuverability of all types of flight vehicles, rockets and projectiles with the aid of the so-called g-forces. G-force, just as velocity, value is vector.

Under the vector of the g-force of rocket \overline{r} , is understood the relation to the resultant of all acting on it forces, except gravitational force, to the weight of rocket, i.e.,

$$\overline{n} = \frac{\overline{F_a + \overline{F_r}}}{mg} , \qquad (9.32)$$

where F_a - resultant of all aerodynamic frices;

 $F_r - resultant$ of all gas-dynamic forces, including the engine thrust.

The mcdule/modulus of the vector of g-icrce n is the value of dimensionless, which is made it convenient icr comparative evaluations. G-forces are determined by energy resources of rocket, by the possibilities of its aerodynamic configuration, by the control effectiveness and control system as a whole. The total vector of g-force can be determined its those comprise along the axes of the adopted system of coordinates.

Tangential g-force characterizes a change in the velocity in

value, and the g-forces, undertaken along the normal to velocity and called normal, a change in the sense of the vector of the velocity of rocket, i.e., the maneuver of the latter. Euring the analysis of the conditions, which ensure the maneuverability of the rockets, are examined usually only normal load factors.

Let us establish communication/connection between the form of the completed by rocket maneuver and the g-forces, necessary for its accomplishment.

During the design of the control systems of initial space, is conducting the detailed analysis whose target/purpose - to establish the required forms of trajectories (i.e. the maneuvers), which will allow the designed rocket to satisfy the stated operational requirements.

Figure 9.13 shows the trajectory of approach of rocket for the maneuvering in vertical plane target, constructed for the method of guidance (on pursuit curve) accepted according to the predetermined trajectory of target/purpose and velocity of rocket.

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A similar trajectory, without opening the system of forces, that

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caused its form, makes it possible to establish the characteristic which makes it possible to pass to the determination of the acting on recket forces. This kinematic characteristic of the maneuver of rocket is the radius of curvature of trajectory P.

Figure 9.13 shows that the presented in it trajectory of the approach of rocket for target/purpose is characterized by a gradual increase in the curvature cr, that analogoualy, by the decrease of radius of curvature from value r_0 at initial point in the trajectory to value 'B at the collision point of rocket for target/purpose. In connection with the half-speed coordinate system, it is possible to write following expressions for radii of curvature:

$$r_{\nu^{*}} = \frac{dS}{d\theta} = v \frac{dt}{d\theta}; \qquad (9.33)$$

$$r_{z^{*}} = -\frac{dS}{d\Psi} = -v \frac{dt}{d\Psi}. \qquad (9.34)$$

In this case, it was accepted that r > C in the case, the code its depicting cut, directed from point in the trajectory toward center of curvature, coincides with positive direction of the corresponding coordinate axis.

Entering expressions (9.33) and (9.34) angular velocities $\frac{d\theta}{dt}$ and $\frac{d\Psi}{dt}$ can be expressed by the appropriate g-forces on the basis of the system of the differential equations, which describe the motion of the center of mass of rocket.



Fig. 9.13./ Change of the radius of curvature of trajectory during the approach of the guided missile for tarcet/purpose along pursuit trajectory.

Fig. 0.14. Schematic of cccrdinates during determination of g-forces.

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Let the plane of angle of attack a be vertical, and the plane of angle of slip β - to it is perpendicular; then on Fig. 9.14 it is possible to write the system of equations of the spatial motion of the center of mass of the guided missile in the half-speed coordinate system. In this case, control forces is combined with aerodynamic forces of X, Y*, Z*; the left and right sides of the equations it is subdivided by weight of rocket Q=mg.

As a result we will chtain

A state of the second second

$$\frac{1}{g} \frac{dv}{dt} = \frac{P \cos a \cos \beta - X}{mg} - \sin \theta;$$

$$\frac{v}{g} \frac{dh}{dt} = \frac{P \sin a + Y^*}{mg} - \cos \theta;$$

$$(9.35)$$

$$-\frac{v}{g} \cos \theta \frac{dV}{dt} = \frac{-P \cos a \sin \beta + Z}{mg}.$$

Brom the written equations, according to definition, let us isclate g-forces and it is simplified expressions for them, after replacing as a result of the smallness of the angles

 $\sin a \approx a; \quad \sin \beta \approx \beta; \quad \cos a \approx \cos \beta \approx 1.$

Then the projections of the vector of c-force on the half-speed coordinate axes are equal to

$$n_{x} = \frac{P \cos \alpha \cos \beta - X}{mg} \approx \frac{P - X}{mg};$$

$$n_{y^{*}} = \frac{P \sin \alpha + Y^{*}}{mg} \approx \frac{P \alpha + Y^{*}}{mg};$$

$$n_{z^{*}} = \frac{-P \cos \alpha \sin \beta + Z^{*}}{mg} \approx \frac{-P \beta + Z^{*}}{mg}.$$
(9.36)

Inserting the projections of the vector of g-force into system (9.35), let us write

$$n_x = \frac{1}{g} \frac{dv}{dt} + \sin \theta; \quad n_y = \frac{v}{g} \frac{d\theta}{dt} + \cos \theta;$$
$$n_{z^0} = -\frac{v}{g} \cos \theta \frac{d\Psi}{dt}. \quad (9.37)$$

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Hence

$$\frac{d\theta}{dt} = \frac{g}{v} \left(n_{y^*} - \cos \theta \right); \quad \frac{d\Psi}{dt} = -\frac{g}{v \cos \theta_i} \left(n_{z^*} \right). \quad (9.38)$$

After substituting with respect (9.38) into equations for radii of curvature (9.33) and (9.34), we will obtain

$$n_{y^{0}} = \frac{1}{r_{y^{0}}} \frac{v^{2}}{g} + \cos \theta, \qquad (9.39)$$

$$n_{z^{0}} = \frac{1}{r_{z^{0}}} \frac{v^{2} \cos \theta}{g}. \qquad (9.40)$$

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The parameters of trajectory v, θ and Ψ determine the value and the sense of the vector of speed, but then derivatives and g-forces characterize the ability of flight vehicle to change value and direction of the flight speed of the center of mass.

Bependences (9.39) and (9.40) show that the rocket can satisfy the maneuver, which is characterized by radius of curvature r, only in such a case, when will be created the necessary g-forces $n_{t'}$ and $n_{t'}$. moreover maneuver with smaller radius of curvature, i.e., sharper, can be realized, other conditions being equal, because of an increase in the normal load factors. Euring the investigation of g-forces, determine the sc-called required g-forces, necessary for obtaining of the trajectories of the assigned form, and the available
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g-forces, which can be actually obtained by rocket. G-forces required and arrange/located are compared between themselves.

Rocket can move over programmed trajectory or over the trajectory of the guidance only when the necessary for its obtaining required g-forces will be less or (in the extreme case) equal to the available g-forces which can be created by the rocket

 $\boldsymbol{n_{y^*\pi} \leqslant n_{y^*p}}; \quad \boldsymbol{n_{z^*\mu} \leqslant n_{z^*p}}. \tag{9.41}$

But given conditions (9.41) it is not possible to consider sufficient. The motion of rockets under actual conditions is always accompanied different kind by the short-term or prolonged disturbance/perturbations which deflect/divert rocket from calculated trajectory. For the stabilization of the motion of rocket, its control system must, as already mentioned that to possess possibility to parry these disturbance/perturbations.

Consequently, during setting of communication/connection between the required and available g-forces it is recessary to compulsorily provide for the reserve of the normal load factors $n_{p^{*0}}$ and $n_{p^{*0}}$, which is necessary for realizing assigned rocket flight under the effect on of its random disturbances.

Taking into account this condition (9.41) they take the form

 $n_{p^{\alpha}n} + n_{p^{\alpha}s} \leqslant n_{p^{\alpha}s}; \quad n_{s^{\alpha}n} + n_{s^{\alpha}s} \leqslant n_{s^{\alpha}p}. \tag{9.42}$

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It is necessary to hear in wird, that setting the maximum sizes cf the g-force, both normal and axial, sust be conducted taking into account the careful analysis of the effect of g-forces on the work of the caboard measuring and controlling equipment, strength of housing and assemblies of rocket.

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STUDY OF TRAJECTORIES AND THE CONCEET OF THE OFTIMUM SOLUTIONS OF THE FEGELEMS OF EXTERNAL BAILISTICS.

The problems of extra-fallistic investigations are very many-sided and depend on the designation/purpose of rocket or projectile and concrete/specific/actual posing of the question. The most frequently encountered investigations can be combined into following basic groups.

1. Investigations, dedicated to setting common/general/total properties of missile trajectories and projectiles in air and vacuum.

2. Investigations, dedicated to finding optimum solutions of problems of theory of rocket flight and projectiles.

3. Investigations of conditions of three-dimensional/space rendezvous of two bodies, i.e., conditions of interception of target/purpose (aircraft, rocket) by zerith or micraft rockets or defeat of it by artillery shells. DEC = 78107113 PIGE 48 771

4. Considerable place in Fallistic investigations occupy questions, connected with determining of characteristics of scattering trajectories and accuracy of firing (these investigations will examine we in chapter XIII).

Almost for all named groups of extra-lellistic investigations, it is necessary to apply the numerical methods of solutions with the aid of appropriate computer technology; only the part of the simple problems can be solved analytically.

§1. General properties of missile trajectories and of the projectiles of constant mass moving in the atmosphere.

Let us examine this question in connection with the unguided rockets and the projectiles of class "surface - surface". The most important motion characteristics it is possible to consider the form of trajectory, the velocity of the center of mass and g-force.

The velocity of the center of mass of kody depends on the acting on it forces and with the constant/invariable composition of the system of forces changes successly. EBC = 78107113 PAGE 49 772

Bigures 10.1 shows the curve/graphs of a change in the rate of motion along the trajectory of two types of rockets - wingless (curve 1) and winged missile (curve 2).

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As can be seen from figure, the velocity of the motion of wingless rocket by active section sharply grow/rises, reaching in its end/lead of the greatest value; then on the free-flight phase (when the system of forces changed) it decreases with the approach/approximation of rocket to peak of the trajectory; after peak of the trajectory, the velocity again grow/rises, and at the end of the trajectory, it again sharply decreases as a result of rocket traking by the dense layers of the atmosphere.

For winged missiles the velocity rapidly grow/rises by the launching phase, and on march it is little affected, but all the same it does not remain constant as a result of a change of the mass of the rocket because of burnout in the operation of engine, nonuniformity of the feed of fuel/propellast into the combustion chamber, etc. During the motion of the rocket in the vacuum, its velocity in the engine operation will continuously grow/rise. In accordance with formula K. F. Tsiclkovskiy the limiting value of velocity is determined by the weight of rocket, by the fuel reserves

and by his energy characteristics (7.146).

Let us examine the properties of the trajectories of the projectiles of constant mass, driving/moving in the atmosphere, characteristic for ground-lessed artillery pieces.

Let us take the first equation of system (5.8), let us substitute in it value of E from (5.5) and then

$$\frac{d\boldsymbol{z}}{d\boldsymbol{x}} = -cH(\boldsymbol{y})\boldsymbol{G}(\boldsymbol{v}).$$

Since c. H(y) and G(v) are positive, then

$$\frac{du}{dx} < 0 \tag{10.1}$$

and, therefore, the horizontal projection of speed decreases along trajectory.

Let us take the second equation of system (5.8) $\frac{dp}{dx} = -\frac{g}{u^2}$ and replace in it $dx = \frac{dy}{p}$; then $pdp = -\frac{g}{u^2} dy$. Let us integrate this equation twice; one time on the ascerding branch of trajectory from point $y_{p,i}$ to peak of the trajectory S and for the second time - on the descending branch of trajectory from the apex/vertex that of the point $y_{p,i}$:

$$\int_{p_{0}}^{0} pdp = -g \int_{y_{0}}^{y_{0}} \frac{dy}{x_{0}^{2}}; \quad \int_{0}^{p_{0}} pdp = -g \int_{y_{0}}^{y_{0}} \frac{dy}{x_{0}^{2}}$$

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Fig. 10.1. The circuit of a change in the rate of motion along trajectory for: 1 -wingless tallistic missile: 2 -winged missile.

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After integration we can write

$$\frac{p_{ul}^2}{2} = g \int_{\mu_{ul}}^{\mu_{S}} \frac{dy}{u_{u}^2}; \quad \frac{p_{ul}^2}{2} = g \int_{\mu_{ul}}^{\mu_{S}} \frac{dy}{u_{u}^2}.$$

With the decreasing horizontal projection of speed for any pair of points, which are located on one height/altitude y_i on the ascending and descending branches of trajectory, $u_b > u_m$, therefore

$$\int_{\nu_{n,i}}^{\nu_{S}} \frac{dy}{u_{n}^{2}} < \int_{\nu_{n,i}}^{\nu_{S}} \frac{dy}{u_{n}^{2}}$$

and, therefore,

$$p_{\mathfrak{s}i} < |p_{\mathfrak{w}i}| + \theta_{\mathfrak{s}i} < \theta_{\mathfrak{w}i}. \tag{10.2}$$

Thus, the descending tranch of trajectory than steeper ascending and angle of incidence is more angle of departure

$$|\theta_c| > \theta_{\bullet}. \tag{10.3}$$

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Let us find the change in the kinetic energy of the projectile of constant mass, which corresponds to its cisplacement/movement from point y_{ni} on the upward leg into point $y_{ni}=y_{ni}$ on the descending branch of trajectory. At the identical height/altitude of the selected points, a change in the kinetic energies is determined by the action only of air resistance F.

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$$\frac{Q}{2g}\left(\boldsymbol{v}_{\pi}^{2}-\boldsymbol{v}_{\pi}^{2}\right)=\int_{V}^{S_{r}}R\cos\left(R\boldsymbol{v}\right)dS.$$

Since $\cos(Rv) = -1$, that $v_{\pi i}^2 - v_{\pi i}^2 < 0$ and, therefore,

 $\boldsymbol{v}_{n\,l} > \boldsymbol{v}_{n\,l}. \tag{10.4}$

Thus, at points in the trajectory with identical height/altitudes $y_{ni}=y_{ni}$ the velocity of projectile on the descending tranch is smaller than the velocity on the upward leg and the velocity of projectile in impact point of the lesser velocity in release point

$$\boldsymbol{v}_{\boldsymbol{C}} < \boldsymbol{v}_{\boldsymbol{\theta}}. \tag{10.5}$$

Let us examine velocity change along trajectory for the

projectiles of constant mass. In accordance with the first equation
of system (5.3) it is possible to write

$$\frac{dv}{dt} = -\frac{X}{m} - g\sin\theta.$$

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Since all values, entering the right side of the last/latter equation, are positive, ther, chvicusly, in the beginning of trajectory the velocity decreases, since dv/dt < C. The minimum of velocity will be under condition dv/dt = C, when is correct the equality

$$\frac{x}{m} = -g\sin\theta. \tag{10.6}$$

It is obvious, equality (10.6) can be fulfilled only on the descending branch of the trajectory where $\sin \theta < 0$, i.e., the minimum of velocity will be after reak of the trajectory: after this the velocity of projectile will begin to grow/rise, since will occur inequality $g\sin\theta>X/m$. In the lower layers of the atmosphere, at high air densities, the velocity again will begin to fall (Fig. 10.2). The presented curve/graph is characteristic for high-angle tajectories. Per low trajectories, depending on initial conditions, the maximum on the right side of the curve/graph car not be formed to impact point; with direct fire, the velocity of projectile at trajectory continuously decreases.

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Fig. 10.2. Curve/graph of a change in the velocity of the projectile of constant mass during its motion in the stmosphere.

§2. General properties of the trajectories of the projectiles of constant mass in a vacuum.

Let us give the properties of the trajectory of the motion of projectile, obtained on the basis of parabolic theory (chapter VII \$1).

The horizontal projection of speed is constant along trajectory, i.e.,

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\boldsymbol{\mu} = \boldsymbol{v}_{\boldsymbol{\theta}} \cos \boldsymbol{\theta}_{\boldsymbol{\theta}} = \text{const.} \quad (10.7)
```

Parabolic trajectory is symmetrical in form and with respect to velocity change along trajectory relative to ordinate y_0 , the passing through the half complete distance. Argle of incidence is equal to angle of departure, final velocity is equal to the initial velocity

 $|\boldsymbol{\theta}_{\boldsymbol{C}}| = \boldsymbol{\theta}_{\boldsymbol{\theta}}; \quad \boldsymbol{\nabla}_{\boldsymbol{C}} = \boldsymbol{\nabla}_{\boldsymbol{\theta}}. \tag{10.8}$

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Let us present (7.1) in the form

$$y = x p_0 - \frac{g x^2}{2v_0^2} (1 + p_0^2), \qquad (10.9)$$

where $r_0 = tg\theta_0$ - parameter of family of curves, described by equation (1(.9).

We will obtain equation by envelope family of curves. For this, let us take derivative of expression (10.9) from parameter p_0

$$x - \frac{gx^2}{v_0^2} p_0 = 0$$

let us find that $p_0 = \frac{v_0^2}{gx}$. Substituting p_0 is equation (10.9), we will chain the equation of envelope

$$y = \frac{v_0^2}{2g} - \frac{g_{x^2}}{2v_0^2} \,. \tag{10.10}$$

Last/latter equation contains argument x only to positive even degree (x^2) and, therefore, that envelopes - the parabola, symmetrical relative to y axis.

If point (target/purpose) is located out of envelope, i.e., if

$$y > \frac{v_0^2}{2g} - \frac{gx^2}{2v_0^2}$$

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then this point is struck be it cannot. Therefore envelope (10.10) they call the parabola of safety. The solution of equation (10.9) relative to $F_0 = tg\theta_0$ with

$$y < \frac{v_0^2}{2g} - \frac{gx^2}{2v_0^2}$$

has two real roots, i.e., target/purpose, contained the parabola of safety, can be struck during two adjustments of sight. Target/purpose, which is located on the parabola of safety, can be struck during one adjustment of sight [59]. Last/latter two conclusion/derivations are valid also for enveloping the families of the trajectories in air, which have v_0 =const and parameter p_0 =tg θ_0 .

Bet us find a decrease in the trajectory under line of elevation. Let us substitute (7.4) In second term of right side (7.1) and after conversion we will obtain:

$$\frac{gt^2}{2} = x \operatorname{tg} \theta_0 - y. \tag{10.11}$$

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This there will be lowering the trajectory under line of elevation (Fig. 10.3). In trrn,

$$oa = \frac{x}{\cos \theta_0} = v_0 t. \tag{10.12}$$

It is obvious, with firing with one and the same velocity at any angle of elevation the giver moment of time it will answer one and the same lowering under line of elevation, which does not depend on angle of elevation. The lines, which connect the points of lowering the trajectories under lines of shots and which correspond to identical flight times, are called isochrores. Within the framework of the parabolic theory of isochrone, will represent by themselves the circumferences, carried cut by a radius v_0t from the center, cmitted from the origin of coordinates along the axis Oy to value $gt^2/2$. The isochrones of the motion of the projectiles of different designation/purpose in air can be constructed according to the numerical calculations of the family of trajectories.

Certain interest for the theory of corrections represents the

dependence, which determines relative transit time by the projectile of the cuts of the ascerding and descending branches of trajectory, limited by ordinates y_i and y_{i-1} (Fig. 10.4).

Let us elevate formula (7.6) into square and is piecemeal divided it into (7.8). As a result we will citain simple formula for calculation of total flying time of projectile along the trajectory

$$t_c = \sqrt{\frac{3}{g}} y_s. \qquad (10. 13)$$

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Rig. 10.3. Lovering the trajectory under lire of elevation.



Pig. 10.4. Dividing circuit of trajectory into layers to derivation **of dependence** for "weight of layer".

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Transit time by the projectile of a layer of the space, limited by ordinates y_i and y_{i-1} , can be found as difference of the missile flight time in trajectory, that has initial ordinates y_i and y_{i-1} .

$$t_{C(l-1)} - t_{Cl} = \int \frac{\frac{8}{8} [(y_s - y_{l-1}) - \sqrt{\frac{8}{8} (y_s - y_l)}]}{\frac{8}{8} [(y_s - y_{l-1}) - \sqrt{\frac{8}{8} (y_s - y_l)}]}$$

The relative retention time of projectile in a layer is equal

$$\frac{i_{C(l-1)} - i_{Cl}}{i_{C}} = \sqrt{\frac{(y_{s} - y_{l-1})}{y_{s}}} - \sqrt{\frac{(y_{s} - y_{l})}{y_{s}}}.$$
 (10. 14)

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For integer n of layers of the equal thickness a, to which is broken the trajectory, it is possible to write

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 $\frac{y_{i-1}}{y_s} = \frac{i-1}{n} \frac{y_i}{y_s} = \frac{i}{n}.$ (10.15) **Key:** (1). and.

Then the conditional "weight of a layer", proportional to the retention time of projectile in the i layer, is equal to

$$\eta_{i} = \frac{t_{C(i-1)} - t_{Ci}}{t_{C}} = \frac{1}{\sqrt{n-i+1} - \sqrt{n-i}}.$$
 (10. 16)

In last/latter formula can be comprised the table with entries n and i. this table or formula (10.16) can be used for calculating the seights of layers during the approximate (estimated) determination of the effect of meteorological factors on the results of firing.

Within the framework of the elliptical theory, which examines the motion of projectile in central gravitational field without the account of air resistance, the trajectory of projectile is described by the equation of ellipse (7.14). Form trajectory is determined by

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eccentricity e. In §2 chapters VII, are examined the trajectories, which correspond to the cases

e=0; 0 < e < 1; e=1; e>1.

Let us give several ccmmon/gereral/totel properties of the elliptical trajectory, which is closed on the surface of the Earth.

The analysis of equation (7.14) shows that the elliptical trajectory has an axis of symmetry which coincides with radius-vector $\bar{r}_{max}=\bar{r}_{S}$, turned relative to the beginning of trajectory to vectorial angle φ_{B} Consequently, in the points, arrange/located on the ascending and descending branches of trajectory is symmetrical relative to radius-vector \bar{r}_{S} (i.e. the focal axis of ellipse), will be identical radii r and angles of arrival (on module/modulus).

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Substituting in (7.16) identical values of r and 0, that correspond to the points of the ascending erd descending branches of trajectory, we can be convirced of the fact that the velocities in these points are also equal on **module/modulus**, i.e., elliptical trajectory is symmetrical relatively \vec{r}_s not only in form, but also according to the distribution of velocity along trajectory. The value

cf a radius $r_s = r_{max}$ and corresponding to it angle φ_s are determined formulas (7.14): if we take $\varphi = \varphi_s$, then

$$r_s = r_{max} = \frac{p}{1 - e};$$
 (10. 17)

if we take $q=q_n=0$ and $r=r_n$, that

$$\varphi_{s} = \arccos \frac{r_{u} - p}{er_{u}} \quad (10.18)$$

§3. Kinematic methods of the study of the trajectories of guidance.

In the theory of flight to the kinematic methods of study, it is accepted to relate the methods, which examine the motion of the center of the masses of flight vehicle without the explicit account of the acting on it forces on the assumption that the velocity of its center of mass in the function of time is known. Most frequently kirematic methods are applied during the study of the motion of the guided missiles, which are aimed to moving targets or which are aimed to target/purpose from mobile guidance staticms.

We investigate the possible conditions of the encounter of the rocket with target with the different methods of guidance. In the general case for providing the rendezvous, distance r between the rocket and the target/purpose must decreases, but this means that must be fulfilled the equality

$$\frac{dr}{di} < 0. \tag{10. 19}$$

and the second second

Bor an example let us examine the case of guidance with fixed-lead angle $\alpha_p = \alpha_{p0} = \text{const}$ and will proceed from formula (7.91). For providing inequality (10.19) it is necessary to have

 $p \cos \alpha_{p0} > \cos \alpha_{u}$.

Utilizing (7.88) and transforming, we will obtain

$$p^{a}(1-\sin^{a}\alpha_{p0}) > (1-p^{a}\sin^{a}\alpha_{p0}).$$
 (10.20)

Consequently, with the method of guidance in guestion for providing the encounter of rocket with target it is necessary to have p > 1, i.e., the velocity of rocket must be more than target speed.

The normal acceleration of the rocket with the method of guidance in guestion is determined by formula (7.98). For providing the rendezvous of rocket for target/furgose a_{α} must approach $a_{\alpha B}$. Then the acceleration limit will be equal

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 $\left\{ \begin{array}{c} npk \ k < 2 & \lim a_{np} = 0; \\ npk \ k > 2 & \lim a_{np} = \infty; \\ npk \ k = 2 & 0 < \lim a_{np} < \infty. \end{array} \right\}$ $\left\{ \begin{array}{c} (10, 21) \\ k \in y; \quad (1), \quad \text{with.} \end{array} \right\}$

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In practice, obviously, can be used only case $k \leq 2$. The analysis of formula (7.58) gives the range of the possible values p depending on a_{p0} (Fig. 10.5). As can be seen from figure, the range of the theoretically possible conditions of the rendezvous (it is cross-hatched) with fixed-lead homing is extremely limited.

For the investigation of the possibilities of parallel approach method, let us conduct the kinematic investigations of the conditions of rendezvous at the constant velocities of the motion of target/purpose and rocket (p = const).

For case of P>1 rectilinearly driving/noving target/purpose can be intercepted in any relative attitude of rocket and target/purpose during time interval Δt since at angles a_p and q_q in

placed only the limitation of ideal advance, prevention (Fig. 10.6a). In the case of p < 1 range of the positions of the rockets, with which is feasible the interception of target/purpose, considerably it is reduced. For providing the interception, must be fulfilled the equality $\Delta t_i = \frac{l_p i}{v_{pl}} = \frac{l_{ul}}{v_{ul}}$, however, in addition to this, the rocket before the interception must be located in the space, limited by the cone whose angle $\mu = \arctan \sin p$ (Fig. 10.6b).

If in initial position rocket is located on the surface of cone with angle μ , then by the only direction, which ensures interception, it will be the direction of motion, perpendicular generatrix of cone. If rocket is located within cope, then possible different initial lead angles a_{p1} and to a_{p2} , moreover the extreme trajectories of the motion of rocket $\overline{O_{3}A}$ and $\overline{O_{3}B}$ will be rectilinear. The trajectories of the motion of rocket, which lie between $O_{2}A$ and $O_{2}B$, for providing the rendezvous must be curvilinear. With the maneuvering of target/purpose, the range, is which is feasible its interception, still decreases. This question requires additional study.

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Fig. 10.5. Dependence of the relative velocity of rocket on fixed-lead angle. The shaded range corresponds to the conditions of the encounter of the rocket with target with the finite quantity of normal accelerations.

Key: (1). Rendezvous is impossible. (2). It is infinite. (3). large.
(4). upon rendezvous. (5). Bendezvous is impossible.

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CCBBUNICATION/CONNECTION between the normal accelerations of rocket and target/purpose is set by formula (7.106). It is obvious that for the case p > 1 normal accelerations of rocket will be less than the normal accelerations of target/purpose.

For case of p < 1, if the rocket before the approach is found on the boundary of the region of possible interception - on by generatrix cone with angle μ , we have $a_p = \frac{\pi}{2}$, $\cos a_p = 0$ $\mu \frac{(p)}{MS} (7.105)$ $\frac{d_{np}}{d_{nu}} = \infty.$ Key: (1). and from.

Obviously that this case cannot be used. If we designate the ratio of accelerations that is acceptable in practice through

$$\zeta = \frac{a_{np}}{a_{nk}},$$

the initial angle an must be equal to

$$a_{u0} = \arcsin \sqrt{\frac{p^2(z^2 - 1)}{z^2 - p^2}}$$
 (10.22)

It is obvious that in this case $\alpha_{m0} < \mu$, a this means that the range of the rational positions of the rocket before its approach for target/purpose is less than the range, determined from the condition of the possibility of interception.

We investigate the conditions of the encounter of the rocket with target with pursuit guidance. From formula (7.107) it is evident that for providing the encounter of rocket with target it is becessary to observe inequality $p > \cos \alpha_n$; with this dr/dt < 0 and value r in the process of guidance will decrease.

23.63





Fig. 10.6. Schematic of the interception of the rectilinearly driving/moving target/purpose with r = const ard parallel method of afficach.

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For the driven out target/purpose from (7.112) it is evident that when $\sin a_{\mu} \rightarrow 0$ the distance between the model the target/purpose also vanishes. Is chvicus, r = 0 when $\sin a_{\mu} = 0$.

For the target/purpose, which flies towards, on (7.116) value r decreases with tendency a_n toward *. When $a_n=\pi$ let us have r = 0. Let us determine the limits of the possible normal accelerations of the recket during its approach for purpose according to pursuit curve. For the driven cut target/purpose, or the basis of (7.121), with 1 < p < 2 we will obtain

$$\lim_{T \to 0} a_{n_{P}} = -\frac{v_{P}^{2}}{p \cdot k} \lim_{T \to 0} (\sin \gamma)^{2-p} (1 + \cos \gamma)^{p} = 0,$$

[es

with
$$\mathbf{F} = 2$$

$$\lim_{\gamma \to 0} a_{\pi p} = -\frac{v_p^2}{p \cdot k} \lim_{\gamma \to 0} (1 + \cos \gamma)^2 = -\frac{4v_p^2}{p \cdot k};$$

with p > 2

$$\lim_{\gamma \to 0} a_{np} = -\frac{v_p^2}{p \cdot k} \lim_{\gamma \to 0} \frac{(1 + \cos \gamma)^p}{(\sin \gamma)^{p-2}} = -\infty.$$

Bcr the target/purpose, which flies towards, on the basis of (7.122) it is possible to obtain:

при () 1 $при <math>p = 2 \lim_{T \to 0} a_{np} = -\frac{4v_p^2}{pk'};$ при $p > 2 \lim_{T \to 0} a_{np} = -\infty.$ Key: (1). with.

Since the real rocket with p > 2 cannot move with infinite normal accelerations, then during approach for target/purpose it will go not along calculated trajectory and can fly wide of the wark. Thus, for providing the reliability of rendezvous with ideal pursuit guidance one should have the condition

2 > p > 1, (10.23)

If we by firing regulations exclude the possibility of rough maneuvers of rocket, then also with p > 2 it is possible to strike target/purpose with the acceptable sizes of the g-force.

We investigate guidance according to the method of coincidence.

On the basis of formula (7.139), let us make several common/general/total derivations. Furing the approach of rocket for target/purpose r_p it approaches r_n and upon menderwous $r_p=r_n$, therefore

$$\left(\frac{y_{\rm m}p}{\sin^2\gamma}\right)^2 - r_{\rm p}^2 = \left(r_{\rm m}\frac{p}{\sin\gamma}\right)^2 - r_{\rm p}^2 = r_{\rm m}^2\left(\frac{p^2}{\sin^2\gamma} - 1\right). \quad (10.24)$$

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And the second second

In the case of p < 1 when at some γ possibly infinite normal acceleration of rocket. Thus, during the design of rocket it is necessary to observe condition p > 1. At the same time a_{np} it is directly proportional to the square of target speed, grows with increase in p and at high values v_n and p can render/show not

admitted. Furthermore, the normal acceleration of the rocket inversely proportional to distance of target/purpose $\frac{y_{\pi}}{\sin \gamma} = r_{\pi}$ and with low firing distances can also render/show exaggerated. the enumerated factors limit the applicability of guidance method on three-point curved. As a rule, it is applied for missile targeting to comparatively low-speed targets, for example - for the antitank missiles (projectiles), controlled from fixed and mobile guidance stations. The instrument realization of three-point guidance method are usually the command methods of control with the aid of radars or wire communication lines.

The tendency to utilize the best properties also of the command methods of control and homing/self-induction led to the appearance of the combined methods. The trajectory of the motion of the rocket with the combined method of control consists of the separate cuts, which correspond to the method of guidance accepted for each of them. For example, command predicted point guidance can be combined with boming/self-induction during the constant-bearing approach. For the increase of the accuracy of guidance desirally more possible smooth conjugation of the individual sections of trajectories not only in angle 6, but also in angular velocity $\frac{d\hbar}{dt}$.

The selection of one or the other method of control and method of guidance to target/purpose is conducted in each specific case by

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the design of rocket. On the tasis of the study of trajectories, are constructed the zones (spatial domains) of the possible attacks of target/purposes by the projectiles of class "surface - air" and "air - air". Being located at the moment of launchirg/starting within the zone of possible attacks, projectile can strike target/purpose. Kill protability depends on the technical flight characteristics of target/purpose and rocket. Similar investigations are conducted during ballistic design.

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§4. Concept of the optimum solutions of the problems of external tallistics.

Setting the most advantageous (cptimum) conditions of motion is one of the most important and complex problems of the theory of flight. As an example of optimum problem, it is possible to give the determination of angle of departure at which is reached the greatest horizontal flying range at the assigned initial velocity. The obtained from these sine conditions is called the angle of maximum range $f_{\rm trans}$. Most simply the argle of maximum range is determined for the parabolic trajectory of motion (i.e. if we do not consider the air resistance and to accept $\overline{g} = \text{const}$).

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Let us take formula (7.5)

$$x_{C} = \frac{v_0^2 \sin 2\theta_0}{g}.$$

Naximum range $x_c = x_{c,max}$ will be obtained when $\sin 2\theta_0 = 1$. Consequently, the angle of saxisus range $\theta_{cx_{max}} = 45^\circ$.

A characteristic example of the optimal solution is also the determination of the angle of maximum range within the limits of elliptical theory when $\bar{g}_{T} \neq \text{const.}$ Figures 7.5 shows that the maximum range will answer the greatest value of angle φ_{8} , determined on formula (10.18). From Keplerian equations it is possible to obtain the formula, which connects angles φ_{8} and θ_{10} .

$$tg \varphi_{s} = \frac{1}{2} \frac{r_{\mu} v_{\mu}^{2} \sin 2\theta_{\mu}}{g_{\tau 0} R_{3}^{2} - r_{\mu} v_{\mu}^{2} \cos^{2} \theta_{\mu}}.$$
 (10.25)

Let us find the maximum of function, differentiate $\frac{d}{d\theta_{H}}(^{t}g \Psi_{S})$ and after making its equal to zero. After conversions we will obtain the formula, which determines the angle of the maximum range:

$$\sin \theta_{mmax} = \sqrt{\frac{g_{10}R_3^2 - r_{\rm H}v_{\rm H}^2}{2g_{10}R_3^2 - r_{\rm H}v_{\rm H}^2}}.$$
 (10.26)

Sigures 10.7 shows curve/graph $\theta_{mmax} = f(v_m)$, comprised for the case $r_m = R_3$. From curve/graph it is evident that at the low initial

velocities the angle of maximum range is close to 45°. With an increase in the velocity, the angle of maximum range decreases, reaching zero at orbital velocity.

Is of interest solution by finding of the minimum initial velocity of the projectile of the constant mass, necessary for the overcoming of the assigned flying range with known $r_{\rm H}$. Problem is reduced, thus, to the determination of angle $\theta_{\rm H, OHT}$, by which will occur $\chi_{\rm H, MH}$. It differentiated equation (7.20) for $\theta_{\rm H}$ and after making obtained expression equal to zero, effer a series of mathematical conversions, we will obtain the following expression:

$$\theta_{n,our} = \frac{1}{2} \operatorname{arctg} \left(\frac{\sin 2\psi}{\frac{r_n}{R_3} - \cos 2\psi} \right).$$
(10.27)

The trajectory, which corresponds to angle $\theta_{n,our}$, calls the trajectory of minimum speed. The dependence of cptimum angle cr range angle for several values $\theta_{n,our}$ is visible from Fig. 10.8.

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If we consider air resistance, then problem regarding the angle of maximum range it is not solved in analytical form without

sufflementary simplifications. Simplest path - consecutive conducting of the large number of trajectory calculations for different values θ_{e} and graphing $r'_{c}=f(\theta'_{e})$. Value $\theta_{m_{max}}$ can be determined either from curve/graphs or by reverse/inverse interpolation. Even for the simple case of moving the body of constant mass in air (projectile of carnom-type artillery) the angle of maximum range depends not only on the imitial velocity, but also on hore, the weight and the form of projectile, united by the formula of ballistic coefficient. Figures 10.9 shows that depending on bore the angle of maximum range can change over wide limits - from ~30° to ~60°. Great difficulties are encountered during the determination of the optimum parameters, which ensure the greatest firing distance with the complex trajectories of the gwided and unguided missiles and projectiles.

Let us take the complex trajectory of the unguided rocket projectile (see Fig. 0.15.1). The determination of the initial angle of the trajectory of maximum range ε_0 and the connection points of jet engine (point N) in the trajectory of the unguided projectile, the passing in dense layers atmosphere, represents by itself multiparametric complex problem. Complete distance depends on

Ph. 6. Ch. 7 m. 9 m. 9 m. enclynamic drag coefficient (or ballistic coefficient $c_{m,n}$), the thrust **P** and of the operating time of engine on the phase of trajectory N - K. The distance, which corresponds to the trajectory phase from point K to encounter with target, is determined by values v_{n}, e_{n}, y_{n} and c_{n} .





Fig. 10.7. Dependence of the angle of maximum range Winks on

velccity " Key: (1) m/s.



Rig. 10.8. Dependence of optimum initial angle $\theta_{u,onv}$ on range angle 2v.

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Thus, total distance will depend on eleven discrete values and two functions $c_n(M)$ and P (y). It is obvious that the identification of

garameters, determining under these conditions the greatest firing
distance, represents by itself the very latoricus problem, the only
method of solution of which - calculation of the family of
trajectories.

With the relatively small firing distances, characteristic for tarrel artillery pieces, total distance is determined in essence by the second inactive leg. This makes it possible to simplify comparative calculations regarding the initial (gun) angle of maximum range.

For conducting comparative calculations, let us make following assumptions. A velocity increment for the time of the engine operation let us determine from formula K. F. Tsiolkovskiy and the total velocity in the combined point N-K to take as

$v_{\rm R} = v_{\rm R} + v_{\rm pu}$

where "pa-velocity, determined by formula K. F. Tsiolkovskiy;

 $v_{\rm H}$ - velocity of projectile at the scient of firing engine.

The torque/moment of firing engine in trajectory let us select on the angle of maximum range for the second inactive leg. For medium

and high calibers and for velocities v_{K} , the not exceeding ~600 m/s, without large error it is possible to take $\theta_{H}=45^{\circ}$.

Let us designate the over-all payload ratic of the rocket charge through

$$\mathbf{a} = \frac{Q_p}{Q_0}, \qquad (10.29)$$

where Q_p - weight of rocket charge;

 Q_0 - initial weight of projectile.

Then on formula K. E. Isiclkovskiy the velocity

$$\boldsymbol{v}_{pa} = \boldsymbol{w}_e \ln\left(\frac{1}{1-\alpha}\right). \tag{10.30}$$

Considering that the factor of the form of projectile will not change with the combustion of rocket charge, it is possible for the second passive section to accept

$$c_{n} = c_{0} \frac{1}{1-a}$$
 (10.31)



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Fig. 10.9. Dependence of the angle of maximum range $\theta_{m_{max}}$ on the initial velocity and the bore of projectile. Key; (1) m/s.

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With the adopted assumptions the initial argle of the trajectory of maximum range can be found via the comparison of the results of the calculations, conducted employing following procedure. At the assigned values v_0 to a for different θ_0 is calculated the first inactive ley before the achievement of point with angle $\theta_0 = 45^\circ$. The calculation of the second passive section with initial conditions

 $v_{\rm H} = v_{\rm H} + v_{\rm P,H}; \ \theta_{\rm H} = \theta_{\rm H} = 45^\circ; \ c_{\rm H}$ leads to the determination of flight range $x_{\rm C}$. Then calculations are repeated with variation by values

vo. co. a. Using the described procedure, we have comprised the tables of the fundamental characteristics of trajectory, which correspond to the point of firings of engine during which one should expect obtaining firing distance, close to the greatest. Tables gives LOC = 78107114 PAGE -26 403

gun angle of ϵ levation - θ_0 ;

the coordinate of the point of firing engine - $x_{\rm H}$ and $y_{\rm H}$;

time delay of firing ergine cr the time of the motion of projectile on the first inactive leg - $t_{\rm m}$;

the velocity of projectile at the somert of firing engine $-v_{\rm H}$.

Bor each of these values, are comprised independent tables. The entry into tables are the values:

wo within the limits of 50-600 m/s;

co within limits of 0.0-1.5;

a within limits of C.05-0.30.

Tables gives in appendix.

Data points for the compilation of the tables are designed by the method of numerical integration. The data of tables can be used
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as initial for the comparison of the diverse variants of trajectories. Work on tables is reduced to linear interpolation on three entries.

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It is necessary to keep in wind that the curve $x_{Cmax} = f(\theta_0)$ within the limits of the angles, close to those given in tables, has flat to Maxim machine gun. Therefore is admissible certain deviation frcs the values, given in tables, since it does not lead to the noticeable decrease of firing distance. At the low values co and vo and time t_n they prove to be small $(x_n \text{ and } y_n - a_n)$ va lue Xe. Vu crder of several ten meters and even it is less). These numerals are given in tables for the generality of solution in all range of the intake parameters. Taking into account the complexity of ballistic solution, one ought not to select low values 1, x, and y, since in this case engine will be included on the phase of trajectory where are still considerable angular flutter speed and the amplitude of nutation angle, but this will lead to an increase in scattering trajectories. Furthermore, engine must be included at safe distance frce gun crew and on materiel.

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The sclution of extreme problems in rocket engineering beginning with the so-called second task of "sicleoveliy which consists in the

determination of the law of a change in the mass of rocket and its velocity depending on time, with which it is possible to expect the greatest height/altitude of the rise of rocket. In the setting of Tsiclkowskiy, the problem is solved for vertical ascent of rocket without the account of the air resistance when \overline{g} = const.

Besearch showed that for the named conditions the maximum climbing range of rocket increases with decreased time of the ccsbustion of the fuel reserve. This law is disrupted upon consideration of the air resistance and variability of the acceleration of gravity with height/altituce. Ard air resistance, and gravitational force decrease with height/altitude; however, the energy losses of rocket, caused by the action of these forces, depend cn different factors. The losses, which depend on the air resistance, are proportional to certain rate of speed of motion and for their decrease it is necessary to approach the limitation of flight speed. The losses, determined by the action of gravitational force, are prepartional to the time of motion and it must be decreased, i.e., to approach the decrease of the burn-up time of fuel/propellant and the fastest achievement of larger flight speed. The version of the solution of problem during which total losses will be smallest, determines optimum state of motion and, therefore, greatest climbing range of rocket.

Brem the given examples it is evident that finding the optimum sclutions of the theory of flight is reduced both to determining of the geometric characteristics of the flight trajectories (angles of departure, form of trajectory, etc.) and to the determination of the states of motion of rockets (rate of motion, laws governing the fuel computition, etc.). The common/general/total formulation of the problem assumes the solution of the problem form of trajectory and flight conditions. The practical solution of this complex problem ercounters great difficulties. By especially complex is represented optimization according to several motion characteristics.

Usually are solved simpler problems with introduction under their conditions of supplementary simplifications. In this respect characteristic are two large groups of problems - determining the geometric characteristics of optimum trajectories under given conditions of flight (ergine power rating) and determination of optimum flight conditions under the assigned form of trajectory or the equivalent conditions, specifying geometric characteristics. The substantiated selection of the optimum versions of the solution of different problems in rocket engineering is based on the mathematical methods of variation calculus.

Variation calculus began to be developed from end/lead the XVII

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century. As his the fourder rightfully they consider the member of the russian Academy of Scierces of L. Euler. The object/subject of variation calculus are investigations for the extremum (maximum or the mirimum) of the separate magnitudes, called functionals.

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Functional - variable value whose value is determined by the selection of one or several functions. For example, the area of certain surface is functional, since it is determined by the selection of the function, entering the equation of surface of z =f(x, y). Resistance of medium to the driving/moving in it with the definite velocity body is also functional, since it depends on the function, which lays out of the surface of the driving/moving hody. One of the first tasks, solved the method of variation calculus, was the problem of the curve of the faster slope, to the so-called brachistochrone. In this problem were required to determine the form plane curve, the connecting two points, arrange/located on the different height/altitude, on which the body would roll down in shortest time. If we do not consider resistance of medium and friction, then such curved proves to be cycloid.

Let us call/name the typical variational problems of rocket engineering. For the rockets of class "surface - air", "air - air"

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and "air - surface" typical are brachistrectrene problems by choice of the program of motion, causing the minimum time, necessary for flight from point with origin coordinates x_1 , y_1 and velocity v_1 to point with the final coordinates x_2 and y_2 . The assignment to velocity v_2 in end point considerably complicates solution. Problems can be solved both for the climbing flight and for dive. Are of interest variational problems with the assigned final conditions feast the minimum fuel consuprior.

For the rockets of class "surface - surface" the most important variational problem it is possible to consider the problem of the selection of the program of maximum range. For tallistic missiles optimum solution is connected with the selection of value and line of force of the engine thrust. With the assigned magnitude of thrust, the variational problem can be brought to the selection of the program of pitch angle, which ensures maximum flying distance.

Setting the program of maximum range is direct-connected with the problems, which appear during the design of rockets. One Of most important is a question concerning setting of the optimum construction of rocket and conditions/mode of its motions, during which the payload (or warhead) will be supplied up to the assigned distance by the rocket of minimum launching weight. In this respect most characteristic is the problem of optimum weight distribution

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between the step/stages of staged rccket ard ergine power ratings of each step/stage. As evaluation criteria of solution is usually received the minimum total weight of rccket, which ensures satisfaction of conditions for final velocity or flying ranges of the ncse come of the assigned weight.

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For winged missiles characteristic variational problem it is possible to consider the problem of the programming of the thrust of starting and sustainer engines. The problem of the programming of thrust is also characteristic for the high-altitude rockets, intended for the sounding of the atmosphere. Wide distribution received variational methods during the optimization of trajectories and states of motion of space vehicles. Mary of the named problems are solved by Soviet scientists [2], [31], [44], [45]. Foreign authors's works are generalized in collection [24].

Let us examine the schematic of solution of one of the variational problems of rocket engineering. Let be required to find the function, which determines the consumption of fuel (change in the mass of the rocket) in time, during realization of which the heigh#/altitude of vertical ascent of rocket will be greatest. It will use the equation of Meshcherskiy for vertical ascent of rocket

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(1.9), after replacing for simplification in the investigation the entering it sum of values $p_r - \frac{dm}{dt} w_{orm}$ by term $-\frac{dm}{dt} w_r$, where $w_r = \text{const} - \text{effective discharge velocity.}$

Seplacing also \dot{x} and $\dot{\dot{x}}$ on v and $\frac{dv}{dt}$, let us have

$$m \frac{dv}{dt} = -mg - \frac{dm}{dt} w_e - R(v). \qquad (10.32)$$

Let the mass of rocket change on dependence $m=m_0$, where f the function, which characterizes a change of the mass of rocket (fuel consumption) in the process of the ergine operation. In the beginning of motion f(0) = 1.

After replacing in equation (10.32) value of m on m_0f and after dividing all the terms into constant value m_0 , we will obtain

$$f\frac{dv}{dt} = -fg - \frac{df}{dt} \boldsymbol{w}_e - \frac{R(v)}{m_0} . \qquad (10.33)$$

Let us conduct the replacement of variables, after introducing the value of the elementary path of rocket dS

$$\frac{dv}{dt} = \frac{dv}{dS} \cdot \frac{dS}{dt} = v \frac{dv}{dS}; \quad \frac{df}{dt} = \frac{df}{dv} \cdot \frac{dS}{dt} = f_v \cdot v \frac{dv}{dS}$$

Then

$$f \cdot \nabla \cdot \frac{dv}{dS} = -fg - \nabla f_{\sigma} \Theta, \frac{dv}{dS} - \frac{R(v)}{m_0}$$

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After conversion and integration, we will obtain expression for the functional

$$S[f(v)] = \int_{-\infty}^{v} \frac{(f+f_{v}\cdot v_{c})vdv}{gf+\frac{1}{m_{0}}R(v)}.$$
 (10.34)

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The target/purpose of the subsequent solution - to find this function f(v), with which the value of integral will be greatest, i.e., to determine this law of mass charge from velocity, during which climbing range of the rocket will be maximum.

The determination of this function is called the investigation of integral for the extremum. Investigation must be conducted with specific boundary conditions; at the start of the studied segment the velocity of rocket $v = v_0$, and of f = 1, i.e., mass $n = n_d$; at the end of the section respectively $v = v_H$, and $m = m_H$. These conditions are determined by design features of the type in question of rockets.

The formulated problem of variation calculus is solved with the

aid cf the differential equations cf Euler, which determine necessary the condition of the extremum of the functional, which has the general view

$$L[y(x)] = \int_{x_{y}}^{x} F[x, y(x), y_{x}] dx, \qquad (10.35)$$

where F - the assigned function of the arguments x, y(x) and $y'_x = \frac{dy}{dx}$.

Let be found function y = f(x), that ensures the extremum of functional. For the determination of conditions which must answer this function, is introduced into examination curve, close to f(x), called the curve of comparison. Function \overline{y} , which determines the curve of the comparison of the same family of curves, as f(x), little from it differs

$$y = f(x) + \operatorname{sd} y(x),$$

where e a low number, and $\delta y(x)$ - the arbitrary function, which turns into zero at the end/leads of the interval/gap of integration, i.e.,

 $\delta y(x_0) = \delta y(x_1) = 0.$

When e=1 the difference between furctions $\delta y(x) = \overline{y}(x) - y(x)$ is called in terms of increase or a variation in argument y(x) of furctional L [y(x)].

Understanding of variation (δy) of argument y(x) of functional

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L[y(x)] it differs dsignificantly from an increase Δx in argument x of function f(x). An increase in the argument is connected with a change in it for the assigned furction, and variation δy is increments of coordinates y because of a change in the form of the function at the fixed value of argument x. In variation calculus it is proven, that the necessary condition of extremum is the inversion in mere variation in the functional. If function f(x) = F[x, y(x), y_x within limits from x_0 to x_1 it is single-valued, three times differentiated it is continuous itself and are continuous it partial derivatives, then a variation in the functional it turns into zero, if the unknown function answers the equation of Euler

$$\frac{\partial F}{\partial y} - \frac{d}{dx} \left(\frac{\partial F}{\partial y'} \right) = 0. \tag{10.36}$$

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This condition is necessary. Integral curve equations of Euler y = $y(x_0 \ C_1, \ C_2)$ are called extremals. Crly cr extremals can be reached the extremum of functional. For determining the function during which car be obtained the extremum, one should integrate the equation of Euler. Arbitrary constants are located from the boundary conditions $y(x_0) = y_0$ and $y(x_1) = y_1$.

The difficulties of prectical solution corrections in the fact that the differential equation of Buler is second order equation and its

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sclution not always can be obtained in final form. If solution takes the final form, then it is must it additionally to check to sufficient conditions. In each specific problem must be stipulated or somehow additionally the defined class of functions within limits of which searches for the extremal.

Binding extremum becomes complicated for the functionals, which comptain several functions of independent variable. In this case is comprised and is solved the system of differential equations, in which the number of equations corresponds to the number of functions. Utilizing the aforesaid, let us find the extremum of functional (10.34). In our case the integrand takes the form

$$F = \left[\frac{(f+f_{\varphi}\mathbf{v}_{e})\mathbf{v}}{gf + \frac{1}{m_{0}}R(\mathbf{v})}\right].$$
 (10.37)

For a substitution in the equatica of Fuler, let us find

 $\frac{\partial F}{\partial f} \cdot \frac{\partial F}{\partial f_{v}} \stackrel{f}{H} \frac{d}{dv} \left(\frac{\delta F}{\partial f_{v}} \right),$ Key: (1). and

bearing in mind that variables are v, F(v) and function f, which depends on v, but value g = ccnst. Then

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$$\frac{\partial F}{\partial f} = \frac{v}{gf + \frac{1}{m_0} R(v)} - \frac{(f + f_v m_0) gv}{\left[gf + \frac{1}{m_0} R(v)\right]^2};$$

$$\frac{\partial F}{\partial f_v} = \frac{v_e v}{gf + \frac{1}{m_0} R(v)};$$

$$\frac{d}{v} \left(\frac{\partial F}{\partial f_v}\right) = \frac{v_e}{gf + \frac{1}{m_0} R(v)} - \frac{v_e v \left(\frac{1}{m_0} \frac{\partial R(v)}{\partial v} + gf_v\right)}{\left[gf + \frac{1}{m_0} R(v)\right]^2}$$

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Substituting right sides of the writter equations in (10.36) and sultiplying after substitution everything by $\left[gf + \frac{\dot{R}(v)}{m_0}\right]^2$, we will obtain

$$f = f(v) = \frac{1}{gm_0 w_e} \left[(v - w_e) R(v) + v w_e \frac{\partial R(v)}{\partial v} \right]. \quad (10.38)$$

Fage 4C8.

As a result of solution, is cleained the equation of extremal during which occurs the maximum of rath S. The obtained dependence cannot be directly used for the solution of our problem, since we obtained function f depending on the velocity of the motion of rocket v, but the majority of systems of equations describes the motion of the rocket with independent by the variable t.

let us present

$$\frac{df}{dt} = \frac{df}{dv} \cdot \frac{dv}{dt} = f'_v \frac{dv}{dt}$$

and, after conducting the appropriate replacement in (10.33), we will chain

$$(f+f'_{\mathbf{v}}\mathbf{w}_{\mathbf{r}})\frac{d\mathbf{v}}{dt}=-\left[gf+\frac{1}{\mathbf{m}_{0}}R(\mathbf{v})\right],$$

whence

$$t = \int_{v}^{v_{e}} \frac{\left(f + f_{v}^{\top} \cdot w_{e}\right) dv}{\pi f + \frac{1}{m_{0}} R(v)}.$$
 (10.39)

For taking of integral, it is necessary to substitute in it right side (10.38) and derivative f_v , which we will preliminarily obtain also from (10.38)

 $\dot{f_{v}} = \frac{d}{dv} (f) = \frac{1}{gm_{0}w_{e}} \left[R(v) + v \frac{\partial R(v)}{\partial v} + v w_{e} \frac{\partial^{2} R(v)}{\partial v^{2}} \right].$

Accepting, as tefcre g = ccrst, we will citain from (10.39) working formula for t

$$t = \frac{v_0 - v}{g} + \frac{w_e}{g} \int_{-\infty}^{\infty} \frac{\left(\frac{\partial R(v)}{\partial v} + w_e \frac{\partial^2 R(v)}{\partial v^2}\right) dv}{R(v) + w_e \frac{\partial R(v)}{\partial v}} . \quad (10.40)$$

At the concrete/specific/actual value of function R (v), that determines the air resistance, integral (10.40) there can be taken numerically, after which will be determined the dependence (curve/graph) t(v). Dependence f(v) is located from (10.38) after substitution into it derived $\frac{\partial R(v)}{\partial v}$ of the concrete/specific/actual function R(v). Having dependences f(v) and t(v), it is possible to establish the unknown dependence f(t), that determines a change in the mass of the rocket in time, with which the path of the rocket will be maximum. We focus attention on the fact that the overall consumption of mass (fuel/propellart) remains constant, appears itself only the optimum version of its expenditure during flight. The concrete/specific/actual form of the function f(t), that connects the current and initial masses of the rocket in time function f(t), that connects the stipulated assumptions depends on curve/graph, determining R(v).

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If we introduce for R(V) analytic function during which there

can be taken integral in expression (10.40), then dependence t(v) will obtain final form. For example, in [31] during the determination of the air resistance are accepted by constants air density and drag coefficient. Then with ρ = const and $c_x = const$ we obtain

$$R(\mathbf{v}) = \frac{1}{2} \mathbf{v} \mathbf{v}^{\mathbf{s}} c_{x} = \beta \mathbf{v}^{\mathbf{s}}; \quad \frac{\partial R(\mathbf{v})}{\partial \mathbf{v}} = 2\beta \mathbf{v}; \quad \frac{\partial^{2} R(\mathbf{v})}{\partial \mathbf{v}^{2}} = 2\beta.$$

Carrying out the replacement of functions under integral in (10.40), let us have

$$t = \frac{1}{g} \left[\boldsymbol{v}_{0} - \boldsymbol{v} - \boldsymbol{w}_{e} \int_{v}^{v_{e}} \frac{2\left(\boldsymbol{v} + \boldsymbol{w}_{e}\right) d\boldsymbol{v}}{\boldsymbol{v}\left(\boldsymbol{v} + 2\boldsymbol{w}_{e}\right)} \right].$$
(10.41)

Integration gives the known fcrmula, which connects time with the velocity of the motion of the rocket

$$t = -\frac{1}{g} \left[v_0 - v - w_e \ln \frac{v_0 (v_0 + 2w_e)}{v (v + 2w_e)} \right].$$
(10.42)

Gecall that this relatively simple solution of variational problem is obtained with assumptions g = ccrst, $\rho = const$, $c_x = const$ and $R(v) = Bv^2$. Simultaneous failure of these assumptions leads to the problem, virtually not solved at present within the framework of classical variation theory.

In the examined above example the curves among which one should

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search for the solutions of variational problem, had the fixed/recorded end points, determining integration limits of the functional. More great possibilities have variation of solution under alternating/variable boundary conditions. They frequently assume that the end-points are located on the specific lines or even surfaces. In these cases the determination of arbitrary constants during the solution of the equation of Fuler requires supplementary conditions. These conditions are called transversality conditions.

The extremum of functional can be reached on extremals with foints of inflection. Between points of inflection separate smooth cuttings off must be the integral curves of the equation of Euler. The coordinates of points of inflection must satisfy Weierstrass-Erdmann's supplementary conditions. Eesides those named, were citained the considerable propagation and other necessary conditions, for example, of the condition of Clebsch, Jacobi, etc.

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Many of the problems of the optimization of the states of motion of rockets can be referred to the class of the problems, called in variation calculus isoperimetric. This name is appropriated to the problems, in which is placed supplementary isoperimetric condition in the form of an auxiliary functional. As example of the similar

rrotlem in the overall theory of variation calculus to usually serve problem for finding of the cecmetric figure of maximum area with the assigned perimeter. A characteristic example of the isoperimetric problem of rocket dynamics can be obtained, after using the given above formulas, which describe vertical ascent of rocket. It is necessary to supply supplementary condition about the minimization of the time of the motion of rocket from point C to point K, which lies on of direct/straight vertical climb. On the basis of (10.39), let us write expression for the functional, which determines the minimum time of the motion of rocket from point C to point K:

$$T[f(v)] = \int_{v_v}^{v_e} \frac{(f + f_v w_e) \, dv}{gf + \frac{1}{m_0} R(v)}.$$
 (10.43)

Froblem consists in finding of this furction f(v), that determines the fuel consumption, by which the rocket will move from point C into point K for minimum time. The unknown function must answer the supplementary condition, writter in accordance with (10.34) and to that determining the path of rocket from point C to point K

$$S_{\text{ex}} = \int_{v_{\text{f}}}^{v_{\text{f}}} \frac{(f + f'_{\text{v}} \cdot w_{\text{f}}) \, vdv}{gf + \frac{1}{m_0} \, R(v)} \,. \tag{10.44}$$

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Furthermore, as before must is maintaired boundary conditions. In the beginning of the section (with $v = v_0$) teing investigated f =1; at the end of the mection (at print K, with $v = v_0$) $f = f_m$ t.e. the mass of rocket $m = m_m^2$. The solution of a minilar problem is given in [31]. The fundamental difficulties, which are encountered when molving variational problems by classical methods, and the imperative need for practical solutions led to the appearance of different approximation methods, to which can be attributed finite-difference method, a Ritz's method and a series of others.

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Becently in connection with the development of electronic digital computers for the solution of variational problems, won acceptance the method of dynamic programming [64].

We focus reader's attertion to the fact that in practical work on the creation of the specimen/samples of nocket and ordnance theoretical variational methods are always combined with design studyings and the comparison of the design/projected specimen/samples with emisting well recommended themselves by systems. ECC = 78107114 PACE 46 722

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Chapter XI.

COBFFCTION FORMULAS OF EXTEENAL EAILISTICS.

The basic problems of external hallistics, examined above, they were solved with the initial data, corresponding to technical specifications for rocket (projectile) and to the characteristics of standard atmosphere.

The trajectory, designed at the "normal" values of its specifying factors indicated, is called basic or that not disturbed. Luring the description of the complex process of flight on mathematical means for simplification in the calculations, the part of the acting factors is not considered, but some of them are taken by average values. However, the conditions of firing in the majority of the cases differ from calculated theoretical. As an example of the nonconformity of calculated and actual conditions, it is convenient to exile to the account of the effect of weather factors. As is known, the problem of external ballistics is solved for the normal (standard) atmosphere, and the meteorological conditions with firing, as a rule, differ from the standard. Calculations are conducted for still air (dead calm), but in actuality very frequently the atmosphere is not calm and wind can substantially change the results COC = 78107114 PAGE 47 823

cf firing. They affect the parameters of trajectory and deviation of cther weather factors from their normal values.

Desiring to establish effect on the trajectory of any cell/element, than not earlier examine/considered, it is necessary to comprise the new system of differential equations, including the which interests us value. For example, solving one time the system of equations of the motion of the center of mass of projectile, comprised without the account of the curvature of the Earth and its rotation, but for the second time - comprised with their account, it is possible to rate/estimate the effect of the ramed factors on the motion characteristic of the center of mass of projectile. It is necessary to comprise the new systems of differential equations for the account of the deviations of some weather factors, for example for the account alternating/variable with respect to height/altitude ard wind direction.

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In many instances the effect of the perturbation factors can be considered without the compilation of new differential equations. Most simple this to make when the which interests us factor is already taken into account in fundamental differential equations and it is necessary to establish the effect of its change to the results

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cf calculation or firing. If the parameter leing investigated changes substantially, then the effect of this change to the results of performance calculation of motion and trajectory elements should find by the solution of the basic system of differential equations with the new changed data. The comparison of the results of the solutions, cbtained with those changes and "nermal" data, gives allowance.

In certain cases it proves to be possible to introduce into the results of the solution of interference correction of the factors, which do not contain in the basic system of differential equations, and without comprising the new system of equations, which includes the which interests us value. For example, effect on the distance of the form of the Earth can be established by indirect method from geometric considerations, without solving the system of the differential equations of model of the Earth.

In practice, as a rule, it is necessary to meet the small deviations of the determining parameters from their normal (drawing rccms and standard) values. In the majority of the cases, the low deviations of parameters lead to small changes in the trajectory elements. This makes it possible to establish the effect of the perturbation factors on the characteristics of the undisturbed trajectory by determining the corrections in these cell/elements with EOC = 78107114 EAGE 49 425

the aid of various short-cut methods and dejendences.

Corrections are called changes in the sotion characteristics or trajectory elements, which correspond to the deviations of its determining parameters. Corrections, as a rule, are calculated on the motion characteristics of the center of mass of rocket or projectile for any fixed/recorded point in the trajectory. Figures 11.1 shows active sections not disturbed 1 and the disturbed 2 trajectories of the unguided rocket. The corrections, which correspond to the torque/moment of the end/lead of the engine operation, caused by the deviation of any determining parameter or group of the parameters, will be: δx_n — correction for actual conditions into coordinate x_n ;

 $\delta y_{\rm H}$ — a correction coordinate $y_{\rm H}$; $\delta \sigma_{\rm H}$ — a correction into the velocity of the motion of the center of mass of projectile; $\delta t_{\rm H}$ — correction during the operation of engine and so forth





Fig. 11.1. A change in the trajectory of the motion of the center of mass of rocket because of a change in the determining parameter: 1 - undisturbed trajectory; 2 - disturbed trajectory.

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Corrections can also be determined for the point, assigned by any motion characteristic (by time, velocity, the abscissa, ordinate, etc.), identical for basic and that agitated of trajectories. For example, to point b of the undisturbed trajectory from the condition of the constancy of flight time will correspond point e of the trajectory of the disturbed motion; in this case, corrections for a trajectory in point b will be values $\delta_{X_i}, \delta_{y_i}, \delta_{v_i}$ and so forth. During the assignment of condition y = const to point e corresponds point c DOC = 78107114

and for a trajectory at point a we will obtain corrections δx_y , δv_y

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If we take condition x = const, then to point a corresponds point d, moreover corrections for a trajectory in point a are equal to by, bu, dr, and the like. For the projectiles of barrel terrestrial artillery and surface-to-surface missiles usually is determined correction into complete firing distance and the deviation of the impact point in the projectile in side direction. When conducting of direct/straight ballistic calculations, i.e., during the solution of the first problem of external ballistics, correction they are introduced into the cell/elements of the undisturbed trajectory, designed under standard conditions (drawing rooms and meteorological). During processing of the results of the firings, obtained under actual conditions, the corrections are introduced into experimental data with the fact in order to lead them to standard conditions.

In the latter case the sign of correction will be reverse/inverse to the sign, determined when conducting of direct/straight ballistic calculations.

§1. Correcting formulas and correction factors.

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In general form, for the trajectory element either motion characteristic it is possible to write

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 $A = f(\xi_1, \xi_2, \xi_3, \dots, \xi_n),$ (11.1)

where A - a trajectory element or motion characteristic;

& - : specifying parameter.

The deviations of the parameters from computed values, designated **%**, **%**, **...**, **%**, will cause a charge in the trajectory element A which let us designate 6A. In accordance with the connection will be determined by formula (E.11). The number of terms of expansion, held in calculations, depends on the required accuracy of the determination of correction. Most frequently during the solution of the practical problems of the theory of corrections, hold the linear terms of expansion. The obtained with this formula corresponds to the formula of total differential from functional dependence (11.1).

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If cne assumes that the parameters, united by functional dependence (11.1), do not depend on each other and in the formula of total

differential infinitesimal increases to replace with certainly low, then it is possible to obtain

$$\delta A = \frac{\partial f}{\partial \xi_1} \, \delta \xi_1 + \frac{\partial f}{\partial \xi_2} \, \delta \xi_1 + \ldots + \frac{\partial f}{\partial \xi_n} \, \delta \xi_n. \tag{11.2}$$

In last/latter formula are not considered the terms of expansion of higher orders, than the first. The error in components/specific/actual calculations, which depends on neglect of the remainders of expansion, is determined by special investigations.

Correcting formulas of type (11.2) are called differential, and the specific by them values δA are called the corrections, calculated according to the method of differentials.

Ballistic derivatives $\frac{\partial f}{\partial \xi_i} = \frac{\partial A}{\partial \xi_i}$ in the theory of corrections are called correction factors for a cell/element A of trajectory to the deviation of the parameter ξ_i . The correction factor (ballistic derivative) it is numerical equal to a change in the trajectory element in question with an increase in the corresponding determining parameter by the unit of its measurement accepted. In the theory of the corrections of cannon-type artillery the correction factors, which characterize changes in the distance curing the deviations of tallistic coefficient $-\frac{\partial x_c}{\partial c}$; the initial velocity $-\frac{\partial x_c}{\partial c_0}$ and angle of departure $-\frac{\partial x_c}{\partial c}$, is conventionally designated as basic correction coefficients.

As an example let us examine citaining correcting formulas for a complete horizontal firing distance from artillery piece under conditions of the vacuum. In the named case complete distance to be determined, as is known, by dependence (7.5)

$$x_c = \frac{\sigma_0^2 \sin 2\theta_0}{g} \, .$$

We will obtain correcting formula taking into account the terms of the expansion of the first and second orders. Taking into account that in the last/latter formula only two arguments (v_0 and θ_0), in general designations on (8.11) let us have

$$\begin{aligned} \mathbf{k}_{C} &= \frac{1}{11} \left(\frac{\partial}{\partial \xi_{1}} \mathbf{k}_{1} + \frac{\partial}{\partial \xi_{2}} \mathbf{k} \right) f(\xi_{0}, \xi_{0}) + \\ &+ \frac{1}{21} \left[\frac{\partial}{\partial \xi_{1}^{2}} (\mathbf{k}_{1})^{2} + 2 \frac{\partial}{\partial \xi_{1} \partial \xi_{2}} \mathbf{k}_{3} \mathbf{k}_{4} + \frac{\partial}{\partial \xi_{2}^{2}} (\mathbf{k}_{0})^{2} \right] f(\xi_{0}, \xi_{0}). \end{aligned}$$
(11.8)

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In the designations of formula (7.5) we will obtain

$$\begin{aligned} \mathbf{\delta} \mathbf{x}_{C} &= \frac{\partial \mathbf{x}_{C}}{\partial v_{0}} \, \mathbf{\delta} \mathbf{v}_{0} + \frac{\partial \mathbf{x}_{C}}{\partial \mathbf{0}_{0}} \, \mathbf{\delta} \mathbf{0}_{0} + \\ &+ \frac{1}{2} \left[\frac{\partial^{2} \mathbf{x}_{C}}{\partial v_{0}^{2}} \, (\mathbf{\delta} \mathbf{v}_{0})^{2} + 2 \, \frac{\partial^{2} \mathbf{x}_{C}}{\partial v_{0} \partial \mathbf{0}_{0}} \, \mathbf{\delta} \mathbf{v}_{0} \mathbf{\delta} \mathbf{0}_{0} + \frac{\partial^{2} \mathbf{x}_{C}}{\partial \mathbf{0}_{0}^{2}} \, (\mathbf{\delta} \mathbf{0}_{0})^{2} \right]. \end{aligned}$$

Same And

Crening the values of partial derivatives, let us have

$$h_{C} = \frac{2v_{0}\sin 2\theta_{0}}{g} \delta v_{0} + \frac{2v_{0}^{2}\cos 2\theta_{0}}{g} \delta \theta_{0} + \frac{\sin 2\theta_{0}}{g} (\delta v_{0})^{2} + \frac{4v_{0}\cos 2\theta_{0}}{g} \delta v_{0} \delta \theta_{0} - \frac{2v_{0}^{2}\sin 2\theta_{0}}{g} (\delta \theta_{0})^{2}.$$
(11.4)

Last/latter three terms are determined by quadratic terms of expansion and are the values of the higher order of smallness, than two first, determined by the linear terms of expansion. In the majority of the cases, are utilized only linear terms and correction is determined so

$$\delta x_{c} = \frac{2v_{0} \sin 2\theta_{0}}{g} \delta v_{0} + \frac{2v_{0}^{2} \cos 2\theta_{0}}{g} \delta \theta_{0}, \qquad (11.5)$$

where the ballistic derivatives (ccrrection factors) are equal to

$$\frac{\partial x_{\rm C}}{\partial v_0} = \frac{2v_0\sin 2t_0}{g}; \quad \frac{\partial x_{\rm C}}{\partial t_0} = \frac{2v_0^2\cos 2t_0}{g}. \quad (11.6)$$

Having a value of correction factor, it is easy to find correction in equivalent component of the trajectory

$$\delta A_i = \frac{\partial A_i}{\partial \xi_i} \, \delta \xi_i. \tag{11.7}$$

If one assumes that changes only initial velocity, then

$$\delta x_{Cv} = \frac{\partial x_C}{\partial v_0} \delta v_0. \tag{11.8}$$

Stilizing (7.5) and first formula (11.6), we will obtain

$$\frac{kx_{CV}}{x_C} = 2 \frac{kv_0}{v_0},$$

i.e. with the assumptions, which correspond to parabolic theory, the low relative deflection of the initial velocity causes the doubled relative change in the distance. Freceeding in like manner, it is presible to obtain correcting formulas also for other cell/elements of the trajectory.

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The derivation of the formulas of corrections and formula of the ballistic derived for trajectories different types of rockets and projectiles is considerably more complex then this is shown higher based on simple example, and it requires special methods.

§2. Qualitative effect of the determining parameters and the signs of tallistic derivatives.

The sign of ballistic derivative (correction factor) for the complete flight distance of the projectiles of rocket and artillery pieces is set depending on the effect of an increase in the determining parameter by firing distance. If an increase in the parameter leads to an increase in the firing distance, then correction factor has positive value; if firing distance decreases with an increase in the determining parameter, then correcting coefficient has negative sign. In many instances the sign of correction factor can be established that or the basis of the gualitative effect of the parameter on distance, before conducting of bcc = 78107115

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calculations according to its determination.

Let us conduct qualitative analysis recarcing the signs of basic correction factors. The initial velocity of the unguided projectile of constant mass - artillery shell and the velocity of nose come in the beginning of inactive leg they are some of the basic parameters, which estimate distance of firing. With an increase in the initial velocity, the distance crows; therefore correction factor has positive value. For rockets with inclined start, the velocity of descent from starter (the initial velocity) is, as a rule, small the part of the velocity of the center of mass of projectile at the end of powered flight trajectory $-v_{\rm M}$. An increase in the velocity of descent from guides leads to increase of the ordinates of an entire trajectory and an increase $v_{\rm M}$. Thus other conditions being equal an increase in the initial velocity of projectile leads to an increase in the firing distance, therefore, $\frac{\partial x_{\rm C}}{\partial x} > 0$.

The lift-drag ratios of projectile affect the firing distance through the area of midsection S and the drag coefficient $c_x(M)$. An increase in the midsection an increase in the integral value of drag coefficient $\tilde{c}_x(M)$ for the time of motion along trajectory will lead to the decrease of firing distance.

It is obvious that

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$$\frac{\partial x_c}{\partial S} < 0$$
 and $\frac{\partial x_c}{\partial \tilde{e}_x(M)} < 0$.

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If we substitute (2.95) into the equation

$$\frac{dv}{dt} = -\frac{X}{m} - \int \sin \theta$$

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that we will obtain

$$\frac{dv}{dt} = -cH(y)F(\Phi) - g\sin \theta.$$

Besignating integral characteristics for the time of the sotion of projectile along trajectory through H(y) and P(v), let us have

$$\frac{\partial x_c}{\partial H(y)} < 0$$
 and $\frac{\partial x_c}{\partial F_s} < 0$.

For the projectiles of the constant mass c = const and one of the basic correction factors $\frac{\partial x_c}{\partial c} < 0$.

Sor the unguided projectiles of variable and constant mass, initial angle of departure is one of the basic determining parameters. The angle of arrival in the beginning of the inactive leg of unguided rocket flight also significantly affects the firing distance. Fig. 11.2 shows the dependence of complete distance for angle θ_{m} and the dependence of correction factor for an initial ECC = 78107115

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engle of departure $\mathbf{A}_{\mathbf{r}}$. For larrel systems and unguided rockets $\mathbf{A}_{\mathbf{r}} = \mathbf{A}_{\mathbf{r}}$. For the inactive legs of the guided and unguided missiles $\mathbf{A}_{\mathbf{r}} = \mathbf{A}_{\mathbf{r}}$.

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Fig. 11.2. Curve/graph of a change of the correction factor $\frac{\partial x_c}{\partial x_u}$ in function from initial angle of departure.

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If θ_n is lesser than the angle of maximum range, then is correction factor $\frac{\partial x_c}{\partial \theta_n} > 0$, if θ_n more angle θ_{mmax} , then $\frac{\partial x_c}{\partial \theta_n} < 0$. In general form it is possible to write $\frac{\partial x_c}{\partial \theta_n} \ge 0$. The sign of coefficient changes in the dependence from value θ_n and the angle of maximum range θ_{mmax} .

Of the fixed/recorded initial velocity $v_{\rm H}$ correcting coefficient $\frac{\partial x_{\rm C}}{\partial u_{\rm H}} = 0$ corresponds to maximum mange and angle of departure, with which this distance is obtained. With firing with the angles, close to the argle of maximum distance small changes in the angle $\theta_{\rm H}$ barely affect the firing distance.

The cell/elements of the end/lead of powered flight trajectory

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 $x_{\rm H}, y_{\rm H}, v_{\rm H}$ and $\theta_{\rm H}$ can be accepted as the independent parameters, which estimate distance of firing. Effect $v_{\rm H}$ and $\theta_{\rm H}$ on firing distance the projectiles of constant mass is examined above. An increase in the abscissa of powered flight trajectory $x_{\rm H}$ when $y_{\rm H}={\rm const}$ displaces trajectory to the right; with this ar increase $x_{\rm H}$ and firing distances $x_{\rm C}$ are equal, i.e., $\frac{\partial x_{\rm C}}{\partial x_{\rm H}}=1$.

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An increase in the crdinate y_{R} leads to ar increase of the distance as a result of the motion of rocket in the higher and less dense layers of atmosphere and certain elorgation of final trajectory. Correction factor $\frac{\partial x_{c}}{\partial y_{R}} > 0$.

A change in the mass of projectile affects differently the firing distance. For the projectiles of constant mass from equation v= -X/m - g sin θ is evident that with an increase in the mass decreases the acceleration of projectile from the air resistance X/m, the velocity of projectile decreases more slowly and firing distance increases; consequently,

$$\frac{\partial x_c}{\partial u} > 0.$$

The artillery shell of smaller weight under the identical conditions of throwing (identical charge) from instrument obtains high initial velocity v_0 . However, in flight light/lung projectile faster loses its velocity than identical to it in form heavy projectile. The total effect of a change in the mass (weight) of $\mathbf{EOC} = 78107115$

projectile to firing distance from artillery instrument is set by the special correcting formula (see further).

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For rockets on powered flight trajectory

FIGE

 $\dot{v} = \frac{P-X}{R} - g \sin \theta_i$

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with F > X with an increase in the mass, the acceleration decreases, which, other conditions heirg equal, will hing to smaller v_m , and, consequently, also to smaller distance. Thus, the sign of correction factor must be determined during calculation according to concrete/specific/actual data.

A change in the weather factors will ertail a change in the air resistance. An increase in the air dersity increases velocity head and decreases firing distance; consequently,

 $\frac{\partial x_c}{\partial q} < 0.$

An increase of the tarcmetric pressure at launch point and in trajectory leads, in accordance with (2.45), to an increase in the air density; consequently,

 $\frac{\partial x_c}{\partial b_0} < 0$ and $\frac{\partial x_c}{\partial b_c} < 0$.
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Complex proves to be the temperature effect of air on firing distance. In accordance with (2.45) an increase in the temperature **leads to the decrease of air density. simultaneously an increase** τ **leads to an increase in the speed of sound in air and a change in Mach ausber.** An increase in Mach number car lead to an increase of function $c_{\pi}(M)$ in the range of increase curve $c_{\pi}(M)$, and can give decrease $c_{\pi}(M)$ on descending leg of a curve $c_{\pi}(M)$. The temperature effect of air on firing distance and, consequently, also the sign of correction factor they are set for artillery projectiles of constant mass on correcting formulas and special tables; for rockets - by one of the methods, presented below.

For the gyroscopically stable projectiles of constant and variable mass, the tailwind increases firing distance and

 $\frac{\partial x_c}{\partial x_c} > 0.$

§3. Methods of calculating the corrections and of ballistic derivatives.

The methods of determining of Fallistic derivatives and ccrrections substantially change depending on the form of trajectory and designation/purpose of calculation.

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Find a use method of the direct/straight sclution of the systems of differential equations of motion, method of the solution of the differential equations of corrections, method of calculating the corrections with the aid of the corrections with the aid of the equations, method of determining the corrections with the aid of the law of similarity of trajectories, different analytical methods of calculating of corrections and the correction factors, difference and takular methods. For the complex trajectories, subdivided during calculations into individual sections (for example, active and passive the controlled and unguided flight) proves to be advisable the application/use of different combined methods.

3.1. Determination of corrections and correction factors by the integration of the systems of equations of action.

Method is applied during a substantial charge in the parameter, which determines the trajectory element being investigated, or if necessary to determine correction for the parameter, which does not LCC = 78107115

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contain in the basic system of equations, comprised for the undisturbed trajectory.

In the first case tasic system is solved with n the discrete values of the determining parameter on at n the functional dependences, correction for change in which it is possible to determine. Let, for example, as a result of structural/design changes in exterior form of rocket on its control organ/controls change the form of the curve/graphs of drag coefficient. Let us designate functional dependence for the undisturbed trajectory c_x (H, Re) and for the missile trajectory of the changed form c_{xt} (H, Re). For determining the effect of a change in the form of rocket to the cell/elements of the end/lead of powered flight trajectory, we will use the system of equations (3.59), after directing thrust along the axis of rocket and after drop/omitting control forces. For the undisturbed trajectory the system can be solved at the values

$$X = qSc_x(M, \text{ Re}); \quad Y = qSc_x(M, \text{ Re})a.$$
(11.9)

For the rocket whose form changed,

$$X_{I} = qSc_{st}(M, Re); \quad Y_{I} = qSc_{st}(M, Re) =$$
(11.10)

system of equations will take the form

$$\begin{array}{c} \dot{y} = \frac{P - X_i}{m} - g \sin \theta; \quad \dot{\theta} = \frac{P + Y_i}{mv} = -\frac{f}{v} \cos \theta; \\ \dot{y} = v \sin \theta; \quad \dot{x} = v \cos \theta; \quad m = m(t); \quad a = \theta_{ab}(t) - \theta. \end{array} \right\} (11.11)$$

If during a change in the form of the rocket (or the construction of control devices) besides X and Y changes even axial

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ccmponent of the engine thrust, then during the solution of last/latter system one should use the changed dependence P_{i} . The comparison of the results of solutions for the undisturbed and disturbed trajectories will give allowance for a change in the form of rocket (its aerodynamic characteristics).

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Are possible the cases, when the parameter, which determines the nctica characteristics of rocket, is not produced change other initial conditions of basic system of equations. Por example, let with the firing the unguided recket charge the angle of departure θ_{0} . In this case the corresponding system of equations - (3.80) or (3.81) - is solved the necessary number once at the changed values θ_{01} . The number of solutions depends on the boundaries of change θ_0 and of the character of value being investigated change depending on θ_0 . If xc=f(0)it changes smoothly and within the limits of the section being investigated from θ_{W} to $\theta_{W(+1)}$ this dependence can be accepted for linear, are sufficient two solutions at the extreme values θ_0 . If third sclution, which corresponds, approximately, to the middle of interval θ_0 , will give acticeable deviation from straight line, then the number of solutions must be increased. The constructed according to the results of sclutions curve/graph or the smoothed tabular dependence $x_c = f(\theta_0)$ will make it possible to determine correction for

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distance Δx_c , corresponding to the deviation of angle of departure ΔG_0 . The advantage of the described method is the possibility of the account of mutual and indirect effect on the allowance of a change in the values, which depend on the deviation of the basic garameter and presented in fasic system differential equations. Furthermore, method makes it possible to determine corrections for each computed motion characteristic in any assigned point in the trajectory.

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The determination of correction factors according to the results of the solutions of the differential equations of motion of the center of mass is expedient when the dependence of the value being investigated from the determining parameter can be accepted as linear within the limits of the expected charge in the determining parameter. In this case at first the system of equations is integrated under standard conditions and for a second time - at the changed value of the parameter whose effect is investigated. After this is computed the unknown correction factor, for example, for determination correction factor. For example, for determining the correction factor into the firing distance of larrel system to the deviation of the initial velocity is must by the solution of the corresponding system of equations to find firing distance at value v₀, accepted as normal, and with v₀₁ = v₀ + Δv_0 , after which to find the correction factor

(11.12)

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This method requires conducting calculations with high accuracy (i.e. by the provision for a large quantity of accurate significant digits), since with low Au₀ are possible the large errors in the determination of difference for and, therefore, correction factor.

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Figures 11.3 gives examples of the graph/diagrams of the dependences $\frac{dx_c}{dx_b}$ on v_0 for a barrel system. From curve/graphs it is evident that the value of correction factor can depend substantially on the determining parameter and the use of discrete values of correction factors is admissible only within the narrow limit of a change in the determining parameters.

The system of equations, which describes the undisturbed motion, and the system of equations, written taking into account the perturbation factors, they must be comprised depending on specific problem. The systems of equations, comprised with the frequently utilized assumptions, are obtained in chapter III. The perturbation factors can be represented in the form of additional accelerations along the axes of the coordinates, in which is comprised basic system COC = 78107115

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cf equations. For the inactive legs of the unguided rockets and trajectories of artillery stells with independent alternating/variable t (time) as the system of equations, which describes the undisturbed flight, car be undertaken system (5.7) with the addition of one equation, which describes motion in side direction along coordinate z

 $\ddot{x} = -E\dot{x}; \quad \ddot{y} = -E\dot{y} - g; \quad \ddot{z} = -E\dot{z}.$ (11.13) The additional accelerations along the axes, caused by the effect of the perturbation factor, let us designate respectively $\mathbf{s}_{n}, \mathbf{s}_{y}$ and \mathbf{s}_{n} . Then the system of equations, written in general form taking into account the influence of the perturbation factor, will

take the form

$\ddot{x} = -E\dot{x} + \epsilon_{x}; \quad \ddot{y} = -E\dot{y} - g + \epsilon_{p}; \quad \ddot{z} = -E\dot{z} + \epsilon_{p}.$ (11. 14)

For example, with the wish to consider the rotational effect of the Earth on rocket flight, it is necessary to compare (2.33), (2.35) and (3.75). After comparison we will obtain

$s_x = \Omega^0 x - 2\Sigma v_x; s_y = 0; s_y = \Omega^0 x + 2\Omega v_y.$

Bor the systems in which on the left side of the equations do rot stand directly the acceleration, the terms, which consider the perturbation factors, will be different from s_m , s_y and s_m .

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Fig. 11.3. Curve/graphs $\frac{\partial x_c}{\partial w_0} = f(v_0)$ for the Earrel system whose projectile has the Eallistic coefficient of c = 0.2: 1 - at $\theta_0 = 25^\circ$; 2 - at $\theta_0 = 55^\circ$.

Key: (1). u/(u/s). (2). u/s.

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For example, with independence alternating/variable x as the system of equations, which describes the undisturbed flight, let us take system (5.8)

$$B'_{x} = -E; p'_{x} = -\frac{g}{g^{2}}; y'_{x} = \beta; t'_{x} = -\frac{1}{g}.$$

The system of equations written taking into account the rerturtation factors, will take the form

$$u'_{x} = -E + \epsilon_{ab} \quad p'_{x} = -\frac{g}{a^{2}} + \epsilon_{pb} \quad y'_{x} = pb \quad t'_{x} = \frac{1}{a} \quad (11.15)$$

We convert values u'_x and p'_x with consideration of (11.14)

 $u'_{x} = xt'_{x} = \frac{x}{a} = -E + \frac{t_{x}}{a};$ $p'_{x} = pt'_{x} = \frac{d}{dt} \left(\frac{y}{x}\right) \frac{dt}{dx} = \frac{yx - xy}{x^{3}}.$

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Remembering that x = u and by utilizing the first two equations of system (11.14), we will citain

$$p'_{g} = -\frac{g}{g^{2}} + \frac{1}{g^{2}} (\epsilon_{g} - \epsilon_{g} p).$$

Thus, from the comparison of the obtained equations with (11.15) we have

$$\mathbf{e}_{\mathbf{g}} = \frac{\mathbf{e}_{\mathbf{g}}}{\mathbf{e}} \text{ and } \mathbf{e}_{\mathbf{g}} = \frac{1}{\mathbf{e}^{\mathbf{g}}} (\mathbf{e}_{\mathbf{g}} - \mathbf{e}_{\mathbf{g}} \mathbf{p}).$$
 (11.16)

Ecr example, for a planar trajectory the terms, which consider correction for a change in value and direction of the acceleration of gravity in accordance with [9] are approximately equal to

$$e_{y} = -\frac{g}{\pi} \frac{x}{R_{y}} \text{ and } e_{y} = \frac{1}{R_{y}} (2y + xp) \frac{g}{R_{y}}.$$
 (11.17)

3.2. Bifferential equations of corrections.

In the case of the low deviations of the determining parameters, the correction factors can be determined during the solution of differential equations of corrections. For the compilation of the equations of corrections, we will use the general method of the linearization of the differential equations, which describe the motion of the center of mass of rocket or projectile (Chapter VIII). Usually are considered only first members of expansion in Taylor

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series and then for the linearization of equations, it is possible to use formula (4.75). the basic system of differential equations is selected depending on specific problem.

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As an example we will obtain the differential equations of corrections for the powered flight trajectory of the unguided rocket. In the named case we utilize a system of equations (3.79)

 $\dot{v} = \frac{P-X}{m} - g\sin\theta; \quad \dot{\theta} = -\frac{g\cos\theta}{v}; \quad \dot{y} = v\sin\theta; \quad \dot{x} = v\cos\theta.$

Let us designate right part one equation through *a*, and will discover the entering it values. Let us consider that the drag coefficient depends only from Mach number and does not change with height/altitude. Let us also assume that for design data point in the trajectory a change in thrust P does not depend on change in altitude y

$$a_{\varphi} = \frac{1}{m_{\varphi} - |\vec{m}| t} \left[|\vec{m}| J_{1g} - \frac{S \varrho v^{2}}{2} c_{x}(M) - g \sin \theta \right]. \quad (11.]18)$$

last/latter equality can be rewritten in the form of the functional , dependence

$$a_{\bullet} = f_1(v, \theta, y, J_1, S, \varrho, c_x(M), m_{\bullet}, |\dot{m}|).$$

Designating right Fart Two equation through and let us write the functional dependence

$$a_{\mathbf{0}} = f_{\mathbf{0}}(v, \mathbf{0}).$$

Entering similarly, we can write for the third and fourth

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equations

 $a_y = f_0(v, 0)$ and $a_x = f_4(v, 0)$.

Utilizing formulas of linearization (E.15), we will obtain the system of the differential equations

$$\frac{d}{dt}(\partial v) = \frac{\partial a_{v}}{\partial v} \partial v + \frac{\partial a_{v}}{\partial \theta} \partial \theta + \frac{\partial a_{v}}{\partial y} \partial y + \frac{\partial a_{v}}{\partial J_{1}} \partial J_{1} + \frac{\partial a_{v}}{\partial \varrho} \partial \varrho + + \frac{\partial a_{v}}{\partial c_{x}} \partial c_{x} + \frac{\partial a_{v}}{\partial m_{0}} \partial m_{0} + \frac{\partial a_{v}}{\partial |\dot{m}|} \partial |\dot{m}|;$$

$$\frac{d}{dt}(\partial \theta) = \frac{\partial a_{y}}{\partial v} \partial v + \frac{\partial a_{y}}{\partial \theta} \partial \theta;$$

$$\frac{d}{dt}(\partial y) = \frac{\partial a_{y}}{\partial v} \partial v + \frac{\partial a_{y}}{\partial \theta} \partial \theta;$$

$$\frac{d}{dt}(\partial x) = \frac{\partial a_{x}}{\partial v} \partial v + \frac{\partial a_{x}}{\partial \theta} \partial \theta.$$
(11.19)

The determination of tallistic derivatives $\frac{\partial u_1}{\partial j}$ in many instances represents although not complex, laborious problem.

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For the adopted by us system we will citain

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1.
$$a_{vv} = \frac{\partial a_v}{\partial v} = -\frac{S_{0v}}{m} \left[c_x(M) + \frac{M}{2} \frac{\partial c_x(M)}{\partial M} \right];$$

2. $a_{v0} = \frac{\partial a_v}{\partial 0} = -g \cos \theta;$
3. $a_{vv} = \frac{\partial a_v}{\partial 0} = -g \cos \theta;$
4. $a_{vJ_1} = \frac{\partial a_v}{\partial J_1} = \frac{P}{m} \cdot \frac{1}{J_1};$
5. $a_{v0} = \frac{\partial a_v}{\partial 0} = -\frac{X}{m} \cdot \frac{1}{\varrho};$
6. $a_{vc_x} = \frac{\partial a_v}{\partial c_x} = -\frac{X}{m} \cdot \frac{1}{\varrho};$
7. $a_{vm_0} = \frac{\partial a_v}{\partial c_x} = -\frac{P-X}{m} \cdot \frac{1}{m};$
8. $a_{v+|\vec{m}|} = \frac{\partial a_v}{\partial m_0} = -\frac{P-X}{m} \cdot \frac{1}{m};$
9. $a_{vv} = \frac{\partial a_0}{\partial v} = \frac{g \cos \theta}{v^2};$
10. $a_{v0} = \frac{\partial a_0}{\partial \theta} = \frac{g \sin \theta}{v};$
11. $a_{vv} = \frac{\partial a_v}{\partial \theta} = \sin \theta;$
12. $a_{v0} = \frac{\partial a_v}{\partial \theta} = \cos \theta;$
13. $a_{xv} = \frac{\partial a_x}{\partial \theta} = -v \sin \theta.$
(11. 20)

After this the system of the differential equations of corrections or as occasionally referred to as, system of equations in deviations let us write in this form:

$$\frac{d}{dt} (\mathbf{0} \mathbf{v}) = a_{yy} \mathbf{0} \mathbf{v} + a_{yy} \mathbf{0} + a_{yy} \mathbf{0} \mathbf{y} + a_{yy} \mathbf{0} \mathbf{y}_1 + a_{yy} \mathbf{0} \mathbf{z}_1 + a_{yy} \mathbf{0} \mathbf{z}_2 + a_{yy} \mathbf{0} \mathbf{z}$$

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System of equations in deviations with independent alternating/variable t, suitable for the calculation of corrections and correction factors for passive phases of trajectories of the unguided rockets and trajectories of the projectiles of barrel systems, we will obtain from system of equations (11.21). With the constant mass of projectile $P = J_1 = |\hat{n}| = 0$ and the first equation of system (11.19) can be written thus:

$$\frac{d}{dt}(\delta v) = \frac{\partial a_y}{\partial v} \delta v + \frac{\partial a_y}{\partial 0} \delta 0 + \frac{\partial a_y}{\partial y} \delta y + \frac{\partial a_y}{\partial 0} \delta Q + \frac{\partial a_y}{\partial v} \delta v + \frac{\partial a_y}$$

Respectively in the formulas of coefficients (11.20) it is necessary to take

$$a_{vm_0} = \frac{\chi}{m_0^2}$$

and in the first, third, by the heel, the sixth formulas to equate $m = m_0$. The remaining formulas of coefficients will remain without change. The system of the differential equations of corrections

taking into account this will take the form

$$\frac{d}{dt} (\delta v) = a_{yy} \delta v + a_{yy} \delta \theta + a_{yy} \delta y + a_{yy} \delta \theta + a_{yy} \delta v + a_{yy} \delta v + a_{yy} \delta \theta + a_{yy} \delta \theta;$$

$$\frac{d}{dt} (\delta \theta) = a_{yy} \delta v + a_{yy} \delta \theta;$$

$$\frac{d}{dt} (\delta x) = a_{yy} \delta v + a_{yy} \delta \theta;$$

$$\frac{d}{dt} (\delta x) = a_{yy} \delta v + a_{yy} \delta \theta;$$
(11.22)

The will obtain the system of equations in deviations, suitable for determining the corrections into the trajectory elements of the projectiles of constant mass with independent alternating/variable x. As tasic system let us take system (5.6), which describes the motion of the center of mass of artillery shell or unguided rocket on inactive leg in the dense layers of the atmosphere

$$u'_{x} = -E; p'_{x} = -\frac{g}{u^{2}}; y'_{x} = p; \quad t'_{x} = \frac{1}{u}.$$

Let us recall that $E=cH_{\tau}(y)O(v_{t})$,

where

 $a_{\tau} = x \sqrt{1 + p^2} \sqrt{\frac{v_{eV}}{\tau}}.$ Since $H_{\tau} = f(y)$ and $\tau = f_1(y)$, the E = f(u, F, Y).

Fage 427.

Let us designate by analogy with previous

 $a_{x} = f_{1}(x, p, y); \quad a_{p} = f_{2}(x); \quad a_{y} = f_{3}(p); \quad a_{t} = f_{4}(x),$

and then

 $u'_{s} = a_{s}; \quad p'_{s} = a_{p}; \quad y'_{s} = a_{p}; \quad t'_{s} = a_{t}.$

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Carrying out the linearization of the equations of system, accepted by us as basic, we will obtain the system of differential equations in the deviations

PAGE

$$\frac{d}{dx}(\delta p) = +a_{\mu\nu}\delta x + a_{\mu\nu}\delta p + a_{\mu\nu}\delta y;$$

$$\frac{d}{dx}(\delta p) = a_{\mu\nu}\delta x; \quad \frac{d}{dx}(\delta y) = a_{\mu\nu}\delta p; \quad \frac{d}{dx}(\delta t) = a_{\mu\nu}\delta x.$$
(11.23)

Let us find first the values of the coefficients a_{ij} , entering the first equation of system (11.23),

$$a_{uu} = \frac{\partial a_u}{\partial u} = \frac{-\partial E}{\partial u}; \quad a_{up} = \frac{\partial a_u}{\partial p} = \frac{-\partial E}{\partial p}; \quad a_{uv} = \frac{\partial a_u}{\partial y} = \frac{-\partial E}{\partial y}.$$

The values of tallistic derivatives let us find, utilizing (5.6)and (11.23). Differentiating L on u, from (5.6) we will obtain

$$\frac{\partial E}{\partial u} = \frac{\partial E}{\partial v_{\tau}} \cdot \frac{\partial v_{\tau}}{\partial u} = cH_{\tau}(y) Q'(v_{\tau}) \sqrt{1+p^2} \sqrt{\frac{\tau_{oN}}{\tau}} \cdot (11.24)$$

Hultiplying numerator and denominator on $\mu G(v_*)$ and designating

$$\frac{G'(\boldsymbol{v}_{q})}{G(\boldsymbol{v}_{q})} \boldsymbol{v}_{q} = \boldsymbol{f}(\boldsymbol{v}_{q}), \qquad (11.25)$$

we will obtain

$$a_{aa} = -\frac{g}{a} f(\mathbf{v}_{a}). \tag{11.26}$$

Differentiating E cn p, we will ottain

$$\frac{\partial E}{\partial p} = \frac{\partial E}{\partial v_{x}} \cdot \frac{\partial v_{x}}{\partial p} = cH_{x}(y)G'(v_{x}) \equiv \frac{p}{\sqrt{1+p^{2}}} \sqrt{\frac{\tau_{eN}}{1^{x}}} \cdot$$

After multiplying numerator and denominator on $G(v_i) \bigvee \overline{1+p}$, we will obtain

$$a_{np} = -\frac{k_p}{1+p^2} \cdot f(\mathbf{e}_n). \qquad (11.27)$$

acre laborious proves to be determination a_{up} since E) depends on y through values $H_{q}(y)$ and $a_{v} = f(y)$:

$$\frac{\partial E}{\partial y} = \frac{\partial E}{\partial [H_{\gamma}(y)]} \cdot \frac{\partial [H_{\gamma}(y)]}{\partial y} + \frac{\partial E}{\partial u_{\gamma}} \cdot \frac{\partial v_{\gamma}}{\partial y} \cdot$$

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From dependence for E, let us have $\frac{\partial E}{\partial [H_{q}(y)]} = cO(q_{q}) = \frac{E}{H_{q}(y)}.$

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For determination $\frac{\partial [H_{q}(y)]}{\partial y}$ we will use formula (2.54). after substituting it into expression for $H_{q}(y)$

$$H_{\tau}(y) = \sqrt{\frac{\tau_{gN}}{\tau}} e^{-\frac{1}{R} \int_{0}^{\frac{R}{2}} \frac{dy}{\tau}}.$$
 (11.28)

Let us take the logarithm expression (11.28):

$$\ln H_{\tau}(y) = \frac{1}{2} \ln \tau_{eV} - \frac{1}{2} \ln \tau - \frac{1}{R} \int \frac{dy}{\tau}.$$

Eifferentiating on y converting, we will obtain

$$\frac{\partial \left[H_{\tau}(y)\right]}{\partial y} = -H_{\tau}(y) \left(\frac{\tau_y}{2\tau} + \frac{1}{R\tau}\right). \quad (11.29)$$

Jurthermore,

$$\frac{\partial E}{\partial v_{\tau}} = cH_{\tau}(y)G'(v_{\tau}) = \frac{E}{v_{\tau}}f(v_{\tau}); \qquad (11.30)$$

$$\frac{\partial v_{q}}{\partial y} = x \sqrt{1+p^{2}} \left(\frac{-\tau_{g}}{2\tau} \sqrt{\tau_{gW}} \right) = -v_{q} \frac{\tau_{g}}{2\tau}. \qquad (11.31)$$

Substituting the value of partial derivatives, we will obtain after the conversions

$$a_{xy} = -\frac{\partial \mathcal{E}}{\partial y} = \frac{\mathcal{E}}{\tau} \left(\frac{f(v_q) + 1}{2} \tau'_y + \frac{1}{R} \right). \quad (11.82)$$

$$a_{pu} = \frac{2g}{u^2}; a_{pp} = 1; a_{pu} = -\frac{1}{u^2}.$$
 (11.33)

3.3. Determination of correction factors by the integration of differential equations of corrections.

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The integration of equations in deviations makes it possible to obtain correction factors and corrections into the motion characteristics, presented in the left sides of the equations, for any fixed/recorded point in the trajectory. Without supplementary simplifications the system of equations of corrections in the general case is solved only by rumerical integration or in the analog electionic mathematical computers of continuous action.

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Coefficients a_{ij} are calculated from the data of the undisturbed trajectory.

As an example let us examine the solution of system (11.21). Let us determine correction factors for the end, lead of powered flight trajectory in the following hypothetical initial data, necessary for the solution of assigned mission:

midsection - $S = C.2289 p^2$;

 $\dot{\mathbf{x}}$ initial mass - \mathbf{m}_0 = 177.18 kg•s²/s;

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the consumption of mass $-|\hat{\mathbf{s}}|= 23.2 \text{ kg} \cdot \epsilon^2 / \epsilon \cdot \epsilon;$

 $\pm hrust - P = 45.4 t;$

the operating time of engine - $t_{n}=2,4$ s;

the initial velocity - $v_0 = 35.5 \text{ m/s}$;

initial angle of departure $\theta_0 = 25^{\circ}00^{\circ}$;

spot height of start - $y_0 = 0$.

Cell/elements of the undisturbed trajectory, which correspond to initial data, placed in Table 11.1.

For convenience in the introduction of some deviations the first equation of system (11.21) let us convert after isolating in its right side separate dimensionless quantities - ratios of deviation of the determining parameter to its complete value:

 $\frac{W_1}{J_1}: \frac{u_2}{e}: \frac{u_2}{e_n}: \frac{u_n}{u_n} \text{ and } \frac{U(n)}{(n)}.$

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$$(2/2)$$
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Table 11.1. Cell/elements of the undisturbed trajectory.

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Ref: (1). on pores. (2). Cell/element. (3). B/E.

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In this case, instead of the coefficients a_{ij} into equation, will enter the following values:

instead of $a_{vJ_1} f_{vJ_1} = a_{vJ_1} J_1 = \frac{P}{R}$;

and instead of $a_{v_Q} \neq a_{v_{c_x}}$ $f_{v_Q} = a_{v_Q} = f_{v_{c_x}} = a_{v_{c_x}} c_x = -\frac{x}{m};$

justead of a_{vm} , a_{vm} , $m_0 = f_{vm} = -\frac{P-X}{m} \frac{m_0}{m}$;

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instead of $a_{v|\vec{m}|} = a_{v|\vec{m}|} |\vec{m}| = f_{v|\vec{m}|} = \frac{P}{m} + \frac{|\vec{m}|t(P-X)}{m^2}$.

Taking into account this equation will obtain the form $\frac{d}{dt}(\delta v) = a_{vv}\delta v + a_{vv}\delta t + a_{vv}\delta y + f_{vJ_1}\frac{\delta J_1}{J_1} + f_{vv}\frac{\delta q}{\varrho} + f_{vc_x}\frac{\delta c_x}{c_x} + f_{vm}\frac{\delta m_0}{m_0} + f_{v|m|}\frac{\delta |m|}{|m|}.$

Ccefficients (11.20) of the differential equations of corrections taking into account new values h_{ij} let us present in the form of curve/graphs and tables, which contain the average values of ccefficients on the section of approximation. Since the cell/elements of supporting trajectory usually are assigned on argument t, and partial derivatives it proves to be necessary to calculate according to argument M or y, then it is necessary to introduce into examination intermediate derivatives. For example, for the determination

$$a_{vv} = \frac{\partial a_v}{\partial v} = -\frac{S_{\mathcal{C}v}}{m} \left[c_x(\mathcal{M}) + \frac{\mathcal{M}}{2} \frac{\partial c_x(\mathcal{M})}{\partial \mathcal{M}} \right]$$

it is necessary to calculate

$$\frac{\partial c_x(M)}{\partial M} = \frac{\partial c_x(M)}{\partial t} \cdot \frac{\partial t}{\partial M}$$

for determining the partial derivatives from t, we will use the formulas of counted differentiation (6.28)

$$\left(\frac{\frac{\partial e_{g}(M)}{\partial t}}{\frac{\partial e_{g}(M)}{\partial t}}\right)_{g} = \frac{e_{g}(M)_{g+1} - e_{g}(M)_{g-1}}{2h_{t}};$$
$$\left(\frac{\partial M}{\partial t}\right)_{g} = \frac{M_{g+1} - M_{g-1}}{2h_{t}},$$

where $h_t - a$ space on time.

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For determining the values of derivative at the first and last/latter points $\left(\frac{\partial M}{\partial t}\right)_0$ and $\left(\frac{\partial M}{\partial t}\right)_{12}$ it is necessary to use Newton's first and second interpolation formulas.

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The results of calculation $\frac{\partial e_x(M)}{\partial t}$ and $\frac{\partial M}{\partial t}$ are given in Table 11.2. An example according to the calculation of coefficients a_{vv} and a_{xv} is given in Table 11.3.

In terms of the calculated values a_{ij} we construct the curve/graphs of coefficients and carry cut stepped approximation through equality the areas, limited by section curved curve/graph and by straight line. The averaged values of coefficients for a numerical example in question are given in Table 11.4. The characteristic curve/graphs of coefficients are given to Fig. 11.4-11.9.

The values of the voltages of electron analogue by which with transition to machine system of equations are replaced the real values, are connected with each other by the appropriate scale factors, for example: ECC = 78107115

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 $U_{10} = \mu_{10} \partial v, \quad U_{1} = \mu_{11} \partial \theta, \dots, \text{ where }$

Po V/I/S, P_{00} V/rad. - scales on voltages for a velocity increment and flight path angle.

The numerical values of the scales of sufferting/reference values are selected from the condition of citaining the maximum voltage, which characterizes real value, are not more than 80-100 V.

(/) № по пор.	ŧ	<u> </u>	<u>dM</u> dt	$\frac{\partial c_x(\mathbf{M})}{\partial \mathbf{M}}$	
0	0	0	0,724	0	
1	0,2	0	0,762	0	
2	0,4	0,0015	0,785	0.00191	
3	0,6	0,0300	0.802	0.0374	
4	0,8	0,1585	0,822	0,193	
5	1.0	0,6882	0,845	0,815	
6	1.2	0,6205	0,865	0,718	
7	1,4	0,0042	0,882	0,00477	
8	1,6	0,1918	0,908	-0,211	
9	1,8	-0,2125	0,930	-0,2284	
10	2,0	0,1412	0,945	0,1494	
11	2,2	-0,1425	0,970	-0,1469	
12	2,4	-0,3810	1,022	-0,373	

Table 11.2. Derivatives of $c_x(M)$ and 'f on time.

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Key: (1). cn pores.

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The numerical values of the scales of the variable coefficients of the realizable with the aid of units variable coefficients, must not exceed 1.0. For further use scales it is expedient to round off to the nearest convenient runter.

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Let us rewrite system of equations (11.21) in the form of machine system with the introduction of stresses in dimensional values, bearing in mind that functions $f_{vJ_1}, f_{vm_0}, f_{v|m|}, f_{vo}$ and f_{vr_x} is our case represent by themselves sufforting/reference values, but functions $a_{vv}, a_{vv}, a_{vv}, a_{vv}, \dots a_{xv}$ - variable coefficients when by, by, by and so forth

$$\frac{d}{d\tau} (U_{1v}) = -k_{vv} |a_{vv}^{*}| U_{1v} - k_{vs} |a_{vs}^{*}| U_{1s} + k_{v} a_{vv}^{*}| U_{1v} + k_{v} J_{1} U_{1v} |\frac{bJ_{1}}{J_{1}} - k_{vv} |J_{1vol}| \frac{\Delta v}{q} - k_{v} |J_{1vol}| \frac{\delta v}{q} + k_{v} |m| U_{1vol}| \frac{\delta v}{q} - k_{v} |J_{1vol}| \frac{\delta v}{q} + k_{v} |m| U_{1vol}| \frac{\delta v}{q$$

In this system $a_{ij}^* = \mathbf{p}_{e_{ij}}a_{ij}$.

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C 8	cte	11.	s. sramp	Te or th	ue carcu	19110	u cr	
Ne no mop	t	<u>M</u> 2	$\frac{M}{2} \frac{\partial c_x(M)}{\partial M}$	$+\frac{M}{2}\frac{\partial e_x(M)}{\partial M}$	$ \begin{array}{c} \Psi \left[c_{x} \left(\mathbf{M} \right) + \\ \mathbf{M} \left[c_{x} \left(\mathbf{M} \right) \\ + \frac{\mathbf{M} \left[c_{x} \left(\mathbf{M} \right) \\ 2 \end{array} \right] \\ \end{array} \right] $	n = no n t	a _{pp} !	i 4x9
0	0	0,052	0	0,3060	10,86	177,18	0,00173	15,00
1	0,2	0,127	0	0,3060	26,51	172,54	0,00438	33,93
2	0,4	0,204	0,00039	0,3064	42,6	167,90	0,00726	52,55
3	0,6	0,284	0,0105	0,3172	61,3	163,26	0,0107	71,02
4	0,8	0,365	0,0704	0,3884	96,5	158,62	0,0174	89,39
5	1,0	0,448	0,3650	· 0,7350	224,3	153,98	0,0416	108,1
6	1,2	0,534	0,3838	0,9771	355,0	149,34	0,068	126,8
7	1,4	0,622	0,0030	0,6212	262,8	144,70	0,0518	145,7
8	1,6	0,710	-0,15	0,4450	215,0	1 40,0 6	0,0438	164,7
9	1,8	0,803	0,1835	0,3580	195,7	135,42	0,0413	184,4
10	2,0	0,896	0,1338	0,3762	229,4	130,78	0,0502	203,9
n	2,2	0,992	0,1455	0,3395	229,0	126,14	0,0519	228,9
12	2,4	1,090	0,406	0,0470	· 34,9	121,50	0,00822	244,4

Table 11.3. Example of the calculation of coefficients $|a_{av}|$ and $|a_{xv}|$

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Table 11.4. Averaged values of coefficients () and fij

	-		-										_
t	c	0-0,2	0,2 0,4	0,4 <u>-</u> 0,6	0,6— 0,8	0,8—1,0	1,01,2	1,2—1,4	1,4-1,6	1,6-1,8	1,82.0	2,0—2,2	2,2-2,4
a _{vv}	1/c	0,003	0,00565	0,00884	0,01 37	0,0278	0,0578	0,0605	0,0468	0.0422	0,0444	0,0516	0, 0435
<i>a</i> x•	₩ <u>/</u> C	24	43	61,6	80	99	117,6	1 3 7	155	174,5	194	214	234
ate - 10 ⁵	1/m	375	75	- 84	19	12	8	6	4,5	3,5	3	2,5	1,8
ا <i>1</i> سو ا	M/c ²	263	279	293	308	328	847	362	381	403	429	457	487
fol mi	H /c ²	260	275	290	310	330	350	370	400	420	440	470	510
a	-	0,9138	0,923	0,928	0,932	0,934	0,936	0,938	0,939	0,940	0,941	0,943	0,944
a _v ,	m /c ²	8,978	9,057	9,105	9,138	9,164	9,184	9,200	9,216	9,230	9,242	9,252	9,259
a _{wy} · 10 ⁸	1/c ²	0,00	0,03	0,06	0,20	0,38	0,56	0,84	1,10	1,40	1,80	2,50	3,40
f_{yJ_1}	m /c ²	259	266	274	282	29 0	299	308	317	328	341	353	367
<i>a_{y1}</i>	M/C	56	101	152	206	258	312	363	425	476	545	601	687
a		0,425	0,386	0,375	0,365	0,356	0,351	0,346	0,342	0,339	0,335	0,332	0,330
fog = = fog	#/ c ²	0,10	0, 30	0,76	1,40	2,40	4,80	8,80	12,25	15,55	1 8,7 0	22.00	26,00
e.,	1/e	0,0005	0,0345	0,022	0,016	0,0125	0,0105	0,0085	0,0075	0,0065	0,0055	0,0050	0,0045

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Fig. 11.4. Stepped approximation of function 4...(?)

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Fig. 11.5. To approximation of function des(!)

Rey: (1). m/s.

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Fig. 11.6. To approximation of function $a_{-9}(t)$

Rey: (1). m/s².

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Fig. 11.7. To approximation of functions.(!)

and the

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Fig. 11.8. To approximation of function $f_{m}(t)$

Key: (1). m/s.

Fig. 11.9. To approximation of function 🖏 💔

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Ccefficients k_{ij} establish communication/connection between scales with transition from physical quantities to voltages. With some values of the scale factors, determined for an example in question, $n = 16^{-1}$, $n = 10^{(2)}$ B.

			$\mu_{e_{yy}} = 10 \frac{1}{(1/c)}$	
			$\mathbf{p}_{a_{\mathbf{v}\mathbf{y}}} = 200 \frac{1}{0 1/c^2}$	$: \mu_{\delta y} = 10 \frac{B}{(3)_{M}} ; $
			$\mu_{f_{\Psi}f_{\Psi}} = 0.25 \frac{B}{(2)_{W/c^2}}$; µ _l =5.
K€y:	(1).	1/s.	(2) . V/(1/5)	. (3). ¥∕∎.

The coefficients, which establish ccrounicaticr/connection between scales, will be equal to

$$k_{ev} = \frac{1}{\mu_{ev}\mu_{t}} = \frac{1}{16 \cdot 5} = 0.012 \frac{1}{c};$$

$$k_{ev} = \frac{\mu_{\delta v}}{\mu_{ev}\mu_{\delta y}\mu_{t}} = \frac{10}{200 \cdot 10 \cdot 5} = 0.001 \frac{1}{c};$$

$$k_{ev} = \frac{\mu_{\delta v}}{\mu_{ev}\mu_{\delta y}\mu_{t}} = \frac{10}{0.25 \cdot 5} = 8 \frac{1}{c}.$$

On the basis of the obtained system of equations, is comprised

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the block diagram of the set of problem in the simulating electronic computer. Figures 11.10 depicts one of the possible versions of the block diagram of solution on widespread Soviet model MPT-9-2. The block diagram of the solution of problem is installed in the setting field of machine. Initially assigning (possible deviations of the parameters), let us determine the effect of each of them on change v_m x_m y_m θ_w individually. Let us take the initial deviations:

FIGE 39

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 $\partial x_0 = \partial y_0 = \partial J_1 = \partial m_0 = \partial |\dot{m}| = \partial c_x = \partial \varrho = 0;$

 $\delta v_0 = 5$ m/s, which corresponds to voltage $U_{ba} = 50$ V;

 $\delta\theta_0 = 0.458 \ deg \approx 0.46 \ deg$, which corresponds to voltage $U_{10,} = 0.458 \ deg \approx 0.46 \ deg$, which corresponds to voltage $u_{10,} = 0.458 \ deg \approx 0.46 \ deg$, which corresponds to voltage $u_{10,} = 0.458 \ deg \approx 0.46 \ deg$, which corresponds to voltage $u_{10,} = 0.458 \ deg \approx 0.46 \ deg$, which corresponds to voltage $u_{10,} = 0.458 \ deg \approx 0.46 \ deg$, which corresponds to voltage $u_{10,} = 0.458 \ deg \approx 0.46 \ deg$, which corresponds to voltage $u_{10,} = 0.458 \ deg \approx 0.46 \ deg$, $u_{10,} = 0.458 \ deg$, $u_{10,}$

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Table 11.5.

(()	2						<u> </u>	
Паремотр	Uter B	15	U6.5.2	b.r _x ₩	U	Ngu H	Uat _a	Mage TREAT
6496 20 2696,46 rjat	47 1,3	4,7 0,10	**	8,8 2,8	75	75 5,8	56 82	0,82 9,47

Key: (1). Farameter. (2). m/s. (3). deg.

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Fig. 11.10. Unit schematic of solution of system of equations in deviations on electron analogue.

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Using the data of Table 11.5 let us determine correction factors. Let us cite the calculation of some of them

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$$\frac{\partial v_{x}}{\partial v_{0}} \approx \frac{\delta v_{x}}{\delta v_{0}} = \frac{4.7}{5.0} = 0.94 \text{ M/C/M/C};$$

$$\frac{\partial x_{x}}{\partial v_{0}} \approx \frac{\delta x_{x}}{\delta v_{0}} = \frac{8.8}{5.0} = 1.76 \text{ M/M/C};$$

$$\frac{\partial y_{x}}{\partial v_{0}} \approx \frac{\delta y_{x}}{\delta v_{0}} = \frac{7.5}{5.0} = 1.5 \text{ M/M/C};$$

$$\frac{\partial \theta_{x}}{\partial v_{0}} \approx \frac{\delta \theta_{x}}{\delta v_{0}} = \frac{0.32}{5.0} = 0.064 \text{ rpaA/m/C};$$

$$\frac{\partial v_{x}}{\partial \theta_{0}} \approx \frac{\delta v_{x}}{\delta \theta_{0}} = -\frac{0.16}{0.46} = -0.35 \text{ M/C/rpaA};$$

$$\frac{\partial x_{x}}{\partial \theta_{0}} \approx \frac{\delta x_{x}}{\delta \theta_{0}} = -\frac{2.0}{0.46} = -4.36 \text{ M/rpaA};$$

$$\frac{\partial y_{x}}{\partial \theta_{0}} \approx \frac{\delta y_{x}}{\delta \theta_{0}} = \frac{5.8}{0.46} = 12.65 \text{ M/rpaA};$$

$$\frac{\partial \theta_{x}}{\partial \theta_{0}} \approx \frac{\delta y_{x}}{\delta \theta_{0}} = \frac{0.47}{0.46} = 1.02 \text{ rpaA/rpaA}.$$

PIGE #2

Key: (1). E/S/E/S. (2). E/E/S. (3). degree/E/S. (4). E/S/deg. (5). E/deg. (6). deg/deg.

Similarly can be found correction factors, also, to other determining parameters. The obtained values are utilized for calculating of corrections and characteristics of scattering trajectories.

3.4. Conjugated/combined system of equations of corrections and its solution.

The system of the nonhomogeneous linear differential equations

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group of which includes the systems of the differential equations of ccrrections, can have irterconnected circuit of linear homogeneous equations. The number of variables conjugate/combined and basic of systems they coincide. For the rule of the compilation of interconnected circuit of linear homogeneous equations, the ccefficients of the first equation are taker equal to the coefficients of the first sembers of lasic system of equations, the coefficients of the second equation are taken equal to the coefficients of the second members of fundamental equations, etc. In new system all coefficients are taken with cffcsite signs with respect to basic. On the basis of the determination of homogeneous differential equations, in interconnected circuit let us dror/cmit the absolute terms, available into basic. For an explanation let us take the system of the differential equations of corrections (11.23) and let us rewrite three first joint of equation, after introducing terms & and &, the considering disturbance/perturbations.

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For generality let us write the terms, which have zero coefficients; ccefficients a_{ij} let us take by absolute values, after introducing intc equations their signs in an explicit form. DOC = 78107115 PAGE # 874

$$\frac{d}{dx}(\delta x) = -a_{xx}\delta x - a_{xy}\delta p + a_{xy}\delta y + \epsilon_{x}\delta$$

$$\frac{d}{dx}(\delta p) = a_{yx}\delta x + 0\cdot\delta p + 0\cdot\delta y + \epsilon_{p};$$

$$\frac{d}{dx}(\delta y) = 0\cdot\delta x + a_{yy}\delta p + 0\cdot\delta y.$$
(11. 34)

Let us accept system (11.34) for tasic, Dependent variables in it are u, p and y. Let us write the conjugated/combined with (11.34) system of equations, after designating variables through λ_1 , λ_2 and λ_3 :

$$\frac{d\lambda_{1}}{dx} = a_{\mu\nu}\lambda_{2} - a_{\mu\nu}\lambda_{3} - 0\lambda_{3};$$

$$\frac{d\lambda_{2}}{dx} = a_{\mu\nu}\lambda_{1} - 0\lambda_{3} - a_{\mu\nu}\lambda_{3};$$

$$\frac{d\lambda_{3}}{dx} = -a_{\mu\nu}\lambda_{1} - 0\lambda_{3} - 0\lambda_{3}.$$
(11.35)

In the general case the physical sense of variables λ_i , depends on the content of the coefficients of fundamental equation a_{ij} and of terms a_{ij} and b_{jj} . Let us establish communication/connection between variables basic and by that conjugate/combined systems of equations for our case. Let us multiply the equation of basic system respectively by λ_1 , λ_2 , and λ_3 , but the equation of interconnected circuit on δu , δp and δy let us add them. After conversions let us have

$$\lambda_{1} \frac{d}{dx} (\partial x) + \lambda_{2} \frac{d}{dx} (\partial p) + \lambda_{3} \frac{d}{dx} (\partial y) +$$
$$+ \partial x \frac{d\lambda_{1}}{dx} + \partial p \frac{d\lambda_{2}}{dx} + \partial y \frac{d\lambda_{3}}{dx} = \lambda_{1} e_{x} + \lambda_{2} e_{p}$$

and, further,

$$\frac{d}{dx}(\lambda_1 \mathbf{a} + \lambda_g \mathbf{a}_p + \lambda_g \mathbf{a}_y) = \lambda_1 \mathbf{a}_g + \lambda_g \mathbf{a}_p, \qquad (11.36)$$

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Let us integrate the right and left of the part of the last/latter equation from x_1 to x_2 . Then

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 $\lambda_{13}\delta u_{1} + \lambda_{23}\delta p_{1} + \lambda_{33}\delta p_{2} - \lambda_{21}\delta u_{1} - \lambda_{32}\delta p_{1} - \lambda_{32}\delta p_{1} =$ $= \int_{x_{1}}^{x_{2}} (\lambda_{2}\epsilon_{x} + \lambda_{3}\epsilon_{y}) dx. \qquad (11.37)$

The cttained equation is ccsmon/general/total and it is correct under any initial conditions.

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For the basic trajectory in question integration limits of interconnected circuit can be selected in the manner that this is convenient for the solution of the specific problem of the theory of corrections. In many instances it proves to be advisable to integrate conjugate system of differential equations, beginning with final point in the trajectory.

For example, during the determination of corrections for the inactive legs of the unguided rockets of class "surface - surface" and the trajectories of the projectiles of terrestrial artillery interdonnected circuit (11.35) is integrated from the impact point in the projectile to the beginning of the trajectory phase in question. Let us write equation (11.37) for the passive section or ground-based
missile trajectory

$$\lambda_{1c} \partial u_{c} + \lambda_{9c} \partial p_{c} + \lambda_{9c} \partial y_{c} - \lambda_{1u} \partial u_{u} - \int_{s_{u}}^{s_{c}} (\lambda_{1} e_{u} + \lambda_{9} e_{p}) dx.$$

$$(11.3)$$

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In order to obtain from (11.38) the formula, which determines correction into distance, it is necessary to select one of the coefficients λ_{ic} so that it would cortain δx_c , the remaining coefficients of interconnected circuit they are taken equal to zero. For an impact point (Fig. 11.11) it is possible to take

$$w_c = \frac{w_c}{|P_c|};$$

after comparison with the third term of equation (11.38) we will obtain

$$\lambda_{\rm sc} = \frac{1}{|P_{\rm c}|}.$$

Furthermore,

$$\lambda_{yc} = 0, \quad Prom (11.38) \quad \text{let us have} \\ \frac{\lambda_{yc}}{\lambda_{xc}} = \lambda_{yc} \frac{\lambda_{yc}}{\lambda_{xc}} + \lambda_{yc} \frac{\lambda_{yc}}{\lambda_{yc}} + \lambda_{yc} \frac{\lambda_{yc}}{\lambda_{xc}} + \lambda_{yc}$$

The physical sense of values λ_{1m} , λ_{0m} and λ_{0m} is explained, after saking equal to zerc e_{u} and e_{p} , and in pairs deviations δu_{m} , δp_{m} , δy_{m} ; for example, by taking $\delta p_{u} = \delta y_{u} = 0$ and $\delta u_{u} = 1$, let us find

$$\lambda_{\rm pe} = \frac{\delta x_{\rm c}}{\delta u_{\rm c}} \, .$$

Similarly we will obtain

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Fig. 11.11. The cuts of trajectories, adjacent to impact point: 1 - ret disturbed (calculated); 2 - deformed.

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Instantaneous values λ_1 , λ_2 , λ_3 in function from x, including λ_{18} , λ_{28} and λ_{80} , are obtained by the numerical integration (by ETSVM or with manual count) of interconnected circuit (11.35) under the initial conditions

$$\lambda_{1C} = 0; \quad \lambda_{9C} = 0; \quad \lambda_{9C} = \frac{1}{|P_C|}.$$

The space of integration for x is taken with minus sign, since integration is realized from one x_c to x_n .

During the determination of correction into distance from (11.39) values δu_{R} , δp_{R} and δy_{R} they sust be determined during the study of the preceding/previous trajectory thase or be specified by assignment. **Dec = 78107115**

If we assume $\delta u_{R} = \delta p_{R} = \delta y_{R} = 0$, then $\delta x_{C} = \int^{x_{C}} (\lambda_{2} e_{a} + \lambda_{3} e_{p}) dx.$ (11.40)

FIGE 47

Last/latter formula makes it possible to determine correction into distance for the factors whose effect is considered through e_u and e_p (solutional effect of the earth/ground, a change in the meteorological conditions, etc.).

Euring determination δx_c for the trajectories of the projectiles of ground-tased cannon-type artillery the integration of interconnected circuit (11.35) it is necessary to carry out from x_c to x = 0. Furthermore, for the point of flight it is possible to accept $\delta y_0 = 0$ and then from (11.39)

 $\delta x_{c} = \lambda_{10} \delta x_{0} + \lambda_{30} \delta p_{0} + \int_{0}^{x_{c}} (\lambda_{1} \epsilon_{s} + \lambda_{3} \epsilon_{p}) dx. \qquad (11.41)$

Last/latter formula is ccmmcn/general/tctal correcting formula for the ccmplete horizontal firing distance ground-lased artillery piece.

3.5. Betermination of correction factors from the similarity of trajectories.

For calculating the corrections into the trajectory elements of the projectiles of constant mass, frequently are utilized the dependences, obtained from similarity conditions of trajectories.

Let us take the first equation of system (7.141) and let us

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determine correction factors for complete distance for a change in pressure and temperature of air is the point of flight (and further along trajectory). If we assume π - π , then π , $-\pi$, and $x_c = \Phi_1(\pi, c^0, \theta_0)$. (11.42)

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After recalling that $c^* = c \frac{h_0}{h_{0N}}$, and, considering v_0 and θ_0 in (11.42) constant/invariable, let us take alternately derivative of x_c on c and cn h₀.

$$\frac{\partial x_C}{\partial e} = \frac{\partial \Phi_1}{\partial e} \frac{\partial e^*}{\partial e} = \frac{\partial \Phi_1}{\partial e^*} \frac{h_0}{h_{0N}};$$

$$\frac{\partial x_C}{\partial h_0} = \frac{\partial \Phi_1}{\partial e^*} \frac{\partial e^*}{\partial h_0} = \frac{\partial \Phi_1}{\partial e^*} \frac{e}{h_{0N}};$$

Comparing two last/latter equalities, we will obtain

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$$\frac{\partial x_c}{\partial h_0} = \frac{\partial x_c}{\partial e} \frac{e}{h_0} . \qquad (11.43)$$

If to accept $h_0 = h_{0N}$, $\tau_0 \neq \tau_{0N}$, the first equation of system (7.141) will take the form

$$x_{C} = \frac{\tau_{0}}{\tau_{0N}} \Phi_{1}(\sigma_{10}, c, \theta_{0}). \qquad (11.44)$$

Accepting in last/latter equality θ_0 constant/invariable and differentiating alternately on v_0 and r_0 , we will obtain after the conversions

$$\frac{\partial x_{C}}{\partial v_{0}} = \frac{1}{v_{0}} \left(x_{C} - \frac{1}{2} v_{0} \frac{\partial x_{C}}{\partial v_{0}} \right). \quad (11.45)$$

Frcm (11.43) and (11.45) it is evident that the correcting coefficients, which consider effect or the firing distance of the deviation of barometric pressure fice given standard atmosphere and the temperatures of air in release point (and further along DQC = 78107115

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trajectory), can be expressed through lasic correction factors to the deviations of the ballistic coefficient and the initial velocity.

3.6. Analytical method can be applied in the case when the basic value being investigated is determined by the function, which allow/assumes obtaining derivatives in analytical form. Method, as a rule, is utilized during the study of the trajectories of the projectiles of constant mass. an example of the use of the named method have examined we in §1 present chapter with the assumptions, which correspond to the parabolic theory of the motion of projectile [11.6].

Ottaining hallistic derivatives in correction with the elliphical theory of the action of the projectiles of constant mass is examined in detail in works [2], [22], [35], [46]. It is presented here the basis of the common/general/total procedure of obtaining correcting formulas and correction factors.

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Let us take the equation of elliptical trajectory, which contains in an explicit form the parameters, which correspond to the beginning of inactive leg and determining all the motion characteristics, including firing distance [2], and let us write it for point C* with DOC = 78107115

a radius $r=r_{C'} > R_{a}$, accepting it for the beginning of the entry of projectile into the dense layers of the atsceptere $\begin{bmatrix} 2r_{C'} (1+tg^{a}\theta_{a}) - \frac{r_{B}\theta_{B}}{k} (r_{a}+r_{C'}) \end{bmatrix} tg^{a}\phi - \frac{r_{B}\theta_{B}}{k} [2r_{C'} (1+tg^{a}\theta_{a}) - \frac{r_{B}\theta_{B}}{k} (r_{a}+r_{C'})] tg^{a}\phi - \frac{r_{B}\theta_{B}}{k} (r_{a}+r_{C'}) dt^{a}\phi + \frac{r_{B}\theta_{B}}{k} (r$

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 $-\frac{r_{e}r_{e}^{2}}{k} [2r_{C} tg \theta_{n} tg \phi + (r_{n} - r_{C})] = 0. \quad (11.46)$ The written equation can be presented in the form of the functional dependence

 $F(\theta_{u}, v_{u}, r_{u}, r_{c'}, 2\phi)=0$

and to write

$$\begin{bmatrix} 2r_{\mathbf{C}'} \left(1 + \mathbf{tg}^{\mathbf{0}} \,\mathbf{\theta}_{\mathbf{x}}\right) - \frac{r_{\mathbf{u}} \sigma_{\mathbf{u}}^{2}}{k} \left(r_{\mathbf{u}} + r_{\mathbf{C}'}\right) \end{bmatrix} \mathbf{tg}^{\mathbf{0}} \,\mathbf{\phi} - \frac{2r_{\mathbf{u}} \sigma_{\mathbf{u}}^{2}}{k} r_{\mathbf{C}'} \,\mathbf{tg} \,\mathbf{\theta}_{\mathbf{u}} \,\mathbf{tg} \,\mathbf{\phi} - \frac{r_{\mathbf{u}} \sigma_{\mathbf{u}}^{2}}{k} \left(r_{\mathbf{u}} - r_{\mathbf{C}'}\right) = F\left(\mathbf{\theta}_{\mathbf{u}}, \,\,\mathbf{\phi}_{\mathbf{u}}, \,\,r_{\mathbf{u}}, \,\,r_{\mathbf{C}'}, \,\,2\mathbf{\phi}\right).$$

Hence it follows that $2\mathbf{\psi} = f(\mathbf{\theta}_{\mathbf{u}}, \,\,\mathbf{\psi}_{\mathbf{u}}, \,\,r_{\mathbf{u}}, \,\,r_{\mathbf{C}'})$.

In connection with the spherical model of the Earth, communication/connection between angular and by linear circular by distances is determined by the relationship/ratio

 $L = l_{\rm B} + 2R_{\rm B} \,\phi. \tag{11.47}$

It is chvious,

$$\mathcal{L} = \Psi(\boldsymbol{v}_{n}, \boldsymbol{\theta}_{n}, \boldsymbol{r}_{n}, \boldsymbol{r}_{C'}, \boldsymbol{l}_{n}), \qquad (11.48)$$

and the correcting formula, obtained taking into account only first terms of expansion, in accordance with (11.2) it will take the form

$$\delta L = \frac{\partial L}{\partial v_n} \partial v_n + \frac{\partial L}{\partial l_n} \partial l_n + \frac{\partial L}{\partial r_n} \partial r_n + \frac{\partial L}{\partial r_{C'}} \partial r_{C'} + \frac{\partial L}{\partial l_n} \partial l_n.$$

Ballistic derivatives $\frac{M}{M_1}$ in accordance with (11.47) and (11.48) will te equal to

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$$\frac{\partial L}{\partial v_{n}} = \frac{\partial (2\psi)}{\partial v_{n}} R_{gi} \qquad \frac{\partial L}{\partial v_{n}} = \frac{\partial (2\psi)}{\partial v_{n}} R_{gi}$$
$$\frac{\partial L}{\partial v_{n}} = \frac{\partial (2\psi)}{\partial v_{n}} R_{gi} \qquad \frac{\partial L}{\partial v_{n}} = \frac{\partial (2\psi)}{\partial v_{n}} R_{gi}$$
$$\frac{\partial L}{\partial v_{n}} = 1.$$

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Equation (11.46) is written in implicit form and, without isolating the determining parameters, we will obtain ballistic derivatives (correcting ccefficients), using the obvious formula

$$\frac{\partial (2\psi)}{\partial \xi_i} = -\frac{\partial F/\partial \xi_i}{\partial F/\partial (2\psi)} \,.$$

Let us give as example the derivation of the dependence for a correction factor $\frac{\partial L}{\partial v_{\mu}}$.

Using (11.46), we will obtain

$$\frac{\partial F}{\partial v_{n}} = -\frac{2r_{n}v_{n}}{k} (r_{n} + r_{C'}) \operatorname{tg}^{0} \phi - - - 4 \frac{r_{n}v_{n}}{k} r_{C'} \operatorname{tg} \theta_{n} \operatorname{tg} \phi - 2 \frac{r_{n}v_{n}}{k} (r_{n} - r_{C'}); \qquad (11.49)$$

$$\frac{\partial F}{\partial (2\psi)} = \left[2r_{C'} (1 + \operatorname{tg}^{n} \theta_{n}) - \frac{r_{n}v_{n}^{2}}{k} (r_{n} + r_{C'}) \right] \operatorname{tg} \psi \frac{1}{\cos^{2}\psi} - - \frac{r_{n}v_{n}^{2}}{k} r_{C'} \operatorname{tg} \theta_{n} \frac{1}{\cos^{2}\psi} . \qquad (11.50)$$

Cale,(11.49) to (11.50), let us have

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 $\frac{\frac{\partial (2\psi)}{\partial u_{n}}}{\left[\frac{2r_{c}}{k}\left[(r_{n}+r_{C'})\sin^{2}\psi+r_{C'} \log \theta_{n}\sin(2\psi)+(r_{n}-r_{C'})\cos^{2}\psi\right]}{\left[\frac{2r_{C'}}{(1+tg^{0}\theta_{n})-\frac{r_{u}\sigma_{n}^{2}}{k}(r_{u}+r_{C'})\right]tg\psi-\frac{r_{u}\sigma_{n}^{2}}{k}r_{C'} tg\theta_{n}}}.$ (11.51)

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Formula for the calculation of correction into firing distance during the deviation only of initial velocity takes the form

$$\delta \mathcal{L}_{\mathbf{v}_{B}} = \frac{\partial \mathcal{L}}{\partial v_{B}} \delta \mathbf{v}_{g} = \frac{\partial \langle \mathbf{2} \mathbf{i} \rangle}{\partial v_{B}} \mathcal{R}_{g} \delta \mathbf{v}_{g}, \qquad (11.52)$$

By form can be written correcting formulas also for

ctler determining parameters

$$\begin{aligned} & \mathbf{k}_{r_{g}} = \frac{\sigma(2\theta)}{\sigma_{u}} R_{g} \mathbf{k}_{x}. \end{aligned} (11.53) \\ & \mathbf{k}_{u} = \frac{\sigma(2\theta)}{\sigma_{u}} R_{g} \mathbf{k}_{u}. \end{aligned} (11.54) \\ & \mathbf{k}_{u} = \frac{\sigma(2\theta)}{\sigma_{u}} R_{g} \mathbf{k}_{u}. \end{aligned} (11.55) \end{aligned}$$

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Frequently they introduce into examination value $x = rv^2/k$ which for initial conditions is equal to $x_n = \frac{r_n v_n^2}{k}$. Utilizing value x_n , it is necessary to bear is mind, that $x_n = f(r_n, v_n)$ and

$$\delta \alpha_{n} = \frac{\partial f}{\partial r_{n}} \delta r_{n} + \frac{\partial f}{\partial v_{n}} \delta v_{n}. \qquad (11.56)$$

Let us give without derivation the ballistic derivatives, expressed through x_m [2]. Then $r_C = R_s$

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$$\frac{\partial L}{\partial r_{\pi}} = R_{3} \frac{\frac{2R_{3}}{r_{\pi}} (1 + ig^{2}\theta_{n}) \sin^{2}\psi ig\psi}{r_{\pi}(r_{\pi} - R_{3} + R_{3}ig\theta_{n}ig\psi)};$$

$$\frac{\partial L}{\partial v_{n}} = \frac{4R_{3}^{2}}{v_{\pi}} \frac{(1 + ig^{2}\theta_{n})\sin^{2}\psi ig\psi}{r_{\pi}(r_{\pi} - R_{3} + R_{3}ig\theta_{n}ig\psi)};$$

$$\frac{\partial L}{\partial \theta_{n}} = 2R_{3}^{2} \frac{(1 + ig^{2}\theta_{n})(r_{\pi} - R_{3} + R_{3}ig\theta_{n}ig\psi)}{r_{\pi}(r_{\pi} - R_{3} + R_{3}ig\theta_{n}ig\psi)}.$$
(11.57)

For a symmetrical trajectory in the case $r_{c'}=r_{u'}$, we will obtain

$$\frac{\partial (2\psi)}{\partial r_n} = \frac{x_n + 2\sin^2\psi \cdot (1 + tg^2\theta_n)}{x_n r_n tg\theta_n};$$

$$\frac{\partial (2\psi)}{\partial v_n} = \frac{4\sin^2\psi \cdot (1 + tg^2\theta_n)}{x_n v_n tg\theta_n};$$

$$\frac{\partial (2\psi)}{\partial \theta_n} = \frac{(x_n - 2tg\theta_1 tg\psi)(1 + tg^2\theta_n)\sin(2\psi)}{x_n tg\theta_n}.$$
(11.58)

Bange angle in case when $r_{C} = R_{3}$, can be represented in the form of the functional dependence

$2\phi = F_+(\sigma_n, \theta_n, h_n),$

where A_{n} — a height/altitude of the initial point of the elliptical trajectory above the surface of the spherical acdel

$h_{\rm H}=r_{\rm H}-R_{\rm H}$

earth/ground. In this case, a change in the range angle taking into account only linear terms of expansion in terms of (11.2) is equal

$$\delta(2\phi) = \frac{\partial(2\phi)}{\partial u_{\mu}} \delta u_{\mu} + \frac{\partial(2\phi)}{\partial u_{\mu}} \delta u_{\mu} + \frac{\partial(2\phi)}{\partial u_{\mu}} \delta u_{\mu}. \qquad (11.59)$$

Utilizing (7.19), I. J. Lysenko was converted (11.59) and channed the equation, connecting in inflicit form of deviation $\delta\sigma_m \, \delta\theta_x$ and δA_n [18].

4

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$$\frac{2\mathfrak{b}\sigma_{\mathfrak{n}}}{\sigma_{\mathfrak{n}}} \left(\frac{1-\cos 2\psi}{\mathfrak{x}_{\mathfrak{n}}\cos^{2}\theta_{\mathfrak{n}}} \right) + \frac{\mathfrak{b}\mathfrak{a}_{\mathfrak{n}}}{R_{3}} \left[1 + \frac{R_{3}\left(1-\cos 2\psi\right)}{\mathfrak{x}_{\mathfrak{n}}^{\prime}\mathfrak{c}\cos^{2}\theta_{\mathfrak{n}}} \right]_{\mathfrak{m}} = \delta\left(2\psi\right) \left[\frac{\sin 2\psi}{\mathfrak{x}_{\mathfrak{n}}\cos^{2}\theta_{\mathfrak{n}}} - \frac{\sin\left(\theta_{\mathfrak{n}}+2\psi\right)}{|\cos\theta_{\mathfrak{n}}|} \right] - \delta\theta_{\mathfrak{n}} \left[\frac{\sin\left(\theta_{\mathfrak{n}}+2\psi\right)}{\cos\theta_{\mathfrak{n}}} - \frac{\sin\left(\theta_{\mathfrak{n}}+2\psi\right)}{\cos\theta_{\mathfrak{n}}} - \frac{\sin\left(\theta_{\mathfrak{n}}+2\psi\right)}{\cos\theta_{\mathfrak{n}}} - \frac{\sin\left(\theta_{\mathfrak{n}}+2\psi\right)}{\cos^{2}\theta_{\mathfrak{n}}} \cos\left(\theta_{\mathfrak{n}}+2\psi\right) - \frac{2\sin\theta_{\mathfrak{n}}}{\mathfrak{x}_{\mathfrak{n}}\cos^{2}\theta_{\mathfrak{n}}} \left(1-\cos2\psi\right) \right]. \quad (11.60)$$

Besentering that $\delta L = R_3 \delta(2\psi)$, from (11.6C) we will obtain when $\delta \theta_n = \delta h_n = 0$:

$$\frac{\partial L}{\partial v_{\rm m}} = 2 \frac{R_{\rm m}}{v_{\rm m}} \frac{1 - \cos 2\psi}{\sin 2\psi - x_{\rm m} \cos \theta_{\rm m} \sin (\theta_{\rm m} + 2\psi)} \, .$$

If one assumes that the elliptical trajectory is symmetrical and tegins on the surface of the Earth, i.e., $r_{\rm H} = r_{C'} = R_{\rm S}$ and projectile it bas velocity $v_{\rm H} = v_{\rm O}$, then

$$\frac{\partial L}{\partial v_0} = 2 \frac{R_3}{v_0} [\sin 2\psi + \operatorname{ctg} \Theta_0 (1 - \cos 2\psi)]. \quad (11.61)$$

Figures 11.12 gives curve/graphs $\frac{\partial L}{\partial v_0} = f_{v_0}(\theta_0)$ with different 2. 2. From curve/graphs it is evident that in all cases the ballistic derivative $\frac{\partial L}{\partial v_0}$ has positive value. With identical angular firing distance, it decreases with an increase of the flight path angle in the beginning of passive elliptical section.

From equation (11.60) accepting $\delta v_m = \delta h_m = 0$, we will obtain $\frac{\partial L}{\partial h_0} = 2R_0 \left[\frac{\sin 2(\theta_0 + \psi)}{\sin 2\theta_0} - 1 \right].$ (11.62) Dependence $\frac{\partial L}{\partial h_0} = f_{A_0}(\theta_0)$ is given to Fig. 11.13. with an increase in the angle of departure, ballistic derivative $\frac{\partial L}{\partial h_0}$ decreases, transfer/converting from positive values to negative. The zero value of derivatives corresponds to the optimum flight path

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angles in the beginning of passive section. With assigned the flying range the zero value of derivative arswers the trajectory, obtained at the minimum initial velocity v_0 .

The ballistic derivative of a charge in the distance in beight/altitude gave inactive leg was equal to

 $\frac{\partial L}{\partial h_{u}} = 2 \operatorname{ctg} \theta_{u} - \frac{\operatorname{ess} (\theta_{u} + 3\phi)}{\sin \theta_{u}} . \qquad (11.63)$ The curve/graphs of dependence $\frac{\partial L}{\partial h_{u}} = f_{A}(\theta_{u})$ are given to Fig. 11.14. In all cases the derivative has positive value.

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Fig. 11.12. Change of ballistic derivatives $\frac{\partial L}{\partial v_0}$ in function of angle of departure $\theta_{\mu} = \theta_0$ at different values of range angle 2ψ

Key: (1). km•s/m.



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Fig. 11.13. Change of ballistic derivatives $\frac{\partial L}{\partial h_1}$ in function of angle of departure $\theta_{n=0}$, at different values of range angle

Key: (1). (km/m•rad). (2). m•rad.

Fig. 11.14. Change of tallistic derivatives $\frac{\partial L}{2h_{\rm H}}$ in function of angle of departure $\theta_{\rm H}=\theta_{\rm H}$ at different values of range angle 2 //.

Key: (1). km/m.

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With identical distance it decreases with ar increase in angle On-

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The analytical method of determining the correction factors can be also used in connection with different approximate solutions of the problem of external ballistics of the projectiles of constant mass, driving/moving in the dense layers of the atmosphere. Let us examine obtaining formulas and correcting coefficients for the method of pseudovelocity. We utilize formulas (7.63) and (7.65), the connecting complete firing distance with the parameters, which determine the trajectory:

$$x_{c} = \frac{1}{c'} (D_{c} - D_{0}); \qquad (11.64)$$

$$\sin 2\theta_{0} = \frac{1}{c'} \left(\frac{A_{c} - A_{0}}{D_{c} - D_{0}} - J_{0} \right). \qquad (11.65)$$

Substituting (11.64) in (11.65), we will obtain the formula, which connects $x_c \in c'$, θ_b and v_c :

$$c'x_{c}(c'\sin 2\theta_{0} + J_{0}) = A_{c} - A_{0}.$$
 (11.66)

Bor obtaining the differential correcting formula of form (11.2) let us differentiate last/latter equality, considering variables x_c, c', v_0, θ_0 . It is obvious that the terms, which contain u_c , will also be variables

$$c'x_{c}d(c'\sin 2\theta_{0}+J_{0})+(c'\sin 2\theta_{0}+J_{0})d(c'x_{c})=dA_{c}-dA_{0}.$$
(11.67)

Utilizing a basic ccrclusion of the method of pseudovelocity, let us write

Then the right side of equation (11.67) car be represented in the form

 $dA_c - dA_s = J_c dD_c - J_dD_s$

Value dD_c can be expressed by the initial parameters, it differentiated (11.64); then

$$IA_c - dA_o = J_c d(c'x_c) + J_c dD_o - J_o dD_o.$$

Carrying out the replacement of right side in (11.67), after ccnversion we will obtain $c'x_{cd}(c'\sin 2\theta_0 + J_0) + [c'\sin 2\theta_0 - (J_c - J_0)] d(c'x_c) =$

$$= (J_c - J_o) dD_o.$$
 (11.68)

Lifference $(J_C - J_0)$ can be determined from the basic conclusion of the method of pseudovelocity. If we in formula (7.60) replace

that we will obtain

$$J_c - J_0 = 2c' \cos^2 \theta_0 (\operatorname{tg} \theta_0 + \operatorname{tg} |\theta_c|) =$$

= c' sin 20_0 + 2c' cos² 0. tg |\theta_c|.

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After substituting $J_c - J_0$ in (11.66), after conversion let us have

$$x_{cd}(c' \sin 2\theta_{0} + J_{0}) - 2\cos^{2}\theta_{0} tg |\theta_{c}| d(c'x_{c}) =$$

= (sin 2\theta_{0} + 2\cos^{2}\theta_{0} tg |\theta_{c}|) dD_{c} (11.99)

Bealizing differentiation and carrying cut the replacement

$$dJ_{0} = -\frac{2gdv_{0}}{v_{0}^{2}G(v_{0})}; \quad dD_{0} = -\frac{dv_{0}}{G(v_{0})}$$

we will obtain

$$\begin{aligned} x_{c} \sin 2\theta_{e} dc' + 2c' x_{c} \cos 2\theta_{e} d\theta_{e} - 2g x_{c} \frac{dw_{e}}{w_{0}^{2} G(w_{e})} - \\ -2 \cos^{2} \theta_{e} tg |\theta_{c}| x_{c} dc' - 2 \cos^{2} \theta_{e} tg |\theta_{c}| c' dx_{c} = \\ = -(\sin 2\theta_{e} + 2 \cos^{2} \theta_{e} tg |\theta_{c}|) \frac{dw_{e}}{G(w_{e})}. \end{aligned}$$

After passing from differentials to finite low increments, we ccavert last/latter equality and we will citair the differential correcting formula

$$\mathbf{k} \mathbf{x}_{C} = \frac{\partial \mathbf{x}_{C}}{\partial \mathbf{r}'} \, \mathbf{b} \mathbf{c}' + \frac{\partial \mathbf{x}_{C}}{\partial \mathbf{n}_{0}} \, \mathbf{b} \mathbf{v}_{0} + \frac{\partial \mathbf{x}_{C}}{\partial \mathbf{n}_{0}} \, \mathbf{b}_{0}.$$

After conversion let us have

$$\begin{aligned} \mathbf{\hat{s}}_{C} &= -x_{C} \left(1 - \frac{\mathbf{tg} \, \mathbf{\hat{s}}_{0}}{\mathbf{tg} \, |\, \mathbf{\hat{s}}_{C}|} \right) \frac{\mathbf{\hat{s}}_{C}'}{\mathbf{\hat{s}}'} + \frac{x_{C} \cos 2 \mathbf{\hat{s}}_{0}}{\cos^{2} \mathbf{\hat{s}}_{0} \, \mathbf{tg} \, |\, \mathbf{\hat{s}}_{C}|} \mathbf{\hat{s}}_{0} + \\ &+ \frac{1}{\mathbf{\hat{s}}' G \left(\mathbf{v}_{0} \right) \mathbf{tg} \, |\, \mathbf{\hat{s}}_{C}|} \left[\mathbf{tg} \, \mathbf{\hat{s}}_{p} + \mathbf{tg} \, |\, \mathbf{\hat{s}}_{C}| - \frac{\mathcal{E} x_{C}'}{\mathbf{v}_{0} \cos^{2} \mathbf{\hat{s}}_{0}} \right] \mathbf{\hat{s}}_{0}. \quad (11.70) \end{aligned}$$

Frequently is introduced substitution $\gamma = \frac{|g|_0}{|g|_c|}$, and then the formulas of basic correction factors will take the form $\frac{\partial x_c}{\partial e'} = -(1-\gamma)\frac{x_c}{e'}; \quad \frac{\partial x_c}{\partial \theta_0} = \frac{x_c \cos 2\theta_0}{\cos^2 \theta_0 |g|_0|};$ $\frac{\partial x_c}{\partial \theta_0} = \frac{1}{e'G(\eta_0)} \left[1 + \gamma - \frac{gx_c}{g_0^2 \cos^2 \theta_0 |g|_0|} \right].$ (11.71) Analytical method is applicable for determining the ballistic

derived and higher orders, than the first.

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Within the framework of parabolic theory, the fallistic derivatives, which characterize principal part of the change in the function, are determined by formulas (11.5). The refinement of correction is determined by three last/latter terms of formula (11.4), from which the second derivatives are equal to

$$\frac{\partial x_c}{\partial q} = 2 \frac{da 2h_0}{g}; \quad \frac{\partial x_c}{\partial q} = 4 \frac{q^2 da 2h_0}{g}; \quad \frac{\partial x_c}{\partial q \partial h_0} = 4 \frac{q da 2h_0}{g};$$

Similarly can be obtained derivatives $\frac{\partial L}{\partial x_{1}^{2}}$, $\frac{\partial L}{\partial x_{2}^{2}}$ and $\frac{\partial L'}{\partial x_{1}\partial x_{2}}$ on the basis of elliptical theory.

However, the mathematical complexities of obtaining the second derivatives, the low value of the standard deviations $(\partial \sigma_n)^2$, $(\partial \Theta_n)^2$ and of product $\partial \sigma_n \partial \Theta_n$ and, as a result, the insignificant refinement of correction itself, given by these terms, lead to the fact that in the practical calculations of corrections are considered the usually only linear terms of expansion.

3.7. Tabular methods of calculating the correction factors.

Under tabular methods let us understand the methods of determining the correction factors with the aid of the various kinds of the precomputed tables. The use of tables won acceptance during

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the calculation of corrections to the trajectory elements of the grojedtiles of cannon-type artillery. Is known the use of ballistic collections (ballistic tables) for calculating the correction factors in the method of differences and the use of tables of quite correction factors. The majorities of ballistic collections contain the cell/elements of characteristic points in the trajectory for normal meteorological conditions depending on the determining garameters c, v_0 and θ_0 .

For zenith trajectories the number of determining parameters, includes time t (see page 286). The tables are comprised as a rule, with the constant space of the determining parameters and this makes it possible to utilize for the calculation of correction factors a known formula of numerical differentiation (6.28):

$$y_{x} = \frac{y_{x+1} - y_{x-1}}{2h}, \qquad (11.73)$$

Alternately substituting in the numerator of last/latter formula the firing distance, undertaken depending on a change in the determining parameter, and into denominator - the tabular interval (space) of the determining parameter, it is possible to obtain the values of basic correction factors for the calculation of corrections into distance.

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For the ground-based cannon-type artillery

$$\frac{dx_{C}}{\partial e} = \frac{x_{C}(e+h_{c}) - x_{C}(e-h_{c})}{2h_{c}};$$

$$\frac{dx_{C}}{\partial v_{0}} = \frac{x_{C}(v_{0}+h_{0}) - x_{C}(v_{0}-h_{0})}{2h_{v_{0}}};$$

$$\frac{dx_{C}}{\partial h_{0}} = \frac{x_{C}(u_{0}+h_{0}) - x_{C}(u_{0}-h_{0})}{2h_{b_{0}}}.$$
(11.74)

Last/latter correction factor in a series of the cases must be calculated when A_{b_0} : undertaken in radians.

For the trajectories of antiaircraft artillery with the assigned ccpstant/invariable time t cf formula for calculating the correction factors, they take the following form:

$$\frac{\partial x}{\partial v} = \frac{x(c+h_c) - x(c-h_c)}{2h_c};$$

$$\frac{\partial y}{\partial v} = \frac{y(c+h_c) - y(c-h_c)}{2h_c};$$

$$\frac{\partial x}{\partial v_0} = \frac{x(v_0 + h_{v_0}) - x(v_0 - h_{v_0})}{2h_{v_0}};$$

$$\frac{\partial y}{\partial v_0} = \frac{y(v_0 + h_{v_0}) - y(v_0 - h_{v_0})}{2h_{v_0}};$$

$$\frac{\partial x}{\partial v_0} = \frac{x(v_0 + h_{v_0}) - x(v_0 - h_{v_0})}{2h_{v_0}};$$

$$\frac{\partial y}{\partial v_0} = \frac{y(v_0 + h_{v_0}) - y(v_0 - h_{v_0})}{2h_{v_0}};$$

$$\frac{\partial y}{\partial v_0} = \frac{y(v_0 + h_{v_0}) - y(v_0 - h_{v_0})}{2h_{v_0}};$$

$$\frac{\partial y}{\partial v_0} = \frac{y(v_0 + h_{v_0}) - y(v_0 - h_{v_0})}{2h_{v_0}};$$

The tables of correction factors can be calculated by one of the described above methods.

Since the trajectory is determined by parameters c, v_0 and θ_0 ,

Same in the second

also the tables of correction factors have the same entries.

Are known the correction cards of artillery academy, calculated by the method of the numerical integration of the differential equations of corrections [59].

In tables for tarrel artillery pieces, they are given:

 $Q_r = \frac{\partial x_c}{\partial r}$ - change in the complete distance with an increase in the temperature of air on 1° along an entire trajectory;

 $Q_{v_0} = \frac{dx_c}{dv_0}$ - change in the complete distance as a result of an increase in the initial velocity on 1 m/s;

 $Q_{\frac{1c}{c}} = \left| \frac{dx_{c}}{dc} \right| \cdot \frac{c}{100}$ - change in the complete distance as a result of an increase in the ballistic coefficient or harometric pressure on 1c/c:

 Q_{v_x} - change in the complete distance as a result of action on the projectile of longitudinal tailwind with a velocity of 1 m/s, constant along an entire trajectory:

 $Q_{v,}$ — lateral deviation of the impact point in the projectile as a result of the action of cross wind at a rate of 1 m/s, constant

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alcrg an entire trajectory.

Practical work on tables is reduced to linear interpolation in terms of the assigned intake values of c_r , v_0 and θ_0 .

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§4. Cprrection into firing distance artillery shells during a change in the initial weight.

The weight of projectile affects both the motion of projectile in the bore of artillery instrument, determining its initial velocity and to the trajectory of its free flight in the dense layers of the atmosphere. Firing distance depends on three values: v_0 , c and θ_0 ; the weight of projectile is determined first two. Consequently, at the fixed angle of departure it is possible to write $x_c = j(v_0, c)$.

Let us make an assumption about independence of action v_0 and c cn firing distance and we will use the differential correcting formula

$$\delta x_{cq} = \frac{\delta x_{c}}{\delta x_{q}} \delta x_{q} + \frac{\delta x_{c}}{\delta t} \delta c. \qquad (11.76)$$

The dependence of the initial velocity on the weight of projectile we will obtain on the correcting formula, known from interior

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tallistics,

 $m_{0} = -l_{0}m_{0}\frac{M}{\Delta}$

where 4- correction factor.

From formula for the ballistic coefficient $c=(id^2/Q)10^3$, accepting product id^2 constant, taking the logarithm of, differentiating and replacing infinitesimal increases by final low, we will obtain f_{1} . Substituting δv_0 and δc in (11.76), let us have



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Since correction factor $\frac{dx_r}{de}$ value negative, is convenient to take it by absolute value and then

$$\mathbf{k} \mathbf{x}_{CQ} = \left[\left| \frac{\partial \mathbf{x}_{C}}{\partial \mathbf{e}} \right| \mathbf{c} - \mathbf{l}_{Q} \mathbf{v}_{0} \frac{\partial \mathbf{x}_{C}}{\partial \mathbf{v}_{0}} \right] \frac{\mathbf{k} \mathbf{Q}}{\mathbf{Q}}, \qquad (11.78)$$

whence correction factor into distance directly by weight of projectile is equal to

$$\frac{\partial x_c}{\partial Q} = \frac{1}{Q} \left[\left| \frac{\partial x_c}{\partial v} \right| c - l_q \sigma_0 \frac{\partial x_c}{\partial v_0} \right]. \quad (11.79)$$

Taking into account this designation we will obtain

 $\delta x_{CQ} = \frac{\delta x_C}{\delta O} \delta Q.$

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Bepending on specific conditions, the correction factor and correction into distance for the deviation of the weight of projectile can have different signs.

§5. Account of the effect of the atmospheric parameters on rocket flight and of artillery stells.

Theoretical studies and firings confirm the essential effect of the deviations of weather factors from their normal values for rocket flight and projectiles. The effect of weather factors is considered during determining of motion characteristics, during the calculation of scattering trajectories and the determination of the accuracy of firing, in the calculations, connected with the design of the system of the flight control, and during calculations for strength.

In ballistics the effect of the deviations of the atmospheric parameters of rocket flight and projectiles can be taken into account by three methods. First method - calculation of corrections according to the ground-based deviations of pressure, humidity and temperature from the normal under the assumption of the validity of hypothesis about the vertical equilibrium of the atmosphere and the preservation/retention/maintaining of the character of a change in

the temperature with the height/altitude of the similar of the function $\tau(y)$, accepted in standard atmosphere. An example of chtaining correction factors into distance to the deviation of ground-based barometric pressure and virtual temperature is placed in §3 present chapters [formula (11.43) and (11.45)].

The second method consists in determination and use of ballistic mean deviations of weather factors - average, conditionally constant with respect to an entire trajectory of the deviation of virtual temperature from normal law and conditionally constant neutral wind. The first and by the second methods are applied most frequently in practical work during preparation of firings and processing of their results.

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Theoretically stricter is the third method according to which into the systems of differential equations of motion are introduced the equations, which directly determine a change of the weather factors in the function of any coordinate (most frequently height/altitude) or in the function of time. With a strict account of the effect of a change in the atmospheric garameters to rocket flight and artillery shells in calculation, must be utilized the concrete/specific/actual (experimental) functions, obtained according EOC = 78107116 PAGE 45 900

to the results of the scurding of the atmosphere.

During forecasting of a change in the weather factors, are used the statistical processings of the results of meteorological investigations. The results of statistical processing can be represented in the form of the random functions, comprised on coordinates and on time. Statistical processing on coordinates is ttilited usually in ballistic calculations, statistical processing on time is utilized predominantly during setting of wind load on flight webicke in stress analyses, in the stability aralyses of motion and flight dynamics in the restless atmosphere.

5.1. Account of a barometric change and temperature.

A barometric change and temperature on height/altitude for a standard atmosphere is examined in charter II, §3. With the wish to consider in ballistic calculations the concrete/specific/actual realization, experimental or calculated, it is necessary to have the data on a barometric change, humidity and temperatures of air on height/altitude, presented in the form of formulas, tables or curve/graphs. Air humidity easily is considered through virtual temperature on formula (2.4%). Fressure and temperature are introduced into calculation according to formulas for an air density (2.43) or (2.51), temperature is considered also in formula for the

speed of sound. During the use of curve/graph $c_x(M)$, comprised for ground-based standard conditions, the effect of a change in the speed of sound in height/altitude for drag can be considered on formula (2.108). A change in the thrust in height/altitude must be calculated for concrete/specific/actual meteorological realization according to common/general/total equation (2.115).

It is obvious, each realization of the charged weather constituents on height/altitude requires the independent solution of the selected system of equations.

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5.2. Equations of motion of the projectile of constant mass upon consideration of a harcmetric change and temperature.

Buring the definition of air drag with the aid of coefficient c.(M) the account of a change in the pressure and temperature is conducted, just as it is described in the preceding/previous section for the projectiles of variable mass. If we for determining the drag use value E (5.6), then the system of differential equations, which describes the motion of the projectile of constant mass taking into account a change in the pressure and temperature, it can be given to the general view of equations (11.14). Let us take changed value

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 $(E+\delta E)$, then

1 and the second second second second

$$\ddot{x} = -E\dot{x} - \frac{NB}{E}E\dot{x}; \quad \ddot{y} = -E\dot{y} - g - \frac{NB}{B}E\dot{y}. \quad (11.80)$$

In accordance with (5.6) it is possible to present $B=f(c, h and \tau)$ and to obtain [9]

$$\frac{\lambda E}{E} = \frac{\lambda c}{c} + \frac{\lambda k}{\lambda} - \frac{f(v_{u}) + 1}{2} \cdot \frac{\lambda \tau}{\tau}.$$
 (11.81)

Value f(o,) is determined from (11.25).

Euring a change only in Eallistic coefficient in accordance with (11.80) and (11.81) we will obtain the following system of differential equations:

$$\ddot{x} = -E\dot{x} - \frac{bc}{c}E\dot{x}; \quad \ddot{y} = -E\dot{y} - g - \frac{bc}{c}E\dot{y}. \quad (11.82)$$

With the constant tallistic coefficient

$$\ddot{x} = -E\dot{x} - \left(\frac{bh}{h} - \frac{f(v_q) + 1}{2} \cdot \frac{b\tau}{\tau}\right)E\dot{x};$$

$$\ddot{y} = -E\dot{y} - g - \left(\frac{bh}{h} - \frac{f(v_q) + 1}{2} \cdot \frac{b\tau}{\tau}\right)E\dot{y}.$$
(11.83)

After integration can be obtained the motion characteristics of the projectile of constant mass during the deviations of pressure δh and of virtual temperature δr from the values, determined by standard atmosphere. If we use hypothesis about the vertical equilibrium of the atmosphere, then the changed motion characteristics can be obtained depending on surface pressure and the function of a change in the temperature with height/altitude [9].

Taking the logarithm of and differentiating equation for h

[barcmetric function w(y)] and transfer/converting from infinitesimal to finite low increments, we will obtain

$$\frac{bb}{b} = \frac{bb}{b} - b \left[\frac{1}{B} \left\{ \frac{dy}{\tau} \right\} \right]$$

cr, by differentiating second term on τ , let us have

$$\frac{bh}{h} = \frac{bh_0}{h_0} + \frac{1}{R} \int \frac{d\tau}{\tau^2} dy. \qquad (11.66)$$

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Introducing replacement in (11.83), we will obtain

$$\begin{aligned} x &= -E\dot{x} - \left(\frac{bd_0}{b_0} + \frac{1}{R}\int_0^{\beta} \frac{d\tau}{\tau^2} dy - \frac{f(u_0) + 1}{2} \frac{b\tau}{\tau}\right) E\dot{x}; \\ y &= -E\dot{y} - g - \left(\frac{bd_0}{b_0} + \frac{1}{R}\int_0^{\beta} \frac{d\tau}{\tau^2} dy - \frac{f(u_0) + 1}{2} \frac{b\tau}{\tau}\right) E\dot{y}. \end{aligned}$$
(11.85)

Last/latter system of equations does not require for its solution of functioning the barometric change with height/altitude and it is convenient for theoretical studies.

5.3. Account of wind effect on the flight ci the unguided rockets and artillery shells.

Basic problems during the study of the action of wind on rocket

flight and artillery shells are: setting the mechanism of the direct action of wind on the driving/moving rocket and projectile, setting metworks of the motion of the masses of air and the proof of the legitimacy of the diagram accepted during the solution of one or the other problem. Meteorological investigations show that for the time interval, which exceeds the time of one group of firings, the motion of air masses in sufficiently large territory can be considered as relatively steady and rectilinear with preferred horizontal flows. In such flows wind velocity gradients in time and horizontal direction prove to be unessential. Figures 11.15 shows an example of a change in wind velocity in horizontal direction.





Fig. 11.15. Change in wind velocity in horizontal direction.

Key: (1). m/s.

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Together with the relatively tranguil flows of air masses, there is chaotic vortex/eddy, intermittent air motion, which is called of gustimess. The investigation of the eddying effect of the atmosphere on rocket flight and projectiles is complex independent problem [20].

Buring ballistic calculations and during the preparation of firing usually are accepted following assumptions about the character of the motion of the atmosphere: they do not consider vertical wind component; they assume that a change in the wind in height/altitude remains constant/invariable within the limits of the entire trajectory in question, i.e., it is assumed that wind does not depend

cn horizontal coordinates. With the adopted assumptions it proves to be most convenient to represent the velocity vector of wind being of two comprising: the constant $\overline{w}_0(y)$ and random $\overline{w}_0(y)$

$$\overline{\Psi}(y) = \overline{\Psi}_0(y) + \overline{\Psi}_c(y). \tag{11.86}$$

Constant component corresponds to the mathematical expectation of upper wind y and characterizes the continuous uniform displacement/movement of the masses of air. Random comprising characterizes a change in the wind from one shot to the next to height/altitude y. When conducting of calculations the velocity vector of wind they expand in the direction of firing (so-called longitudinal wind), also, in standard to it (cross wind).

The special feature/peculiarity of the meticn of flight vehicle to the moved atmosphere consists in the fact that its velocity relative to the earth/ground and relative to the driving/moving masses of air is different. The velocity of flight vehicle (aircraft, recket, projectile) relative to the Earth it is accepted to call ground speed, velocity relative to the atmosphere - air. If the ground speed of projectile (absolute) is designated by vector \overline{v} , the velocity of the motion of the atmosphere relative to the Earth (translational speed - wind velocity) by vector \overline{v} and the airspeed of projectile (relative) through \overline{v} , then

and the velocity of projectile in motion relative to the atmosphere

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FIGE 22

is equal to

The acceleration of projectile in motion relative to the Barth is equal to \vec{v} , and relative to the atmosphere $-\vec{J}_n$. In this case, it is necessary to bear in mind, that the aerodynamic forces and torque/moments are determined by the airspeed, i.e., \vec{s}_n whose module/modulus can be designed on the approximate dependence

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The mechanism of the effect of wind in the general case during the solution of spatial problem taking into account the angles of attack and slip, determined with respect to airspeed, is very complex. Therefore in practical work frequently is examined the action of longitudinal and cross wind separately.

Let us write by analogy with (3.44), (3.49) and (3.50) the systems of equations, which describe the separate longitudinal and yawing motions of rocket, taking into account in them respectively chly longitudinal or only cross wird and counting that thrust \overline{P} is directed along the axis Ox_1 of rocket. For the axial motion of the projection of vector equality (11.68) on the axis of the starting coordinate system, combined with the center of mass of projectile,

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ate equal (Fig. 11.16):

$$\Psi_{rs} = \Psi \cos \theta - \Psi_{s}; \quad \Psi_{rs} = \Psi \sin \theta. \tag{11.90}$$

The flight path angle in relative axial action is determined from the dependence

The fundamental equations, which describe the axial motion of the unguided rocket taking into account the action of wind B_{e} -const, will take the form

$$\frac{1}{2} = \frac{1}{2} \left[P - X, \cos(\theta, -\theta) + Y, \sin(\theta, -\theta) - Q \sin(\theta); \right]$$

$$\frac{1}{2} = \frac{1}{2} \left[P - X, \cos(\theta, -\theta) + Y, \sin(\theta, -\theta) - Q \cos(\theta); \right]$$
(11.92)
$$J_{2} = \sum M_{2}; \quad \theta = 0 \text{ and } 0 \text{ and$$



Fig. 11.16. Schematic of the construction of argle of attack in **relative motion** upon consideration of longitudinal tailwind.

Key: (1). Trajectory.

Fage 460.

Supplementary equation for determining the angle of attack in relative motion

 $\mathbf{a}_r = \mathbf{\hat{v}} - \mathbf{\hat{v}}_r. \tag{11.93}$

In accordance with (2.85), (2.86) and (2.85) aerodynamic forces and torque/moment taking into account the action of longitudinal wind are approximately equal to

$$X_r = \frac{ev_r^2}{2} Sc_x(M_r); \quad Y_r = \frac{ev_r^2}{2} Sc_y^* a_r;$$

$$M_{syr} = \frac{ev_r^2}{2} Slm_{s_x}^* a_r,$$
(11.94)

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where

$$c_x(\mathbf{M}_r) = c_{s0}(\mathbf{M}_r) + c_{st}(\mathbf{G}_r);$$

$$c_{s0}(\mathbf{M}_r) = c_{s0}\left(\frac{v_r}{s}\right).$$

The system of equations, which describes the yawing motion of the unguided rocket taking into account the action of constant cross wind \overline{w} , can be writter in the form

$$\dot{\Psi} = -\frac{1}{m\nu\cos\theta} (-P\beta + Z_r);$$

$$J_{\nu}\ddot{\Psi} = M_{\nu,r}; \quad \phi = \Psi + \beta; \quad \dot{z} = -\nu\cos\theta\sin\Psi;$$
(11.95)

Becall that velocity v must be determined during the solution of the system of equations, which describes axial action.

Buring the determination of the direction of the yawing motion of rocket on powered flight trajectory, has a value the mutual location of the center of mass and resultant pressure of rocket. The effect of cross wind \overline{w}_{2} or rocket will create supplementary aerodynamic force \overline{R}_{0} , applied in resultant pressure (Fig. 11.17). Of finned statically stable rocket the resultant pressure is arrange/located after the center of gravity nearer to tail assembly: therefore under the action of torque/moment from force R_{0} rocket will turn itself by nose section towards wind so that its longitudinal axis will coincide with the vector of airspeed. In this case, will appear lateral thrust component P_{0} , the directed against

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wind and when $P_0>R_0$ rocket will be moved also against wind. The spin-stabilized missiles as a result of derivation, composite action of the force of Magnus and cross wird can be moved on powered flight trajectory in wind direction. On inactive leg with P=0 the rocket just as the projectile of tarrel system, will be moved in the direction of cross wind. Figures 11.18 depicts the schematic of yawing motion against wind of the statically stable fin-stabilized rocket on powered flight trajectory under the assumption that the rocket is inertia-free.

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Let us designate the horizontal projection of the velocity of the center of mass in the starting coordinate system through u=v cos θ , the horizontal projection of the speed in relative motion through ur. For Fig. 11.18

$$\mathbf{z}_{r} = \sqrt{\mathbf{z}^{2} + \mathbf{w}_{r}^{2} + 2\mathbf{z}\mathbf{w}_{r}\sin\Psi}; \qquad (11.96)$$

$$\cos\Psi_{r} = \frac{\mathbf{v}\cos\theta\cos\Psi}{\mathbf{z}_{r}}, \qquad (11.97)$$

sufflementary equation for determining the slip angle in the relative yawing motion

$$\beta_r = \phi - \Psi_r. \tag{11.96}$$

Aerodynamic forces and torque/moment taking into account the action of cross wind will be equal to

$$Z_{r} = \frac{q r_{r}^{2}}{2} Sc_{s}^{*} r_{r}^{*} \qquad M_{0s} = \frac{q r_{r}^{2}}{2} Sim_{p}^{*} \beta_{r}. \qquad (11.90)$$


Fig. 11.17. Rotation of the unguid€d fin-stabilized rocket against wird.



Fig. 11.18. Schematic of construction of slip argles during yawing motion of rocket against wird.

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Figures 11.19 depicts the schematic of yawing motion stabilized by rolling of the rocket (projectile) in powered flight trajectory in the case when lateral threst component acts in wind direction. According to Fig. 11.19

$$u_r = \sqrt{u^2 + w_r^2 - 2uw_r \sin \Psi}; \qquad (11. 100)$$
$$\cos \Psi_r = \frac{v \cos \theta \cos \Psi}{u_r} \qquad (11. 101)$$

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and sufflementary equation

$$\boldsymbol{\beta}_r = \boldsymbol{\phi} - \boldsymbol{\Psi}_r. \tag{11. 102}$$

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For spatial motion with constant wind from vector equality (11.88) we will obtain for modulus of velocity

$\boldsymbol{v}_{r} = \sqrt{(\boldsymbol{v}_{s} - \boldsymbol{w}_{s})^{2} + (\boldsymbol{v}_{g} - \boldsymbol{w}_{g})^{2} - (\boldsymbol{v}_{s} - \boldsymbol{w}_{s})^{2}}, \quad (11.103)$

where v_x , v_y , v_z and w_x , w_y and w_z projectics of the velocity of the center of mass of projectile and projection of wind velocity on the axis of the starting occordinate system, cowhined with the center of mass. Upon consideration only of horizontal component wind

$$\boldsymbol{v}_r = \sqrt{(\boldsymbol{v}_s - \boldsymbol{w}_s)^2 + \boldsymbol{v}_g^2 + (\boldsymbol{v}_s - \boldsymbol{w}_s)^2}. \quad (11.106)$$

Gensidering stable recket as material point of variable mass and taking into account that with wind force vector of thrust coincides with the airspeed of the center of mass, we will obtain the cosines of the angles between the direction of airspeed and the corresponding axes of the starting system of coordinates

 $\frac{\mathbf{v}_{rs}}{\mathbf{v}_{r}} = \frac{\mathbf{v}_{s} - \mathbf{v}_{s}}{\mathbf{v}_{r}}; \quad \frac{\mathbf{v}_{rs}}{\mathbf{v}_{r}} = \frac{\mathbf{v}_{s} - \mathbf{v}_{s}}{\mathbf{e}_{r}}; \quad \frac{\mathbf{x}_{rs}}{\mathbf{v}_{r}} = \frac{\mathbf{e}_{s} - \mathbf{v}_{s}}{\mathbf{v}_{r}}, \quad (11.105)$





Fig. 11.19. Schematic of the construction of slip angles during the yawing motion of rocket in wind direction.

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For a statically stable fin-stabilized rocket cp powered flight trajectory, the system of equations of motion taking into account wind will take the form

$$\boldsymbol{\upsilon}_{x} = \frac{P \boldsymbol{\upsilon}_{rx}}{m \boldsymbol{\upsilon}_{r}} - \frac{Q \boldsymbol{\upsilon}_{r}}{2m} (\boldsymbol{\upsilon}_{x} - \boldsymbol{\upsilon}_{x}) S c_{x} (M_{r});$$

$$\boldsymbol{\upsilon}_{y} = \frac{P \boldsymbol{\upsilon}_{ry}}{m \boldsymbol{\upsilon}_{r}} - \frac{Q \boldsymbol{\upsilon}_{r}}{2m} (\boldsymbol{\upsilon}_{y} - \boldsymbol{\upsilon}_{y}) S c_{x} (M_{r}) - g;$$

$$\boldsymbol{\upsilon}_{y} = \frac{P \boldsymbol{\upsilon}_{rs}}{m \boldsymbol{\upsilon}_{r}} - \frac{Q \boldsymbol{\upsilon}_{r}}{2m} (\boldsymbol{\upsilon}_{x} - \boldsymbol{\upsilon}_{y}) S c_{x} (M_{r}).$$
(11.106)

Euring the study of the action of the cyroscopically stable rocket (TRS) on powered flight trajectory in the right side of the last/latter equation of first term the sign aret be changed by reverse/inverse:

$$\dot{\boldsymbol{v}}_{s} = -\frac{P\boldsymbol{v}_{rs}}{m\boldsymbol{v}_{r}} - \frac{\boldsymbol{\zeta}\boldsymbol{v}_{r}}{2m}(\boldsymbol{v}_{s} - \boldsymbol{w}_{s})\boldsymbol{S}\boldsymbol{c}_{s}(\boldsymbol{M}_{r}). \quad (11.107)$$

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For the projectile of constant mass, system (11.106) will take the form

$$\begin{aligned}
\boldsymbol{v}_{x} &= -\frac{\boldsymbol{\varrho}\boldsymbol{v}_{r}}{2\boldsymbol{m}} \left(\boldsymbol{v}_{x} - \boldsymbol{w}_{x} \right) \boldsymbol{S}\boldsymbol{c}_{x} \left(\boldsymbol{M}_{r} \right); \\
\boldsymbol{v}_{y} &= -\frac{\boldsymbol{\varrho}\boldsymbol{v}_{r}}{2\boldsymbol{m}} \left(\boldsymbol{v}_{y} - \boldsymbol{w}_{y} \right) \boldsymbol{S}\boldsymbol{c}_{x} \left(\boldsymbol{M}_{r} \right) - \boldsymbol{g}; \\
\boldsymbol{v}_{z} &= -\frac{\boldsymbol{\varrho}\boldsymbol{v}_{r}}{2\boldsymbol{m}} \left(\boldsymbol{v}_{z} - \boldsymbol{w}_{z} \right) \boldsymbol{S}\boldsymbol{c}_{x} \left(\boldsymbol{M}_{r} \right).
\end{aligned} \tag{11.108}$$

If we determine the drag through the Fallistic coefficient of c and function $O(v_r)$, then value E in (5.6) rust be calculated through velocity v_{rr}

and the second state

$$\boldsymbol{\mathcal{E}}_{r} = c \boldsymbol{\mathcal{H}}_{\tau}(\boldsymbol{y}) \boldsymbol{\mathcal{G}}(\boldsymbol{v}_{rr}). \tag{11.109}$$

For the projectile of constant mass the system of equations, which considers wind effect, can be writter in the form of common/general/total system (11.14)

$$\ddot{x} = -E_r(\dot{x} - w_s); \quad \ddot{y} = -E_r\dot{y} - g; \quad \ddot{z} = -E_r(\dot{z} - w_s). \quad (11.110)$$

Using (11.104), it is possible to obtain

$$v_r = v_1 \sqrt{1 + \frac{w^2}{s^2} - \frac{2\pi w_g}{s^2} - \frac{2\pi w_g}{s^2}}, \qquad (11.111)$$

where

------ and -----

The velocity of the projectiles of constant mass, as a rule, is considerably more wind velocity. Feject/throwing in (11.111) terms v^2/v^2 and $\frac{2iw_1}{v^2}$, low in comparison with unity, expanding simplified function (11.111) in a binomial series and disregarding in it the terms, which contain relation to the projection of wind velocity

 w_x to the ground speec in larger that by the first, degree, possible, following [9], to obtain

$$\boldsymbol{v}_{r} = \boldsymbol{v} - \boldsymbol{w}_{x} \quad \frac{\dot{x}}{v}; \qquad (11.\ 112)$$

$$G(\boldsymbol{v}_{w}) = G(\boldsymbol{v}_{\tau}) \left[1 - \boldsymbol{w}_{x} \quad \frac{f(\boldsymbol{v}_{\tau})}{v^{2}} \cdot \dot{x} \right]; \qquad (11.\ 113)$$

$$E_{r} = \boldsymbol{E} \left[1 - \boldsymbol{w}_{x} \quad \frac{f(\boldsymbol{v}_{\tau})}{v^{2}} \cdot \dot{x} \right], \qquad (11.\ 114)$$

where $f(\mathbf{v}_{i})$ is determined from (11.25).

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Substituting E_r in (11.110) and disregarding the small second-order guantities, it is possible to obtain

$$\ddot{x} = -E\dot{x} + E\left[1 + \frac{f(v_{s})}{v^{2}}\dot{x}^{s}\right] \Psi_{s};$$

$$\ddot{y} = -E\dot{y} + E\left[\frac{f(v_{s})}{v^{2}}\dot{x}\dot{y}\Psi_{s} - g;$$

$$\ddot{z} = -E(\dot{z} - \Psi_{s}).$$
(11.115)

5.4. Account of wind effect on the flight of the guided missiles.

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For the guided missiles of different types wind, it is one of the basic perturbation factors whose actics is parried by the control system, which holds the center of mass of rocket in the predetermined trajectory. The deviation of projectile fice calculated trajectory under the effect of wind load is the component part of the overall error for the firing the guided missiles. Not all types of the ccntrcl systems parry wind and in certain cases its action must be considered by the methods of ballistics. During preset control only cr the pitch angles, yaw and hank the control system cannot consider and parry the deviations of coordinates x, y and z, caused by the action of wind. Assuming that the control provides obtaining the zero angles of bank and yaw, let us comprise system of equations for determining of the longitudinal-behavior characteristics of the rccket at the assigned tilt angle $\Phi_m(i)$ and with rectilinear bcrizontal displacement/mcvement of the masses of air. As before let us designate calculated values in relative action by index r and from (3.56) will have

$$a_{6,r} = -\frac{2Y_{g}^{*} t_{g} b_{g}}{e \sigma_{r}^{2} S/m_{g}^{*}} = -a_{0,r} b_{g}, \qquad (11.116)$$

after which on (3.58) we will obtain

$$\mathbf{a}_{\mathbf{0},r} = \frac{\mathbf{s}_{\mathbf{0},r} \mathbf{A}_{\mathbf{0}\mathbf{0}} \left(\mathbf{0}_{\mathbf{1}\mathbf{0}} - \mathbf{0}_{r}\right)}{1 + \mathbf{s}_{\mathbf{0},r} \mathbf{A}_{\mathbf{0}\mathbf{0}}}.$$

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In the absolute motion

$$a_6 = \frac{\epsilon_0 h_{0,0} \left(h_{up} - 0 \right)}{1 + \epsilon_0 h_{00}}.$$
 (11.117)

Cn the smallness of angles accepting

$\sin a_6 \approx a_6$; $\sin a_6, \approx a_6$; $\cos a_6 \approx a_6, \approx 1$

and using (3.59), we will citain the system of equations the legitudinal controlled flight, which considers the action by longitudinal component of the wind

$$\dot{v} = \frac{P - X_r \cos(\theta_r - \theta) + Y_r \sin(\theta_r - \theta)}{m} - g \sin \theta;$$

$$\dot{\theta} = \frac{1}{mv} \left[P a_6 - X_r \sin(\theta_r - \theta) + Y_r \cos(\theta_r - \theta) \right] - \frac{g \cos \theta}{v};$$

$$\dot{\theta} = \theta + a_6 \quad (\text{иЛи } \theta = \theta_r + a_6 r);$$

$$\theta_r = \arctan \frac{v \sin \theta}{v \cos \theta - w_r}; \quad X_r = \frac{\theta v_r^2}{2} S c_x (M_r);$$

$$Y_r = \frac{\theta v_r^2}{2} S c_{\theta}^* a_6 r = Y_r^* a_6 r.$$
the tailwind

With the tailwind

 $v_r = \sqrt{v^2 + w_x^2 - 2vw_x \cos \theta}; \quad y = v \sin \theta; \quad x = v \cos \theta;$ $\theta_{mp} = \theta_{mp}(t) \quad H \quad m = m(t).$ and. (11.119)

Maving longitudinal-tehavior characteristics, yawing motion taking into account wind car be calculated from the separately sclued

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 FIGE 35.919

system of equations. On (3.62), it is possible to write

$$\beta_{d,r} = -\frac{2Z_{d}^{p_{1}} t_{d} k_{p_{1}}}{e^{a_{p}^{2} S \pm a_{p_{1}}^{2}}} = -\epsilon_{\phi_{r}} k_{p_{1}}$$

and further

$$\beta_{0,r} = \frac{*_{\phi_{p}} A_{\phi_{p}} (h_{\phi} - \Psi)}{1 + *_{\phi_{p}} A_{\phi_{p}}}.$$
 (11. 120)

Curing the writing of last/latter equality, it was assumed that the rocket has gas-dynamic controls of flight (the jet vanes) and, therefore, control force and torque/moment they do not depend on the effect of wind.

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In absolute motion, as before

$$\beta_6 = -\frac{2Z_{pl}^{\mu_1} i_p \mathbf{k}_{\mu_1}}{q q^2 Sim_{\mu_1}^2} = -\mathbf{s}_{q} \mathbf{\delta}_{\mu_1}$$

and

$$\beta_{6} = \frac{\epsilon_{\phi} k_{0\psi} (\psi_{HP} - \Psi)}{1 + \epsilon_{\phi} k_{0\psi}},$$

Cn the smallness of the angles

 $\sin\beta_{e}\approx\beta_{e};\ \cos\beta_{e}\approx\cos\beta_{e},\approx1;\ \sin\beta_{e},\approx\beta_{e},$

The first equation of system (3.49) taking into account the action of cross wind will take the form

 $\dot{\Psi} = -\frac{1}{m\nu\cos\theta} \left[-P\beta_0 + Z_r^{\mu}\beta_{0,r} \right].$

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If there are no any special considerations, the programmed value of the yaw angle should take equal to zero and system of equations taking into account expressions for β_{i} and β_{i} , will be written as follows:



Since $\psi = 0$, then in yawing motion axis of rocket must be moved in rarallel to itself (Fig. 11.20).



Fig. 11.20. Plow chart of the guided missile upon consideration of the action of cross wind $(\Psi = O)$.

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Buring control on coordinates, wind effect is parried by the control system. the dynamics of motion in the restless atmosphere and the calculation of the cortrol system for wind disturtance/perturbation is examined in the automation of guided missiles [19], [20].

5.5. Ballistic neutral wind and hallistic temperature deflection.

Introduction into the fallistic calculation of the concrete/specific/actual functions of a charge in the temperature and wird on height/altitude is laborious problem. Each meteorological realization requires the independent solution of complex system of equations. Considerably the more simply approximation method, which consists in obtaining of the corrections, designed on differential

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correcting formulas. The application/use of differential correcting formulas for determining the corrections for wind and deviation of temperature of air from normal function $\tau(y)$ proves to be possible, if we introduce into calculation reutral corstant according to an entire trajectory wind and constant deviation of temperature. These constants were called tallistic average. Emilistic wind - w_p and ballistic temperature deflection - $\delta \tau_p$ they are defined from the condition that those caused by their action of the deviation of impact point will be the same as during the real concrete/specific/actual realization of a change in the wind and temperature on height/altitude. If are known tallistic average, then the deviations of impact point from thece designed under normal meteorological conditions will be determined according to simple formulas. Correction into distance for tallistic temperature deflection is equal to

$$\delta x_{C_2} = \frac{\partial x_C}{\partial t} \delta t_g. \qquad (11.122)$$

Ccrrection into distance for the action of longitudinal component of the tallistic wind

$$\delta x_{Cw_{Z}} = \frac{\delta x_{C}}{\delta w_{Z}} \, \varpi_{Z} \, \mathbf{p} \, (11.123)$$

the lateral deviation of impact point from the action mide component of the wind

$$z_{Cw_{g}} = \frac{\partial e_{C}}{\partial w_{g}} w_{g S}. \qquad (11. 124)$$

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The most theoretically substantiated method of calculating the tallistic average, suitable for sciertific research works and the control/check of approximation methods, follows of the condition indicated above of their determination. Bor an example let us examine calculation procedure by longitudinal component of ballistic wind. On the data of the undisturbed nominal trajectory, we determine firing distance $\frac{1}{20}$ and correctics factor to constant in an entire trajectory wind - $\frac{dx_c}{200}$.

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The determination of correction factor we carry out one of precise methods, described above. For example, by the solution of the system of equations, which includes constant wind.

Trajectory calculation is repeated during the practical realization of a change by wind component in height/altitude, is determined x_{cy} , and correction into distance.

 $k_{Cw_s} = x_{Cw_s} - x_c.$ (11.125)

Longitudinal component of tallistic wind for a concrete/specific/actual trajectory and concrete/specific/actual wind function is equal to

$$w_{x\,b} = \frac{bx_{Cu_x}}{bx_C bw_x}.$$
 (11.126)

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A similar method can be used for determining side component of tallistic wind - w_{rg} and tallistic temperature deflection δr_{g} .

Bor a practical work are applied more idle time, but less precise method, connected with the determination of the weights of layers. For determining the weights of layers and ballistic the median trajectory it is divided on height/altitude into a series of layers (usually an equal thickness). The thickness of each layer is taken depending on trajectory height from 200 to 800 m, multiple of 100 m/ For each of layers, are determined the correction factors

 $\left(\frac{\partial x_{c}}{\partial \tau}\right); \left(\frac{\partial x_{c}}{\partial w_{c}}\right); \left(\frac{\partial x_{c}}{\partial w_{c}}\right).$

First two determine charge in the complete distance with an increase in the temperature on 1° K and wind velocities on 1 m/s only in the i layer. The third correction factor determines the lateral deviation of the impact point in the projectile with cross wind 1 m/s only in the i layer. The calculation of each correction factor for the i layer is conducted independently; with a change in the temperature or the account of wind in the i layer in remaining layers, is assumed normal law $\tau(y)$ and dead cals. For each trajectory it is necessary to conduct n of the calculations where n - minimum number of whole layers, in the sum of height/altitudes of which is placed the trajectory height. Ballistic temperature deflection is determined from the equality

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$$\left(\frac{\partial x_{c}}{\partial t}\right)_{1} = \left(\frac{\partial x_{$$

where $\delta \tau_i$ deviation of concrete/specific/actual mean temperature in the i-th layer from mean temperature in the i-th layer, determined by the normal law of a change in the temperature on height/altitude.

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Ballistic temperature deflection

$$\delta \tau_{\rm B} = \frac{\sum_{i=1}^{n} \left(\frac{\partial x_{\rm C}}{\partial \tau}\right) \delta \tau_{i}}{\frac{\partial x_{\rm C}}{\partial \tau}}.$$
 (11.128)

The relation

$$q_{\tau l} = \frac{\left(\frac{\partial x_{C}}{\partial \tau}\right)_{l}}{\frac{\partial x_{C}}{\partial \tau}} \quad (11.\ 129)$$

is called the weight of the i-th layer according to temperature. Consequently,

$$\delta \tau_{\rm b} = \sum_{l}^{a} q_{\rm u} \delta \tau_{l}. \quad (11.130)$$

The calculation of the weights of layers of ballistic wind is hinder/hampered by the fact that wind is vector quantity and can change module/modulus and direction depending on the number of a layer. Furthermore, one-type rocket and artillery pieces, having the DOC = 78107116

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identical weights of layers, will have different ballistic wind depending on the line of fire. Because of this the calculation of ballistic wind is conducted for the conditional line of fire with zero azimuth, i.e., for a direction in rorth, with the subsequent conversion in the real line of fire [18]. Ite line of fire is determined by the azimuth of firing A_{er} by the angle, calculated clockwise off direction in north before direction in target/purpose. Jzimuth of wind A_{ei} - the angle, calculated also clockwise off direction in north to the vector of wind, directed toward the firing position (Fig. 11.21). They have plus sign the longitudinal wind, directed toward target/purpose (i.e. tailwind), and the cross wind, which blows to the right from the line of fire.





Fig. 11.21. Azimuth of firing Act and the azimuth of wind Are

Key: (1). North. (2). West. (3). Fast. (4). Wind direction. (5). Line of fire.

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According to determination for the incidental fallistic wind, dimected from north to south, it is possible to write the equality:

$$\delta x_{Cw_{x}} = \sum_{1}^{n} \left(\frac{\partial x_{C}}{\partial w_{x}} \right)_{i} \Psi_{(C-10)i} = \frac{\partial x_{C}}{\partial w_{x}} \Psi_{b(C-10)i},$$

whence the longitudinal kallistic wind

$$P_{S(C-10)} = \sum_{1}^{n} q_{\Psi_{n}i} \Psi_{(C-10)i},$$
 (11.131)

where

$$q_{\mathbf{v}_{x}^{l}} = \left(\frac{\partial x_{\mathbf{C}}}{\partial \mathbf{v}_{x}}\right)_{l} / \frac{\partial x_{\mathbf{C}}}{\partial \mathbf{v}_{x}} - (11.132)$$

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- weight of a layer on longitudinal component of ballistic wind.

The lateral deviation of projectile under the action wind component, which blows from the east to west, is equal

$$z_{Cw_{g}} = \sum_{l}^{n} \left(\frac{\partial z_{C}}{\partial w_{s}} \right)_{l} w_{(3-3)l} = \frac{\partial z_{C}}{\partial w_{s}} w_{3(3-3)l}$$

Hence side component the velocities of the fallistic wind

$$\Psi_{\mathbf{5}}(\mathbf{s}_{-\mathbf{3}}) = \sum_{1}^{n} \varphi_{\Psi_{g}l} \Psi_{(\mathbf{s}_{-\mathbf{3}})l}, \qquad (11.\ 133)$$

where

$$q_{w_z l} = \left(\frac{\partial z_C}{\partial w_z}\right)_l \left/ \frac{\partial z_C}{\partial w_z} - \text{BCC}\right)_l$$

a layer on side component wind velocity.

The direction of the action of fallistic wind is determined from the acgle

$$a_{w_{B}} = \arccos t_{g} \left| \frac{w_{B} (n-3)}{w_{B} (C-10)} \right|.$$
 (11.134)

If ballistic wind is directed toward the firing position from the first fourth, then A_{m_1} if from the second fourth, then A_{m_2}

300°- a_{op} ; if from the third fourth, then $a_{op} = 100°$ (a_{op}) and if from the fourth, as shown in Fig. 11.21, then $a_{op} = 100°$ (a_{op}).

Hodulus of velocity of the ballistic wind

For the real line of fire, longitudinal and side components of

tallistic wind are determined from the formulas

 $w_{5z} = -w_5 \cos(A_{er} - A_{w_5}); w_{5z} = w_5 \sin(A_{er} - A_{w_5}).$ (11.185)

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The weights of layers $q_{vl}, q_{v_sl}, q_{v_sl}$ for cne and the same layer it is various in value and depends on the number of a layer. If we in formula (11.127) assume $a_{1}=a_{2}=\cdots=a_{n}=a_{n}$, then $q_{v_1}+q_{v_2}+\cdots+q_{v_n}=1$; is analogous



Equality to the unit of the sum of the weights of layers is testing the correctness of their calculation. For celculating the corrections into complete flying range for wind and deviation of temperature in connection with artillery shells the weight of layers in the first approximation, can be designed on obtained in parabolic theory formula (10.16).

For different systems and the trajectories of one system, the weight of layers will be different and it must be calculated previously. During the preparation of concrete/specific/actual firings at the data of meteorological bullettin are calculated from layers of deviation on me-ax, after which are calculated ballistic temperature deflection and ballistic wind.

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5.6. Approximation correcting formulas to longitudinal and lateral constant wind.

The set-forth method is developed for the account of the separate effect of constant longitudinal and crosswind on the motion of the center of mass of artillery shell. Vertical component wind velocities is not considered.

Let us examine first crly axial motion. The special feature/peculiarity of method consists in the fact that relative motion of the center of mass of projectile is examined in the coerdinate system, which accomplishes translational motion relative to the Earth in wind direction with its velocity. Complete firing distance relative tc) (in absolute motion Farth with longitudinal tailwird w_x is equal to

 $x_{Cm_2} = x_{Cr} + w_{AS},$ (11.137)

where sc- distance in relative scticn;

4c-total flying time, identical Ecth in relative and in absclute motions.

Besignating sc-distance in absolute motion with dead calm,

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we will obtain correction for the longitudiral wind

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Bren (11. 138)

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Eifference x_{0} is usually small for its determination it is possible to use differential correcting formula, bearing in mind that the trajectory of the projectile of constart mass is determined by values v_0 , θ_0 and c

$$\delta x_{Cw_{x}} = \frac{\partial x_{C}}{\partial w_{0}} \delta v_{0} + \frac{\partial x_{C}}{\partial \theta_{0}} \delta u_{0} + \frac{\partial x_{C}}{\partial \theta} \delta c + w_{x} t_{C}. \quad (11.139)$$

Bifferences in the initial velocities and angles of departure in absolute and relative actics are equal to

 $\delta v_0 = v_{0r} - v_0; \quad \delta \theta_0 = \theta_{0r} - \theta_0. \tag{11. 140}$

Bifference in the fallistic coefficients &c=0.

The relative initial velocity in axial motion and the angle of departure along Fig. 11.22 are equal to

After the determination of basic correction factors $\frac{\partial x_r}{\partial y}$, $\frac{\partial x_c}{\partial y}$ and of the time of motion f_r along the data of nominal trajectory,

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it is necessary to calculate why and be the

Correction for longitudinal wind will te determined according to formula (11.139). For high initial velocities it is possible to use with formula (11.112), after replacing in it x/v by cos θ_0 ; then $\theta_0 = \theta_0 - \theta_0 = -\theta_0 \cos \theta_0$. (11.368)

Let us present (11.142) in the form

 $\frac{1}{2} \frac{1}{2} - \frac{1}{2} \frac{1}{2} \frac{1}{2} - \frac{1}{2} \frac{1}{2}$



Big. 11.22. Schematic of the determination of the initial velocity and angle of departure in relative plane motion under the effect of constant tailwind.

Fig. 11.23. Schematic of determination of initial velocity and angle of departure in relative yawing motion under effect of constant cross wind.

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Tecompcse/expanding the value, included into trackets, in a binomial series and omitting the members of the second and higher than the crder of smallness, it is possible to obtain:

Ccunting approximately

$$\delta \operatorname{tg} \theta_{p} = \operatorname{tg} \theta_{p} - \operatorname{tg} \theta_{p} \approx \frac{\delta \theta_{p}}{\cos^{2} \theta_{p}},$$

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we will obtain

 $\mathbf{W}_{\mathbf{p}} \approx \frac{\mathbf{v}_{\mathbf{r}}}{\mathbf{n}} \sin \theta_{\mathbf{p}}.$ (11. 144)

Let us substitute (11.143) and (11.144) is (11.139) and after conversions we will obtain the approximation correcting formula

$$\delta x_{Cw_{g}} = w_{g} \left[t_{C} - \frac{\partial x_{C}}{\partial u_{0}} \cos \theta_{0} + \frac{\partial x_{C}}{\partial \theta_{0}} \frac{\sin \theta_{0}}{u_{0}} \right], \quad (11.145)$$

where $\frac{\partial x_c}{\partial y}$ it is taken in the dimensionality s/rad.

The lateral deviation of the center of mass of projectile under the action of constant cross wind in absolute action is equal

$$\boldsymbol{z}_{\boldsymbol{w}_{g}} = \boldsymbol{z}_{r} + \boldsymbol{w}_{g} \boldsymbol{z}. \tag{11. 146}$$

In the system of coordinates **0, x, y, z**, driving/seving with the speed of cross wind in the direction of its action, the velocity of the center of mass of projectile is equal to

CI.

$$\mathbf{v}_{\mathbf{r}} = \mathbf{v}_{\mathbf{r}} \sqrt{\frac{1+\left(\frac{\mathbf{v}_{\mathbf{r}}}{\mathbf{v}_{\mathbf{r}}}\right)^{2}}{1+\left(\frac{\mathbf{v}_{\mathbf{r}}}{\mathbf{v}_{\mathbf{r}}}\right)^{2}}}.$$
 (11. 147)

For high initial velocities (1,

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From the same figure 11.23

$$\frac{v_{0} \sin \theta_{0}}{V d \cos^{2} t_{0} + v_{1}^{2}} = \frac{\frac{v_{0}}{v_{0}}}{\sqrt{1 + \frac{v_{1}^{2}}{v_{0}^{2} \cos^{2} t_{0}}}}.$$
 (11. 148)

For relatively low angles of departure and large v_0 can be counted **igh_wigh** and $\delta \theta_0 = 0$. With the adopted assumptions lateral deviation 2- is determined by the rotation of trajectory plane of relative motion relative to the system of coordinates $0_{r,T_r,Y_r,Z_r}$ around axis y, to angle Ψ_{v_s} and then for an inject point

$$z_{C'} = -x_C \operatorname{tg} \Psi_{\Psi_2} = -\frac{\Psi_{S} x_C}{v_0 \cos \theta_0}.$$
 (11. 149)

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The lateral deviation of projectile of relatively terrestrial starting coordinates for current point in the trajectory with respect to (11.146) is equal

$$z_{w_s} = w_s t - \frac{w_s x}{v_0 \cos \theta_0} \qquad (11.150)$$

and for an impact point

$$z_{Cw_s} = w_s \left(t_C - \frac{x_C}{v_0 \cos v_0} \right). \qquad (11.151)$$

Let us note that value $tg \Psi_{w_{p}} = \frac{\Psi_{s}}{v_{0}\cos t_{0}}$ is citained without the acccuat longitudinal component wind velocity. If we consider longitudinal component of wind, ther

$$\mathbf{tg} \, \boldsymbol{\Psi}_{\mathbf{w}_{g}} = \frac{\mathbf{w}_{t}}{\mathbf{w}_{t} \cos \mathbf{t}_{t} - \mathbf{w}_{x}}.$$
 (11, 152)

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As a result of the made simplifications in formula (11.145) and (11.151) give noticeable errors at the low initial velocities and the large angles of departure. Acceptable results can be obtained on (11.145) and (11.151) with $v_0>250$ m/s and $6_c<45^\circ$.

§6. Determination of corrections during the calculation of complex trajectories.

Complex trajectories are subdivided into the individual sections which are distinguished by the character of effect on rocket flight and projectiles of the basic and perturbation power factors. For example, under the effect of cross wind on the active-reactive statically stable unrotated mine it will twice change the direction of yawing motion (Fig. 11.24). On the first inactive leg, the mine will be moved in wind direction, on the second section (active) the yawing motion of mine will be directed against wind and on the third section (second passive) the mine will be again moved in the direction of cross wind. Value and the sign of drift of rocket from plane of reference of firing on the first and second sections _____ and 'a DCC = 78107116





Fig. 11.24. Schematic of the lateral displacement/movement of active-reactive mine under the effect of cross wind.

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For each of the sections of the motion characteristic of rocket, they are determined by the imitial conditions: the coordinates of transition point to the changed character of motion, by the sense of the vector of speed and by the modulus of velocity of the center of mass at the moment of transfer/transition. Furthermore, for each of the sections acts its system of forces. The inertia properties of rocket (projectile) determine smooth transition from one section to the nest; the horizontal projection of trajectory in Fig. 11.24 does not have acute angles, and the maximum values z₁ and z₂ do not correspond to the points N and K on elevation. EOC = 78107116 PIGE 54 938

Coordinate definition of the end points of complex trajectories - the point of shell burst with antiaircraft fire and of the impact point in the projectile with contact firing - it is possible to conduct in the manner that this was described above. Systems of equations for each of the sections must include basic forces and the perturbation factors whose effect on the results of firing is assumed to establish/install. Motion characteristics at the end point of the preceding/previous section will be initial conditions for that follow.

Buring the use of differential correcting formulas, the correction factors must be determined separately for each of the sections; the obtained corrections must subsequently be summarized taking into account their signs. It is necessary to keep in mind that the method of differential correcting formulas in application to complex trajectories is not considered transient processes from one section to the next. It is assumed that the new system of forces immediately, without transient process, charges the direction of the motion of the center of mass of rocket or projectile. Upon this setting smooth curve of the horizortal projection of trajectory in fig. 11.24 will be converted into troken line with salient points at the end/leads of the sections. Let us examine, for an example, the calculation of the lateral deviation of the impact point in the unquided rocket from plane of reference of ining under the effect of comptant cross wind (Fig. 11.25).

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fig. 11.25. Circuit of lateral displacement of projectile of constant mass under action of cross wind.

Key: (1). Impact point. (2). Plane of reference of casting. (3).
Firing position.

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Pcint projection of the point of the end/lead of powered flight trajectory on horizontal plane. Distance from point K_r on the projection of the plane of casting on horizontal plane (line of fire) - m_r Let us designate through z_{0R} - deviation of the impact point in the projectile from the line of fire, determined by deviation M_R . The

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 FIGE 57 940

lateral displacement of projectile, determined by the deviation of velocity vector of angle $\delta \Psi_{\rm R}$ in horizontal plane, let us designate – Tow The lateral displacement of projectile, determined by the deviation of velocity acdulus, let us designate through $z_{\rm Cox}$ and the lateral displacement of projectile, determined by the effect of wind, let us designate $z_{\rm Coy}$. For the case when the deviation of determining parameters $\delta \Psi_{\rm R} > 0$; $\delta z_{\rm R} < 0$; $\delta \sigma_{\rm R} < 0$ and $w_z < 0$, in accordance with Fig. 11.25 we obtain the total lateral deviation

$$z_{c} = -z_{c_{x}} - z_{c_{x}} - z_{c_{x}} + z_{c_{x}}. \qquad (11.153)$$

Utilizing a differential correcting formula, we will obtain

$$z_{\rm C} = -\frac{\partial z_{\rm C}}{\partial z_{\rm x}} \partial z_{\rm x} - \frac{\partial z_{\rm C}}{\partial \Psi_{\rm x}} \partial \Psi_{\rm x} - \frac{\partial z_{\rm C}}{\partial w_{\rm x}} w_{\rm s} + \frac{\partial z_{\rm C}}{\partial w_{\rm x}} \partial v_{\rm x}. \quad (11.154)$$

Corrections for wind w_r and for a change in the velocity at the end of the operation of engine δv_n are determined by one of previously examined methods.

With the adopted assumptions the deviation of a squivalent displacement relative to axis Ox of the projection of an entire trajectory on horizontal plane, i.e.,

Correction factor for a charge in the angle $\partial \Psi_R$ will be determined from geometric considerations. Let the velocity vector of $\tilde{\sigma}_R$ be ECC = 78107116

deflected from the plane, parallel to plane of reference of firing and passing through point K, by angle **but i** (in inclined plane); then

Frcs Big. 11.25

 $z_{\Psi_z} = L_z \sin W_z = -\frac{1}{2}$

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Here L_{e} — length of inactive leg when M_{e} On the smallness of angle, it is possible to accept M_{e} and M_{e} and then

Se - M. W. - M.

where the correction factor

Fage 477.

Let us examine the procedure of the calculation of the corrections, which consider the curvature of the Earth. For the spherical model of the Earth in the central gravitational field of the plane of two trajectories, agitated and not disturbed, they comprise the dihedral angle whose fin/edge traverses the conditional center of the Earth. The value of difedral angle will be determined by the lateral deviation of point K of value $dz_{\rm H}$. Let us designate the calculated end/lead of the engine operation with point $K_{\rm H}$, the real

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end/lead of the engine operation - by a point K. The projections of points K, and of K on the surface of the spherical model of the Farth let us designate K_{ps} and K_s ; respectively, impact points let us designate C, and C. The fin/edge of dihedral angle is located from the radius-vectors, which connect points K, and K, with the conditional center of the Earth, on angle w/2. It is obvious, $e_{st} =$ as this was obtained earlier for low firing distances with the nomintersecting planes basic and that agitated of trajectories. Let us designate in Fig. 11-26 intersection of the fin/edge of dihedral angle with the surface of the Earth by point - a. If we do not consider the difference between e_{st} at beight/altitude y_{st} and e_{st} on the surface of the Earth, then it is possible to use similar spherical triangles Cond Cond Formation for the spherical trigonometry

Satisfication of the state

and, which means, that



Fig. 11.26. Lateral displacement of the projectile of constant mass s. depending on the lateral linear deflection of initial point dea

Fig. 11.27. Lateral displacement of projectile of constant mass $z_{cw_{R}}$ depending on lateral angular deflection of velocity vector δW_{R} .

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With the low range angle

cos 20 = 1 and zer = ier

In the particular case, when $2\phi = \frac{\pi}{2}$, $s_{CR} = 0$.

Let us examine correction for angular ceflection of with the

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spherical model of the Earth. In Fig. 11.27 sides of spherical triangle $C_{p}K_{3}$ and CK_{3} are projections on the surface trajectories of calculated and disturbed Earth. The lateral deviation of impact point from trajectory plane, passing through points K and C_{p} , determined by angular deflection $\delta \Psi_{p}$, we will obtain from spherical triangle $C_{p}K_{3}C$

$$z_{CV_{HH}} = \frac{R_3 \sin 2\psi}{\cos t_{H}} \frac{\partial Y}{\partial Y_{HH}} = \frac{\partial z_C}{\partial Y_{HH}} \frac{\partial Y}{\partial Y_{HH}}.$$
 (11.159)

The correction factor

$$\frac{\partial r_c}{\partial T_{th}} = \frac{R_0 \sin 2\phi}{\cos k_c}.$$
 (11. 160)

The great value correction $z_{CV_{min}}$ will have with range angle $2\phi = \frac{\pi}{2}$; with distance $2\phi = 2\pi$ the correction will become equal to zero. With short distances $\sin 2\phi \approx \frac{L_{\pi}}{2}$ and then we will obtain already known to us formula (11.158). EOC = 78107116

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Charter XII.

Initial conditions of shot.

Type and the designation/furfose of rocket or projectile, their construction, aerodynamic and ballistic date, the construction of projectile installation and meteorological conditions, by which is conducted the firing, determine the initial conditions of shot for the assigned distance. Of the unguided rockets and projectiles, initial conditions lay out of trajectory.

The characteristics of the spatial meticn of the unguided rocket can be determined, after solving system of equations, obtained from equations (3.7)-(3.12), and (3.14)-(3.23), efter placing in them equal to zero control forces and torgue/moments.

After simplifications the system will take the form

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$$\dot{\Psi} = \frac{1}{m} (P_x - X + Q_y); \quad \dot{\Psi} = \frac{1}{m\Psi} (P_x + X + Q_y);$$

$$\Psi = -\frac{1}{m\Psi} (P_x + Z^{*});$$

$$J_{x_1} \bullet_{x_1} + (J_{x_1} - J_{y_1}) \bullet_{y_1} \bullet_{y_1} = \sum M_{y_1};$$

$$J_{y_1} \bullet_{y_1} + (J_{x_1} - J_{x_1}) \bullet_{x_2} \bullet_{x_1} = \sum M_{y_1};$$

$$J_{y_1} \bullet_{y_1} + (J_{y_1} - J_{x_1}) \bullet_{x_2} \bullet_{y_1} = \sum M_{y_1};$$

$$J_{y_1} \bullet_{y_2} \sin \psi + \phi_y \cos \psi;$$

$$H = \sum M_{y_1};$$

$$J_{y_2} \bullet_{y_1} - J_{y_1} = \sum M_{y_1};$$

$$J_{y_2} \bullet_{y_2} - J_{y_1} \bullet_{y_2} \bullet_{y_2} = \sum M_{y_1};$$

$$J_{y_2} \bullet_{y_1} - J_{y_1} \bullet_{y_2} \bullet_{y_2} \bullet_{y_1} = \sum M_{y_1};$$

$$J_{y_2} \bullet_{y_1} - J_{y_1} \bullet_{y_2} \bullet_{y_2} \bullet_{y_1} = \sum M_{y_1};$$

$$J_{y_2} \bullet_{y_2} - J_{y_1} \bullet_{y_2} \bullet_{y_2} \bullet_{y_1} = \sum M_{y_1};$$

$$J_{y_2} \bullet_{y_2} - J_{y_1} \bullet_{y_2} \bullet_{y_2} \bullet_{y_1} = \sum M_{y_1};$$

$$J_{y_2} \bullet_{y_2} - J_{y_1} \bullet_{y_2} \bullet_{y_2} \bullet_{y_1} = \sum M_{y_1};$$

$$J_{y_2} \bullet_{y_2} - J_{y_1} \bullet_{y_2} \bullet_{y_2} \bullet_{y_1} = \sum M_{y_1};$$

$$J_{y_2} \bullet_{y_2} - J_{y_1} \bullet_{y_2} \bullet_{y_2} \bullet_{y_1} = \sum M_{y_1};$$

$$J_{y_2} \bullet_{y_2} - J_{y_1} \bullet_{y_2} \bullet_{y_2} \bullet_{y_1} = \sum M_{y_1};$$

$$J_{y_2} \bullet_{y_2} - J_{y_1} \bullet_{y_2} \bullet_{y_2} \bullet_{y_1} = \sum M_{y_1};$$

$$J_{y_2} \bullet_{y_2} - J_{y_1} \bullet_{y_2} \bullet_{y_2} \bullet_{y_1} = \sum M_{y_1};$$

$$J_{y_2} \bullet_{y_2} - J_{y_1} \bullet_{y_2} \bullet_{y_2} \bullet_{y_1} = \sum M_{y_1};$$

$$J_{y_2} \bullet_{y_2} - J_{y_1} \bullet_{y_2} \bullet_{y_2} \bullet_{y_2} \bullet_{y_1} = \sum M_{y_1};$$

$$J_{y_2} \bullet_{y_2} - J_{y_1} \bullet_{y_2} \bullet_{y_2} \bullet_{y_2} \bullet_{y_1} = \sum M_{y_1};$$

$$J_{y_2} \bullet_{y_2} - J_{y_1} \bullet_{y_2} \bullet_{y_2} \bullet_{y_2} \bullet_{y_1} \bullet_{y_2} \bullet_{y_2} \bullet_{y_1} \bullet_{y_2} \bullet_{y_2} \bullet_{y_2} \bullet_{y_2} \bullet_{y_1} \bullet_{y_2} \bullet_{y_2} \bullet_{y_2} \bullet_{y_1} \bullet_{y_2} \bullet_{y_2} \bullet_{y_2} \bullet_{y_2} \bullet_{y_1} \bullet_{y_2} \bullet_{y_2} \bullet_{y_2} \bullet_{y_2} \bullet_{y_1} \bullet_{y_2} \bullet_$$

Fage 480.

Ber the solution of the written system, it is necessary to know the iditial values of the following values: the initial value of mass mo; the initial values of the moments of inertia $J_{x_{10}}$, $J_{y_{10}}$, $J_{z_{10}}$, the initial values of all forces and torque/moments, written in right part one of six equations of system. Furthermore, it is mecessary to know the initial values of fifteen motion characteristics v_0 , θ_0 , Ψ_0 , x_0 , y_0 , x_0 , ϕ_0 , ϕ_0 , $\phi_{z_{10}}$, $\phi_{z_{10}}$, $\phi_{z_{10}}$. COC = 78107116 PAGE -64 947

If we instead of equations (3.14) - (3.16) utilize three equations (3.13), then instead of $\bullet_{x_1}, \bullet_{y_1}$ and \bullet_{z_1} it is necessary to know $\bullet_{b_1} \bullet_{b_2}$ The enumerated initial conditions next be determined for the torque/moment of the loss of tight coupling of the unguided rocket with projectile installation. For determining the effect of scattering initial conditions for scattering of trajectories (or impact points) it is necessary to know the stochastic characteristics of the named initial values. The solution of assigned mission represents great difficulties; therefore the effect of initial conditions for the results of firing is estimated in the simplified setting with one or the other assurptions.

Effect on the firing distance of deviations from computed values of the initial angle of departure, of initial velocity and meteorological conditions is considered with the aid of correcting formulas and is examined in chapter XI. Let us here show the effect of the design features of projectile installation on the formation of the initial conditions of firing.

A change in the initial conditions of shot can occur because of:

a) the motion of the carrier of armament;
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b) the oscillation/vibration of the carrier of armament and strictly projectile installation as a result of the elastic properties of constructions;

c) the loss of the axial alignment of rccket and launching racks with the descent of rocket from guides:

d) the effect of gas flows, which is exhibited with the descent of the rocket from launcher or on leaving of the projectile of artillery instrument from tore.

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§1. Effect of the motion of the carrier of armament on the initial conditions of shot.

The motion of the carrier of armament is solid body can be divided into the displacement/movement of its center of mass and oscillation/vibration relative to the center of mass. The displacement/movement of the center of mass is determined by two motions - by basic motion of the carrier of armament under the action of motors and the oscillatory motion of the center of mass under the

effect of environmental disturbances (for example, the orbital tossing of ship, caused sea rating).

The oscillation/vibrations of the carrier of armament relative to the center of mass, by analogy with the tossing of ship, it is possible to divide into the rolling motion - rolling of carrier around its longitudinal axis; keel tossing - fluctuation of carrier of its relatively vertical exis.

The total initial velocity of projectile in earth-based cccrdinate system without the account of the rotational effect of the Farth is equal to

$\vec{v}_0 = \vec{v}_{0r} + \vec{v}_{1} + \vec{v}_{2}, \quad (12.2)$

where \overline{v}_{0} — velocity of projectile of relatively projectile installation at the moment of the loss of tight coupling with guides by the latter (for the projectile of artillery instrument - muzzle velocity):

 $\mathbf{F}_{\mathbf{n}} \rightarrow \mathbf{a}$ speed of running of the carrier of armament (ship, tank, aircraft);

Fr- the supplementary initial velocity, caused by envirgemental disturbance, in which soves the carrier (for example, ECC = 78107116 FACE 57 950

the tossing of ship-launch vehicle).

The greatest difficulties are encountered during determination F_{n} . If we do not consider the effect of medium and to equate $v_{n}=0$, then the value of the velocity of rocket (projectile) in module/modulus and direction in earth-lased coordinate system will be determined by following manner. Let us join the Leginning of earth-based coordinate system with the center of mass of launch vehicle and it is directed axis Gx_{3} then so that it would coircide with the velocity vector F_{n} (Fig. 12.1). The sense of the vector of the velocity of the motion of the center of mass of projectile relative to launch vehicle F_{n} let us assign by angle of departure in relative motion h_{n} and by the heading/course angle of firing ϕ_{n} . Then, bearing in mind that v_{n} translational speed, we will obtain for the projections of the speed of projectile on the axis of earth-tased coordinate system the following expressions:

(12.3)

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Fig. 12.1. Heading/course angle, the initial velocity and angle of departure, obtained taking irtc account the velocity of carrier.

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Mcdulws of velocity of projectile in the absolute motion

$$v_0 = \sqrt{v_{u_3}^1 + v_{u_3}^1 + v_{u_3}^2}.$$

Substituting equalities (12.3), we will obtain after the conversions:

$$\boldsymbol{v}_{0} = \boldsymbol{v}_{0r} \sqrt{1 + 2 \frac{\boldsymbol{v}_{n}}{\boldsymbol{v}_{0r}} \cos \theta_{0r} \cos q_{re} + \frac{\boldsymbol{v}_{n}^{2}}{\boldsymbol{v}_{0r}^{2}}}.$$
 (12.4)

Beading/course angle of firing in the absolute action

$$q_{\rm e} = \arg\left(\frac{c_{\rm Qr}\cos\theta_{\rm er}\sin q_{\rm re}}{v_{\rm Qr}\cos\theta_{\rm er}\cos q_{\rm re} + v_{\rm m}}\right). \tag{12.5}$$

Angle of departure in the absolute action

$$\theta_{\theta} = \arccos\left(\frac{u_{\theta r}\cos\theta_{r}\cos\theta_{r}}{u_{\theta}\cos\theta_{r}}\right). \quad (12.6)$$

Three Last/latter formulas are obtained on the assumption that projectile - material print.

For determining initial missile attitude (rocket) relative to the Earth it is necessary to consider the effect of the oscillation/vibrations of the carrier of armament and launcher.

Let us examine the effect of the cscillation/vibrations of launch vehicle relative to the center of mass on a change in the parameters of the motion of rocket at descert from launching rack. We utilize five systems of coordinates (Fig. 11.2). The oscillation/vibration of the carrier of armsmert let us examine relative to earth-based coordinate system $Q_1 x_1 y_2 x_3$.

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Fig. 12.2. Coordinate system for case of missile takeoff from unstabilized launcher of driving/mcving carrier.

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The origin of coordinate system $0_{m}x_{m}y_{m}z_{m}$, is rigid, connected with carrier, let us place in the center of mass of carrier 0_{m} . The rotation of the system of coordinates $x_{m}y_{m}z_{m}$ relative to system $x_{3}y_{3}z_{3}$ let us define that as usual, by three angles: in horizontal plane – by angle ψ_{m} in vertical plane – by angle ψ_{m} : lateral rotation relative to axis $0_{m}x_{m}$ – by angle ψ_{m} . In the center of oscillation of launcher 0_{m} let us place the beginning of two coordinate systems: the systems of coordinates $0_{m}x_{m}y_{m}z_{m}$, of the rigidly connected with launcher. EOC = 78107117

The axes of coordinates x_{n}, y_{n}, z_{0} , are collinear to axes $x_{n}y_{n}z_{n}$. The rotation of launcher relative to carrier is determined by two argles: by the heading/course angle of firing q_{re} - between axes $0_{n}x^{n}$, and $0_{n}x_{n}$, and the angle q_{m} , those determining the slope/inclination of launcher to the plane of axes $0_{n}x_{n}$, and $0_{n}x_{n}$, by rigidly connected with carrier. The fifth coordinate system is the system $0_{1}x_{1}y_{1}z_{1}$, connected with rocket at the moment of its descent with guides; the axis $0_{1}x_{1}$ of this system coincides with the line of fire.

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If we do not consider the elastic vibrations of the

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ccnstructions of launch vehicle and of launcher and disturkance/perturbations of rocket at the moment of its descent from guides, then, by using the table of transfer cosines from earth-based coordinate system to the coordinate system, connected with rocket carrier (by table, similar 2.3d), we will obtain equations for determining the deviations of the pitch angles, yaw and bank of rocket at the moment of start depending on the angles of the wibration of launch vehicle.

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Bor case $\psi_n = \varphi_n = \varphi_n = 0$ and the adopted by us assumptions at the schent of the descent of rocket from guides, let us, obviously, have $\psi_n = \varphi_n$ (from direction Θr_n) and $\gamma = 0$.

In other case, when 4440, 4440, and 4440, the rocket will descend from guides at angles 44 and 7. Comparing, as this made V. P. Kazakowtsev [18], cell/elements of transition at the moment of the start of rocket from the system of coordinates 44400, 4500,

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DOC = 77207412 PAGE Solution **Accepting, as a result of the smallness of angles,** $\sin \psi_{\mu} = \psi_{\mu}, \quad \sin \psi_{\mu} = \psi_{\mu}, \quad \sin \gamma_{\mu} = \gamma_{\mu},$ $\cos \psi_{\mu} = \cos \psi_{\mu} = \cos \gamma_{\mu} = 1,$

we will obtain

$$\Delta \psi = \arctan\left[\frac{-\psi_{\rm H}\cos\varphi_{\rm rc} + \gamma_{\rm H} tg\varphi_{\rm H} + \sin\varphi_{\rm rc}}{\cos\varphi_{\rm rc} - \varphi_{\rm H} tg\varphi_{\rm H} + \psi_{\rm H}\sin\varphi_{\rm rc}}\right] - q_{\rm rc}; \quad (12.7)$$

$$\Delta \vartheta = \arcsin\left[\varphi_{\rm H}\cos q_{\rm rc}\cos\varphi_{\rm H} + \sin\varphi_{\rm H} - \gamma_{\rm H}\cos\varphi_{\rm H}\sin q_{\rm rc}\right] - \varphi_{\rm n}; \quad (12.8)$$

$$\Delta \gamma = \arctan\left[\frac{\varphi_{\rm H} tgq_{\rm rc} + \gamma_{\rm H}}{\varphi_{\rm H}\sin\varphi_{\rm H} - \frac{\cos\varphi_{\rm H}}{\cos\varphi_{\rm rc}} - \gamma_{\rm H}\sin\varphi_{\rm H} tgq_{\rm rc}}\right]. \quad (12.9)$$

The projections of the angular velocity vector of the rotation of lawnch vehicle of relatively earth-lasec occrdinate system on the connected with it coordinate axes, similar (3.13), will be written in this form:

$\boldsymbol{\omega}_{x \mu} = \dot{\boldsymbol{\psi}}_{\mu} \sin \boldsymbol{\varphi}_{\mu} \perp \dot{\boldsymbol{\gamma}}_{\mu};$	(12. 10)
$\boldsymbol{\omega}_{\boldsymbol{\mu}} = \dot{\boldsymbol{\varphi}}_{\boldsymbol{\mu}} \cos \boldsymbol{\varphi}_{\boldsymbol{\mu}} \cos \boldsymbol{\gamma}_{\boldsymbol{\mu}} + \dot{\boldsymbol{\varphi}}_{\boldsymbol{\mu}} \sin \boldsymbol{\gamma};$	(12.11)
$\mathbf{w}_{\mathbf{x}} = -\dot{\mathbf{\phi}}_{\mathbf{x}} \cos \mathbf{\varphi}_{\mathbf{x}} \sin \mathbf{\gamma}_{\mathbf{x}} + \dot{\mathbf{\varphi}}_{\mathbf{x}} \cos \mathbf{\gamma}_{\mathbf{x}}.$	(12. 12)

Since we consider laurch vehicle and launcher as rigid and rigidly connected Ecdies, then it is a choice that the vector of the instantaneous angular velocity of launching racks will coincide with the vector of the instantaneous angular velocity of launch vehicle.

If we introduce the assumption that the rocket during motion along launching racks completely receives the angular displacements of guides, which corresponds to the stiffening joint between them, then the vector of the instantaneous angular velocity of rocket will be equal to the vector of the instantaneous angular velocity of guides. On the basis of the aforesaid, it is possible to easily find

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the projection of the vector of the instantaneous angular velocity of rocket, caused by the rotion of launch vehicle, on the axis of the connected with rocket syster of coordinates $0_1 x_1 y_1 z_1$. Taking into account dependence (12.10), (12.11) and (12.12), and also the direction cosines of passage from the syster of coordinates $0_2 x_2 y_2 z_2$ to the syster of coordinates $0_1 x_1 y_1 z_1$, let is have

 $\mathbf{e}_{x_1} = \mathbf{e}_{x_2} \cos \varphi_{\pi} \cos q_{x_2} + \mathbf{e}_{y_2} \sin \varphi_{\pi} + \mathbf{e}_{x_2} \cos \varphi_{\pi} \sin q_{r_{e^*}} (12.13)$ $\mathbf{w}_{y_1} = -\mathbf{w}_{x_2} \sin \varphi_{\pi} \cos q_{r_e} + \mathbf{w}_{y_1} \cos \varphi_{\pi} - \mathbf{w}_{x_2} \sin \varphi_{\pi} \sin q_{r_{e^*}} (12.14)$ $\mathbf{e}_{x_1} = -\mathbf{e}_{x_2} \sin q_{r_e} + \mathbf{e}_{x_2} \cos q_{r_e} + \mathbf{e}_{x_3} \cos q_{r_{e^*}} = -\mathbf{e}_{x_3} \sin q_{r_3} \sin q_{r_3$

The obtained equations make it possible to determine the angular velocities of rocket at the soment of start from the nonstabilized lawncher of the oscillating launch vehicle (without the account of disturbance/perturbations with descent).

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Let us examine the disturbance/perturbations of rocket with descent from the directing stabilized launcher, caused only by the motion of launch vehicle. The automatic machine of the angular stabilization of launcher provides the constancy of the angular position of the guides with rocket in space. However, the rocket will have the initial disturbances along the position of the center of mass and rate of motion with start, determined by the motion of launch vehicle, by the nonuriformity of course and by the orbital tossing of its center of mass and by the vibration of carrier

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relative to the center of mass. The necessary coordinate systems are shown on Fig. 12.3. Fixed (terrestrial) coordinates let us designate $0_3 x_3 y_3 z_3$: the coordinates, rigidly connected with launch vehicle, let us designate as before $0_{1}x_{1}y_{2}z_{3}$. Let us introduce with the suffort systems of coordinates $0_{2}x_{2}y_{3}z_{3}$, $0_{2}x_{2}y_{3}z_{3}z_{3}$, axis of which is directed collinearly appropriate by the axes of basic earth-based coordinate system, and beginning let us place: 0_{2} - to the center of mass of launch vehicle; 0_{2} - to the conditional center of oscillation of launcher; 0_{20} - to the center of mass of rocket at the moment of its descent from launching rack.

The axes of the system of coordinates $O_{mx} = are turned$ relative to the system of coordinates $O_{mx} = are turned$ y_{st} , the determining vibrations of the carrier of armament relative to the center of mass. Let us introduce another coordinate system whose beginning let us place into the point O_{co} of the position of the center of mass of rocket at the moment of its descent from guides.

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EAGE



Fig. 12.3. Coordinate system for the case the stria of rocket from the stabilized launcher.

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is directed along axis of rocket with its descent from Axis Sim. guides, axis Que it is arranged in rance plane, while axis Que will supplement system to right. Since launcher is stabilized, the with the adopted assumption direction of the axes of the system of 0coxcoycozco coordinates is constant/invariable in space and range plane coincides with coordinate plane Ocercollos. The initial angle cf the line of fire and the anyle of elevation of launching racks let us designate through ψ_0 and ϕ_0 (Fig. 12.3). Without examining the disturbance/perturbation of the rocket with descent from guides, let

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us consider that initial pitch angle $a_0 = a_0$. From the center of base ground coordinates 0_3 let us conduct ground coordinates $0_{2n}x_{2n}y_{2n}x_{2n}$ of axis of which it is directed collinearly to axes $0_{0n}x_{2n}y_{2n}x_{2n}$. Purthermore, in Fig. 12.3 it is marked: \bar{r}_{m} - radius-vector, carried out from point 0_3 (conditionally selected and fixed on the Earth) to the center of mass of launch vehicle: \bar{r}_{m} - radius-vector, carried out from the center of mass of launch vehicle to the conditional center of oscillation of launcher (IskPO): \bar{r}_{1} - radius-vector, carried out from the center of oscillation of launcher into the point, occupied by the center of sass of incoded at the moment of its descent from guides. In the process of moving the carrier of armament \bar{r}_{n} it changes in module/modulus and direction, \bar{r}_{n} - changes only in the direction, \bar{T}_{1} is constant in the value (if we do not consider scattering the time of the motion of rocket along guides to the loss with them of power interaction) and does not change its direction in

sface.

Let us designate: \overline{s}_{30} - velocity vector of the center of mass of the rocket (relative to the Earth) at the moment of its descent from guides with the driving/moving carrier; \overline{v}_0 - velocity vector of the rocket at the moment of descent from guides with fixed carrier (i.e. relative to earth-based coordinate system); \overline{v}_{50} , \overline{v}_{50} , \overline{v}_{50} - projection \overline{v}_{50} on the axis of earth-tased coordinate system \overline{v}_{50} , \overline{v}_{50} . DOC = 78107117 FAGE = 96/

The deviations of the velocity of rocket, determined by the motion of carrier, will be equal to

 $\Delta v_{r_{20}} = v_{r_{20}} - v_{ci} \quad \Delta v_{r_{20}} = v_{r_{20}}, \quad \Delta v_{r_{20}} = v_{r_{20}}, \quad (12.16)$

since \overline{v}_0 it is directed along the axis $0_{ab}x_{ab}$ collinear axis . $0_{ab}x_{ab}$

On Fig. 12.3 taking into account the stipulated conditions $\bar{v}_{m} = (\bar{v}_{n} + A \bar{v}_{n}) + [\bar{v}_{n} \times (\bar{r}_{n} + \bar{r}_{1})] + \bar{v}_{n}$ (12.17)

Here \bar{p}_{R} - velocity of the undisturbed motion of the center of mass of carrier;

 $\Delta \tilde{\nu}_m$ - additive velocity of the forward motion of the center of mass of carrier, caused by the nonuniformity of its motion;

 $\tilde{w_n}$. - angular velocity vector of the rotary motion of carrier of its relatively center of mass.

Radius-vector \overline{r}_1 , as value low in comparison with a radius \overline{r}_m , subsequently consider we will not.

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It is consistent axis Or, with velocity vector the then

v. = v. Ke.

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It is expressed additional velocity AF through projections on the axis of earth-based coordinate system

 $\Delta \overline{v}_{1} = x_{3} \overline{x_{3}} + y_{2} \overline{y_{2}} + x_{3} \overline{x_{3}}$

Here \dot{x}_3 , \dot{y}_3 , \dot{z}_3 - the projection of vector $\Delta \overline{y}_1$ on the axis of earth-based coordinate system $\dot{0}_3 x_3 y_3 z_3$; \ddot{x}_3 , \ddot{y}_3 , \ddot{z}_3 - the unit vectors of ground coordinates.

Now let us rewrite (12.17)

$$\vec{x}_{30} = (\sigma_{n} + \dot{x}_{3}) \vec{x}_{3} + \dot{y}_{3} \vec{y}_{3} + \dot{z}_{3} \vec{z}_{3} + + \begin{vmatrix} \vec{x}_{n}, & \vec{y}_{n}, & \vec{z}_{n} \\ \mathbf{e}_{x n}, & \mathbf{w}_{y n}, & \mathbf{e}_{x n} \\ x_{n}, & y_{n}, & z_{n} \end{vmatrix} + \sigma_{0} \vec{x}_{30}.$$
 (12. 18)

Here ω_{xB} , ω_{yB} , ω_{zB} - projection $\overline{\omega}_{B}$ on the axis of the system of ccordinates $0_{n}x_{B}y_{B}z_{B}$; x_{B} , y_{B} , z_{B} - projection \overline{r}_{B} on the axis of the system of coordinates $0_{n}x_{B}y_{B}z_{B}$; \overline{x}_{B} , \overline{y}_{B} , \overline{z}_{B} - the corresponding unit vectors.

Utilizing cosines of the angles of transfer/transition (h_{ij}) from the axes of system $Q_0 x_3 y_3 z_3$ to axes $Q_{30} x_{30} y_{30} z_{30}$, and also the cosines of the angles of transfer/transition firm the axes of the system of coordinates $Q_{30} x_{30} y_{30} z_{30}$ to the system of coordinates $Q_{30} x_{30} y_{30} z_{30}$ (h_i), we will obtain the projections of vector equality (12.18) on the axis of coordinates $Q_{30} x_{30} y_{30} z_{30}(h_{i})$.

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After substituting (12.19) into equations (12.16), let us have

$$\Delta v_{s_{30}} = v_{s_{30}} = (v_{s_{30}} + z_{3}) n_{s_{30}} + y_{3} n_{s_{30}} + \dots;$$
(12.20)
$$\Delta v_{s_{30}} = v_{s_{30}} = (v_{s_{30}} + z_{3}) n_{s_{30}} + y_{3} n_{s_{30}} + \dots;$$
(12.20)

Complete writing of equations (12.20) at the opened values of transfer cosines () and () is sufficiently bulky.

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Following V. P. Kazakovtsev, it is simplified them [18]. In view of the smallness of the angles of the vibration of carrier Ψ_m , Ψ_m , Ψ_m screover

 $\sin \phi_{a} \approx \phi_{a}; \sin \phi_{a} \approx \phi_{a}; \sin \gamma_{a} \approx \gamma_{a}$

and

$\cos \phi_n \approx \cos \phi_n \approx \cos \gamma_n \approx 1.$

Let us disregard the products of low values $\sin \phi_{m} \sin \phi_{m} \approx 0$; and $\sin \phi_{m} \sin \phi_{m} \approx 0$ of so forth. The projecticss of the vector of the instantaneous angular velocity of carrier we take is the form

 $\bullet_{x_1} \approx \dot{\gamma}_{y_1}; \quad \bullet_{y_1} \approx \dot{\phi}_{y_1}; \quad \bullet_{y_1} \approx \dot{\phi}_{y_2}.$

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Taking into account the named assumptions let us write (12.20) in abbreviated form [18]

$$\Delta \nabla_{x_{30}} = (\nabla_n + \dot{x}_3) \cos \theta_0 \cos \phi_0 + \dot{y}_3 \sin \theta_0 + \dot{x}_3 \cos \theta_0 \sin \phi_0 + + \dot{\phi}_n (z_n \cos \theta_0 \cos \phi_0 - x_n \cos \theta_0 \sin \phi_0) + + \dot{\phi}_n (x_n \sin \theta_0 - y_n \cos \theta_0 \cos \phi_0) + + \dot{\gamma}_n (y_n \cos \theta_0 \sin \phi_0 - z_n \sin \theta_0);$$

$$\Delta \nabla_{y_{30}} = -(\nabla_n + \dot{x}_3) \sin \theta_0 \cos \phi_0 + \dot{y}_3 \cos \theta_0 - - \dot{z}_3 \sin \theta_0 \sin \phi_0 + \dot{\psi}_n (x_n \sin \theta_0 \sin \phi_0 - z_n \sin \theta_0 \cos \phi_0) + + \dot{\phi}_n (x_n \cos \theta_0 + y_n \sin \theta_0 \cos \phi_0) - - \dot{\gamma}_n (y_n \sin \theta_0 \sin \phi_0 + z_n \cos \theta_0);$$

$$\Delta \nabla_{z_{30}} = -(\nabla_n + \dot{x}_3) \sin \phi_0 + \dot{z}_3 \cos \phi_0 - \dot{\phi}_n (x_n \cos \phi_0 + + z_n \sin \phi_0) + \dot{\phi}_n y_n \sin \phi_0 + \dot{y}_n \cos \phi_0.$$

(12. 21)

in the written equations of value v_n , wo and θ_G , are constant and do not depend on time. Variable values will be: the projections of additional speed z_1, y_2, z_3 ; the angles of rotation of carrier v_n, v_n, v_n ; \wedge the angular velocities of the rotation of carrier of its relatively center of mass.

We will obtain changes in the mcdulus of velocity, angles of attack and slip, determined by the motion of carrier at the moment of the descent of rocket from the directing stabilized launcher. Let us consider that at the moment of the descent of rocket from control surface of its symmetry will be vertical and coincides with the plane of earth-based coordinate system, determined by axes Only, and Omer.

Euring the motion of carrier, the axis of rocket at the moment of its descent from guides will coincide in the direction with axes θ_{20} , π_{20} and θ_{00} , a vector a the total velocity of rocket \overline{r}_{20} will be deflected from the axis of rocket (Fig. 12.4). Line θ_{30} B - projection \overline{r}_{30} on the plane of the symmetry of rocket.

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Angle of attack is defined as angle between the projection of velocity vector on the plane of symmetry and the axis of rocket; angle of slip - as angle between vectors of speed and its projection on the plane of vertical symmetry. On Fig. 12.4 at the positive values of those composing velocities Δv_{xyy} , Δv_{yyy} , Δv_{yyy} , we will obtain a positive value of a change in the angle of attack $\Delta \alpha_{00}$. In accordance with Fig. 12.4

$$\sin \left(\Delta \beta_0 \right) = \frac{\Delta \sigma_{x \ge 0}}{\left| \overline{\sigma_{x \ge 0}} \right|},$$

either

$$\Delta \beta_{0} = \arctan \frac{\Delta v_{x_{20}}}{\sqrt{(v_{0} + \Delta v_{x_{20}})^{2} + \Delta v_{x_{20}}^{2} + \Delta v_{x_{20}}^{2}}}; \quad (12.22)$$

$$\operatorname{tg}(\Delta \alpha_{0}) = -\frac{\Delta v_{x_{20}}}{v_{0} + \Delta v_{x_{20}}},$$

CI

$$\Delta a_0 = \operatorname{arcig}\left(-\frac{\Delta \sigma_{\mu_{20}}}{a_0 + \Delta \sigma_{a_{20}}}\right). \qquad (12.23)$$

A change of the modulus of velocity of rocket at the moment of descent from guides, caused by the action of launch vehicle, we will obtain on the following dependence:

$$\Delta \mathbf{u}_{i} = \mathbf{u}_{pe} - \mathbf{u}_{e} = \sqrt{(\mathbf{u}_{e} + \Delta \mathbf{u}_{app})^{2} + \Delta \mathbf{v}_{app}^{2} + \Delta \mathbf{v}_{app}^{2}} - \mathbf{u}_{e}. \quad (12.24)$$

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§2. Notion of rockets along launching racks.

One of the most widely used trigger circuits of the rockets of low and average distance is the schematic of missile takeoff from launching rack. There are differences in the character of the motion of rocket along the guides depending on the simultaneity of the descent of the tags of rocket from guides. Let us examine first the case of moving the rocket with the simultaneous descent of tags.

Easic forces and the torque/memory, which act on the rocket during its motion along guides, shown on Fig. 12.5.

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Fig. 12.4. Changes in the angles of attack and slip, determined by the motion of carrier.

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Cn rocket acts thrust P, longitudinal X₁ arc normal Y₁ aerodynamic forces, longitudinal X_{rl} and normal Y_{rl} gas-dynamic forces, gravitational force Mg, normal reactions F_{Nl} , F_{N2} and frictional force

 $F_{\tau l}, F_{\tau 2}$ in the tags. On figure is also shown the starting system of coordinates $O_{\sigma} x_{\sigma} y_{\sigma}$ and the system of coordinates $O_{\sigma} x_{\sigma} y_{\sigma}$. The beginning of this coordinate system (point O_{m}) coincides with the initial position of the center of mass of rocket prior to start. Axis coincides with axis of rocket and to parallel guides: axis $O_{\sigma} y_{\pi}$ is perpendicular to axis $O_{\sigma} x_{\pi}$ and lie/rests at the longitudinal plane of the symmetry of rocket.

Will write the equations of motion of rocket along the guides in

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the system of coordinates $0_{ax_{ay_{m}}}$ which we we consider fixed; in this case, let us consider launcher absclutely rigic body with rectilinear guides. Also let us consider that the clearances between the tags of recket and guides are equal to zero. Then

 $m \frac{dx_n}{dt} = P - X_1 + X_m - F_m - F_m - mg \sin \theta_i;$ $m \frac{d^{2}y_{n}}{dt^{2}} = Y_{1} + Y_{r_{1}} + F_{N_{1}} + F_{N_{2}} - mg\cos\theta_{0} = 0; \qquad (12.25)$ $J_{s_{1}} \frac{d\theta_{s_{1}}}{dt} = M_{s_{2}} + M_{rs_{1}} + F_{rs_{2}} - F_{rs_{2$

Equality zero second and third equaticas of system (12.25) is caused by the introduced assumptions. In this form these equations make it possible to determine power load or the tage of rocket.

Buring the motion of rocket along the open guides of the value of axial aerodynamic and gas-dynamic forces, are low in comparison with thrust. So are small frictional forces. Therefore during the calculation of the undisturted motion of rocket along launching macks, it is possible these forces to disregard. The second and third equations of system (12.25) are actually the equations of relation, but not by equations of motion.

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Fig. 12.5. Circuit of the forces, which act on the rocket during its solution along guides.

Key: (1). c.m.

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Hence it follows that instead of the system of equations (12.25) it is possible to be restricted to one equation of the axial motion of rccket along guides

$$\frac{d^2 x_{\rm m}}{dt^2} = \frac{P}{m} - g \sin \theta_{\rm s}.$$
 (12.26)

Integrating this equation from t=0 to the time of the descent of rocket t_2 , we will obtain the parameters of the undisturbed motion of rocket along guides. For integration it is recessary to know the thrust and mass of rocket in function from time. The time of the descent of rocket t_2 is determined in the process of integration, when the instantaneous value of coordinate x_n becomes equal to the

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length of guides l_{H} . At this same torgue/meant determine the velocity of the motions of the rocket which will be the initial velocity for the powered flight of the rocket

$$v_0 = x_{n_{(l-1_0)}}$$
. (12.27)

Frequently the numerical integration of equation (12.26) is replaced by the analytical solution which is instituted on the introduction of any supplementary assumptions. Let us examine one of most widely used analytical solutions [18].

It is known that during motion along guides the rocket expend/consumes the small mass of fuel/propellant, in this case, the time of motion along guides is sufficiently small. Consequently, the mass of rocket can be taken by constant, equal to its average value on the section of the motion of rocket along guides (m_{op}). A change in the thrust in time is shown on Fig. to 12.6, from which it is evident that during the motion of rocket along directing force of thrust on initial section increases to value P_{on} and ther it remains constant. For analytical solution let us replace the curve P(t) on the initial section of straight line:

Here 4 - time of the activation of the motor.

On the second section the value of thrust will be constant

 $t_{\mathbf{y}} \leq t \leq t_{\mathbf{y}} \cdot P(t) = P_{\mathbf{y}} = \text{const.}$ (12.29)

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Let us examine the motion of rccket on the first section of its displacement over guides. Equation of motion taking into account (12.28) takes the following form:

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Fig. 12.6. Curve/graph of a change in the thrust during the motion of rocket along guides.

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Let us integrate twice equation (12.30), remembering that with t=0 $x_n=v=0$, $x_n=0$. Then we are have

$$v = \dot{x}_{n} = \frac{P_{0}}{2m_{cp}t_{p}} t^{a} - g \sin \theta_{q} t^{a}, \qquad (12.31)$$
$$x_{n} = \frac{P_{0}}{6m_{cp}t_{p}} t^{a} - \frac{1}{2} g \sin \theta_{q} t^{a}. \qquad (12.32)$$

Let us substitute into equations (12.31) and (12.32) value $t=t_p$; then we will obtain dependences for determining of path and velocity at the end of the initial section of the motion of the rocket

$$v_{p} = \left(\frac{P_{0}}{2m_{cp}} - g \sin \theta_{0}\right) t_{p}; \qquad (12.33)$$
$$x_{n,p} = \left(\frac{P_{0}}{6m_{cp}} - \frac{1}{2} g \sin \theta_{0}\right) t_{p}^{2}. \qquad (12.34)$$

These values will be initial data for the calculation of the second section of the motion of rocket alcng guides. The equation of motion of the rocket in this case, accordingly (12.39), takes the following form:

 $\frac{d^2 x_g}{dt^2} = \frac{dv}{dt} = \frac{P_0}{m_{ep}} - g\sin\theta_p. \qquad (12.35)$

Let us integrate twice equation (12.35)

$$v = x_{u} = v_{p} + \left(\frac{r_{0}}{m_{up}} - g\sin\theta_{0}\right)(t - t_{p}); \quad (12.36)$$

$$x_{u} = x_{u,p} + v_{p}(t - t_{p}) + \frac{1}{2}\left(\frac{\mu_{0}}{m_{up}} - g\sin\theta_{0}\right)(t - t_{p}); \quad (12.37)$$

For the torque/moment of the descent of rocket from guides $l=l_3, l_4=0$ and $n=l_6$. Consequently, it is possible to write

$$v_{0} = v_{p} + \left(\frac{P_{0}}{m_{ep}} - g\sin^{2}\theta_{0}\right) \phi_{0} - t_{p}; \qquad (12.38)$$

$$l_{n} = x_{n,p} + v_{p} (t_{0} - t_{p}) + \frac{1}{2} \left(\frac{P_{0}}{m_{ep}} - g\sin^{2}\theta_{0}\right) (t_{0} - t_{p})^{2}, \qquad (12.39)$$

Let us re-group equation (12.3) relative to the time of the motion of rocket along guides to descent. As a result of simple conversions, we will obtain

$$t_{2} = \frac{1}{\frac{P_{0}}{m_{ep}} - g \sin \theta_{0}} \times \left[\sqrt{v_{p}^{2} + 2(l_{o} - x_{a,p}) \left(\frac{P_{0}}{m_{ep}} - g \sin \theta_{0} \right) - v_{p}} \right] + l_{g} (12,40)$$

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Obtained equations (12.33), (12.34), (12.38) and (12.40) make it ressible to expect velocity of rocket v_{p_1} rath x_{m_2} the time of motion along the guides t_2 , and also velocity of the rocket with descent from the guides v_0 .

Let us examine now the motion of rocket along guides with the

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nensimultaneous descent of the tage of rocket. In this case it is necessary to distinguish two sections of motion.

Birst section - motion of rocket to the torque/moment of the descent of the first tag from guides. The motion of rocket on this section in no way differs from the motion of rocket in the first case. Therefore the equations, obtained above, are completely suitable for the calculation of the first section. The time of the motion of rocket along by guides to the descent of the first tag let us designate t₁.

The second section (schetimes it they call the section of cutput) motion it begins after the first and terminates at the moment of the descent of the second tag of rocket from guides. This section is characterized by the fact that the rocket obtains supplementary degrees of freedom, namely - to swivel feature around rear tag.

Basic forces and the tcrque/mcments, which act on rocket on the second section of its motion along guides (for conditional rotation counterclockwise) are shown on Fig. to 12.7. Let us comprise the equations of motion of the rocket relative to the axes of the system of coordinates $O_n x_m y_n$, in this case let us consider guides fixed rigid body. Let us accept also, that $F=F_n$. Then

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 $m_{X_{R}} = P_{0} \cos \Delta \theta - X_{1} \cos \Delta \theta - Y_{1} \sin \Delta \theta - - -F_{v_{R}} - mg \sin \theta_{0};$
 $m_{Y_{R}} = P_{0} \sin \Delta \theta - X_{1} \sin \Delta \theta + Y_{1} \cos \Delta \theta + + F_{MR} - mg \cos \theta_{0};$
 $J_{z_{1}} \frac{d\omega_{z_{1}}}{dt} = M_{e_{T}} - F_{M} f_{z} \cos \Delta \theta - F_{v_{R}} \frac{d}{\theta} \cos \Delta \theta + \frac{1}{\theta} + \frac{1}{2} \int_{0}^{\infty} \frac{d\omega_{z_{1}}}{dt} = M_{e_{T}} - F_{M} f_{z} \cos \Delta \theta + \frac{1}{\theta} \int_{0}^{\infty} \frac{d\omega_{z_{1}}}{dt} = M_{e_{T}} - \frac{1}{2} \int_{0}^{\infty} \frac{d\omega_{z_{1}}}{dt} = \frac{1}{2} \int_{0}^{\infty} \frac{d\omega_{z_{1}}}}{dt} = \frac{1}{2} \int_{$

Fig. 12.7. Circuit of the forces, which act cn rocket on the section of descent from guides.

Kej: (1). c.m.

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To these equations it is necessary to add the equation of relation, caused by the displacement of the certer of mass of the rocket with its turning on the second teg

 $y_{\mathbf{z}} = l_{\mathbf{z}} \Delta \mathbf{\hat{e}}. \tag{12.42}$

The studies of the motion of rocket on the section of output shawed that for always of motion the value of angle AD does not exceed ten angular minutes. Therefore it is possible for trigonometric functions to accept the following values: $\cos \Delta^0 = 1; \sin \Delta^0 = \Delta^0.$

It is simplified the third equation of system (12.41) on the

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basis of the fact that the angle A8 is low; for rockets usually $L_{2,2}$ 10d and force $F_{2,2}$ where the coefficient of the friction of steel in steel fx0.2. We will obtain

$$J_{s_{1}} \frac{ds_{s_{1}}}{dt} = M_{s_{1}} - F_{N_{s}} \left[l_{3} - \frac{d}{2} \Delta \theta + f \frac{d}{2} + f l_{3} \Delta \theta \right] = M_{s_{1}} - F_{N} d_{3},$$
(12.43)

since three last/latter terms in bracket for the values of f indicated, d and $\Delta \vartheta$ in comparison with \mathbf{I}_2 prove to be values low and them can be disregarded. The velocity of the motion of rocket along guides does not usually exceed 70 m/s. In this case aerodynamic forces and torque/moments are obtained by sufficiently low, and them in equations (12.41) can be disregarded.

Taking into account entire afcresaid alove the system of equations (12.41) is led to the following simplified form:

$$m\ddot{x}_{\mu} = P_{\phi} - mg \sin \theta_{c};$$

$$m\ddot{y}_{\mu} = P_{\phi}\phi - mg \cos \theta_{\phi} + F_{mp};$$

$$J_{\mu}\phi = -I_{p}F_{Mp}.$$
(12.46)

Here for convenience in the recording, is marked $\Delta \theta = \phi$.

In that obtained of systems, the first equation connected with the second and third equations. Consequently, it it is possible to integrate separately. This equation coincides in form with (12.26). Integration of equation gives the velocity of the motion of the center of mass of rocket along the guides is the function of time. Fhysical sense of the possibility of integration separately of the

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first equation of system (12.44) censists is the fact that the lateral divergences barely affect the velocity of motion.

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Us interests the rotation of the rocket during its motion on the section of output. Therefore let us examine the second and third equations of system (12.44). Let us introduce replacement $y_1 = h_0$, let us re-group the second equation of system relatively F_{M2} let us substitute the obtained value into the third equation. Taking into account the equation of relation (12.42) we will obtain

 $(J_{s_1} + ml_2)\ddot{\varphi} = P_{d_2} - mgl_2 \cos \theta_0.$ (12.45)

let us introduce the designations

 $J'_{z_1} = J_{z_1} + ml_{z_1}^{a_1} + \frac{p_{d_2}}{J'_{z_1}} = a_1.$

Let us substitute them into equation (12.45) and after conversion we will obtain

$$\Psi - a_1 \Psi = -\frac{mgl_2 \cos \theta_0}{J_{g_1}}$$
 (12.46)

Buring the integration of equation (12.46) the mass of the rocket can be taken by average value on the section of conclusion. In this case the right side of equation (12.46) is a constant value, and equation itself - by linear beterogeneous with constant coefficients. This equation it is possible to integrate in analytical form. Without giving intermediate unpacking/facings, let us write the obtained results of the integration COC = 78107117 PIGE 259 978

 $\Delta \theta_{0} = \eta_{0} = -\frac{m_{opf}\cos\theta_{0}}{2P_{0}} \left[e^{\sqrt{a_{1}}(t_{0}-t_{1})} + e^{-\sqrt{a_{1}}(t_{0}-t_{1})} - 2 \right]; \quad (12.47)$ $b_{0} = \phi_{0} = -\frac{m_{eq}g\cos b_{0}}{2} \sqrt{\frac{b_{1}}{m_{e}}} \left[e^{\sqrt{a_{1}} (b_{e} - c_{1})} - e^{-\sqrt{a_{1}} (b_{e} - c_{1})} \right], (12.48)$

where $t_2 - time$ of the descent of the second tag from launching rack;

 $\Delta \vartheta_0$ - angle of rotation of the axis of rocket at the end of the section of output;

 δ_0 - angular velocity cf the axis cf rccket at the end of the section of output.

The constructions of the starting/laurching moved settings up of ground-based rockets are different. They can have guides of framework construction in the form of the shared leass of tubular type, etc.

Launchers for the rcckets, started from aircraft, usually have either tubular guide or simply light/lurg farm/trusses for the sumpension of rockets.

Let us examine the basic special feature/peculiarities of the action of rockets at the actent of the start when there is interaction between the rocket and the starter.

§3. Motion of the rockets during laurching/starting from launching

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rad and from shaft/mine.

The isolation/evolution of rocket from laurching pad occurs as instantly at that torque/mcment when thrust begins to become more than the weight of rocket.

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For the calculation of further trajectory of the motion of rocket, it is necessary to calculate the parameters of the rocket with its treakaway from starting table. In essence this is related to the determination of the initial mass of rocket \mathbf{r}_0 , since the part of the fuel/propellant will be spent to the engine operation from firing point to breakaway torque of rocket of launching pad (i.e. for time t_{er}).

The mass of rocket we define, as refore cr the formula: $m_0 = m_{er} - \int_{0}^{t_{er}} |\dot{m}|(t) dt$

where m_{cr} - a mass of recket prior to the beginning of ignition;

(t) - the consumption of mass, which in the process of the launching of rocket is the value of variable depending on time.

Thus, for the calculation of the powered flight trajectory of

DOC = 78107117 FIGE 25 980

tallistic missile as the initial parameters we have $v_0 = x_0 = y_0 = 0; \quad 0, = 0_{and} = 90^\circ.$

Let us examine now the motion of the rocket on the launching phase during starting/laurching from shaft/mine.

Bigures 12.8 shows the flat/plane schematic of the action basic forces and torque/acments during the action of rocket from shaftymine. From the figure one can see that besides the thrust P and of the weight of rocket Hg on rocket act even gas-dynamic forces Xri, Yri and torque/moment Mrs. Gas-dynamic forces and torque/moments appear because of the limited volume of shaft/mine. The gases, which are formed with the combustion of fuel/propellant, escape/ensue from engine nozzle and fall into the limited space. In spite of gas-bleeding channels in this space is forsed the zone of elevated pressure. Part of the gas bursts open between the walls of rocket and shaft/mine, affecting side walls of rccket. Space after the bottom of recket because of the action of rocket always changes. In connection with this they change and the parameters of gas, which is located teyond the bottom of rocket, therefore, continuously change also gas-dynamic forces and the torque/scments, which act on rocket. The effect of gas-dynamic forces and torque/moments on rocket ceases only after its output from shaft/mine.



Fig. 12.8. Circuit of the action of forces on the rocket with start from shaft/mine.

Key: (1). center of gravity.

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The calculation of gas-dynamic forces and torque/moments is extremely hinder/hampered due to the unsteady character of a change of the parameters of gas in shaft/mire. Aerodynamic forces and torque/moments in this case are virtually alsept, since the velocity of the rocket during motion in shaft/mire and cutput from shaft/mine

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 FIGE 337982

dces act exceed several ten meter« per second. The perceptible effect of aerodynamic forces and torque/moments on the motion of rocket appears only at the velocities more than 50-70 m/s. Therefore for the calculation of motion rocket in the period of cutput from shaft/mine aerodynamic forces and torque/moments it is possible not to consider.

According to the schematic of the action of forces and torque/moments, let us comprise the equations of motion of rocket in the process of its output from shaft/mine. The equations of motion of the center of mass let us write in projections on the axis of the starting coordinate system. During the axisymmetric construction of rocket, the problem can be solved in flat/plane setting and system of equations will take the following form:

$$my = (P + X_{P1}) \sin \vartheta + Y_{P1} \cos \vartheta - mg;$$

$$mx = (P + X_{P1}) \cos \vartheta - Y_{P1} \sin \vartheta;$$

$$J_{z_{1}} \dot{\omega}_{z_{1}} = M_{r} z_{1} + M_{P} z_{1};$$

$$\vartheta = \omega_{z_{1}}; \quad m = m_{cT} - \int_{\vartheta}^{t} |\dot{m}|(\ell) d\ell;$$

$$\vartheta_{TD} = 90^{\circ}; \quad \nabla = \sqrt{x^{2} + y^{2}}.$$

(12.49)

For the calculation of the garameters of the motion of rocket, the system of equations (12.49) is integrated to that torque/moment until rocket leaves the shaft/mine and to it will cease to act gas flow. As a result of integration, we obtain the initial parameters of motion for a powered flight trajectory: $v_0 \neq 0$: $x_0 = y_0 = 0$: ϑ_0 , and also

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value w_0 ; in this case, one should consider the fact that during motion in shaft because of cas-dynamic forces and torque/moments the rocket will be deflect/diverted along the pitch angles, yaw and bank.

§4. Supplementary factors, which determine the initial conditions of shot.

Buring the motion of rocket along guides, the gas flow, coming out from nozzle units, washes the parts of launcher. In this case, is exhibited the supplementary action of gases on launcher and rocket. Various kinds deflectors and reflectors can be the reason for the appearance of secondary gas flows, which frequently act on rocket unsymmetrically.

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As a result of the elastic properties of rocket, of launcher and scil, the initial conditions of moving the rocket differ from the calculated. The theoretical calculation of gas-dynamic forces and torque/moments, which act on the rocket, driving/moving along launching racks, is frequertly difficult. It is difficult to also theoretically describe the vibrations of launcher and rocket during their interaction during start. The diversity of initial conditions with shot is one of the reasons, which determine scattering the
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unguided rockets. The experimental study of the dynamics of start makes it possible to determine the initial conditions of shot and to introduce corrections into calculation data.

With shot from artillery instrument as a result of the elastic properties of shaft, gun carriage and deformation of the soil of the firing position, the initial conditions of shot also differ somewhat from the conditions, establish/installed with aiming. On leaving of projectile from bore the unsymmetric action of cas flows on the outgoing projectile. Of succth-bore systems the unsymmetric action of gas flow on tail assembly with initial angle A8 can lead to the considerable scattering of angular initial conditions (Fig. 12.9).

As a result of the vibrations of weapon and deformation of the soil of the firing position, the argle of elevation of instrument ϕ_0 , establish/installed with aising, does not coincide with real angle of departure - θ_0 .

An angular difference $\gamma = \theta_0 - \phi$ is called argle of jump which can be both the positive and negative. The numerical value of angle of jump for this instrument depends on many factors: the elastic properties of shaft and gur carriage, soil of the firing position, tallistics of instrument, which determines power loads, the angle of elevation, rate of fire, etc. The theoretical determination of angle

of jump causes great difficulties; therefore as a rule, angle of jump it is determined experimentally [59]. at a distance of x from the mussle end face of target adjustable gamel.





Fig. 12.9. Change of the initial argle of departure in the period due to games.

Fig. 12.10. Schematic of determination of argle of jump with firing from ganel.

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Instrument is aimed on the cross lines, adjustable on muzzle and treech faces, into the mark, plotted/applied on panel so that the shaft before the shot would have (on quadrant) an angle of elevation ϕ_0 (Fig. 12.10).

As a result of the named reasons the projectile will fly not in the direction OA, but at the initial memont it will be deflected from OA by angle of jump γ . With small x it is possible not to consider the effect of air resistance on a decrease in the projectile the hearth of the lines of shot and to determine decrease according to ECC = 78107117 PAGE 35 986

formula $gt^2/2$. Then, on the basis of Fig. 12.10, we obtain

$$\gamma = \arctan\left[\frac{1}{x}\left(y + \frac{x^{\alpha}}{2}\right)\right] - \gamma_{0}. \qquad (12.50)$$

Time t can be determined either experimentally or approximately on the known initial velocity of projectile v_0 .

$$t \approx \frac{x}{v_0 \cos t_0}$$
.

At the zero angle of the increase

$$\gamma = \arctan\left(\frac{y}{x} + \frac{gx}{2v_0^2}\right). \tag{12.51}$$

The determination of the mean statistical value of angle of jump requires repeated firings. For determining the angle of jump of rocket systems, are conducted so-called "rocking testings" during which it is simulated the motion of rocket elong guides also on the initial trajectory phase.

In conclusion of chapter, let us note that the study of the dynamics of the start of concrete/specific/actual rocket system, just as the initial conditions of shot from artillery piece, it represents by itself complex composite problem. DOC = 78107117 FAGE -29 987

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Charter XIII.

Errors for firing, missile dispersion and of projectiles.

The errors of firing it is possible to divide into rough miscalculations and errors, constant or systematic errors and errors random. Rough miscalculations are the consequence of poor training/preparation of which operates system calculation and can be removed by the increase of skill and combat training.

The source of systematic errors acts or all shots equally. For example, longitudinal constant tailwird increases firing distance against calculated, and contrary decreases. To systematic errors can te attributed also the errors in the determination of the coordinates of target, etc. If we by the methods of the theory of corrections consider in calculations also during the preparation of firing the effect of the corresponding factor, systematic errors can be, if are not excluded entirely, then to a considerable degree are decreased. It is necessary to reveal/detect the reasons for systematic errors and to consider them during the preparation of firing. With the firing the rockets, prepared according to some drawings and technical

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specifications, from the guides of one and the same launcher, from one launching site, during one and the same settings up of the sight mechanisms of missile trajectory they will not coincide with each other. With the firing identical projectiles from barrel artillery piece during identical charges and the adjustments of sight of the trajectory of separate projectiles, also they do not coincide.

The noted phenomenon calls scattering trajectories. It is explained by the effect on rocket flight and projectiles of the random errors, which are the consequence of the combination of random changes in the separate values.

Entire/all rocket (or projectile) in collection, its separate structural/design units and parts are made with the appropriate tolerances in size, the weight and other parameters. Change of different values within the limits of allowances and diversity in the effect of weather factors which it is difficult to consider as systematic error, and is the source of the random errors, which lead to scattering of trajectories and impact points in the rockets (projectiles).

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The group of the trajectories, which correspond to identical

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initial data, is called the heam (cr sheaf) of trajectories (Fig. 13.1).

With firing at ground targets, as shown in Fig. 13.1, the deviation of impact point from target/purpose is determined by ccordinates x and z.

The characteristics of scattering are determined with firing at horizontal or vertical place, and scattering itself is called scattering on plane.

With firing at the flying target/purposes with the so-called remote projectiles with the proximity fuses or surface-to-air missiles, the places of the explosions of warheads are arrange/located in certain space around the calculation predicted collision point, i.e., occurs volumetric scattering (Fig. 13.2). During volumetric scattering the points of discontinuity is determined by coordinates X, y, z. In this case the coordinate plane, on which are determined coordinates y and z, is conducted through the predicted collision projectile for target/purpose, it is perpendicular to the velocity vector of the center of mass of target/purpose in the predicted calculation collision point. Coordinate x is determined in the sense of the vector of the velocity of the center of mass of target/purpose.



Fig. 13.1. Sheaf of fire.

Rey: (1). Sheaf of fire. (2). Flace of start. (3). Target.

Fig. 13.2. Three-dimensional/space scattering with firing remote projectiles.

Key: (1). Plane of figure. (2). Fredicted collision point.

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The random errors, which determine the scatter of points of an incidence/drop in the projectiles or plane or volumetric scattering with time fire at air or underwater targets, appear without the determined order, the concrete/specific/actual reasons for their DOC = 78107117 FAGE # 991

appearance are known sometimes only qualitatively, and sometimes also are entirely unknown. Ic judge the accuracy of firing at the data of any shot would be incorrectly, since same this result was accidental.

For evaluating the accuracy of firing, are utilized the characteristics, which reflect the properties of random variables and random functions and lass, by which they are subordinated. The mathematical basis of the determination of scattering trajectories and of the points of the operation of varheads (on plane and in space, on coordinates and on time) are the protability theory, the theory of errors and mathematical statistics.

The flight of projectile, examine/considered taking into account the random disturbances, initial and which act in the process of motion, can be considered as stochastic process, described by the random function, concrete/specific/actual realization of which is this trajectory. The study of the random character of missile trajectories and of projectiles is based on the special section of the probability theory - theory of random functions or, otherwise, the theory of random or stochastic processs. The serious obstructions of theoretical and calculated order are encountered during the stochastic investigations of the trajectories of the guided missiles and of projectiles, whose number of factors, which affect the deviation of motion characteristics from the calculated (ideal), is especially great. EQC = 78107117

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In practice during the study of the scatter of points of the rendervous of the rocket (projectile) with flat/plane tarrier/obstacle or the volumetric scatter of points of the operation of remote projectiles in space for simplification in the solution of problem utilizes the sections of the probability theory, which examine random variables.

The region of the stochastic studies of scattering trajectories and of the isolated points of the operation of the warheads of the rockets, projectiles, min and aircraft homes is extremely vast. Many guestions require the detailed specialized investigations. In connection with firing stochastic problems can be divided into two large groups.

The first group of problems is characteristic for stages of the design of means armament, their firal adjustment and testing. One of the basic problems of the first group is study of the action of the perturbation factors and calculated determining of the expected characteristics of scattering, accuracy of firing and, in the final analysis, the effectiveness of the action of the design/projected means armament.

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The second group covers the problems, connected directly with the use of the already available means armament. Are this involved, for example, the development of rules and methods of use combat of technology, the determination of the consumption of the resources armament for accomplishing of one or the other tactical mission.

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These questions usually are examined in independent courses "Theory of firing", the "Theory of tombing", etc.

Both named groups of stochastic investigations are tightly interdennected, at their tasis lie/rest the mathematical methods of the theory of probablity and the generalized statistical experiment in the form of the characteristics of scattering and accuracy of the work of articles as a whole (rockets, projectiles, etc.) or the separate structural/design units, close in designation/purpose and construction (autopilots, gyroscopes, acceleration pickups, etc.). The combination of mathematical investigations with statistical processing of experimental information was called experimental-theoretical methods of the evaluation of the characteristics of the missile dispersion and projectiles. Experimental-theoretical methods, used during design, are divided on the sc-called analytical methods and the methods of statistical COC = 78107117

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testings. The realization of the latter practical is possible only during use EVM [computer]; these methods in ballistic practice were called the conditional the methods of mathematical or electronic firing.

§1. Analytical methods of the evaluation of the characteristics of scattering.

Determining the characteristics of scattering for the newly design/projected systems of rocket or artillery armament only possible experimental-theoretical method. Let us examine one of the varieties of this method - analytical method.

For using this method, it is necessary to know well construction of the design/projected system, technology of its manufacture and physical nature of the perturbation random factors. Only in this case the effect of each of the random factors in question can be determined by calculation. Let us examine for an example determining the characteristics of scattering for any point in the trajectory, for example, for the point, which corresponds to the end/lead of the operation of the engine of target/purpose. Trajectory elements at the peints indicated will be random variables. For example, the distance x_c of the firing projectile from harrel artillery piece can EQC = 78107117 PAGE ## 995

te represented as function of random variables v_0 , θ_0 , m_0 , c and of sc forth.

Let us examine randes variable A, which can be presented as function of several random arguments in b.....in

$$f(\mathbf{i}_1, \mathbf{i}_2, \dots, \mathbf{i}_n). \tag{13.1}$$

It is decomposed (13.1) in Taylor series in the vicinity of the value of function A, which corresponds to the mathematical expectations of arguments $m_{i_1}, m_{i_2}, \dots, m_{i_n}$ after preserving in expansion the members not higher than second order.

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Then stochastic dependence (13.1), adhering to the designations of formula (8.9), can be preserted in this form:

$$A = f(m_{l_1}, m_{l_2}, \dots, m_{l_n}) + \sum_{l=1}^n \left(\frac{\partial f}{\partial l_l}\right)_m \mathbf{k}_l + \frac{1}{2} \sum_{l=1}^n \left(\frac{\partial^2 f}{\partial l_l^2}\right)_m (\mathbf{k}_l)^2 + \sum_{l=1}^n \left(\frac{\partial^2 f}{\partial l_l}\right)_m (\mathbf{k}_l)^2 + \sum_{l=1}^n \left(\frac{\partial^2 f}{\partial$$

where a_i and a_j . - in the case in question the contered random variables:

$\delta t_i = t_i - m_{t_i}$ $k_j = t_j - m_{t_j}$

Partial derivatives in (13.2) during the calculation of the **missile dispersion** and projectiles correspond to the ballistic

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derivatives the procedures of determination of which were examined in Chapter XI. Index m shows that during the calculation of the values of derivatives the arguments ξ_i must be undertaken equal to their mathematical expectations m_{ξ_i} .

Applying the general methods of determining the numerical characteristics of the distribution of the function of random arguments, let us write [10]

 $m_A = f(m_{i_1}, m_{i_2}, \dots, m_{i_n}) + \frac{1}{2} \sum_{i=1}^n \left(\frac{m_i}{M_i}\right) D_{i_1}$

where $D_{ij} = \sum_{i=1}^{n} A_{ij} - A_{ij} = dispersion of random argument =$ $<math>K_{ij} = \sum_{i=1}^{n} \sum_{j=1}^{n} A_{ij} - A_{ij} = covariance of the ir pairs undertaken$ arguments = and = an

In the simpler case when have are not correlated, the preceding/previous formula will take the form

For function many alternating/variable A the formula, which makes it possible to determine its dispersion D_A , is obtained complex and its practical application/use encounters the great difficulties of theoretical and calculation orders.

FIGE # 997 DOC = 78107117

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Forsula considerably is simplified, if one assumes that arguments **b**, **b**... **b** are functionally independent and pot correlated. Then

$$D_{A} = \sum_{l=1}^{n} \left(\frac{\partial f}{\partial \xi_{l}}\right)_{m}^{2} D_{\xi_{l}} + \frac{1}{4} \sum_{l=1}^{n} \left(\frac{\partial f}{\partial \xi_{l}^{2}}\right)_{m}^{2} \left(\mu_{A}\left[\xi_{l}\right] - D_{\xi_{l}}^{2}\right) + \\ + \sum_{l < l} \left(\frac{\partial^{2} f}{\partial \xi_{l} \partial \xi_{l}}\right)_{m}^{2} D_{\xi_{l}} D_{\xi_{l}} + \sum_{l=1}^{n} \left(\frac{\partial f}{\partial \xi_{l}}\right)_{m} \left(\frac{\partial^{2} f}{\partial \xi_{l}^{2}}\right)_{m} \mu_{B}\left[\xi_{l}\right], \quad (13.5)$$

where $\mathbf{m}_{i}[\mathbf{k}] = \sum_{i=1}^{n} p_i (\mathbf{k}_i - m_{\mathbf{k}_i})^n$ and $\mathbf{m}_{i}[\mathbf{k}] = \sum_{i=1}^{n} p_i (\mathbf{k}_i - m_{\mathbf{k}_i})^n$. - the respectively third and fourth central moments; $\sum_{i < j}$ - sign, which means that is conducted the summation of all possible pairwise combinations of random variables \mathbf{k}_i .

For the normal law of the distribution

 $\mu_{p}[i_{t}] = 0; \quad \mu_{e}[i_{t}] = 3\sigma_{t}^{4} = 3D_{t}^{4}$

and then formula (13.5) is converted as follows:

$$D_{A} = \sum_{t=1}^{n} \left(\frac{\partial f}{\partial t_{i}}\right)_{m}^{2} D_{t_{i}} + \frac{1}{2} \sum_{t=1}^{n} \left(\frac{\partial f}{\partial t_{i}}\right)_{m}^{2} D_{t_{i}}^{2} + \sum_{t < f} \left(\frac{\partial^{2} f}{\partial t_{i} \partial t_{i}}\right)_{m}^{2} D_{t_{i}} D_{t_{i}},$$
(13.6)

As is evident, formulas (13.3) and (13.5) are very complex: simplified formulas (13.4) and (13.6) during their practical use also will lead to bulky and complex calculations. Therefore during determining of the characteristics of scattering at the stage of

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tallistic designed calculations, they are limited usually in the formula of expansion (8.9) only by linear terms of a series. Then random function (13.1) can be represented in this form:



Applying the general methods of determining the numerical characteristics of distribution for linear functions, we will obtain for mathematical expectation and the dispersion of random function λ the following expressions:



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Transfer/converting to root-mean-square deviation, we will obtain

$$\sigma_A^2 = \sum_{l=1}^n \left(\frac{\partial f}{\partial t_l}\right)_m^2 \sigma_{t_l}^2 + 2 \sum_{l < l} \left(\frac{\partial f}{\partial t_l}\right) \left(\frac{\partial f}{\partial t_l}\right)_m r_l \sigma_{t_l} \sigma_{t_l}, \quad (13.9)$$

where r_{ij} - a coefficient of correlation of value ξ_i and ξ_i

$$r_{ij} = \frac{K_{ij}}{a_{l_i} a_{l_j}}.$$

Galculations according to formula (13.4) can be carried out, if are known the particular derived $\left(\frac{\partial f}{\partial t_i}\right)_m$ and numerical characteristics of distribution for the arguments of the stochastic system:

mathematical expectations $m_{i_1}, m_{i_2}, \dots m_{i_n}$ and the correlation matrix/die

$$\|K_{ij}\| = \begin{vmatrix} k_{11} & k_{22} & \dots & k_{2n} \\ & k_{22} & \dots & k_{2n} \\ & & \ddots & \ddots & \ddots \\ & & & k_{nn} \end{vmatrix}.$$

recall that the cell/elements of the correlation matrix/die, arrange/located along principal diagonal, they represent by itself to the dispersion of arguments. For calculations according to formula (13.9) it is necessary to have, besides a_{i_i} and a_{i_j} , the standardized/normalized correlation matrix/die, which consists of the coefficients of correlation r_{i_i} .

If random arguments are not correlated, then all cell/elements of correlation matrix/die, except diagonal, are equal to zero

 $(k_{ij}=0$ with $i\neq j$) and then from formulas (13.8) and (13.9) let us have

$$D_A = \sum_{l=1}^{n} \left(\frac{\partial f}{\partial \mathbf{t}_l} \right)_m^2 D_{\mathbf{t}_l}, \qquad (13.10)$$

CE

$$\mathbf{e}_{A}^{2} = \sum_{l=1}^{n} \left(\frac{\partial f}{\partial \mathbf{t}_{l}} \right)_{m}^{2} \mathbf{e}_{\mathbf{t}_{l}}^{2}. \tag{18.11}$$

It is necessary to keep in mind that in the examined method of determining the numerical characteristics of scattering the functions of random variables are not establish/installed the laws of their distribution. For the determination of the laws of the distribution of the functions of random arguments - the density of distribution and of distribution function - are required special LOC = 78107118 FAGE 100/

experimental-theoretical studies.

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As indicate many theoretical studies and statistical testings, total scattering of artillery shells it is subcriminated to the normal law of distribution; scattering the determining parameters (v_0 , θ_0 and c) is also subordinated to the normal law of distribution. This means that the normal law of distribution property of statility according to which it is possible to expect that the function, obtained as a result of the combinations of the random arguments, which are subcriminated to the normal law of distribution, will be distributed also according to normal law.

Let us assume we have the function of n-independent random variables, distributed according to the normal law

$$A = f(\mathbf{t}_1, \mathbf{t}_2, \dots, \mathbf{t}_d).$$

It is chvious, the mathematical expectation of value A will be equal

$$m_{\lambda} = \sum_{l=1}^{n} A_{l} \rho_{l}.$$
 (13. 12)

In conformity with the property of stalility, let us have for A the normal law of distribution with the roct-mean-square deviation,

determined with respect to formula 13.11,

$$f(A) = \frac{1}{\bullet_A V =} e^{-\frac{(A-\bullet_A)^*}{2\bullet_A^*}}.$$

For the normal law of distribution, the roct-mean-square deviation is connected with the middle error E through the constant factor

 $E = 0,6745 \sigma$ (13.13)

and then

$$\boldsymbol{E}_{A}^{\mathbf{s}} = \sum_{i=1}^{s} \left(\frac{\partial f}{\partial t_{i}} \right)_{m}^{s} \boldsymbol{E}_{i_{i}}^{t}$$
(13. 14)

If we designate mean deviation, which characterizes that comprise of the complete scattering of function A because of scattering of random factor &, through

$$\boldsymbol{\boldsymbol{\mathcal{E}}}_{\boldsymbol{A}_{j}} = \left(\begin{array}{c} \boldsymbol{\boldsymbol{\mathcal{A}}}_{j} \\ \boldsymbol{\boldsymbol{\mathcal{K}}}_{i} \end{array} \right) \boldsymbol{\boldsymbol{\mathcal{E}}}_{\boldsymbol{\mathbf{Y}}_{j}}, \qquad (13.15)$$

that the middle error, which characterizes the complete scattering of value A, will be equal to

$$E_{A} = \sqrt{E_{A_{1}}^{2} + E_{A_{2}}^{3} + \dots + E_{A_{n}}^{3}}.$$
 (13. 16)

Hean deviation of the determining parameters frequently designate through $r_{\rm f}$.

For example, for the barrel system

$$r_{\bullet}$$
 - mean deviation v_0 ;
 r_{\bullet} - mean deviation θ_0 ;
 r_e - mean deviation e .

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If we introduce 76 that

$$E_{A} = \sqrt{\sum_{l=1}^{n} \left(\frac{\partial A}{\partial t_{l}} r_{l}\right)^{2}}.$$
 (18, 17)

Fcr our example

$$E_{x_{c}} = \sqrt{\left(\frac{\partial x_{c}}{\partial v_{0}}r_{v_{0}}\right)^{2} + \left(\frac{\partial x_{c}}{\partial v_{0}}r_{b_{0}}\right)^{2} + \left(\frac{\partial x_{c}}{\partial c}r_{c}\right)^{2}}.$$
 (13. 18)

The property of stability possess not all laws of distribution. For example, combination of the values, which ckey the law of uniform density, gives new law [10]. Special complexities are encountered during the determination of the laws of the distribution of the DOC = 78107118 EAGE \$ 1004

functions, which are the consequence of the continuations of the arguments, which have different laws of distribution. This fact decreases the accuracy of the examined method during its applidation/use for determining scattering the trajectories of new criginal controlled rockets and projectiles.

Let us examine performance calculation of scattering the unguided rockets by analytical method.

The contemporary unguided rockets of class "surface-surface" with engine, usually by worker on solid fuel, are utilized for a firing to the comparatively short distances (it is not more than 5C-100 km). The trajectory height of such rockets, as a rule, does not exceed 30-50 km, i.e., entire/all missile trajectory passes in the sufficiently dense layers of the atmosphere, and air resistance significantly affects rocket flight entire trajectory. By the rockets of this class they conduct firing mainly at the target/purposes, arrange/located on the surface of ground or sea; therefore the greatest interest represents the scatter of points of incidence/drop.

The weak interconnection of longitudinal and yawing motions makes it possible to separately examine longitudinal scattering (on distance) in the direction of χ -axis of the starting coordinate system and lateral - in the direction of z exis.

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The characteristics of scattering complex trajectories for each of the sections are calculated separately, just as the correction (see Chapter XI §6). The characteristics of scattering trajectory elements at the end of the preceding/previous section serve as initial characteristics for the subsequent section. Let us examine determining the characteristics of scattering separately for active and inactive legs. For the calculation of longitudinal scattering on active section, we utilize a system of equations (3.79)

 $\dot{v} = \frac{P-X}{\pi} - g\sin\theta; \quad \dot{v} = -\frac{g\cos\theta}{2}; \quad \dot{g} = g\sin\theta; \quad \dot{x} = g\cos\theta.$

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The integration of this system makes it possible to obtain the metion characteristics of rocket as material point. Let us write the right sides of equations (3.79) in functional form.

$$\vec{v} = f_1(P, X, m, v, 0); \quad \vec{v} = f_0(v, 0); \vec{y} = f_3(v, 0); \quad \vec{x} = f_4(v, 0).$$
 (13. 19)

Scattering the cell/elements of trajectory at the end of the active section will be determined by the probable deviations of the arguments of system (13-19) and by scattering the initial conditions for which it is possible to accept the random vector of the velocity EOC = 78107118 FAGE \$ 1006

cf the center of mass at the screet of the descent of rocket from guides, that is changed in sodule/scdulus and direction. Scattering the operationates of initial point in the trajectory with fixed guides can be disregarded.

Thrust P and drag X in turn, depend or a rumber of factors: therefore expedient to open their values, for which we will use previously obtained dependence (11.18).

$$v = f_1(v, \theta, y, J_1, S, \varrho, c_x(M), m_{q_1} | \dot{m} |).$$

tanb the unit/single momentum/impulse/pulse J₁ depends mainly on fuel heating value and in smaller measure on the design features of engine. Contemporary solid fuels represent by themselves, as a rule, the mixture of different components. The composition of mixture ideally accurately for all rockets cannot be maintained; therefore unit/single momentum/impulse/pulse for each rocket will differ somewhat from the nominal. To the value of unit/single scmentum/impulse, has effect the initial temperature of fuel/propellant. This fact also leads to the spread of unit/single scmentum/impulse/pulse, since to attair the identical temperature field of charge in all rockets is virtually impossible.

Instead of the consumption of mass during setting of the scatter of trajectory elements at the end of the active section, they prefer **EGC =** 78107118

to examine the operating time of ergine is with sufficient accuracy communication/connection between them is expressed by the dependence

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101 m

where m. - mass of fuel/propellant.

The operating time of engine l_m also differs from the nominal, since they occur the scatter of combustion chanter pressure and the dispersion of the nozzle threat area, that lead to the scatter of fuel consumption per second.

The spread of unit/single momentum/impulse/pulse will affect mainly scattering of the velocity of rocket at the end of the operation of engine , while time jitter of burning - scattering of coordinates and the angle of the slope of velocity vector the

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Thrust represents by itself the vector, determined, besides mcdule/modulus, even by direction. The law of a change in the sense of the vector of thrust also does not remain identical for the rockets of one party/batch. They occur eccentricity of thrust, i.e.,

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the displacement of the thrust line relative to the center of mass of rocket, and the misalignment of the thrust line when it is not parallel to axis of rocket. This leads to the fact that the axis of rocket differs from tangent to trajectory, rocket moves with the angles of attack and slip. As a result of initial disturbances and the disturbance/perturbations, which act in trajectory, even in the attack and slip are not equal to zero. The presence of these angles determines the composing the thrusts in transverse direction, deforming trajectory. Trajectory or active section heaves or is cmitted, it is displaced to the right or to the left.

System of equations (3.79) is comprised for normal meteorological conditions. With the wish to consider the atmospheric disturtances it is necessary functional dependence (13.19) to add as arguments pressure, the temperature of air and the wind (see Chapter XI). The more complete account of aerodynamic forces will require passage from system (3.79) to another basic system of equations, which includes necessary those comprise of aerodynamic drag. Let us designate the atmospheric perturbation factors through includes.

Initial conditions let us consider angle θ_0 , by angular velocity $\dot{\theta}_0$ and by initial velocity v_0 .

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For simplicity of writing, let us designate trajectory elements at the end of the active section $x_{\rm H}, y_{\rm H}, v_{\rm H}$ and $\theta_{\rm H}$ through $A_{\rm H},$ then, taking into account (13.19), (11.18) and the considerations presented, we can write stochastic dependence for $A_{\rm H}$

 $A_{xl} = f_l(J_1, t_x, e_r, \delta_r, S, c_x(M), m_0, \theta_0, \dot{\theta}_0, v_0, \xi_{ur}). \quad (13, 20)$

In first part one four argument determines scattering thrust; arguments S and $c_x(M)$ determine scattering of drag; argument π_0 determines scattering initial mass (or weight Q_0); arguments θ_0 , $\dot{\theta}_0$ and v_0 determine scattering initial conditions and value ξ_{er} - total effect of the atmospheric disturbances.

Let all the random arguments is not correlated and are subordinated to the normal law of distribution; then the total scattering of random function A_{ni} is also subordinated to the normal law of distribution and is determined by formula (13.11).

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For simplicity let us write separately average guadratic deviation

 $\bullet_{A_{Bl}}$, determined by thrust, air resistance, by weight, initial conditions and the atmospheric disturbances. From scattering of thrust vector, we will obtain

$$(\sigma_{A_{\underline{\kappa}\,\underline{i}}})_{\underline{\tau}} = \sqrt{\left(\frac{\partial A_{\underline{\kappa}\,\underline{i}}}{\partial J_{1}}\sigma_{J_{1}}\right)^{2} + \left(\frac{\partial A_{\underline{\kappa}\,\underline{i}}}{\partial t_{\underline{\kappa}}}\sigma_{\underline{i}\,\underline{\kappa}}\right)^{2} + \left(\frac{\partial A_{\underline{\kappa}\,\underline{i}}}{\partial t_{\underline{\tau}}}\sigma_{d_{\underline{\tau}}}\right)^{2} + \left(\frac{\partial A_{\underline{\kappa}\,\underline{i}}}{\partial t_{\underline{\tau}}}\sigma_{\underline{i}\,\underline{\tau}}\right)^{2}}.$$
(13.21)

The roct-mean-square deviation, determined by scattering drag, is equal

$$(\sigma_{A_{x,i}})_{R} = \sqrt{\left(\frac{\partial A_{x,i}}{\partial S}\sigma_{s}\right)^{2} + \left(\frac{\partial A_{x,i}}{\partial c_{x}}\sigma_{c_{x}}\right)^{2}}.$$
 (13.22)

For the calculation of scattering, it is presible to take constant for entire powered flight trajectory value a_{r_x} , expressed in percentages.

Boot-mean-square deviation from scattering of the initial mass

$$(\mathbf{e}_{\mathbf{A}_{\mathbf{g}}})_{\mathbf{g}} = (\mathbf{e}_{\mathbf{A}})_{\mathbf{g}} = \left(\frac{\partial \mathbf{A}_{\mathbf{g}}}{\partial \mathbf{Q}_{\mathbf{g}}}\right) \mathbf{e}_{\mathbf{g}_{\mathbf{g}}}.$$
 (13.23)

From scattering of the initial conditions

$$(\sigma_{A_{w,l}})_{0} = \sqrt{\left(\frac{\partial A_{w,l}}{\partial \sigma_{0}} \circ_{\sigma_{0}}\right)^{2} + \left(\frac{\partial A_{w,l}}{\partial \sigma_{0}} \circ_{\sigma_{0}}\right)^{2} + \left(\frac{\partial A_{w,l}}{\partial \sigma_{0}} \circ_{\sigma_{0}}\right)^{2}}.$$
 (13.24)

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The root-mean-square deviation, determined by the atmospheric disturbances, is equal

 $(\mathbf{e}_{A_{\mathbf{k},i}})_{a\tau}^{2} = \sum_{i=1}^{n} \left(\frac{\partial A_{\mathbf{k},i}}{\partial \mathbf{k}_{i}}\right)^{2} \mathbf{e}_{i}^{2}.$ (13.25)

In the analytical method of calculation of scattering in guestion to directly consider the chance of the process of changing the weather factors with height/altitude and time is not represented presible. It is expedient preliminarily from a change in the weather factors as random functions to pass to scattering of ballistic wind, the tallistic temperature deflection and barometric pressure, after describing by their corresponding root-mean-square deviation from standard conditions, determined by standard atmosphere.

Grand average standard deviation, which characterizes scattering the cell/elements of motion at the end of the engine operation, will te determined according to the formula

$$\mathbf{e}_{A_{K}} = \sqrt{(\mathbf{e}_{A_{K}})_{1}^{2} + (\mathbf{e}_{A_{K}})_{1}^{2} + (\mathbf{e}_{A_{K}})_{0}^{2} + (\mathbf$$

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Scattering trajectories on passive section is determined: by scattering initial for a passive section motion characteristics, by

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scattering aerodynamic drag, by scattering weight and by scattering meteorclogical factors. From aerodynamic characteristics let us as tefore consider scattering only drag. The characteristics of scattering the values of trajectory elements at the point of the end/lead of the engine operation, determined on formula (13.26), let us designate respectively: a_{y_x} , a_{y_x} , a_{x_x} . If we consider that the scatter of points of an incidence/drop in the rocket was determined only by scattering initial conditions, ther we will obtain the sc-called latent scattering for which

$$a_{s_{C^{3}}} = \sqrt{\left(\frac{\partial x_{C}}{\partial a_{x}}a_{s_{x}}\right)^{2} + \left(\frac{\partial x_{C}}{\partial a_{x}}a_{s_{x}}\right)^{2} + \left(\frac{\partial x_{C}}{\partial y_{x}}a_{s_{x}}\right)^{3} + \left(\frac{\partial x_{C}}{\partial x_{x}}a_{s_{x}}\right)^{3} + \left(\frac{\partial x_{C}}{\partial x_{x}}a_{s_{x}}\right)^{3}}$$
(13. 27)

The effect of scattering the drag coefficient and weight of rocket on passive section can be registration/accounting through the appropriate root-mean-square deviation of drag coefficient $\mathbf{v}_{r,\mathbf{v}}$ and the root-mean-square deviation of the weight of rocket $\mathbf{v}_{r,\mathbf{v}}$. However, is more convenient on passive section to introduce to examination tallistic coefficient $\mathbf{c}_{\mathbf{H}} = \frac{d\mathbf{e}}{Q_{\mathbf{h}}} \mathbf{10}^{\mathbf{e}}$ and to determine the immediately root-mean-square deviation of tallistic coefficient - $\mathbf{c}_{\mathbf{h}}$

The root-mean-square deviation, which characterizes the atmospheric disturbances on passive section, let us determine according to the formula, similar (13.25), after introducing the

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The determination of root-mean-square deviation must be conducted in accordance with the considerations, presented in connection with formula (13.25). On trajectories which pass in the relatively dense layers of the atmosphere, from weather factors the most essential effect exerts scattering wind velocity. Taking into account basic factors for the unguided rockets, average quadratic range error will be determined according to the formula

 $\sigma_{x_{c}} = \frac{1}{\sqrt{\left(\frac{\partial x_{c}}{\partial a_{t}} \cdot \sigma_{x_{c}}\right)^{2} + \left(\frac{\partial x_{$

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Scattering in side direction on inactive leg is determined in essence: by scattering angle Ψ_{R} , characterizing the sense of the vector of the speed of relatively plane of reference of casting; by scattering angular velocity Ψ_{R} , by scattering coordinate z_{R} and by scattering the velocity of lateral ballistic wird. The total lateral scattering of trajectories is characterized by value

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$$\frac{\partial a_{c}}{\partial r_{x}} = \sqrt{\left(\frac{\partial a_{c}}{\partial r_{x}} \circ v_{x}\right)^{2} + \left(\frac{\partial a_{c}}{\partial r_{x}} \circ v_{x}\right)^{2} + \left(\frac{\partial a_{c}}{\partial r_{x}} \circ v_{x}\right)^{2} + \left(\frac{\partial a_{c}}{\partial r_{x}} \circ v_{x}\right)^{2}}.$$
 (13.30)

In the more detailed study of problem, besides the named factors, it is necessary to still consider eccentricity of masses and aercdynamic eccentricity, caused by the technological inaccuracies, for example, by the misalignment of stabilizer fins. As a result of eccentricity and misalignment of the axis of thrust of aerodynamic eccentricity and eccentricity of masses, and also the disturtance/perturtations of the rocket with descent from guides, arrears scattering the angles of attack and slip. The more detailed account of all affecting factors (including technological) will considerably complicate sclution. Upper consideration only of basic rerturbation factors, the examined method makes it possible to rate/estimate the characteristics of scattering the design/projected rccket and to determine the advisability of further development of the assumed construction. It is necessary to keep in mind that when the problem is solved in the first approximation, it is difficult, and sometimes also virtually it is not possible to consider the correlation communication/connections between the separate rerturtation factors, it is difficult to previously establish/install the laws of the distribution of random arguments and the random furction of these arguments. Frequently without sufficient

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substantiation it is necessary to select the normal law of distribution.

Special difficulties are encountered during the calculated determination of scattering the guided missilem of class "surface-surface". Despite the fact that similar rockets are equipped by the control systems, to completely remove their scattering is not represented possible. Steering functions of type (9.9) and (9.10) are comprised with the specific assumptions. In the majority of the cases, are considered only first terms of expansion in a series, since the account of guadratic terms of expansion leads to the considerable complication of the control system. The noncoincidence of the right and left sides of equalities (5.7) or (9.10) at the cutoff of engine will always give the error, determined in general form by formula (9.11).

The cessation of the operation of engine (resetting to zero of thrust) is realized not instantly [38].

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The momentum/impulse/pulse of the aftereffect of thrust, after the delivery of the main command to its cutoff, will have certain spread, which will give scattering velocity $v_{\rm M}$. If the rocket is controllable

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cnly on active section, ther on the descending branch of trajectory they will cccur of the disturbance/perturbations, determined by the passage of the rocket through the dense layers of the atmosphere. The enumerated errors, caused by the method of control accepted, are called systematic.

Besides systematic errors, to the flight of the guided missiles have effect the instrument errors, which are inherent in steering devices. The gyrostabilized platforms, on which are establish/installed accelerometers, have attendance/departures, in consequence of which the axis of the sensitivity of accelerometers in the course of time differ from given one on the start of the direction of measurement, yes even direction itself, measurements on start is assigned with error. But this leads to the fact that the projections of the speed of rocket and its coordinate are determined with errors. The formation of steering function ϕ and its comparison with computed value ϕ_{π} for the end/lead of powered flight trajectory are conducted also with errors.

tajk average quadratic range errors for the class of rockets indicated can be designed by analytical method of statistical testings, examined subsequently paragraph. If is applied analytical method, then calculation formula takes the form DOC = 78107118 EAGE #9- 1017

 $\bullet_{L} = \sqrt{\bullet_{L}^{2} \bullet_{L}^{2} \bullet_{L}^{2} \bullet_{L}^{2} \bullet_{L}^{2} \bullet_{L}^{2} \bullet_{L}^{2} \bullet_{L}^{2} \bullet_{I}^{2} + \left(\frac{\partial L}{\partial J_{ax}} \sigma_{J_{ax}}\right)^{2} + \sum_{i} \left(\frac{\partial L}{\partial \xi_{i}} \sigma_{\xi_{i}}\right)^{2}_{ax}}, \quad (13.31)$

where σ_{L_0} - rms range error as a result of the assumptions, accepted during introduction to steering function:

^{OLENCTD} - THE TANGE EFFCE AS A RESULT OF the presence of instrument errors;

σ_{J_{RA}} - the root-mean-square deviation of the **BCBENTUB/INFULSE**/pulse of aftereffect.

The third term (13.31) expresses effect on the dispersion in distance of the spread of the momentum/impulse/pulse of the aftereffect of thrust, and last/latter term characterizes the effect on σ_L of the disturbance/perturbations, which act on the section of the atmospheric entry.

Scattering long range fallistic missiles in side direction first of all will be determined by scattering trajectory elements at the end of the active section (i.e. of the cutoff point of the thrust). The lateral scatter of points of incidence/drop as a result of this reason in linear approach/approximation is determined by the formula PAGE = 1018

 $\frac{\mathbf{e}_{ac} = \sqrt{\left(\frac{\delta \mathbf{e}_{c}}{\delta \mathbf{e}_{c,c}} \mathbf{e}_{\mathbf{e}_{c,c}}\right)^{2} + \left(\frac{\delta \mathbf{e}_{c}}{\delta \mathbf{e}_{c,c}} \mathbf{e}_{\mathbf{e}_{c,c}}\right)^{2} + \left(\frac{\delta \mathbf{e}_{c}}{\delta \mathbf{e}_{c,c}} \mathbf{e}_{\mathbf{e}_{c}}\right)^{2} + \left(\frac{\delta \mathbf{e}_{c}}{\delta \mathbf{e}_{c}} \mathbf{e}_{c}\right)^{2}$

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Bost essential effect on value a_{c} have terms $\frac{\partial z_{c}}{\partial a_{k_{s}}} a_{e_{k,s}}$ and $\frac{\partial z_{c}}{\partial a_{k}} a_{e_{k}}$. Thrust cutoff is realized in function value, governing flying range, because of this to utilize a selection of the cutoff point of thrust for obtaining the sinisum of lateral deviation is not represented possible. Therefore for decreasing the scattering in side direction, it falls on all powered flight trajectory to attempt via control to decrease values $a_{e_{k}}$ and $a_{e_{k}}$.

There is an error, which the autonomous system control not at all can remove. This error is sighting error along azimuth. Acceptable accuracy in side direction in the autonomous control system it is possible to actieve only in the case of small sighting error. Sighting error along azimuth can be even if is not removed entirely, then is substantially decreased with tracking rocket flight on ground tracking stations and on conducting the correction of trajectory.

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Together with the deviations of trajectory elements at the end of the active section of scattering in side direction definite effect have the disturbance/perturbations, which act on the rocket upon its entry into the dense layers of the atmosphere.

The scattering, caused by the examined above factors (i.e. depending only on rocket itself), they frequently call technical.

Essides technical scattering occurs arother the scattering, connected with the errors for the preparation of firing. To the errors for the preparation of firing are related the errors in determination of geographical reference of the launching point and target/purpose, error in the determination of the atmospheric parameters, error in the determination of the temperature of fuel/propellant, etc. The errors for training/preparation for this group of launching/startings are more less are identical, but they noticeably are distinguished for different groups. The errors for training/preparation lead to scattering of the centers of the grouping of projectiles and rockets in different firings. If the laws of technical scattering and errors for trairing/preparation are identical, then the characteristics of complete scattering will be equal to

 $\bullet_{s_{C}} = \sqrt{\bullet_{a_{C}}^{*} + \bullet_{a_{max}}^{*}}; \quad \bullet_{a_{C}} = \sqrt{\bullet_{a_{C}}^{*} + \bullet_{a_{max}}^{*}};$ (13, 33)
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Bespectively under the normal laws of the distribution of technical scattering and errors for training/preparation, it is presible to write for mean deviation:



where B_{RT} and $B_{0.7}$ - middle, cr probable, range errors and in side direction, that characterize technical scattering;

 $B_{\text{R,mox}}$ and $B_{6,\text{mox}}$ - middle, or probable, the deviations, which characterize the errors for training/preparation.

For setting of kill probability to target/purpose, the ammunition consumption and time of fire for effect to target/purpose, the account of the errors for trairing/preparation is necessary.

§2. Determination of the characteristics of scattering by the method cf statistical testings.

The method of statistical testings - cre cf the most universal

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methods of determining the probabilistic characteristics of the results of the large number of solutions of the systems of the differential equations, which describe physical process, in which the initial data are assigned as random variables. Method found application/use in many areas of science and technology, including in tallistics [8], [18].

In the latter case is performed the calculation of a large quantity of trajectories or computers. Those of the parameters the effect of scattering the which is assumed to consider that they are represented in the form of random runters. The functions the effect cf scattering the which is assumed to consider, for example, a change in the weather factors with height/altitude ((), (), (), they are represented in the form of random functions. Refore conducting of statistical testings (calculations) is conducted the careful study of all parameters and functions, which affect the motion of rockets and **projectiles**, are establish/installed the laws of their distribution and numerical characteristics of these laws. Then is comprised the scst complete, for giver specific conditions, system of equations, which includes the random parameters and the functions whose effect is assumed to consider. Utilizing the obtaired values of the random parameters and functions, is performed calculation ("experiment") on computers.

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The random parameters during the j testing are taken in this form:

 $\mathbf{t}_{1} = \mathbf{M} [\mathbf{t}_{1}] + \mathbf{N}_{1} \mathbf{v}_{1};$ ╸M┡╢┿*Ňⅆ℄*℠ $\mathbf{E}_{\mathbf{I}} = \mathbf{M} \left[\mathbf{E}_{\mathbf{I}} \right] + \mathbf{N}_{\mathbf{I}} \mathbf{e}_{\mathbf{I}}.$

Here g_{ij} - value of the i parameter during the j testing (celculation);

M(E) - mathematical expectation of the i parameter or its resinal value:

 N_{ij} - the random number, which characterizes the value of the i estimater during the j testing; for example, during the first testing - N_{ii} , cn the second - N_{ij} and so forth.

The random functions, introduced into fallistic calculation, are assigned either in the form of experimental specific realizations citained during testings, or in the form of the realizations, cbtaimed during the canonical expansion of the random functions,

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which describe a change in each of the characteristics.

As is known, the canonical expansion of random function is represented in the form

$$X(t) = m_{\alpha}(t) + \sum_{i=1}^{n} V_{i} \varphi_{i}(t),$$
 (13.37)

where $m_x(t)$ - mathematical expectation of render function;

 $\varphi_1(t), \varphi_2(t), \dots, \varphi_m(t) - \text{coordinate functions; } V_1, V_2, \dots, V_m$ uncorselated random variables with the mathematical expectations, equal to zero.

In ballistic calculations are introduced the random functions, which reflect a change of the weather factors depending on height/altitude.

Buring the use of cancrical expansions, it is possible to write for deviation of temperature from the mornal law

$$V_{(g)} = \sum_{i=1}^{n} V_{i} (ig),$$
 (13.38)

where V_{τ} and φ_{τ} - random variable and coordinate function, that characterize deviation of temperature.

Bandom function for wind, for reasons, presented in Chapter XI,

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 FIGE $\frac{26}{1024}$

it is expedient to present in the form of two functions. In N S direction morth-south:

$$w_{C-iD}(y) = \tilde{w}_{C-iD}(y) + \sum_{i=1}^{2} \tilde{v}_{iC-iD}(y) = (13.39)$$

Eage 518.

E W In disection east-west:

 $\Psi_{\mathbf{B}\to\mathbf{S}}(y) = \widetilde{\Psi}_{\mathbf{S}\to\mathbf{S}}(y) + \sum_{l=1}^{n} V_{\Psi_{\mathbf{B}\to\mathbf{S}}l} \varphi_{\Psi_{\mathbf{B}\to\mathbf{S}}l}(y).$ (13.40)

Here $\widetilde{\mathbf{w}}_{\mathbf{C}-\mathbf{D}}(\mathbf{y})$ and $\widetilde{\mathbf{w}}_{\mathbf{b}-\mathbf{S}}(\mathbf{y})$ - change in the average values of the projections of wind velocity in height/altitude; $V_{\mathbf{w}_{\mathbf{C}-\mathbf{D}}}, V_{\mathbf{w}_{\mathbf{S}-\mathbf{S}}}$ - random variables; $\varphi_{\mathbf{w}_{\mathbf{C}-\mathbf{D}}}(\mathbf{y}), \varphi_{\mathbf{w}_{\mathbf{S}-\mathbf{S}}}(\mathbf{y})$ - cccrdinate functions, which determine random comprising velocity of wind according to height/altitude in the appropriate direction.

In each concrete/specific/actual trajectory calculation, it is utilized on one of the realizations of random functions $\delta T(y), w_{C-D}(y)$ and $w_{D-3}(y)$. Bandom variables and coordinate functions are determined as a result of the very laborious probabilistic analysis of large quantity of the experimental data, obtained with the meteorological sounding of the atmosphere. With ballistic rockets the number of component in right sides expansions of random function (13.38)-(13.40) is determined by the available experimental data, COC = 78107118 FIGE $\frac{27}{1025}$

their reliability and a quantity. In the majority of the cases, prove to be possible to take 10-15 terms of expansion.

Let us examine the exemplary/approximate order of the use of expressions (13.38)-(13.40) during performence calculation of scattering of rockets. The characteristics of random variables V_1 and coordinate functions $\varphi_1(y)$ we consider another. Let us assume that available there are on n of coordinate functions for determination δI , w_{C-D} and w_{D-3} and from n of the tables of random coefficients. After taking according to one random number V_{τ} of each table, after multiplying each of them by its (coinciding in number) coordinate function it summed up the obtained terms, is found concrete/specific/actual curve (realization), which characterizes the law of a change in the deviation of temperature with height/altitude at the first "launching/starting".

Then similarly is found the law of a change in wind velocity with height/altitude, characterized by functions $w_{C-D}(y)$ and $w_{D-D}(y)$. For the facilitation of calculations, they project wind velocity in the line of fire and the side direction, as a result of which is obtained $w_{x}(y)$ and $w_{x}(y)$. The obtained realizations of the deviations of the atmospheric parameters from the standard $\delta T(y)$, $w_{x}(y)$, $w_{x}(y)$ are utilized during the solution of the system of equations of the motion of nocket for the first "launching/starting". ICC = 78107118 PAGE -28 1026

In exactly the same way are calculated realizations $\delta T(y)$, $\mathbf{w}_{r}(y)$ and $\mathbf{w}_{r}(y)$ for the 2nd, 3rd and so forth of "launching/startings".

Utilizing the obtained values of the render parameters and the random functions, are carried cut trajectory calculations ("experiments") on ETSVE [SUBM- digital computer] or AVM. Analog computers give results with large errors; however, they are favorable when to describe mathematically phenomeron is not completely represented possible and it is necessary to join up of machine the real assemblies of article.

The results of calculation, cltained in the first "testing", are introduced into table.

Then thus is conducted the second "testing", the third and so up to the latter.

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When are carried out all "experiments", from table is extracted the which interests us value (for example, distance, flight altitude, LCC = 78107118

etc.), are establish/installed the law of its distribution and the numerical characteristics of law. Thus, as a result of "testings" is obtained the complete description of studied value from the protabilistic point.

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The reliability of results during the application/use of a method of statistical testings depends to a considerable extent on the number of the "experiments conducted" and cp the accuracy of the mathematical description of phenomenon; for obtaining the reliable results, are required usually hundred and thousand "experiments".

By basic advantage of the method of statistical testings was a sufficiently complete description rocket flight as of random process, determination of all characteristics of this process.

To deficiency/lacks in the method, one should relate the long time, required for conducting such "testings", and high consumption in comparison with analytical method.

By the described method are determined the characteristics of complete scattering. In order to determine the effect of scattering the separate parameter or function, it is recessary this parameter or the function to represent as random, and remaining determining parameters and functions to take or rating (or on mathematical

ECC = 78107118 FIGE 35 /028

expectation). Similar investigations are connected with the calculation of a large guantity of trajectory and the corresponding expenditures of the machine count time on computers.

§3. Determining the characteristics of scattering according to the results of firing.

Birect/straight experiment (firing) they make it possible most to connectly estimate the characteristics of scattering. Firing is conducted in the final development stages of missile or artillery complex. With an increase in the cost/value of rockets (projectiles), it is logical, they descend the possibility of obtaining sufficient statistical material by conducting the firings. Therefore the results of firing always are estimated in combination with theoretical calculations. The basic difficulty of the solution of this problem with rocket firings consists in the limitedress of statistical material or, in other words, in the low number of rocket launchings, and also in the fact that is previously unknown the law of distribution. DOC = 78107118



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Fig. 13.3. Scattering the coordinates of the impact points in the projectiles in locality.

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It connection with this in the process of the calculation of numerical characteristics according to the results of experiment, it is necessary to speak not about precise of their value, but only about the average statistical values of the obtained quantities. The average statistical values of rumerical characteristics usually are designated with line above. For example $M[x] = M_x$ — average statistical mathematical expectation of random variable;

 $D[x]=D_x$ - average statistical dispersion of random variable, etc.

let us examine the basic stages of processing the results of

COC = 78107118 FIGE = 1030

firing. Let us assume that with "n" rocket laurchings we have "n" of the points of their incidence/drop on plane xoz (Fig. 13.3). The coordinates of impact points it is most expedient to determine in the starting coordinate system. They first of all extract consecutively the coordinates (x; z) of each impact point in second and third columns in Table 13.1.

Further are determined the values of average statistical mathematical expectations according to coordinates x and z

 $\overline{M}_{z} = \frac{\sum_{l=1}^{n} z_{l}}{a};$ $\overline{M}_{z} = \frac{\sum_{l=1}^{n} z_{l}}{a}.$ (18.41) $\overline{M}_{z} = \frac{\sum_{l=1}^{n} z_{l}}{a}.$ (18.42)

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Table 13.1.

(/) 1000000 10072	x _i	81	x;M.s	sı-Me	$(x_i - \overline{M}_x)^2$	(<i>s</i> 1—M ₈) ²	$(x_{i} \rightarrow \overline{M}_{z}) \times \times (x_{i} - \overline{M}_{z})$
1 2							
•							
ž						-	

Kef: (1). Number of experiment.

Fage 521.

column of Then are filled the remaining 13.1 for all s Fatle experiments and are determined average statistical dispersions $(D_x; D_y)$ and covariance (I_{xx}) according to the following dependences:

$$\overline{D}_{s} = \frac{\sum_{i=1}^{n} (x_{i} - \overline{M}_{s})^{s}}{n-1}; \qquad (13.43)$$

$$\overline{D}_{s} = \frac{\sum_{i=1}^{n} (x_{i} - \overline{M}_{s})^{s}}{n-1}; \qquad (13.44)$$

$$\overline{R}_{ss} = \frac{\sum_{i=1}^{n} (x_{i} - \overline{M}_{s})(x_{i} - \overline{M}_{s})}{n-1}; \qquad (13.45)$$

n-1

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Further are calculated average statistical values of root-mean-square deviation $(\overline{\sigma}_x; \overline{\sigma}_z)$ and of the coefficient of correlation (\overline{r}_{xx}) :

$$\overline{e_x} = \sqrt{\overline{D_x}}; \qquad (13.46)$$

$$\overline{e_s} = \sqrt{\overline{D_y}}; \qquad (13.47)$$

$$\overline{r_{ss}} = \frac{\overline{R_{ss}}}{\overline{e_s} \cdot \overline{e_s}}. \qquad (13.48)$$

Passage from dispersions to root-mean-square deviation usually is made on that reason, that the dimensionality \cdot is equal to the dimensionality of random variable, while the dimensionality of dispersion it is equal to the dimensionality of the square of random variable. By correlation coefficient also to more usually conveniently use than covariance. The correlation coefficient is dimensionless and changes from 0 to 1. If $f_m=0$, then this means that between random variables x and z there is no correlation. These values are independent from the point of the protability theory. Difference f_m from zero is the sign/criterion of existence of the stochastic dependence between random variables x and z are completely dependent on each other.

EGC = 78107118 FIGE 25 /033

For evaluating the reliability of the obtained average statistical values of runerical characteristics, calculate for them the so-called confidence intervals. Let us examine as an example the determination of confidence interval for the average statistical mathematical expectation of random variable x. 12

Let as a result of processing data of firing we obtain that $\overline{M_x}$ and it is necessary to rate/estimate error ' $\overline{M_x}$ - M_x , where M_x precise value of the mathematical expectation of random variable x.

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Let us assign certain sufficiently large probability β (for example, $\beta=C.9$), such so that the event with probability β it would be possible to consider virtually reliable, and let us find such value ϵ , for which

$$\rho(|\overline{M}_x - M_x| < \epsilon) = \beta. \qquad (13.49)$$

Then the range of the virtually possible values of the errors, which appear during replacement M_x on \overline{M}_x , will be $\pm x$. Large in absolute value errors will appear with the low possibility

$$\mathbf{c} \neq \mathbf{i} - \boldsymbol{\beta}. \tag{13.50}$$

let us rewrite equality (13.49) in this fors:

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$$\rho(\overline{M}_x - \epsilon < M_x < \overline{M}_x + \epsilon) = \beta. \tag{13.51}$$

Citained equation (13.51) scans that with pickability β the unknown **precise value** M_{α} falls into following interval (Fig. 13.4)

$$J_{g} = (\bar{M}_{x} - \epsilon; \bar{M}_{x} + \epsilon).$$
 (18,52)

Value J_{β} is called of confidence interval, and β - confidence coefficient. In our case β - the probability of the fact that the coefficience interval J_{β} will cover value M_{α} .

Let us determine confidence intervals for the average statistical mathematical expectations of the random variables of ccordinates x and z. For this, let us assign value β , from which let us find value to [10]. The relative boundaries of confidence intervals let us determine according to the formulas

()m,-4 / 5. () ()m,-4 / 5. ()

Consequently, confidence intervals for average statistical sathematical expectations will be equal to



 $(J_0)_{H_0} = [M_0 - (a)_{H_0}, M_0 + (a)_{H_0}] + (12.50)$ $(J_0)_{H_0} = [M_0 - (a)_{H_0}, M_0 + (a)_{H_0}] + (12.50)$

Fig. 13.4. Confidence interval /, cn number scale axis.

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The relative boundaries of confidence intervals for the average

statistical dispersions of random variables x and x can be determined by following formulas [10]:

$$(t)_{D_x} = t_{\phi} \sqrt{\frac{2}{n-1}} \overline{D}_x; \qquad (13.57)$$

$$(t)_{D_x} = t_{\phi} \sqrt{\frac{2}{n-1}} \overline{D}_x. \qquad (13.58)$$

In this case, the confidence intervals of average statistical dispersions will be equal to

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$$(J_{\emptyset})_{\mathcal{D}_{x}} = \begin{bmatrix} \overline{\mathcal{D}}_{x} - (\varepsilon)_{\mathcal{D}_{x}}; \quad \overline{\mathcal{D}}_{x} + (\varepsilon)_{\mathcal{D}_{x}} \end{bmatrix}; \qquad (13.59)$$
$$(J_{\emptyset})_{\mathcal{D}_{x}} = \begin{bmatrix} \overline{\mathcal{D}}_{s} - (\varepsilon)_{\mathcal{D}_{s}}; \quad \overline{\mathcal{D}}_{s} + (\varepsilon)_{\mathcal{D}_{s}} \end{bmatrix}. \qquad (13.60)$$

If we designate

$$a_{1x} = \sqrt{\overline{D}_x - (\epsilon)_{D_x}}; \quad a_{3x} = \sqrt{\overline{D}_x + (\epsilon)_{D_x}},$$

then it is possible to chtain confidence intervals for average statistical standard deviations

$$(J_{g})_{e_{g}} = (c_{1x}; c_{g_{g}});$$
 (13.61)
 $(J_{g})_{e_{g}} = (c_{1x}; c_{g_{g}}),$ (13.62)

The following development stage of firings is the construction of the histogram in form of which is introduced the hypothesis about the possible character of the law of distribution. For the construction of histogram, are carried out following process/operations. They divide entire range of the obtained values x and z for the discharges (intervals), limited by values x; x₁₁₁ and z₁; x₁₁₁ (for the i interval), and compute a quantity of values of random variable m₁₀ that being necessary for each i-th discharge. For convenience in the construction of histogram, take not true values x₁ and x₁, ***

100

PAGE -35 1037 COC = 78107118

mathematical expectations $\frac{1}{2}$ and $\frac{1}{2}$. Usually rumerical length is equal to six, eight and it is thinner - ten. Generally a quantity of discharges k depends on the number of the experiments conducted. The greater the experiments (r), the greater numerical length can be taken. The length of discharge it is convenient to take equal to root-mean-square deviation. The selected discharges and a quantity of values of coordinate (ma), that being necessary for each discharge, will bring in into the first two lines of Tetle (13.2), which is made for random variable x (for an example it is undertaken of six discharges, on three discharges to each side from zero). Similar table it is necessary to make, also, for random variable z.

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On findings are constructed the histograms both for the random variable x and for random variable z, plot/depositing along the axes of abscissas discharges, while along the axes of ordinates - the corresponding to them guantities of values (m_i) of random variables.

Analyzing the obtained histograms, is introduced hypothesis on the probable character of the law of the distribution of given random variable. For example, on the histogram, presented in Fig. 13.5, it is possible to introduce hypothesis about the sormal law of random rumber distribution x. COC = 78107118 FAGE #0 1038

The introduced hypothesis or the law of distribution of given random variable must be checked on goodness of fit. Nost frequently for this is applied Pearson's so-called criterion χ^2 , who makes it possible to rate/estimate the degree of the coordination introduced of theoretical and statistical laws of distribution.

Value 🛫 is determined according to the dependence



where p_i - the hit probability into the i-th discharge, designed according to the introduced theoretical law of distribution.

Tables 13.2.

1	() Разряды	x3;x2	x ₂ ;x ₁	_x: 0	0; x ₁	x ₁ ; x ₂	x ₂ ; x ₃
2	m i						
3	$\hat{\Phi} \left \frac{x_{l+1}}{\varrho \sqrt{2} \mathfrak{a}_x} \right $						
4							
5	$p_i = \frac{1}{2} 3-4 $						
- 6	n p _i					1	
7	m _i —np _i					[
8	(<i>m₁-np₁</i>) ²	· · ·		Į	ļ		
9	8:6			ļ		1	



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Fig. 13.5. Histogram of processing test results.

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and the state of the

For example, for the normal law of hit probability distribution of random variable x into this discharge is computed according to the formula

$$p_{l} = \frac{1}{2} \left[\Phi \left| \frac{\dot{x}_{l+1}}{\varrho \sqrt{2} \epsilon_{x}} \right| - \Phi \left| \frac{\dot{x}_{l}}{\varrho \sqrt{2} \epsilon_{x}} \right| \right], \qquad (13.64)$$

where \dot{x}_i, \dot{x}_{i+1} - boundary values of the i discharge; $Q\sqrt{2}=0.6745$; $s_x=s_x$ - root mean square value of given random variable, found earlier from experimental data; $\hat{\Phi} \left| \frac{\dot{x}_i}{Q\sqrt{2}s_x} \right|$ - value of the given function of Taplace.

Calculation of χ^2 for the normal law of distribution is convenient to conduct with the aid of Table 13.2.

Further is defined the number of degrees of freedom r as numerical length k minus the number of conditions (communication/connections) S, superimposed for the theoretical law of the distribution

$$r = k - S.$$
 (13.65)

accepted.

In our case the number of superimposed conditions

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(ccmmunicatics/connections) S is equal to three:

it is necessary that the sum of frequencies would be equal to
 cge;

2) is necessary the coincidence of the theoretical and average statistical of mathematical expectations;

3) is necessary the coincidence of the theoretical and average statistical of dispersions.

Further according to those found χ^3 and r determine probability F of the fact that the law of distribution accepted does not contradict experimental data [10]. If probability p is low, then hypothesis about the introduced law of distribution is reject/thrown as unlikely. But if the obtained probability (p) is relatively great, then hypothesis can be recognized by the not contradictory experimental data.

Is how low must be probability p, in order to reject/throw a hypothesis concerning the introduced law of distribution, a question not defined. It cannot be solved from mathematical considerations. In practice, if p<0.1, then it is necessary either experiment to repeat cr to attempt to find the more adequate/approaching law of distribution.

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It should be noted that with the aid c1 critericn χ^2 (or any cther goodness of fit) at the high values c1 p it is possible only to establish that the hypothesis accepted about the law of distribution does not contradict experimental data.

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The experimental characteristics of scattering are compared with calculated, together they are estimated. If necessary are introduced the corrections into the calculated law of distribution and characteristic of scattering.

The scatter of points of an incidence/oreg in the artillery shells and unguided rockets they are subordinated to the normal law of distribution; in this case, the impact points are arrange/located on plane within the limits of the ellipse, called the ellipse of scattering. The center of ellipse coincides with center of dispersion (or by the center of grouping). With the fixing the artillery rotating shells as a result of right derivation, the center of the ellipse of scattering is displaced to the right from the direction of plane of reference of firing. With firing at locality, the axis of ellipse in the direction of firing is equal to 88. In side direction

the axis is equal to 8B₆. With the firing at panel the projectiles, which have low trajectory, the ellipse of scattering it approaches a circumference. Probable (middle) deviation in the direction of vertical axis is designated B_m, a in side direction, just as with firing at locality, is designated B₆. Batics $\frac{B_{\pi}}{x_{C}}$, $\frac{B_{0}}{x_{C}}$ and $\frac{B_{m}}{x_{C}}$ in artillery is conventionally designated as the characteristics of closely grouped fire.

With firing at locality by the unguided fir-stabilized missiles at small angles of increase, just as with the firing artillery shells, $B_x > B_6$; with firing with the angles, close to the angles of maximum range, $B_6 > B_x$ and transverse is directed perpendicularly to the line of fire (Fig. 13.6).

Eepending on the combination of the acting random factors, which determine complete scattering, the longitudinal axis of ellipse can be indlined toward the line of fire.

Let us examine dependence for the calculation of certain characteristics of scattering the projectiles of cannon-type artillery, which proceeds mainly as a result of change from one shot to the next m, k, c. The effect of scattering weather factors for one short-term firing it is possible not to consider. As a rule, the reasons, calling change m, k, and c, they act independently one

from another. Let us designate mean deviation v_0 through $r_{v_0}, 0$ through r_{v_0} and c through r_c . Hiddle range errors in the case of acting only one reason, are equal to:

$$E_{v_1} = \frac{\partial x_c}{\partial v_0} r_{v_1}; \qquad E_{v_1} = \frac{\partial x_c}{\partial v_0} r_{v_1}; \qquad E_{v_1} = \frac{\partial x_c}{\partial v_0} r_{v_1}.$$

where $\frac{dx_c}{dy}$, $\frac{dx_c}{dy}$, $\frac{dx_c}{dz}$ already to us tasic correction factors.

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Since each of cause acts independently one from another, the total range probable error is determined from the formula

$$B_{a} = \sqrt{\left(\frac{\partial x_{c}}{\partial v_{0}}r_{\bullet,c}\right)^{2} + \left(\frac{\partial x_{c}}{\partial \theta_{0}}r_{\bullet,c}\right)^{2} + \left(\frac{\partial x_{c}}{\partial e}r_{c}\right)^{2}}.$$
 (13.66)

According to the characteristics of complete scattering, it is possible to calculate the characteristics of scattering of one of the determining values. For example, if we shoot the barrel system at angle of elevation, close to the angle of maximum range, and to determine experimental B_{R} , then, knowing that at the angles, close to the angle of maximum range $\frac{\partial x_{C}}{\partial t_{0}} \approx 0$, it is possible from formula (13.66) to find value

$$r_{c} = \frac{\partial c}{\partial x_{c}} \sqrt{B_{a}^{0} - \left(\frac{\partial x_{c}}{\partial v_{0}} r_{v_{a}}\right)^{2}}.$$

FIGE # 1045 DCC = 78107118

Value r_{0} , is determined according to the results of ballistic firings with the use of the formula

 $r_{m} = 0.6745$ $\frac{\sum_{l=1}^{n} (\Delta v_{l})^{2}}{1 - 1}$, (18.67)

where $\Delta v_i = v_i - v_{cp}$ - a deviation of the initial velocity on separate shots v_i from arithmetic mean velocity in the group



For evaluating the scattering with the time fire the rockets or the projectiles of classes the "earth/grourd - air" and "air - air" can be used three probable deviation B_R , B_6 and B_n , since the points of discontinuity will occupy the area of space, limited by the volume of dispersion. In this case it is necessary to determine the rotation of the principal axes of the volume of dispersion of the relatively starting coordinate system. One of the axes it is expedient to guide tangentially toward trajectory (value its $\in B_R$), the second axis along the normal to trajectory (value its $\in B_R$), the third - on binormal (value its $\otimes B_6$).

The volume of dispersion is shown on Fig. 13.7. The coordinates

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cf center of dispersion will be Mm Mp Mb



Fig. 13.6. Ellipses of scattering with the firing the unguided rocket projectiles at different arcles of elevation.

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When with firing at locality either vertical barrier/obstacle protable deviations in different directions are identical, is introduced into examination probable circular deviation, frequently calling it simply circular deviation or circular error.

Circular probable deviation is called a radius of circle with center in the center of dispersion, the hit probability to which is equal to 0.5. It is obvious, circular error will be the more than

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appropriate deviations in the direction. The dependence between the probable circular deviation E_{xp} (by circular error) and the probable deviation in the direction takes the form

$$E_{\rm KD} \approx 1,75 \, E$$
 (13.68)

cr, in connection with firing at locality,

 $r_{\rm sep} = 1,75 \, {\rm B_g} = 1,75 \, {\rm B_6}.$

It is obvious, when $B_{\pi} \approx B_{6} \approx B_{\pi}$ it is possible to introduce into examination the sphere of scattering, also, for its characteristic spherical probable deviation. Under spherical probable deviation is understood a radius of the sphere, the hit probable deviation the point of discontinuity into which is equal to 0.5. Spherical probable deviation can be determined through one of probable deviations in the direction of formula

$$E_{c\phi} \approx 2,28 E.$$
 (13.69)

Fig. 13.7. Volume of dispersion.

§4. Errors of firing with induction to target/furpose.

FIGE == 1048

The accuracy of firing and scattering missile trajectories (projectiles) with induction to target/purpose depend on the designation/purpose of rocket, its construction, principle of operation and equipment/device of the control system, i.e., on the method of guidance, maneuverability of projectile, inertness of control and instrument errors [16] accepted. For the index of the accuracy of guidance, is taken the value of the error, under which is understood the vector guantity, which corresponds to the minimum distance between the projectile and the target/purpose (h). With the

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error, equal to zero, let us have the sc-called direct hit.

For the total characteristic of the accuracy of guidance, it is necessary to know the mathematical expectations of components of vector of error h_x , h_y , h_z and the corresponding to them dispersions D_{h_x} , D_{h_y} , D_{h_y} .

Fage 529.

Letermining the characteristics of the accuracy of firing at the three-dimensional random vector of error represents by itself very complex problem; therefore in practice error determines two those comprise, lying in the plane, passing through the vital structural/design assembly of target/purpose so that the vector of error would possibly more correctly characterize the accuracy of guidance. For the approximate estimate of error, it is possible to be restricted to its module/modulus.

Scattering the trajectories of homing missiles is caused by a series of the reasons: by errors, determined by the inertness of control, by the errors, determined by the limitedness of the maneuver of rocket and which depend on the method of guidance and maneuvers of target/purpose and rocket, and by the instrument errors, which are exhibited in the process of the guidance and in the so-called "dead LOC = 78107118 PAGE 52 1050

zcme" cf control, in which the control system for a number of reasons ceases to act.

Among instrument errors first of all cre should note the measuring error of the angular coordinates of target/purpose relative to rocket. Instrument errors, which determine the angular coordinates of target/purpose (coordinators), lead to the fact that missile targeping is realized not on target/purpose, but on the fictitious point, arrange/located on certain distance irom target/purpose. The greater the error for coordinator, the greater the value of error. To correct the error for coordinator guidance system not can, since it accepts the direction, indicated by coordinator, for true.

To the errors for coordinators are devoted at present sufficiently many works both in Soviet and in foreign literature.

Scattering missile trajectories with remote control in the case when measuring device is located or model (for example, television head), is determined by the same sources of errors that and with homing/self-induction, with this vectoring error it does not depend on the range of the control system. In the case of remote control with the arrangement/position of measuring devices (goniometers, radars) on control post scattering trajectories is caused by the errors for determining the coordinates of model with the aid of

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radar, With command and remote control the value of error, caused by instrument errors, increases with an increase in the range of remote-control system. To instrument errors can be also referred the errors, which appear in computer and in command radio link.

While relatively small target ranges and its maneuverings essential effect on error have the errors, caused by the inertness of rocket (limited maneuverability) and of the errors, caused by the inertness of control.

The guidance dispersions as a whole just as purely instrument vectoring errors, are examined in the specialized literature, for example, in [16].

Fage 530.

Let us here examine only dynamic errors, cornected with the inertness of rocket and the motion characteristics of target/purpose and rocket. The action of the dynamic errors, determined by the inertness of rocket, is exhibited in the fact that the rocket reacts to the command/crews of the control system with certain delay, that, it is logical, it leads to about Mach. Dynamic errors depend substantially on the guidance method which determines required normal load factors, and from the character of the maneuvering of target/purpose. ECC = 78107118 PAGE # 1052

Let us examine the error, which depends or maneuvering characteristics of rocket. Maneuverability can be determined either by the smallest possible radius of curvature of the trajectory of the motion of the center of mass or by maximally possible transverse acceleration and, on which depend the normal load factors.

The minimum radius of curvature, frequently utilized during the ccmstructions of the trajectories of the maneuver of rocket and during definition of lethal areas, can be determined by the known formula of the kinematics

$$\varrho = \frac{v_p^2}{a_{np}}.$$
 (13.70)

The normal acceleration of rocket on formula (4.11) is equal

$$a_{np} = v_p \left(\frac{da_p}{dt} + \frac{d\gamma}{dt} \right);$$

angular velocities $\frac{da_p}{dt}$ and $\frac{d\gamma}{dt}$ are determined by the motion characteristics of target/purpose and by guidance method.

If we proceed from the kinematics of the guidance in one plane, then with error function r(t) must have a minimum, and the derivative dr/dt be equal to zero. Figs (4.8) condition dr/dt=0 for an error is CC = 78107118

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fulfilled with

 $v_{\rm g} \cos a_{\rm g} = v_{\rm g} \cos a_{\rm g}$

Bor an example let us cherve the order of value determination of error with missile targeting to target/purpose from the method of pursuit, described in Chapter IV. Let us examine the kinematics of the motion of rocket on the last/latter section of the path before its rendez vous for target/purpose, accepting p=const. A change in the distance between the target/purpose and the rocket and the rate of the rotation of the line of sighting with pursuit guidance to the target/purpose, driving/moving towards, we will obtain that on the tasis of formula (7.109). Let us replace in (7.169) dy/dt from formula (7.114) and we will obtain

$$r = \frac{v_p v_n \sin u_n}{u_n p}.$$
 (13.71)

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In the general case r_{min} it is possible to determine, after applying on right side (13.71) the condition

$$\frac{dr}{dt} = \frac{d}{dt} \left(\frac{v_{\rm p} v_{\rm h} \sin e_{\rm h}}{a_{\rm h,p}} \right) = \frac{d}{dt} \left(\frac{de_{\rm h} \sin e_{\rm h}}{dt} + \frac{d\gamma}{dt} \right) = 0. \quad (13, 72)$$

While maneuvering of target/purpose in the direction and velocity.

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the problem is solved only numerically.

Similarly it is possible to derive the formulas, which determine the error, which depends or maneuvering characteristics of rocket, and for other methods of guidance to target/purpose.

It is necessary to keep in mind that the camage to target is determined not only accuracy of guidance. The calculation of the effectiveness of firing must be carried out taking into account all acting factors, including the factors, which depend on construction and work of the warhead of the rocket. This is the large problem, coming out beyond the scope of this book. ICC = 78107119

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Fage 532.

Charter XIV.

EXPERIMENTAL METHODS OF EXTERNAL FAILISTICS.

The methods of conducting hallistic testings, their crganization, metering equipment and methods of processing test results depend on the construction of the experience/tested projectile and designation/purpose of firing. Extra-ballistic testings depending on designation/purpose can be divided into two large groups - into laboratory testings, conducted, as a rule, with firing from special projectile settings up or artillery instruments, and to the range tests of rockets and artillery shells.

Eaboratory tests are conducted in the specially equipped dashes or on polygon ballistic routes. Firings can be carried out both on those opened and in closed (decompression) routes. The separate types of testings are firings for determining the initial velocity of projectile and characteristics of scattering the initial velocity.
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firing for determining of the aerodynamic characteristics of full-scale specimen/samples and models, firing for determining of the stability characteristics of motion and dynamic qualities of full-scale specimen/sample or model.

Pelygon firings from artillery instruments and by rockets are carried cut for purpose of the determination of the initial conditions of shot, ranging of firing and characteristics of the scatter of points of incidence/drop. When conducting of firings from rifled artillery weapons, can be provided experimental determination of derivation. Each form of the named polygon firings can be carried cut independently in the process of creation and final adjustment of the specimen/sample of armament or composite in the process of subsequent testings and performing work on the creation of firing tatles.

Independent value have the trajectory observations of rockets and projectiles. Trajectory observations in the form of the fulfilled problems can be divided into three groups. The first group is cornected with testing of rockets and projectiles.

Fage 533.

Second group - with trajectory prediction, the detection of the

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places of the missile takeoffs and of firing positions of artillery instruments, the determination of the predicted points of impact in the rockets or proposed targets. The third group of trajectory observations is connected directly with the interception of the driving/moving rockets and projectiles of different designation/purpose.

In the process of creation and final adjustment of the new specimen/samples of the rockets and other flight vehicles final stage are flight tests. They serve for obtaining the most complete information about the work of missile complex as a whole, and also its separate parts.

Flight tests are only testings which make it possible to rate/estimate the behavior of flight veticle directly in flight and to define such parameters of its motion as linear coordinates, velocity, angular displacement, g-force, etc.

As a rule, any the newly created specimen/sample of rocket passes two types of testings - check-cut and combat.

The first are interded for performance checkout of different systems, placed on board rocket, and the flight characteristics of rocket as a whole. Test specimen is equipped with a large quantity of LOC = 78107119

inspection and measurement and other special equipment which is placed on board the rocket instead of some assemblies and the units whose work does not undergo investigations, and their absence does not interfere with the rormal functioning of all remaining cell/elements and units of rocket. So enter, for example, with by warhead and the equipment, which ensures its detriment/blasting.

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All obtained on board rocket information is transferred at tracking station where cocurs its treatment. These stations conduct also external measurements for parameter determination of the motion of rocket.

Combat tests are intended for testing of the military characteristics of rocket. They are conducted for the evaluation of scattering and effectiveness of action on the target/purpose of the newly created specimen/sample. During service firing usually special setering equipment onboard for the rocket is not establish/installed and in the process of testings are conducted only external measurements, which can be fulfilled with the aid of ground-based optical and radio engineering (rader) equipment.

Depending on the method of measurement of the parameters of trajectories, extra-trajectory radio engineering measurements are constructed most frequently according to active and passive DOC = 78107119

FIGE 7069

principles.

The active principle of measurements is characterized by the fact that entire/all metering equipment is arrange/located at tracking station, and the flight vehicle cply reflects the sent to it from station signals.

Eage 534.

This principle of measurements is utilized for the target detection and tracking their motion along trajectories, as a result of which are determined (for example, for kallistic missiles) launching points, the predicted trajectories and the impact points in the acckets.

With the passive principle of external reasurements, basic menitoring-measuring equipment is placed on the Earth and fixes the position of flight vehicle according to the signals of the radio transmitter, establish/installed on board the latter.

§1. Measurement of speed of motion of body on tallistic route.

Gne Of the most widely used methods of determining the rate of the motion of body along trajectory is the method, instituted on the DCC = 78107119

measurement of time t of passage by the body of the section of the route of the specific length 1. In this case, after making assumption about linear change of speed, is calculated its unknown value from the dependence

7060

FAGE

 $v = \frac{1}{2} \mathbf{M}_{\mathcal{K}}^{\mathcal{S}}$

The obtained value of velocity v is related to point in the trajectory A, which coincides with the middle of the measuring section 1 (Fig. 14.1).

Ber decreasing the errors, caused by the averaging of velocity, the length of section 2 selects possible lower, how this allows metering equipment. As it is clear from the essence of the examined method, its realizing metering equipment must consist of two interdependent component/links. The first component/link - these are chronograph, instrument for measuring the transit time by the body of the trajectory phase. The second component/link - these are the looking apparatus, adjustable on the end/leads of the measuring phase of trajectories, at consecutive signals of which occurs the starting/launching and the cessation of the computer of chronograph.

Let us examine in more detail the equipment, utilized for velocity measurement.

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PIGE 7/061

Fig. 14.1.

Fig. 14.2.

Fig. 14.1. For determination of average speed on section of ballistic route,

Fig. 14.2. Block diagram of electronic chronograph.

Rey: (1). V.

Fage 535.

Chrcnograph.

The most widely used at present type of the chronographs, utilized on fallistic routes, is the electronic chronograph (EKh), working according to the principle of the comparison of the measured

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time intervals with the sum of the periods of oscillations of the high-frequency crystal oscillator of the electrical signals. The block diagram, which elucidates device work of one of such chronographs, is given in Fig. 14.2.

In this schematic:

BU1 and BU2 - respectively more front/leading and rear locking affaratus: UPSB - amplifier-convertor of the signals of blocking; EP - electronic breaker (switch); KGECh - quartz high-frequency cscillator; ESU - electronic computer; VS - controlled rectifier.

Chronograph is supplied through the unit VS from the grid/network of alternating current with voltage/stress ~220V.

The measurement of time by chronograph is conducted as follows. In transit through EU1 the body whose velocity must be measured appears the electrical signal, which is appropriately converted and is amplified by the unit LESE and it passes to unit EP.

The high speed electronic interrupter (triggering time its ~1.10⁻⁶ s) at the signal from blocking EU1 closes the circuit, which connects computer with crystal oscillator. In this case, alternating current from KGVCh with period of T = 1/f begins to pass into ESU,

DOC = 78107119 FIGE \$1063

which computes a quantity of entered it momentum/impulse/pulses. At the moment of the cutput of body from measuring section at the signal from EU2 again wear/operates the unit FF, but already disconnecting KGVCh from ESU. Thus, for the time of the motion of body along the measuring phase of trajectory the FSU will record the specific quantity of the entered it periods 1. If the number of periods is equal to n, then the unknown time

t=sT.

In the chronographs, utilized during fallistic measurements, are applied KGVCh, that develop current with frequercy not less than f=10.05 Hz, i.e., with the cscillatory period, it is not more

T=1-1.10-5

Error due to the instability of the frequency of KGWCh they virtually disregard, since it does not exceed 0.010/0.

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Error from a reading error the measured time interval comprises nct more than one period, since the teginning of the measured time interval can either coincide with the momentum/impulse/pulse of generator or render/show in spacing tetween pulses. LCC = 78107119

PAGE 10-10-4

Error $\Delta t=T$ is for FKh maximum over entire range of the measured time intervals.

The computer of chronograph can take a reading of momentum/impulse/pulses either on decimal or along binary system; can be used the combined system. Figures 14.3 shows the schematic of the work of ESU in the decimal system of count. Consists this ESU of several identical computing decades, assembled on electron tubes or transistors.

Buring the measurement of time, the current from KGVCh with frequency f proceeds to the tenth momentum/impulse/pulse from decade No. I it goes to decade No. II which counts already ten periods, and so on. The passage of the signals through decades is noted on display unit from mean bulbs for visible reading (indicators can be and arrow type).

At the moment of the cessation of ESU at the signal BU2 on indicator of each decade, is fired the bulk, which corresponds to the rumber of pasts through this decade of periods.

Thus, for instance, in the position of the cessation of count, shown on Fig. 14.3 (underkered small diroles - the burning neon bulks), computer fixed a quantity of momentum/impulse/pulses n=3851; ICC = 78107119

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this corresponds to the measured flight time

$t=nT=3851\cdot 10^{-5}=0.03851$

The lisplay system of the chronograph whose computer is instituted on the use of a binary count, is shown on Fig. 14.4. It includes 12 cells, equiffed each by the necr bulb (number of cells can be other). Ignition with the cessations of the chronograph of any neon bulb corresponds to the passage through this cell of the computer of the quantity of momentum/impulse/pulses, equal to number $n_i=2^{m-i}$; where N - number of cell. Number this (n_i) calls the "value" of cell. The "values" of each of 12 cells are given to Fig. by 14.4.

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Fig. 34.3. Schematic of recording the measured time interval with decimal system of count.

Key: (1). s. (2). From. (3). Hz.

Fage 537.

Thus, total number of mcmentum/impulse/pulses, which entered the chronograph for the time of the motion of Ecdy, will be determined from burning neon bulbs taking into account their "value". For example, in the position, shown in Fig. 14.4, computer recorded the number of periods

 $n = 2^{0} + 2^{1} + 2^{2} + 2^{5} + 2^{7} + 2^{0} + 2^{10} + 2^{11} = 3851,$

that with f=10⁵ Hz gives the flight time

t = nT = 0.03851 g.

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Iccking apparatus.

The forms of locking apparatus known much - are applied electrostatic, capacitive, photoelectric, accustic inertia, etc. of blocking. Let us examine for an example some of the blocking cell/elements.

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Prames-targets. Frame-target (Fig. 14.5) represents by itself the flat/plane frame, usually square. The size/dimension of frame can change depending on bore d of model whose velocity will be measured. To frame isolated from it is wound, as shown in figure, fine/thin wire with a diameter of $d_{mp}=0.20-0.25$ **so** (tirsel).

Frames-targets are establish/installed on the route so that their planes would be perpendicular to the trajectory of the motion of body, and their windings are included in the unit UPSB of chronograph and are supplied by direct current. With passage through the plane of frame-target, the body breaks its winding, what is the signal for starting/launching or cessation of ESU of chronograph.

For providing the reliable interruption with the driving/moving body of the winding of frame-target the distance between adjacent turns of wire 1 is taken from condition $1 \leq 0.25d$, and the winding/coil of wire is conducted with certain interference in order to decrease the possibility of its separation into sides and drawings. For the same purpose during measurements v of the bodies of the low caliber of the wire of frame-targets, they stick or fime/thin paper.

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Fig. 14.4. Schematic of recording of measured time interval in the binary system of count.

Key: (1). cell. (2). Value cf cell.

Fage 638.

Tc frame-targets are characteristic the following operational deficiency/lacks:

- is necessary to restore the completeness of their windings after each measurement;

- occurs certain nonuniformity in the interruption of the wires of winding, which leads to the unavoidable errors in value v.

Nevertheless frame-tarcets, in view of their reliability and simplicity, are utilized very widely.

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Sclenoid blocking. The sensing cell/element of the given blocking is the solenoid with double winding, placed in metal casing (Fig. 14.6). one winding of sclencid is supplied by direct current, creating constant magnetic flux. With passage within the sclencid of the metallic body, which possesses magnetic properties, changes the magnetic flux (as a result of change) of magnetic permeability) and in inducing winding of sclenoid, connected with UPSB, it is induced the pulse signal of currert.

This signal - weak and in the unit of LESE it is amplified and ccqverted into the signal with steep wave ficnt, convenient for the cperation of unit EP of chronograph.

Sclenoid blocking possesses a series of the advantages:

- allows prolonged regeated use, since do not require any process/operations on the restoration/reduction of their completeness;

- does not distort the flight of the tody teing investigated;

- it gives a precise and uniform time mark of passage by the body of the plane of solenoid.

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however, the application/use of sclencids as looking apparatus requires conducting firings by the magnetized lodies and it is conjugate/combined with the complication of the units UPSB of chronographs.

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Fhotoelectric blocking. Recently, in connection with the appearance of low-inertia photocells, for laboratory ballistic routes they will begin to use extensively photoelectric blocking. The exemplary/approximate schematic of the separate assembly of photo-blocking is shown on Fig. to 14.7.



FAGE

Fig. 14.5.

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Fig. 1446.

ween a ser

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Pig. 14.5. Wire frame, target,.

Key: (1). To chronograph.

Fig. 14.6. Solenoid blocking cell/element.

Eage 539.

It consists of tube L, low-inertia photocell FF and three-stage amplifier with thyratron.

With the passage of the model of the Edy Letween the tube and the photocell, it intersects light ray, causing an instantaneous change in the illumination of photocell. The appearing in this case LOC = 78107119

FAGE

electrical signal, in passing by through the amplifier, triggers the thyratron of electronic relay whose cutput pulse enters the electronic chronograph EKh; including or disconnecting the computer of the latter. Photoelectric blocking provides the high accuracy of time marks.

Inertia blocking. Cn laboratory tallistic routes, mainly for artillery practice from small arms, is utilized inertia blocking.

The construction of the inertia blocking cell/elements can be different. One Of the constructions, the so-called "contact cup", it is shown on Fig. to 14.6. This contact inertia cell/element, connected to the entry of chronograph, is fastered on the metallic or plywood plate, placed at the end of the route. From the impact/shock of the model of body into plate, the latter shudders and contact bar to instant rebounds from the wall of our. The occurring with this phenomenon explosion of electrical circuit, as in frame blocking, it is the signal for a chronograph.

The advantage of the inertia looking apparatus in the fact that the contact in them is restored after each alot automatically (unlike frames-targets). However, to be applied for blocking the route they can only as last/latter cell/element, since sharply is changed the character of the motion of model along trajectory. The length of the ENGE 173

phase of trajectory ; for velocity measurement of bodies with firing from artillery or rifle systeme is selected on the basis of the required accuracy of the determination of velocity and possibilities of chronograph. DOC = 78107119



FIGE 274

¥ig. 14.7.

Fig. 14.8.

Fig. 14.7. Photoelectric blccking cell/element.

Key: (1). Amplifier.

Fig. 14.8. Inertia blocking cell/element.

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Since the velocity in the described sethed is defined as



that the greatest relative error in its determination will be

$$\frac{hv}{v} = \frac{hl}{l} + \frac{hl}{l}.$$

Here 41/1 - relative fault of measurement of base 1; At/t - relative

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measuring error for the flight time of body on chronograph.

Let according to test conditions be required to obtain this accuracy of determination v, in order to $\Delta v_{1}v_{1}c_{2}o_{0}$. This will be, for example, provided in such a case, when $l_{1}l_{1}c_{0}$. 10/0 and $\frac{M}{4} < 0.1$ %.

Since for the electronic chronograph the error M=T=const, we obtain $\frac{T}{r} < 0.1\%$, whence tyl0007. For a special case, with $T=1\cdot10^{-5}$ with tyl0.01 s. Based on this minimum interval of time there is established the length of the measuring section], equal to 1, wt=0.01v to m, where v - expected rate of the motion of body. Knowledge 1 makes it possible to find the permissible error A1 in the setting up of blocking devices from relationship/ratio A1<0.001 2 m. With the observance of two these conditions $\Delta v/v \leq 0.20/0$.

Buring the definition of the initial velocity of the projectile of rifled artillery system, the locking apparatus place along trajectory in the manner that it is shown or Fig. to 14.9. In this case, distance 1 is selected in accordance with considerations presented above, and removal/distance l_0 of first blocking device from the point of the flight of projectile - so that on BU1 would not affect the escaping from projectile setting up gases. Value l_0 depends on the form of projectile setting u_1 , but usually does not enceed 20-30 m (duming frame blocking). The average measured velocity of projectile $u_{ep}-v$ can be referred to the half of the distance between frame-targets. For velocity transformation to mumber of projectile end BOC = 78107119

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face, i.e., for determining the initial velocity v_0 , they use the formula, obtained in the analytical method of pseudovelocity in connection with the short low trajectories

(14. 1) D(+)---D(+) = ex.

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Accepting for a special case in question the linear dependence between the change in the velocity and the change in function D(v), it is possible to comprise the proportion

$$\frac{D(r) - D(r_0)}{r - r_0} = \frac{\Delta D(r)}{10}, \qquad (14.2)$$

where $\Delta D(\mathbf{v}) - \mathbf{a}$ change in function $\Gamma(\mathbf{v})$, that corresponds to a change in argument \mathbf{v} on 10 m/s.

After designating $v-v_0=\delta v$ and comparing (14.1) and (14.2), we will obtain

$$\frac{\Delta D(v)}{10} \Delta v = cx.$$

Opening the value of the fallistic ccefficient of c, we will obtain correction for velocity transformation to the suzzle end face

$$\Delta \nabla = \frac{M^2}{Q} \, 10^4 \, \frac{x}{\Delta D(\tau)} \,, \quad (14.3)$$

where x - a distance between the suzzle end face and the middle of

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the blocked phase of trajectory (Fig. 14.9)0; i - an experimental factor of the form of projectile (kullet). Values $\Delta D(v)$ in connection with the Siacci function of air resistance can be taken on table 31, given in work [59].

§2. Determining aerodynamic characteristics according to the results of ballistic firings.

Cne Of the most important aerodynamic characteristics - drag coefficient $e_m(M)$ it can be determined by firing on ballistic route.

The calculated dependence, which makes it possible to determine the average value of drag coefficient on the final out of trajectory, proceeds from the law of a change in the kiretic energy of projectile and requires the measurement of the velocity of projectile in two points in the trajectory. The diagram of installation of looking apparatus along ballistic route is shown or Fig. **14.10.** If projectile setting up (for example - montar), does not make it possible to have horizontal trajectory, then looking apparatus along trajectory are establish/installed on different height/altitude (Fig. 14.11). Let in the general case be known the parameters of trajectory v, x, y at two different points; then it is possible to write

 $\frac{mn_1^2}{2} - \frac{mn_1^2}{2} = \int X \cos(X, S) dS + \int mg \cos(g, y) dy. \quad (14.4)$

where z - mass of body; X - drag.

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Fig. 14.9. Arrangement/Ecsition of the Electing cell/elements along trajectory for measuring the initial velocity.

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Taking into account that $\cos(x, \bar{x}) = \cos(\bar{x}, \bar{y}) = \cos 180^{\circ} = -1$, we obtain

$$\frac{m}{2} (v_1^2 - v_2^2) = \int_{s_1}^{s_1} X dS + mg(y_2 - y_1). \quad (14.5)$$

Force X from under integral sign they will carry by average value, then

$$\frac{m}{2}(v_1^2 - v_2^2) - mg(y_2 - y_1) = X_{ep}(S_2 - S_1)$$

and further

$$X_{ep} = \frac{m}{2} \cdot \frac{v_1^2 - v_2^2 - 2g(y_2 - y_1)}{S_2 - S_1}.$$
 (14.6)

Usually firing for determination $e_n(M)$ or tallistic route is crganized so that the measured trajectory place would render/show horizontal for the possibility of the exception/elimination of the gravitatonal work (see Fig. 14.10). In this case, obviously, let us have

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Taking into account that indicated

 $X_{-}=\frac{a}{2}\frac{a^{2}-a^{2}}{L}$ (14.7)

Each chtained the data of experiment value \mathcal{X}_{ep} must be referred to a certain value of the velocity of article for which is accepted the average speed at the length of measurement I, equal to

 $v_{ep} = \frac{v_1 + v_2}{2}$

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Fig. 14.10. Setting up of the blocking frames during determination c.(M) by firing.

Fage 543.

For transition from the force of drag X_{ep} to scrodynamic coefficient $c_n(M)$ is utilized the dependence

$$X_{cp} = S \frac{\varrho \sigma_{cp}^2}{2} c_s \left(\frac{v_{cp}}{s}\right), \qquad (14.8)$$

where ρ and a - a respectively mass air density and the speed of scund in air at the moment of experimentation; ε - area of the midsection of body. From the comparison of expressions (14.7) and (14.8) for χ_{cp} it follows

$$\frac{m}{2}\left(\frac{v_1^2-v_2^2}{L}\right)=S\frac{0}{2}\left(\frac{v_1+v_2}{2}\right)^2c_x\left(\frac{v_{cp}}{a}\right).$$

Hence unknown value $c_x(M)$ is equal

$$c_{g}\left(\frac{\mathbf{v}_{cp}}{a}\right) = \frac{4m}{SL_{0}} \frac{\mathbf{v}_{1} - \mathbf{v}_{2}}{\mathbf{v}_{1} + \mathbf{v}_{2}}.$$
 (14.9)

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Final calculated dependence, after combination of all constant values and scale factors, takes the form

$$r_x\left(\frac{v_{up}}{a}\right) = 5,09 \frac{Q}{a^2 L I7} \frac{v_1 - v_2}{v_1 + v_2}.$$
(14.10)

where d - a diameter of the midsection of Ecdy; P - the specific gravity/weight of air.

During calculation the entering the formula values must be taken in the following dimensionality:

The values of the specific gravity/weight of air P for the different values of the barometric pressure h mm Hg and the temperature of air $t^{o}C$ must be determined from the appropriate tables.

The length of base L for measurement c_m must be selected so that is justified the averaging of value X and, at the same time, is provided the noticeable incidence/drop in the velocity of body, necessary for obtaining of the required accuracy of calculation c_m .

Under conditions of range/polygon, usually, base L is taken equal to 200-400 m; under laboratory conditions when the length of tallistic route is limited, it is allow/assumed to take the base of the equal to several of ten meters, depending on the velocity v of the metion of body and possibilities of chronographs.



Fig. 14.11. Blocking inclined trajectory.

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Aerodynamic functions with tilting moment $K_M(\frac{\Psi}{\sigma})$ and under lift $K_N(\frac{\Psi}{\sigma})$ also can be determined by results of firing. For this, during experiment must be measured the cell/elements of the rotary motion of projectile δ_r , $\dot{\delta}_r$, ν_r , ν_r , after which the named coefficients are determined from the appropriate calculation formulas.

Are known two methods of determining argular missile attitude in the process of its motion along trajectory - method of photography in two mutually perpendicular planes and method of firing at cardboards. In the first case the argles of the slope of the longitudinal axis of projectile to the direction of the motion of the center of mass are determined from photographs, in the second - these values they can be obtained by the measurement of holes in the specially processed fine/thin cardboard. During the coircidence of the longitudinal axis ICC = 78107119



cf projectile with velocity vector, the hole in cardboard has a form of cincumference. If nutation angle $6 \neq C$, then hole has oval form (Fig. 14.12). Angle, comprised by the longitudinal axis of oval with vertical line - precession angle w. Recwire the size/dimensions of projectile, according to the size/dimensions of the axes of oval a and a, it is possible to determine rutation angle 6.

With the sufficiently large number of cross sections, which fix missile attitude on each of the periods of a change in the mutation angle 6, it is possible to obtain experimental curve/graphs (major).; Eatween the cross sections, which fix missile attitude, are establish/installed the locking apparatus, which make it possible to measure the transit time of the projectile through the appropriate cross sections. In summation, it is possible to obtain the experimental dependences





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Fig. 14.12. Form of hole ir pastebcard target.

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As an example let us examine determinetice in the assumption that the dynamic axis of equilibrium coincides with the velocity vector of the center of mass of projectile. We will use the formulas, obtained in Chapter VIII, §4. From (6.81) it is possible to write that the half-period of the fluctuation of angle 6 will be equal to

$$\frac{T_1}{2} = \frac{\pi}{e \sqrt{e}}$$

ubence.

$$\mathbf{s} = \frac{4\pi^2}{\mathbf{s}^2 T_1^2} \,. \tag{14.11}$$

FICE (8.79) let us find

β=a⁶(1-σ)

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and, after substituting (14.11), we will citain

$$\beta = e^2 \left(1 - \frac{4e^2}{e^2 T_1^2} \right) \cdot$$

The aperture of value α and β and after replacing r_0 on (8.84), we can write

$$\frac{d^{2}k}{gA} 10^{6}H(y) \nabla^{2}K_{A}\left(\frac{v}{a}\right) = \frac{C^{2}w^{2}v^{2}}{\sqrt{4}} - \frac{4w^{2}}{T_{4}^{2}}$$

With firing at small argles of increase H(y)=1. Then the average value of coefficient $\mathcal{R}_{H}(\frac{v}{e})$ when v also is taken by the average value and the angular velocity of spin r=r_o=corst, we will obtain equal to

$$\widetilde{K}_{AI}\left(\frac{v}{a}\right) = \frac{\pi^{2}q}{a^{2}bv^{2}l0^{2}} \left(\frac{C^{2}a_{0}^{2}}{\lambda q^{2}a^{2}} - \frac{4A}{T_{0}^{2}}\right).$$
(14.12)

Walue h is difficultly defined better theoretical and experimentally and, strictly speaking, it it is alternating/variable during the fluctuations of projectile in the process of motion. Euring the use of known empirical formulas for determining of h, for example, the formula, given in explanation to (2.111), it is accepted b=const. The tables or the curve/graphs, comprised in this case for $K_{M}(\frac{v}{e})$. must contain indication of the method of determination h. DOC = 78107119

Are known also the methods of determination according to the results of firings on the hallistic route of the aerodynamic coefficient of lift and of the coefficient of the damping moment. Methods these are complex and the accuracy of the obtained results is inferior to the accuracy of the results of experiment in wind tunnels. Por familiarization with the methods indicated let us send away the reader to books [9], [59] and the thematic works on hallistic testings.

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§3. Theoretical bases of the methods of determining the position of rocket.

The position of body in space at the any moment of time is determined by six generalized coordinates. Three Linear values determine the position of the center of mass of rocket, while three angular - its orientation in space. These angles frequently call Fuler angles.

Let us examine the possible methods of determining the position of rocket and calculated dependences for calculating the coordinates. DCC = 78107119

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The cocrdinates of rocket in space can be defined as coordinates of the point of intersection of different surfaces. The surface, which determines the possible aircraft attitude at the given instant, frequently calls the surface of position or position surface. Three Geometric surfaces during intersection have one or several common points. Thus, with the aid of three surfaces of position it is possible to fix the location of rocket at any point of space. The type of position surfaces depends on the utilized ground-based metering equipment.

If we simultaneously follow the rocket from three ground-based point/items and to calculate its removal/distances (slant ranges) from each of the stations, then position surfaces will be spheres with centers in the point/items of the tracking, described by the radii; equal to slant ranges.

The method position finding of flight vehicle in space as of four of intersection of three spheres is called ranging.

We will obtain formulas for coordinate determination of the rocket in of the terrestrial system Cxyz from the measured from three point/items of tracking slart ranges. Let the tracking stations be arrange/located in points $C_1(x_1, y_1, z_1)$. $\bigoplus O_1(x_2, y_2, z_2)$ and $O_1(x_3, y_3, z_3)$ (Fig. 14.13). Let us designate the distances between tracking stations and the beginning of the system of coordinates j_1 , l_2 , l_3 . Bith sespect slant ranges to point in the trajectory $\mathbb{E}(x, y, x) =$ through ρ_1 , ρ_2 , ρ_3 , distance OM - through ρ_4 .

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Fig. 14.13. Ranging method of position finding of rocket.

Ref: (1). Trajectory.

Eage 547.

From analytical geometry it is known that



and

Analogously are determined values ρ_2 , ρ_3 and l_2 , l_3 .

The length of cut CM can be represented in the form

(14. 15)

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Let us discover equation (14.13), after substituting in it equations (14.14) and (14.15)

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$$q_1 = \sqrt{l_1^2 + q_2^2 - 2(x_1x + y_1y + x_1z)}.$$
 (14. 16)

Dependences for determining the values p_2 and p_3 take the same form, only of the entering in them values index 1 is replaced by index 2 or 3 respectively. After comprising expressions for differences $p_{1}^{2}-p_{2}^{2}$, $p_{1}^{2}-p_{3}^{2}$ and $p_{2}^{2}-p_{3}^{2}$ and after re-grouping terms, we will obtain

$$(x_{1}-x_{0})x + (y_{2}-y_{0})y + (z_{1}-z_{0})z = c_{1};$$

$$(x_{1}-x_{0})x + (y_{2}-y_{0})y + (z_{1}-z_{0})z = c_{0};$$

$$(14. 17)$$

$$(x_{2}-x_{0})x + (y_{0}-y_{0})y + (z_{0}-z_{0})z = c_{0};$$

where

$$c_{1} = \frac{1}{2} (\vec{a} - \vec{a} + \vec{l} - \vec{a});$$

$$c_{2} = \frac{1}{2} (\vec{a} - \vec{a} + \vec{l} - \vec{a});$$

$$c_{3} = \frac{1}{2} (\vec{a} - \vec{a} + \vec{l} - \vec{a}).$$

System of equations (14.17) further is converted as follows:

where

$$\begin{array}{c} \mathbf{a}_{1} = \frac{\mathbf{a}_{1} + \mathbf{a}_{2} + \mathbf{a}_{3} + \mathbf$$

FAGE US

Page 548. ▮

For determining the coordinate y, it is possible to utilize the dependence, obtained from equation (14.13) after the substitution of values x and z from expressions (14.18)

$$(a^{2}+b^{2}+1)y^{2}+2(x_{1}a-y_{1}+z_{1}b-ac_{1}-be_{2})y-$$

-($e_{1}^{2}-e_{1}^{2}-e_{1}^{2}-e_{2}^{2}+2x_{1}e_{1}+2z_{1}e_{2})=0.$ (14. 19)

Usually they attempt to arrange/locate all tracking staticns on the same level, i.e., in plane xCz. In this call $y_1=y_2=y_3=a=b=0$ and the coordinates of rocket are determined from the formula

$$x = e_{1};$$

$$y = \sqrt{e_{1}^{2} - l_{1}^{2} - e_{1}^{2} - e_{2}^{2} + 2x_{1}e_{1} + 2z_{2}e_{3}};$$

$$z = e_{1}.$$

$$(14.20)$$

Buring external measurements frequently is utilized the so-called total ranging method when the position of rocket in space is defined as point of intersection three ellipscids of revolution. DOC = 78107119 EAGE

It is known that the point, which belongs to ellipse, possesses that property that the sum of distances from it to foci remains constant; any point of ellipsoid possesses this same special feature/peculiarity.

Fig. 14.14 illustrates the application, use of a total ranging method for the determination of the position of rocket on plane.

We will obtain dependences for coordinate determination of rocket in earth-based coordinate system Oxy2, after using Fig. 14.13. Let us consider that in the teginning of coordinates is located sc-called master station; and three wirgmer (receiving) are arrange/located in points C_1 , O_2 and C_3 .

In the process of tracking the recket there are determined the sums of the slant ranges

Qa=Q+Q1; Qa=Q+Q; Qa=Q+Q; (14.21)
DOC = 78107119 EAGE 1097



Fig. 14.14. Total ranging sethed of pesitics finding of rocket.

Key: (1). Trajectory.

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After substituting into the left side of equation (14.16) value r_1 from equations (14.21), we will ditain

$$x_1 x + y_1 y + z_1 z - 0.0 u = f_1, \qquad (14.22)$$

where

$$f_1 = \frac{1}{2} (t_1^2 - e_1^2).$$

Otilizing two second equations (14-21), analogously we can write

$$x_{s}x + y_{s}y + z_{s}z - Q_{s}Q_{ss} = f_{s}$$

$$x_{s}x + y_{s}y + z_{s}z - Q_{s}Q_{ss} = f_{s},$$
(14.23)

where

$$f_2 = \frac{1}{2} (l_2^2 - q_{02}^2); \quad f_3 = \frac{1}{2} (l_3^2 - q_{03}^2).$$

In three equations (14.22) and (14.23) - four unknown values x, y, z and ρ_{0} . For their determination these equations must be solved together with equation (14.15).

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Are given below expressions for calculating the coordinates of fount M in the case wher leading and slave stations are arrange/located in plane xCz

$$\begin{aligned} x &= \frac{1}{A} \left[f_1 (z_g \varrho_{00} - z_g \varrho_{(0)}) - z_1 (f_g \varrho_{00} - f_g \varrho_{00}) - \\ &- \varrho_{01} (f_2 z_3 - f_3 z_3) \right]; \\ y &= \sqrt{\varrho_{0}^2 - x^3 - z^5}; \\ z &= \frac{1}{A} \left[x_1 (f_g \varrho_{00} - f_g \varrho_{03}) - f_1 (x_g \varrho_{02} - x_g \varrho_{03}) - \\ &- \varrho_{01} (x_2 f_8 - x_8 f_3) \right]; \\ \varrho_0 &= \frac{1}{A} \left[x_1 (z_2 f_8 - z_6 f_8) - z_1 (x_6 f_6 - x_8 f_8) + \\ &+ f_1 (x_g z_0 - x_g z_3) \right], \end{aligned}$$
(14.24)

where

$$A = x_1 (z_{gQ_{10}} - z_{gQ_{10}}) - z_1 (z_{gQ_{10}} - z_{gQ_{10}}) - Q_{44} (x_{g} z_{g} - x_{g} z_{g})$$

Method, instituted on the definition of the position of rocket as points of intersection of three hyperbolcids of rotation, is called differential ranging. On plane the location of rocket with the aid of this method is defined as point of intersection of two hyperbolas (Fig. 14.15), property of which is the fact that a



difference in the distances from foci to the point, which lies on bypertcla, there is a constant value.

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Same of the second

Betermination of coordinates in differential ranging method is conducted on formulas (14.24) by replacement ew on Aev, where

$\Delta \varrho_1 = \varrho_0 - \varrho_1; \quad \Delta \varrho_2 = \varrho_0 - \varrho_2; \quad \Delta \varrho_3 = \varrho_0 - \varrho_3.$

The position of rocket can be also unarbiguously defined as intersection of two ellipsoids and sphere, two hyperboloids and sphere, two spheres and ellipsoid, etc.

Frequently the location of rocket is defined as point of intersection of sphere, cone and plane (Fig. 14-16). In this case the position of rocket will be determined by the spherical coordinates: elant range p, by the bearing angles A and of place E. Transfer/transition from them to the coordinates of rectangular system is realized on the dependences

$x = e \cos \epsilon \cos A; y = e \sin \epsilon; z = e \cos \epsilon \sin A.$ (14.25)

Ecsides spherical cccrdinates, for determining the three-dimensional/space aircraft attitude scmetimes there is applied cylindrical system. In this system they are measured the bearing

FIGE 45

angle of object A, the projection of cut OF on plane xOz - f and removing of point H from this plane - h (Fig. 14.17).

Dependences for transfer/transition from cylindrical coordinates to rectangular take the form

$x = l \cos A; y = h; x = l \sin A,$ (34.90)

Wide acceptance will citain the direction-finding method of coordinate determination of object. It lies in the fact that from two tracking stations 0_1 and 0_2 with the aid of special equipment are measured angular coordinates of body A_i and a_i but according to the bearing angles and place of object and the known base distance ietween B stations are determined the coordinates of point M (Fig. 14.18).



FAGE 1096

Fig. 14.15.

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Fig. 14.16.

Pig. 14.15. Differential ranging method of position finding of rocket.

Keg: (1) - Trajectory.

Fig. 14.16. Position finding of rocket in spherical coordinates.

Fage 551.

Frequently for convenience in the calculations instead of the azimuths, are determined angles on calculated off direction in one of the tracking stations, to the laurching point of the rocket or to ary another characteristic I turn/sharpen the earth's surface. Transfer/transition from these angles to azimuths (and vice versa) does not represent difficulties and is fulfilled on simple analytical COC = 78107119

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dependence. For example, in connection with Fig. 14.18

 $\bullet_l = \mathbb{A}_l - \mathbb{A}_{s_l}$

where A_i - angle, calculated from local meridian to direction in target/purpose: A_{bi} - angle between the local meridian and direction from point/item 0_1 in point/item 0_2 .

We will obtain formulas for coordinate determination of rocket in connection with the direction-finding method of external measurements. Let the point/items of tracking be arrange/located on plane x0z in the points 0_1 and 0_2 , the distance between which is equal to B (Fig. 14.19) 1.

FCCTNOTE 1. The topographic excess of the measuring posts above plane ucz into our conclusions it is not considered. FNDFOOTNOTE.

As main direction for the reading of horizortal angles, let us select the line, determined by the axis 0_1x of earth-based coordinate system which passes through points C_1 and C_2 .

For simplicity we consider that axis () coincides with direction sever - Yug; in this case, the measured angles will be azimuths.

Let us designate distance (0,0), through 1_1 and (0,0), through 1_2 . From (0,0), we have





Fig. 14.17.

Fic. 14.18.

Fig. 14.17. Position finding of rocket in cylindrical coordinate system.

Fig. 14.18. Angular coordinates, used in direction-finding method for rcsiticn finding of rocket.

Key: (1). North. (2). Scuth.

Fage 552.

Taking into account this for the coordinates of point M in system $O_1 x y z_0$ we will obtain

$$x = \frac{B}{\sin (A_2 - A_1)} \sin A_3 \cos A_1;$$

$$y = \frac{B \sin A_2}{\sin (A_2 - A_1)} \operatorname{tg} e_1 = \frac{B \sin A_1}{\sin (A_2 - A_1)} \operatorname{tg} e_3;$$

$$x = \frac{B}{\sin (A_2 - A_1)} \sin A_3 \sin A_2.$$
(14. 27)

14

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F & G E

Values x, y, z can be given to another form, if one considers that $\sin A = \frac{2}{3} \tan A$

$\sin A_1 = \frac{1}{y} \log a_1;$	(14. 28)
sin A ₂ == - # tg eg.	(14. 29)

After substituting (14.28) and (14.29) into formula (14.27), we will cbtain



Together with the direction-finding method of determining the position of rocket, is applied also the contined ranging-direction-finding method, in which they are measured slant range and angular coordinates - angles of azimuth and place of target/purpose.



PAGE 51100

Fig. 14.19. Schematic tc the derivation of relationship/ratios for a direction-finding method.

Key: (1). Trajectory. (2). South. (3). North.

Fage 553.

Thus, all the existing methods of the external measurements are reduced in essence to the measurement of slent range and the angular coordinates, which determine direction in rocket. At tracking stations, these measurements can be conducted with the aid of optical and radio equipment.

§4. Optical measurements.

When conducting of external measurements, very widely is

FAGE SEIJOI

utilized optical equipment (cinethecdolites and cinetelescopes, tallistic cameras, apparatuses for high-speed/velocity photography, etc.), which for this purpose was adopted considerably earlier than radio engineering.

The basic advantages of optical measurements in comparison with radio engineering are their clarity, since optical equipment makes it possible directly visually to control the process of moving the flight vehicle and to photograph it for the subsequent analysis (for example, with missile takeoff, stage separation, the interception of target/purpose, etc.), and the high accuracy of the determination of angular coordinates of of cliect.

Therefore optical equipment part is utilized for calibrating the radio engineering measuring systems. Thus, for instance, interferometric system "Minitrek" is calibrated by optical instruments with an accuracy to 2" [16]. At the same time the work of optical equipment lends itself worse to automation and it depends substantially on the state of the atmosphere at the moment of the measurements which, as a rule, are conducted by comparatively small distances to flight vehicle. In view of this optical measurements do not eliminate radio engineering; they successfully supplement each other. CCC = 78107119

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For conducting the external measurements on the individual sections of the missile trajectories of long-range, for the photographing of short trajectories, for secring of air-to-air missiles and the "earth/ground - air" is applied the cinetheodolite method which makes it possible to find not only three-dimensional/space occidinates of flight vehicle, but also its velocity, but in certain cases also acceleration and a series of other parameters of motion. It realizes the direction-finding method of measurements and lies in the fact that ficm two (or three) cinetheodolite posts they simultaneously conduct tracking the motion of one object, they photograph it and determine its angular coordinates (azimuth and argle of elevation) in the function of time. As a rule, in cinetheodolite station enter three posts of tracking. One of them fulfills auxiliary function and car be used for the control of coordinate determination of cipect.

Eag€ 554.

Furthermore, simultaneous tracking from three cinetheodolite stations makes it possible to exclude random measuring errors with the cut of cbject at acute angles, and also during photography against the sur-

Basic part of any cinetheodolite - the sain thing 1 and sighting 2 telescopes (Fig. 14.20) whose optical axes are parallel. Sighting DCC = 78107119

FAGE S

telescores (sights) play auxiliary role and serve for the facilitation of the process of the guidance of the optical axis of main (measuring) telescore for the objective of photography.

Usually cinetheodolite for the increase of the accuracy of the tracking is serviced by the operators one of which aims main telescope at flight vehicle along azimuth, and another - on angle of elevation. The position of the object of the relatively optical axis of main telescope with the aid of mention is continuously is detented to motion picture film. The gates of the camera/chambers of all cinetheodolites of one station are synchronized between themselves with the aid of special electrical circuit with high degree accuracy, and entire/all photography it is conducted automatically. Thus, the profilem of operators is reduced only to visual tracking the object and the retertion of object on the optical axis of main telescope (in the center of orcess lines).

The photographed motion picture film includes entire basic information about the motion of flight vehicle, written automatically in the process of tracking.





Fig. 14.20. Cinethecdolite.

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Cn each exposure are recorded the conditional index of the cinetheodolite, from which is conducted the observation, the number of frame for joining it to time, the reference grid (or cross lines), which determines the position of the optical axis of main telescope, the image of object and the goniometrical scales with the fixed values of azimuth and argle of elevation, which determine the direction of the optical axis of main telescope.

Figures 14.21 gives exposure, photographed in the process of

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cinetheodolite measurements. In left lower angle is photographed the genicmetrical scale for the reading of azimuth, halfway - the scale for the reading of angles of elevation, while is right lower to angle are given the number of exposure and the index of theodolite. The reading of angles is conducted on the indicators which are establish/installed against the appropriate divisions of the genicmetrical scales. As a rule, exposure counter initially to zero it is not establish/installed; therefore for the designation of the beginning of the synchronous working of all three cinetheodolites, is applied the special mark of personrel/frames.

FIGE 50 /

At the moment of the "grip/capture" of object by the main telescope of cinetheodolite, and also in the process of tracking the fast moving target/purposes due to the possible errors for operators the image of object and the center of cross lines of reference grid, as a rule, do not coincide. This speaks about the fact that the object of photography is displaced relative to the optical axis of telescope and for the precision determination of its angular coordinates with the interpretation of film should introduce corrections into the bearing angles and place.



EAGE 57/104

Fig. 14.21. Exposure, photographed with the aid of cinetheodolite. Fage 556.

The signs of corrections depend on the location of object of relatively reference grid. Thus, for instance, if object is located in the first fourth, angular corrections have plus sign, if in the third, then corrections must be deducted from readings of the genicmetrical scales, etc.

For decreasing the following errors, the guidance of cimetheodolites to moving object can be realized and it is automatically - with the aid of radar stations with those errors which characterize radar methods of external trajectory measurements.

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The higher accuracy of tracking provide the operators, which correct the rotation of main telescope with the aid of hand drives. In a number of cases there is also applied the preset quidance of cinethecdolites to target/purpose. The accuracy of the cinethecdolite method of measurements depends substantially on the synchronism of the work of all theodolites and frequercy stability of photography. For the possibility of the introduction of corrections for lack of synchronation, is conducted recording the torque/moments of the complete opening of the gates of cinetleodolites with the aid of tigh-precision chronographs.

PIGE 58 51/67

Very laborious process/cperation is the decoding of the photographed motion picture films, which consists of removal/taking of readings from the goniometrical scales for the bearing angles and place, and also in the determination of connections for the removincidence of the image of object with the optical axis of cinetheodolite. Decoding is frequently fulfilled by hand with the aid of the instrument, called comparator. It consists of microscope and two measuring grids. The measuring grids are installed to the carriage which can be moved in two mutually perpendicular directions. The position of measuring grids relative to the exposure is determined with the aid of the special scales which can be graduated in the portions of degrees.

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Those considered the gonicmetrical dial/limbs of each frame of the value of azimuths A_{af} and of the argles of elevation e_{af} , and also of correction (taking into account sign) into these angles

PIGE 55

 $\Delta A_{\pi i}$ and $\Delta e_{\pi i}$ for the noncoincidence of the image of object the optical axis of main telescope (with the center of cross lines) usually will be brought in into the special service record (table 14.2), in which are calculated also the values of true azimuths $A_i = A_{\pi i} + \Delta A_{\pi i}$ and of angles of elevation $e_i = e_{\pi i} + \Delta e_{\pi i}$.

ab For accelerating the process of the decoding of motion picture films, are applied also the methods of the semiautomatic and automatic reading of readings of the gonicmetrical scales (in the presence of special equipment), in this case, the angular coordinates are automatically recorded on punched cards or on magnetic tape.

In terms of the values of angles $A_t(t)$ and $e_t(t)$, smoothed with the aid of the method of least squares [10], and with known base B can be designed on formulas (14.27) or (14.30) the values of coordinates x(t), y(t), and z(t). Similar calculation is conveniently fulfilled on form, presented below (table 14.3).

The comprising velocities of the motion of rocket along the axes of coordinates Ox, Cy, Cz are located by the numerical differentiation

 $v_x = \frac{\Delta x}{M}; \quad v_y = \frac{\Delta y}{M} \quad u \quad v_y = \frac{\Delta x}{M}.$

Full speed is calculated from the dependence $v = \sqrt{v_s + v_s + v_s}$

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Fage 557.

Table 14.2. The angular coordinates, obtained with the aid of cinetheodolites No. 1 and Nc. 2 1 .

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FCOINGTE 1. In an example of calculation (table 14.2 and 14.3) is used conditional assay. ENDFOCTNOTE.

				(1)	Пуск Ј	¥	
(1) Ле вонотео- долита	Ма кайра (м) утлы	421	422	423	424		750
	A _{a1}	42°17′					
	ΔΑ,1	+2'					
	A 1	42°19′					,
1	e ^{aj}	27°49′					
	Δε _{s1}	3'					
	6 1	27*46'	ĪĪ				
	≜ at	109*4′					
	4A,2	-1'		ĺ		1	
9	A2	109°3′					
2	¢.#2	36* 26′					
	<u>ل</u> ه ع	2'					
•	e3	35*24'					

Key: (1). Launching/starting. (2). cinethecdclite. (3). frame. (4). angles.

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Fage 558.

Analogously can be found the acceleration of object as derivative of velocity. However, repeated differentiation does not provide the necessary accuracy of calculations and therefore, as a rule, it is not applied. For determination of acceleration, more frequently are utilized the direct methods of measurements with the aid of special mensors. The obtained values of the full speed v and its comprising v_x , v_y , v_z make it possible to determine the angular position of the velocity vector of rocket of relatively earth-based coordinate system. Angles 0 and Ψ are found by the formulas

cbtaimed from (2.4).

In principle it is possible to find also such parameters of motion as g-forces, angles of attack and slip, but the accuracy of their determination will be low.

Cinetheodolite measurements are applied for determining the parameters of the motion of objects at short distances. The possibilities of optical measurements can be expanded during the use of the telephotocine cameras.

PAGE 65

table 14.3. Calculation of the coordinates of clject.

	Б=6000 м	(1)	Пуск У	A	
(H) 34 строни	t c	0	<i>t</i> 1	12	t _n
	A2-A1	66°44′			
2	$\sin(A_2 - A_1)$	0.9186			
3	Б:[2]	6533			
4	sin A ₁	0.6732			
5	cos A ₁	0,7394	1		
6	sin A ₂	0,9452			
7	tg =1	0.5265			
8	tg 12	0.7373			
9	[3].[6]	6175			
10	$x = \{9\} \cdot \{5\}$ M	4564			
11	$z = [9] \cdot [4] +$	4157	1		
12	$y_1 = [9] \cdot [7] $ M	3251			
13	$\mu_{2} = [3] \cdot [4] \cdot [8] \times$	8243	1		
14	$y = \frac{1}{2} [y_1 + y_2]$ m	8247			

Here numerals in the brackets conditionally designated the values, which stand in the appropriate table rows.

Key: (1). Launching/starting. (2). line.

Fage 559.

Such telescopes are establish/installed on special base/roots and have more powerful optics. They are applied for recording the processes of the stage separation, interception and other phenomena, which cocur at considerable removal/distance from observation

PIGE STILLY

station. Figures 14.22 gives the general view of the American cinetelescope ROTI, which makes it possible to measure the angles with accuracy (20-30) ".

Offical measurements widely are utilized for tracking the artificial Earth satellites, the space stations and the ships. For this, is conducted the photographing of object against the background of the stars whose position on celestial sphere is known with high degree of accuracy. For the photographing of such strongly distant objects, can be applied the cinetelescopes and the ballistic camera/chambers which, unlike the first, are fixed, but they have the large field of view.

Bith the aid of the criented on stars fallistic camera/chambers it is possible to determine angles with accuracy 3" 💼 -5".

Purther development of optical measurements occurs over the path of an increase in the distarce of photography, accuracy of determination of angular coordinates and automation of readout.



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Fig. 14.22. Cinetelescope.

Fage 560.

Becently together with optical equipment increasingly more widely begins to be utilized infrared technology. The instruments of infrared vision are applied when is hinder/tampered the use of optical or electronic equipment - for example, for the study of the initial moment of missile takeoff when the usual photographing of rocket is impossible due to a large guantity of dust and smoke; they successfully are utilized for coordinate determination of object, entering the dense layers of the atmosphere, and in a series of other ECC = 78107119

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cases.

The methods of trajectory measurements with the aid of radio-technical resources, the methods of parameter determination of the trajectories of ballistic missiles from the data of radar measurements, the methods of measuring the error on the flying target/purpose and the determinations of the errors for external measurements are presented in works [25] and [61]. The generalization of the named questions is partially carried out in work [18]. ECC = 78107119

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Fages 561-577.

Application Appendix.

Tables of the characteristics of the point of firing engine on the first passive phase of complex trajectory.

				(I) Suar	FORMER (o night o	100 M/	<u>(</u> 5)			
c 0	50	100	150	200	250	300	350	400	450	500	550	600
				<u></u>	·	<u> </u>		<u> </u>	<u> </u>	·	<u> </u>	<u> </u>

a=0,05] 0,0 45,2 45,5 45,7 45,9 46,0 46,1 0,1 46,1 46,1 46,1 46,1 46,1 46,1 46,1 45,5 46,1 46,4 46,3 47,1 47,3 47,3 47,3 42,3 47,3 47,3 47,3 47,3 0.2 45,8 46,6 47,1 47,7 48,1 48,4 48,4 48,4 48,4 0.3 48,4 48,4 48,4 46,0 47,0 47,8 48,5 49,2 49,6 0,4 49,6 49,6 49,6 49,5 49,6 49,6 0.5 46,3 47,5 48,5 49,4 50,2 50,7 50,7 50.7 50.7 50.7 50.7 50.7 0,6 46,5 0,7 46,8 0,8 47,0 0,9 47.1 47,1 49.0 50.7 52.3 53,5 54,2 54,2 1.0 54,2 54,2 54,2 54,2 54,2 47,1 49,0 50,7 52,3 53,5 54,2 54,2 54,2 54,2 54,2 54,2 1,1 54,2 54,2 1,2 1,3 1.4 1,5 a=0,10

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COC = 78107112

				0	Buares		alla c	~ P /c					
¢0	50	100	1 50	200	250	300	350	400	450	500	550	600	
0,6	47,9	50,4	52,7	54,7	56,3	56,9	56,9	57,0	57,0	57.0	57,1	57,1	
0,7	48,3	51,1	53,7	55,8	57,6	58,3	58,3	58,4	58,4	58.4	58,4	58,4	
0,8	48,5	51,6	54,4	56,7	58,5	59,4	59,4	59,4	59,4	59,4	59,4	59,4	
0,9	48,6	51,9	54,8	57,3	59,1	60,0	60,0	60,0	60,0	60,0	60.0	60,0	
1,0	48,7	52,1	55,0	57,6	59,4	60,2	60,2	60,2	60,2	60,2	60,2	60,2	
1,1	48,7	52,1	55,0	57,6	59,4	60,2	60,2	60,2	60,2	60,2	60,2	60,2	
1,2	48,7	52,1	55,0	57,6	59,4	60,2	60,2	60,2	60,2	60,2	60,2	60,2	
1,3	48,7	52,1	55,0	57,6	59,4	60,2	60,2	60,2	60,2	60,2	60,2	60,2	
1,4	48,7	52,1	55,0	57,6	59,4	60,2	60,2	60,2	60,2	60,2	60,2	60,2	
1,5	48,7	52,1	55,0	57,6	59,4	60,2	60,2	60,2	60,2	60,2	60,2	60,2	
					α=	=0,15							
0.0 45,0													
0,1	45.7	46,2	46,7	47,1	47,4	47,4	47,5	47,6	47,6	47,7	47,7	47,8	
0,2	46,3	47,4	48,4	49,3	49,8	49,9	50,0	50,1	50,2	50,3	50,4	50,6	
0,3	47,0	48.7	50,2	51,5	52,2	52, 3	52,4	52,6	52.8	53,0	53,1	53,3	
0,4	47,6	49,9	51,9	53,6	54,5	54,6	54,8	55,1	55,4	55,5	55,7	55,9	
0,5	48,3	51,0	53,6	55,6	56,8	57,0	57,2	57,5	57,7	57,9	58,1	58,3	
0,6	48,9	52,0	55,0	57,3	58,7	58,9	59 ,1	59,3	59,4	59,6	59,8	59,9	
0,7	49,4	52.9	56,1	58,6	60,1	60,3	60,4	60,5	60,6	60,7	60,8	60,8	
0,8	49,8	53,6	56,9	59,4	61,0	61,1	61,2	61,2	61.3	61,3	61.8	61,3	
0,9	50,0	54,0	57.5	59,9	61,4	61,5	61,5	61,5	61,5	61,5	61,5	61,5	
1,0	50,0	54,2	57,8	60,2	61,5	61,6	61,6	61,6	61,6	61,6	61,6	61,6	
1,1	50,0	54,2	57,8	60,2	61,5	61,6	61,6	61.6	61,6	61,6	61,6	61,6	
1,2	50,0	54,2	57,8	60,2	61,5	61,6	61,6	61,6	61,6	61,6	61,6	61,6	
1,3	50,0	54,2	57,8	60,2	61,5	61,6	61,6	61,6	61,6	61,6	61,6	61,6	
1,4	50.0	54,2	57.8	60,2	61,5	61,6	61,6	61,6	61,6	61,6	61,6	61,6	
1,5	50, 0	54,2	57,8	60,2	61,5	61,6	61,6	61 ,6	61,6	61,6	61,6	61 .6	
					G=	0,20							

0,0	45,0	45,0	45,0	45.0	45,0	45,0	45,0	45,0	45,0	45,0	45,0	45,0
0,1	45,7	46,3	47,0	47,5	47;6	47,9	48,1	48,3	48,4	48,4	48,6	48,8
0,2	46,4	47,7	49,0	49,9	50,3	50, 8	51,1	51,5	51,7	51,8	52,1	52,6
0,3	47,1	49,1	50,9	52,3	52,9	\$3,5	53,9	54,4	54,7	55,0	55,4	56,1
0,4	47,9	50,5	52,8	54,5	55,8	56,0	56,5	57,0	57,5	57,9	58,4	80, 1

FIGE FIGE

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				Q	Jue ve	m 4		. A						
•0	50	160	150	200	250	300	850	400	450	500	550	600		
0,5	48,6	51,8	54,6	56,5	57,5	58,2	58,8	59,3	59,8	60,3	60, 3	61,3		
0,6	49,3	53,0	56,2	58,3	59,4	6 0,0	60,6	61,0	61,5	61,8	62,1	62,3		
0,7	50,0	54,0	57,5	5 9, 7	6 0,8	61,3	61,8	63,1	62,4	62,6	62,7	62,8		
0,8	50,5	54,8	58,5	60,7	61 ,8	62,3	62,6	62,7	62,9	62,9	62,9	63,0		
0,9 50,9 55,3 59,2 61,3 62,4 62,9 63,0 63,0 63,1 63,0 63,0 63,1														
1,0 51,0 55,5 59,5 61,5 62,6 63,1 63,1 63,1 63,1 63,1 63,1 63,1 63														
1,1	51,0	55,5	59 ,5	61,5	62, 6	63,1	63,1	63,1	63,1	63,1	63,1	63,1		
1,2	51,0	55,5	59,5	61,5	62,6	63,1	63,1	63,1	63,1	63,1	63,1	63,1		
1,3	51,0	55,5	59,5	61,5	62, 6	63,1	63,1	63,1	63,1	63,1	63,1	63,1		
1,4	51.0	55,5	59,5	61,5	62,6	63,1	63,1	63,1	63,1	63,1	63,1	63,1		
1,5	51,0	55,5	59,5	61,5	62,6	63,1	63,1	63,1	63,1	63,1	63,1	63,1		
		I		l	ļ		1	(I	i	I			
					a =	-0,25								

0,0 0,1 45.6 46,2 46,8 47,2 47,8 48,0 48,3 48,7 49,2 49,6 50,3 51,5 0,2 46,3 47,5 48,6 49,5 50,5 51,1 51,6 52,3 53,2 54,1 55,4 57,7 0,3 47,0 48,8 50,4 51,7 53,2 54,2 54,8 55,8 56,8 58,1 52,5 61,5 0,4 47,6 **50,0 52,2 54,0 55,8 57,2 57,8 58,9 59,9 61,0 62,1 63,5** 0,5 48,3 51,3 54,0 56,2 58,2 59,7 60,4 61,2 62,0 62,8 63,5 64,3 0,6 49,0 52,5 55,8 58,4 60,3 61,6 62,2 62,8 63,2 63,7 64,1 64,3 0.7 49,7 53,8 57,4 60,2 62,0 63,0 63,3 63,7 63,9 64,1 64,3 64,3 0,8 63,1 63,9 64,0 64,2 64,3 64,3 50,3 54,9 58,7 61,5 64,4 64,3 0,9 50,8 55,7 59,4 62,3 63,7 64,4 64,4 64,4 64,5 64,4 64,4 64,3 1,0 51,0 56,0 59,8 62,6 64,0 64,6 64,6 64,5 64,5 64,4 64,4 64,3 1,1 51.0 56,0 59,8 62,6 64,0 64,6 64,6 64,5 64,5 64,5 64,4 64,4 64,3 1,2 51,0 56.0 59.8 62.6 64.0 64.6 64.6 64.5 64.5 64.4 64.4 64.4 1,3 51,0 56,0 59,8 62,6 64,0 64,6 64,6 64,5 64,5 64,4 64,4 64,4 64,3 1,4 51,0 59.6 02.6 64.0 64.6 64.6 64.5 64.5 64.4 64.4 64.3 56,0 1.5 50.0 62.0 64.0 64.6 64.6 64.5 64.5 64.4 64.4 64.3 51,0 56,0



E,

¢ ₀	50	100	150	200	250	300	350	400	450	500	550	600		
					a =	= 0,3 0				~				
		· ·								1		1		
0,0	45,0	45,0	45,0	45,0	45,0	45,0	45,0	45,0	45,0	45,0	45,0	45 ,0		
0,1	45,8	46,5	47,2	47,7	48,3	49,1	49,4	49,8	50,2	51,2	52,5	54,0		
0,2	46,6	48,0	49 ,3	50,2	51,6	53,0	53,6	54,4	55,1	56,7	58,9	61,5		
0,3	47,4	49,5	51,4	52,9	54,8	56,4	57,4	58,4	59,4	61,1	63,2	65,4		
0,4	48,2	51,0	53,5	55,5	57, 7	59,3	60,5	61,6	62,6	64,0	65,6	67,1		
0,5	49,0	52,4	55,4	57 ,9	60,2	61,7	62,7	63,3	64.7	65,8	66,8	67,9		
0,6	49,8	53,8	57,1	60,0	62,2	63,6	64,3	65,1	65,8	66,6	67,3	67,9		
0,7	50,6	54 ,9	58,5	61,6	63,7	64,8	65,3	65,8	66,4	66,9	67,5	67, 9		
0,8	51,3	55,7	59,5	62,7	64,6	65,5	65,8	66,4	66,7	67,1	67,5	67,9		
0,9	51,8	56,2	60,0	63,1	65,0	65,8	66,1	66,5	66,9	67,2	67,5	67,9		
1,0	52,0	56,3	60,2	63,2	65,1	65,9	66,2	66 ,5	66,9	67,2	67,5	67,9		
1,1	52,0	56,3	60,2	63,2	65, 1	65,9	66,2	66,5	66,9	67,2	67,5	67,9		
1,2	52,0	56,3	60,2	63,2	65,1	65 ,9	66,2	66,5	66,9	67,2	67,5	67,9		
1,3	52,0	56,3	60,2	63,2	6 5,1	65,9	66,2	66,5	66,9	67,2	67,5	67,9		
1,4	52,0	56,3	60,2	63,2	65,1	65,9	66,2	66,5	66,9	67,2	67,5	67,9		
1,5	52,0	56,3	60,2	63,2	65,1	65,9	66,2	66,5	66,9	67,2	67,5	67,9		
				Зна	чения	Щci	aph v _i	м/с						
<i>c</i> 0	50	100	150	200	250	30 0	350	400	450	500	550	600		
					a:=	-0,05								
0.0	0.01	0.0	0.01	0,0	0.0	0,0	0,0 (0,0	0,0	0,0	0,0	0,0		
0.1	0.0	0,1	0,2	0,5	0,6	0,8	1,0	1,1	1,2	1,4	1,5	1,7		
0.2	0.2	0.5	0.8	1,1	1,5	1,8	2,2	2,4	2,7	2,9	3,2	3,5		
0.3	0.6	1.0	1.4	2,0	2,5	2,9	3,4	3,8	4,1	4,4	4,7	5,1		
0.4	0.7	1,4	2.0	2,7	3,3	3,8	4,4	4,9	5,8	5,7	6,1	6,6		
0,5	0,8	1,6	2,4	3,2	3,9	4,5	5,0	5,6	6,0	6,6	7,1	7,6		
0,6	0,8	1,7	2,5	3,3	4,1	4,8	5,3	5,9	6,5	7,1	7,6	8,2		
0,7	0.9	1,8	2,6	3,5	4,3	5,0	5,6	6,1	6,8	7,4	8,0	8,6		
0,8	0,9	1,9	2,7	3,6	4,5	5,2	5,8	6,3	7,0	7,6	8,3	9,0		
0,9	1,0	2,0	2,9	3,8	4,7	5,5	6,1	6,6	7,2	7,8	8,5	9,2		
1,0	1,0	2,0	3,0	4,0	4,9	5,7	6,8	6,8	7,4	8,0	8,6	9,3		

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	() Susvenue & c apu vu arc											
Cu	50	1 0 0	150	200	250	300	35 0	400	450	50 U	550	600
1,1	1,1	2,1	3,1	4,2	5.1	6.0	6,6	7,0	7.5	8,0	8,6	9,3
1,2	1,1	2,2	3,2	4,4	5,3	6,3	6,8	7,2	7.6	8,1	8,5	9,2
1,3	1,2	2,3	3,4	4,5	5,5	6,5	7,1	7,5	7,8	8,1	8,5	9,0
1,4	1,2	2,4	3,5	4,7	5,7	6,7	7,3	7.7	7,9	8,2	8,4	8,8
1,5	1,3	2,5	3,7	4,9	5,9	7,0	7,6	7,9	8,0	8,2	8.3	8,5
	· ·	•			α=	- = 0,10	•					
0.0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
0,1	0	0,2	0,5	0,8	1,3	1,6	1,8	2,1	2,4	2,6	2.9	3,2
0,2	0,4	0,9	1,5	2,0	2,7	3,3	4.0	4,5	4,9	5,3	5,7	6,2
0,3	0,8	1,7	2,4	3,2	4,1	5,0	6,0	6,5	7,1	7,5	8,0	8,7
0,4	1,1	2,2	3,2	4,3	5,2	6,4	7,4	8,0	8,7	9,3	10,1	10,8
0,5	1,3	2,5	3,7	5,0	6,1	7.3	8.3	9,1	10,0	10,8	11,5	12,3
0,6	1,4	2,7	4,1	5,4	6,7	7,9	9,0	9,9	10,8	11,7	12,5	13,3
0,7	1,5	2,8	4,3	5 ,7	7,1	8.4	9,5	10,5	11.4	12,3	13,1	14,0
0,8	1,5	2,9	4,5	5,9	7,4	8.8	9,9	10,9	11,8	12,7	13,5	14,4
0,9	1,6	3,1	4,6	6,1	7,6	9,1	10,2	11,2	12,1	12,9	13,8	14,6
1,0	1,7	3,2	4,8	6,3	7,9	9.4	10.5	11,4	12,2	13,0	13,9	14,7
1,1	1,8	3,3	4,9	6,5	8,2	9,7	10,8	11,6	12,2	13,0	13,9	14,6
1,2	1,8	3,4	5,1	6,8	8,4	10,0	11,1	11,7	12,3	13,0	13,8	14,4
1,3	1,9	3,5	5,3	7,0	8,7	10,4	11,3	11,9	12,4	12,9	13,6	14,1
1,4	1,9	3,7	5.4	7,2	8,9	10.7	11,6	12.1	12,4	12,9	13,4	13,8
1,5	2,0	3,8	5,6	7,4	9,2	11,0	11,9	12,2	12,5	12,8	13,3	13,4
	1	1	1			1	1	1	1		ł	l
					a.	-0,15						
0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
0,1	0,1	0,3	0,6	1,1	1,5	1,8	2,2	2,6	2,9	3,4	3,7	4,2
0,2	0,5	1,0	1,6	2,3	3,0	3,7	4,3	5,0	5,7	6,3	7,1	8,4
0,3	1,0	1,9	2,7	3,6	4,8	5,5	6,3	7,3	8,2	9,2	10,1	11,5
0,4	1,3	2,5	3,5	4,8	6,0	7,0	8,1	9,3	10,3	11,4	12,6	13,8
0,5	1,5	2,8	4,1	5,5	6,9	8,2	9,4	10,6	11,7	12,9	14,0	15,2
0,6	1,6	8,0	4,5	6,0	7,5	9,0	10,0	11.4	12,5	18,7	14,7	15,9
0,7	1,6	3,2	4,7	6,3	7,9	9,5	10,4	11,9	12,9	14,1	15,1	16,2
0,8	1,7	3,3	4,9	6,6	8,2	9,9	10,8	12,1	13,1	114,3	15,3	16,4

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-34 7162

	Q Значения /н Спри vo нос												
e 0	50	100	150	200	250	300	350	400	450	500	550	600	
0,9	1,7	3,4	5,1	6,8	8,5	10,3	11,1	12,3	13,3	14,3	15,3	16,3	
1,0	1,8	3,5	5,2	7,0	8,7	10,5	11,4	12,4	13,3	14,3	15,2	16,2	
1,1	1,8	3,6	6,3	7,2	8,9	10,7	11,6	12,4	13,3	14,2	15,0	16,0	
1,2	1,9	3,7	5,4	7,3	9,1	10,9	11,8	12,4	13,2	14,0	14,7	15,6	
1,3	1,9	3,8	5,5	7,4	9,2	11,1	11,9	12,5	13,1	13,8	14,4	15,2	
1,4	2,0	3,8	5,7	7,6	9,4	11,3	12,0	12,5	13,0	13,6	14,1	14,7	
1,5	2,0	3,9	5,8	7,7	9,6	11,5	12,1	12,5	12,8	13,3	13,7	14,2	
				4 .									

a=0,20

:

0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
0,1	0,1	0,3	0,8	1,3	1,7	2,2	2.7	3,3	3,8	4,3	5,0	5,7
0,2	0.4	0,9	1,8	2,6	3,4	4,4	5,4	6,4	7,4	8,3	9,3	10,9
0,3	0,9	1,7	2,8	4,1	5,2	6,4	7,6	8,8	10,2	11,5	12,8	14,6
0,4	1,3	2,4	3,7	5,3	6,5	7,9	8,4	10,7	12,1	-13,6	15,1	16,7
0,5	1,5	3,0	4,5	6,1	7.7	9,3	10,7	12,1	13,6	15,0	16,5	18,0
0,6	1,7	3,3	4,9	6,7	8,4	10,2	11,7	13,0	14,5	15,7	17,1	18,4
0,7	1,8	3,5	5,3	7,1	8,9	10,8	12,3	13,5	14,8	16,0	17,2	18,5
0,8	1,9	3,7	5,5	7,4	9,3	11,3	12,6	13,8	14,9	16,1	17,1	18,4
0,9	2,0	3,8	5,7	7,6	9,6	11,6	12.8	13,9	14,9	16,0	17,0	18,2
1,0	2,0	3,9	5,9	7,8	9,8	11,8	12,8	13,8	14,8	15,6	16,8	17,8
1,1	2,0	3,9	5,9	7,9	9,9	11,9	12,8	13,7	14,6	15,5	16,5	17,5
1,2	2,0	4,0	6,0	7,9	9,9	11,9	12,7	13,6	14,4	15,2	16,1	17,0
1,3	2,0	4.0	6,0	8,0	10,0	12,0	12,7	13,5	14.2	14,9	15,7	16,4
1,4	2,0	4,0	6,0	8,0	10,0	12,0	12,6	13,3	13,9	14,6	15,2	15,9
1,5	2,0	4,0	6,0	8,0	10,0	12,0	12,6	13,1	13,6	14,2	14,8	15,3
											-	

page 1121

					34240	nmi t _n	chipe	00	c	_		
c ₀	50	100	150	200	250	300	350	400	450	50e	550	600
				<u>. </u>					·			

a= 0,25

0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
0,1	0,1	0,3	0,7	1,1	1,8	2,3	2,9	3,7	4,8	5,8	7,3	9,8
0,2	0,4	1,0	-1,6	2,6	3,7	4,7	5,9	7,1	8,6	10,5	12,7	16,0
0,3	1,0	2,9	2.9	4,2	5,6	5.9	8,4	9,9	11,6	14,0	16,0	18,7
0,4	1,5	2,8	3,1	5,7	7,2	8,7	10,4	12.2	14,0	16,1	18,1	20,3
0,5	1,7	3,4	5,1	6,8	8,5	10.3	12,0	13,8	15,6	17,4	19,1	21,0
0,6	1,9	3,8	5,6	7,5	9,4	11,3	12,9	14,7	16,5	18,0	19,4	21,1
0,7	2,0	4,0	6,0	8,0	10,1	12,0	13,5	15,1	16,8	18,1	19,3	2 0, 9
0,8	2,1	4,2	6,3	8,4	10,5	12,5	13,9	15,3	16,8	17 , 9	19,1	20,5
0,9	2.2	4,3	6,4	8,6	10,7	12.8	14,1	15,3	16,6	17,6	18,8	19 ,9
1,0	2,2	4,3	6,5	8,7	10,8	13,0	14,1	15,1	16,2	17,2	18,3	19,3
1,1	2,2	4,3	6,5	8,7	10,8	13,0	14,0	14,9	15,9	16,8	17.8	18,7
1,2	2,2	4,3	6,5	8,7	10,8	12,9	13,9	14,7	15,5	16,4	17,3	18,1
1,3	2,1	4,3	6,4	8,6	10,7	12,8	13,7	14,4	15,2	16.0	46.8	17,5
1,4	2,1	4,2	6,3	8,5	10,6	12,7	13,4	14,1	14,8	15.5	$16,2^{\rm i}_{\rm f}$	16,9
1,5	2,1	4,2	6,2	8,3	10,4	12,5	13,1	13,7	14,4	15,0	15,6	16,3

α=0,30

0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
0,1	0,1	0,4	0,8	1,4	2,1	3,1	3,9	4,8	5,9	7,8	10,3	13,5
0,2	0,6	1,3	2,2	3,1	4,5	6,0	7,4	9,0	10,8	13,0	15,8	18,6
										{	1	

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Page 1122

	Знатения ta стри vo arc														
<i>c</i> 0	9 0	100	150	200	250	300	350	400	450	500	550	600			
0.3	1,2	2,5	3,9	5.2	6.9	8,7	10.4	12.3	14.5	16.8	19.3	22,0			
0,4	1,8	3,4	5,1	6,9	8,8	10,7	12,6	14.5	16,8	18,9	21,4	23,8			
0,5	2,0	3,9	5,9	7,8	9,8	11,8	13,8	15,9	18,0	20,1	22,2	24,3			
0,6	2,1	4,2	6,4	8,4	10,6	12,7	14,5	16,5	18,4	20,4	22,0	23,7			
0,7	2,2	4,4	6,6	8,8	11,0	13,2	14,9	16,7	18,4	20,1	21,4	23,1			
0,8	2,3	4,5	6,7	9,0	11,3	13,5	15,1	16,6	18,2	19,6	20,9	22,5			
0,9	2,3	4,6	6,8	9,1	11,4	13,7	15,0	16,5	17,9	19,1	20,4	21,9			
1,0	2,3	4,6	6,8	9,1	11,4	13,7	14,9	16,2	17,5	18,7	19,9	21,2			
1,1	2,3	4,6	6,8	9,0	11,3	13,7	14,7	15,9	17,1	18,2	19,4	20,6			
1,2	2,3	4,5	6,7	8,9	11,2	13,5	14,5	15,6	16,7	17,8	18,9	20,0			
1,3	2,2	4,4	6,6	8,8	11,0	13,3	14,2	15,2	16,2	17,3	18,4	19,4			
1,4	2,2	4,3	6,5	8,7	10,7	13,1	13,9	r4,9	15,8	16,9	17,8	18,8			
1,5	2,2	4,2	6,3	8,5	10,5	12,8	13,6	14,5	15,4	16,4	17,3	18,2			

C 0		У Значения Un м/с при vo м/с													
•0 —	50	100	150	200	250	300	350	400	450	500	550	600			
					α=	=0,05									
0,0	50	100	150	200	250	300	350	400	450	500	550	600			
0,1	49	98	146	195	243	292	336	381	425	471	517	562			
0,2	48	96	143	190	237	234	323	364	404	446	488	530			
0,3	47	94	139	185	231	276	312	349	387	425	463	502			
0,4	46	93	136	181	225	268	302	336	372	407	442	478			
0,5	45	91	134	177	220	262	293	325	359	39 2	424	458			
0,6	44	89	131	174	215	255	285	315	347	379	409	441			
0,7	43	87	129	170	211	248	278	305	337	368	396	427			
0,8	42	85	126	167	207	242	271	299	329	358	38 5	415			
0,9	41	83	124	165	203	237	265	293	322	350	377	406			
1,0	40	81	122	163	200	232	260	288	316	344	371	400			
1,1	39	79	120	160	197	229	256	283	312	340	367	396			
1,2	38	78	118	157	193	226	253	280	309	337	364	393			
1,3	38	76	116	155	190	224	252	278	307	335	362	391			
1,4	37	75	114	152	188	223	251	277	305	333	361	389			
1,5	36	74	113	150	186	222	250	276	304	832	360	386			

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	Successor v. Ste spe vo ste												
C _U	50	100	150	200	250	300	350	400	450	500	550	600	
					a =	=0,10							
0,0	50	100	150	200	250	1 300	350	i 400	450	1 500	550	60 0	
0,1	46	94	141	189	236	283	323	362	397	440	476	5 15	
0,2	43	88	133	179	224	267	299	330	358	390	420	447	
0,3	41	83	126	170	213	252	279	303	325	349	374	3 94	
0,4	39	79	120	162	203	238	261	280	298	316	336	35 3	
0,5	37	76	115	155	194	225	244	260	275	290	305	3 20	
0,6	36	73	110	149	186	214	229	243	256	268	279	29 2	
0,7	35	71	106	143	179	204	217	228	239	250	257	268	
0,8	34	68	103	138	171	195	207	215	224	234	239	248	
0,9	33	66	99	133	165	187	198	204	211	220	225	232	
1,0	32	64	96	128	160	180	190	195	201	208	214	220	
1,1	31	62	93	124	155	175	185	189	194	201	206	213	
1,2	30	60	91	121	151	172	180	186	190	196	201	208	
1,3	29	58	89	118	148	169	177	182	187	193	199	20 5	
1,4	29	57	87	116	145	166	174	180	185	191	198	204	
1,5	28	56	85	114	142	165	172	178	184	190	197	203	
					a =	0,15							
0,0	50	100	150	200 (250 (300 (350 (400	450	500 t	5 50 (600	
0,1	46	93	138	185	233	277	317	355	394	432	465	502	
0,2	43	86	128	172	217	257	288	317	347	374	398	426	
0,3	41	80	120	161	203	239	263	285	308	329	346	3 67	
0,4	39	75	113	152	191	223	242	259	276	292	306	322	
0,5	37	7 1	107	144	180	210	225	238	250	263	275	287	
0,6	35	68	102	137	171	199	211	221	230	241	251	260	
0,7	33	65	98	131	163	189	200	207	215	234	233	241	
0,8	31	62	94	125	156	181	190	196	203	211	219	227	
0,9	29	60	91	120	151	174	182	187	194	201	208	216	
1,0	28	58	88	116	146	168	175	180	187	193	200	206	
1,1	27	56	86	113	142	163	170	175	182	187	194	200	
1,2	26	55	84	110	139	160	166	171	178	183	190	196	
1,3	26	54	82	108	136	157	163	168	175	181	187	193	
1,4	25	53	80	105	134	155	161	106	173	179	185	191	
1.5	25	52	78	195	182	153	159	165	171	177	183	189	

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MARK AND A



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CALENDARY PARTY AND -

1. Sec. 2.

		Значения Un 11/с 10/н Uu 11/с													
e ₀	50	100	150	200	250	300	350	400	450	500	550	600			
					a=	= 0,2 0									
0,0	50	100	150	200	250	300	350	400	450	500	550	600			
0,1	45	91	138	184	231	276	311	349	384	419	452	483			
0,2	41	84	128	170	214	255	280	307	330	355	375	395			
0,3	38	78	119	158	199	237	254	274	288	306	319	334			
0,4	36	73	111	148	186	221	233	247	257	270	280	292			
0,5	34	68	104	139	175	205	216	226	235	244	254	263			
0,6	32	65	98	131	165	193	202	210	219	226	236	243			
0,7	31	62	93	124	157	183	190	198	20 6	213	221	228			
0,8	30	59	89	118	150	174	181	188	195	202	209	216			
0,9	29	56	86	113	143	166	173	180	186	193	1 9 9	207			
1,0	28	54	83	1 0 9	137	159	166	173	179	186	192	199			
1,1	27	53	81	106	133	153	160	167	173	180	186	1 92			
1,2	26	52	79	104	130	149	156	162	168	175	181	187			
1,3	25	51	77	102	128	147	154	159	165	172	1 78	183			
1,4	25	50	76	101	126	146	152	158	163	170	175	181			
1,5	24	50	75	100	125	145	151	157	162	168	173	179			
					a =	: (^,25									
0,0	50	101	150	20 0	250	300	350	400	450	500	550	600			
0,1	45	91	137	183	228	275	309	339	365	39 1	412	431			
0,2	41	83	126	168	209	252	276	293	306	321	334	347			
0,3	- 38	77	116	155	193	232	248	258	266	275	285	293			
0,4	35	72	107	144	180	214	225	232	239	246	253	259			
0,5	33	67	100	134	168	1 9 8	207	213	220	225	232	238			
0,6	31	62	94	125	157	185	193	199	206	211	217	225			
0,7	29	58	89	117	147	174	181	188	195	200	207	215			
0,8	27	55	84	111	139	165	171	178	186	191	199	206			
0,9	26	52	80	106	133	157	163	170	178	183	192	199			
1,0	25	50	76	103	128	150	157	164	171	178	185	193			
1,1	25	49	74	100	125	145	152	158	165	172	179	187			
1,2	24	48	73	96	123	142	1 49	154	161	167	174	181			
1,3	24	46	72	97	122	149	146	152	158	164	170	176			
1,4	28	47	71	96	121	189	144	150	155	161	167	172			
1,5	23	47	71	95	120	138	148	146	153	159	164	160			



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	Surfexant vy T'c Apri vo T'c												
e ₀	50	100	150	200	250	300	350	400	450	50 0	550	600	
					a:	=0,30							
0,0	50	100	150	200	250	300	350	400	450	500	550	600	
0,1	44	88	133	177	221	259	285	307	329	353	377	405	
0,2	40	78	119	159	196	228	244	254	265	280	396	310	
0,3	36	70	108	144	180	204	215	221	228	237	245	254	
0,4	33	64	99	132	165	187	194	199	204	211	218	222	
0,5	30	60	92	122	153	175	180	185	190	196	201	206	
0,6	28	57	87	115	145	167	172	177	182	188	194	200	
0,7	27	54	83	110	139	160	166	171	176	183	189	195	
0,8	26	52	80	106	134	155	161	166	171	176	184	189	
0,9	25	51	. 77	103	129	151	157	162	167	175	178	184	
1,0	24	50	75	100	126	148	153	158	163	168	173	178	
1,1	24	49	74	98	123	145	149	154	158	163	167	172	
1,2	23	48	73	96	121	142	146	150	154	158	162	166	
1,3	23	47	71	95	119	139	143	146	149	153	156	160	
1,4	22	46	70	94	117	137	140	142	145	148	150	154	
1,5	22	46	69	9 3	116	135	137	139	141	143	145	148	
	1			3	начен	MR Xn	м при	00 N	c				
¢j	50	100	150	200	250	300	350	400	450	500	550	600	
		·			Q ==	0,05							
0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	U,0,	0.0	. 0 ,0 j	0,0	
0,1	1	9	30	60	110	170	230	300	390	480	580	690	
0,2	20	40	100	150	200	300	460	670	830	1100	1300	16 00	
0,3	60	100	170	250	340	490	700	930	1200	1550	20 50	2420	
0,4	100	150	230	340	450	630	860	11 3 0	1410	1760	2250	2640	
0,5	110	180	280	410	540	700	920	1170	1440	1770	2130	2500	
0,6	120	210	320	450	58 0	740	93 0	1150	1390	16 90	2020	2350	
0,7	120	240	350	480	610	770	940	1140	1340	1620	1920	22 20	
0,8	130	260	380	510	640	800	950	1130	1300	1560	1830	2100	
0,9	1 3 0	270	400	540	670	830	960	1120	1280	1510	1750	19 9 0	
1,0	140	280	420	560	69 0	850	970	1120	1270	1470	1680	1 9 00	
1,1	150	29 0	430	570	710	860	98 0	1120	1260	1440	1630	1820	
1,2	150	300	440	58 0	720	870	990	1130	1260	1420	1580	1750	
1,3	160	300	45 0	590	730	880	1000	1140	1260	1400	1540	1680	
1,4	170	310	460	600	740	890	1020	1150	1270	1390	1500	1610	
1,5	170	310	460	600	750	900	1040	1170	1280	1380	1480	1550	

	T	Sufferent se x for vo for													
48	50	100	150	200	250	300	360	400	450	500	550	600			
					a =	=0,10									
0,0	1 0, 0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0			
0,1	2	16	50	110	210	320	440	580	730	900	1090	1290			
0,2	30	70	150	300	490	630	820	1070	1300	1 76 0	2200	2700			
0,3	90	180	330	520	730	950	1200	1600	1900	2500	3120	3620			
0,4	140	270	450	670	890	1170	1450	1810	2250	2670	3150	3720			
0,5	170	320	510	720	950	1220	1520	1850	2240	26 20	30 50	3580			
0,6	170	340	530	740	970	1240	1520	1840	2200	2560	2950	3440			
0,7	180	360	540	760	990	1250	1520	1830	2160	2510	2860	3310			
0,8	180	370	550	770	1000	1260	1520	1820	2130	2450	2770	3180			
0,9	180	380	560	780	1010	1270	1520	1800	2090	2390	2690	3060			
1,0	180	380	570	790	1020	1280	1520	1780	2050	2330	2610	29 50			
1,1	180	380	570	800	1030	1280	1510	1760	2010	2270	2530	2840			
1,2	180	390	580	810	1040	1290	1510	1740	1970	2210	2460	2740			
1,3	180	400	600	820	1050	1290	1500	1720	1930	2150	2390	2640			
1,4	190	410	620	840	1060	1300	1490	1700	1890	2100	2320	2540			
1,5	210	430	650	860	1080	1300	1 48 0	1680	1860	2060	2250	2450			
					6-	•0,15									
0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0 _l	0,0			
0,1	3	20	70	140	260	370	520	710	890	1140	1380	1700			
0,2	40	110	230	360	550	700	1000	1300	1730	2070	2600	3550			
0,3	120	250	430	630	830	1100	1440	1800	2600	8030	8530	4060			
0,4	170	350	590	830	1090	1400	1720	2080	2520	2940	3400	8900			
0,5	180	870	570	790	1060	1380	1680	2040	2440	2840	3290	87 50			
0,6	170	350	550	770	1040	1350	1640	1990	2330	2740	3160	3690			
0,7	170	340	540	760	1020	1820	1610	1950	2290	2640	3030	8430			
0,8	160	330	530	760	1010	1300	1590	1910	2220	2550	2910	8280			
0,9	160	330	520	750	1000	1280	1570	1870	2160	2470	2800	8130			
1,0	160	330	520	750	990	1270	1550	i 830	2100	2390	2690	8000			
1,1	170	320	530	760	1000	1260	1540	1790	2050	2320	2590	2880			
1,2	180	340	550	770	1010	1260	1530	1760	2000	2250	2500	2760			
1,8	190	370	500	790	2000	1270	1590	1740	1000	2100	9490	2650			
1,4	200	410	••••	630	1000	1300	1880	1720	1000	2140	2000	2000			
1,5	30	-	660	880	1100	1300	1690	1700	1990	2100	2390	3469			

·	Τ			3				v.	ę –			
c 0	50	100	150	200	250	300	350	400	450	500	550	600
	··		-		G :	-0,20			<u> </u>	<u> </u>	·	<u></u>
0,0	0,0	0,0	0,0	0,0	, 0, 0	0,0	0,0	0,0	0,0	0,0	0.0	0,0
0,1	3	20	80	170	290	440	640	880	1150	1420	1810	2270
0,2	50	130	270	450	700	1000	1320	1800	2500	3050	3800	4600
0,3	150	300	520	760	1000	1340	1700	2250	2900	3400	3920	4420
0,4	190	400	670	920	1200	1530	1 970	2340	2770	3220	3700	4160
0,5	210	460	710	989	1250	1550	1870	2240	2640	3050	3500	3930
0,6	210	430	670	930	1190	1770	1770	2130	2510	2890	3310	3720
0,7	200	410	630	880	1130	1400	1690	2030	2390	2740	3130	3530
0,8	190	390	600	840	1080	1350	1630	1940	2280	2610	2970	3350
0,9	180	380	580	810	1050	1310	1580	1860	2180	2490	2830	3180
1,0	180	370	570	790	1030	1270	1540	1800	2090	2380	2700	3 020
1,1	180	360	570	780	1020	1240	1500	1750	2020	229 0	2590	2880
1,2	180	360	580	770	1010	1220	1480	1710	1970	2210	2480	2760
1,3	190	370	590	780	1020	1230	1470	1690	1940	2160	2390	2650
1,4	200	390	610	820	1050	1250	1490	1710	1930	2140	2330	2550
1,5	210	430	650	880	1100	1310	1530	1740	1940	21 3 0	2300	2470
	•		•		a -	=0.95		•				
0.0			0.0		0.0	0.0	0.01	n 01	0.0	0.01		0.0
0,0	3	20	70	150	300	450	690	000	1400	1870	2570	3660
0,1	60	120	250	430	750	1130	1600	2300	3000	3680	4540	5760
0.3	130	280	570	010	1250	1750	2150	2800	3250	3700	4340	5030
0.4	200	500	830	1110	1510	1840	2220	2600	3020	3440	3060	4440
05	220	520	790	1080	1400	1720	2080	2430	2810	3030	3660	4100
0.6	220	480	720	1010	1300	1610	1050	2200	2650	3040	3430	3850
0.7	210	440	670	950	1220	1510	1830	2160	2500	2960	3220	3620
0.8	190	410	630	890	1140	1420	1720	2040	2370	2000	3040	3400
0.9	180	390	600	840	1080	1340	1630	1030	2010	2560	2880	3210
1.0	180	370	570	790	1030	1270	1550	1840	2140	2440	2000	3050
1.1	180	360	560	770	990	1240	1500	1780	2060	2340	2630	2900
1.2	180	370	550	760	940	1220	1470	1740	2000	2010	2520	2770
1.3	100	200	560	770	990	1290	1400	1730	1970	2210	2440	2650
1.4	190	400	600	810	1000	1250	1480	1730	1980	2180	2390	2560
1,5	210	430	660		1100	1810	1540	1770	1980	2170	2350	2500
-	1 1			· . !		•		•	•			

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-1990 V
1128

	Surveying Xa H april Up afc												
¢ij	50	100	150	200	250	300	350	400	450	500	550	600	
a=0,30													
0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	
0,1	4	30	80	180	350	610	880	1250	1690	2440	3460	4770	
0,2	50	130	270	460	840	1150	1660	2400	3310	3970	4800	6100	
0,3	130	260	540	850	1250	1650	2200	2660	3140	3840	4360	6060	
0,4	210	430	770	1070	1420	1820	2120	2510	2930	3420	3860	4450	
0,5	240	490	750	1030	1330	1670	1990	2350	2730	3140	3560	4000	
0,6	220	460	700	970	1250	1560	1870	2220	2560	2930	3300	3710	
0,7	210	430	650	910	1180	1470	1770	2090	2400	2740	3090	3460	
0,8	190	400	610	860	1120	1 39 0	1680	1970	2250	2570	2900	3250	
0,9	180	380	590	820	1070	1320	1600	1860	2130	2420	2730	3070	
1,0	180	370	570	790	1030	1270	1530	1780	2030	2300	2590	2900	
1,1	180	360	56 0	780	1000	1230	1490	1730	1960	2220	2 48 0	2750	
1,2	180	360	560	770	990	1210	1460	1700	1930	2160	2400	2640	
1,3	190	380	570	780	1000	1220	1450	1690	1920	2130	2350	2560	
1,4	200	400	600	820	1040	1250	1470	1720	1930	2130	2330	25 10	
1,5	210	430	650	880	1100	1310	1530	1760	1970	21 60	2340	2490	
				3	начен	u ya	м При	v0 \$	с,				
	50	100	150	200	250	300	350	400	450	500	550	600	
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0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0.0	0,0	
0,1	1	9	30	60	110	180	29 0	310	400	490	590	70 0	
0,2	20	40	100	150	210	310	480	690	850	1130	1320	1620	
0,3	60	100	170	260	360	510	720	960	1240	1580	2080	2450	
0,4	100	150	240	350	47 0	660	890	1170	1470	1800	2290	2690	
0,5	110	180	290	430	570	740	96 0	1220	1520	1810	2190	2570	
0,6	120	220	330	480	610	790	980	1210	1480	1740	2100	2450	
0,7	120	250	370	520	670	830	1000	1210	1420	1680	2020	2350	
0,8	130	270	400	550	710	860	1020	1210	1400	1640	1950	2260	
0,9	130	280	420	580	730	900	1030	1220	1400	1610	1890	2180	
1,0	140	/290	440	590	750	920	1050	1240	1410	1610	1840	2110	
1,1	150	300	460	600	770	940	107 <u>0</u>	1250	1410	1600	1810	2060	
1,2	150	310	470	620	780	960	1090	1260	1410	1600	1780	2030	
1,3	160	320	400	640	800	970	1110	1280	1420	1600	1770	1980	
1,4	170	330	490	660	820	990	1140	1310	1470	1620	1700	1940	
1,5	170	330	500	670	830	1000	11 70	1340	1500	1640	1770	1900	

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¢9	50	100	150	200	250	300	8 50	400	450	500	550	500
α=0,10												
0,0	0,0	0,0	0,0	0 ,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
0,1	2	16	50	110	220	340	460	600	760	930	1130	1340
0,2	50	120	200	340	520	750	930	1190	1470	1810	2300	2850
0,3	110	250	410	640	900	1120	1360	1720	2200	2600	3240	3820
0,4	160	360	560	800	1100	1360	1670	2100	2520	2970	3460	4030
0,5	180	400	650	900	1190	1500	1850	2270	2700	3190	3670	4250
0,6	190	430	680	940	1240	1570	1950	2370	2790	3240	3770	43 30
0,7	200	440	710	980	1290	1630	2020	2440	2850	3340	3830	4380
0,8	210	460	740	1020	1330	1680	2070	2480	2890	3370	3840	4390
0,9	220	480	760	1050	1370	1720	2100	2500	2910	3370	3830	4360
1,0	230	500	780	1080	1 40 0	1750	2120	2500	2900	3350	3800	4300
1,1	240	520	800	1110	1420	1770	21 30	2500	2880	3300	3730	4220
1,2	250	540	820	11 30	1440	1780	2140	2480	2840	3240	364 0	4110
1,3	260	560	840	1150	1460	1750	21 3 0	2460	2800	3170	3540	3960
1,4	270	570	860	1170	1470	1800	2120	2430	2750	3090	3430	3790
1,5	280	58 0	880	1190	1480	1800	2090	2390	2690	300 0	330 0	3600
					a= :	0,15						
0,0	0,0	0,0	0,0	0,0	0.0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
0,1	3	20	70	150	270	38 0	540	740	940	1200	1450	1790
0,2	40	120	240	380	570	720	1030	1340	1800	2280	2920	3620
0,3	120	280	440	650	860	1140	1520	2000	2730	3380	4200	50 40
0,4	180	370	600	860	1130	1460	1880	2420	3060	3700	4340	50 70
0,5	210	440	700	1000	1300	1650	2070	2600	3150	3730	4320	5000
0,6	230	490	770	1080	1400	1750	2180	2670	3160	3720	4290	4930
0,7	250	530	820	1140	1480	1820	2250	2710	3170	3710	4250	4850
0,8	260	560	860	1190	1530	1880	2300	2730	3180	3690	4200	476 0
0,9	270	580	890	1230	1570	1920	2330	2740	3170	3650	4140	4660
1,0	280	600	920	1260	1600	1950	2350	2750	3150	3600	4070	4550
1,1	280	620	940	1280	1620	1970	2360	2750	3130	3540	3990	4440
1,2	290	630	950	1290	1630	1980	2360	2730	3090	3480	3900	4320
1,8	800	630	960	1300	1640	1980	2350	2700	3050	3420	3510	4200
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1,5	300	630	900	1900	10.00	1900				diciti.		

		Зание и и при vo и/с										
٥٢	50	100	150	200	250	300	350	400	450	500	550	600
a=0,20												
0,0	0,0	0, 0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
0,1	3	20	80	180	300	460	670	940	1280	1510	1 940	2430
0,2	70	150	280	480	730	1070	1400	1950	2600	3230	4270	6100
0,3	i60	320	540	800	1080	1440	1820	2450	3140	4000	5120	6300
0,4	200	420	700	970	1300	1 690	2110	2720	3400	4250	5150	6100
0,5	220	480	760	1080	1440	1850	2300	2880	3540	4260	5000	5800
0,6	220	49 0	790	11 30	1520	1 970	2430	2970	3570	4210	4850	5530
0,7	230	510	820	1180	1590	2060	2520	3020	3580	4150	4720	5310
0,8	240	530	850	1220	1640	2110	2570	3050	3570	4080	4600	512 0
0,9	250	550	870	1250	1680	2140	2590	3060	3540	4000	4480	4950
1,0	260	560	900	1280	1710	2150	2600	3040	3480	3920	4360	480 0
1,1	270	570	920	1310	1720	2140	2580	3000	3430	3840	4250	467 0
1,2	280	590	940	13 3 0	1730	2130	2550	2950	3370	3750	4140	4540
1,3	290	610	960	1340	1720	2110	2510	2900	3290	3660	4040	4420
1,4	300	630	980	1350	1710	2090	2460	2830	3200	3560	3930	43 10
1,5	310	660	1000	1360	1700	2050	2 40 0	2750	3100	3460	3820	4200
					6 -	.0,25						
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1	4	30	90	200	380	660	720	1150	1680	2540	3560	4870
0.2	80	160	300	560	860	940	1700	2480	3390	4300	5140	7100
0.3	160	350	590	950	1850	1800	2300	3000	4000	4840	6200	7850
0.4	230	480	800	1180	1450	1900	2460	3220	4070	4820	6070	7440
0.5	260	550	860	1220	1620	2050	2620	3280	4000	4700	5550	6600
0,6	270	570	910	1290	1740	2180	2700	3280	3930	4560	5260	6060
0.7	280	590	940	1350	1820	2270	2750	3270	3960	4440	5040	5680
0.8	290	610	960	1390	1870	2330	2770	3260	3790	4320	4860	5400
0,9	300	630	980	1420	1890	2350	2770	3240	3720	4210	4700	5180
1,0	310	640	1000	1440	1900	2850	2760	3200	3650	4100	4550	5000
1,1	320	650	1010	1450	1890	2330	2730	3150	3580	4000	4420	4850
1,2	320	660	1080	1460	1880	2300	2590	3100	3500	3910	4310	4720
1,3	330	670	2040	1400	1886	200	2550	3946	3439	3830	4210	4610
1,4	340		1000	1440	1890	2210	2500	2970	8996	8730	4110	4600
1,5	380	700	1050	1400	1700	2150	2590	2000	2000	3540	4838	4400

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a=0,30														
0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0		
0,1	4	30	90	190	370	650	950	1360	1860	2740	3990	5670		
0,2	80	180	340	500	890	1280	1800	2550	3500	5080	6800	8 300		
0,3	180	430	680	1000	1360	1800	2440	3350	4460	5600	694 0	8440		
0,4	270	570	880	1240	1680	2160	2860	3680	4540	5480	6610	7960		
0,5	300	610	950	1350	1800	23 00	2940	3680	4460	528 0	6200	7350		
0, 6	310	63 0	96 0	1390	1850	2350	2940	3650	4360	5090	5890	681 0		
0,7	320	640	1000	1420	1880	2380	2930	3610	4260	4920	5610	6370		
0,8	330	650	1 0 20	1440	1900	2390	2 93 0	3560	4160	4750	5370	6010		
0,9	340	670	1040	1460	1920	2400	2920	3500	40 50	4600	51 7 0	5730		
1,0	350	680	1060	1480	1930	2400	2900	3430	394 0	4460	499 0	5500		
1,1	350	690	1070	1490	1930	2390	2870	3360	3840	4320	4820	5300		
1,2	360	710	1060	1500	1920	2370	2820	3290	3740	4190	4650	5120		
1,3	370	720	1100	1500	1910	2340	2760	3210	3640	4060	4490	4940		
1,4	370	730	1110	1500	1900	2300	.2700	3120	3540	3930	4340	4770		
. 1,5	300	750	1130	1500	1880	2250	2630	3030	3430	3620	4200	4600		

Key: (1). Values. (2). with. (3). m/m. (4). s with.

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C509	BALLISTIC RES LABS	ī	E408	AFWL	ī		
C510	AIR MOBILITY RED	ī	E410	ADTC	ī		
	LAB/FIO	-	E413	ESD	2		
C513	PICATINNY ARSENAL	1		FTD			
C535	AVIATION SYS COMD	ī		CCN	1		
C591	FSTC	5		ASD/FTD/NICI) 3		
C619	MIA REDSTONE	1		NIA/PHS	1		
D008	NISC	ī		NICD	2		
H300	USAICE (USAREUR)	ī					
P005	ERDA	ī					
P005	CIA/CR6/AD8/SD	ī					
NAVOR	DSTA (50L)	ī					
NASA/	KSI	ī					
AFIT/	LD	ī					
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